Jet quenching

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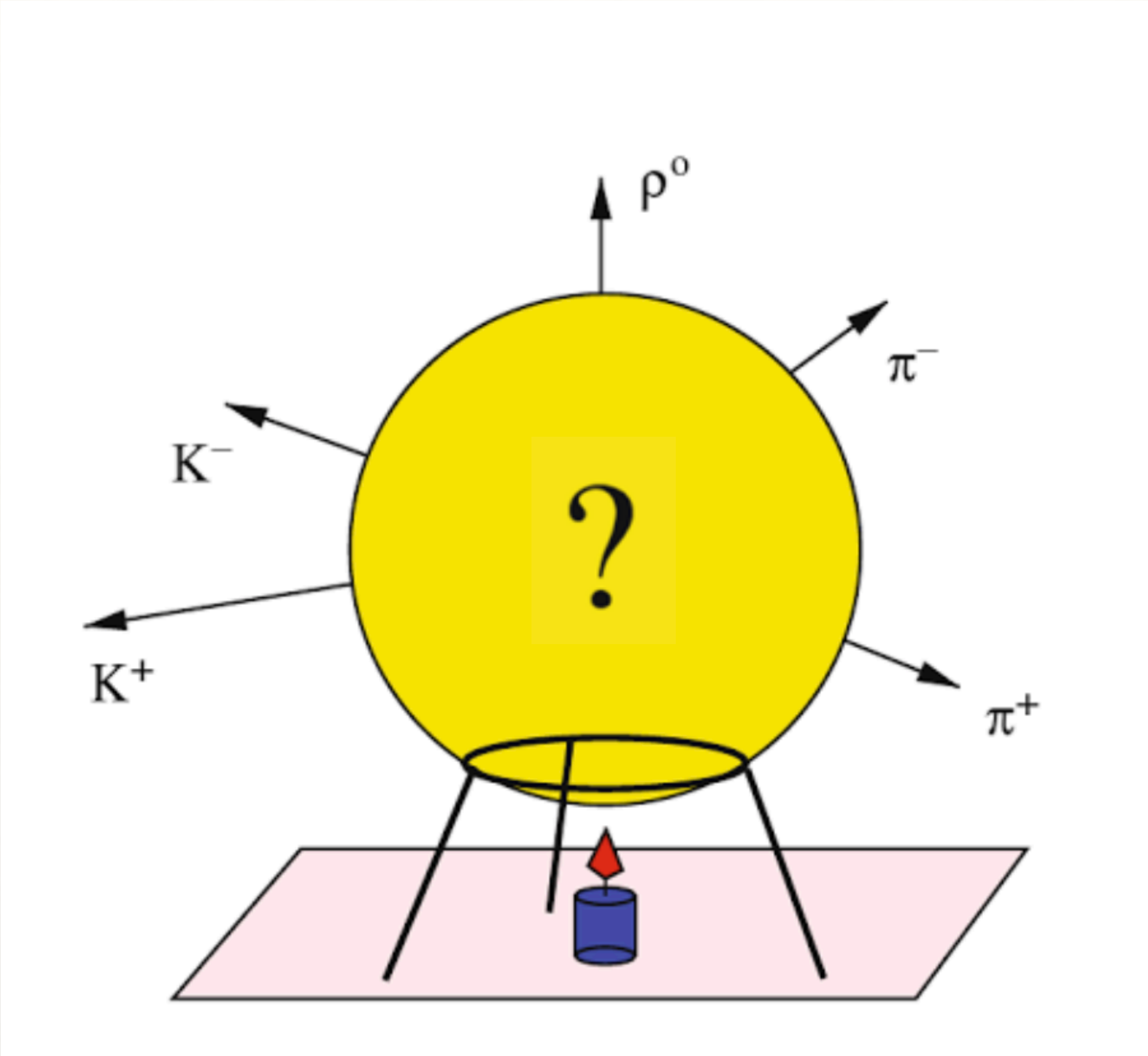
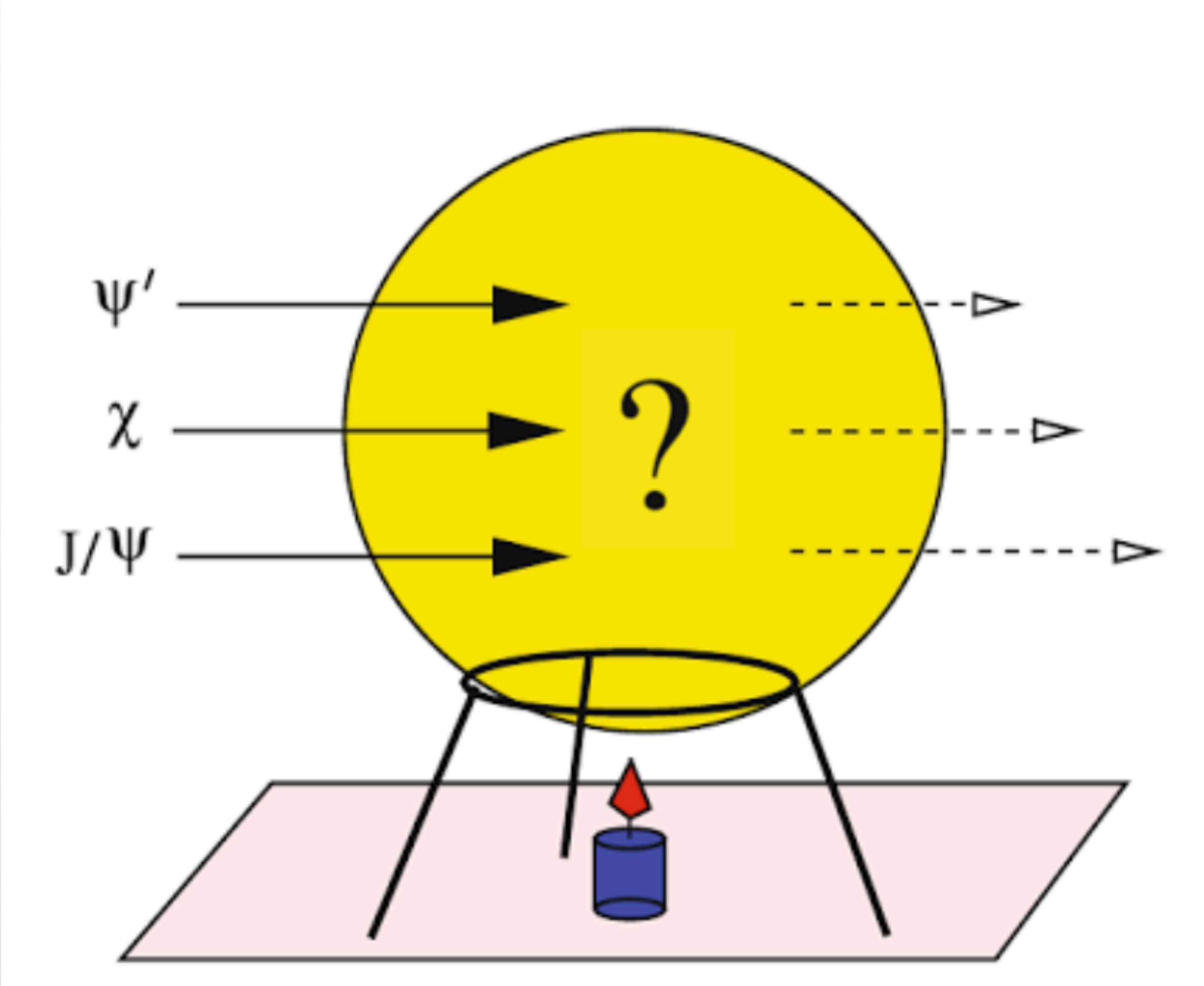
Reference: Rept. Prog. Phys. 84 (2021) 2, 024301

General picture: how to study hot matter with short lifetime



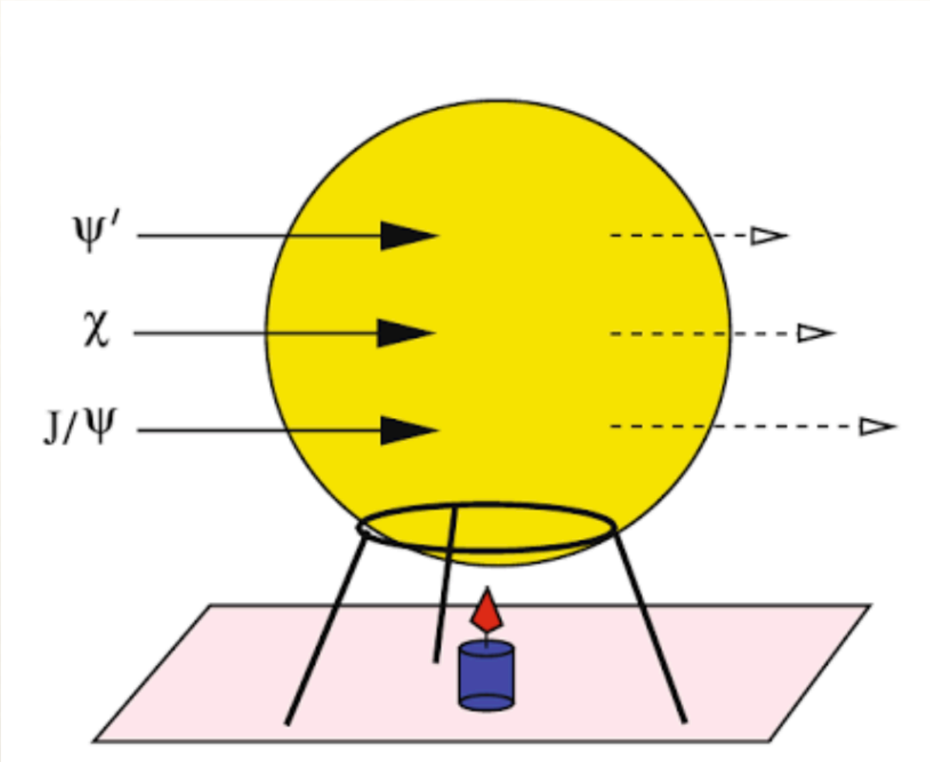


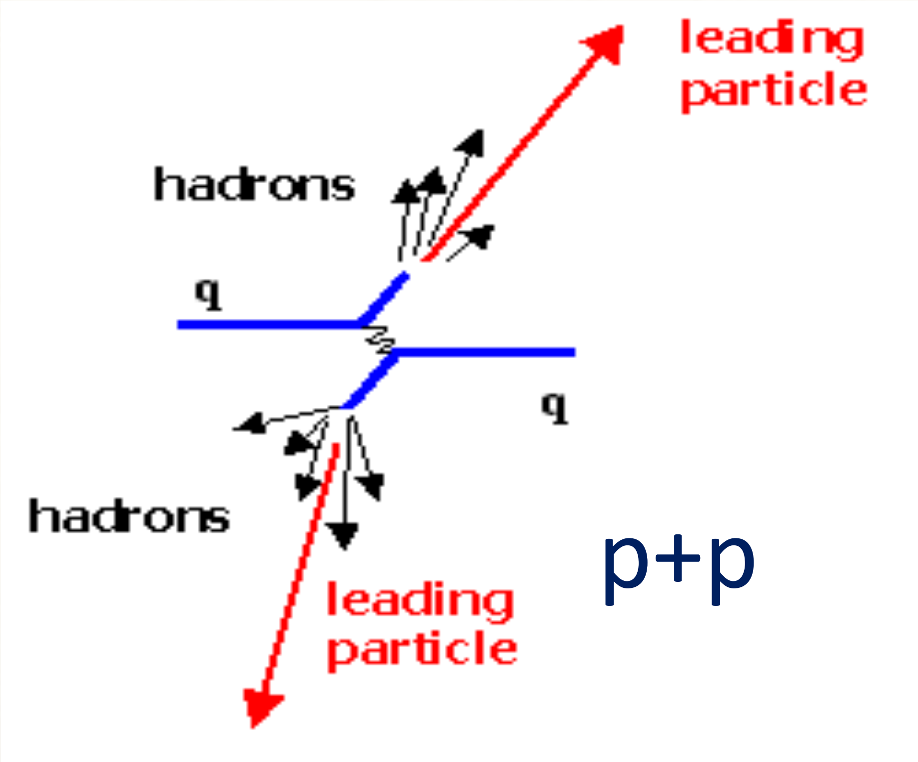
- Observe (thermal/**soft**) particles emitted from the medium

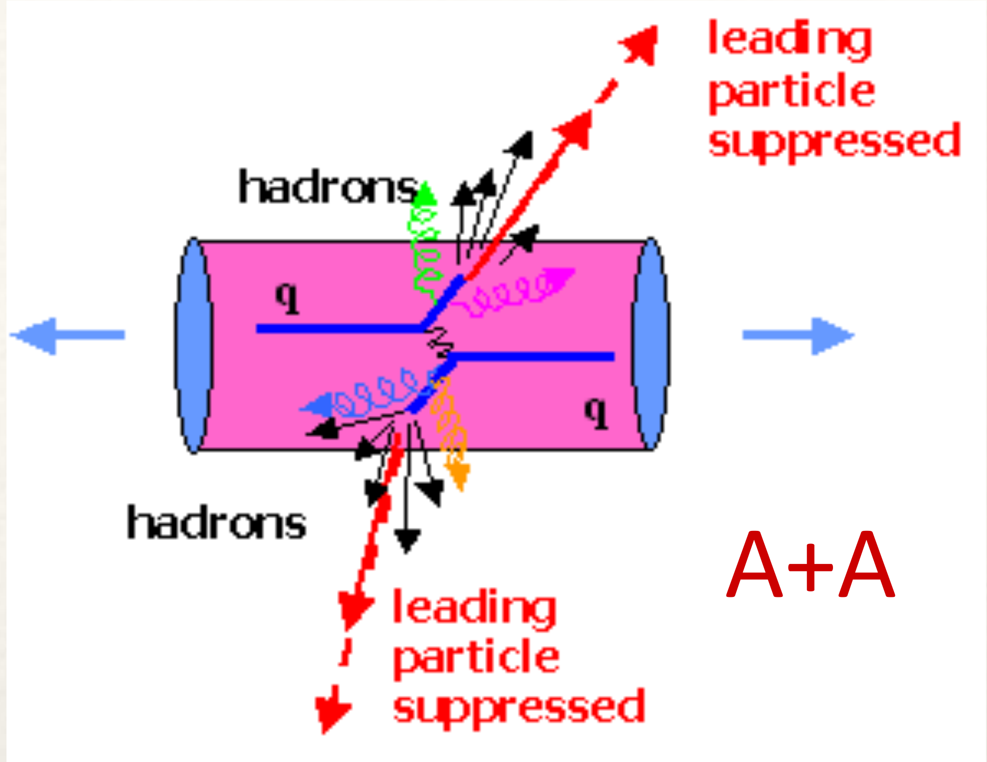
- Send energetic particles (**hard** probes) through the medium

Hard probes

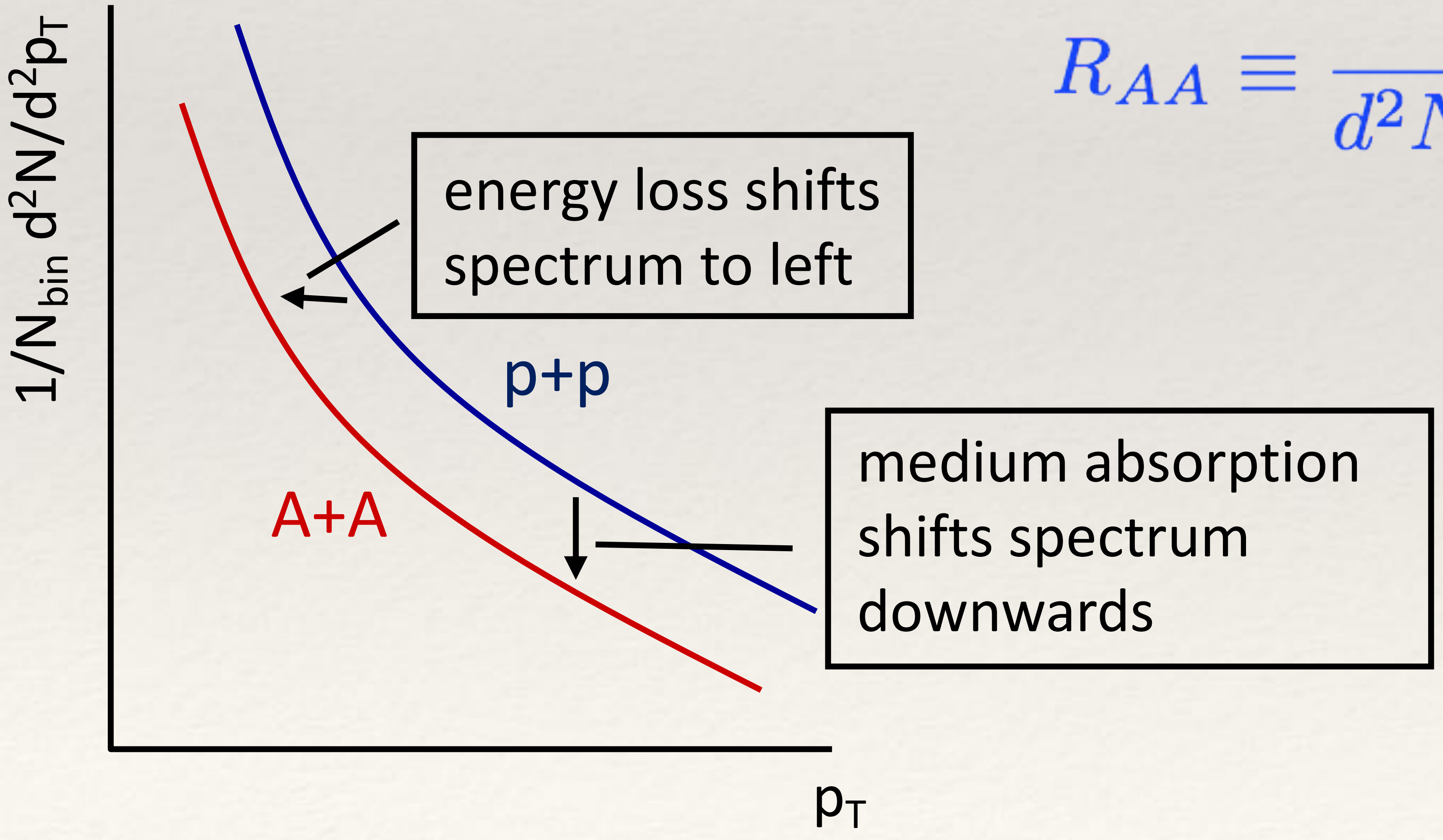
hard probes



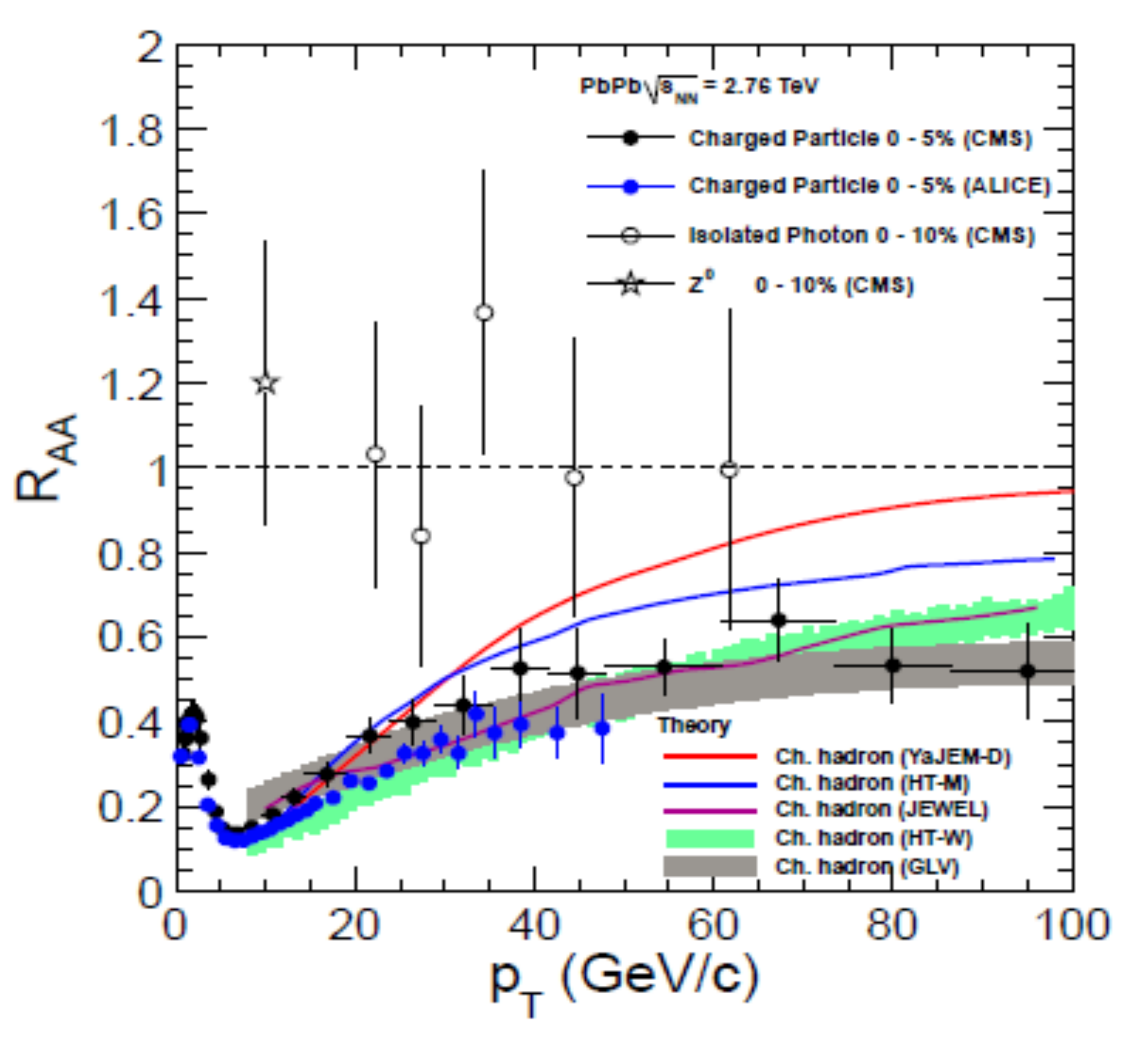




Nuclear modification factor

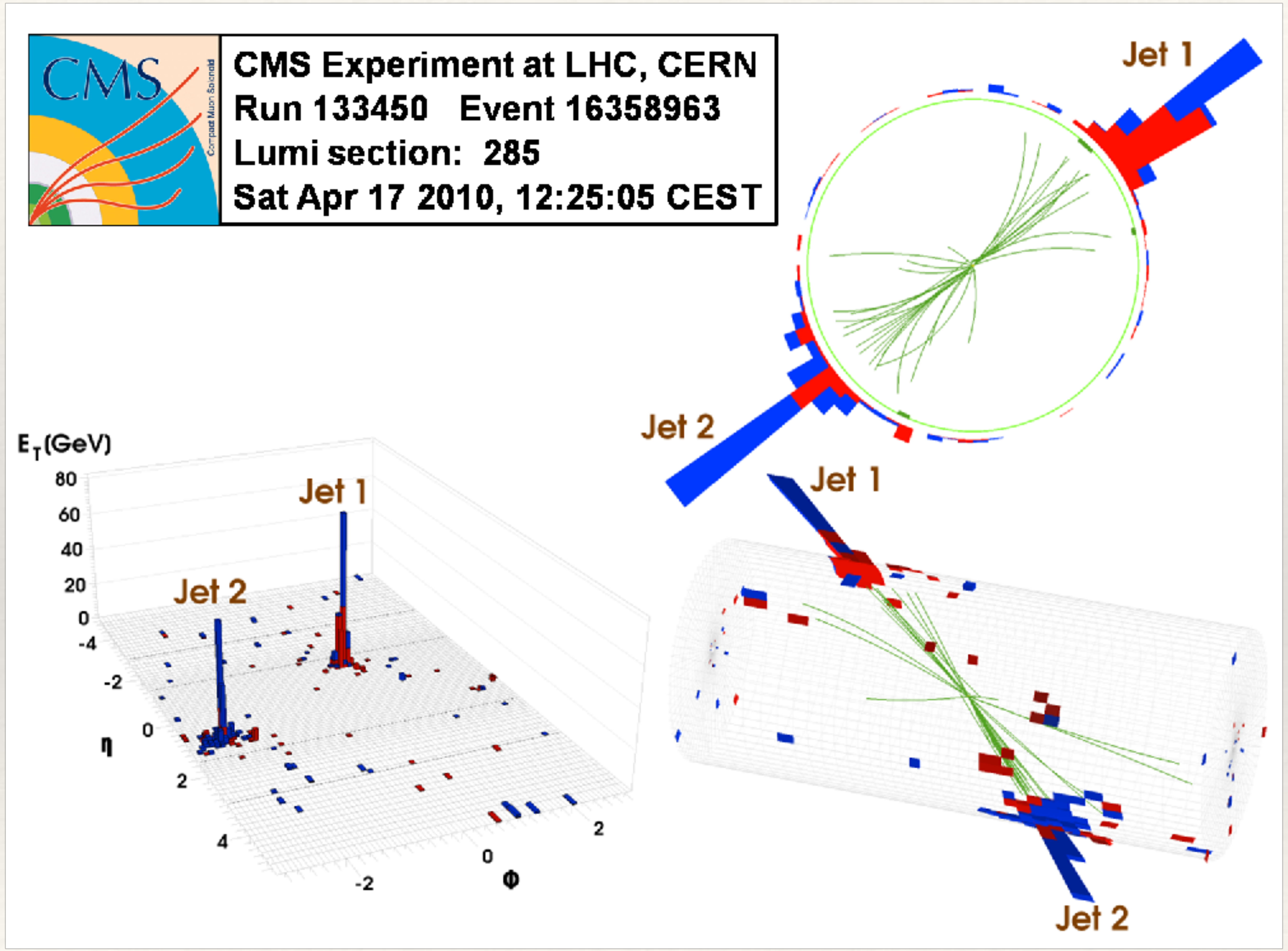


$$R_{AA} \equiv \frac{d^2N^{AA}/dydp_\perp}{d^2N^{pp}/dydp_\perp \times \langle N_{coll}^{AA} \rangle}$$



Mueller *et al.*, Ann. Rev. Nucl. Part. Sci. 62, 361 (2012)

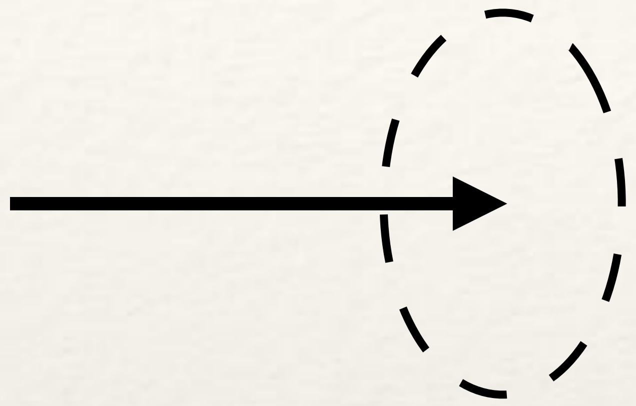
What is jet?



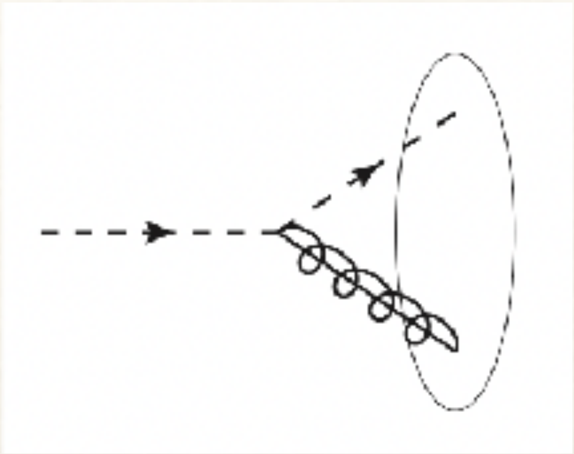
A narrow cone of hadrons and other particles produced by hadronization and decay of quarks and gluons.

What is jet depends on how you define a jet

Theorist: a single particle (LO) or a single particle + one emission within a cone (NLO)



(LO)

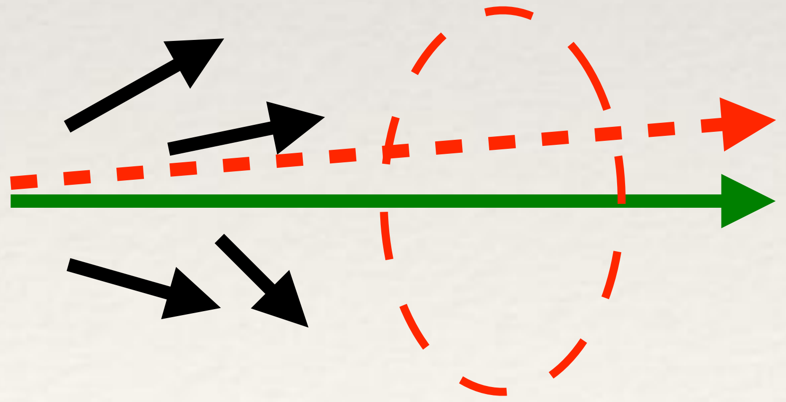


(NLO)

Experimentalists: different jet finding algorithms to construct jets

Cone algorithms (e.g. IC-PR, iterative cone algorithm with progressive removal)

2. Circle with radius R, summation over momenta inside gives jet trial axis

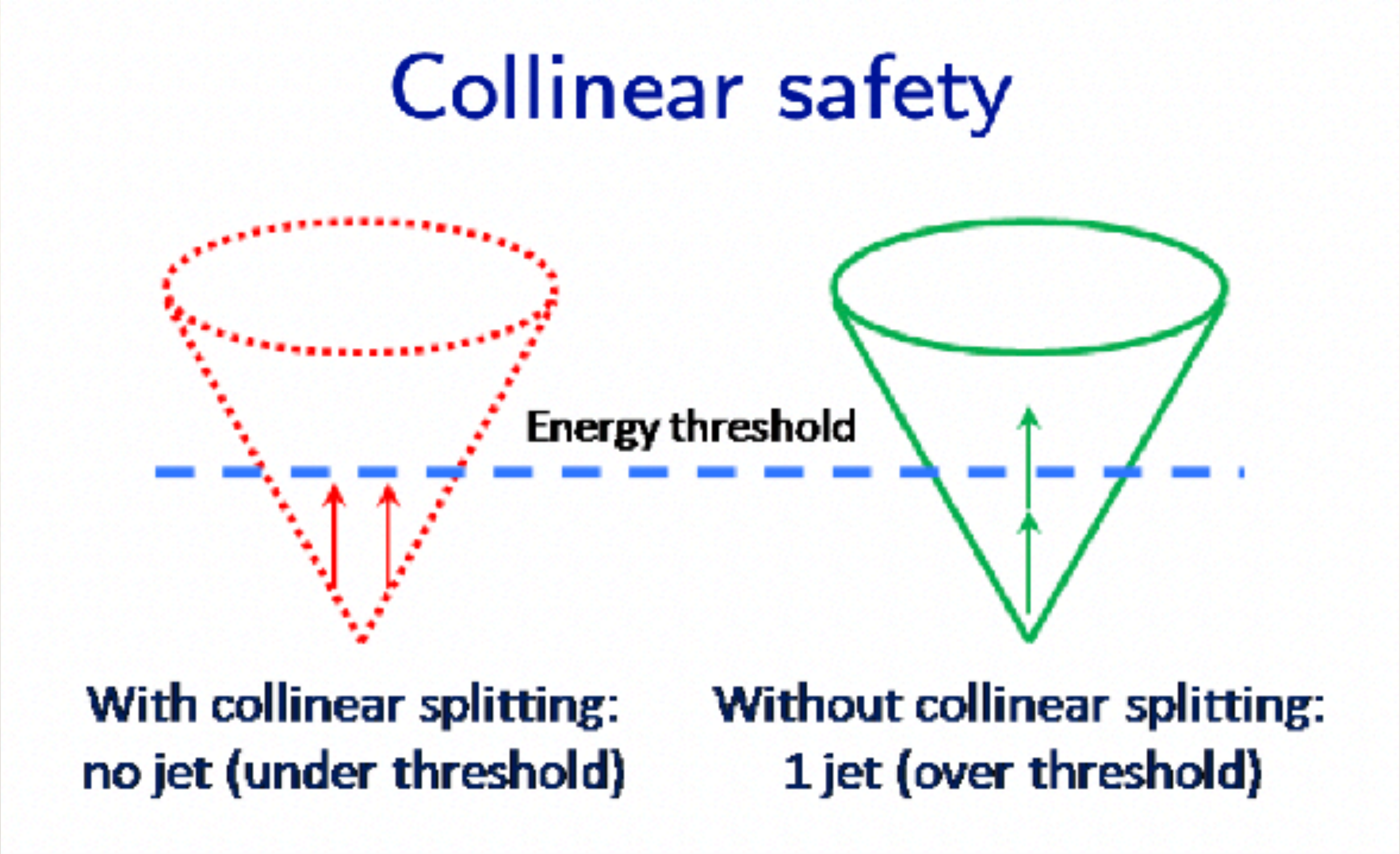


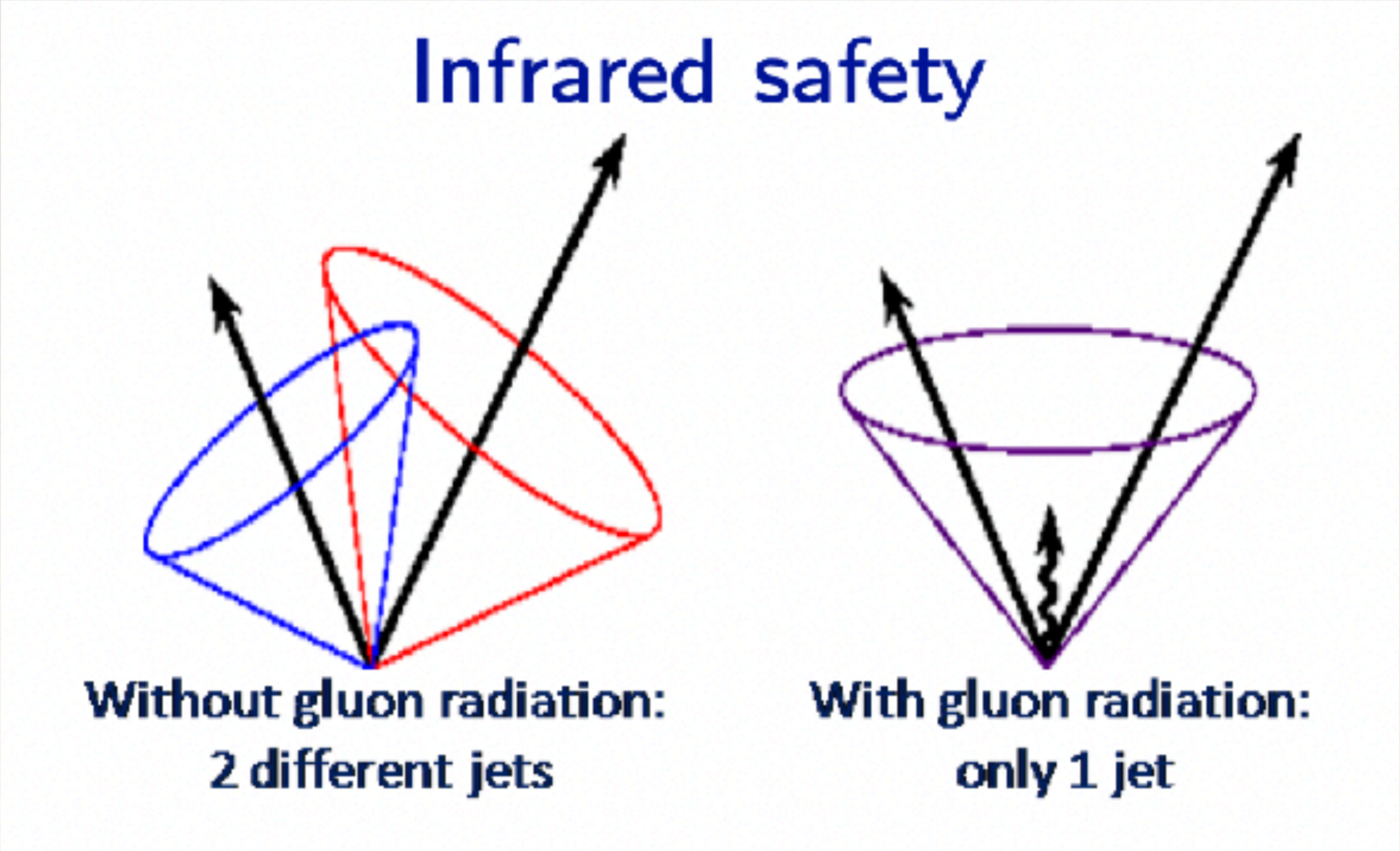
3. If seed axis = jet trial axis, a jet is found, removed from particle list, otherwise set current trial as seed and repeat 2 & 3

1. Highest p_T particle: seed axis

Jet finding algorithms

Requirements





Sequential clustering algorithm (e.g. k_t algorithm)

Define distance

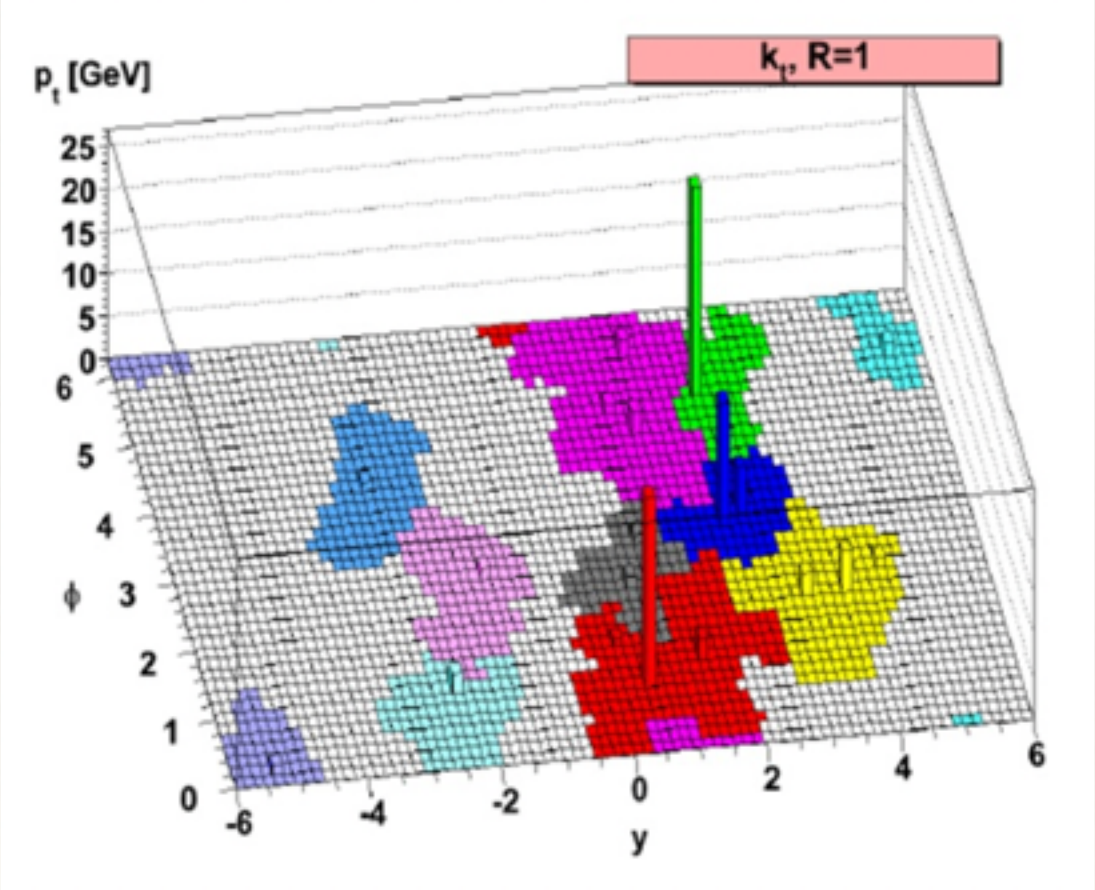
$$d_{ij} = min(p_{ti}^2, p_{tj}^2) \times \frac{R_{ij}^2}{R} \qquad d_{iB} = p_{ti}^2$$

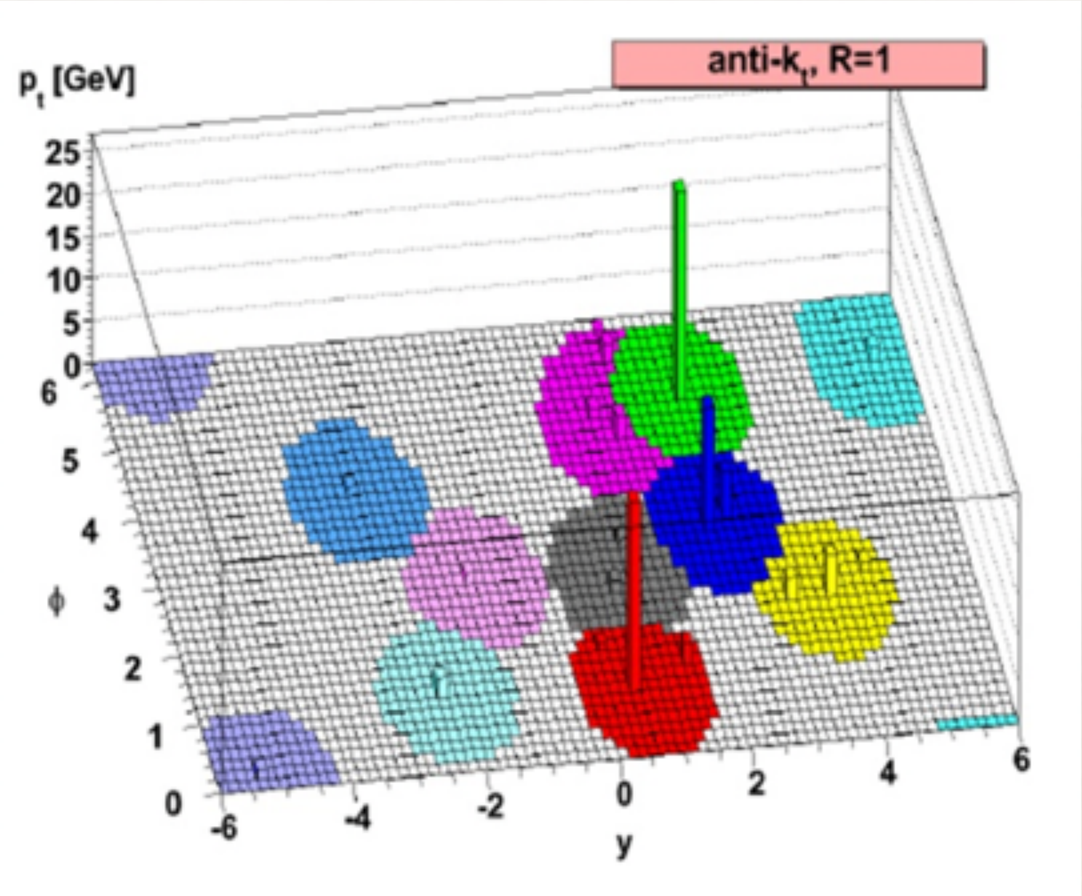
$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$

