CCNU QCD lectures

Lectures on Heavy-Flavor Probes of Quark-Gluon Plasma

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Heavy flavor transport as probes of QGP



Lecture III

heavy quarkonium production in AA collisions

- vacuum properties: spectroscopy, static energy & pNRQCD
- HQ potential vs free energy at finite T
- Equilibrium properties: potential models & T-matrix, reaction rates
- Phenomenology: semi-classical modelling of transport
- Phenomenology: Open quantum system approach

Heavy quarkonium: multi-scale system



Non-relativistic potential model

• use phenomenological static potentials, e.g., Cornell potential

$$V(r) = \sigma r - \frac{\alpha}{r}$$

- long-range scale: confining (non-perturbative QCD), string tension, $\sigma \simeq 0.2 \ {\rm GeV}^2$
- short-range scale: Coulomb-like (pQCD), $\alpha \simeq \pi/12$
- heavy-quarkonium states from non-relativistic Schrödinger equation

$$\left[2m_Q - \frac{1}{m_Q}\Delta + V(r)\right]\Phi_i(\vec{r}) = M_i\phi_i(r)$$

 fit to spin-averaged heavy-quarkonium spectra $\Rightarrow m_c = 1.25 \text{ GeV}, \ m_b = 4.65 \text{ GeV}, \ \sqrt{\sigma} = 0.445 \text{ GeV}, \ \alpha = \pi/12$

from wave function	$\langle r^2 \rangle = \langle \overline{\Phi}, \vec{r} \overline{\Phi} \rangle \rangle$	F		
	$\langle r_i \rangle = \langle \Psi_i \mathbf{I} \Psi_i \rangle 1.$.5 350	240	

• 11		vave	Turre		$\backslash i$	/ - \	$ \Psi_i $	$ \Psi_i $	1.5		350	24)		-	$\sim Oply V(18)$ is
state	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ_b'	Υ″	1	- 550		And a start	- Barris	າ ພ(2S		color-Coulomb
mass [GeV]	3.10	3.53	3.68	9.46	9.99	10.02	10.26	10.36	0.5		J	J	[/ψ,]	$\Gamma(2S)$		
ΔE [GeV]	0.64	0.20	0.05	1.10	0.67	0.54	0.31	0.20	0			- Y(18)		-	• $J/\psi \& Y(2S)$ are bound by
ΔM [GeV]	0.02	-0.03	0.03	0.06	-0.06	-0.06	-0.08	-0.07	-0.5				-40	α/3r+σ	r	confining force
r_0 [fm]	0.50	0.72	0.90	0.28	0.44	0.56	0.68	0.78	1	-		r [fm]			-	
										0.25	0.5	0.75	1	1.25	1.5	

Charmonium spectroscopy



Eichten et al., RMP80,1161(2008)

• Schrodinger equation with spin-dependent potential: $V(r)=V_{Cornell}(r) + V_{SD}(r)$ \Rightarrow fine and hyperfine splitting $V_{SS}(r) = \frac{1}{\sqrt{2}} \nabla^2 V_V(r) = \frac{16\pi \alpha_s}{\delta^3(r)}$

$$V_{SD}(r) = V_{SS}(r) \left[S(S+1) - \frac{3}{2} \right] + V_{LS}(r) (\mathbf{L} \cdot \mathbf{S}) \qquad V_{LS}(r) = \frac{1}{2m_Q m_{\bar{Q}}} \left(3 \frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr} \right) + V_T(r) \left[S(S+1) - 3(S \cdot \hat{r})(S \cdot \hat{r}) \right] \qquad V_T(r) = \frac{1}{6m_Q m_{\bar{Q}}} \left(3 \frac{dV_V^2(r)}{dr^2} - \frac{1}{r} \frac{dV_V(r)}{dr} \right)$$

Eichten & Quigg, PRD49,5845(1995); Soni et al., Eur.Phys. J. C(2018)78:592

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Bottomonium spectroscopy



Eichten et al., RMP80,1161(2008)

- $n=n_r + 1$, $n_r =$ number of nodes of radial wave functions
- P=(-1)^{L+1}, C=(-1)^{L+S}

Vacuum static Q-Qbar potential

• A quark-antiquark color singlet state with gauge link/Wilson line

$$|\phi_{\alpha\beta}^{lj}\rangle \equiv \frac{\delta_{lj}}{\sqrt{3}}\bar{\psi}_{\alpha}^{i}(x)U^{ik}(x,y,C)\psi_{\beta}^{k}(y)|0\rangle$$

Q-Qbar meson correlator

$$G(T) = \langle \phi(\mathbf{x}, 0) | \phi(\mathbf{y}, T) \rangle = \langle \phi(\mathbf{x}, 0) | \exp(-iHT) | \phi(\mathbf{y}, 0) \rangle$$

- → in large Euclidean time limit only ground state survives $G(-iT) = \sum_{n} \langle \phi(\mathbf{x},0) | \psi_n \rangle \langle \psi_n | \phi(\mathbf{y},0) \rangle \exp(-E_n T)$ $\rightarrow \langle \phi(\mathbf{x},0) | \psi_0 \rangle \langle \psi_0 | \phi(\mathbf{y},0) \rangle \exp(-E_0 T) \quad \text{for} \quad T \to \infty$
- ground state energy:

$$E_0 = -\lim_{T \to \infty} \frac{\log G(-iT)}{T}$$

 consider statice Q-Qbar pair (separation r) created at t=0, and annihilated at t=T

$$G_{\beta_1\beta_2\alpha_1\alpha_2}(T) \xrightarrow{m \to \infty} \delta^3(\mathbf{x}_1 - \mathbf{y}_1) \delta^3(\mathbf{x}_2 - \mathbf{y}_2)(P_+)_{\beta_1\alpha_1}(P_-)_{\alpha_2\beta_2}$$

$$\times e^{-2mT} / \operatorname{Tr} \mathbf{P} e^{ig \oint_{T_0} dz_\mu A_\mu(z)} \setminus$$

Wilson loop along a rectangular loop Γ_0

$$W(\Gamma_0) = \operatorname{Tr} \operatorname{Pe}^{ig \oint_{\Gamma_0} dz_{\mu} A_{\mu}(z)}$$

$$V_0(r) \equiv E_0(r) = -\lim_{T \to \infty} \frac{1}{T} \log \langle W(\Gamma_0) \rangle$$

$$U(x,y;C) = P \exp\left\{ig \int_{y}^{x} A_{\mu}(z) dz^{\mu}\right\}$$
$$U'(x,y;C) = \exp\left\{i\theta(x)\right\} U(x,y;C) \exp\left\{-i\theta(y)\right\}$$

Brambilla, 2204.11295

У,

х

t = T

t=0

 \mathbf{x}_{2}

Finite-T Q-Qbar potential (I)

• The Wilson loop obeys the Schrodinger equation

 $i\partial_t W_{\Box}(t, r) = \Phi(t, r) W_{\Box}(t, r).$

Q-Qbar potential defined at late times $V(r) = \lim_{t \to \infty} \Phi(r, t).$

Spectral function of real-time vs imaginary time Wilson loop

$$W_{\Box}(r,t) = \int d\omega e^{-i\omega t} \rho_{\Box}(r,\omega), \qquad W_{\Box}(r,\tau) = \int d\omega e^{-\omega \tau} \rho_{\Box}(r,\omega).$$

➔ Q-Qbar potential

$$V(r) = \lim_{t \to \infty} \frac{\int d\omega \, \omega e^{-i\omega t} \rho_{\Box}(r, \, \omega)}{\int d\omega e^{-i\omega t} \rho_{\Box}(r, \, \omega)} = \lim_{t \to \infty} i \frac{\partial \log W(r, \tau = it, T)}{\partial t}$$

• It is challenging to reconstruct the spectral function from lattice data of Euclidean Wilson loop If time-independent potential description holds for all times: $\Phi(t, r) = V(r) \rightarrow W(t,r) = \exp(-iV(r,T)t)$



Finite-T Q-Qbar potential (II)

• Perturbative evaluation from HTL (static) gluon propagator Laine et al., JHEP03(2007)054

$$D_{00}(\omega=0,\boldsymbol{q}) = \frac{-1}{\boldsymbol{q}^2 + m_D^2} + i \frac{\pi m_D^2 T}{|\boldsymbol{q}| (\boldsymbol{q}^2 + m_D^2)^2}$$

one-loop self-energy of the space-like gluon exchanged between Q-Qbar has an imaginary part → cutting it corresponds to Q scattering off medium q/g: Landau damping

Fourier transform → ReV + ImV

$$V_{\infty}(\mathbf{r}_{1}-\mathbf{r}_{2}) \equiv g^{2} \int \frac{dq}{(2\pi)^{3}} (1-e^{iq \cdot (\mathbf{r}_{1}-\mathbf{r}_{2})}) \left[\frac{1}{q^{2}+m_{D}^{2}}-i\frac{\pi m_{D}^{2}T}{|q|(q^{2}+m_{D}^{2})^{2}}\right]$$

$$= -\frac{g^{2}}{4\pi} \left[m_{D} + \frac{e^{-m_{D}r}}{r}\right] - i\frac{g^{2}T}{4\pi}\phi(m_{D}r)$$

$$\stackrel{\text{HTL ReV identical to free energy (screened Coulomb only) at the same order}}{energy (screened Coulomb only) at the same order}$$

$$\stackrel{\text{ImV} \rightarrow 2^{*}\Gamma_{Q} \text{ when } r \rightarrow \infty$$

$$\begin{array}{c}\text{Blazoit et al., NPA806(2008)312-338}\\\phi(x) \equiv 2\int_{0}^{\infty} dz \frac{z}{(z^{2}+1)^{2}} \left[1 - \frac{\sin(zx)}{zx}\right] \right]$$

Finite-T Q-Qbar potential (III)

• ReV & ImV extracted from lattice data of Eucliean Wilson loop Petreczky et al., PRD105, 054513 (2022)



ReV little screening up to 3-4*Tc; ImV large & increases toward high T



- Q-Qbar potential represent interactions on time-scale 1/E_B or 1/Γ_{diss}, shorter than 1/T→ insignificant screening
 - Q-Qbar free energy F on long time-scale >>1/T
 ➔ significant screening

FIG. 21. Comparison of extracted Ω using subtracted and unsubtracted correlators using Padé pole analysis with the T = 0 effective mass and color singlet free energy at T = 408 MeV

Finite-T Q-Qbar potential (IV)

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DNN extraction of Q-Qbar potential by

lattice data of bottomonia: ٠ little mass shift & large width



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1.0

Extracting HQ potential from bottomoium R_{AA}

- statistical transport analysis of Y data with trial input potential **Du et al. '19**
- trial screened Cornell potential with screening parameters \rightarrow T-matrix to get Y(nS) binding energies \rightarrow quasifree reaction rates (E_B larger \rightarrow interference stronger) \rightarrow rate equation \rightarrow R_{AA}
- K on single-Q collisional rate Γ_Q larger \rightarrow stronger potential \rightarrow larger $E_B \rightarrow$ stronger interference to render Γ_{diss} amenable to R_{AA}





- extracted potential little screening at T<250 MeV
- even at T~ 400 MeV, still significant residual confining force
- consistent with T-matrix solution of potential in the strongly coupled scenario,
- **1.2 & qualitatively consistent with latest lattice results**

Free energy of static Q-Qbar

McLerran, Svetitsky, PRD 24 (81) 450

 $\psi_a^{\dagger}(\tau, x), \ \psi_a(\tau, x)$ -creation annihilation operators for static quarks at time τ and position x

 $\psi_a^{\dagger c}(\tau,x), \ \psi_a^c(\tau,x)$ -creation annihilation operators for static antiquarks at time τ and position x

$$[\psi_a(\tau, x), \psi_b^{\dagger}(\tau, y)]_+ = \delta(x - y)\delta_{ab}$$

$$(-i\partial_{\tau} - gA_0(\tau, x)) \psi(\tau, x) = 0$$
formal solution $\psi(\tau, x) = \mathcal{P} \exp\left(ig\int_0^{\tau} d\tau' A_0(\tau', x)\right) \psi(0, x) = W(x)\psi(0, x)$
lattice : $W(x) = \prod_{x_0=0}^{N_{\tau}-1} U_0(x, \tau)$

Free energy of static quark anti-quark pair

$$Z(\beta)e^{-\beta F(x,y)} = \sum_{s} < s|e^{-\beta H}|s>$$

 $|s\rangle$ denotes any state with a static quark at position x and static anti-quark at position y;

Free energy of static Q-Qbar

Let us denote by $|s'\rangle$ states with no static quarks

$$e^{-\beta F(x,y)} = \sum_{s'} \frac{1}{N_c^2} \sum_{a=a',b=b'} < s' |\psi_a(0,x)\psi_b^c(0,y)e^{-\beta H}\psi_{a'}^{\dagger}(0,x)\psi_{b'}^{\dagger c}(0,y)|s' \ (1)$$

$$= \sum_{s'} \frac{1}{N_c^2} \sum_{a=a',b=b'} < s' |e^{-\beta H}\psi_a(\beta,x)\psi_b^c(\beta,y)\psi_{a'}^{\dagger}(0,x)\psi_{b'}^{\dagger c}(0,y)|s' \ (0,y)|s' \$$

L(x) = TrW(x)- Polyakov loop

→ Singlet Q-Qbar free energy ~ correlator of two Polyakov loops:

 $\exp(-F_1(r,T)/T) = \frac{1}{N} \langle \operatorname{Tr}[L^{\dagger}(x)L(y)] \rangle,$

- LO HTL: screened Coulomb only F<0 ٠ $F_1(r,T) = -\frac{N^2 - 1}{2N} \frac{\alpha_s}{r} \exp(-m_D r) - \frac{(N^2 - 1)\alpha_s m_D}{2N}$
- Lattice: F>0 for T<2*T_c, remnant of ٠ linear confining term
- Singe-Q F=deconfinement order parameter 0.2 for pure YM without dynamical light guarks

 $P = \frac{1}{N} \operatorname{tr} L, \ \langle P \rangle = \exp(-F_Q(T)/T)$



0.9

0.8

0.7

0.6

0.5

0.4

0.3

0.1

0

150 200 250 300 350 400

Heavy quarkonium in QGP



How does the suppression of heavy quarkonium in QGP come about?