1. Start with *i* with the highest p_T
2. Calculate $\{d_{ij}, d_{iB}\}$, if d_{ij} is smallest with some *j*, combine *i* and *j* into one object and remove them from the particle list; repeat this step until d_{iB} is smallest, then *i* is the final jet
3. Repeat from 1 until no particle is left in the list

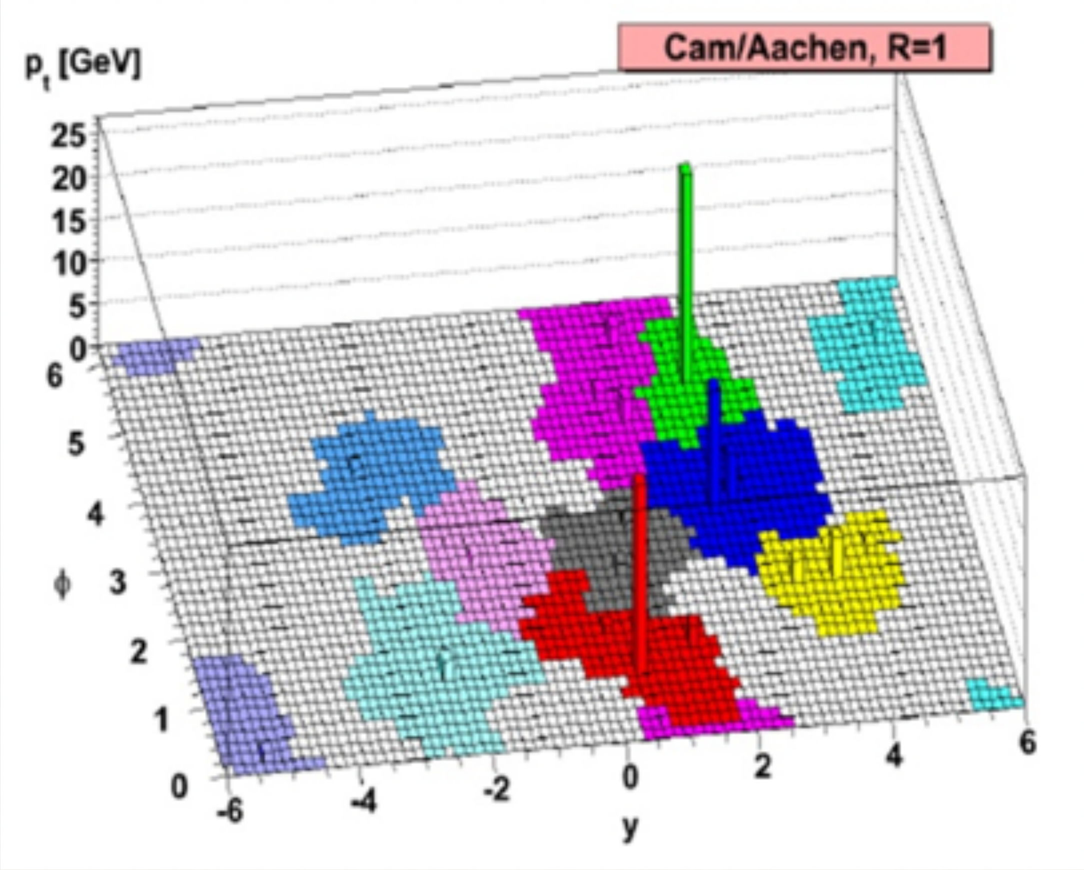
Jet finding algorithms

k_t	anti-k_t	Cambridge/Aachen (C/A)
$d_{ij} = min(p_{ti}^2, p_{tj}^2) \times \frac{R_{ij}^2}{R}$ $d_{iB} = p_{ti}^2$	$d_{ij} = min\left(\frac{1}{p_{ti}^2}, \frac{1}{p_{tj}^2}\right) \times \frac{R_{ij}^2}{R}$ $d_{iB} = \frac{1}{p_{ti}^2}$	$d_{ij} = \frac{R_{ij}^2}{R}$ $d_{iB} = 1$
(combine soft first)	(combine hard first)	



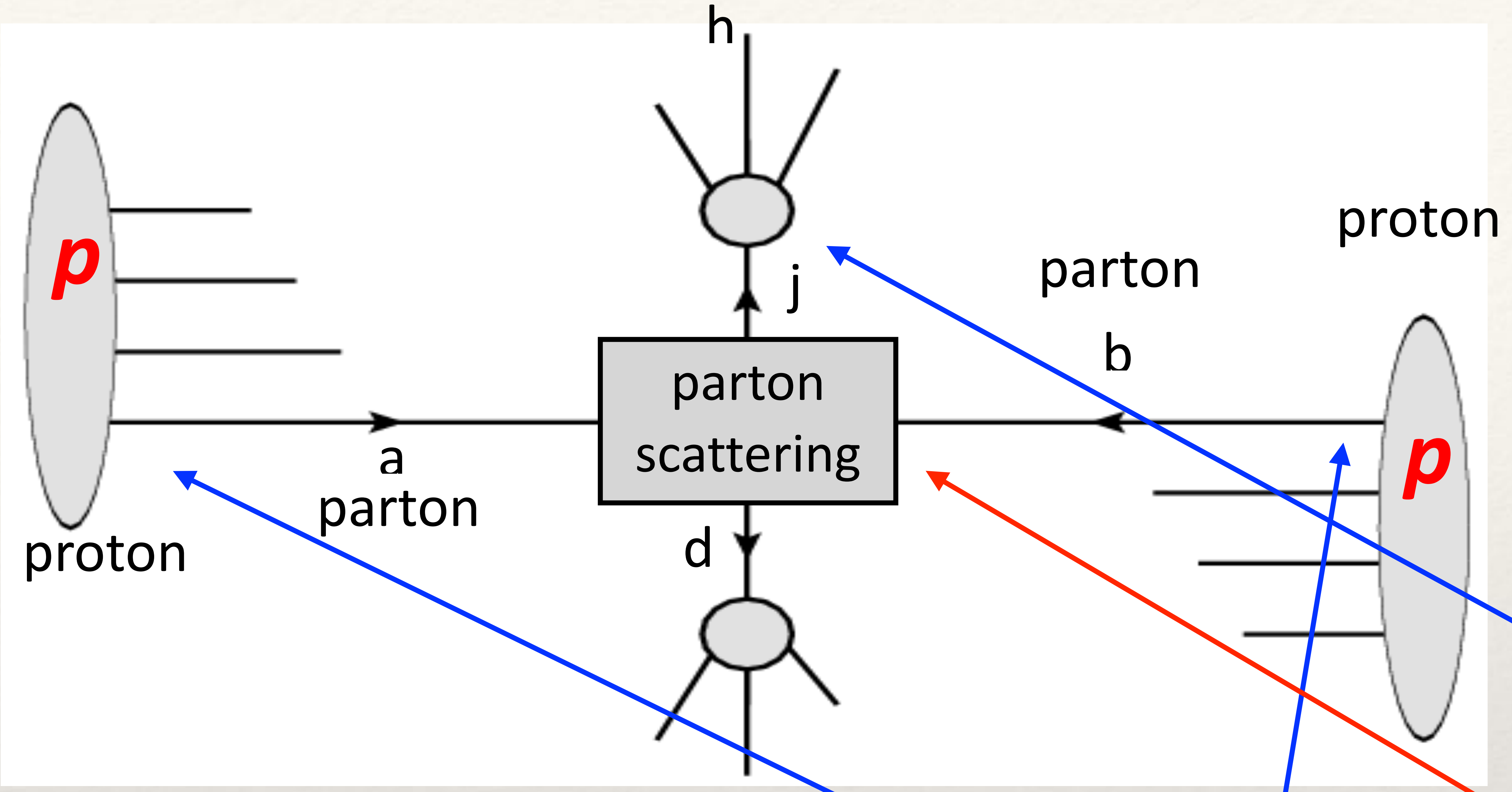


(best for resolving jets)



(best for de-clustering, studying jet substructure)

Theoretical framework for hadrons/jets in pp collisions



factorization into perturbative and non-perturbative parts

$$d\sigma_{pp\to hX} = \sum_{abjd} \int dx_a \int dx_b \int dz_j \, f_{a/p}(x_a, Q^2)\, f_{b/p}(x_b, Q^2)\, d\sigma_{ab\to jd}(Q^2)\, D_{h/j}(z_j, Q^2)$$

$$d\sigma_h = \sum_{abjd} f_{a/p} \otimes f_{b/p} \otimes d\sigma_{ab\to jd} \otimes D_{h/j}$$

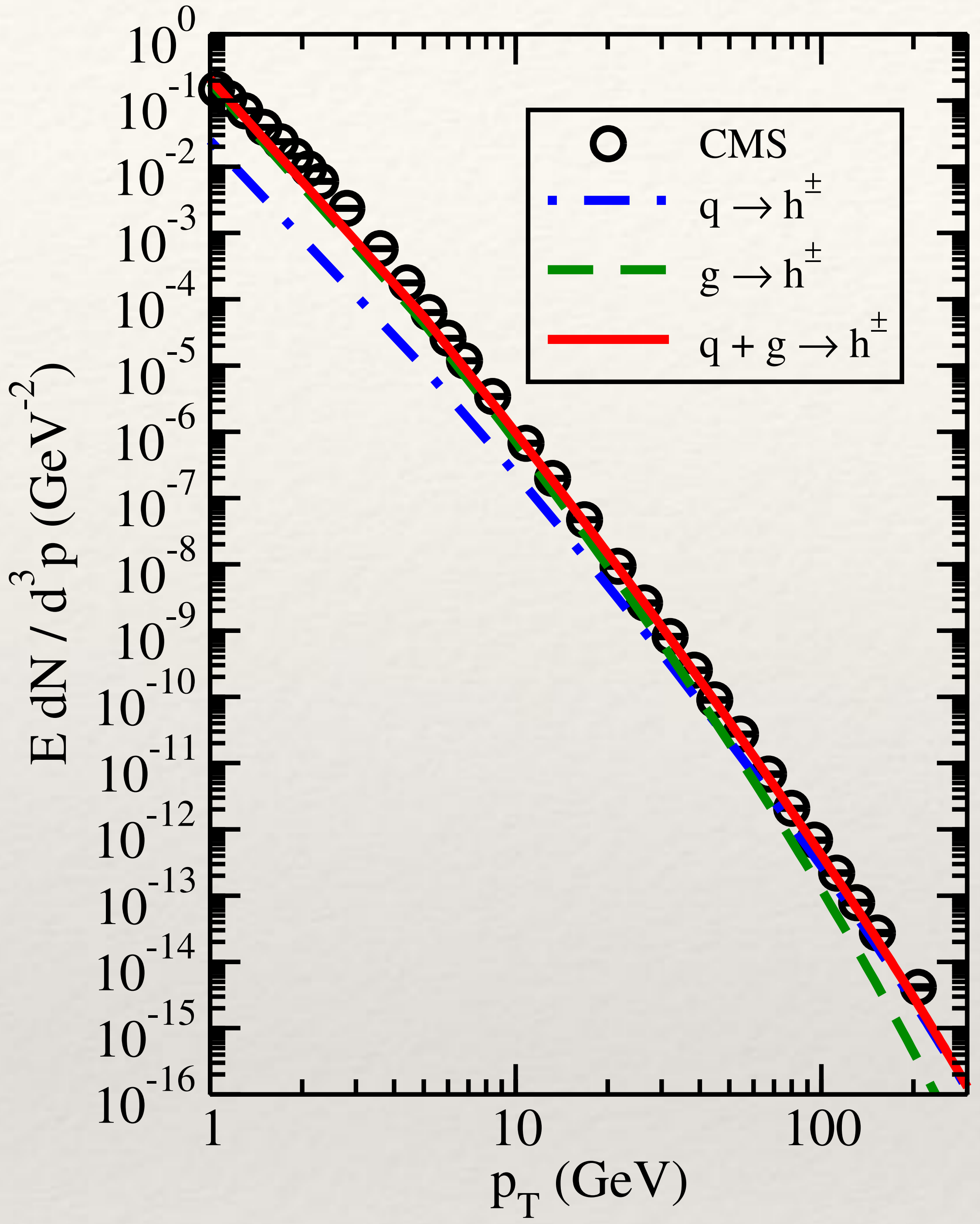
hadron production

$$d\sigma_{\text{jet}} = \sum_{abjd} f_{a/p} \otimes f_{b/p} \otimes d\sigma_{ab\to jd} \otimes J_j$$

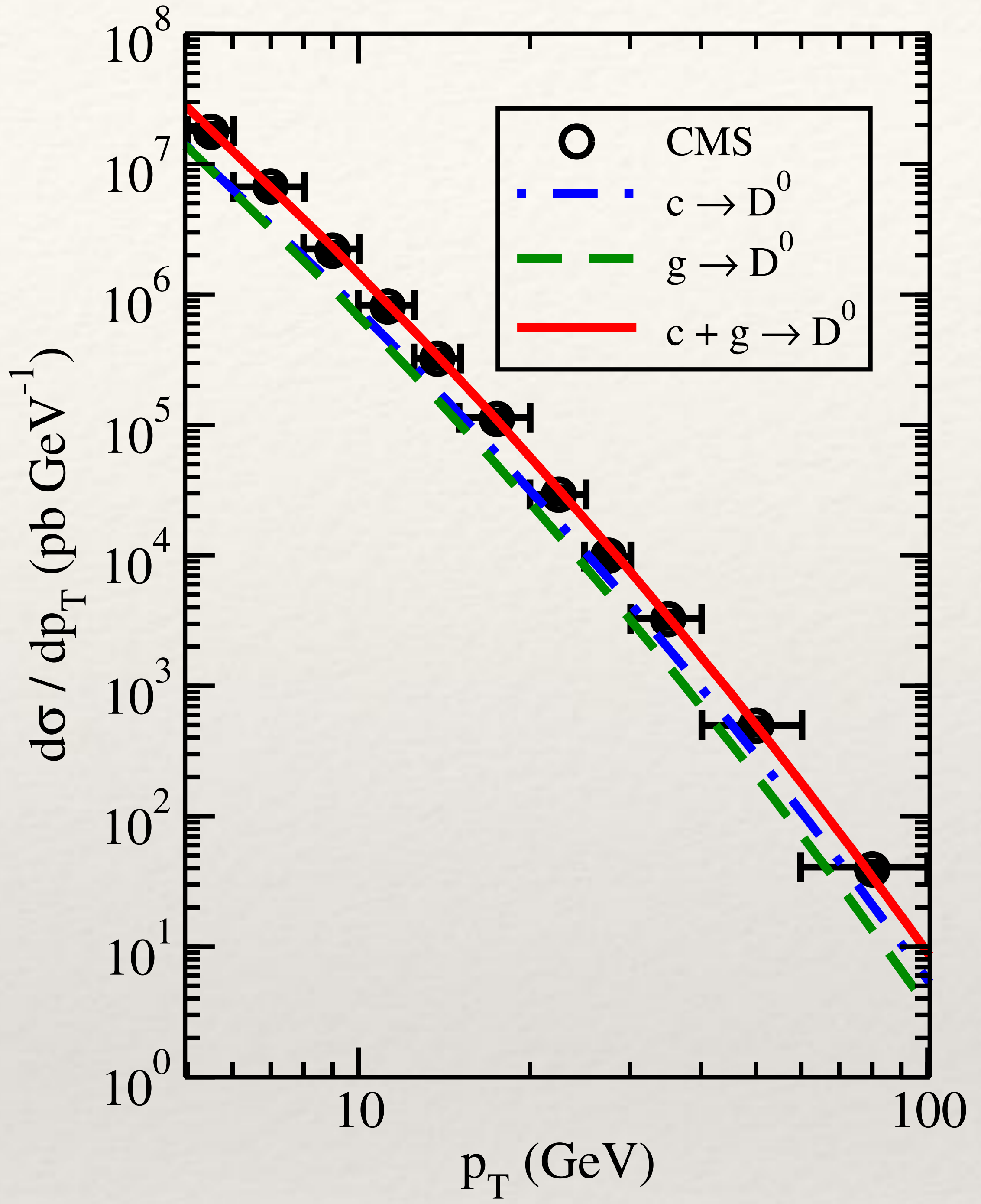
jet production

Hadron spectra in pp collisions

charged hadron



D meson



Within the NLO initial production + fragmentation framework, **gluon fragmentation**

- dominates $h^{\pm}$ production up to 50 GeV
- contributes to over 40% *D* up to 100 GeV

Formation of jet from a single parton

More theoretical studies, e.g. [Kang, Ringer, Vitev, JHEP 10 (2016) 125]

Here we concentrate on numerical modelings/simulations, e.g. physics in Pythia

- Start with a highly virtual parton ($Q^2 = p^2 - m^2$), consider $a \to bc$ splitting

DGLAP equation for FF: $$\frac{\partial}{\partial Q^2} D(z, Q^2) = \frac{\alpha_s}{2\pi} \frac{1}{Q^2} \int_z^1 \frac{dy}{y} P(y) D\left(\frac{z}{y}, Q^2\right)$$

Sudakov form factor: $$\Delta_a(Q^2_{\max}, Q^2_a) = \prod_i \Delta_{ai}(Q^2_{\max}, Q^2_a) = \prod_i \exp\left[-\int_{Q^2_a}^{Q^2_{\max}} \frac{dQ^2}{Q^2} \frac{\alpha_s(Q^2)}{2\pi} \int_{z_{\min}}^{z_{\max}} dz P_{ai}(z, Q^2)\right]$$

Probability of no splitting between $Q_{\max}$ and Q_a

i: splitting channel

P: splitting function

z: fractional energy or momentum

Monte Carlo implementation

- Random number $r \in (0,1)$
- If $r \leq \Delta(Q^2_{\max}, Q^2_{\min})$, particle is stable, no splitting ($Q_{\min}$: minimum allowed virtuality)
- Otherwise, splitting happens
- If $r \leq \Delta(Q^2_{\max}, Q^2_a) = \dfrac{\Delta(Q^2_{\max}, Q^2_{\min})}{\Delta(Q^2_a, Q^2_{\min})}$, no splitting above Q_a, or splitting happens at or below Q_a
- Solve $r = \Delta(Q^2_{\max}, Q^2_a)$ to obtain Q_a, virtuality at which a splits
- Determine the splitting channel, use branching ratio from $\mathrm{BR}_{ai}(Q^2_a) = \displaystyle\int_{z_{\min}}^{z_{\max}} dz P_{ai}(z, Q^2_a)$
- For a given channel, sample z using $P_{ai}(z, Q^2_a)$
- zQ_a and $(1-z)Q_a$ are new $Q_{\max}$'s for determining Q_b and Q_c in $a \to bc$ splitting

Kinematics

- $\left[p^+, \underline{\dfrac{Q_a^2}{2p^+}}, 0\right] \rightarrow \left[zp^+, \underline{\dfrac{Q_b^2 + k_\perp^2}{2zp^+}}, \vec{k}_\perp\right] + \left[(1-z)p^+, \underline{\dfrac{Q_c^2 + k_\perp^2}{2(1-z)p^+}}, -\vec{k}_\perp\right]$
- Conservation of the minus-component gives: $k_\perp^2 = z(1-z)Q_a^2 - (1-z)Q_b^2 - zQ_c^2$

Light cone coordinates $(A^+, A^-, \vec{A}_\perp)$ from cartesian coordinates: (A^0, A^1, A^2, A^3)

$$A^+ = (A^0 + A^3)/\sqrt{2}, \quad A^- = (A^0 - A^3)/\sqrt{2}, \quad \vec{A}_\perp = (A^1, A^2)$$

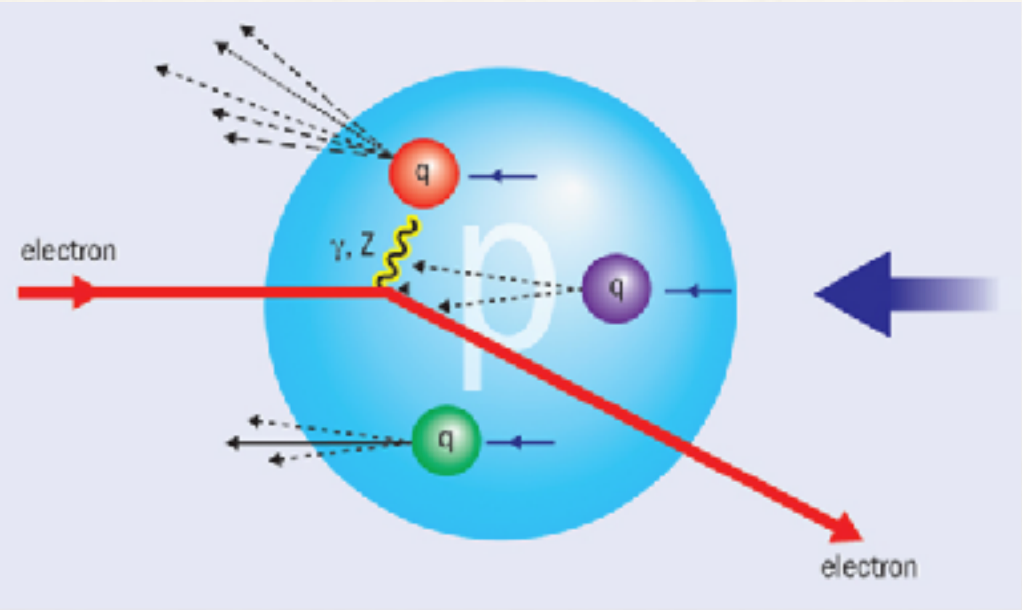
$$A \cdot B = A^+B^- + A^-B^+ - \vec{A}_\perp \cdot \vec{B}_\perp, \qquad A^2 = 2A^+A^- - A_\perp^2$$

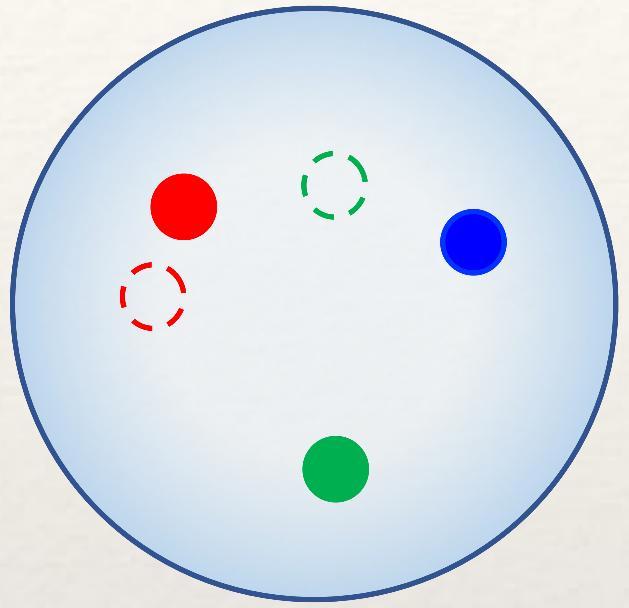
On shell condition: $p^2 = m^2 \;\rightarrow\; p^- = \dfrac{m^2 + p_\perp^2}{2p^+}$

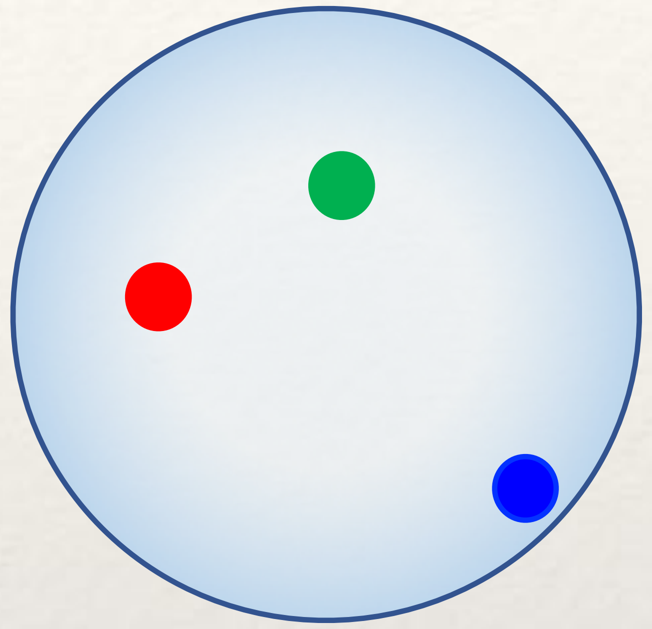
$p = \left(p^+, \dfrac{m^2 + p_\perp^2}{2p^+}, \vec{p}_\perp\right)$ convenient for a large p^+ component

Light cone coordinates

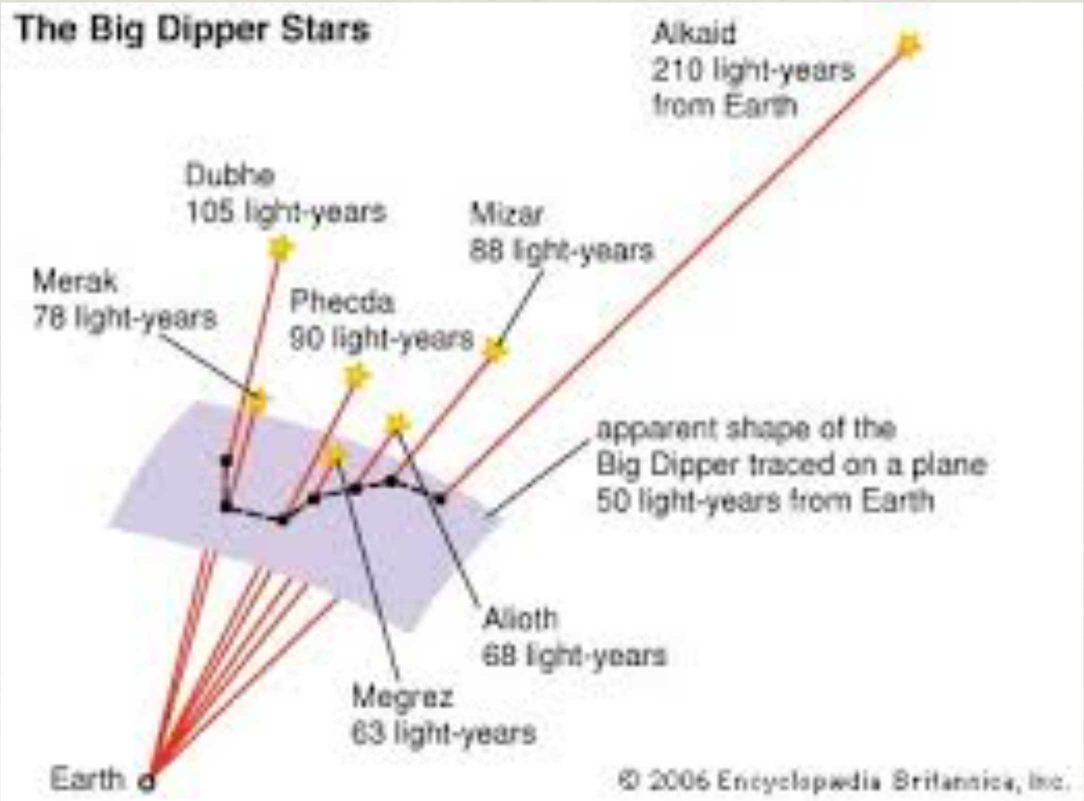
We observe the world with light





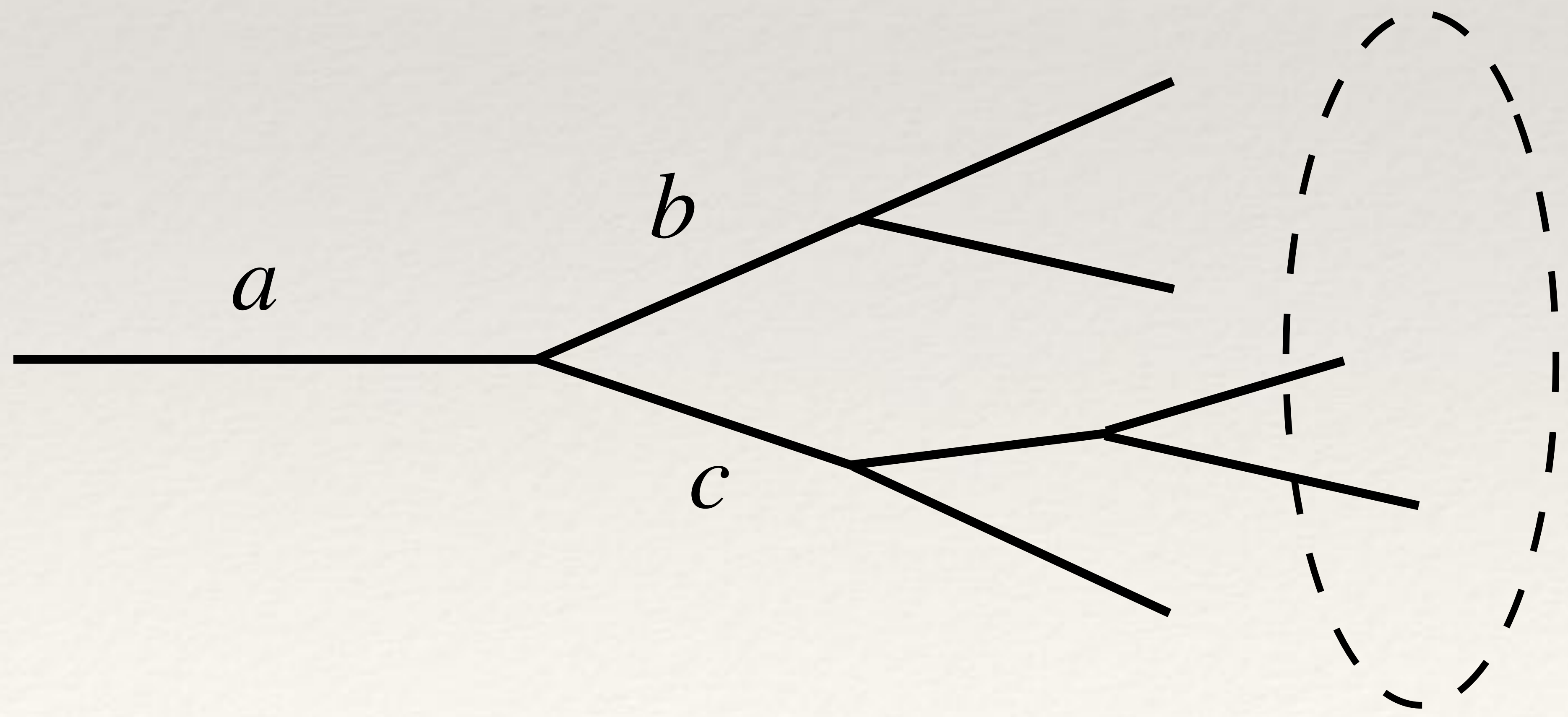


What is the time on the photo? ($t = x^0$ or $t = x^0 + x^3$)

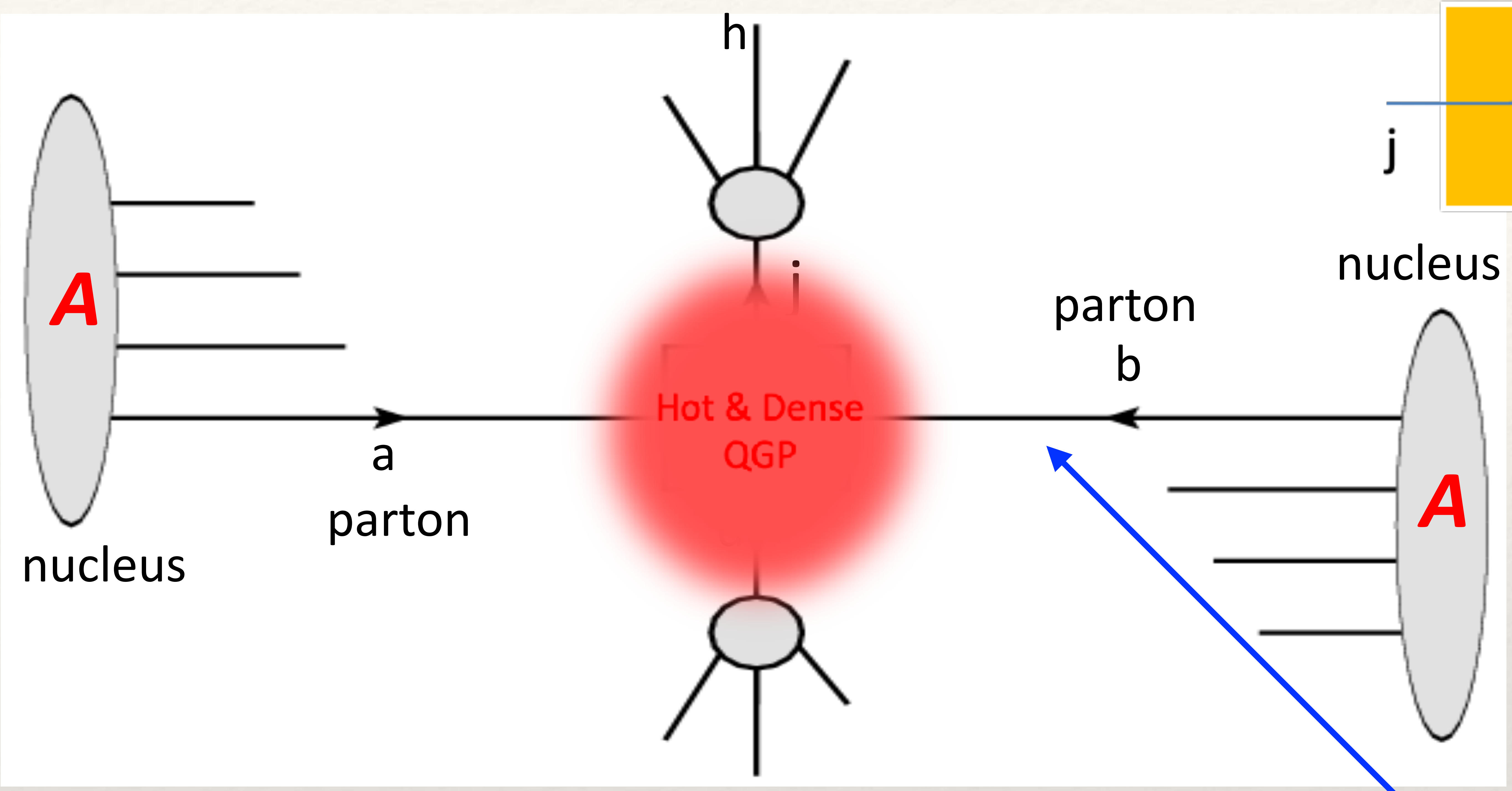


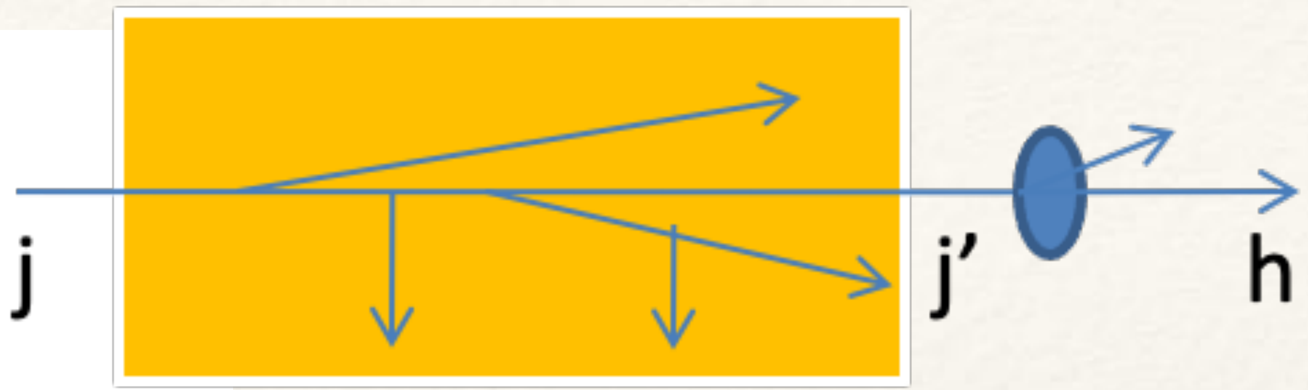
Parton shower

- We know how to determine Q and p^μ for $a \to bc$
- Starting from a $Q_{\max}$ (e.g. Pythia choice $Q^2_{\max} = 4Q^2_{\text{hard}}$), generate one splitting
- Using new Q_b and Q_c, generate secondary splittings
- Iterate until all partons reach $Q_{\min}$ (usually choose 1 GeV in vacuum)
- Hadronization
- Use jet finding algorithm to construct jets



Theoretical framework for hadrons/jets in AA collisions





Could be perturbative or non-perturbative

$$d\tilde{\sigma}_h = \sum_{abjX} \underline{f_{a/A}} \otimes \underline{f_{b/B}} \otimes d\sigma_{ab\to jX} \otimes \underline{\tilde{D}_{h/j}}$$

Cold nuclear matter effects

Medium-modified fragmentation function:

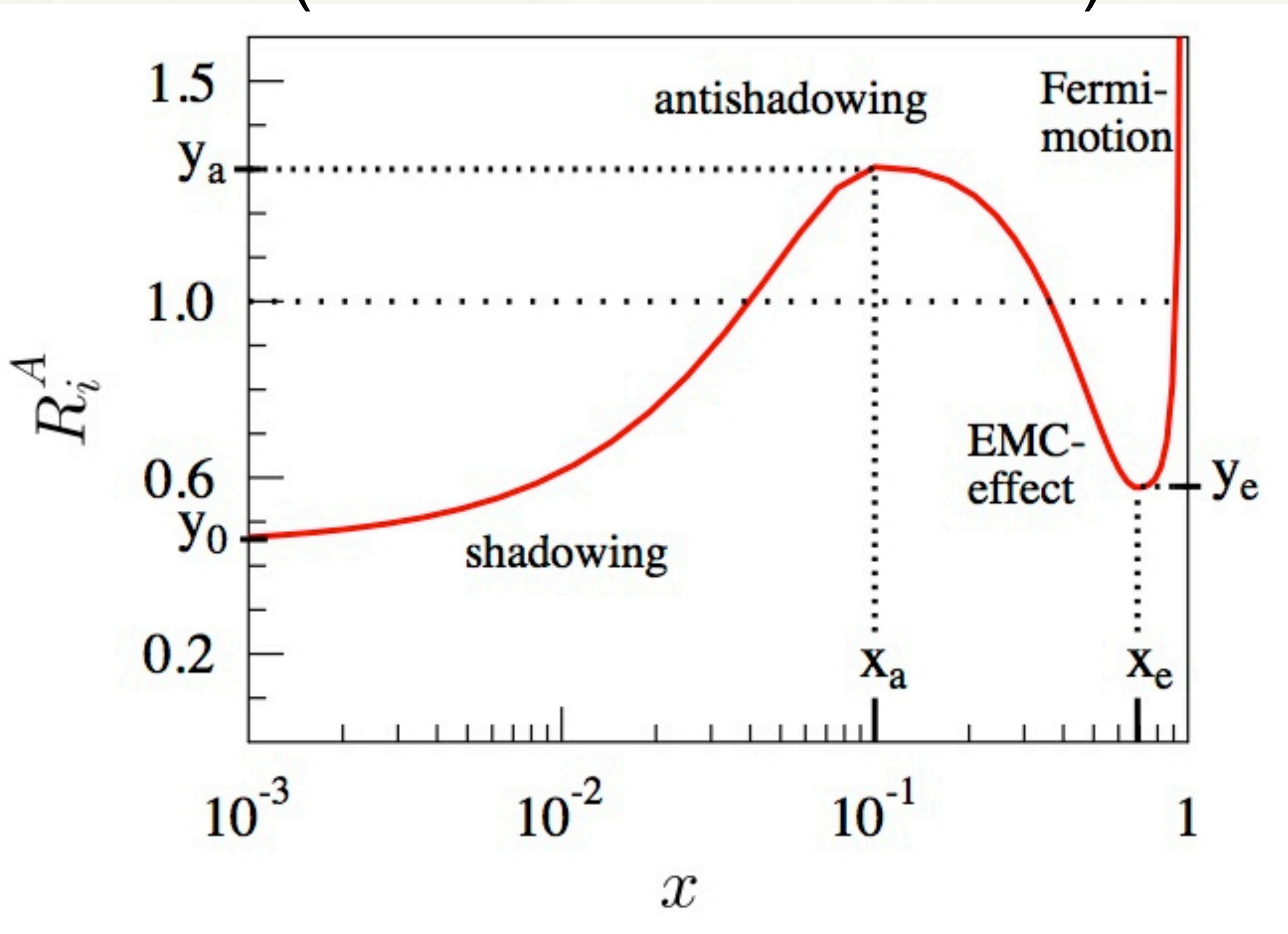
$$\tilde{D}_{h/j} = \sum_{j'} P_{j\to j'} \otimes D_{h/j'}$$

Cold nuclear matter effects

Nuclear shadowing effect

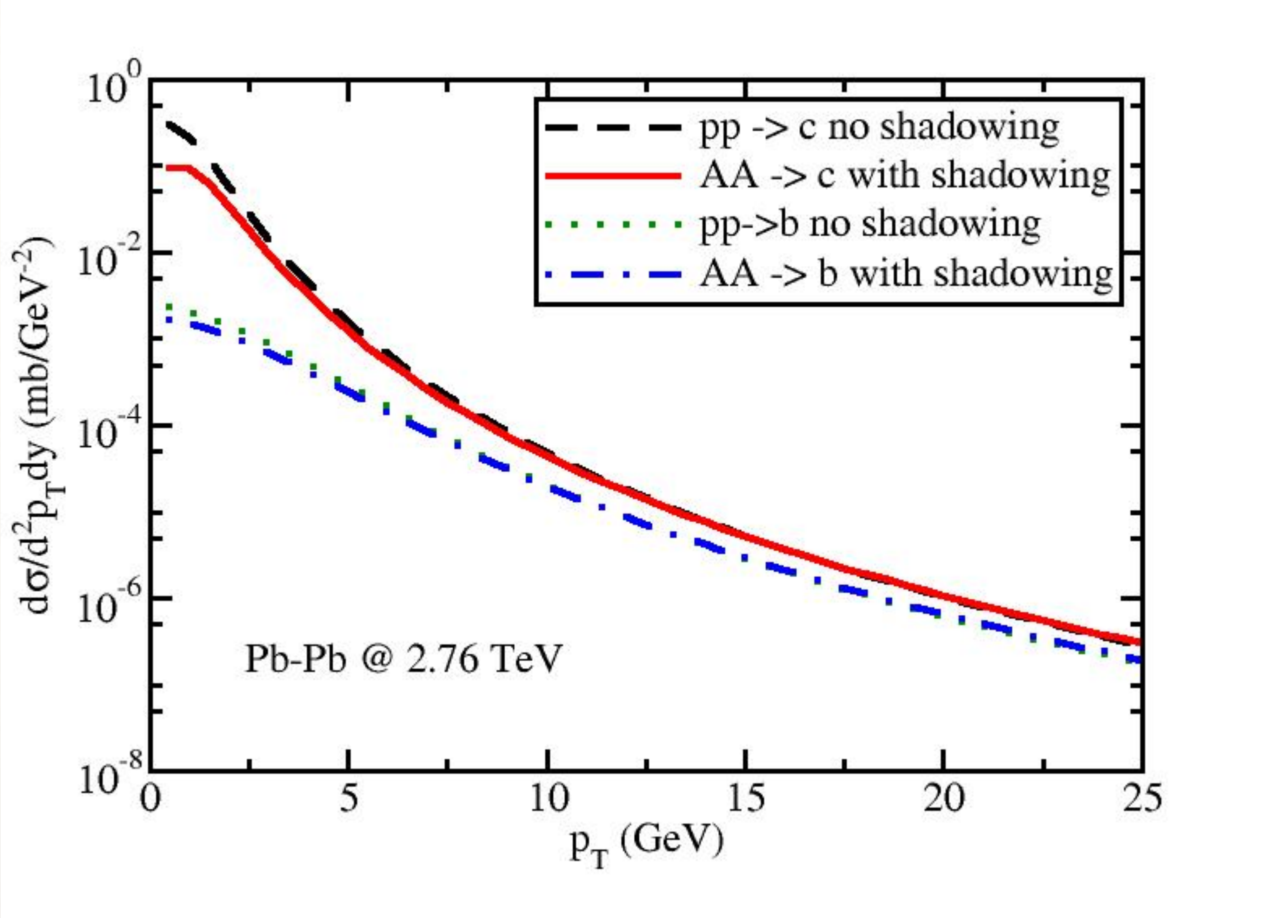
- nucleon in nucleus is “shadowed” by other nucleons at small x
- Momentum conservation requires anti-shadowing at larger x