- static color screening? Classic paradigm, but probably insignificant from state-of-the-art lattice studies
- dynamical inelastic collisional dissociation: inelastic reaction rate
- regeneration of charmonia from abundant near-thermalized charm & anticharm quarks, in particular at the LHC energies

Melting by static screening

• Karsch-Satz seminar work Karsch-Satz, Z.Phys.C37,617-622(1988) vacuum ornell potential → screening of both Coulomb and confining part

$$V(r, \mathbf{O}) = \sigma r - \frac{\alpha}{r}, \qquad V(r, T) = (\sigma/\mu(T))(1 - e^{-\mu(T)r}) - (\alpha/r) e^{-\mu(T)r}$$

$$\Rightarrow V(r \rightarrow \infty, T) = \sigma/\mu(T) = 2^* \triangle m_Q(T) \& m_Q^{\text{eff}}(T) = m_Q^0 + \triangle m_Q(T)$$

• Schrodinger equation for Q-Qbar with T-dependent potential

$$H(r, T) = 2m - \frac{1}{m} \nabla^2 + V(r, T) \qquad [H(r, \mu) - E_{n, l}(\mu)] \Phi_{n, l}(r, \mu) = 0,$$



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Melting by static screening



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Dynamical dissociation: LO

• Peskin's OPE analysis in vacuum: Coupling of heavy quarkonium to external gluons is a short-distance process **Peskin**, **Nucl. Phys. B 156**, **365** (1979)

• This leads to gauge-invariant multipole expansion of the external soft gluons around the small-size Q-Qbar bound state → NR Hamiltonian Yan, PRD22,1652(1980)



Deriving the gluo-dissociation cross section: $g + \psi \rightarrow c + cbar$

• Color E1 transition

--- Weyl gauge

$$H_{E1} = -\vec{d^a} \cdot \vec{E^a}(t, \vec{0}) = \frac{g_s}{2} (\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}) \vec{r} \cdot \frac{\partial \vec{A^a}}{\partial t}$$
with

$$\vec{A^a}(t, \vec{x}) = \sum_{\vec{k}, \lambda} N_{\vec{k}} \vec{\epsilon}_{\vec{k}\lambda} [a^a_{\vec{k}\lambda} e^{i\vec{k}\cdot\vec{x}-i\omega_E t} + h.c.]$$
Peskin'Coulomb
reproduced
---- Fermi's golden rule: 1st order perturbation

$$\sigma_{E_1}^{g_+J/\psi \to c+\vec{c}}(E_g) = \frac{g_s^2\pi}{2 \cdot 9} E_g \frac{V}{(2\pi)^3} \int d^3\vec{p} = \sigma_{E_1,Coulomb} \sigma_{E_1,Coulomb} (E_g) = \frac{2^7}{9} g_s^2 \frac{\epsilon_B^{5/2}}{m_Q} \frac{(E_g - \epsilon_B)^{3/2}}{E_g^5}$$

$$\times | < (c\bar{c})_8, \vec{p}|\vec{r}|J/\psi > |^2 \delta(E_g - \epsilon_B - \frac{\vec{p}^2}{m_Q}), \text{Coulomb appro.}$$
A novel
contribution

$$H_{M1} = -\vec{m^a} \cdot \vec{B^a}(t, \vec{0}) = -\frac{g_s}{2\pi g_s} (\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}) (\frac{\vec{\sigma}}{2} - \frac{\vec{\sigma}'}{2}) \cdot \nabla \times \vec{A^a}(t, \vec{0})$$

$$--- 1^{st} \text{ order perturbation}$$

$$\sigma_{M_1}^{g_+J/\psi \to c+\bar{c}}(E_g) = \frac{g_s^2\pi}{2 \cdot 3} \frac{E_g}{m_Q^2} \frac{V}{(2\pi)^3} \int d^3\vec{p} = \sigma_{M_1,Coulomb} \sigma_{M_1,Coulomb}(E_g) = \frac{2^3}{3} g_s^2 \frac{\epsilon_B^{5/2}}{m_Q^2} \frac{(E_g - \epsilon_B)^{1/2}}{E_g^3}$$

$$\times | < (c\bar{c})_8|J/\psi > |^2 \delta(E_g - \epsilon_B - \frac{\vec{p}^2}{m_Q}).$$
Coulomb appro.

- Selection rules
 - --- E1: $\Delta L = 1$, $\Delta S = 0$; M1: $\Delta L = 0$, $\Delta S = 1$ from singlet to octet transition --- P-wave states $\chi_c \& \chi_b$ also derived

In-medium potential Schrodinger

• In-medium potential model Satz 88

$$V(r,T) = -\frac{\alpha}{r}e^{-m_D(T)r} + \frac{\sigma}{m_D(T)}(1 - e^{-m_D(T)r})$$

$$[H(r,T) - E_{n,l}(T)]\psi_{n,l}(r,\theta,\phi) = 0$$

• Vacuum/in-medium quarkonium wave func., binding $\varepsilon_{B}(T)$





- --- higher T, stronger screening
 --- bound state size grows, binding energy decreases
- --- $T_d(J/\psi) \sim 1.7T_c$, $T_d(Y) \sim 2.6T_c$

Gluo-diss. $g + \psi \rightarrow c + cbar$ cross section



--- Coulomb wave func. + full ε_B differs from the full potential result by ~50% --- M1 overtakes E1 at low energies $\bigstar \Delta L = 0$ s-wave isotropic scattering dominant

• In-medium cross sections



--- as T increases, dipole size grows → E1 increases, M1 most prominent at low T --- more tightly bound Y(1S) much smaller cross section

Gluo-dissociation rates



M1 most prominent for J/ψ at low T, accounting for ~10-25% of the total (E1+M1) rate in T_c --1.2T_c → could be significant as system stays long
at low T, ε_B large, tightly bound J/ψ or Y → gluon sees the bound state as a whole → LO gluo-dissociation sensible → rate increases with T
at high T, ε_B decreases, σ [k²*f_g(k)] shifts toward lower [higher] energy

 \rightarrow phase space mismatched \rightarrow rate drops off fast \rightarrow calling for NLO

Dynamical dissociation: NLO

- LO, i.e. gluo-dissociation --- the incoming gluon sees the Ψ as a whole, which is reasonable for a tightly-bound state (and relatively low-energy, long-wavelength gluon)
 → applicable at low temperatures, efficient when gluon wave-length ~ Ψ size
- At high temperatures, Ψ binding energy drops off and radius grows, the incoming gluon should see individual Q or Qbar in Ψ, opening up new phase space Rapp et al., 2001 & 2011



 Quasi-free: σ(g+Ψ→g+c+cbar)=2*σ(g+c→g+c), with a spectator cbar: off-shellness of Q & Qbar within Ψ & bound state wave function effects are neglected



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NLO dissociation: going beyond quasi-free

• Quasi-free: effects of bound state wave func. completely ignored



• Starting from the QCD Hamiltonian in Weyl gauge A^{0a}=0 $H_{QCD} = \int d^{3}\vec{x}\psi^{\dagger}(t,\vec{x})[\beta m_{q} - i\vec{\alpha} \cdot (\vec{\partial} - ig_{s}\vec{A}^{a}(t,\vec{x})\frac{\lambda^{a}}{2})]\psi(t,\vec{x})$ $+ \int d^{3}\vec{x}[\frac{1}{2}\vec{E}^{a}(t,\vec{x}) + \frac{1}{2}\vec{B}^{a}(t,\vec{x})]$ $= H_{kin} + V_{q\bar{q}g} + V_{Q\bar{Q}g} + V_{3g} + V_{4g}$ with $\vec{E}^{a} = -\frac{\partial\vec{A}^{a}}{\partial t} - \nabla A^{0a} + g_{s}f^{abc}\vec{A}^{b}A^{0c} = -\frac{\partial\vec{A}^{a}}{\partial t}$ $\vec{B}^{a} = \nabla \times \vec{A}^{a} - \frac{1}{2}g_{s}f^{abc}\vec{A}^{b} \times \vec{A}^{c}$