(Taken from Eskola 2009)



$$R_i^A(x) = f_{i/A}(x)/f_{i/p}(x)$$

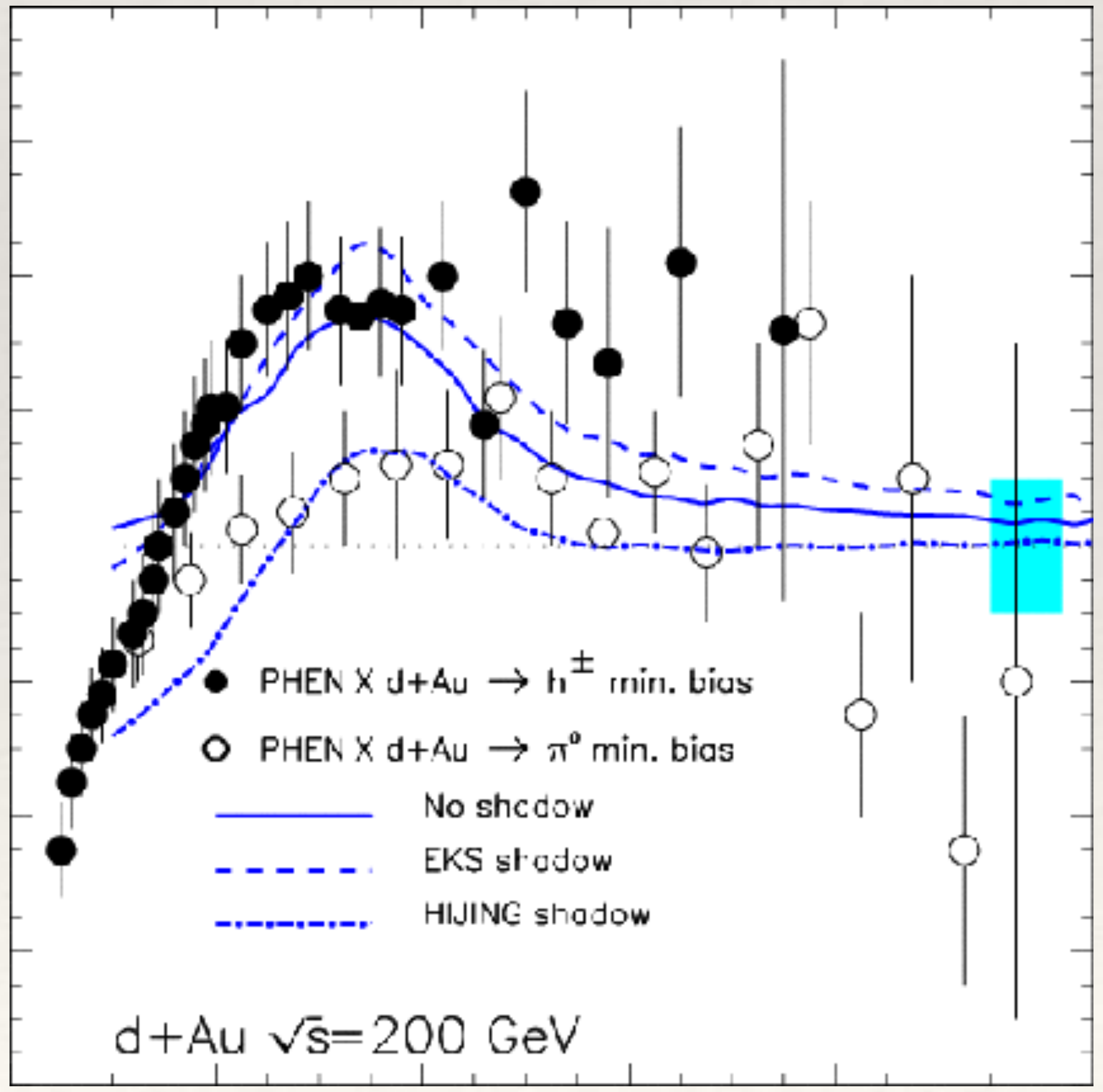
Initial spectra of c/b quarks



Cronin effect: momentum broadening of partons before hard scatterings

Constraints from pA or dA collisions

Prediction from HIJING



Hot nuclear matter (QGP) effects

Different theories/models at different virtuality/energy scales

- **Medium-modified parton shower at high virtuality**

Recall: Sudakov
$$\Delta_a(Q^2_{\max}, Q^2_a) = \prod_i \Delta_{ai}(Q^2_{\max}, Q^2_a) = \prod_i \exp\left[-\int_{Q^2_a}^{Q^2_{\max}} \frac{dQ^2}{Q^2}\frac{\alpha_s(Q^2)}{2\pi}\int_{z_{\min}}^{z_{\max}} dz P_{ai}(z, Q^2)\right]$$

Splitting function: $P_{ai}(z, Q^2) = P^{\mathrm{vac}}_{ai}(z) + P^{\mathrm{med}}_{ai}(z, Q^2)$ (vacuum part + medium-induced part)

Medium-induced splitting function:

(e.g. from higher-twist energy loss calculation)

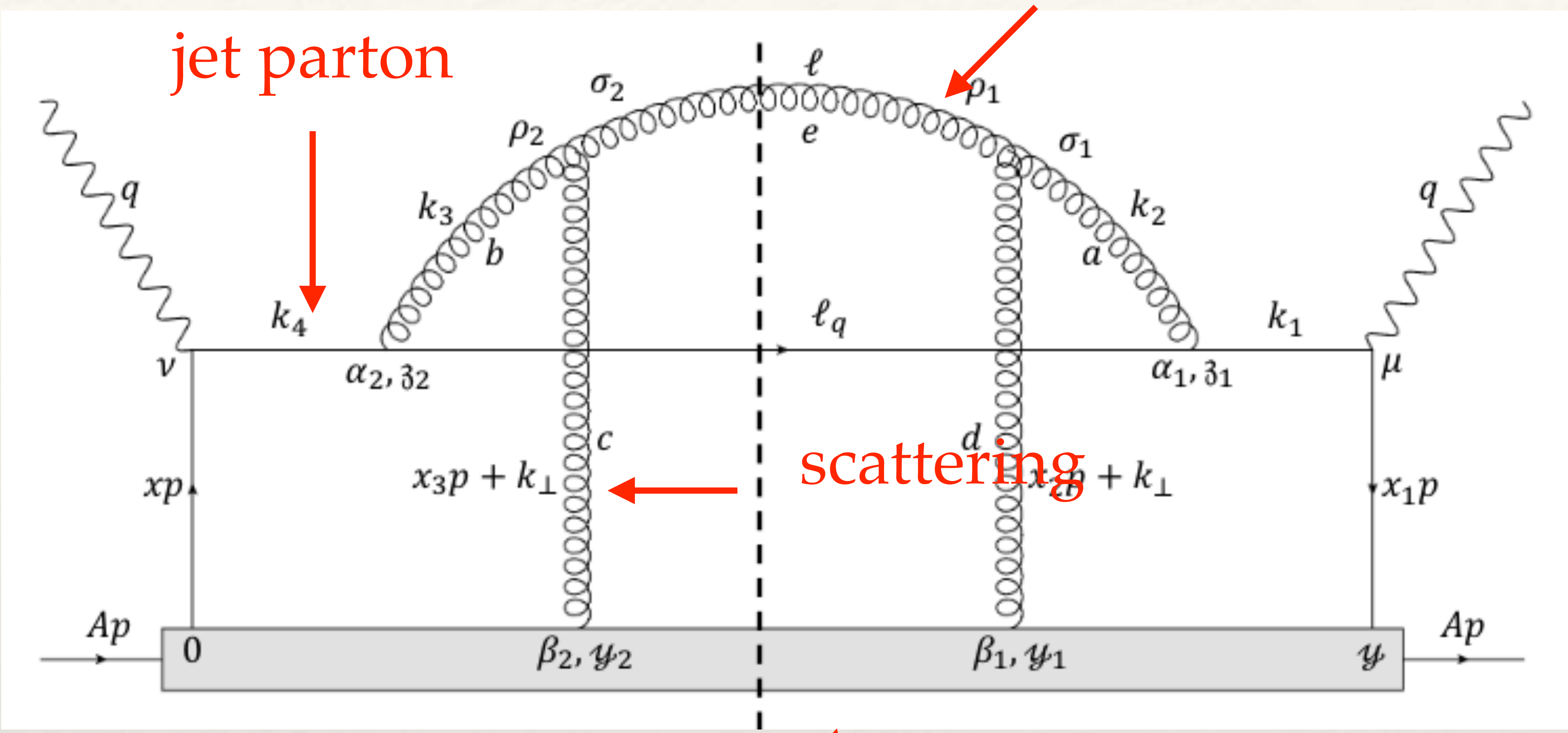
* needs hydrodynamic input

$$P^{\mathrm{med}}_{ai}(z, Q^2) = \frac{C_A}{C_2(a)}\frac{P^{\mathrm{vac}}_{ai}(z)}{z(1-z)Q^2}$$ interference effect

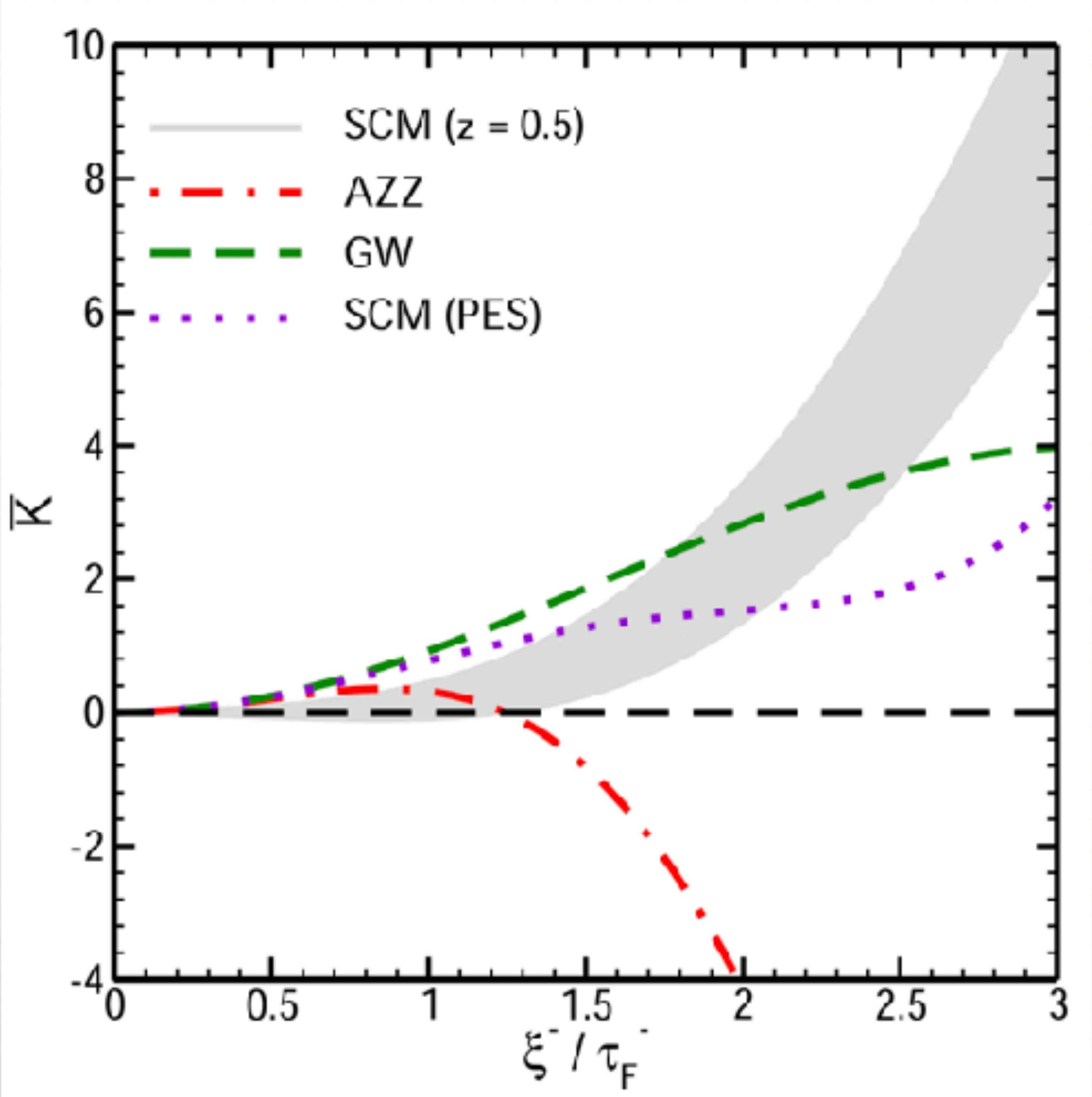
$$\times \int_0^{\tau_f^+} d\zeta^+ \hat{q}_a\left(\vec{r} + \hat{n}\frac{\zeta^+}{\sqrt{2}}\right)\left[2 - 2\cos\left(\frac{\zeta^+}{\tau_f^+}\right)\right]$$

medium inforation location dependence

Medium-induced splitting function



+ 18 other diagrams



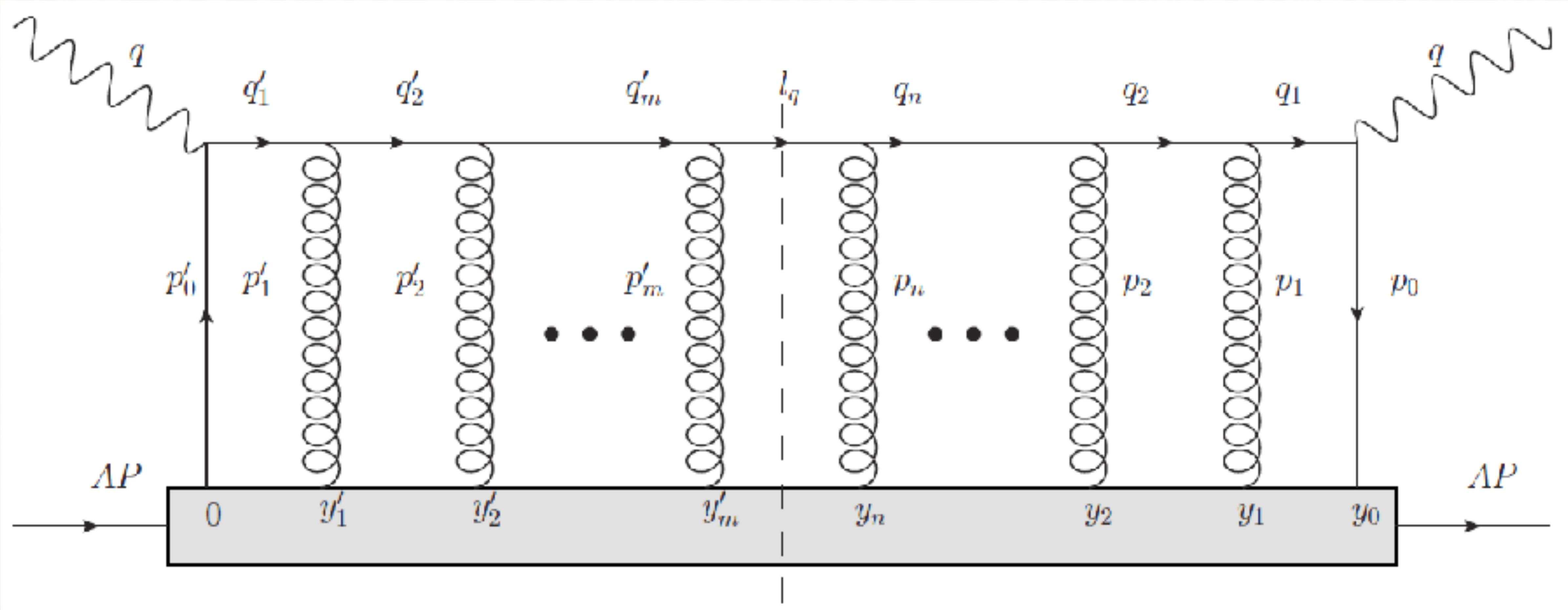
[Phys. Rev. C 105 (2022) 2, 024908]

- GW: ignore phase factors in collinear expansion
- AZZ: consider phase factors, but calculate only one diagram
- SCM: calculate all diagrams with phase factors
- More and more precise description of the length dependence of jet energy loss
- Monte Carlo implementation — MATTER [Phys. Rev. C 101 (2020) 2, 024903]

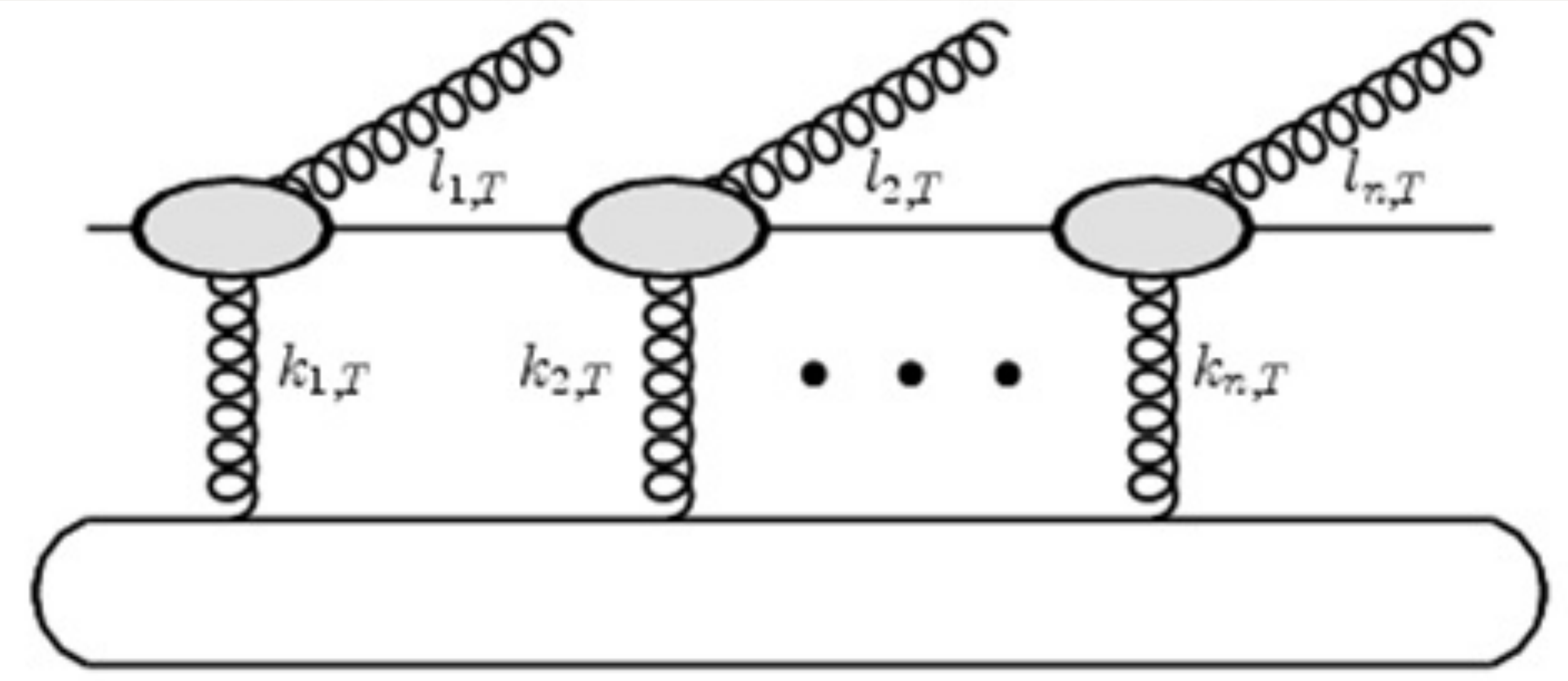
Other challenges

Multiple scatterings

e.g. AMY energy loss formalism
[JHEP 06 (2002) 030]



Multiple emissions



Space-time structure of parton shower

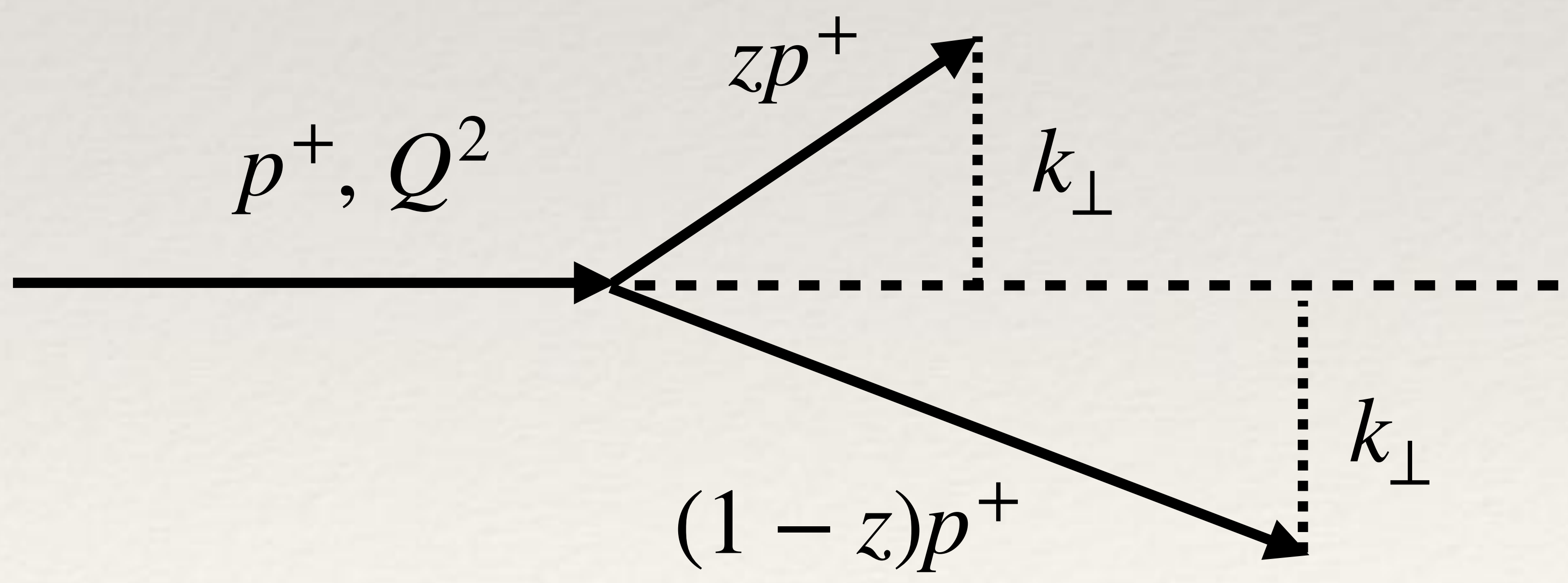
Space-time structure is usually ignored for vacuum shower, but important for its medium modification

$$P_{ai}^{\mathrm{med}}(z,Q^2) = \frac{C_A}{C_2(a)} \frac{P_{ai}^{\mathrm{vac}}(z)}{z(1-z)Q^2} \times \int_0^{\tau_f^+} d\zeta^+ \hat{q}_a \underline{\left(\vec{r} + \hat{n}\frac{\zeta^+}{\sqrt{2}}\right)} \left[2 - 2\cos\left(\frac{\zeta^+}{\tau_f^+}\right)\right]$$

Determined by the initial location, direction of propagation and time of propagation

τ_f^+: formation time (time of propagation before splitting)

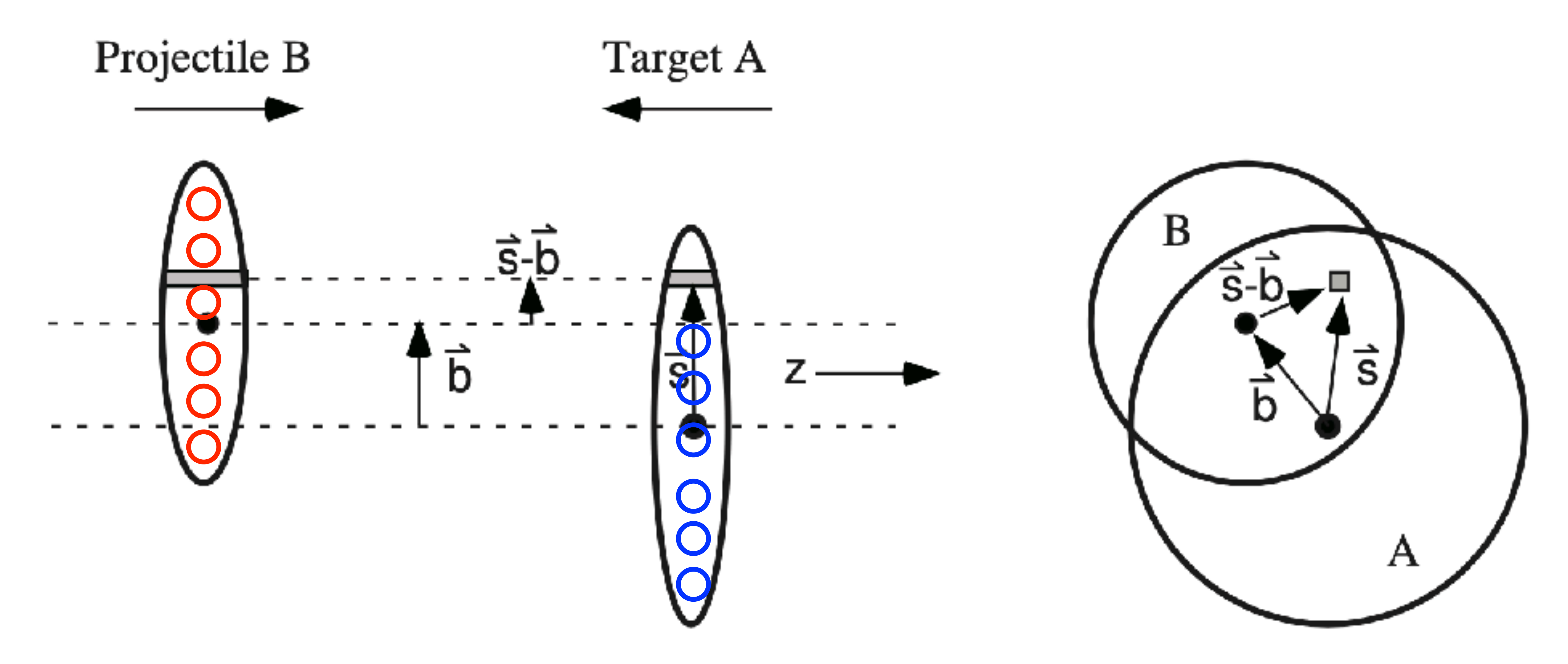
$$\tau_f^+ = \frac{2p^+}{Q^2} = \frac{2p^+ z(1-z)}{k_\perp^2}$$



Formation time

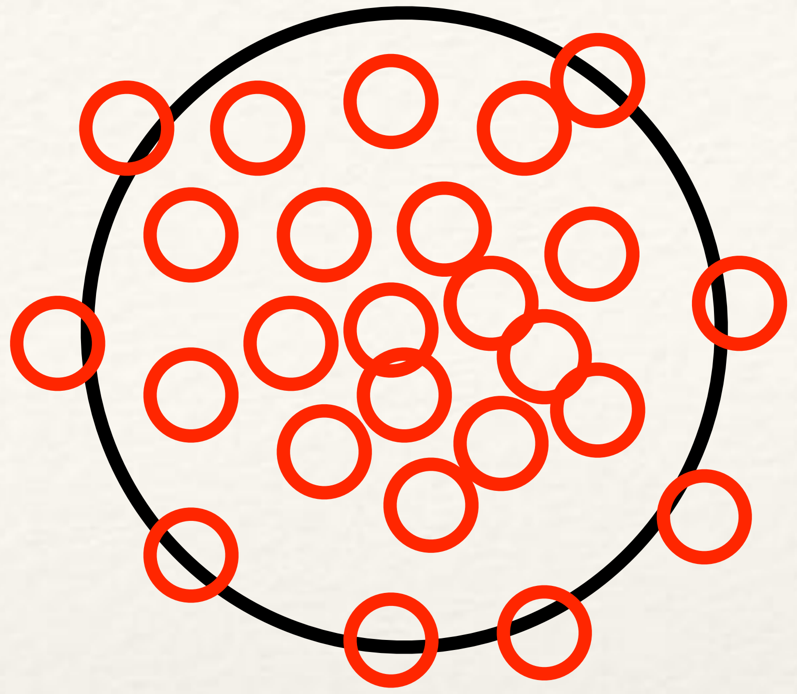
- $\left[p^+, \frac{Q_a^2}{2p^+}, 0\right] \to \left[zp^+, \frac{k_\perp^2}{2zp^+}, \vec{k}_\perp\right] + \left[(1-z)p^+, \frac{k_\perp^2}{2(1-z)p^+}, -\vec{k}_\perp\right], \quad (Q_b, Q_c \ll Q_a)$
- Needs virtuality for p^- conservation, $Q_a^2 = \frac{k_\perp^2}{z(1-z)}$
- Off-shellness is unstable, $\tau_f^+ \sim \frac{1}{\Delta p^-} = \frac{2p^+}{Q_a^2} = \frac{2z(1-z)p^+}{k_\perp^2}$ (assume 0 rest mass)
- Cartesian coordinate: $\tau_f = \frac{2E}{Q_a^2} = \frac{2z(1-z)E}{k_\perp^2}$
- Additional correction is needed for massive particles: $\tau_f = \frac{2E}{Q_a^2} = \frac{2z(1-z)E}{k_\perp^2 + z^2M^2}$, if $M_a = M_c = M$

Initial condition



a) Side View

b) Beam-line View



Thickness function:

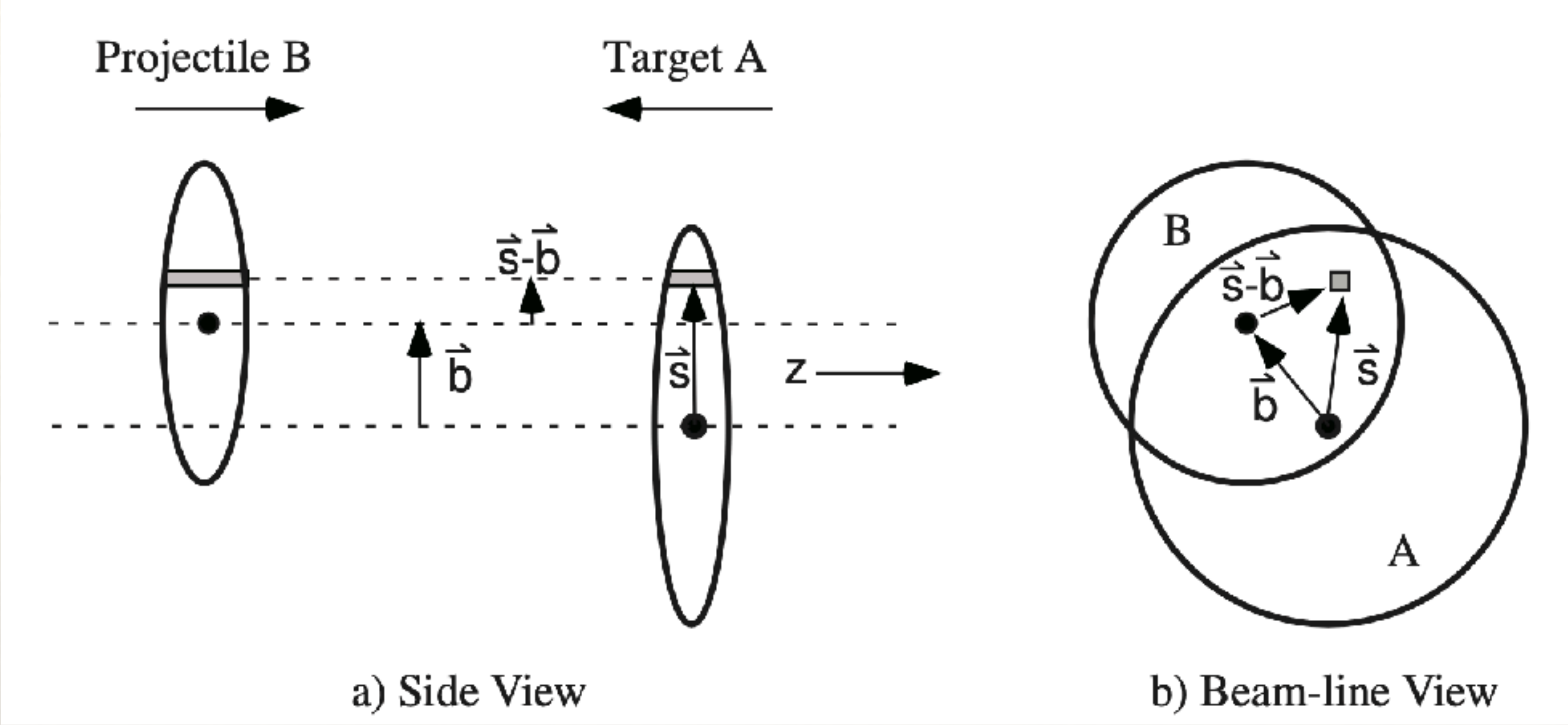
$$T_A(\vec{r}) = \rho(\vec{r}, z_A) dz_A$$

- Glauber model assumption: nucleus-nucleus collision is viewed a superposition of independent interaction of nucleon pairs.
- At high energy, each nucleon moves in a straight line

normalized to 1

- Distribution of nucleons inside a nucleus — Woods-Saxon: $\rho(r) = \rho_0 \dfrac{1 + w(r/R)^2}{1 + \exp[(r-R)/a]}$

Glauber model



$$\hat{T}_{\mathrm{A}}(\vec{s}) = \int \hat{\rho}_{\mathrm{A}}(\vec{s}, z_{\mathrm{A}}) dz_{\mathrm{A}},$$

$$\hat{T}_{\mathrm{B}}(\vec{s}-\vec{b}) = \int \hat{\rho}_{\mathrm{B}}(\vec{s}-\vec{b}, z_{\mathrm{B}}) dz_{\mathrm{B}},$$

Overlap:

$$\hat{T}_{\mathrm{AB}}(\vec{b}) = \int \hat{T}_{\mathrm{A}}(\vec{s}) \hat{T}_{\mathrm{B}}(\vec{s}-\vec{b}) d^2 s.$$

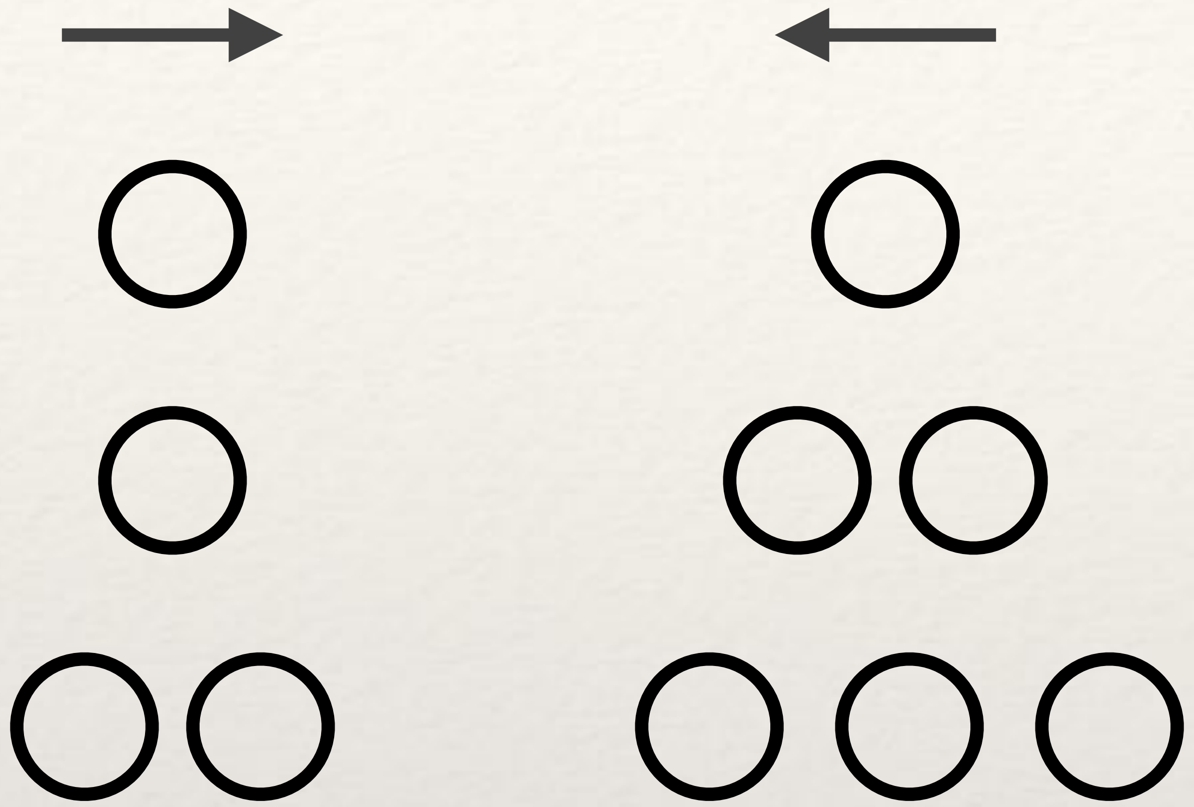
- Superposition of nucleon-nucleon (p-p) collision with given $\sigma_{\mathrm{inel}}^{\mathrm{NN}}(s)$
- The cross section of A-B collision is then

$$\frac{d^2\sigma_{\mathrm{inel}}^{\mathrm{AB}}}{d^2 b} = 1 - \left[1 - \hat{T}_{\mathrm{AB}}(\vec{b})\sigma_{\mathrm{inel}}^{\mathrm{NN}}\right]^{AB}$$

$$\sigma_{\mathrm{inel}}^{\mathrm{AB}} = \int_0^\infty 2\pi b db \left\{1 - \left[1 - \hat{T}_{\mathrm{AB}}(\vec{b})\sigma_{\mathrm{inel}}^{\mathrm{NN}}\right]^{AB}\right\}.$$

Participant number and binary collision number

- Participant and binary collision:



	participant number	binary collision number
	2	1
	3	2
	5	6

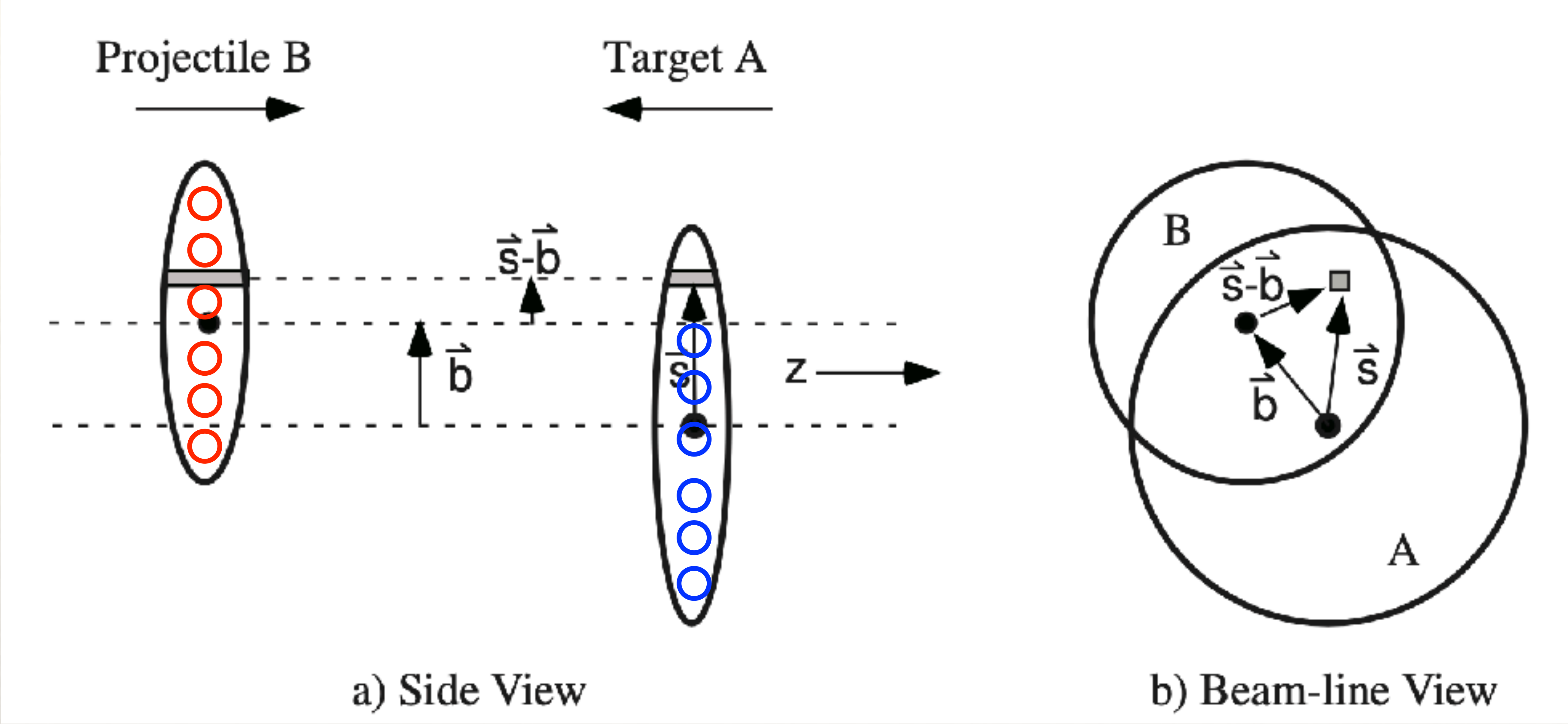
$$N_{\mathrm{part}}(\vec{b}) = A\int \hat{T}_{\mathrm{A}}(\vec{s})\left\{1-\left[1-\hat{T}_{\mathrm{B}}(\vec{s}-\vec{b})\sigma_{\mathrm{inel}}^{\mathrm{NN}}\right]^{B}\right\}d^2s + B\int \hat{T}_{\mathrm{B}}(\vec{s}-\vec{b})\left\{1-\left[1-\hat{T}_{\mathrm{A}}(\vec{s})\sigma_{\mathrm{inel}}^{\mathrm{NN}}\right]^{A}\right\}d^2s,$$

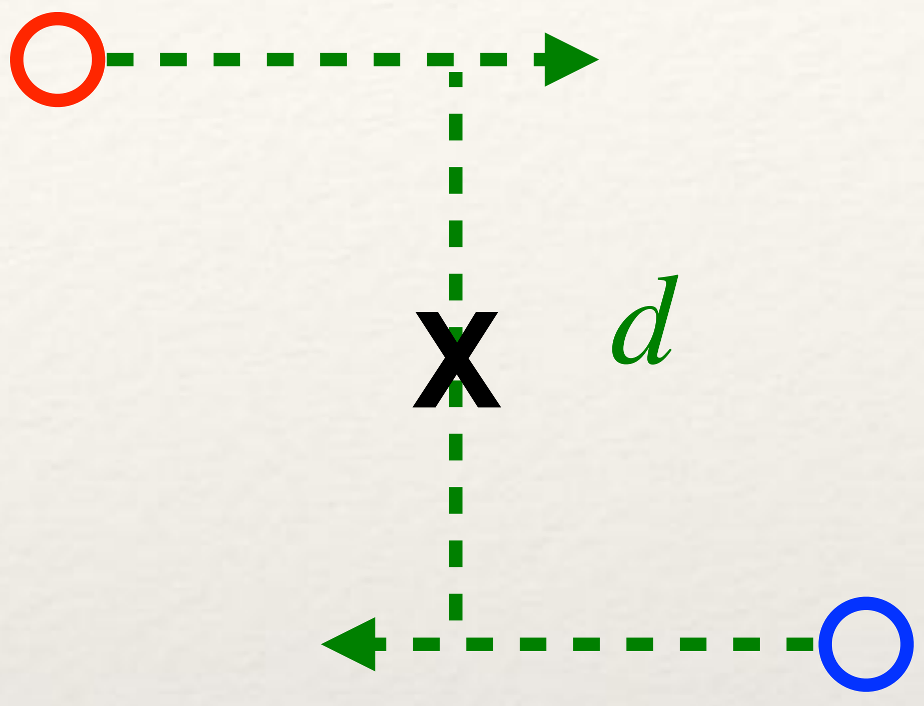
proportional to soft matter density

$$N_{\mathrm{coll}}(\vec{b}) = AB\hat{T}_{\mathrm{AB}}(\vec{b})\sigma_{\mathrm{inel}}^{\mathrm{NN}},$$

proportional to hard probe production

Semi-classical determination of collision locations





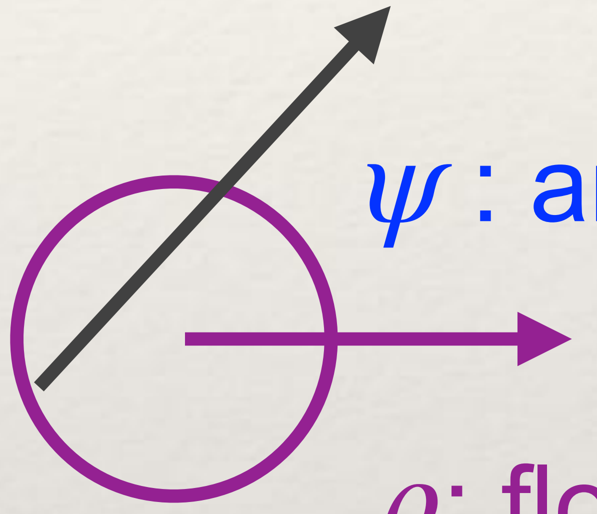
- If $d < \sqrt{\sigma/\pi}$, collision happens between this pair
- Each nucleon contributes to location of participants → initial energy density of QGP
- The middle point of the pair contributes to local of binary collisions → jet production points

Simplified models of medium modification

Modify splitting functions in Pythia

- e.g. $P_{q\to qg}(z) = C_F \dfrac{1+z^2}{1-z} \quad \Rightarrow \quad C_F \left[\dfrac{2(1+f_{\rm med})}{1-z} - (1+z)\right]$
- $f_{\rm med} = K_f \int d\zeta \left[\epsilon(\zeta)\right]^{3/4} \times \left[\cosh\rho(\zeta) - \sinh\rho(\zeta)\cos\psi\right]$

model parameter   energy density   flow effect: $\hat{q}_a = \hat{q}_{a,\rm local} \cdot p^\mu u_\mu / p^0$

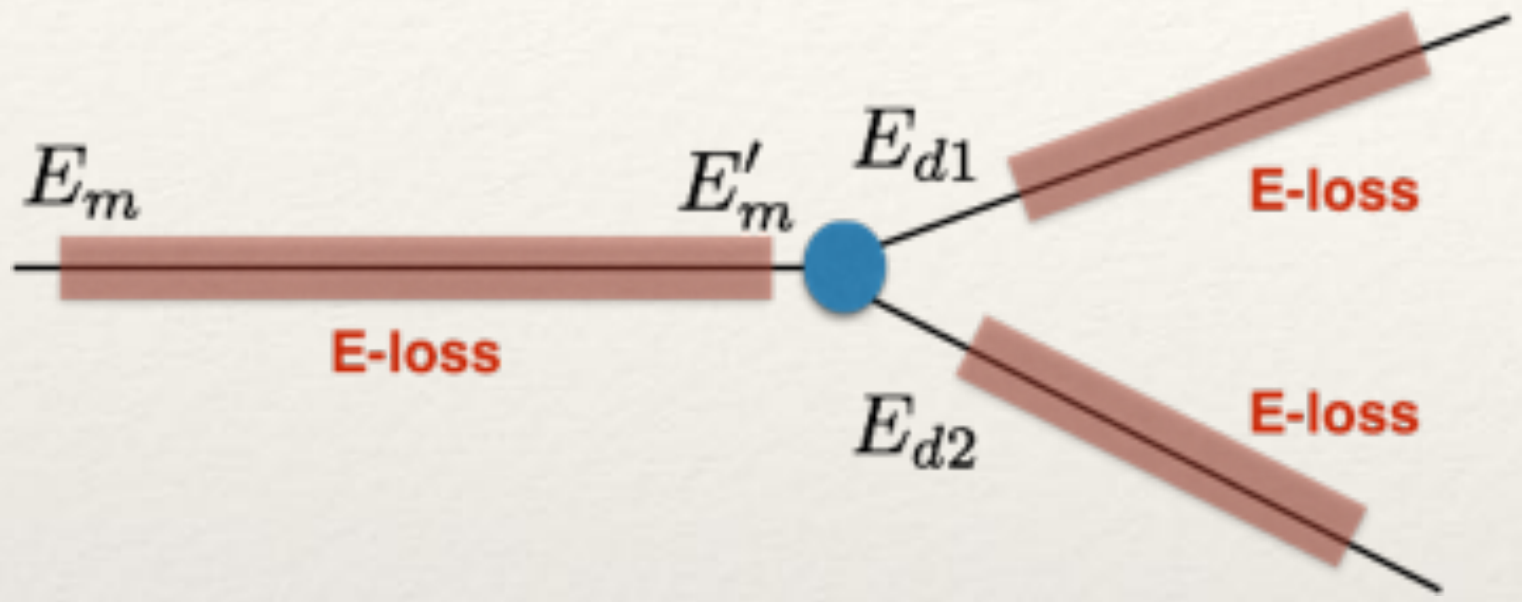


- Implementation YAJEM-FMED, Q-Pythia

Simplified models

Keep vacuum splitting function, modify parton kinematics from Pythia

Example 1: Hybrid model [JHEP 07 (2011) 118, JHEP 03 (2013) 080]



- Apply strongly coupled drag on partons

strong coupling parameter

- $$\frac{1}{E_{\rm in}}\frac{dE}{dx} = -\frac{4}{\pi}\frac{x^2}{x_{\rm stop}^2}\frac{1}{\sqrt{x_{\rm stop}^2 - x^2}},$$ stopping distance: $$x_{\rm stop} = \frac{1}{2\kappa_{\rm sc}}\frac{E_{\rm in}^{1/3}}{T^{4/3}}$$

Example 2: YAJEM [Phys. Rev. C C79 (2009) 054906]

Transport/drag coefficient $\sim T^3$ and $p^\mu u_\mu / p^0$

$$\Delta Q_a^2 = \int_{\tau_a^0}^{\tau_a^0+\tau_a^f} d\zeta \hat{q}_a(\zeta) \quad \text{(YAJEM-RAD)}$$

$$\Delta E_a = \int_{\tau_a^0}^{\tau_a^0+\tau_a^f} d\zeta D_a(\zeta) \quad \text{(YAJEM-DRAG)}$$

Simplified models

Example 3: JEWEL [JHEP 03 (2016) 053, JHEP 10 (2014) 019]

- A radiated gluon can scatter with the medium during $\tau_f \sim 2E/k_\perp^2$
- Scattering increases $k_\perp^2$, therefore also updates τ_f (becomes shorter)
- If scattering (mean free path) is before the new τ_f, keep this medium modification, otherwise reject
- If there are multiple scatterings, the medium-induced gluon is accepted with probability $1/N_{\text{scat}}$, modeling “totally coherent limit”
- Scatterings can raise Q^2 and excite additional splittings

Medium modification at low virtuality: transport theory

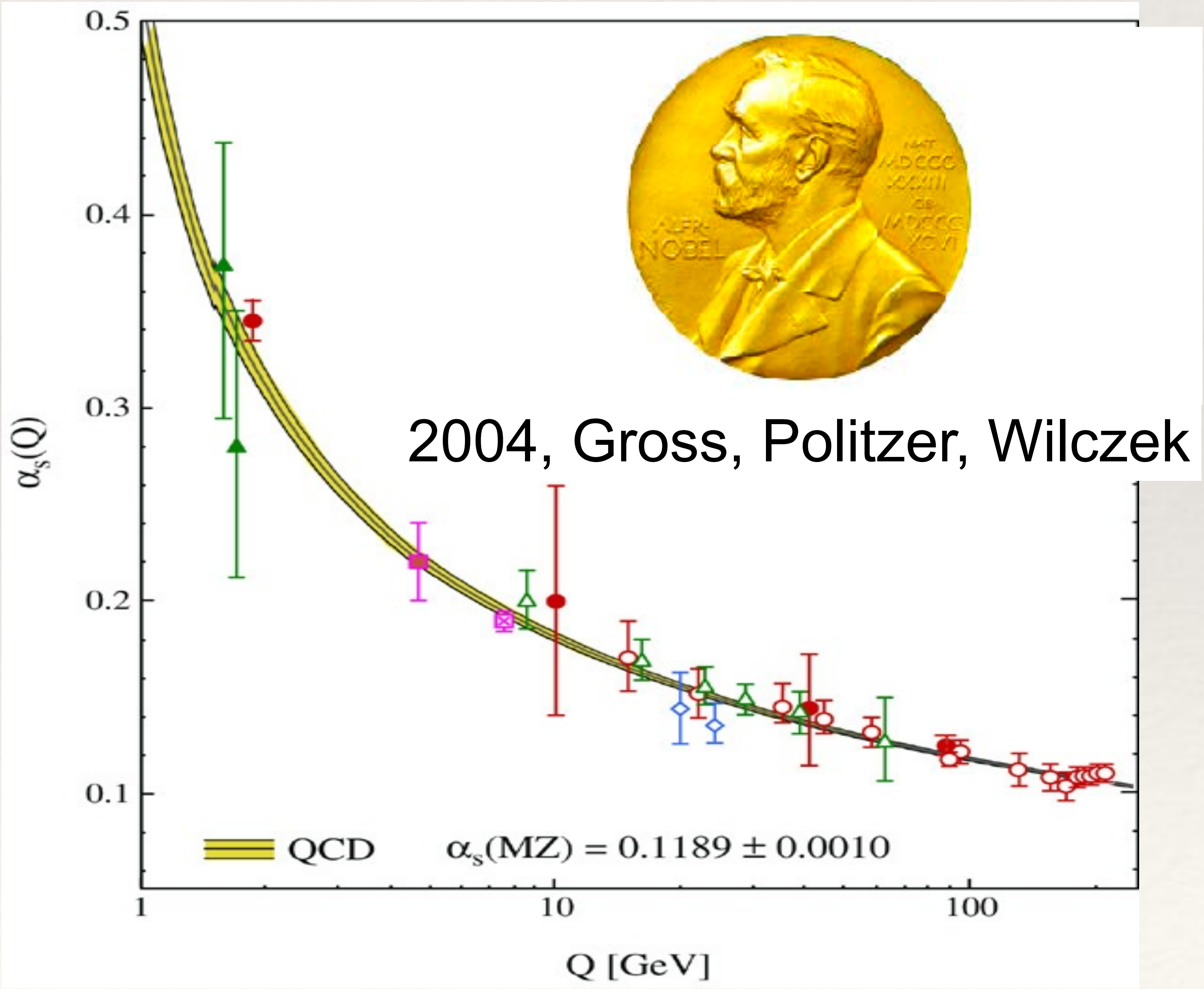
- Parton shower picture fails when virtuality no longer keeps dropping ($Q^2 \sim \hat{q}\tau_f$)
- Microscopic transport model: time-evolution of particle systems by solving transport equations

e.g. Boltzmann equation $$\frac{d}{dt} f_a(t, \vec{x}, \vec{p}) = \left[\frac{\partial}{\partial t} + \frac{p_i}{E_{\vec{p}}}\frac{\partial}{\partial x_i} + F_i \frac{\partial}{\partial p_i}\right] f_a(t, \vec{x}, \vec{p}) = \mathscr{C}[f_a]$$

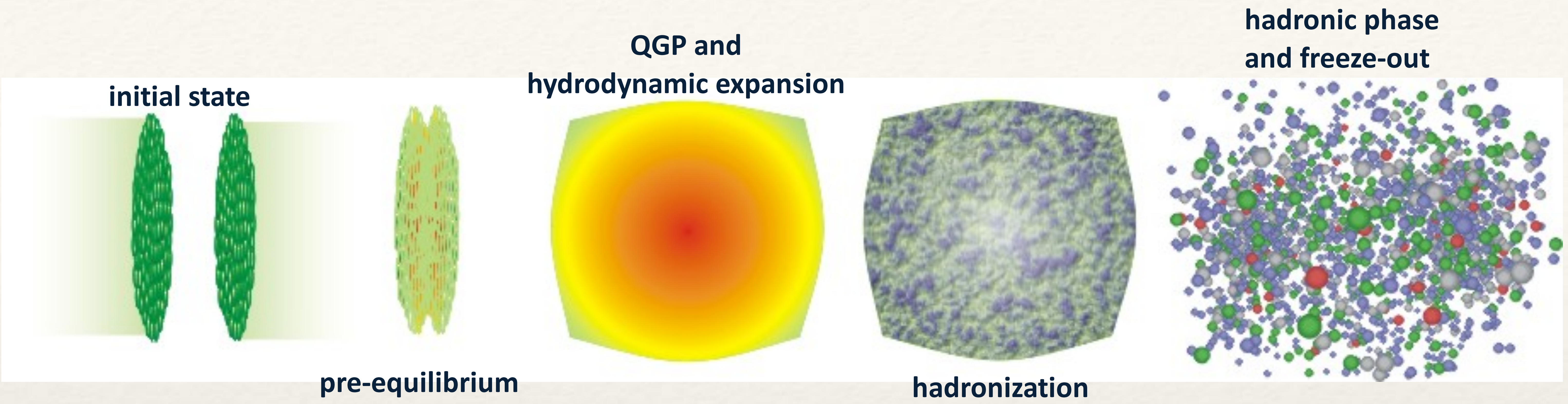
Boltzmann equation applies when $l_{\mathrm{mfp}} \sim \frac{1}{\rho\sigma} \gg \sqrt{\sigma}$

- Dilute quasi-particle system
- Weak interaction

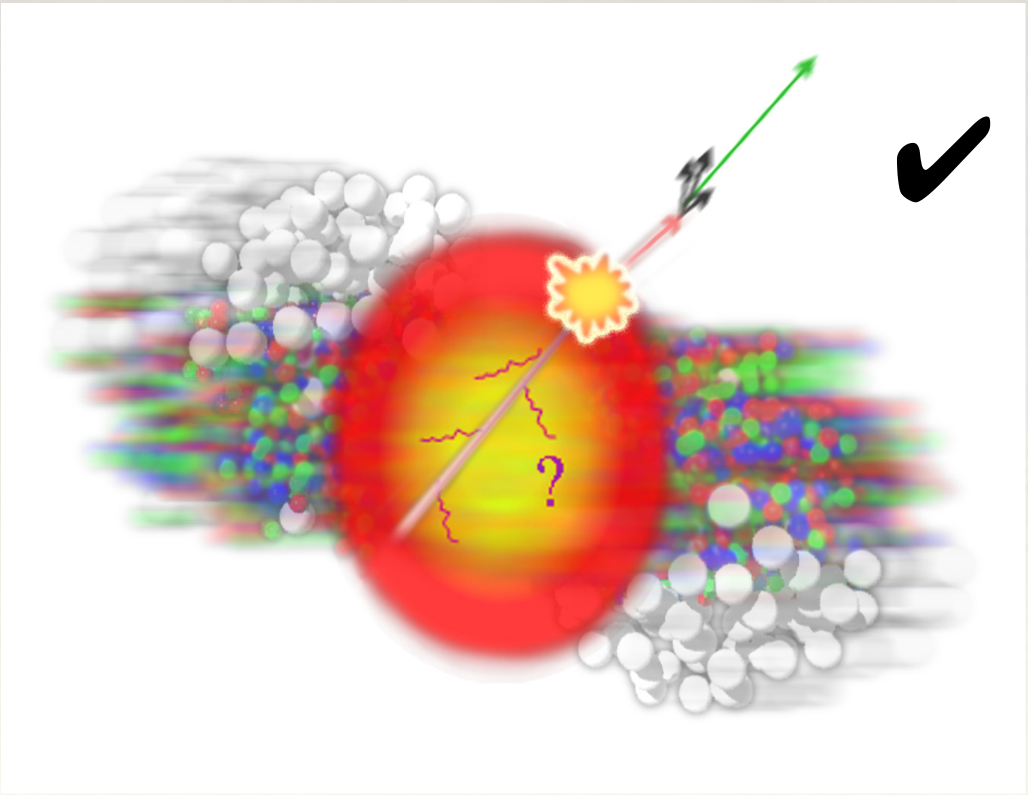
Running coupling of QCD:
Boltzmann Transport is more valid for particles with larger energy scale



Transport theory in heavy-ion collisions



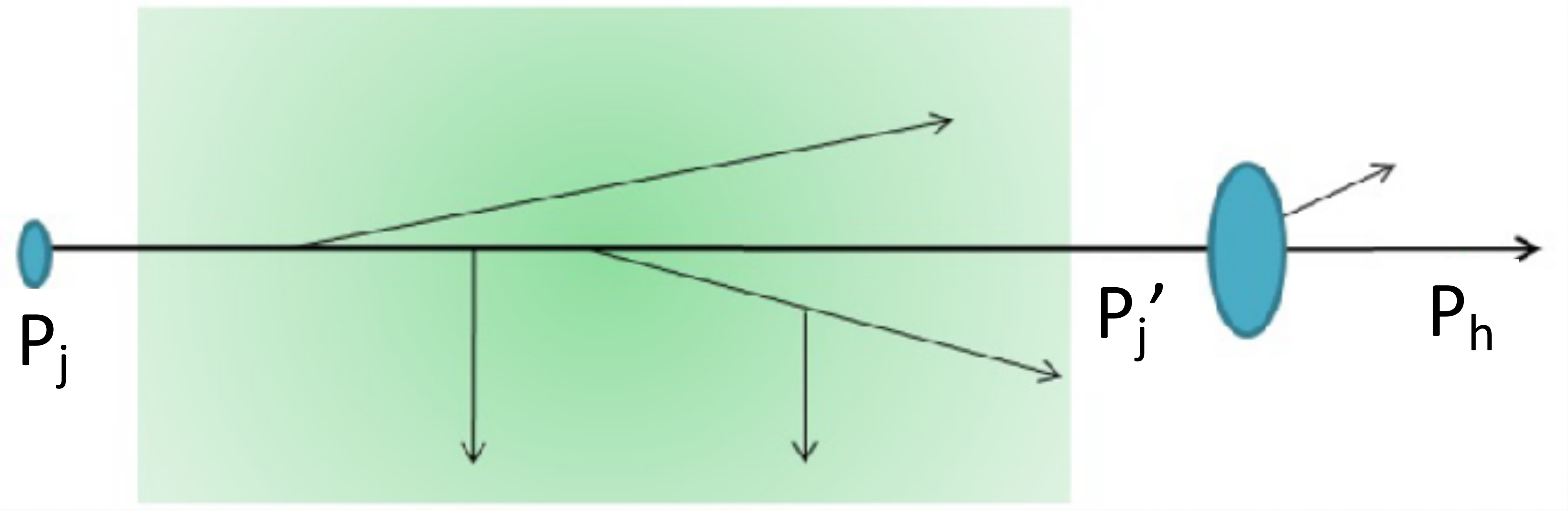
Transport?	✔ energetic particles	? strongly coupled system	✖ non-perturbative	✔ dilute system



Applicable to **jet-medium interaction**:

quarks and gluons (partons) at large energy scale are quasi-particles

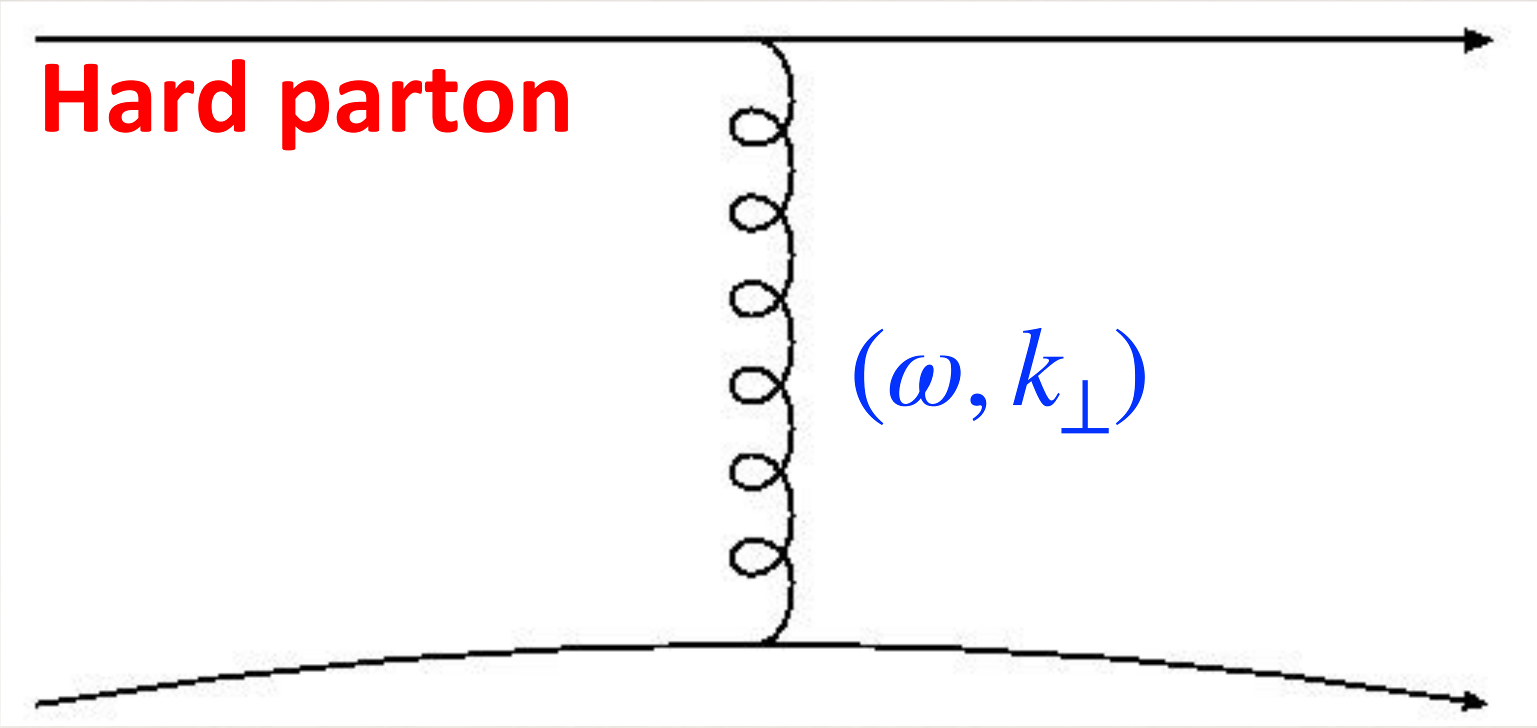
Jet parton interactions with the QGP



$$\frac{d}{dt}f_a(t,\vec{x},\vec{p}) = \left[\frac{\partial}{\partial t} + \frac{p_i}{E_{\vec{p}}}\frac{\partial}{\partial x_i} + \cancel{F_i\frac{\partial}{\partial p_i}}\right] f_a(t,\vec{x},\vec{p}) = \mathscr{C}[f_a]$$

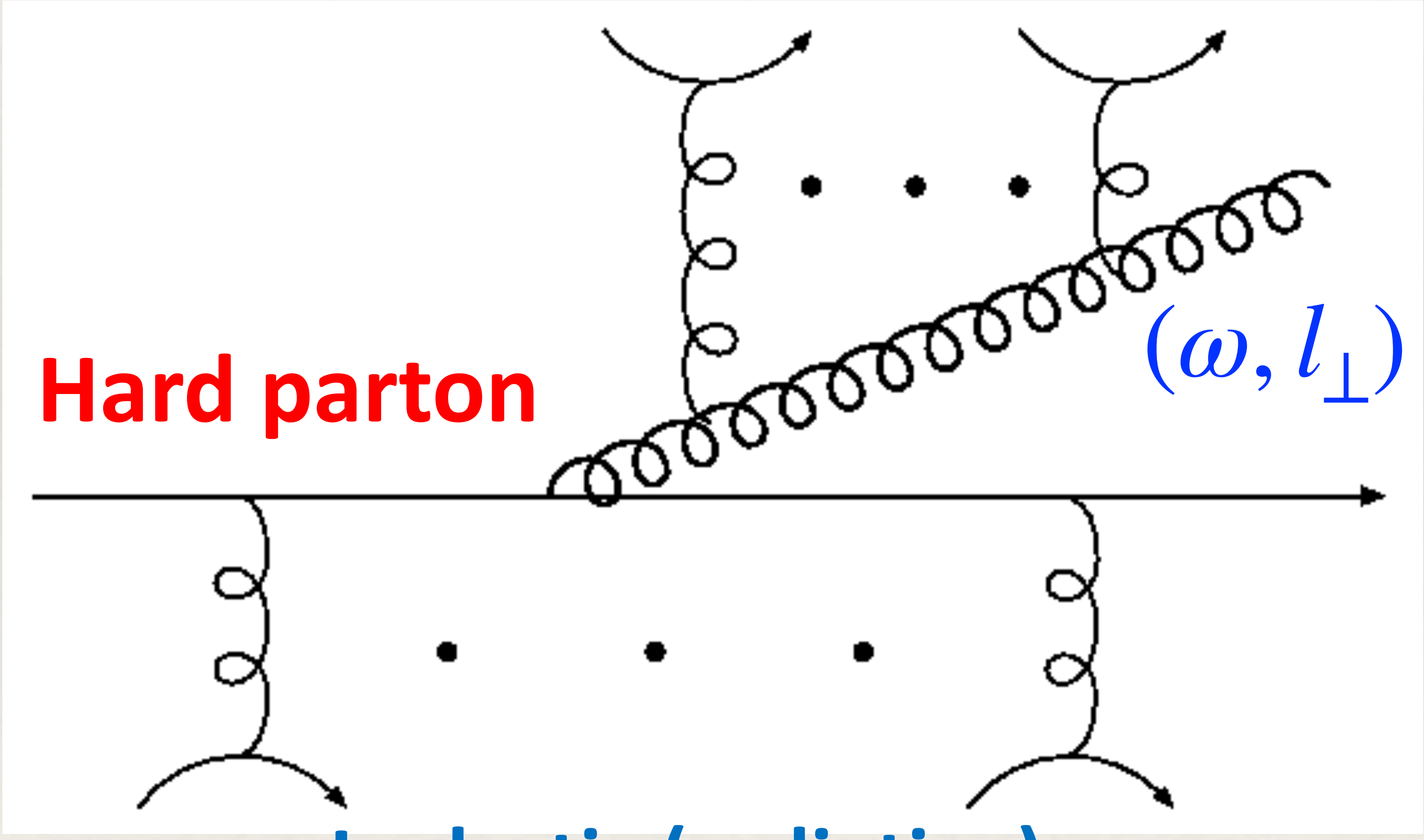
$$p_a \cdot \partial f_a(x_a, p_a) = E_a(\mathscr{C}_a^{\mathrm{el}} + \mathscr{C}_a^{\mathrm{inel}})$$

Elastic and inelastic processes:



Elastic (collisional)

$$\frac{d\Gamma_{\mathrm{coll}}}{d\omega dk_\perp^2}(T, E, \dots) = ?$$



Inelastic (radiative)

$$\frac{d\Gamma_{\mathrm{coll}}}{d\omega dl_\perp^2}(T, E, \dots) = ?$$

Elastic process

Boltzmann equation: $$\frac{d}{dt} f_a(t, \vec{x}, \vec{p}) = \mathscr{C}[f_a]$$

Collision integral: $$\mathscr{C}[f_a] \equiv \int d^3k \left[w(\vec{p} + \vec{k}, \vec{k}) f_a(\vec{p} + \vec{k}) - \underbrace{w(\vec{p}, \vec{k})}_{\text{transition rate}} f_a(\vec{p}) \right]$$ gain - loss

Loss term: $$\frac{d}{dt} f_a = -\int d^3k\, w(\vec{p}, \vec{k})\, f_a = -\Gamma f_a$$

Scattering rate: $$\Gamma = \int d^3k\, w(\vec{p}, \vec{k})$$ number of scattering per unit time

Relating transition rate to cross section: $$w(\vec{p}, \vec{k}) = \gamma_b \int \frac{d^3q}{2\pi^3} \underline{f_b(\vec{q})}\, v_{\mathrm{rel}}\, d\sigma(\vec{p}, \vec{q} \to \vec{p} - \vec{k}, \vec{q} + \vec{k})$$

if f_b is not affected by f_a, linear equation with respect to f_a

Elastic process

Relative velocity:

$$v_{\rm rel} = \left| \frac{\vec{p}}{E_{\vec{p}}} - \frac{\vec{q}}{E_{\vec{q}}} \right| = \frac{\sqrt{(p\cdot q)^2 - (m_a m_b)^2}}{E_{\vec{p}} E_{\vec{q}}}$$

Cross section:

$$v_{\rm rel} d\sigma(\vec{p},\vec{q} \to \vec{p}',\vec{q}') =$$

$$\frac{1}{2E_{\vec{p}} 2E_{\vec{q}}} \frac{d^3p'}{(2\pi)^3 2E_{\vec{p}'}} \frac{d^3q'}{(2\pi)^3 2E_{\vec{q}'}} \frac{1}{\gamma_Q \gamma_{q,g}} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p+q-p'-q').$$

Full expression for elastic scattering rate:

$$\Gamma_a^{\rm el}(\vec{p}_a, T) = \underline{\sum_{b,(cd)}} \frac{\gamma_b}{2E_a} \int \prod_{i=b,c,d} \underline{d[p_i]} f_b \times (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d) |\mathscr{M}_{ab\to cd}|^2$$

$$d[p_i] \equiv \frac{d^3p_i}{2E_i(2\pi)^3}$$

sum over all possible scattering channels

Elastic process

Light flavor (massless):

$ij \to kl$	$\lvert M\rvert^2_{ij\to kl}$
$gg \to gg$	$\frac{9}{2}g_s^4\left(3-\frac{ut}{s^2}-\frac{us}{t^2}-\frac{st}{u^2}\right)$
$gg \to q\bar{q}$	$\frac{3}{8}g_s^4\left(\frac{4}{9}\frac{t^2+u^2}{tu}-\frac{t^2+u^2}{s^2}\right)$
$gq \to gq$ $g\bar{q} \to g\bar{q}$	$g_s^4\left(\frac{s^2+u^2}{t^2}-\frac{4}{9}\frac{s^2+u^2}{su}\right)$
$q_iq_j \to q_iq_j$ $q_i\bar{q}_j \to q_i\bar{q}_j$ $\bar{q}_iq_j \to \bar{q}_iq_j$ $\bar{q}_i\bar{q}_j \to \bar{q}_i\bar{q}_j$	$\frac{4}{9}g_s^4\frac{s^2+u^2}{t^2}, \quad i\neq j$
$q_iq_i \to q_iq_i$ $\bar{q}_i\bar{q}_i \to \bar{q}_i\bar{q}_i$	$\frac{4}{9}g_s^4\left(\frac{s^2+u^2}{t^2}+\frac{s^2+t^2}{u^2}-\frac{2}{3}\frac{s^2}{tu}\right)$
$q_i\bar{q}_i \to q_j\bar{q}_j$	$\frac{4}{9}g_s^4\frac{t^2+u^2}{s^2}$
$q_i\bar{q}_i \to q_i\bar{q}_i$	$\frac{4}{9}g_s^4\left(\frac{s^2+u^2}{t^2}+\frac{t^2+u^2}{s^2}-\frac{2}{3}\frac{u^2}{st}\right)$
$q\bar{q} \to gg$	$\frac{8}{3}g_s^4\left(\frac{4}{9}\frac{t^2+u^2}{tu}-\frac{t^2+u^2}{s^2}\right)$

Heavy flavor:

$$\Sigma\left|\mathcal{M}_{qQ\to qQ}\right|^2=\frac{64}{9}\pi^2\alpha_s^2(M_{\mathrm{T}})\frac{(M^2-u)^2+(s-M^2)^2+2M^2t}{t^2},$$

$$\begin{aligned}\Sigma\left|\mathcal{M}_{gQ\to gQ}\right|^2=\pi^2\alpha_s^2(M_{\mathrm{T}})\Bigg[&\frac{32}{t^2}(s-M^2)(M^2-u)\\&+\frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(s+M^2)}{(s-M^2)^2}\\&+\frac{64}{9}\frac{(s-M^2)(M^2-u)+2M^2(M^2+u)}{(M^2-u)^2}\\&+\frac{16}{9}\frac{M^2(4M^2-t)}{(s-M^2)(M^2-u)}\\&+16\frac{(s-M^2)(M^2-u)+M^2(s-u)}{t(s-M^2)}\\&-16\frac{(s-M^2)(M^2-u)-M^2(s-u))}{t(M^2-u)}\Bigg],\end{aligned}$$

Calculation of the scattering rate

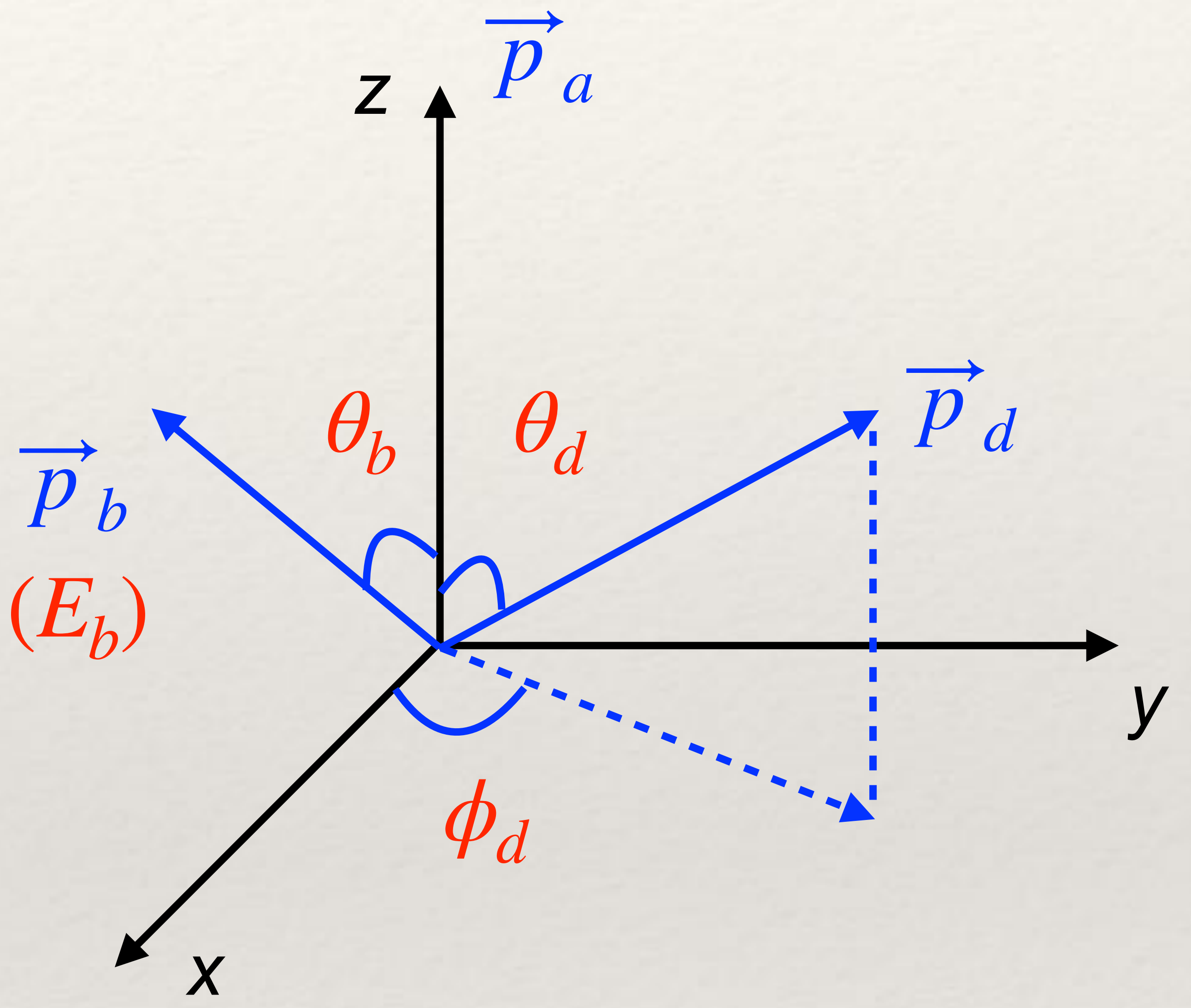
$$\Gamma_a^{\rm el}(\vec{p}_a, T) = \sum_{b,(cd)} \frac{\gamma_b}{2E_a} \int \prod_{i=b,c,d} d[p_i]\, f_b\, \underline{S_2(\hat{s},\hat{t},\hat{u})} \times (2\pi)^4 \delta^{(4)}(p_a + p_b - p_c - p_d)\, |\mathcal{M}_{ab\to cd}|^2$$

Avoid collinear divergence: $S_2(\hat{s},\hat{t},\hat{u}) = \theta(\hat{s} \geq 2\mu_{\rm D}^2)\theta(-\hat{s} + \mu_{\rm D}^2 \leq \hat{t} \leq -\mu_{\rm D}^2)$

Debye screening mass: $\mu_{\rm D}^2 = g^2T^2(N_c + N_f/2)/3$

$$\underline{\Gamma_a^{\rm el}(\vec{p}_a, T)} = \sum_{b,(cd)} \frac{\gamma_b}{16E_a(2\pi)^4} \int dE_b d\theta_b d\theta_d d\phi_d$$

$$\times f_b(E_b, T) S_2(\hat{s},\hat{t},\hat{u})\, |\mathcal{M}_{ab\to cd}|^2 \underline{\frac{E_b E_d \sin\theta_b \sin\theta_d}{E_a - |\vec{p}_a|\cos\theta_d + E_b - E_b\cos\theta_{bd}}}$$



$$\cos\theta_{bd} = \sin\theta_b \sin\theta_d \cos\phi_d + \cos\theta_b \cos\theta_d$$

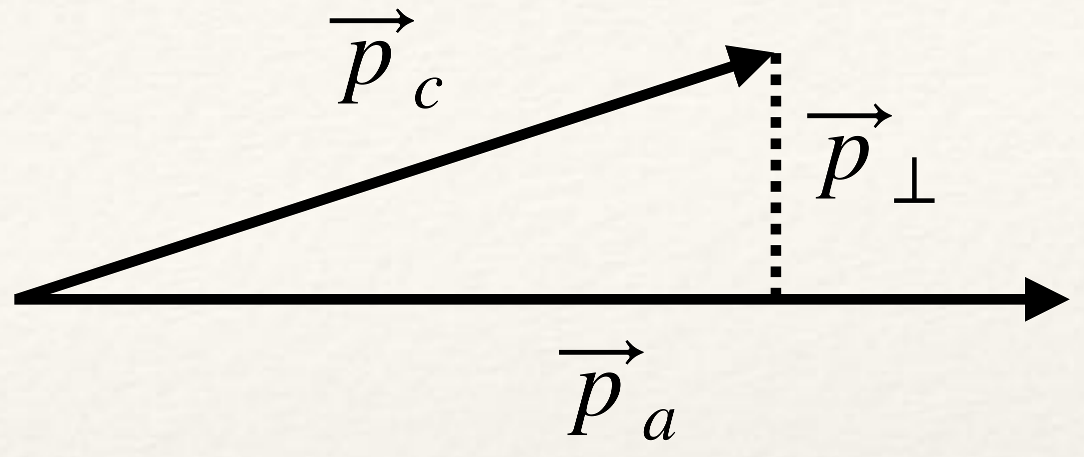
$$E_d = \frac{E_a E_b - p_a E_b \cos\theta_b}{E_a - p_a \cos\theta_d + E_b - E_b \cos\theta_{bd}}$$

Assume *b* massless here

Massive case [Eur. Phys. J. C 82 (2022) 4, 350]

Scattering / momentum broadening / energy loss rate

General form:



$$\langle\langle X(\vec{p}_a, T)\rangle\rangle = \sum_{b,(cd)} \frac{\gamma_b}{16E_a(2\pi)^4} \int dE_b d\theta_b d\theta_d d\phi_d$$

$$\times X(\vec{p}_a, T) f_b(E_b, T) S_2(\hat{s}, \hat{t}, \hat{u}) |\mathscr{M}_{ab\to cd}|^2 \frac{E_b E_d \sin\theta_b \sin\theta_d}{E_a - |\vec{p}_a|\cos\theta_d + E_b - E_b\cos\theta_{bd}}$$

$$\Gamma_a^{\mathrm{el}} = \langle\langle 1\rangle\rangle \qquad \hat{q}_a = \langle\langle \left[\vec{p}_c - (\vec{p}_c\cdot\hat{p}_a)\hat{p}_a\right]^2\rangle\rangle \qquad \hat{e}_a = \langle\langle E_a - E_c\rangle\rangle$$

Small angle approximation:

$$\Gamma_a^{\mathrm{el}} = C_2(a)\frac{42\zeta(3)}{\pi}\frac{\alpha_s^2 T^3}{\mu_D^2} \qquad \hat{q}_a = C_2(a)\frac{42\zeta(3)}{\pi}\alpha_s^2 T^3 \ln\left(\frac{C_{\hat{q}} E_a T}{4\mu_D^2}\right) \qquad \hat{e}_a = C_2(a)\frac{3\pi}{2}\alpha_s^2 T^2 \ln\left(\frac{C_{\hat{e}} E_a T}{4\mu_D^2}\right)$$