Effective Hamiltonian

 QCD multipole expansion for heavy quarkonium system Yan 80, Sumino 14 *V_{QQgg} → H₀ + V_{SO} + V_{OO}

 for the QQbar system bound by internal gluons* H₀ = ^{*p*²}/_{*mQ*} + V₀(*r*) + V₈(*r*)

 One is left with 4 interaction vertices at the order of O(g_s)



NLO: $g + \Psi \rightarrow g + c + cbar$ diagrams

• s- & u-channel diagrams formed out of V_{SO} & V_{OO} analogous to Compton



• t-channel diagrams formed out of V_{SO} & V_{3g} looks like quasi-free diagram



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NLO: 2nd order QM perturbation

• Transition amplitude for $g + \Psi \rightarrow g + c + cbar$

 $T_{fi} = \sum_{m} \frac{\langle f | H_I | m \rangle \langle m | H_I | i \rangle}{E_i - E_m + i\epsilon},$ Chen & MH, PLB786 (2018) 260–267

initial/final state: $|i\rangle = |J/\psi, g(\vec{k}, \lambda, a)\rangle = |(c\bar{c})_8(\vec{p}, b), g(\vec{\kappa}, \sigma, c)\rangle$

• Intermediate states & amplitudes for diagrams (a) & (b)

$$\begin{split} A(p,k) &= \frac{\int r^3 dr j_1(pr) R_{10}(r)}{p(-\epsilon_B + \omega_{\vec{k}} - \frac{\vec{p}^2}{m_Q} + i\epsilon)}, \\ B(p,k) &= \frac{\frac{2p}{m_Q} \int r^3 dr j_1(pr) R_{10}(r)}{(-\epsilon_B + \omega_{\vec{k}} - \frac{\vec{p}^2}{m_Q} + i\epsilon)^2} - \frac{\int r^4 dr j_2(pr) R_{10}(r)}{(-\epsilon_B + \omega_{\vec{k}} - \frac{\vec{p}^2}{m_Q} + i\epsilon)} \\ &= d^{bac} \frac{ig_s^2}{2V} \sqrt{\frac{\pi \omega_{\vec{k}} \omega_{\vec{k}}}{6V}} \vec{\epsilon}_{\vec{k}\lambda} \cdot [A(p,k)\vec{\epsilon}_{\vec{k}\sigma} + (\vec{\epsilon}_{\vec{k}\sigma} \cdot \vec{p})\frac{\vec{p}}{p^2}B(p,k)]. \end{split}$$

wave func., $\Delta L=2$, dipole transition twice

--- (2) $|m\rangle = |(c\bar{c})_{c}(\vec{a}, d)\rangle$

NLO: 2nd order QM perturbation (cont.)

$$\begin{array}{l} ---\left(\mathbf{b}\right) \quad |m >= |(c\bar{c})_{8}(\bar{q},d),g(\vec{k}_{1},\lambda_{1},d_{1}),g(\vec{k}_{2},\lambda_{2},d_{2}) > \\ C(p,\kappa) = \frac{\int r^{3}dr j_{1}(pr)R_{10}(r)}{p(-\epsilon_{B}-\omega_{R}-\frac{\vec{p}^{2}}{m_{Q}}+i\epsilon)} \qquad T_{fi}^{(b)} = -d^{bac}\frac{ig_{s}^{2}}{2V}\sqrt{\frac{\pi\omega_{\vec{k}}\omega_{\vec{k}}}{6V}} \quad \mathbf{C} \quad \mathbf{d}^{abc} \\ D(p,\kappa) = \frac{\frac{2p}{m_{Q}}\int r^{3}dr j_{1}(pr)R_{10}(r)}{(-\epsilon_{B}-\omega_{R}-\frac{\vec{p}^{2}}{m_{Q}}+i\epsilon)} - \frac{\int r^{4}dr j_{2}(pr)R_{10}(r)^{-1}}{(-\epsilon_{B}-\omega_{R}-\frac{\vec{p}^{2}}{m_{Q}}+i\epsilon)} \qquad \times \vec{\epsilon}_{\vec{k}\lambda} \cdot [C(p,\kappa)\vec{\epsilon}_{\vec{k}\sigma} + (\vec{\epsilon}_{\vec{k}\sigma}\cdot\vec{p})\frac{\vec{p}}{p^{2}}D(p,\kappa)] \\ \bullet \quad \mathbf{Intermediate states} \quad \mathbf{\& amplitudes} \quad \mathbf{for \ diagrams} \ (\mathbf{c}) \quad \mathbf{\&} \quad (\mathbf{d}) \\ --\cdot \left(\mathbf{c}\right) \quad |m \rangle = |(c\bar{c})_{8}(\vec{q},d),g(\vec{k}_{1},\lambda_{1},d_{1}),g(\vec{k}_{2},\lambda_{2},d_{2}) > \\ T_{fi}^{(c)} = f^{abc}\frac{(-ig_{s}^{2})}{V}\sqrt{\frac{\pi}{6V\omega_{\vec{k}}\omega_{\vec{k}}}} \{(\vec{\epsilon}_{\vec{k}\sigma}\cdot\vec{p})(\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{\kappa}) \\ + (\vec{\epsilon}_{\vec{k}\sigma}\cdot\vec{k})(\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{p}) - (\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{\epsilon}_{\vec{k}\sigma})\frac{(\vec{k}^{2}-\vec{k}\cdot\vec{\kappa})\vec{k}\cdot\vec{p} + (\vec{k}^{2}-\vec{k}\cdot\vec{\kappa})\vec{k}\cdot\vec{p}]}{(\vec{k}-\vec{\kappa})^{2}} \\ \times \frac{1}{p}\int r^{3}dr j_{1}(pr)R_{10}(r)\frac{1}{-\epsilon_{B}^{J/\psi}-\frac{\vec{p}^{2}}{mc}-\omega(\vec{k}-\vec{\kappa})+i\epsilon}} \\ T_{fi}^{(d)} = f^{abc}\frac{ig_{s}^{2}}{V}\sqrt{\frac{\pi}{6V\omega_{\vec{k}}\omega_{\vec{k}}}} \{(\vec{\epsilon}_{\vec{k}\sigma}\cdot\vec{p})(\vec{\epsilon}_{\vec{k}\lambda}\vec{\kappa}) + (\vec{\epsilon}_{\vec{k}\sigma}\cdot\vec{k})(\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{p}) \\ - (\vec{\epsilon}_{\vec{k}\lambda}\cdot\vec{\epsilon}_{\vec{k}\sigma})\frac{(\vec{k}^{2}-\vec{k}\cdot\vec{\kappa})\vec{k}\cdot\vec{p} + (\vec{k}^{2}-\vec{k}\cdot\vec{\kappa})\vec{k}\cdot\vec{p}]}{(\vec{k}-\vec{\kappa})^{2}} \} \\ \times \frac{1}{p}\int r^{3}dr j_{1}(pr)R_{10}(r)\frac{1}{\omega(\vec{k})-\omega(\vec{k})-\omega(\vec{k})-\omega(\vec{k}-\vec{k})+i\epsilon}. \\ \bullet \ \mathbf{d}^{abc} \mathbf{f}^{abc} = \mathbf{0} = > \ no \ interference \ between \ T_{fi}^{(a)} + T_{fi}^{(b)} \quad \mathbf{k} \quad T_{fi}^{(c)} + T_{fi}^{(d)} \end{aligned}$$

NLO cross section: (a) + (b)