Monte Carlo simulation

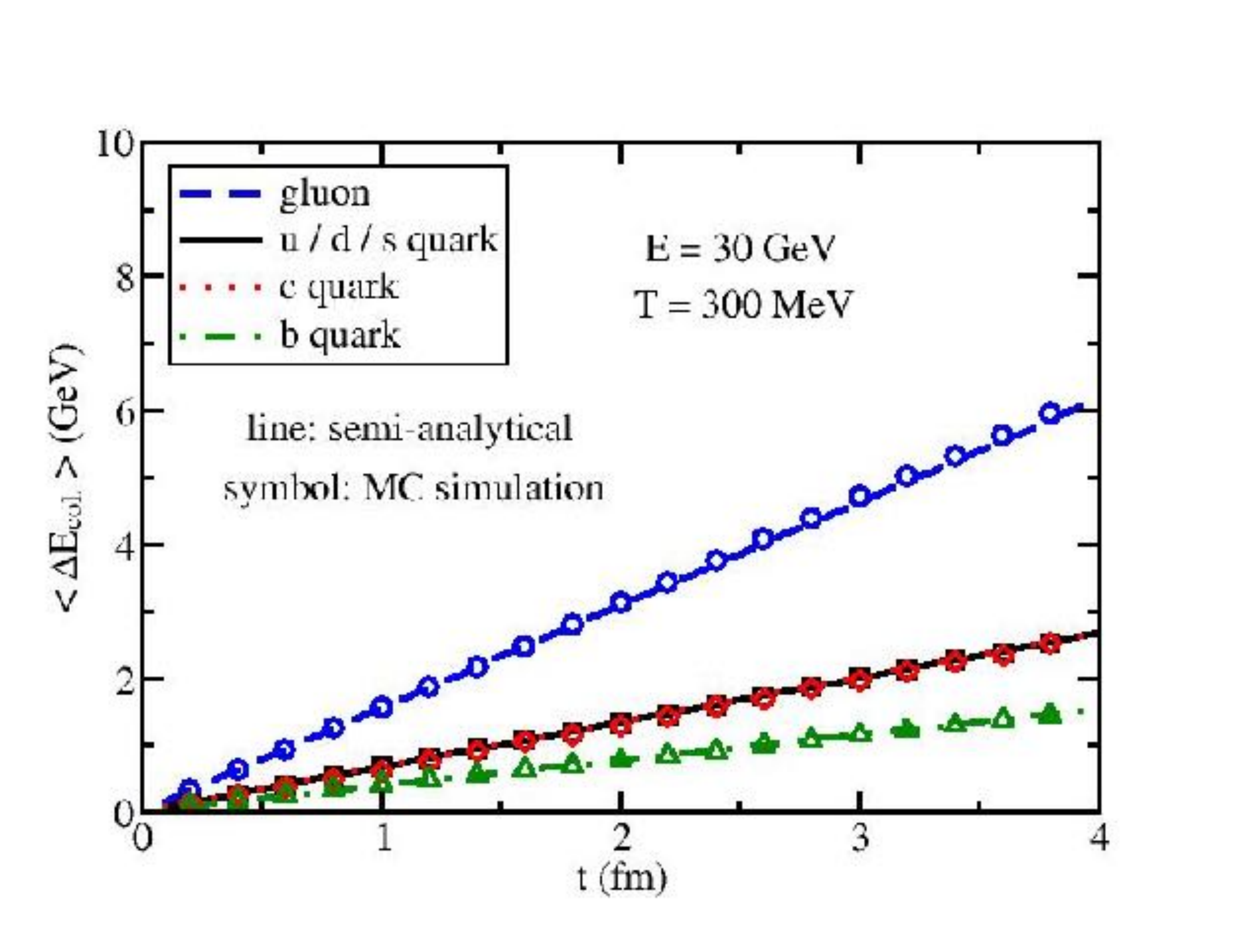
$$\Gamma_a^{\rm el}(\vec{p}_a, T) = \sum_{b,(cd)} \frac{\gamma_b}{16E_a(2\pi)^4} \int dE_b d\theta_b d\theta_d d\phi_d$$

$$\times f_b(E_b, T) S_2(\hat{s}, \hat{t}, \hat{u}) |\mathcal{M}_{ab\to cd}|^2 \frac{E_b E_d \sin\theta_b \sin\theta_d}{E_a - |\vec{p}_a| \cos\theta_d + E_b - E_b \cos\theta_{bd}}$$

1. Use total rate $\Gamma = \sum_i \Gamma_i$ to determine the probability of elastic scattering $P_{\rm el} = \Gamma \Delta t$

2. Use branching ratios Γ_i / Γ to determine the scattering channel

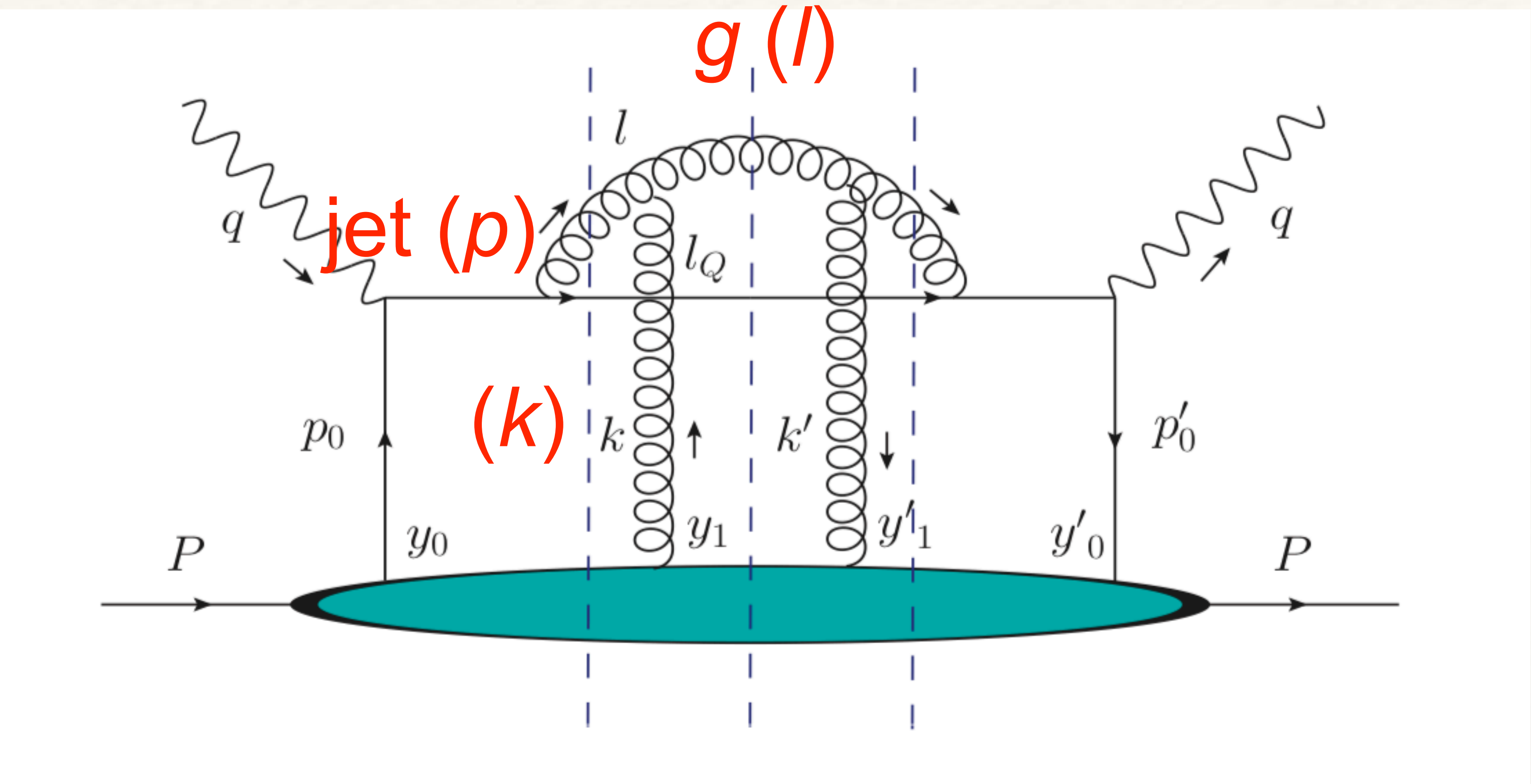
3. Use the differential rate to sample the *p* space of the two outgoing partons



$\Delta E_{\rm col.}$ from our MC simulation agrees with the semi-analytical result.

Inelastic scattering

- Inelastic scattering rate



[Majumder PRD 85 (2012); Zhang, Wang and Wang, PRL 93 (2004)]

- Higher-twist: collinear expansion ($\langle k_\perp^2 \rangle \ll l_\perp^2 \ll Q^2$)

$$\frac{d\Gamma_a^{\mathrm{inel}}}{dz dl_\perp^2} = \frac{dN_g}{dz dl_\perp^2 dt} = \frac{6\alpha_s P(z) l_\perp^4 \hat{q}}{\pi(l_\perp^2 + z^2 M^2)^4} \sin^2\left(\frac{t - t_i}{2\tau_f}\right)$$

- Medium information absorbed in $\hat{q} \equiv d\langle p_\perp^2 \rangle / dt$ — calculated using elastic scattering

- $P_{\mathrm{inel}} = \Gamma_{\mathrm{inel}} \Delta t \rightarrow$ Monte Carlo simulation

Monte-Carlo simulation

Number n of radiated gluons during Δt – Poisson distribution:

$$P(n) = \frac{\langle N_g \rangle^n}{n!} e^{-\langle N_g \rangle}$$

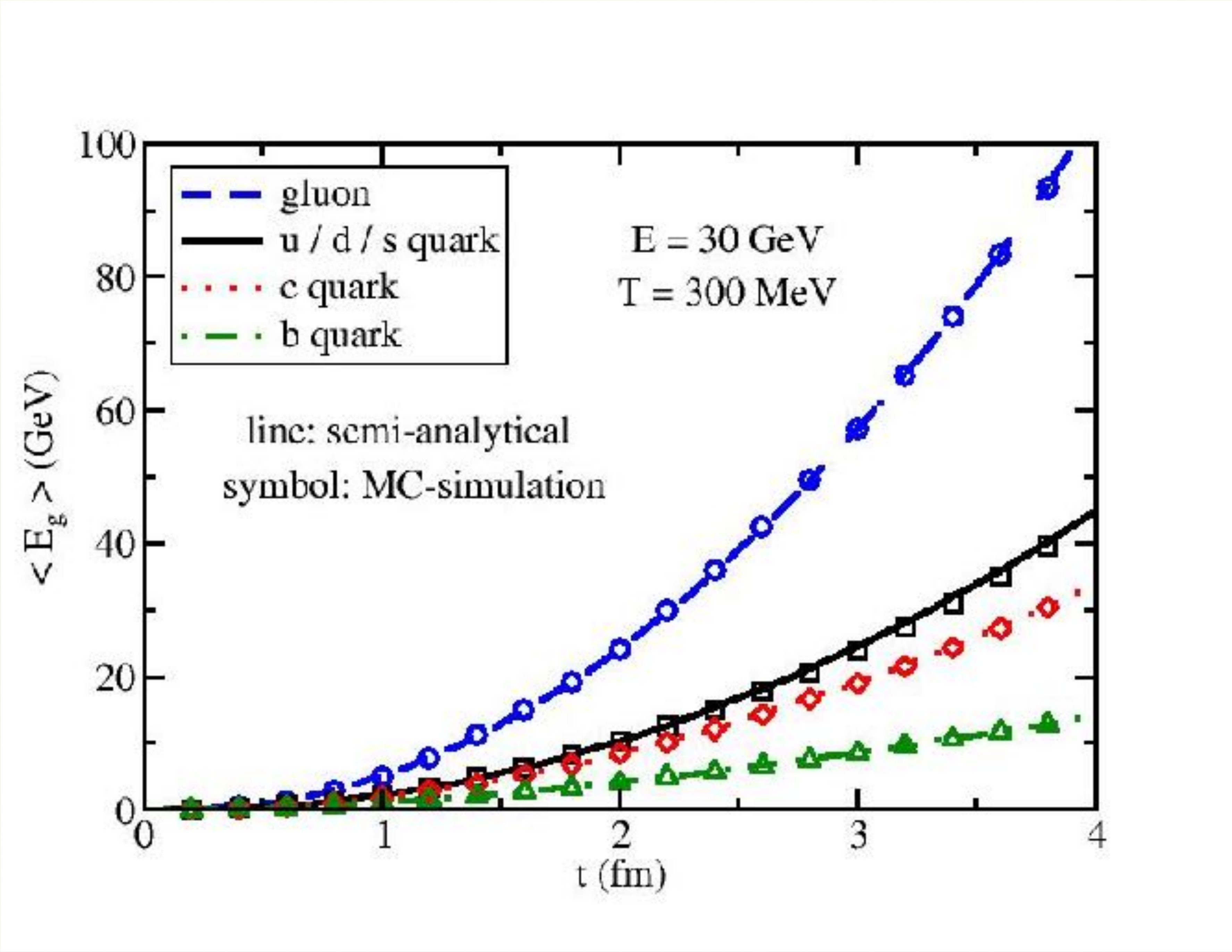
Probability of inelastic scattering during Δt: $P_{\rm inel} = 1 - e^{-\langle N_g \rangle}$

1. Calculate $\langle N_g \rangle$ and thus $P_{\rm inel}$

2. If gluon radiation happens, sample n from $P(n)$

3. Sample E and p of gluons using the differential spectrum

4. Assume 2->2 first and adjust E and p of the 2+n final partons together to guarantee E-p conservation of 2->2+n process



$<E_g>$ from our MC simulation agrees with the semi-analytical result.

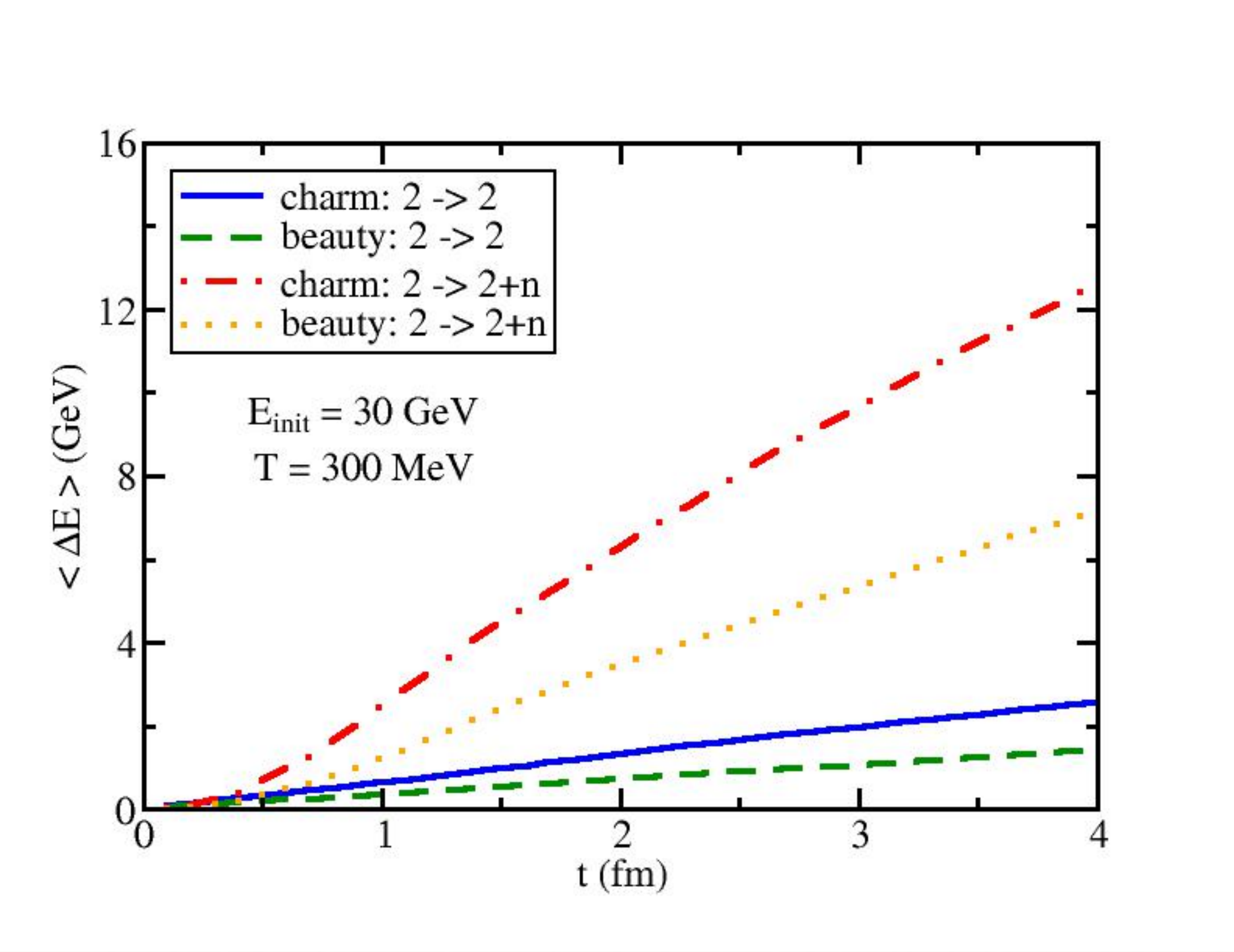
Elastic *vs.* inelastic energy loss

Divide scattering probability of jet parton into two regions

1. Pure elastic scattering without gluon emission: $P_{\rm el}(1 - P_{\rm inel})$
2. Inelastic scattering: $P_{\rm inel}$

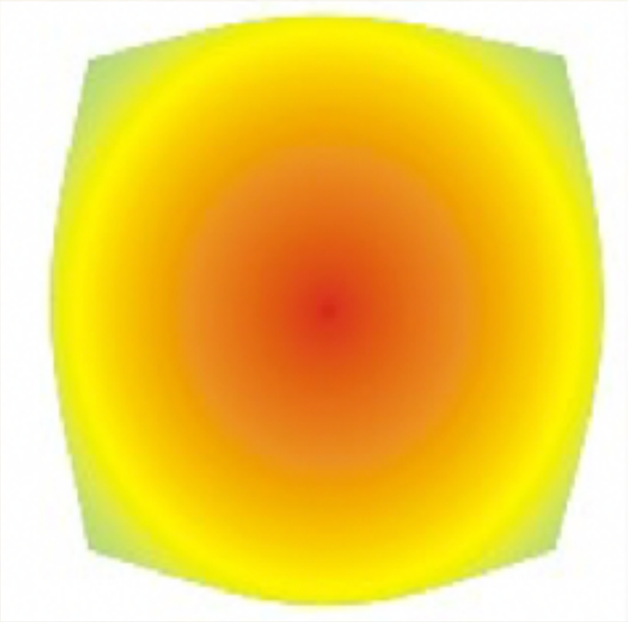
Total probability: $P_{\rm tot} = P_{\rm el} + P_{\rm inel} - P_{\rm el}P_{\rm inel}$

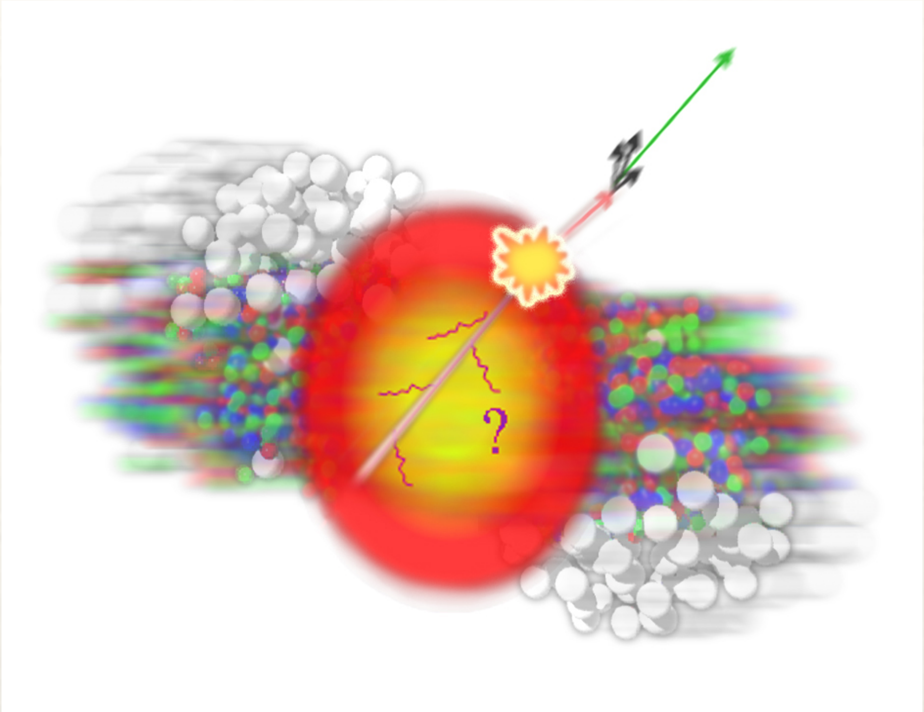
$$\boxed{\begin{aligned} P_{\rm tot} &= 1 - e^{-(\Gamma_{\rm el}+\Gamma_{\rm inel})\Delta t} \\ &= (1 - e^{-\Gamma_{\rm el}\Delta t}) + (1 - e^{-\Gamma_{\rm inel}\Delta t}) \\ &\quad -(1 - e^{-\Gamma_{\rm el}\Delta t}) \times (1 - e^{-\Gamma_{\rm inel}\Delta t}) \end{aligned}}$$



- Elastic and inelastic energy losses are comparable at early time
- Inelastic process dominates at large time

Jet propagation through hydrodynamic medium





local medium information

QGP medium

Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

$\epsilon, T, \mathbf{u} \ \ldots$

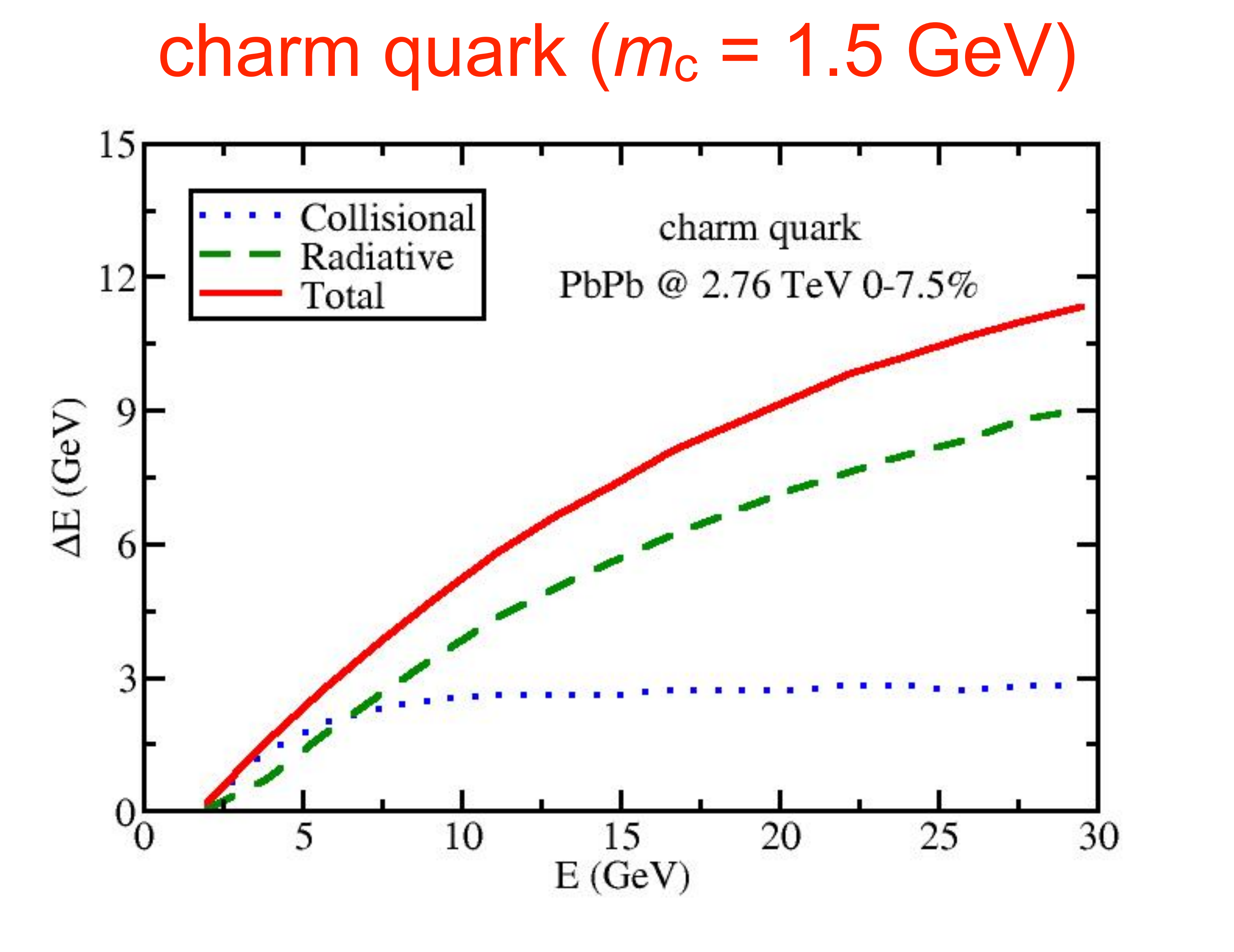
linear approximation

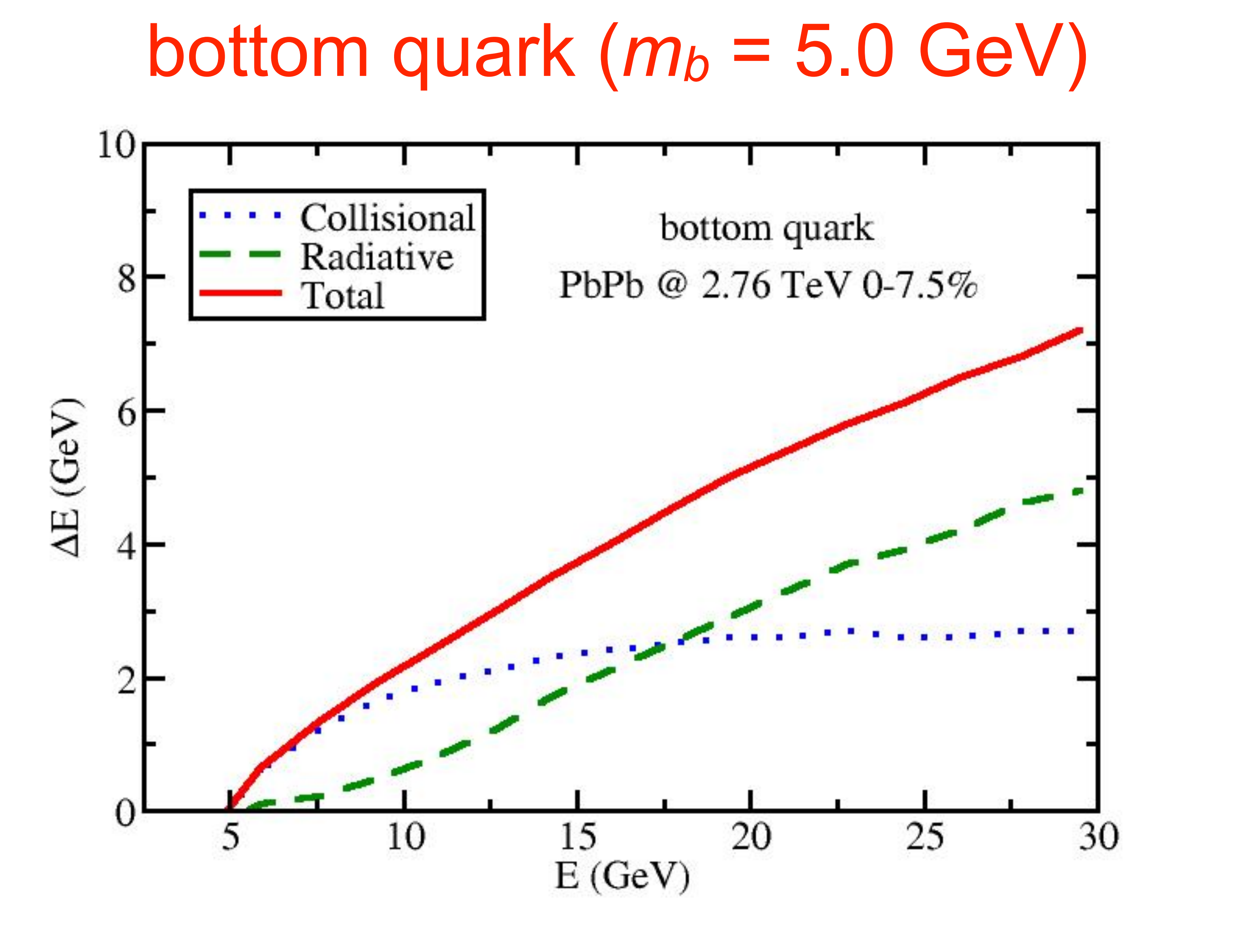
energetic partons

LBT

$$\frac{1}{p \cdot u} p^\mu \partial_\mu f = \mathscr{C}$$

Mass effects on elastic *vs*. inelastic energy loss

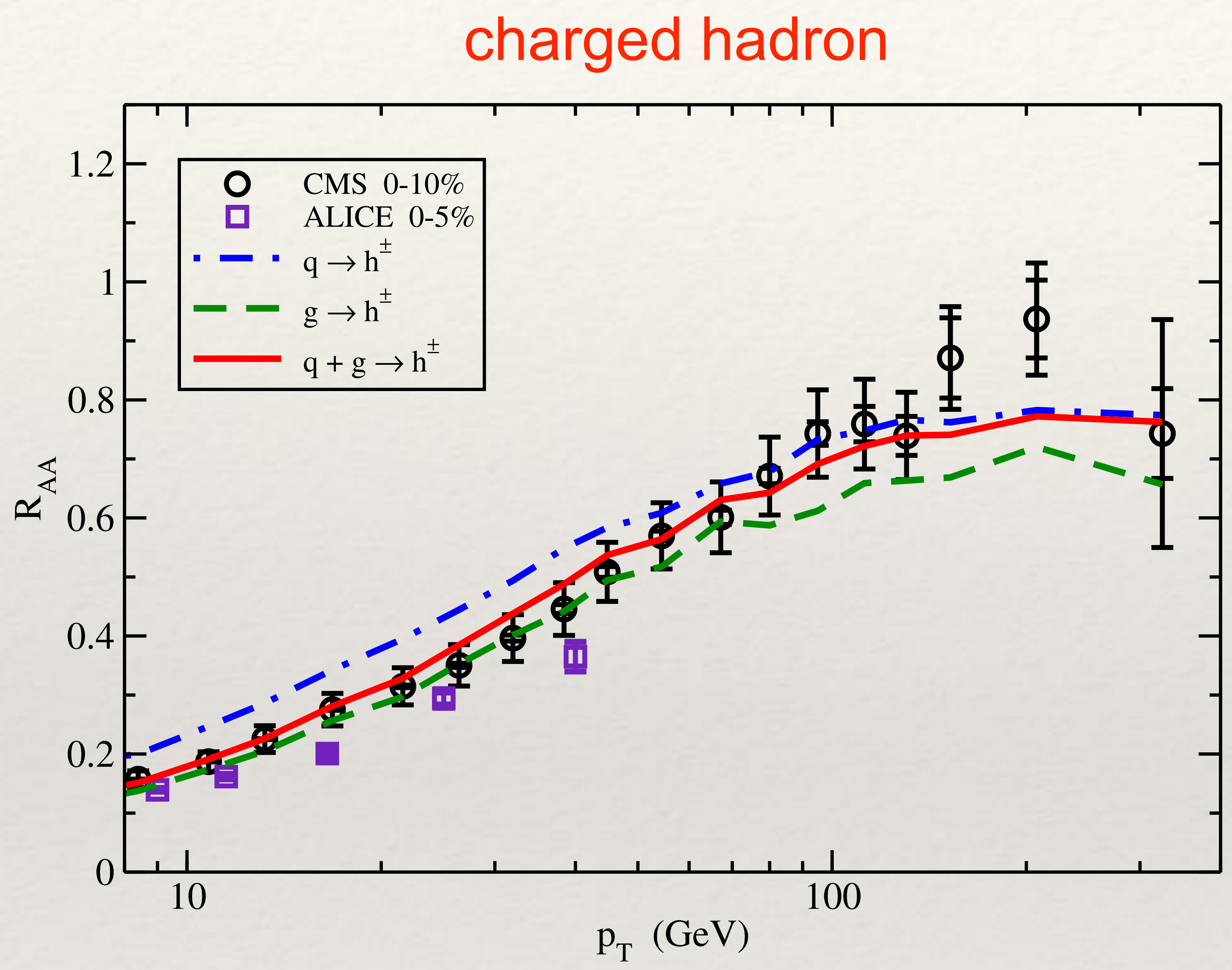


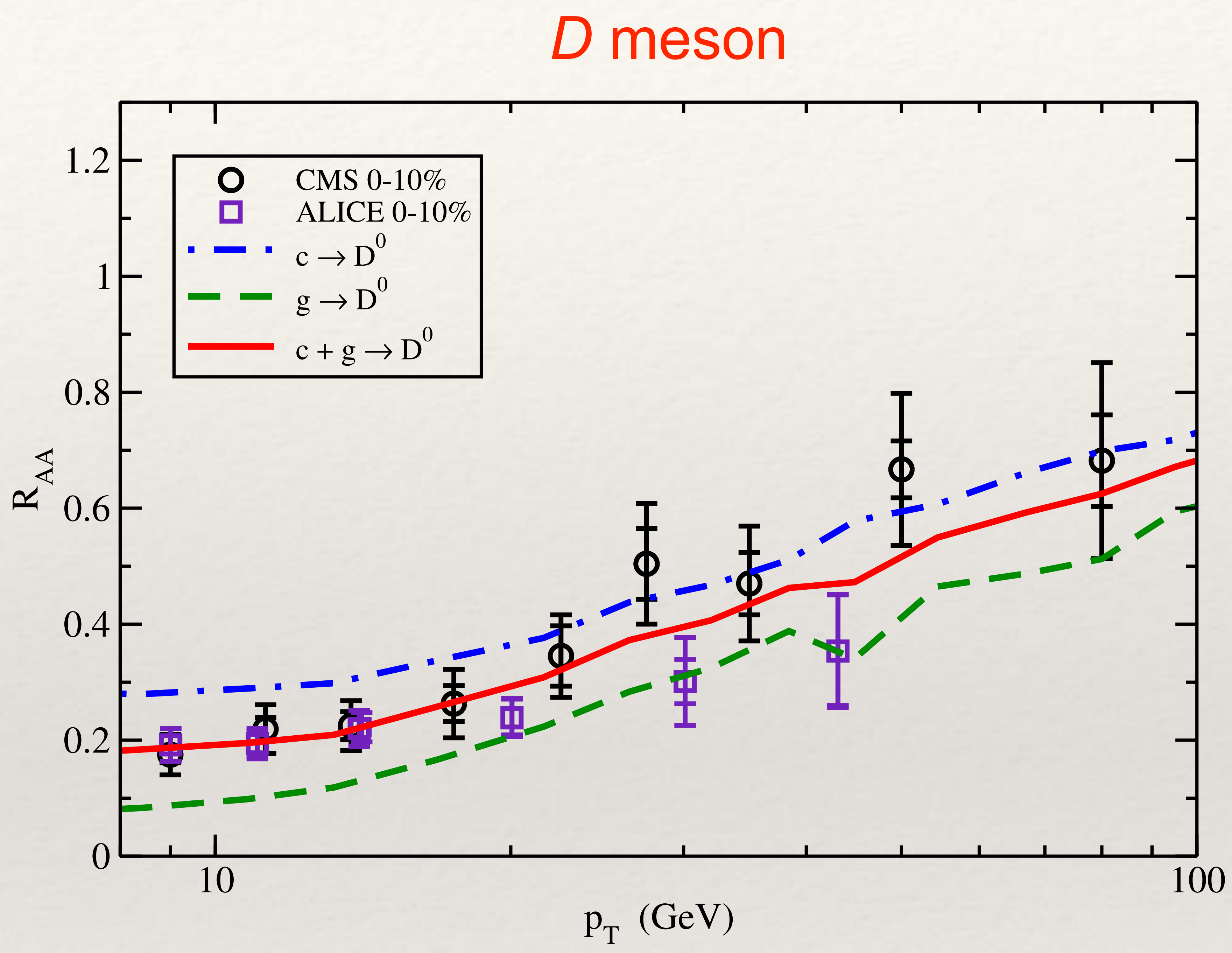


- Collisional energy loss dominates at low energy, radiative dominates at high energy
- Crossing point: 7 GeV for charm quark, 18 GeV for bottom quark
- $\Delta E_b < \Delta E_c < \Delta E_{u/d/s} < \Delta E_g$ holds. Why similar R_{AA} between light hadron, *D* and *B*?

Flavor hierarchy of jet quenching

NLO initial production and fragmentation + Boltzmann transport (elastic and inelastic energy loss) + hydrodynamic medium for QGP

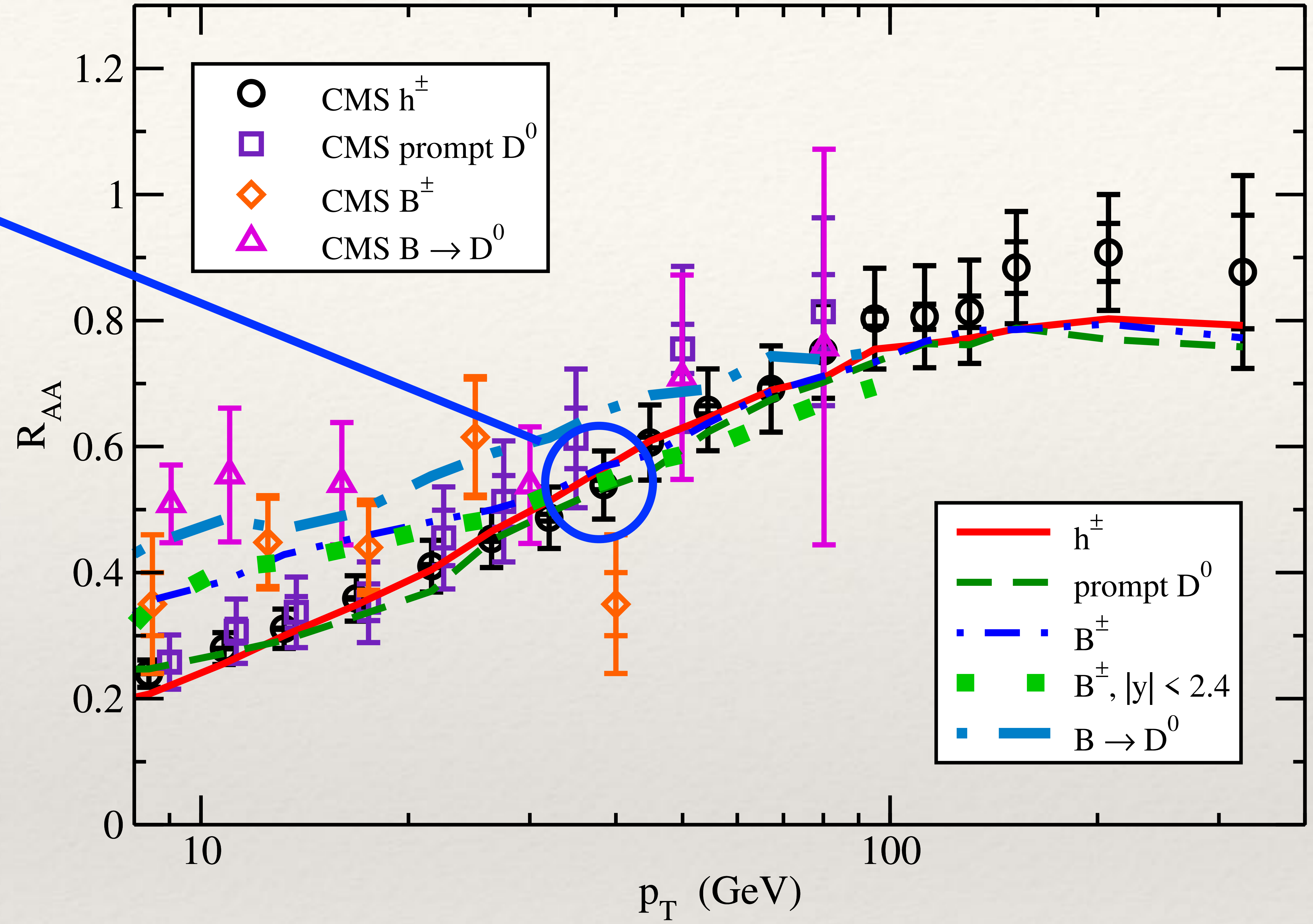




- *g*-initiated *h* & *D* R_{AA} < *q*-initiated *h* & *D* R_{AA} => **$\Delta E_g > \Delta E_q > \Delta E_c$ holds**
- Although R_{AA} (*c*->*D*) > R_{AA} (*q*->*h*), R_{AA} (*g*->*D*) < R_{AA} (*g*->*h*) due to different fragmentation functions => **R_{AA} (*h*) ≈ R_{AA} (*D*)**

Flavor hierarchy of jet quenching

Merging of D and B R_{AA} at $p_T \sim 40$ GeV



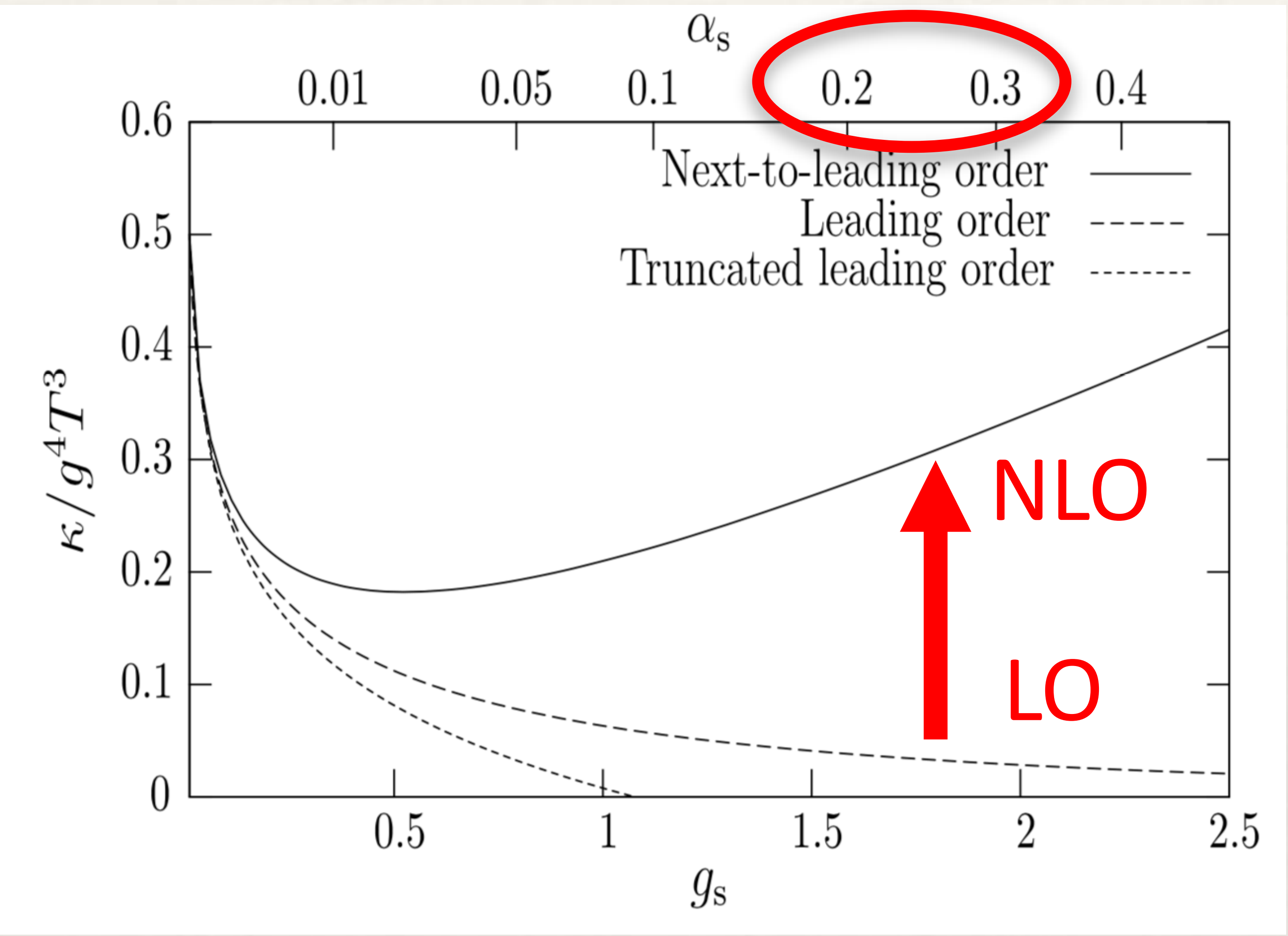
[Xing, Cao, Qin and Xing, Phys. Lett. B 805 (2020) 135424]

- A simultaneous description of charged hadron, D meson, B meson, B-decay D meson R_{AA}'s starting from $p_T \sim 8$ GeV
- Predict R_{AA} separation between B and h / D below 40 GeV, but similar values above – **wait for confirmation from future precision measurement**

Introducing non-perturbative interactions for low p_T heavy quarks

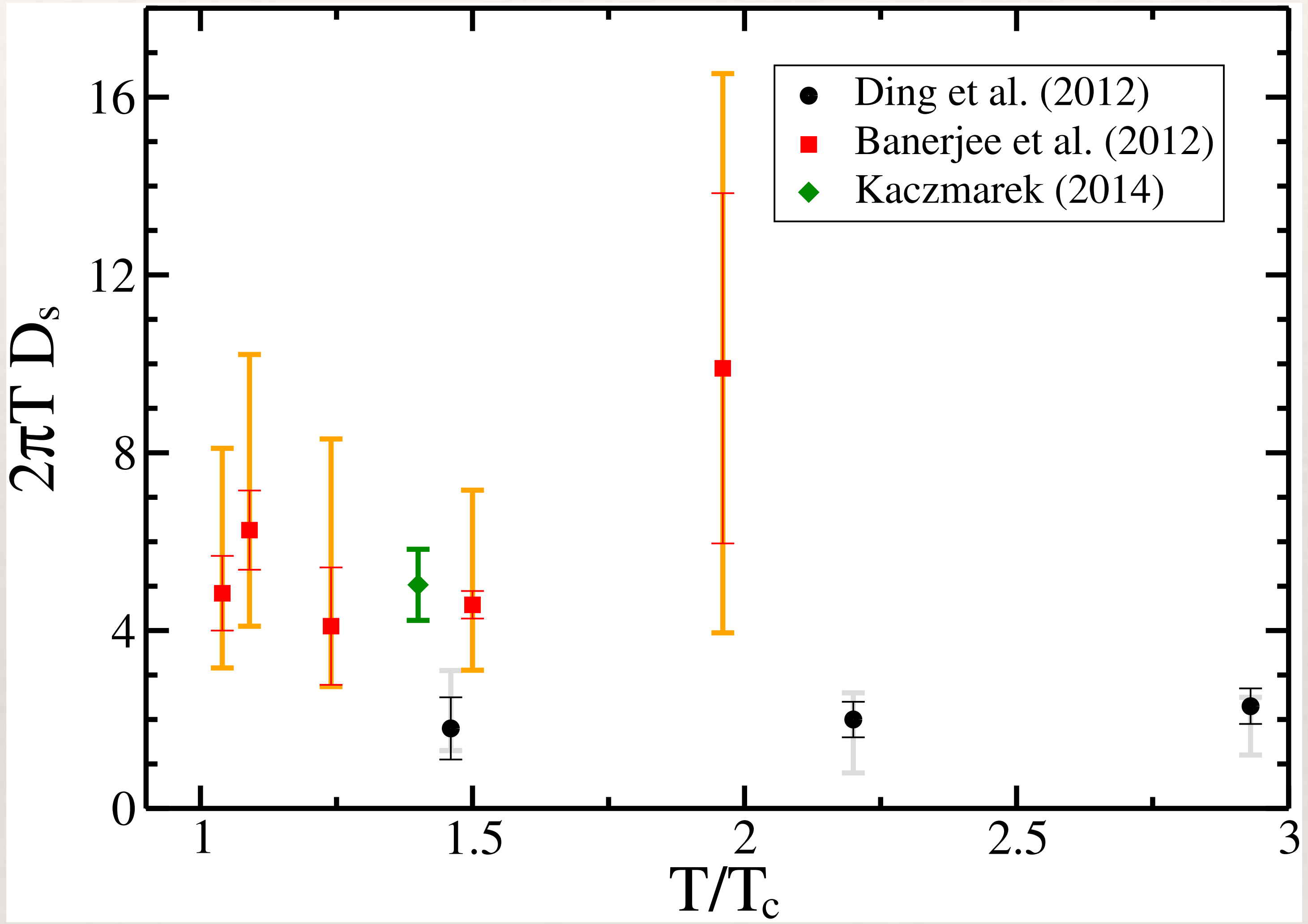
- Suppression of radiative energy loss due to the "dead cone effect"
- Heavy quark diffusion, **diffusion coefficient** κ or D_s as important input into transport models

Perturbation calculation fails at low p_T



- LO: Svetitsky, PRD 37 (1988)
  Moore and Teaney, PRC 71 (2005)
- NLO: Caron-Huot and Moore, JHEP 02 (2008)
- A factor of over 5 increase at NLO

Inputs from lattice calculations



- Uncertainty is still large
- No results for finite momentum HQ yet

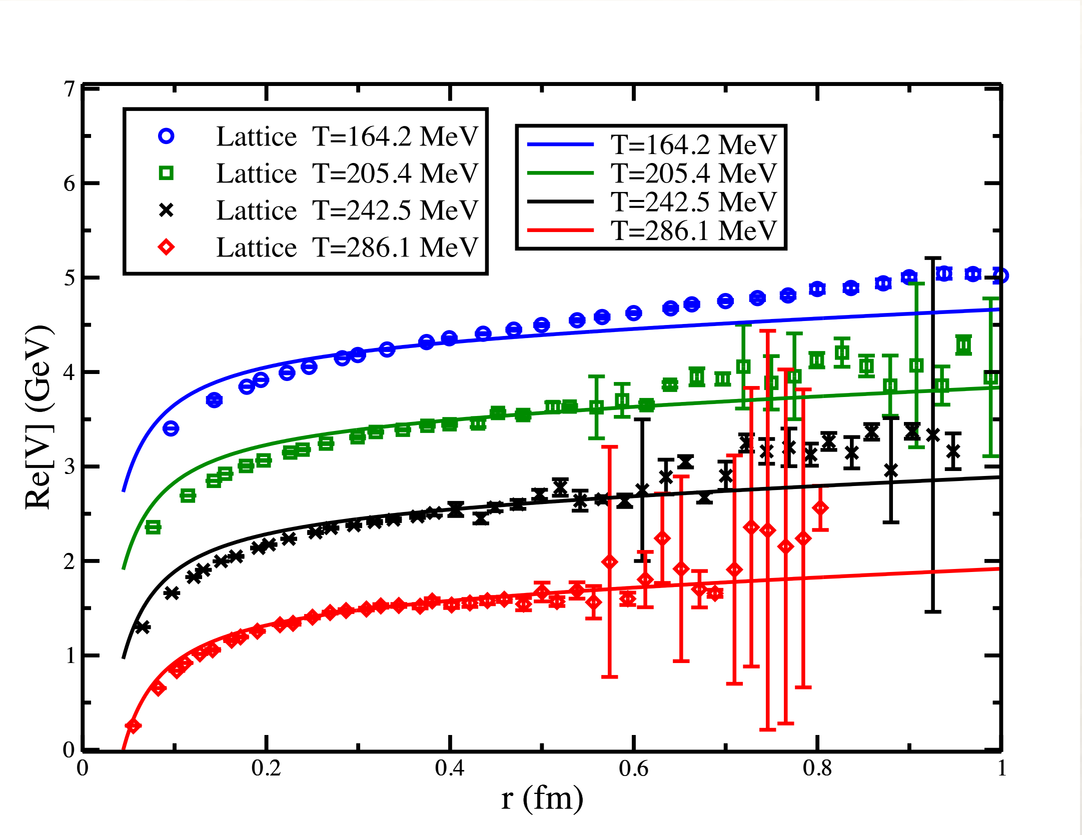
Perturbative calculation with effective propagator approach

- **Parametrization of the heavy-quark-QGP interaction potential:**

Parameters can describe the lattice potential

$$V(r,T) = -\boxed{\frac{4}{3}\alpha_s \frac{e^{-m_d r}}{r}} - \boxed{\frac{\sigma}{m_s} e^{-m_s r}}$$

Yukawa (color coulomb) String



Burnier, Kaczmarek and Rothkopf , Phys. Rev. Lett. 114 (2015) 082001

in which $m_d = a + b * T$ and $m_s = \sqrt{a_s + b_s * T}$ are the respective screening masses, α_s and σ are the respective Yukawa and confining interaction strength.

- **By Fourier transformation,**

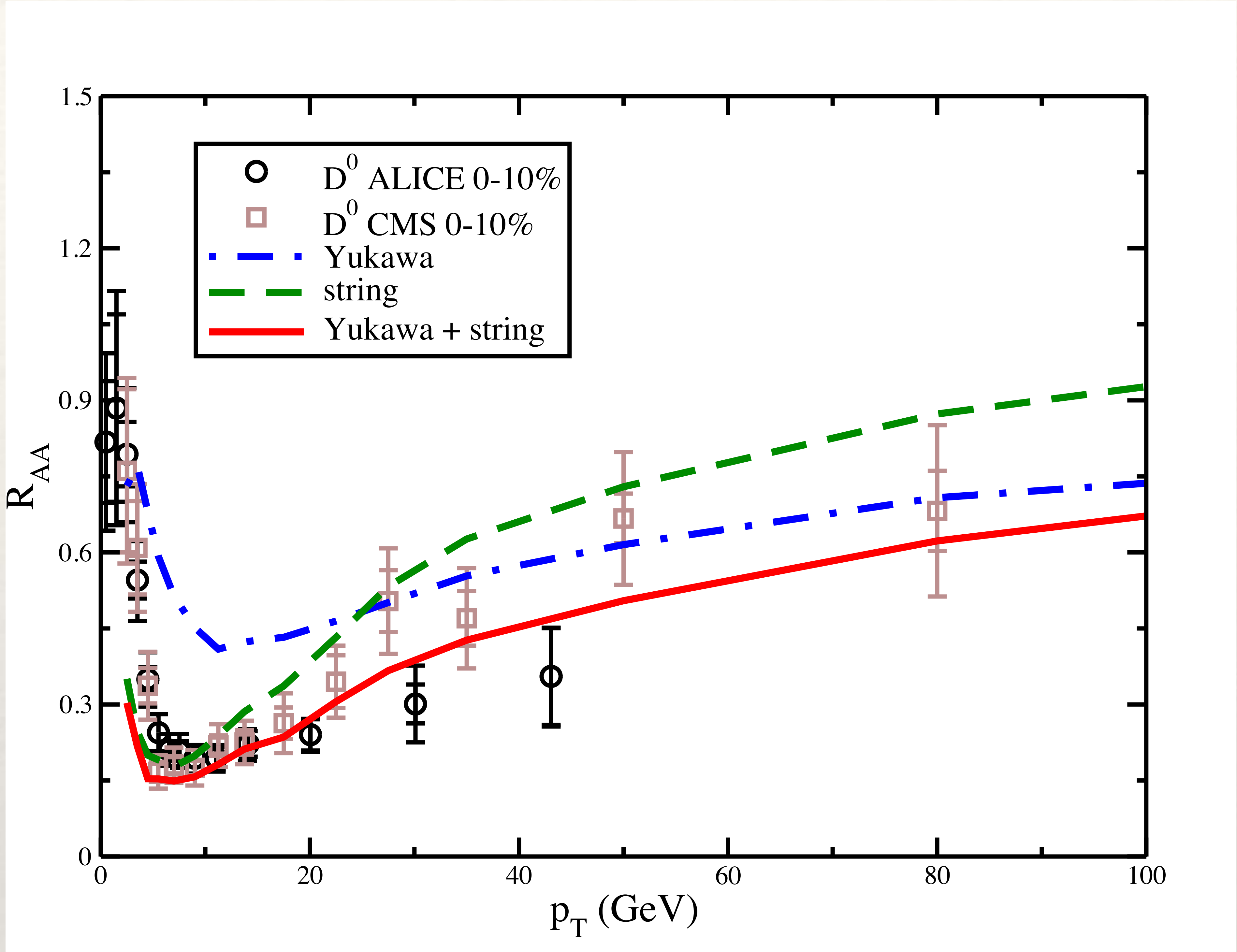
$$V(\vec{q},T) = -\boxed{\frac{4\pi\alpha_s C_F}{m_d^2 + |\vec{q}|^2}} - \boxed{\frac{8\pi\sigma}{\left(m_s^2 + |\vec{q}|^2\right)^2}}$$

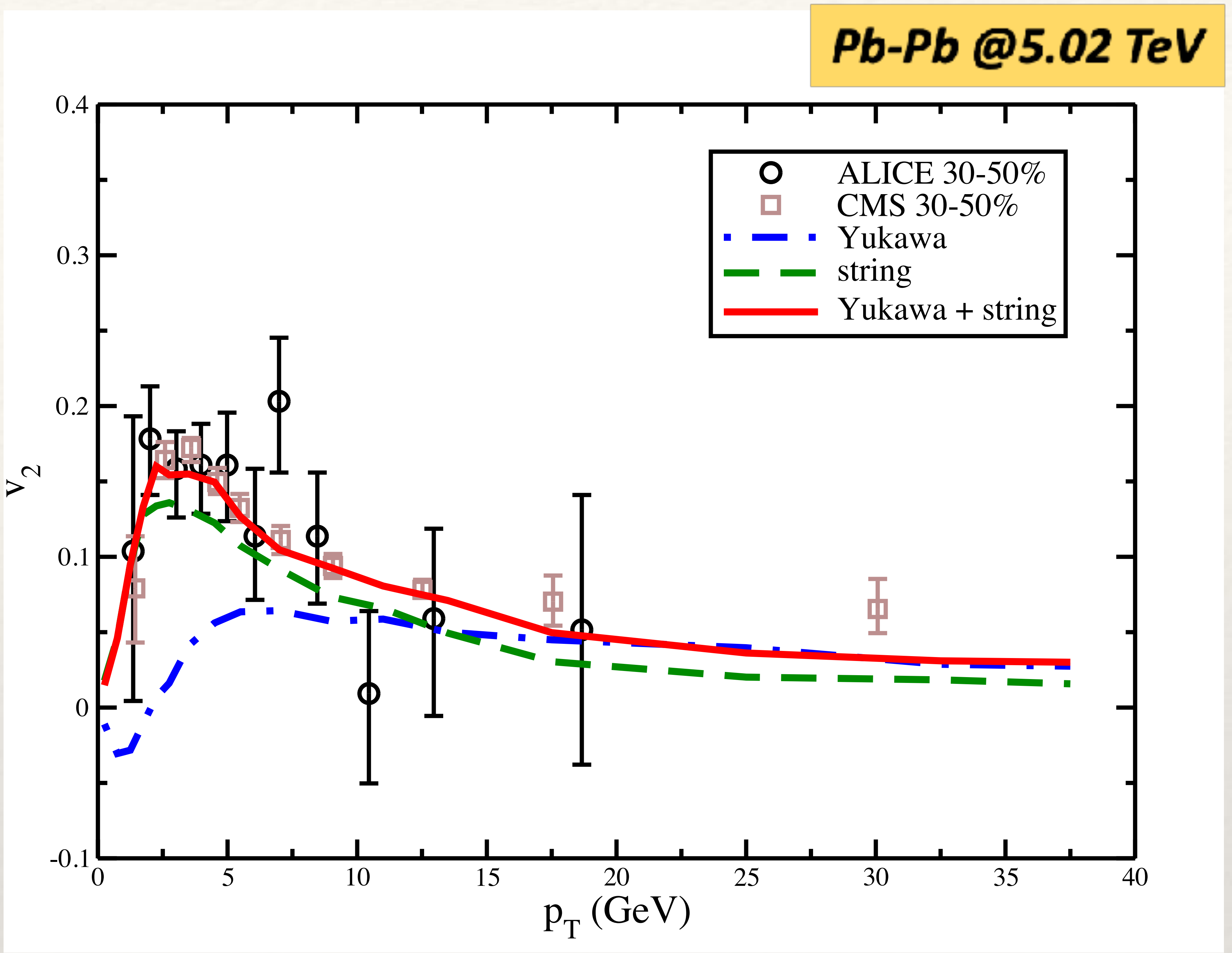
- **For $Qq \longrightarrow Qq$ process, we express the scattering amplitude with effective potential propagator,**

Riek and Rapp, Phys. Rev. C 82 (2010) 035201

$$iM = iM_C + iM_S = \bar{u}\gamma^\mu u V_c \bar{u}\gamma^\nu u + \bar{u}u V_S \bar{u}u$$

R_{AA} and v_2 of D mesons at LHC

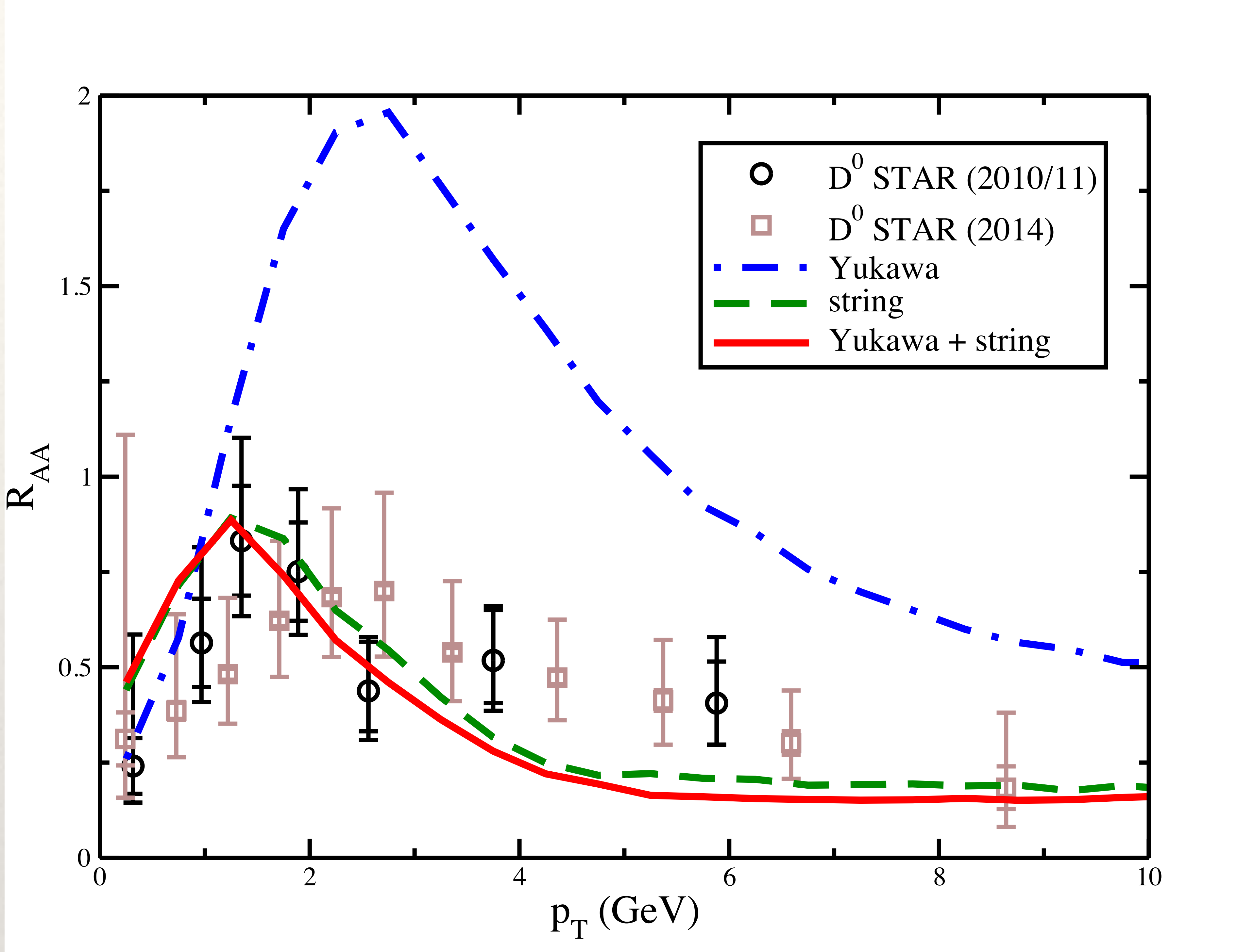


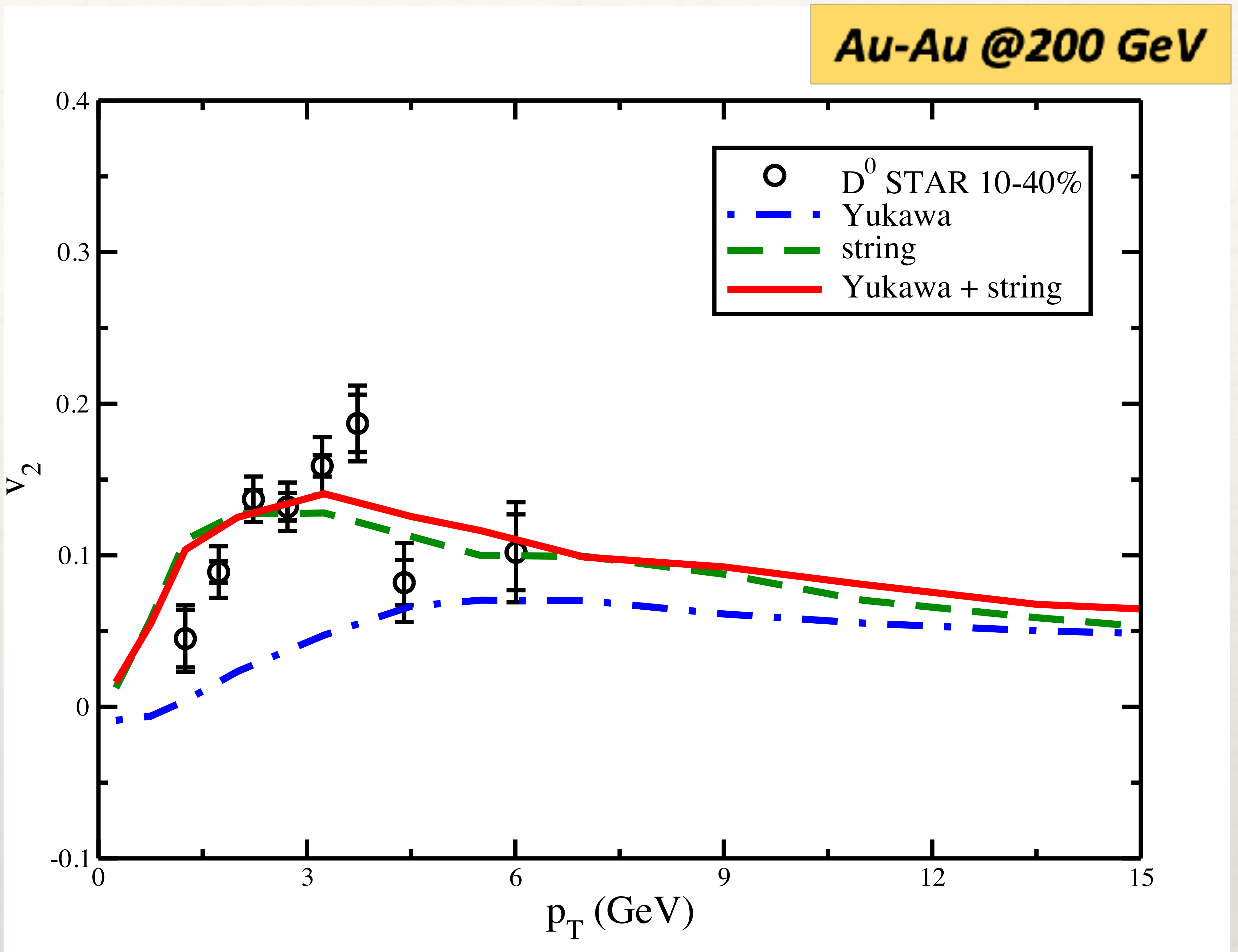


Xing, Qin, Cao, arXiv:2112.15062

- At high p_T, the Yukawa interaction dominates heavy-quark-medium interaction
- At low to intermediate p_T, the string interaction dominates, stronger contribution at later evolution stage (near T_c)

R_{AA} and v_2 of *D* mesons at RHIC



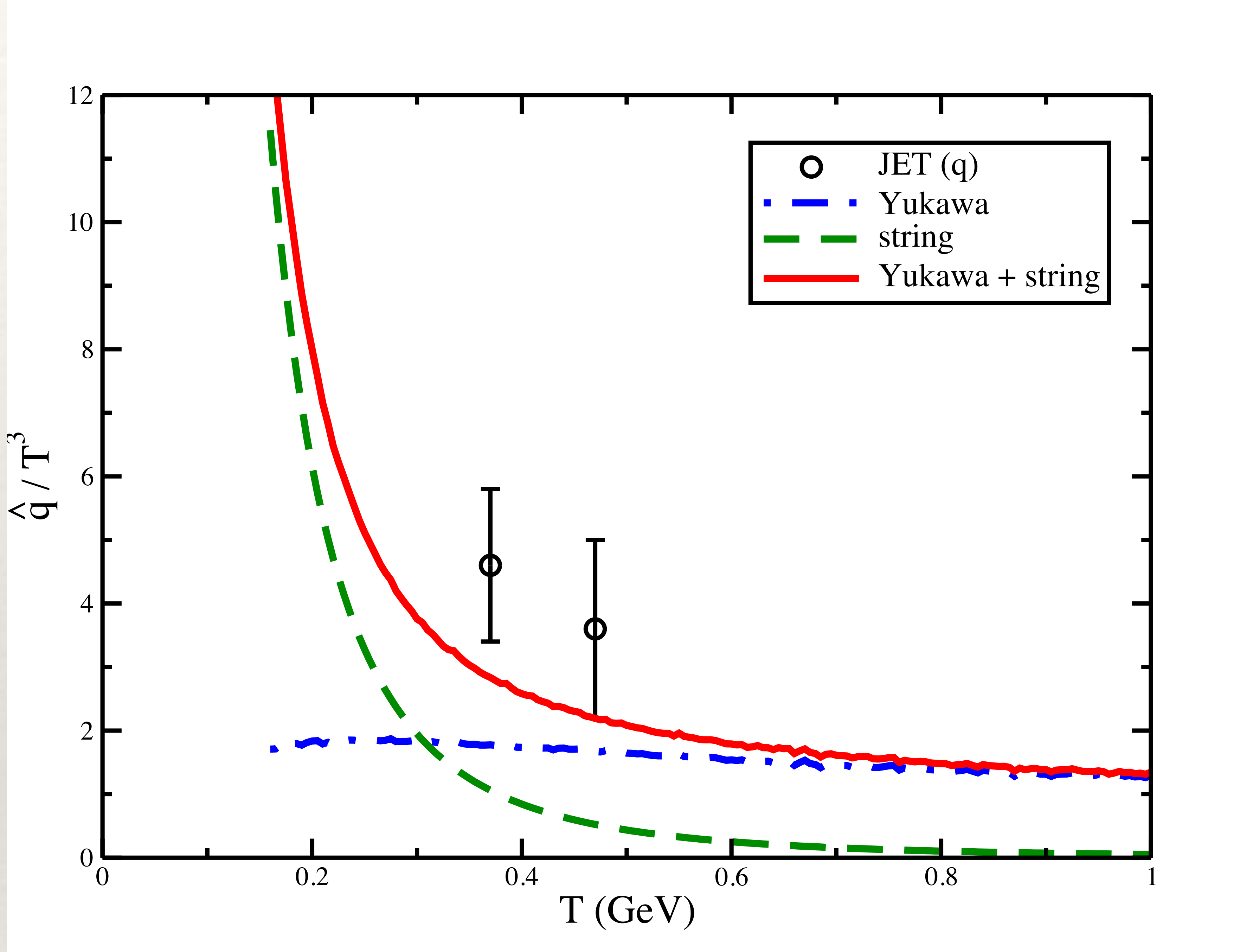


Xing, Qin, Cao, arXiv:2112.15062

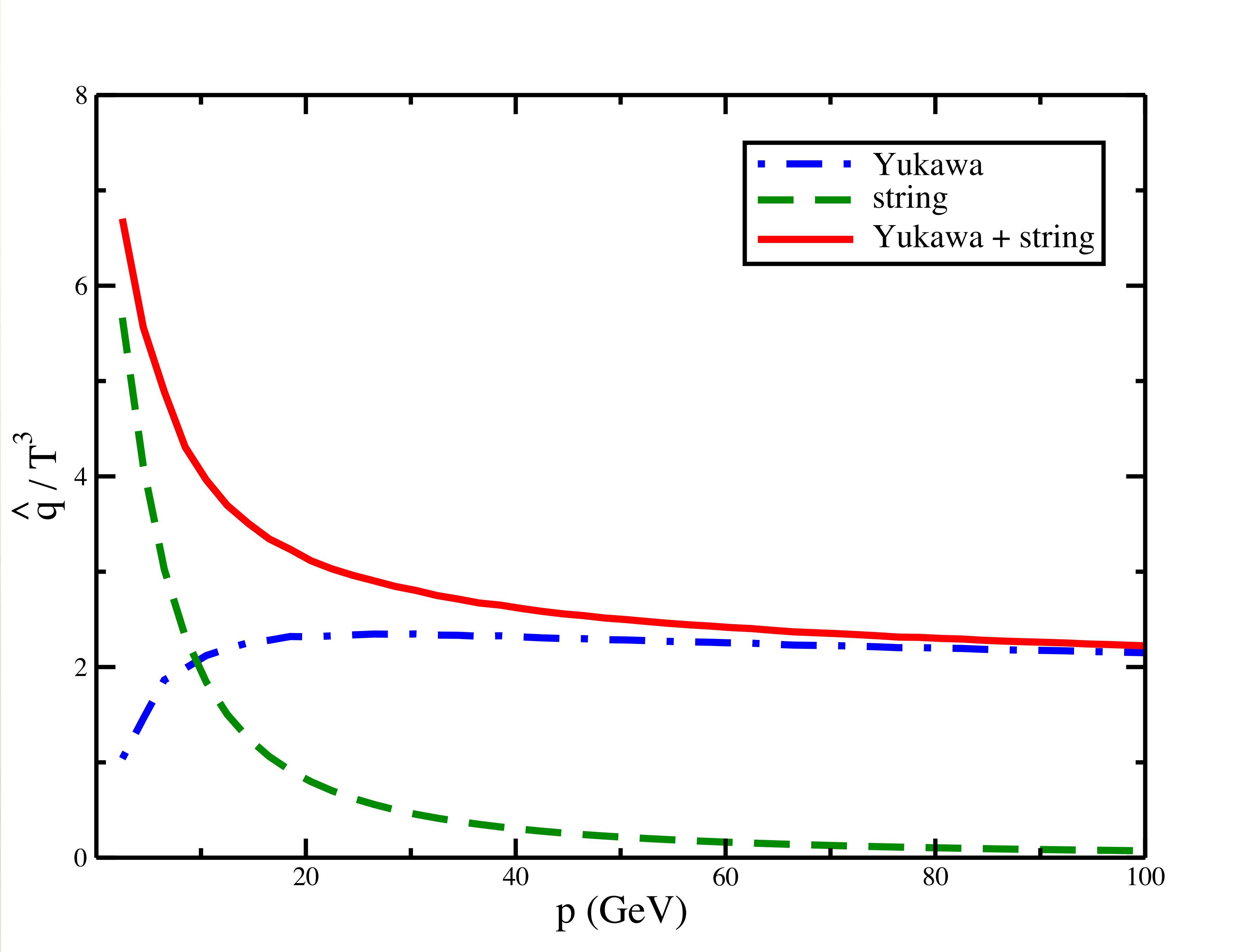
- Effects of string interaction are crucial for the p_T regime studied at RHIC
- Combination of short-range Yukawa and long-range string interactions provide a reasonable description of the *D* meson R_{AA} and v_2

Transport coefficients — $\hat{q}$

Temperature dependence



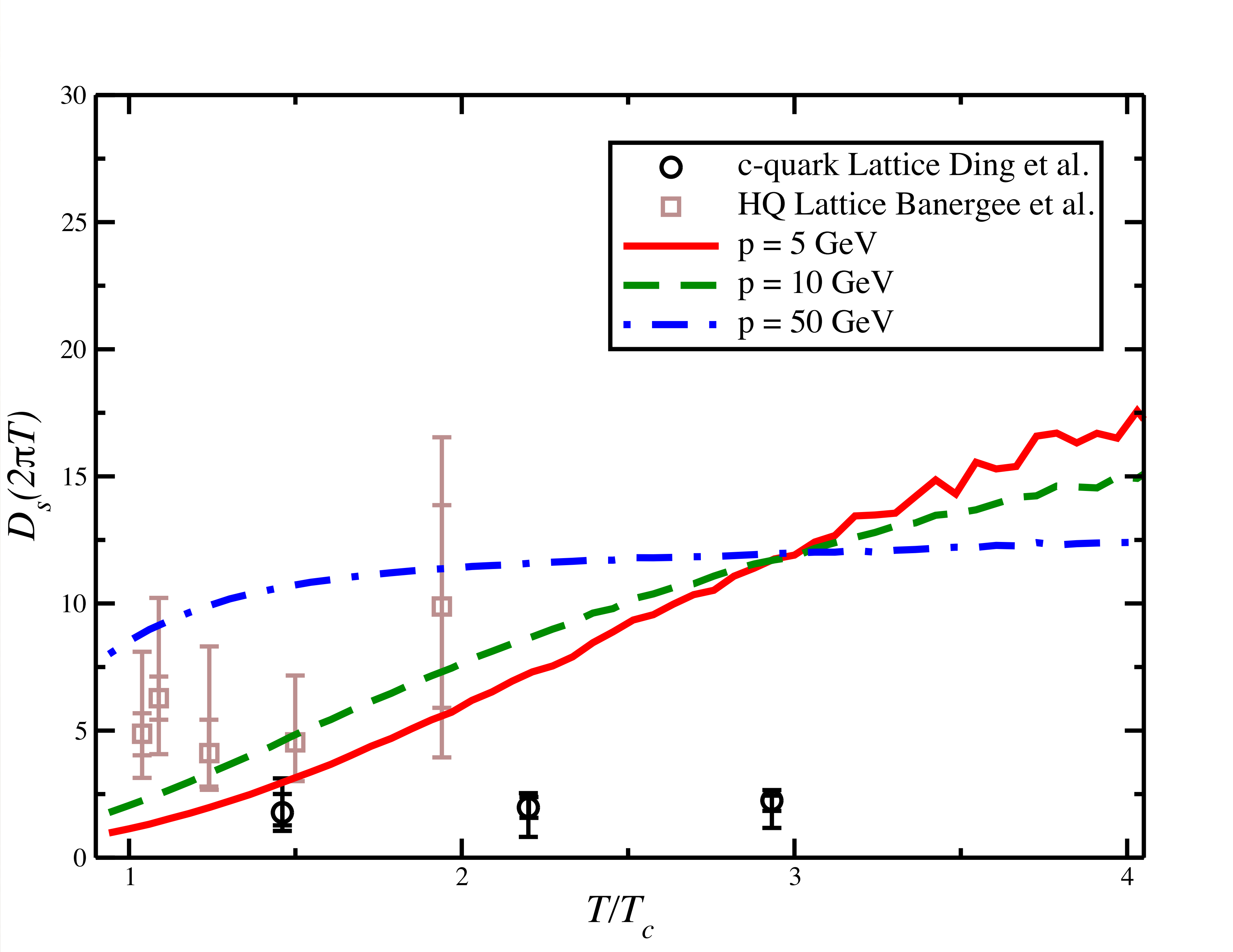
Momentum dependence



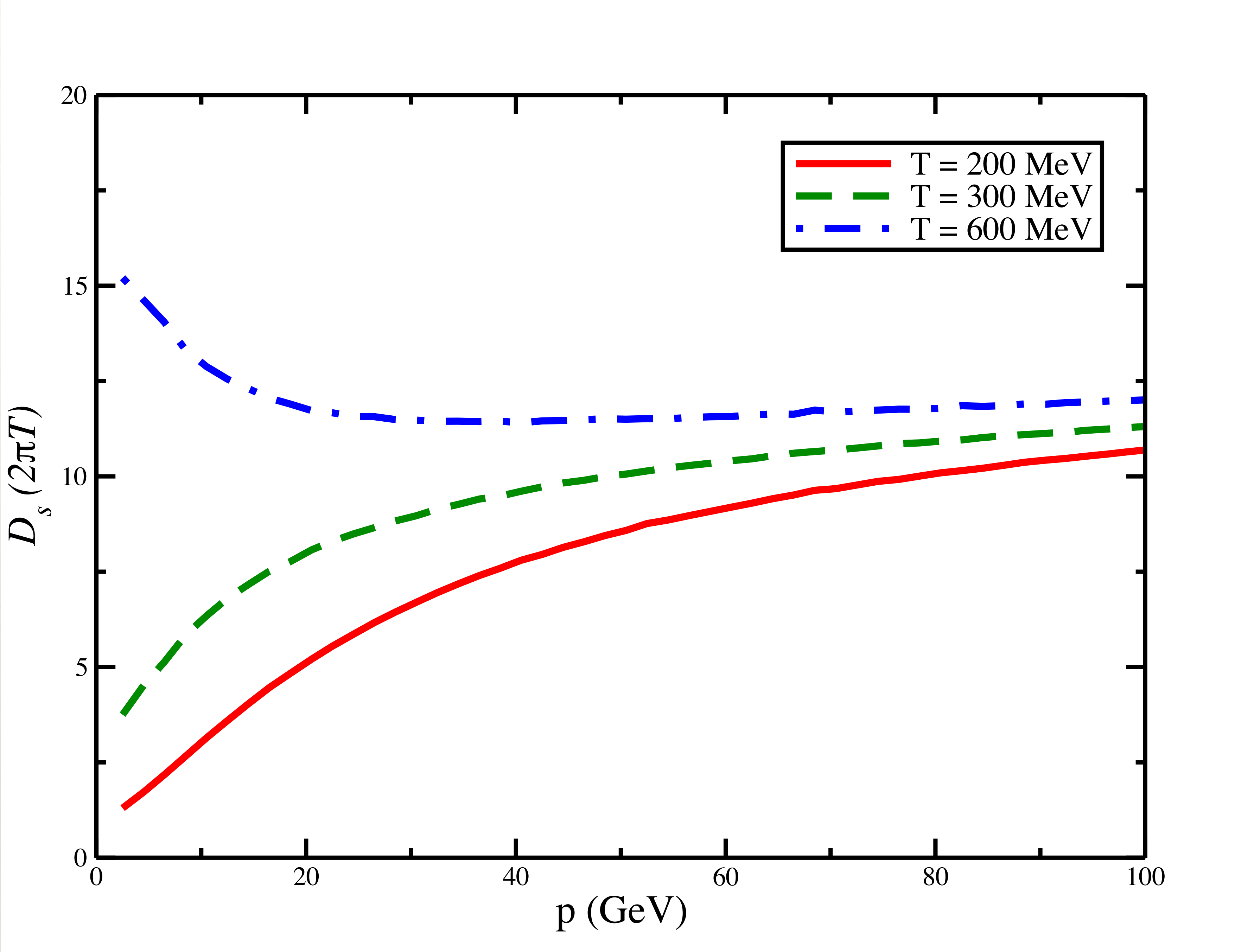
- Yukawa interaction dominates at high temperature and high momentum
- String interactions dominates at low temperature and low momentum

Transport coefficients — D_s

Temperature dependence



Momentum dependence



- Stronger temperature dependence at lower momentum
- Different momentum dependence at different temperature

From Boltzmann to Fokker-Plank equation

Boltzmann equation:

$$\frac{d}{dt} f_Q(t,\vec{x},\vec{p}) = \left[\frac{\partial}{\partial t} + \frac{p_i}{E_{\vec{p}}}\frac{\partial}{\partial x_i} + F_i \frac{\partial}{\partial p_i}\right] f_Q(t,\vec{x},\vec{p}) = C\,[f_Q]$$

Collision integral:

$$C\,[f_Q] \equiv \int d^3k \left[w(\vec{p}+\vec{k},\vec{k}) f_Q(\vec{p}+\vec{k}) - w(\vec{p},\vec{k}) f_Q(\vec{p})\right]$$

Small *k* approximation:

$$w(\vec{p}+\vec{k},\vec{k}) f_Q(\vec{p}+\vec{k}) \approx w(\vec{p},\vec{k}) f_Q(\vec{p}) \qquad |\vec{k}| \ll |\vec{p}|$$

$$+ k_i \frac{\partial}{\partial p_i}\left[w(\vec{p},\vec{k}) f_Q(\vec{p})\right] + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}\left[w(\vec{p},\vec{k}) f_Q(\vec{p})\right].$$

$$\underline{A_i(\vec{p})} = \int d^3k\, w(\vec{p},\vec{k}) k_i,$$

$$\underline{B_{ij}(\vec{p})} = \frac{1}{2}\int d^3k\, w(\vec{p},\vec{k}) k_i k_j.$$

$$C\,[f_Q] \approx \int d^3k \left(k_i \underline{\frac{\partial}{\partial p_i}} + \underline{\frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j}}\right) w(\vec{p},\vec{k}) f_Q(\vec{p})$$

Fokker-Plank equation:

$$\frac{\partial}{\partial t} f_Q(t,\vec{p}) = \frac{\partial}{\partial p_i}\left\{\underline{A_i(\vec{p})} f_Q(t,\vec{p}) + \frac{\partial}{\partial p_j}\left[\underline{B_{ij}(\vec{p})} f_Q(t,\vec{p})\right]\right\}$$

Physical meanings of *A* and *B*

$$A_i(\vec{p}) = \int d^3k w(\vec{p}, \vec{k}) k_i,$$

$$B_{ij}(\vec{p}) = \frac{1}{2} \int d^3k w(\vec{p}, \vec{k}) k_i k_j.$$

Recall: $\Gamma_{12\to 34}(\vec{p}_1) = \int d^3k w_{12\to 34}(\vec{p}_1, \vec{k}).$

Scattering rate: number per unit time

Define: $\langle X(\vec{p}) \rangle = \int d^3k w(\vec{p}, \vec{k}) X(\vec{p})$, average change of *X* per unit time

$$A_i(\vec{p}) = \langle (p - p')_i \rangle \,,$$

$$B_{ij}(\vec{p}) = \frac{1}{2} \langle (p - p')_i (p - p')_j \rangle$$

k_i is substituted by $(p - p')_i$.