• 2nd transition rate

$$\Gamma_{i \to f}^{(a+b)} = \frac{2\pi}{\hbar} \sum_{f} |T_{fi}^{(a)} + T_{fi}^{(b)}|^2 \delta(E_i - E_f)$$
Cross section= rate/flux, using $\frac{1}{4\pi} \int d\Omega_{\vec{k}} \frac{1}{2} \sum_{\lambda=1,2} |\vec{\epsilon}_{\vec{k}\lambda} \cdot \vec{\rho}|^2 = \frac{1}{3} |\vec{\rho}|^2$

$$\sigma^{(a+b)}(E_g) = 2\pi V \frac{V}{(2\pi)^3} \int d^3 \vec{p} \sum_b \frac{V}{(2\pi)^3} \int d^3 \vec{\kappa} \sum_\sigma \sum_c \frac{1}{4\pi} \int d\Omega_{\vec{k}}$$

$$\times \frac{1}{2} \sum_{\lambda} \frac{1}{8} \sum_a |T_{fi}^{(a)} + T_{fi}^{(b)}|^2 \delta(-\epsilon_B + \omega_{\vec{k}} - \frac{\vec{p}^2}{m_Q} - \omega_{\vec{k}}),$$

$$= \frac{5}{216\pi^2} g_s^4 E_g \int p^2 dp \int \kappa^2 d\kappa \omega_{\vec{k}} \times \{\cdots\} \delta(-\epsilon_B + E_g - \frac{\vec{p}^2}{m_Q} - \omega_{\vec{k}})$$
where $\{\cdots\} = [A^2(p,k) + \frac{1}{3}B^2(p,k) + \frac{2}{3}A(p,k)B(p,k)]$

$$+ [C^{-}(p,\kappa) + \frac{1}{3}D^{-}(p,\kappa) + \frac{1}{3}C(p,\kappa)D(p,\kappa)] - 2[A(p,k)C(p,\kappa) + \frac{1}{3}(A(p,k)D(p,\kappa) + B(p,k)C(p,\kappa) + B(p,k)D(p,\kappa))].$$
(21)

--- Infrared divergence from A(p, κ) & B(p, κ) \propto 1/ κ if gluons are massless, to be regularized by finite thermal gluon mass

NLO cross section: (c) + (d)



--- soft-collinear divergence (the square bracket) for massless-k / / κ gluons





NLO cross section: finite temperature

In-medium cross section

--- divergences regularized by gluons' thermal mass $m_g(T) = \sqrt{3/4}g_sT$



--- NLO quickly takes over from LO; no fall-off

- -- NLO (c)+(d): decreasing with T, expanding wave function overcome by decreasing ε_B
- --- NLO (a)+(b): increases fast with T, due to expanding wave function & decreasing ϵ_B
- near T_d, dipole size blows up > gluon wave-length, dipole transition to be invalidated
 near T_d, cross sections may not be quantitatively reliable

NLO dissociation rate



The artifact of LO dropping off toward high T: replaced by NLO increase
 Near T_d, Γ_{diss} ~ GeV: very fast break-up, conceptually consistent with static dissociation by color screening

--- Quantitatively might be questionable as dipole approximation becomes invalidated, but empirically supported/needed by phenomenological transport study, e.g. Strickland '15, $\Gamma_{diss} > 2$ GeV needed for T>=T_d

pNRQCD point of view: **Y** dissociation

pNRQCD: d.o.f. = singlet & octet QQbar states + light quarks & gluons ٠

Sina selfe

$$\mathcal{L}_{\text{pNRQCD}} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\,\mu\nu} + \sum_{i=1}^{n_{f}} \bar{q}_{i} i \not D q_{i} + \int d^{3}r \,\operatorname{Tr} \left\{ S^{\dagger} \left[i\partial_{0} - h_{s} \right] S + O^{\dagger} \left[iD_{0} - h_{o} \right] O + V_{A} \left(O^{\dagger} \mathbf{r} \cdot g \mathbf{E} S + \text{H.c.} \right) + \frac{V_{B}}{2} O^{\dagger} \left\{ \mathbf{r} \cdot g \mathbf{E} , O \right\} + \dots \right\}.$$
(5)

Singlet-to-octet transition ~ gluo-dissociation, singlet 1-loop selfenergy \rightarrow cutting: g+ $\Psi \rightarrow$ c-cbar octet ٠



Landau damping (ImV) ~ parton inelastic collisional dissociation ~ guasi-free Brambilla et al., 2013 ٠

Dissociation rate from Schrodinger with ImV

Solving the eigenstate Schrodinger equation with Q-Qbar complex potential

$$-\frac{\nabla^2}{m_b}\psi_n + [V_R(T,r) + iV_I(T,r)]\psi_n = E_n\psi_n \quad \Rightarrow \quad \Gamma_i = -2\mathrm{Im}[E_i]$$
Strickland et al., PRD97,016017(2018)

complex potential taken from fitting lattice data: Coulomb + string ReV + ImV

$$V_c(r) = -\alpha_s \left[m_{\rm D} + \frac{e^{-m_{\rm D}r}}{r} + iT\phi(m_{\rm D}r) \right]$$

$$\operatorname{Re}V_{s}(r) = -\frac{\Gamma[\frac{1}{4}]}{2^{\frac{3}{4}}\sqrt{\pi}}\frac{\sigma}{\mu}D_{-\frac{1}{2}}(\sqrt{2}\mu r) + \frac{\Gamma[\frac{1}{4}]}{2\Gamma[\frac{3}{4}]}\frac{\sigma}{\mu}$$
$$\operatorname{Im}V_{s}(r) = -i\frac{\sigma m_{D}^{2}T}{\mu^{4}}\psi(\mu r) = -i\alpha_{s}T\psi(\mu r)$$



Summary of dissociation rates



MH, van Hees, Rapp, arXiv:2204.09299

- TAMU: quasi-free with T-matrix binding energies; interference effect (1-sin(tr)/tr) • implemented for Y states
- Tsinghua: gluo-dissociation with geometric scaling with in-medium radius of bound states ٠
- .Kent: computed from Schrodinger eigen-energy with ImV_{OObar} ٠
- \rightarrow Reasonable agreement in values for J/ ψ and Y(2S) but differing considerably for Y(1S) between different groups (although very different underlying assumptions)

Semi-classical transport approaches

• Boltzmann equation for Ψ transport in QGP

$$\frac{\partial f_{\Psi}}{\partial t} + \vec{v}_{\Psi} \cdot \vec{\nabla} f_{\Psi} = -\alpha_{\Psi} f_{\Psi} + \beta_{\Psi} \qquad f_{\Psi}(x, p, t) = \frac{(2\pi)^3 dN_{\Psi}}{d^3 x \, d^3 p}$$
 loss term gain term

• For 2 \leftarrow >2 process: gluo-dissociation $\Psi + g \leftrightarrows c + \bar{c}$,



$$\begin{split} \alpha_{\Psi}(p,x) &= \frac{1}{2E_{\Psi}} \int d\Phi_1(p_g) \ d\Phi_2(p_c,p_{\bar{c}}) \\ &\times (2\pi)^4 \delta^{(4)}(p_c + p_{\bar{c}} - p - p_g) \ d_g \ f_g(p_g) \ |\overline{\mathcal{M}_{\Psi g \to c\bar{c}}(s,t)}|^2 \\ &= \int \frac{d^3 p_g}{(2\pi)^3} \ d_g \ f_g(p_g) \ v_{rel} \ \sigma_{\Psi}(s) = \Gamma(\mathsf{p},\mathsf{T}) \quad \not \rightarrow \text{dissociation rate} \\ \beta_{\Psi}(p,x) &= \frac{1}{2E_{\Psi}} \int d\Phi_1(p_g) d\Phi_2(p_c,p_{\bar{c}})(2\pi)^4 \delta^{(4)}(p_c + p_{\bar{c}} - p - p_g) \quad \not \rightarrow \text{ regeneration via} \\ &\quad \times |\overline{\mathcal{M}_{c\bar{c}} \to \Psi g}(s,t)|^2 \ d_c \ f_c(p_c) \ d_{\bar{c}} \ f_{\bar{c}}(p_{\bar{c}}) \ (1 + f_g(p_g)) \ , \end{split}$$