Decomposition:

$$A_i(\vec{p}) = A(\vec{p}) p_i,$$

$$B_{ij}(\vec{p}) = B_0(\vec{p}) P^{\parallel}_{ij}(\vec{p}) + B_1(\vec{p}) P^{\perp}_{ij}(\vec{p}),$$

$$P^{\parallel}_{ij}(\vec{p}) \equiv \frac{p_i p_j}{\vec{p}^2}, \qquad P^{\perp}_{ij}(\vec{p}) \equiv \delta_{ij} - \frac{p_i p_j}{\vec{p}^2}.$$

Physical meanings of *A* and *B*

$$A_i(\vec{p}) = A(\vec{p})p_i,$$

$$B_{ij}(\vec{p}) = B_0(\vec{p})P^{\parallel}_{ij}(\vec{p}) + B_1(\vec{p})P^{\perp}_{ij}(\vec{p}),$$

$$A(\vec{p}) = p_i A(\vec{p})^i/\vec{p}^2 = \left\langle 1 - \frac{\vec{p}\cdot\vec{p}'}{\vec{p}^2} \right\rangle,$$ Longitudinal drag

$$B_0(\vec{p}) = P^{\parallel}_{ij}B^{ij} = \frac{1}{2}\left\langle \frac{(\vec{p}\cdot\vec{p}')^2}{\vec{p}^2} - 2\vec{p}\cdot\vec{p}' + \vec{p}^2 \right\rangle,$$ Longitudinal diffusion

$$B_1(\vec{p}) = \frac{1}{2}P^{\perp}_{ij}B^{ij} = \frac{1}{4}\left\langle \vec{p}'^2 - \frac{(\vec{p}\cdot\vec{p}')^2}{\vec{p}^2} \right\rangle,$$ Transverse diffusion

Fokker-Plank:

$$\frac{\partial}{\partial t}f_Q(t,\vec{p}) = \frac{\partial}{\partial p_i}\left\{ A_i(\vec{p})f_Q(t,\vec{p}) + \frac{\partial}{\partial p_j}\left[B_{ij}(\vec{p})f_Q(t,\vec{p})\right]\right\}$$

Stochastic realization of Fokker-Plank: Langevin equation

Langevin equation

$$dx_i = \frac{p_i}{E_{\vec{p}}} dt,$$

$$dp_i = -\Gamma p_i dt + \sqrt{dt} C_{ik} \rho^k,$$

$\overrightarrow{\rho}$: Gaussian-normal distributed random (stochastic) variable

$$P(\vec{\rho}) = \left(\frac{1}{2\pi}\right)^3 \exp\left(-\frac{\vec{\rho}^2}{2}\right)$$

Connection to Fokker-Plank:

$$\Gamma(\vec{p}) = A(\vec{p}) + \frac{1}{p_i} \xi C_j^k(\vec{p}) \frac{\partial C_{ik}(\vec{p})}{\partial p_j},$$

$$C_{ik}(\vec{p}) = \sqrt{2B_0} P_{ik}^{||} + \sqrt{2B_1} P_{ik}^{\perp}.$$

Summary of different transport equations:

Boltzmann equation (in the limit of small momentum transfer $|\overrightarrow{k}| \ll |\overrightarrow{p}|$) $\rightarrow$

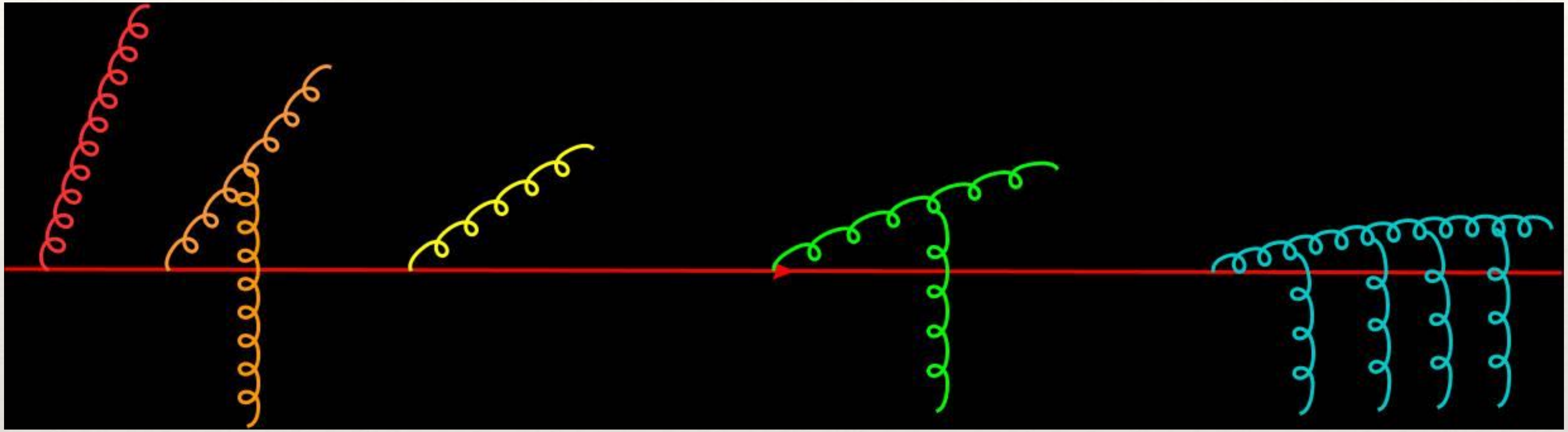
Fokker-Plank equation (in the limit of multiple scatterings) $\rightarrow$

Langevin equation, e.g. heavy quarks inside QGP

Langevin may be used in complex systems where Boltzmann is not applicable (e.g. water)

From single to multi-stage jet evolution

High Q^2 $\rightarrow$ Low Q^2, require a framework that combines different physics of medium modification



- In high virtuality phase (DGLAP/radiation phase): $\hat{q}\tau \ll l_\perp^2 \sim \mu^2$, so rare scatterings
- Scale continues to drop as scattering is rare
- In the low virtuality phase (BDMPS/transport phase) $\hat{q}\tau \sim l_\perp^2 \sim \mu^2$
- Each scattering is equally important and sum of scatterings much larger than vacuum term

Examples of multi-stage approach

Example 1: DGLAP + transport evolution [Cao, Majumder, Qin and Shen, PLB 793 (2019)]

Scale 1 ($Q \gg M_{\mathrm{HM}}$): HQ fragmentation function (FF) is treated with the DGLAP equation

- Input 1: medium-modified splitting function (higher-twist)

$$P(y, Q^2) = P_{\mathrm{vac}}(y) + P_{\mathrm{med}}(y, Q^2)$$

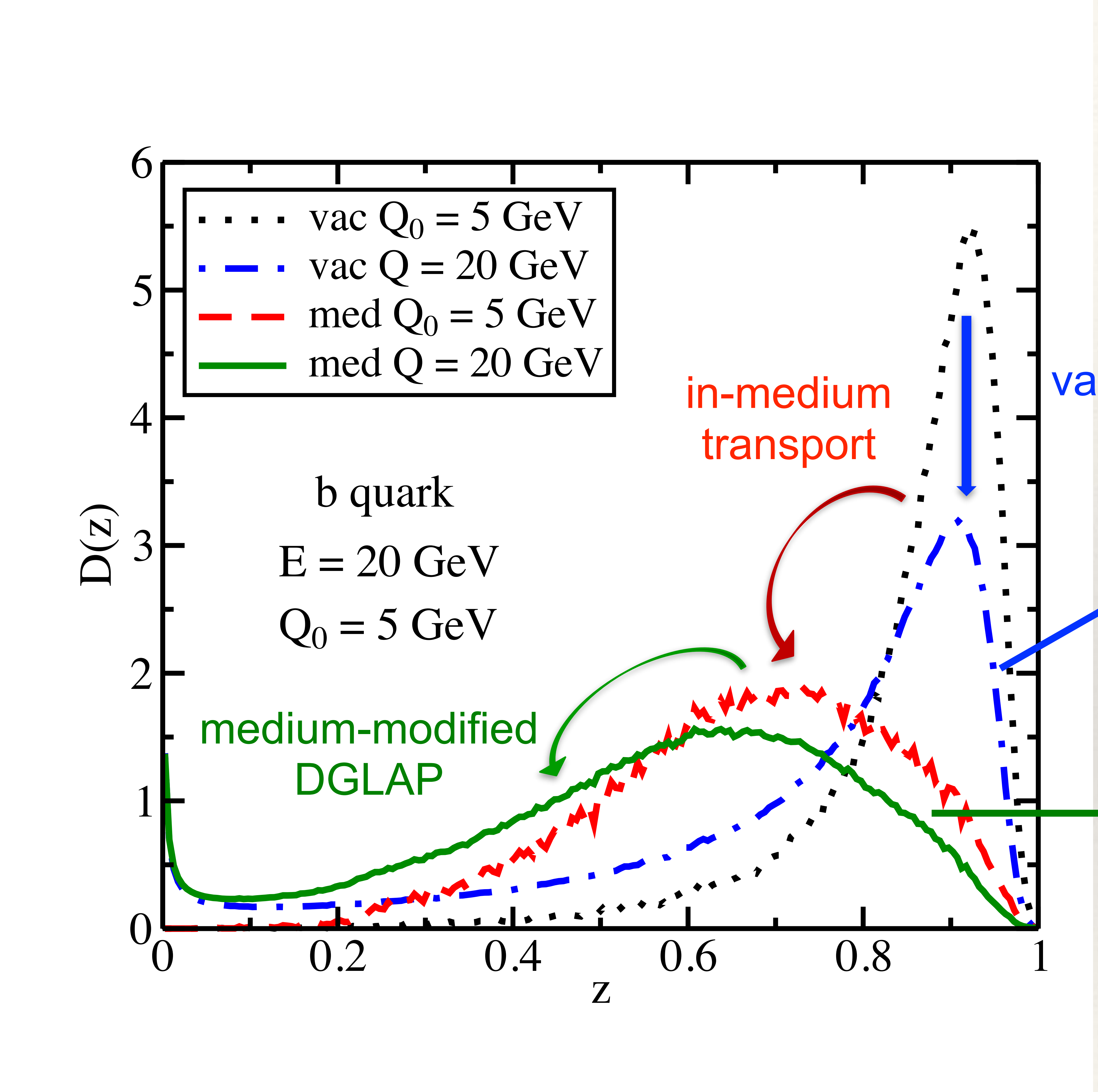
- Input 2: FF at a low scale $D(z, E, Q_0^2)$

Extracted from transport model (in scale 2) — medium modified FF at $Q_0 \sim M_{\mathrm{HM}}$

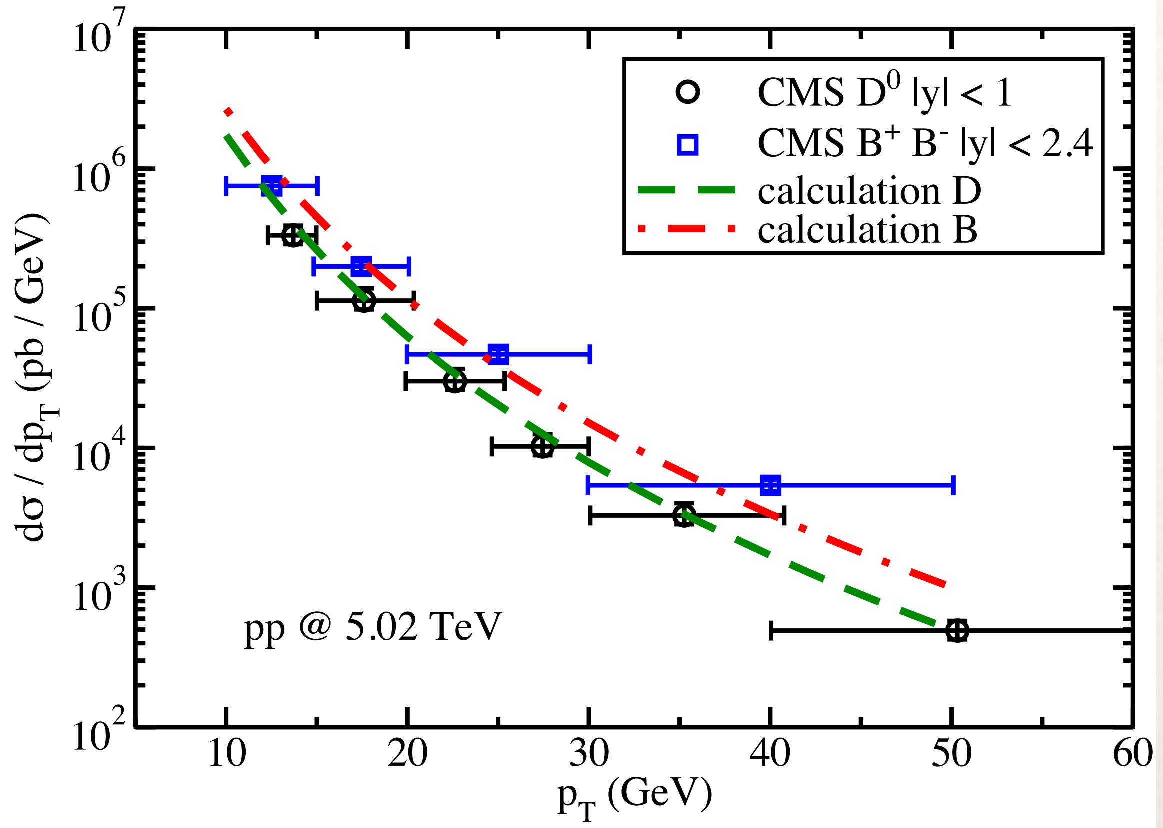
Scale 2 ($Q \sim M_{\mathrm{HM}}$): Transport model with the rate equation (elastic + inelastic)

$$\Gamma^{\mathrm{inel}}(t) = \int dy \int dl_\perp^2 \frac{dN}{dy dl_\perp^2 dt}$$

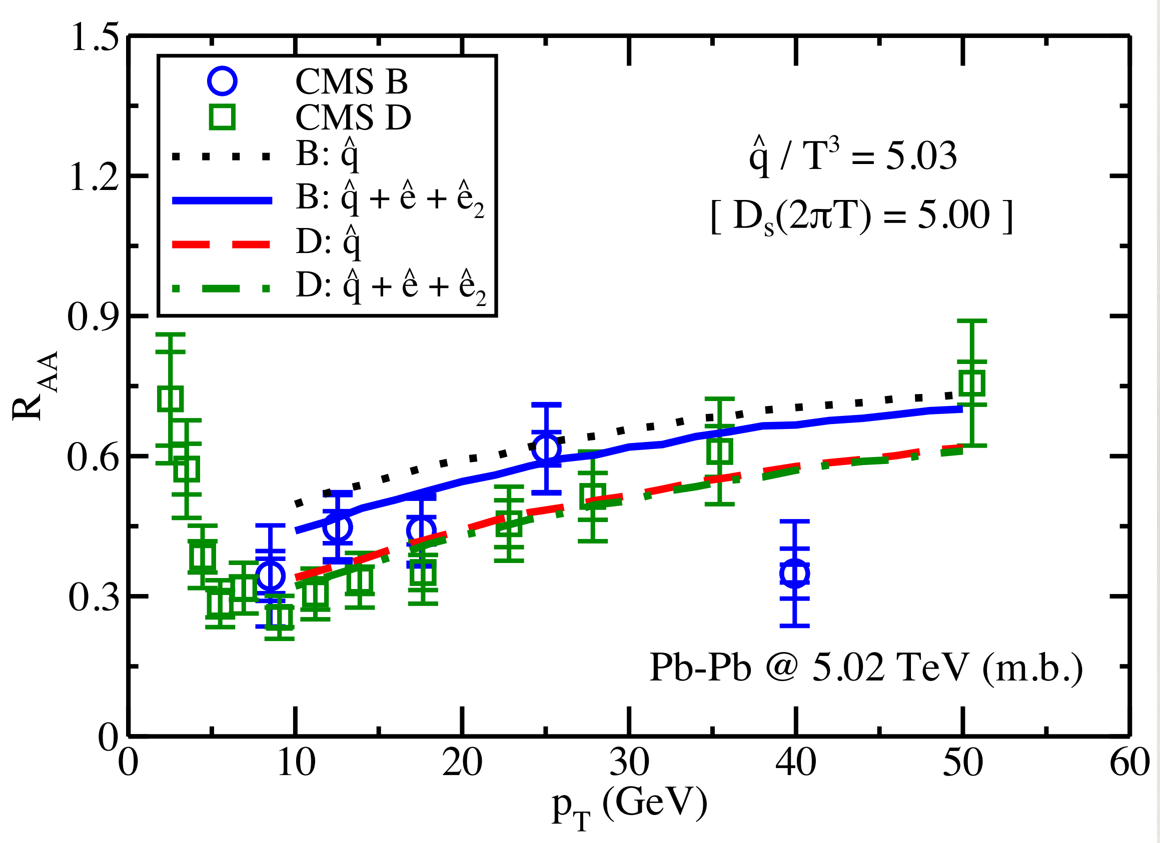
Multi-scale evolution of the *b*-quark fragmentation function



spectra in p-p



spectra in A-A, thus R_{AA}

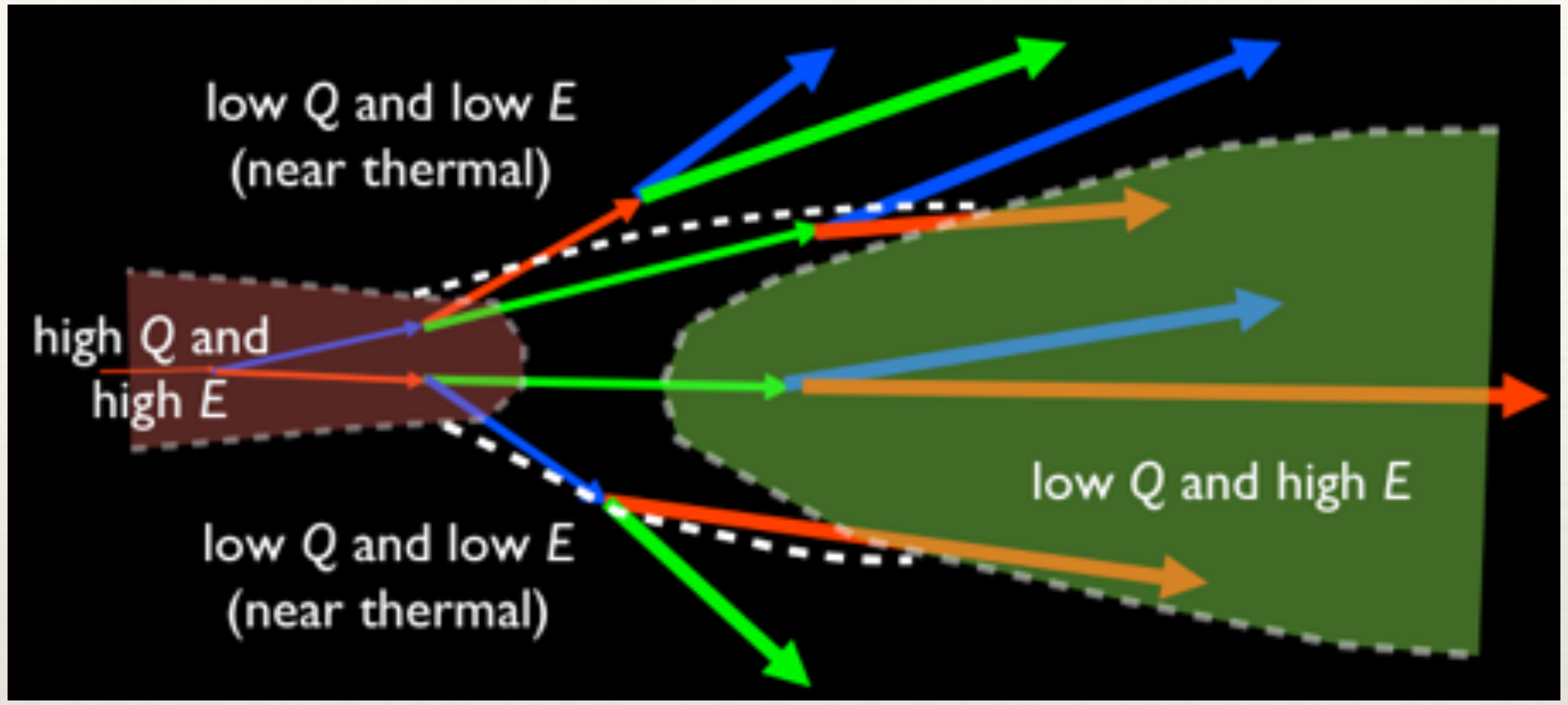


Semi-analytical calculation [Cao et. al., PLB 793 (2019)]

→ full Monte-Carlo simulation — JETSCAPE

Examples of multi-stage approach

Example 2: Full Monte-Carlo simulation — JETSCAPE [https://jetscape.org]

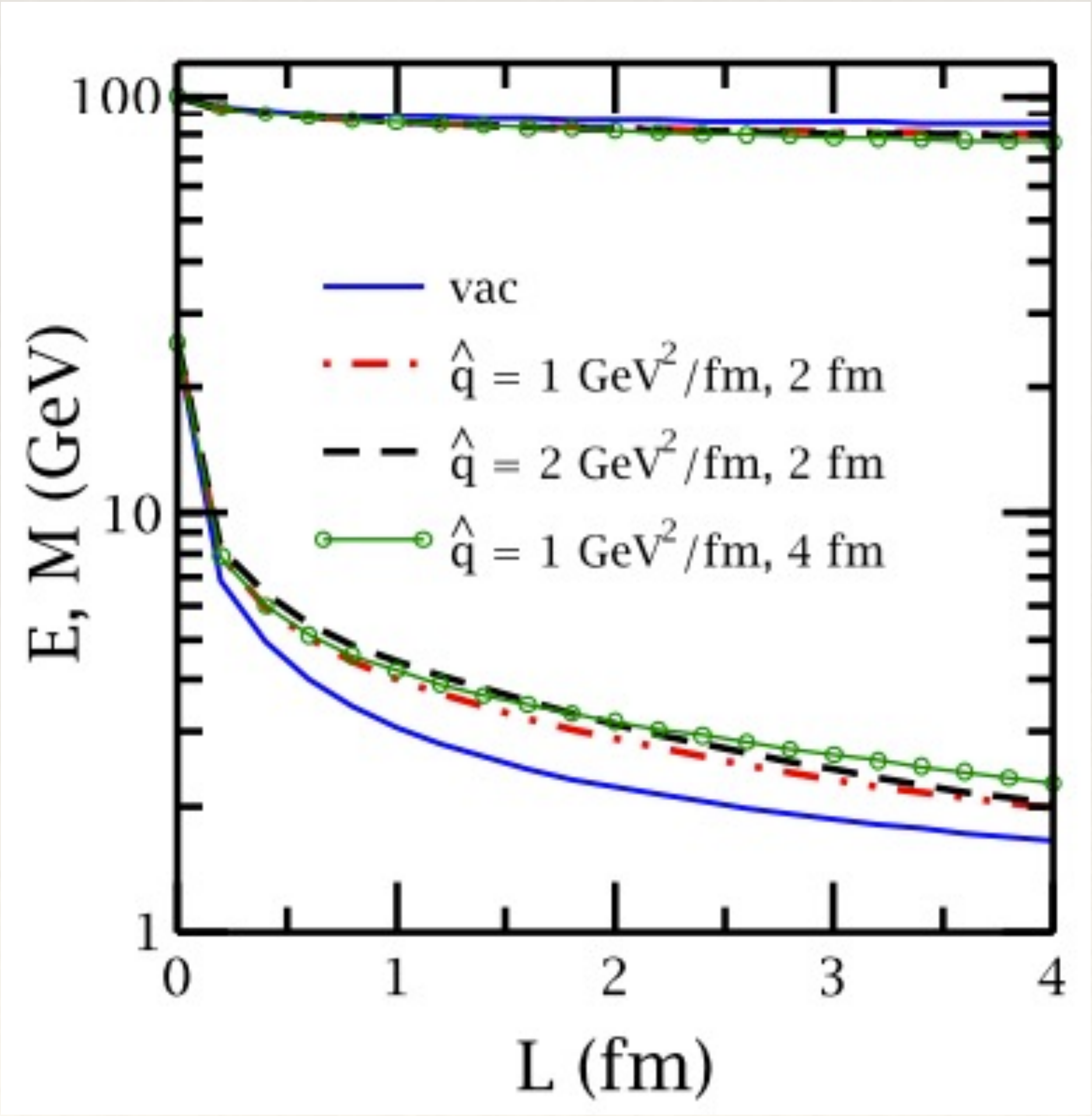


Stage 1: High Q and high E — medium-modified shower (MATTER);

lose Q faster than E [Majumder and Putschke, PRC 93 (2016)]

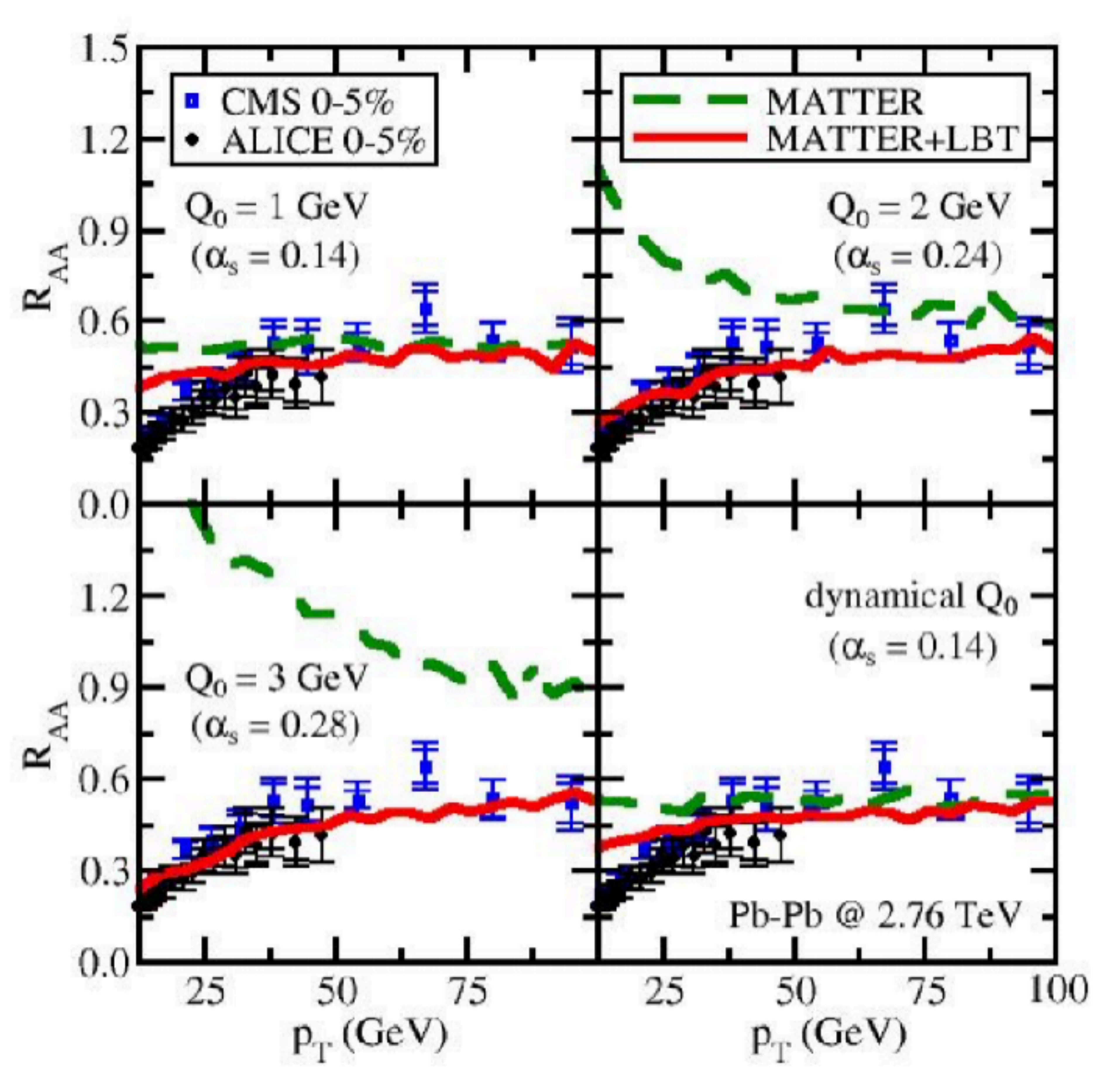
Stage 2: low Q and high E — transport (LBT or MARTINI)

Stage 3: low Q and low E — energy loss (strongly coupled approach)



Effects of different stages on the hadron R_{AA}

Dependence on Q_0 (central Pb-Pb collisions)



- Partons with larger E hit Q_0 later → MATTER dominates high p_T, LBT low p_T
- Larger Q_0 leads to shorter MATTER evolution → LBT contribution is larger
- Setup of dynamical Q_0:

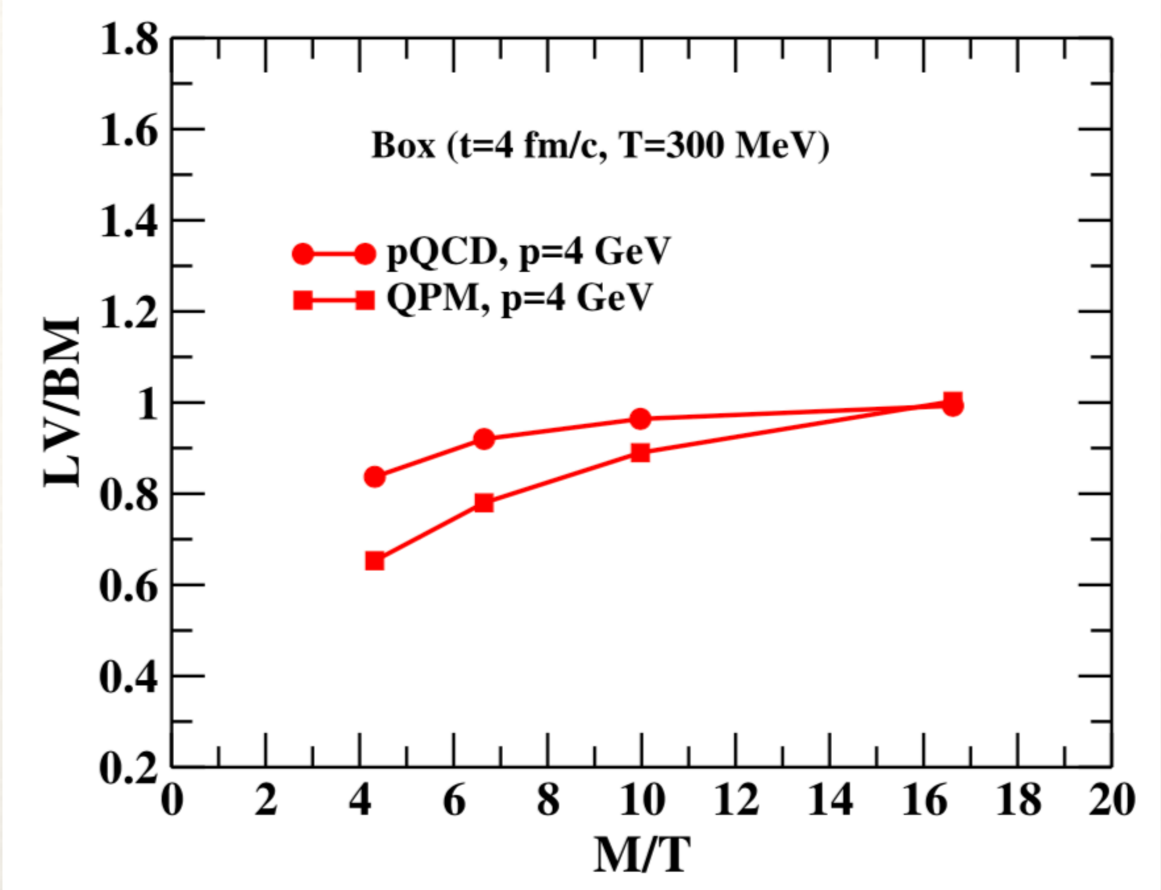
$$Q_0^2 = \hat{q}\tau_f \,, \quad \tau_f = 2E/Q_0^2$$

$$\rightarrow \; Q_0^2 = \sqrt{2E\hat{q}}$$

Examples of multi-stage approach

Example 3: Boltzmann (BM) + Langevin (LV) transport

- BM: scattering between **quasi-particles**
- BM + small momentum transfer (k) => LV
- LV deviates from BM when $k \ll p$ (or $M/T \gg 1$) is not satisfied
- LV can be extended to non-quasi-particle medium where BM does not apply

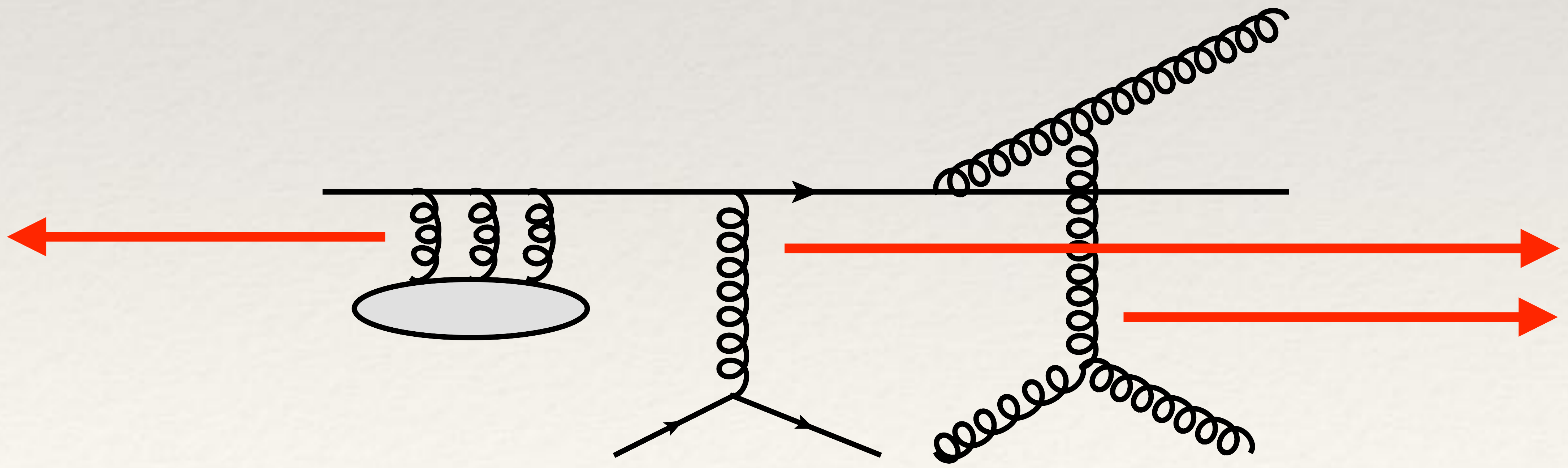


[EMMI, NPA 979 (2018)]

Neither BM nor LV alone is sufficient for HQ interaction with QGP!

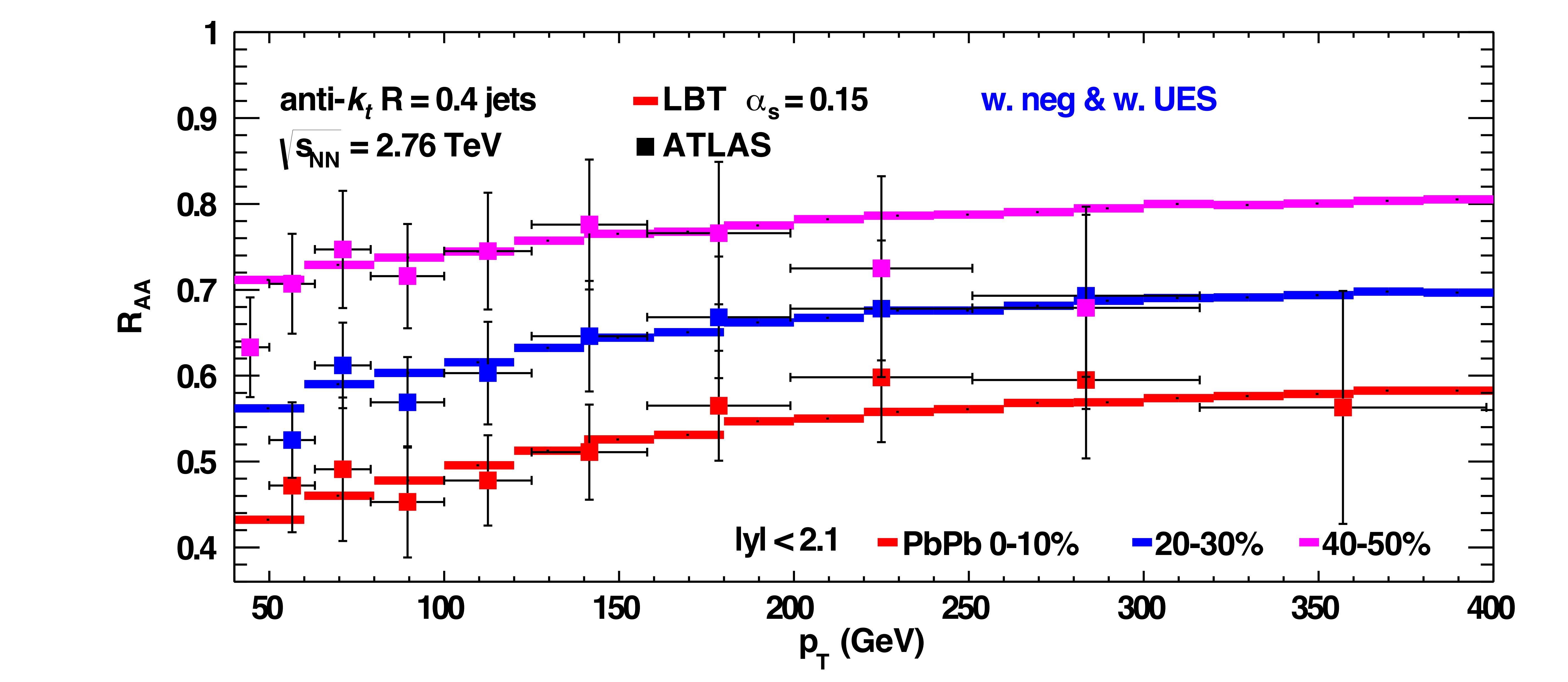
Lido (Linearized Boltzmann with diffusion model) (Duke) [Ke, Xu and Bass, PRC 98 (2018)]

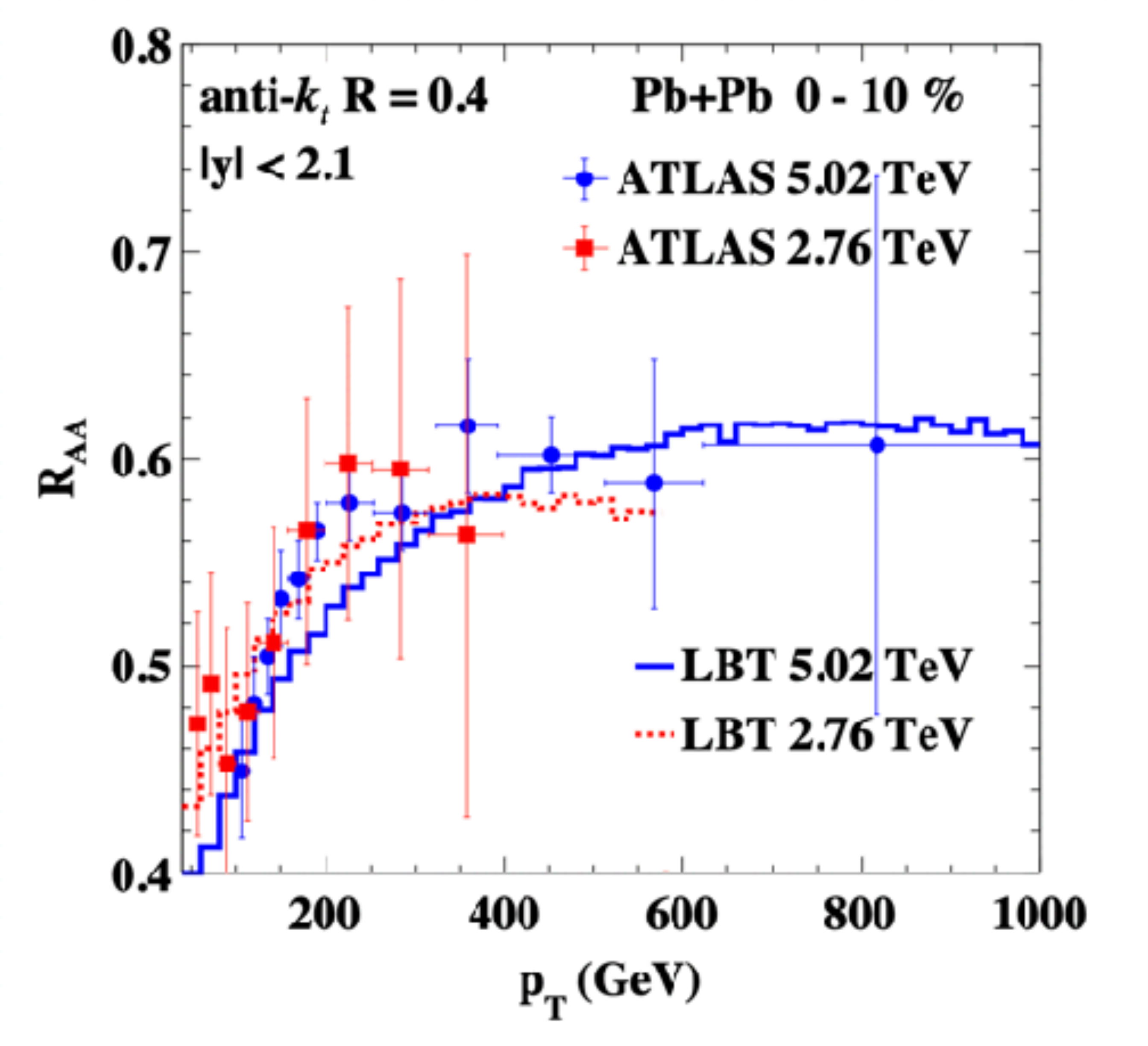
small k ($<k_0$)
HQ cannot "see" quasi-particles
Langevin



large k ($>k_0$)
HQ "sees" quasi-particles
Boltzmann

Single inclusive jet R_{AA}

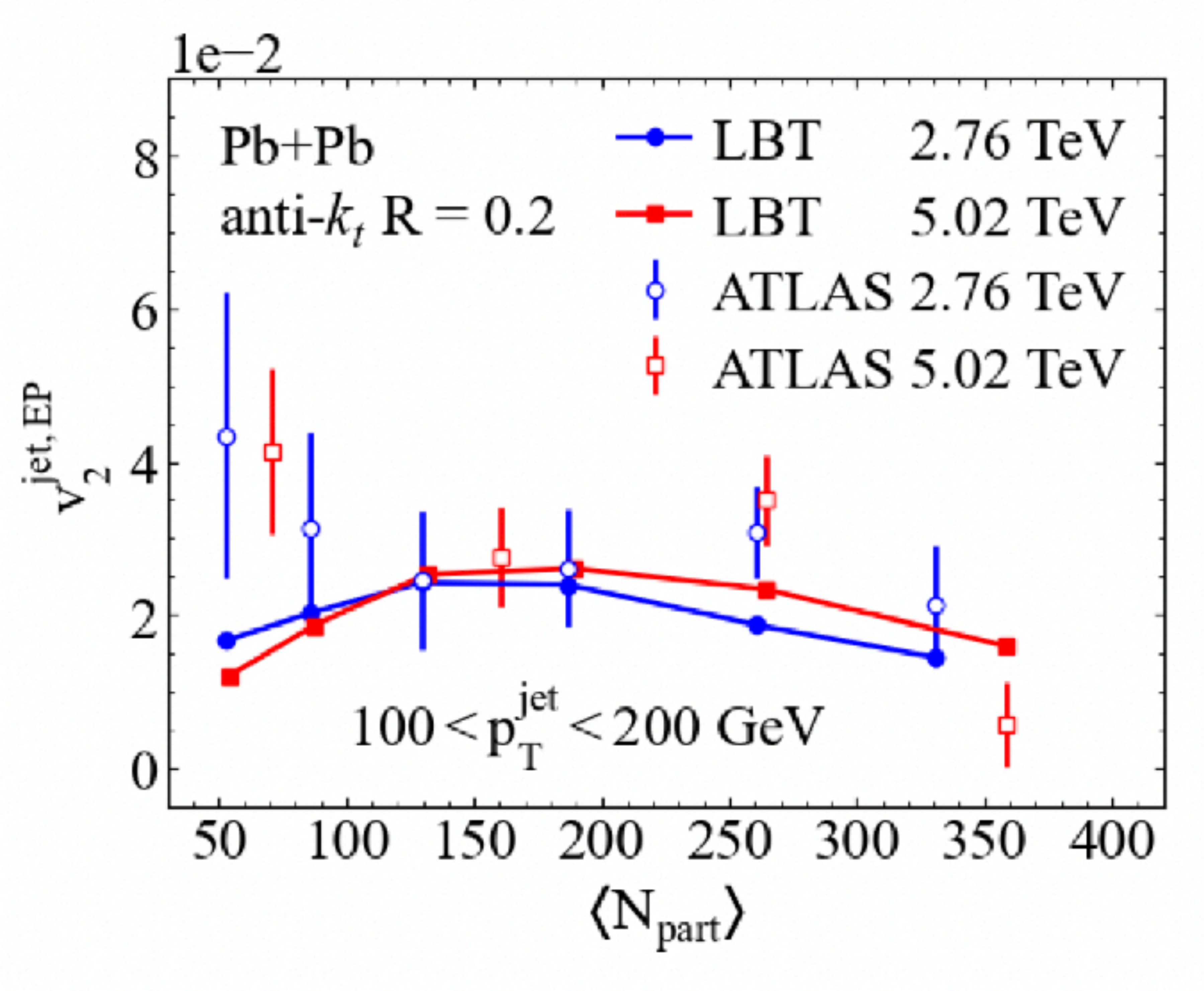




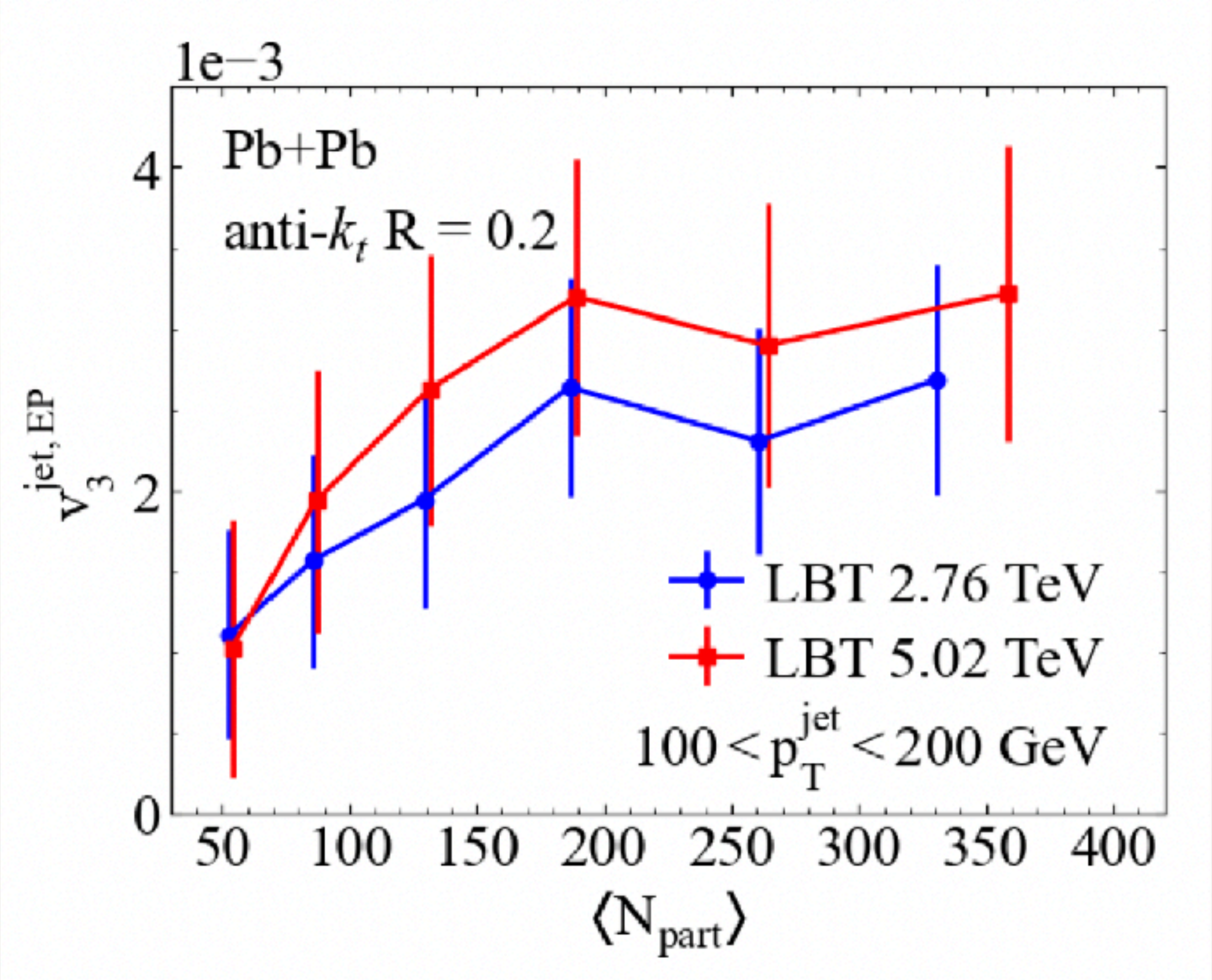
- Jet observables require treatment of secondary partons from splittings and recoiled partons
- Parton showers in pp collisions are simulated with Pythia
- Medium modification is simulated with LBT
- Jets are reconstructed with anti-k_T algorithm at the parton level