Semi-classical Boltzmann

Boltzmann equation for Ψ transport in QGP

$$\frac{\partial f_{\Psi}}{\partial t} + \vec{v}_{\Psi} \cdot \vec{\nabla} f_{\Psi} = -\alpha_{\Psi} f_{\Psi} + \beta_{\Psi} \qquad f_{\Psi}(x, p, t) = \frac{(2\pi)^3 dN_{\Psi}}{d^3 x \, d^3 p}$$

loss term gain term

• For 2 \leftarrow >3 process: quasi-free/Landau damping dissociation $i + \Psi \rightarrow i + c + \bar{c} \; (i=g,q,\bar{q})$



Semi-classical Boltzmann: formal solution

Boltzmann equation for Ψ transport in QGP

$$\frac{\partial f_{\Psi}}{\partial t} + \vec{v}_{\Psi} \cdot \vec{\nabla} f_{\Psi} = -\alpha_{\Psi} f_{\Psi} + \beta_{\Psi} \qquad f_{\Psi}(x, p, t) = \frac{(2\pi)^3 dN_{\Psi}}{d^3 x \, d^3 p}$$
loss term gain term

• Formation solution A.Polleri, arXiv:nucl-th/0303065 (2003); Yan & Zhuang, PRL97, 232301 (2006)

$$f_{\Psi}(p, x, t) = f_{\Psi}(p, x - v_{\Psi}(t - t_0), t_0) e^{-\int_{t_0}^t dt' \alpha_{\Psi}(p, x - v_{\Psi}(t - t'), t')} + \int_{t_0}^t dt' \beta_{\Psi}(p, x - v_{\Psi}(t - t'), t') e^{-\int_{t'}^t dt'' \alpha_{\Psi}(p, x - v_{\Psi}(t - t''), t'')}$$

- 1st term: describing the dissociation/loss of the initially produced heavy quarkonia, at t₀, known as primordial component
- 2nd term: increasing with time, describing the regeneration process of heavy quarkonia from recombination of charm and anticharm quarks from t₀ to t', and their subsequent dissociation from t' to t

Reduction of Boltzmann to rate equation

• When c & cbar (as well as light gluons/quarks) are in full thermal equilibrium,

For 2 \leftarrow >2 process: gluo-dissociation $\Psi + g \leftrightarrows c + \bar{c}$, $E_g + E_{\Psi} = E_c + E_{\bar{c}} \rightarrow e^{-E_g/T} \cdot \gamma_c^2 e^{-E_{\Psi}/T} = \gamma_c e^{-E_c/T} \cdot \gamma_c e^{-E_c/T}$

For 2 \leftarrow >3 process: quasi-free/Landau damping dissociation $i + \Psi \rightarrow i + c + \bar{c} \; (i=g,q,\bar{q})$

$$E_g + E_{\Psi} = E'_g + E_c + E_{\bar{c}} \quad \Rightarrow \quad e^{-E_g/T} \cdot \gamma_c^2 e^{-E_{\Psi}/T} = e^{-E'_g/T} \cdot \gamma_c e^{-E_c/T} \cdot \gamma_c e^{-E_{\bar{c}}/T}$$

detailed balance condition at the level of scattering matrix element squared:

$$\overline{\left|\mathcal{M}_{i\to f}\right|^2} = (g_f/g_i)\overline{\left|\mathcal{M}_{f\to i}\right|^2}$$

→ gain term : $\beta = \Gamma_{\Psi}(p,T) \cdot f_{\Psi}^{eq}(x,p)$

- This leads to (true even when including charm fugacity) $\frac{df_{\Psi}(x,p,t)}{dt} = -\Gamma_{\Psi}(p,T)[f_{\Psi}(x,p,t) f_{\Psi}^{eq}(x,p)]$
- Assume a momentum average dissociation rate Γ_ψ(T), and upon integration over x and p,
 → kinetic rate equation for the integrated yield

$$\frac{\mathrm{d}N_{\Psi}(t)}{\mathrm{d}t} = -\Gamma_{\Psi}(t)[N_{\Psi}(t) - N_{\Psi}^{\mathrm{eq}}(t)]$$
reaction rate Γ_{Ψ} regeneration toward equilibrium

Kinetic rate equation

can be decomposed into two equations for primordial and regenerated component

 $N_{\Psi}(\tau) = N_{\Psi}^{\text{prim}}(\tau) + N_{\Psi}^{\text{reg}}(\tau)$ for primordial component: $\frac{dN_{\Psi}^{\text{prim}}}{d\tau} = -\Gamma_{\Psi}^{\text{diss}}N_{\Psi}^{\text{prim}} \text{ with initial condition: } N_{\Psi}^{\text{prim}}(0) = N_{\Psi}(0)$ for regeneration component: $\frac{dN_{\Psi}^{\text{reg}}}{d\tau} = -\Gamma_{\Psi}^{\text{diss}}(N_{\Psi}^{\text{reg}} - N_{\Psi}^{\text{eq}}) \text{ with initial condition: } N_{\Psi}^{\text{reg}}(\tau < \tau_{0}^{\Psi}) = 0 \quad T(\tau_{0}^{\Psi}) = T_{\Psi}^{\text{diss}}$

Two transport parameters: reaction rate & equilibrium limit

$$N_{q\bar{q}} = \frac{1}{2} \gamma_q n_{\rm op} V_{\rm FB} \frac{I_1(\gamma_q n_{\rm op} V_{\rm FB})}{I_0(\gamma_q n_{\rm op} V_{\rm FB})} + \gamma_q^2 n_{\rm hid} V_{\rm FB} \qquad \qquad N_{\mathcal{Q}}^{\rm eq}(T, \gamma_Q) = V_{\rm FB} \ d_{\mathcal{Q}} \ \gamma_Q(T)^2 \int \frac{d^3 p}{(2\pi)^3} \ f^B(m_{\mathcal{Q}}, T)$$

• Correction to the equilibrium limit, due to off-equilibrium distribution of c & cbar quarks with $\tau_c^{eq}=3-5 \text{ fm/c}$: $N_{\Psi}^{eq} = \mathcal{R}(\tau)N_{\Psi}^{stat}$, $\mathcal{R}(\tau) = 1 - \exp\left(-\tau/\tau_c^{eq}\right)$



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Equilibrium limit: $J/\psi vs \psi(2S)$



• For large-mass particle m>>T: $n \propto (mT)^{3/2} e^{-m/T}$

- But $N_{\psi} < N_D$ $N_{c\bar{c}} \simeq \gamma_c n_{op}^{th} V \Rightarrow \gamma_c \propto (m_D T)^{-3/2} \cdot e^{m_D/T} / V$
- The charmonium equilibrium number

$$N_{\Psi}^{\rm eq} = V \gamma_c^2 \cdot n_{\Psi}^{\rm th} = V \gamma_c^2 (m_{\Psi} T)^{3/2} e^{-m_{\Psi}/T} \propto m_{\Psi}^{3/2} / [m_D^3 T^{3/2} V] \cdot e^{E_B/T}$$

where binding energy $E_B = 2m_D - m_{\Psi}$

Statistical production of charmonia

[Braun-Munzinger and Stachel, PLB 490 (2000) 196] [Andronic, Braun-Munzinger and Stachel, NPA 789 (2007) 334]

- Charm quarks are produced in initial hard scatterings $(m_{c\bar{c}} \gg T_c)$ and production can be described by pQCD $(m_{c\bar{c}} \gg \Lambda_{QCD})$
- Charm quarks survive and thermalise in the QGP
- ► Full screening before *T*_{CF}
- Charmonium is formed at phase boundary (together with other hadrons)
- Thermal model input $(T_{CF}, \mu_b \rightarrow n_X^{th})$

$$N_{c\bar{c}}^{\mathsf{dir}} = \underbrace{\frac{1}{2}g_{c} \mathcal{V}\left(\sum_{i} n_{D_{i}}^{\mathsf{th}} + n_{\Lambda_{i}}^{\mathsf{th}} + \cdots\right)}_{\mathsf{Open \ charm}} + \underbrace{g_{c}^{2} \mathcal{V}\left(\sum_{i} n_{\psi_{i}}^{\mathsf{th}} + n_{\chi_{i}}^{\mathsf{th}} + \cdots\right)}_{\mathsf{Charmonia}}$$

- Canonical correction is applied to nth_{oc}
- With exact charm conservation, charm balance equation: $N_{c\bar{c}}^{dir} = \frac{1}{2}g_c N_{oc}^{th} \frac{I_1(g_c N_{oc}^{th})}{I_0(g_c N_{c\bar{c}}^{th})} + g_c^2 N_{c\bar{c}}^{th} \rightarrow g_c$
- Production yields $N_D = g_c V n_D^{th} I_1 / I_0$ $N_{J/\psi} = g_c^2 V n_{J/\psi}^{th}$
- Charm fugacity determined by the input charm cross section (subject to CNM/shadowing): $N_{ccbar} = \langle T_{AA} \rangle d\sigma^{ccbar}/dy$

$$\sigma_{c\bar{c}} = 1/2 \ [\sigma_{D^+} + \sigma_{D^-} + \sigma_{D^0} + \sigma_{\bar{D}^0} + \sigma_{\Lambda_c} + \sigma_{\bar{\Lambda}_c} \dots]$$

SHM of charmonia

• Typical charm fugacity at LHC energies



• SHM charmonia: prediction of $\psi(2S)/J/\psi \sim 0.05$, a factor 3 smaller than pp integrated R_{AA} sensitive to input charm cross section



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Quarkonium excitation function: R_{AA} **vs sqrt(s)**



- J/ ψ : at SPS, T<=230 MeV (from dilepton data), J/ ψ suppression due to hot dissociation of excited states ($\chi_c \& \psi(2S)$) + nuclear absorption (CNM) [direct J/ ψ not affected]
- J/ψ: at RHIC & LHC: higher T & hot suppression stronger, but regeneration from abundant near-thermalized c-cbar becomes efficient → R_{AA} becomes larger toward LHC energies
- Y(2S) similar vacuum binding energy as J/ψ, but very different R_{AA}, because b-bbar small number and less thermalization → nearly no regeneration → Y(nS) sequential suppression and stronger suppression toward LHC energies (higher T)

$J/\psi \; r_{AA}$ to characterize regeneration



- J/ψ r_{AA} =1.5 (SPS) → 1 (RHIC) → 0.5 (LHC): transition from primordial production at SPS to regeneration production from a nearly thermal source at LHC
- Stronger thermalization (lower mean p_T) of c-cbar enhances regeneration of J/ ψ

J/ψ : suppression vs regeneration



• semi-classical transport: regeneration component p_T -spectrum modeled with a blast wave \rightarrow falling off two fast Du et al. '15, Zhou et al. '14, Ferreiro et al. '14

$$\frac{dN_{\Psi}^{\text{reg}}}{p_t dp_t} \propto m_t \int_0^R r dr K_1 \left(\frac{m_t \cosh y_t}{T}\right) I_0 \left(\frac{p_t \sinh y_t}{T}\right)$$

SHMc: hydrodynamic blastwave spectrum + pp corona

Andronic et al. '19

$$\frac{\mathrm{d}N(h_{hc}^{j})}{\mathrm{d}y} = g_{c}^{2} V n_{j}^{\mathrm{th}} \propto \mathrm{g_{c}}^{2} \leftarrow \mathrm{d}\sigma^{\mathrm{ccbar}}/\mathrm{d}y$$

J/ψ "v₂ puzzle"



Latest SQM22: $J/\psi vs \psi(2S)$



- Sequential regeneration: T_d(J/ψ) > T_d(ψ(2S))
 → regeneration of ψ(2S) is much later (at T~150-160 MeV)
- $R_{AA}(J/\psi) > R_{AA}(\psi(2S))$: production of both of them at low p_T are dominated by regeneration
- ALICE data of ψ(2S)/J/ψ favors transport calculation; SHM is disfavored



Open quantum system approach to bottomonia

- Regeneration of bottomonia is insignificant → more direct window on suppression mechanisms semi-classical transport: Γ(T) vs E_B(T) → a potential much stronger than F is needed Du,MH&Rapp, PRC96, 054901 (2017) Strickland et al., PRD 97, 016017 (2018)
- semi-classicsl transport: well-defined eigenstates (bound states) during in-medium evolution eigenstates being dissociated, but no quantum transition between them
- Evolution of a single b-bbar pair wave-packet: time-dependent Schrodinger equation with complex in-medium Q-Qbar potential $V(r) = V_R(r) + iV_I(r)$

 $V_I(r) = \Im[V(r)] = -C_F \alpha_s T \phi(m_D r)$

$$V_{R}(r) = \Re[V(r)] = \begin{cases} V_{\rm KMS}(r) & \text{if } V_{\rm KMS}(r) \le V_{\rm vac}(r_{\rm SB}) \\ V_{\rm vac}(r_{\rm SB}) & \text{if } V_{\rm KMS}(r) > V_{\rm vac}(r_{\rm SB}) \end{cases} \qquad \qquad V_{\rm KMS}(r) = U = -\frac{a}{r}(1+m_{D}r)e^{-m_{D}r} + \frac{2\sigma}{m_{D}}\left[1-e^{-m_{D}r}\right] - \sigma re^{-m_{D}r}$$

Strickland et al., JHEP03(2021)235

Purely radial potential \rightarrow angular momentum is conserved $u(r, \theta, \phi, t) = r\psi(r, \theta, \phi, t)$

$$u(r,\theta,\phi,t) = \sum_{\ell m} u_{\ell m}(r,t) Y_{\ell m}(\theta,\phi) \qquad \qquad \hat{H}_{\ell} = \frac{\hat{p}^2}{2m} + V_{\text{eff},\ell}(r,t)$$

• Independent time evolution of different l states: $u_{\ell}(r, t + \Delta t) = \exp(-i\hat{H}_{\ell}\Delta t)u_{\ell}(r, t)$

with initial wave-function $u_{\ell}(r, \tau = 0) \propto r^{\ell+1} \exp(-r^2/\Delta^2) \rightarrow$ finally projecting onto vacuum eigenstates

 The total norm of a single b-bbar pair is conserved, but are being redistributed into different bound eigenstates and unbound (dissociated) state due to ImV [if there's only ReV, expansion coefficient c_n(t)=c_n(0)exp(-iE_nt), probabilities not changing]

OQS: reduced density matrix

• b-bbar as an open quantum system interacting with the medium described by a reduced density matrix: $R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$



Probe = heavy-quarkonium state

Medium = light quarks and gluons that comprise the QGP

 Can treat heavy quarkonium states propagating through QGP using an open quantum system approach

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm medium} + I_{\rm probe} \otimes H_{\rm medium} + H_{\rm int}$$

• Total density matrix

$$\rho_{\text{tot}} = \sum_{j} p_j |\psi_j\rangle \langle \psi_j | \longrightarrow \frac{d}{dt} \rho_{\text{tot}} = -i[H_{\text{tot}}, \rho_{\text{tot}}]$$

• Reduced density matrix Brambilla et al. 2018, 2019, 2021; Yao 2021; Akamatsu, 2021

 $\rho_{\text{probe}} = \text{Tr}_{\text{medium}}[\rho_{\text{tot}}] \longrightarrow \text{Evolution equation?}$

OQS: pNRQCD + Lindblad equation

Non-relativistic bottomonium + medium scale hierarchy: marginally satisfied only by Coulombic Y(1S)

 $M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E_{\star}$

environment correlation time τ_{E} , ~1/ π T,

system/bottomonium intrinsic time $\tau_s \sim 1/E$, \rightarrow

system relaxation time 1/ Γ : $\tau_R \sim \frac{1}{\Sigma_s} \sim \frac{1}{a_0^2 (\pi T)^3}$

Brambilla et al. 2017, 2018, 2019, 2021

 $au_R \gg au_{E^*}$

Markovian approximation, Insensitive to prior evolution

- $\tau_S \gg \tau_E$. bottomonium quantum Brownian motion
- Non-relativistic bottomonium: pNRQCD, singlet & octet as d.o.f. + dipole coupling to medium gluons

$$\mathcal{L} = -\frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu,a} + \operatorname{Tr} \left\{ S^{\dagger} \left(i\partial_{0} - \frac{\mathbf{p}^{2}}{m} - V_{s} \right) S + O^{\dagger} \left(i\partial_{0} - \frac{\mathbf{p}^{2}}{m} - V_{o} \right) O \right\}$$
$$+ V_{A} \operatorname{Tr} \left\{ O^{\dagger} \mathbf{r} \cdot g \mathbf{E} S + S^{\dagger} \mathbf{r} \cdot g \mathbf{E} O \right\} \rightarrow \underbrace{\frac{1}{O^{\dagger} \mathbf{r} \cdot g \mathbf{E} S}}_{O^{\dagger} \mathbf{r} \cdot g \mathbf{E} S}$$
$$+ \frac{V_{B}}{2} \operatorname{Tr} \left\{ O^{\dagger} \mathbf{r} \cdot g \mathbf{E} O + O^{\dagger} O \mathbf{r} \cdot g \mathbf{E} \right\} \rightarrow \underbrace{\frac{1}{O^{\dagger} \mathbf{r} \cdot g \mathbf{E} S}}_{O^{\dagger} \mathbf{r} \cdot g \mathbf{E} O}$$

• Lindblad equation for the reduced density matrix (diagonal in color singlet & octet space)

$$\rho = \begin{pmatrix} \rho_s & 0\\ 0 & \rho_o \end{pmatrix} \quad \frac{d\rho}{dt} = -i[H,\rho] + \sum_i (C_i \rho C_i^{\dagger} - \frac{1}{2} \{C_i^{\dagger} C_i,\rho\})$$

OQS: pNRQCD + Lindblad equation

• Lindblad equation for the reduced density matrix (diagonal in color singlet & octet space)

$$H = \begin{pmatrix} h_s & 0\\ 0 & h_o \end{pmatrix} + \frac{r^2}{2} \gamma(t) \begin{pmatrix} 1 & 0\\ 0 & \frac{7}{16} \end{pmatrix} \qquad C_i^0 = \sqrt{\frac{\kappa(t)}{8}} r^i \begin{pmatrix} 0 & 1\\ \sqrt{8} & 0 \end{pmatrix}, \qquad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}} r^i \begin{pmatrix} 0 & 0\\ 0 & 1 \end{pmatrix}$$

• Transport coefficient: heavy quark mom. diffusion coefficient & quarkonium mass shift → lattice QCD

$$\kappa = \frac{g^2}{18} \operatorname{Re} \int_{-\infty}^{+\infty} ds \, \langle \operatorname{T} E^{a,i}(s, \mathbf{0}) \, \phi^{ab}(s, 0) \, E^{b,i}(0, \mathbf{0}) \rangle$$
$$\gamma = \frac{g^2}{18} \operatorname{Im} \int_{-\infty}^{+\infty} ds \, \langle \operatorname{T} E^{a,i}(s, \mathbf{0}) \, \phi^{ab}(s, 0) \, E^{b,i}(0, \mathbf{0}) \rangle$$

Reorganized into

$$\frac{d\rho_{\text{probe}}}{dt} = -iH_{\text{eff}}\rho_{\text{probe}} + i\rho_{\text{probe}}H_{\text{eff}}^{\dagger} + \sum_{n} C_{n} \rho_{\text{probe}} C_{n}^{\dagger}$$
with non-Hermitian effective Hamiltonian with total width $\Gamma \sim \text{ImV}$
 \Rightarrow no mixing between different color & L
$$H_{\text{eff}} = H_{\text{probe}} - \frac{i}{2}\Gamma$$
 $\Gamma = \kappa r^{i} \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_{c}^{2}-2}{2(N_{c}^{2}-1)} \end{pmatrix} r^{i}$
& 6 collapse operators \Rightarrow mixing between different L by
color-dipole singlet-octet transition
$$C_{i}^{0} = \sqrt{\frac{\kappa}{N_{c}^{2}-1}}r^{i} \begin{pmatrix} 0 & 0 \\ \sqrt{N_{c}^{2}-1} & 0 \end{pmatrix}$$
 $C_{i}^{1} = \sqrt{\frac{(N_{c}^{2}-4)\kappa}{2(N_{c}^{2}-1)}}r^{i} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$$I=0$$
isinglet octet

Open quantum system approach to Y states

- ♦ OQS + pNRQCD → Lindblad equation Brambilla et al. '17-21, Yao et al., '21, Blaizot '18 Akamatsu '21, Rothkopf '20, Gossiaux et al. '21 $\frac{d\rho(t)}{dt} = -i[H, \rho(t)] + \sum_{n} \left(C_n \rho(t) C_n^{\dagger} - \frac{1}{2} \{ C_n^{\dagger} C_n, \rho(t) \} \right) M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E.$
 - quantum transition between different states included, lacking in semi-classical
 - regeneration currently limited to diagonal $b\bar{b}$
 - Coulomb potential + transport coefficient κ encoded in C_n



Summary & outlook

- HFs: excellent probes of sQGP structure, transport properties, in-medium force & hadronization
 - a small open HF diffusion coefficient $\boldsymbol{\mathcal{D}}_{s}$
 - recombination/color neutralization important
 - quarkonia melting by large reaction rates
 - \rightarrow connection between open- & hidden-HF, e.g. via J/ ψ regeneration
- ✤ HFs: outlook into Run3 & ALICE3 see ALICE Collab.: Letter of intent for ALICE 3



strong V(r,T)

The following are back-up pages

X(3872) production in HIC

inner structure: compact tetraquark vs loosely bound molecule