Single inclusive jet v_n

v_2



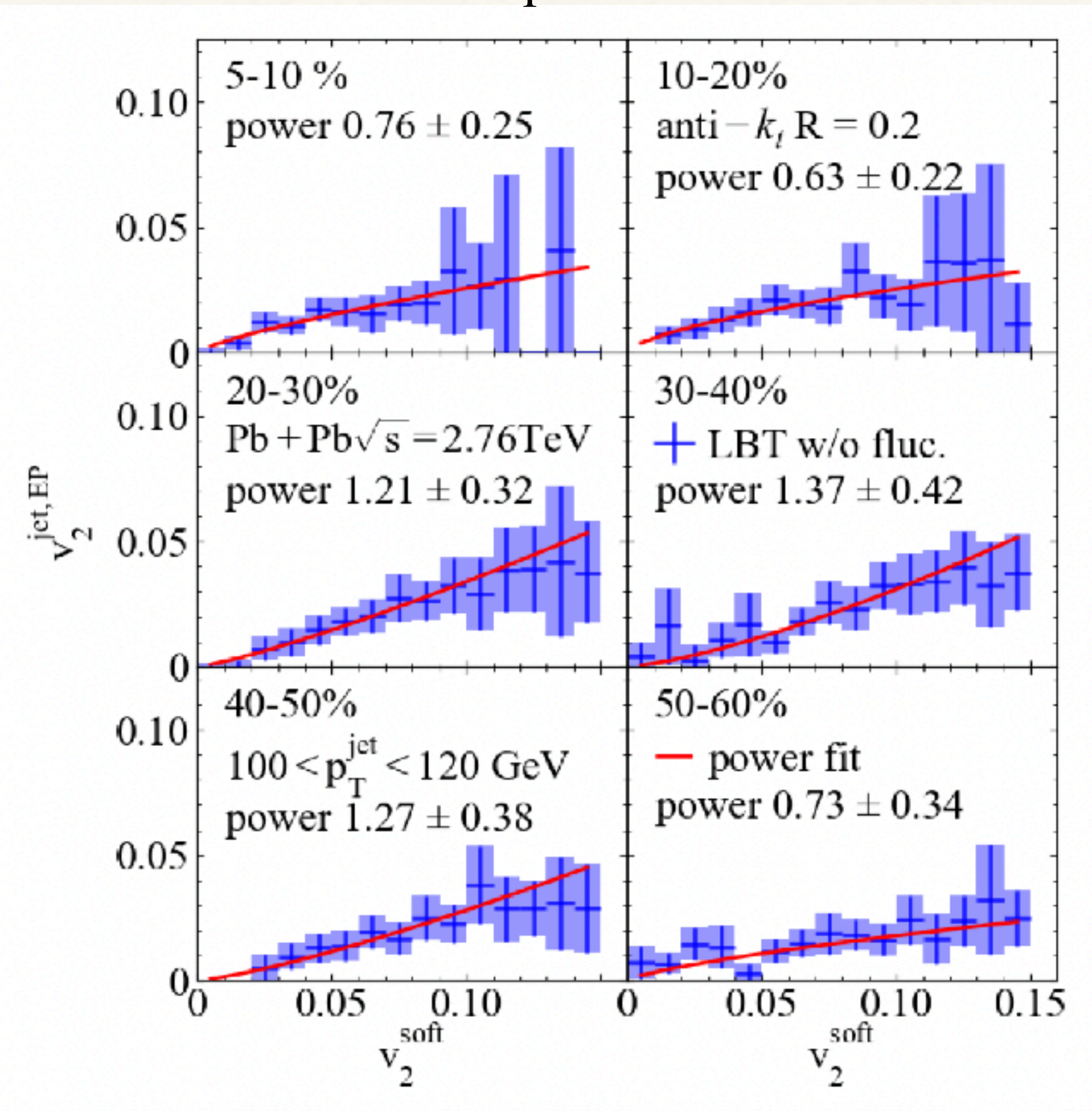
v_3



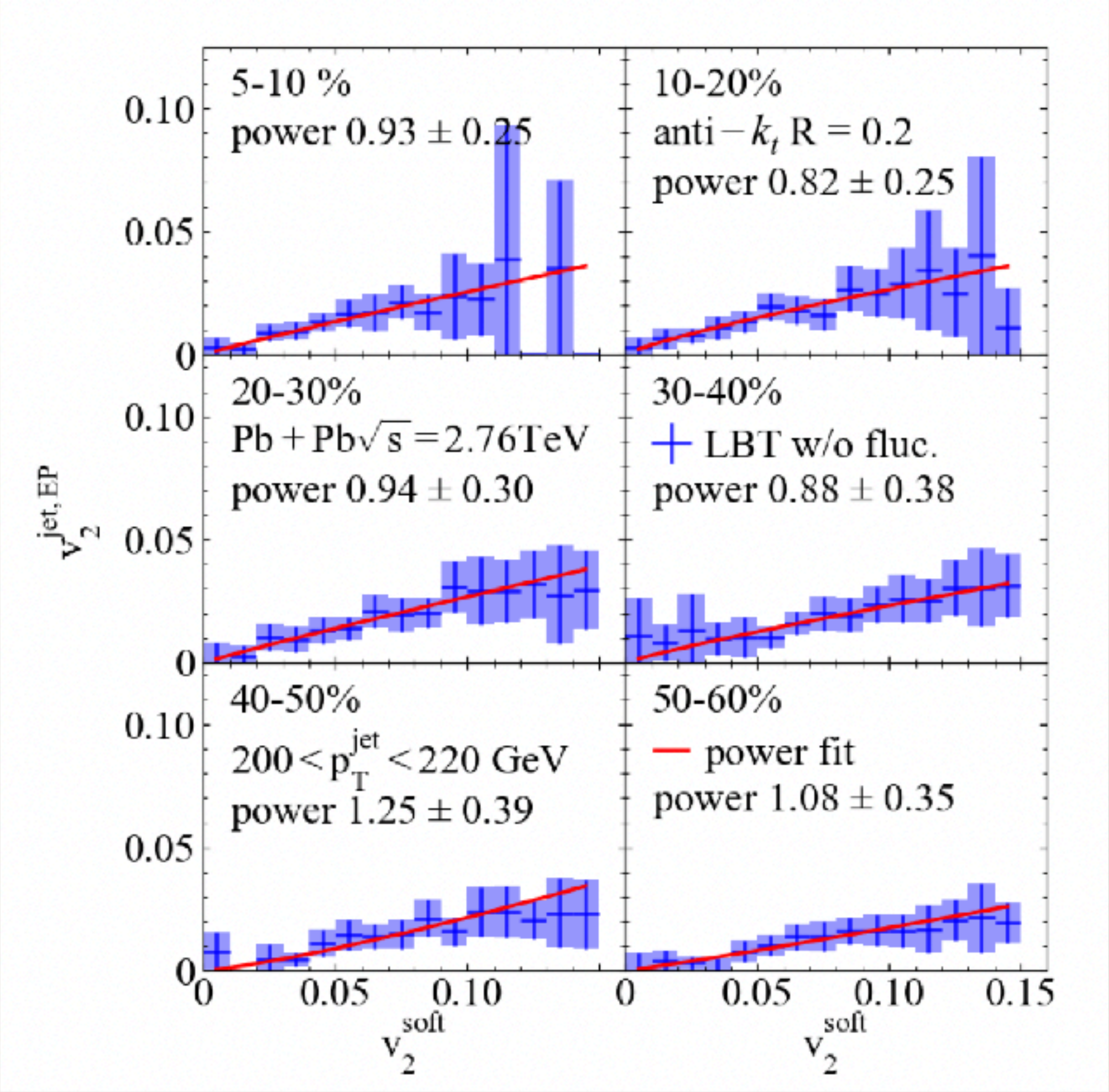
- Effects of energy loss, geometry and fluctuations, etc.

Correlation between soft and hard v_n

$100 < p_T^{jet} < 120$ GeV



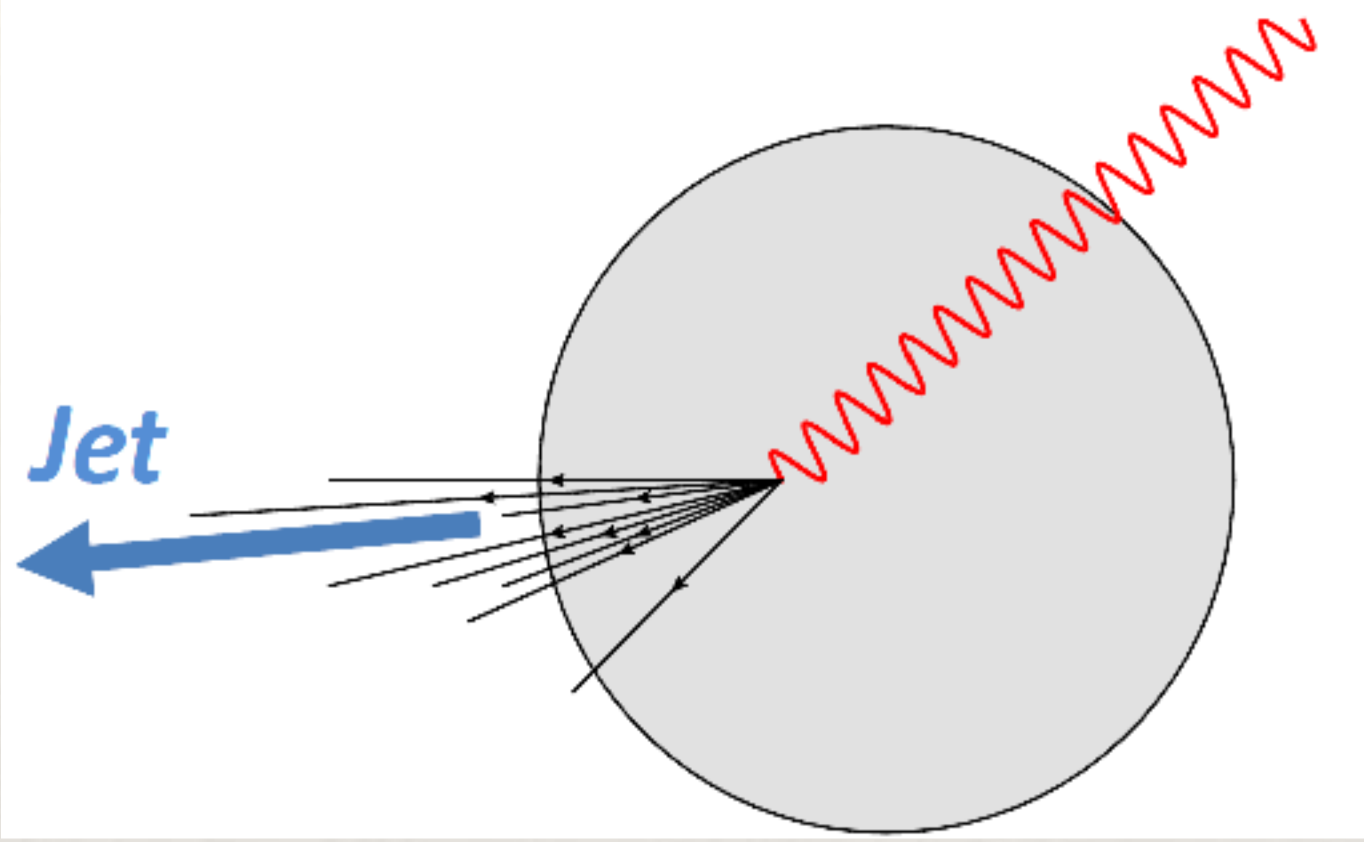
$200 < p_T^{jet} < 220$ GeV



- Possible constraints on the path length dependence of jet energy loss

Gamma-jet yield and asymmetry

The golden channel for studying jets – high p_T photons provide unmodified baselines, no “surface bias” in triggered events that di-jets suffer

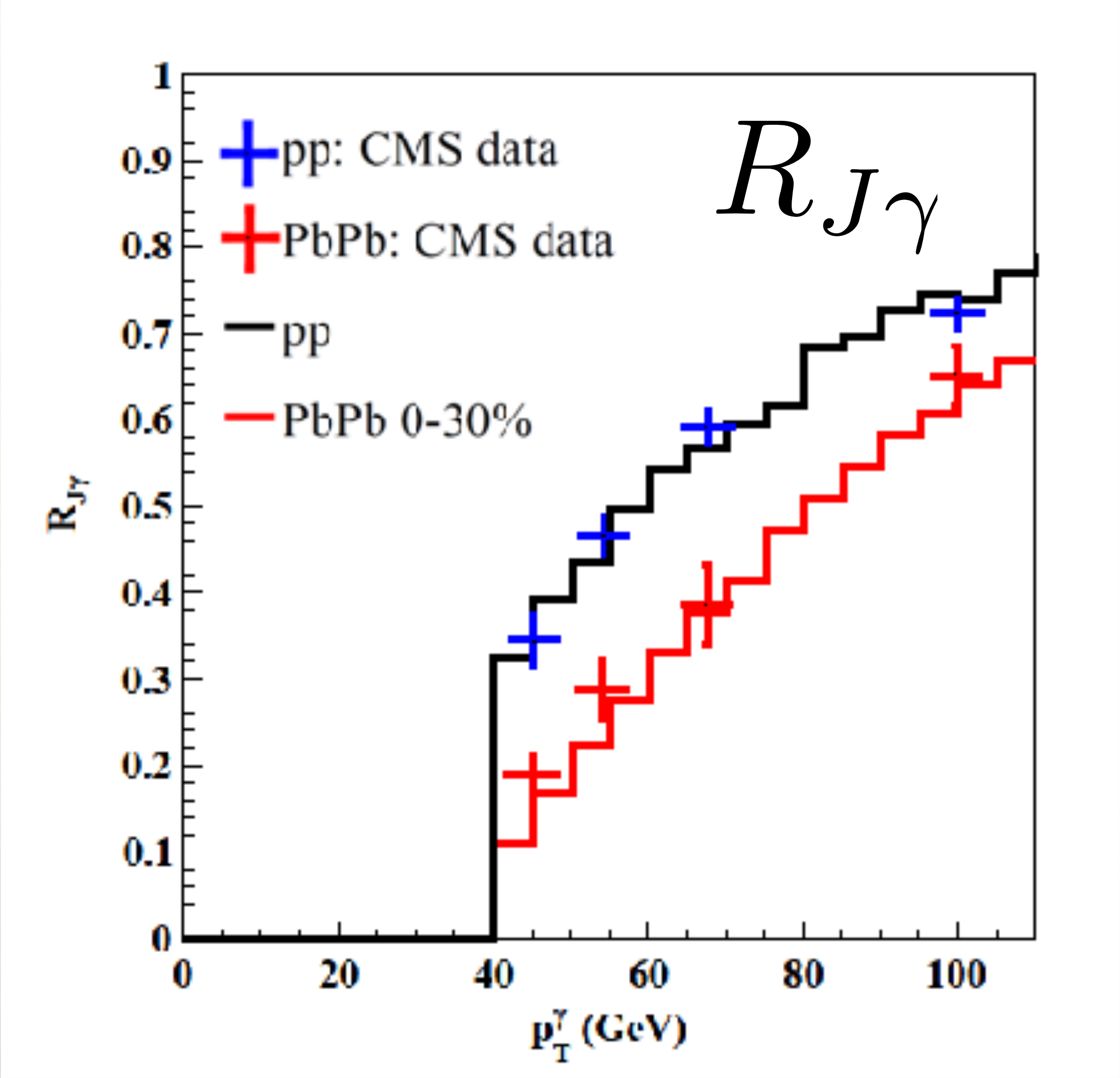


$|\eta_\gamma| < 1.44$

$P_{Tjet} > 30GeV$

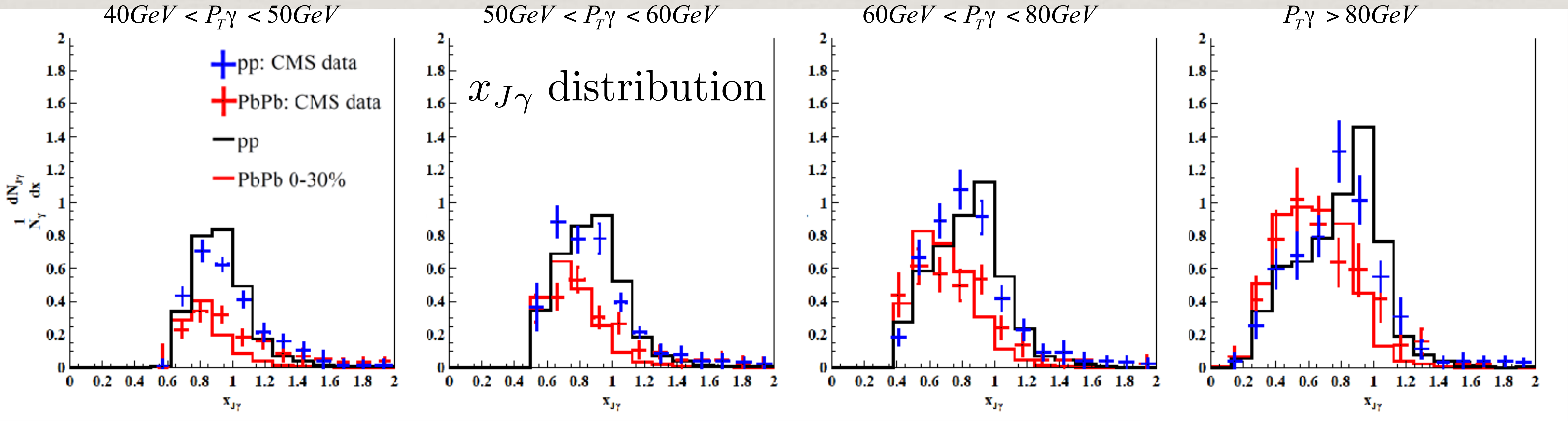
$|\eta_{jet}| < 1.6$

0-30%



$$R_{J\gamma} \equiv \frac{N_{J\gamma}}{N_\gamma}$$

$$x_{J\gamma} \equiv \frac{p_T^J}{p_T^\gamma}$$

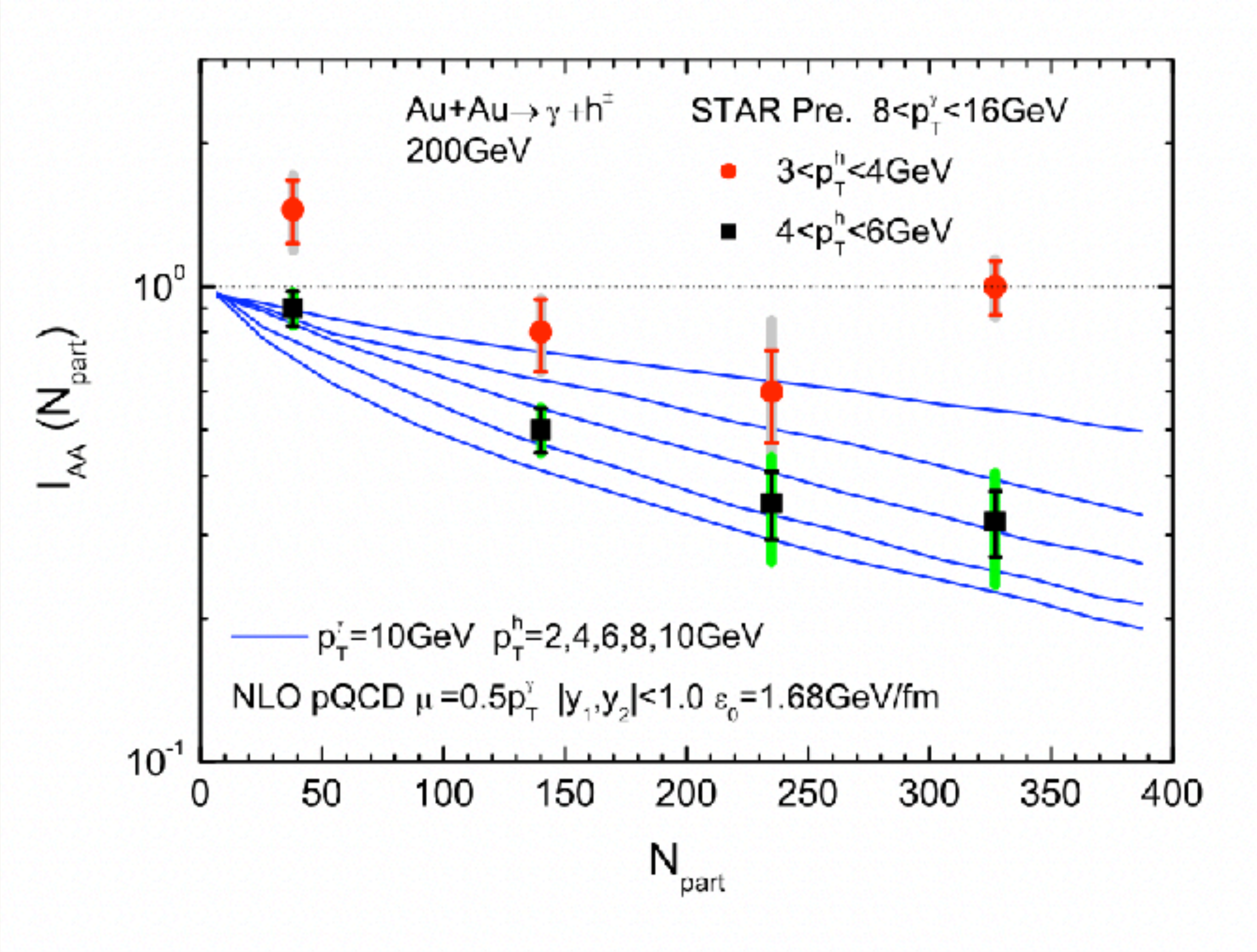


Nuclear modification of hadrons triggered by gamma-jet

$$R_{AA}(p_T, y, \phi) \equiv \frac{1}{\langle N_{coll} \rangle} \frac{\frac{dN_{AA}}{dp_T dy d\phi}}{\frac{dN_{pp}}{dp_T dy d\phi}}$$

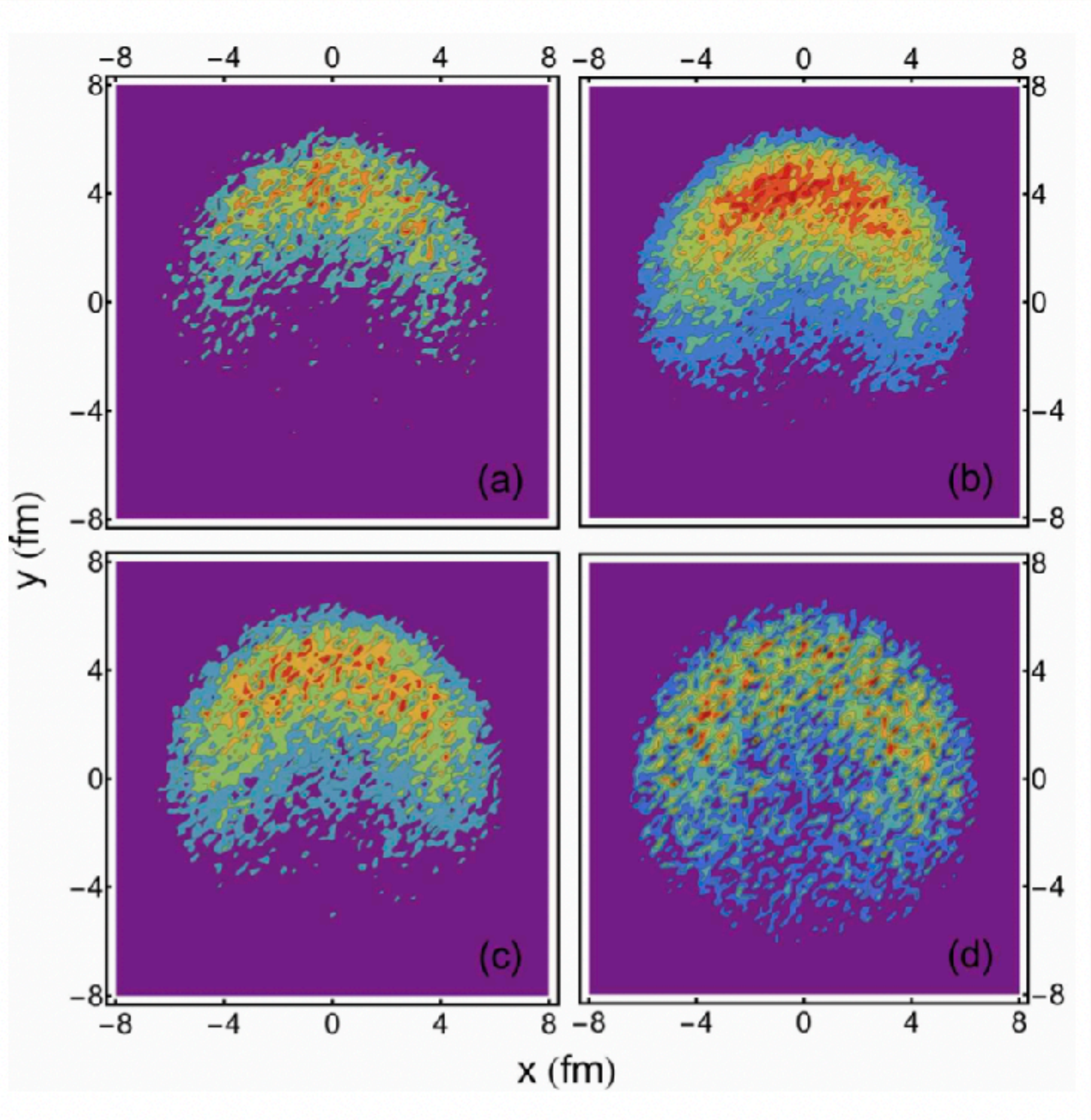
with trigger

$$I_{AA}(z_T) \equiv \frac{\frac{1}{N_{trig}} \frac{dN^{asso}}{dz_T}|_{AA}}{\frac{1}{N_{trig}} \frac{dN^{asso}}{dz_T}|_{pp}}$$



$p_T^\gamma > 10$ GeV, different p_T^h

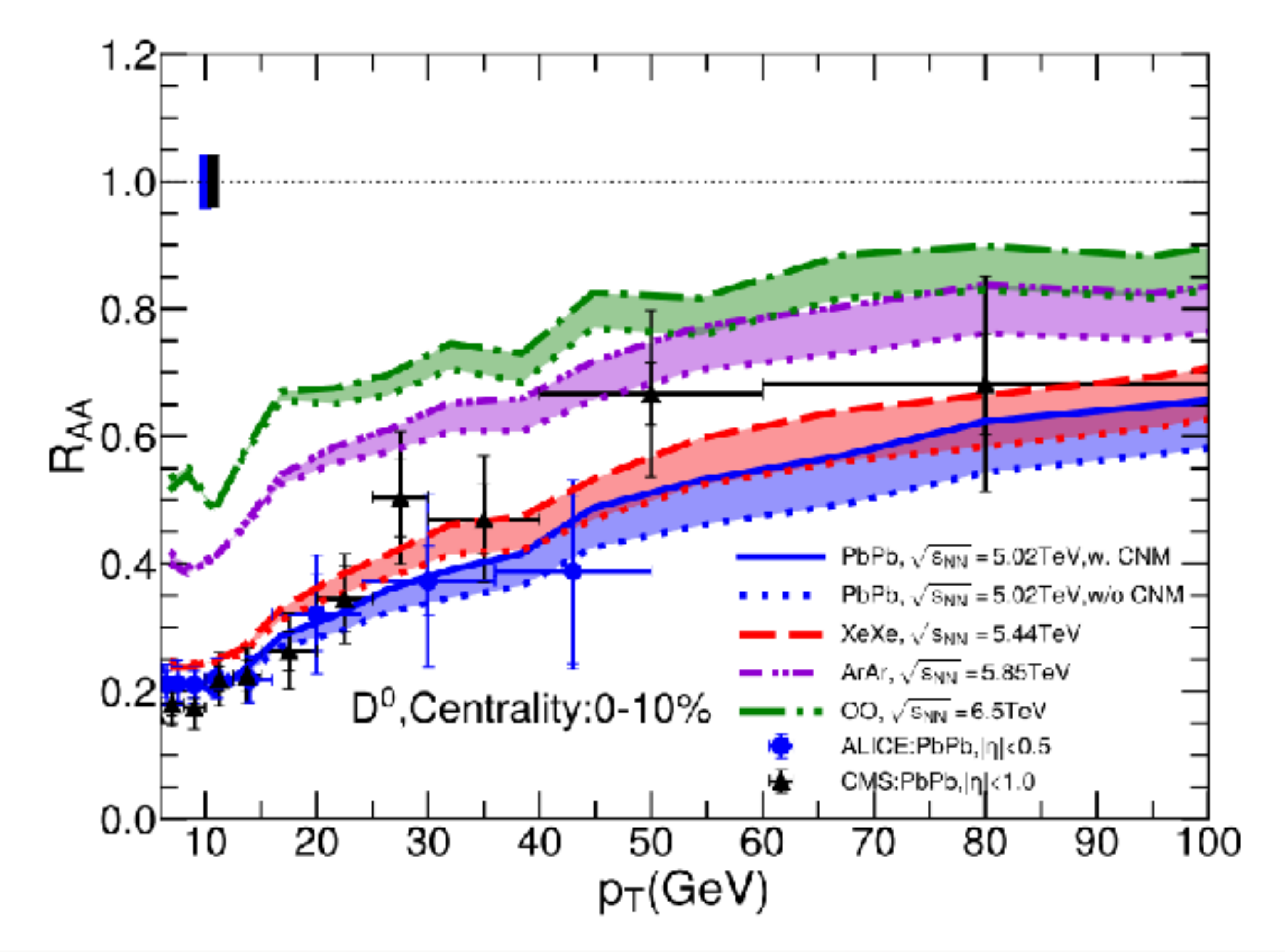
Di-hadron/jet asymmetry

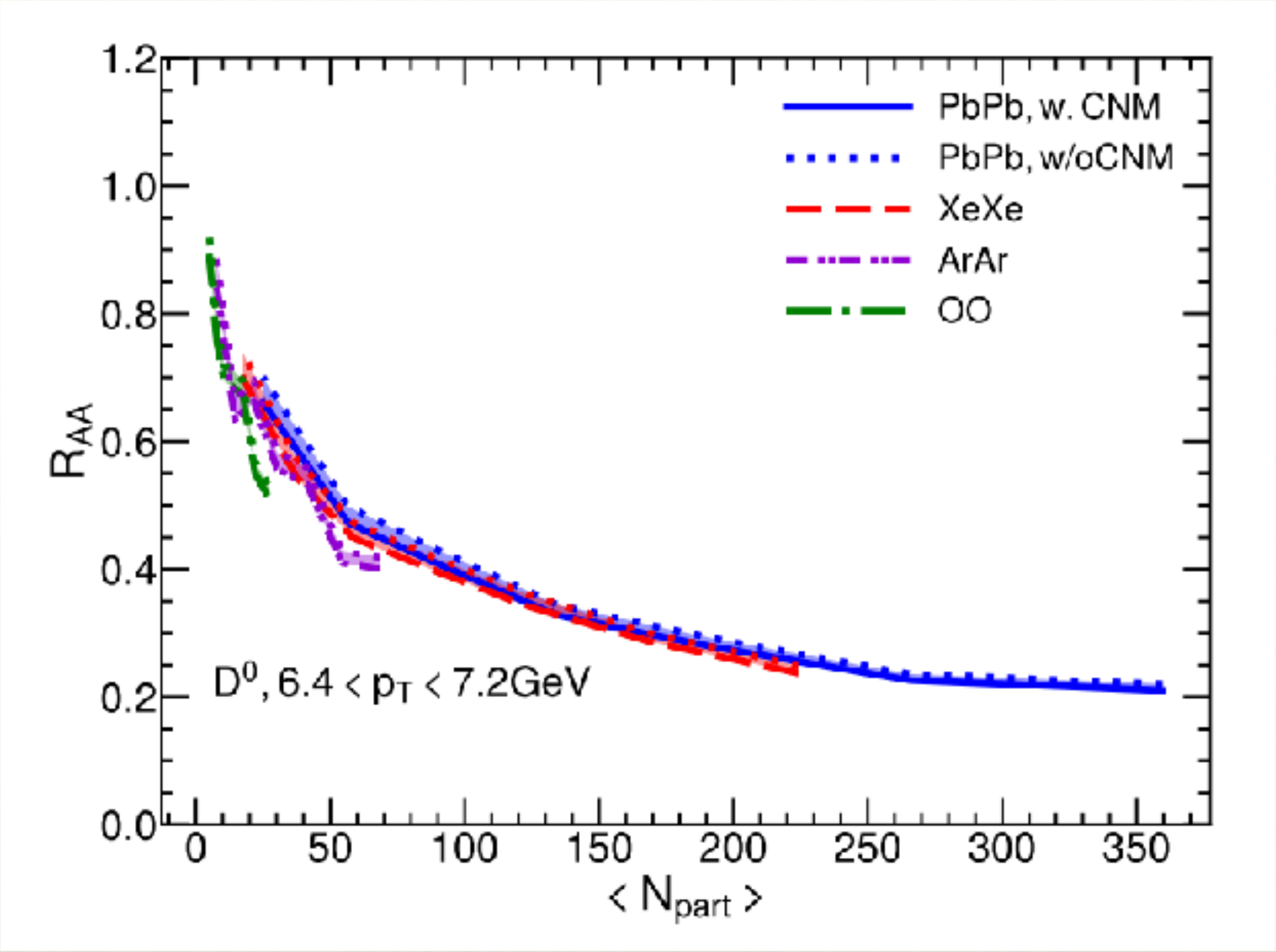


- $x_T = \min(p_T^D, p_T^{\bar{D}}) / \max(p_T^D, p_T^{\bar{D}})$
- Density distribution of the initial $c\bar{c}$ production points: (a) $x_T \in [0.2, 0.4]$, (b) $x_T \in [0.4, 0.6]$, (c) $x_T \in [0.6, 0.8]$, and (d) $x_T \in [0.8, 1.0]$.
- Di-hadron asymmetry helps locate the region of production vertex of hard partons.

D meson R_{AA} in different systems

O-O
Ar-Ar
Xe-Xe
Pb-Pb

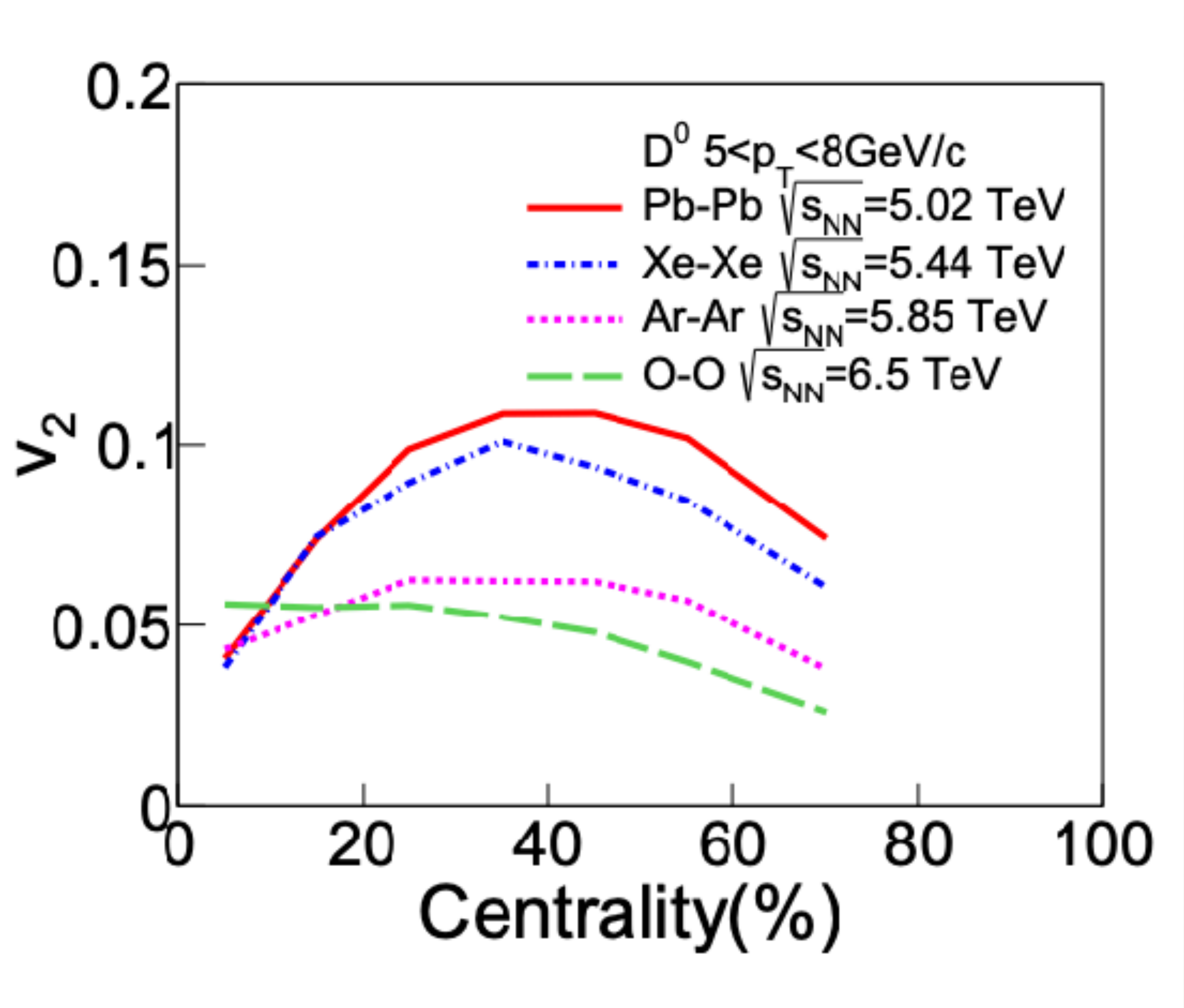


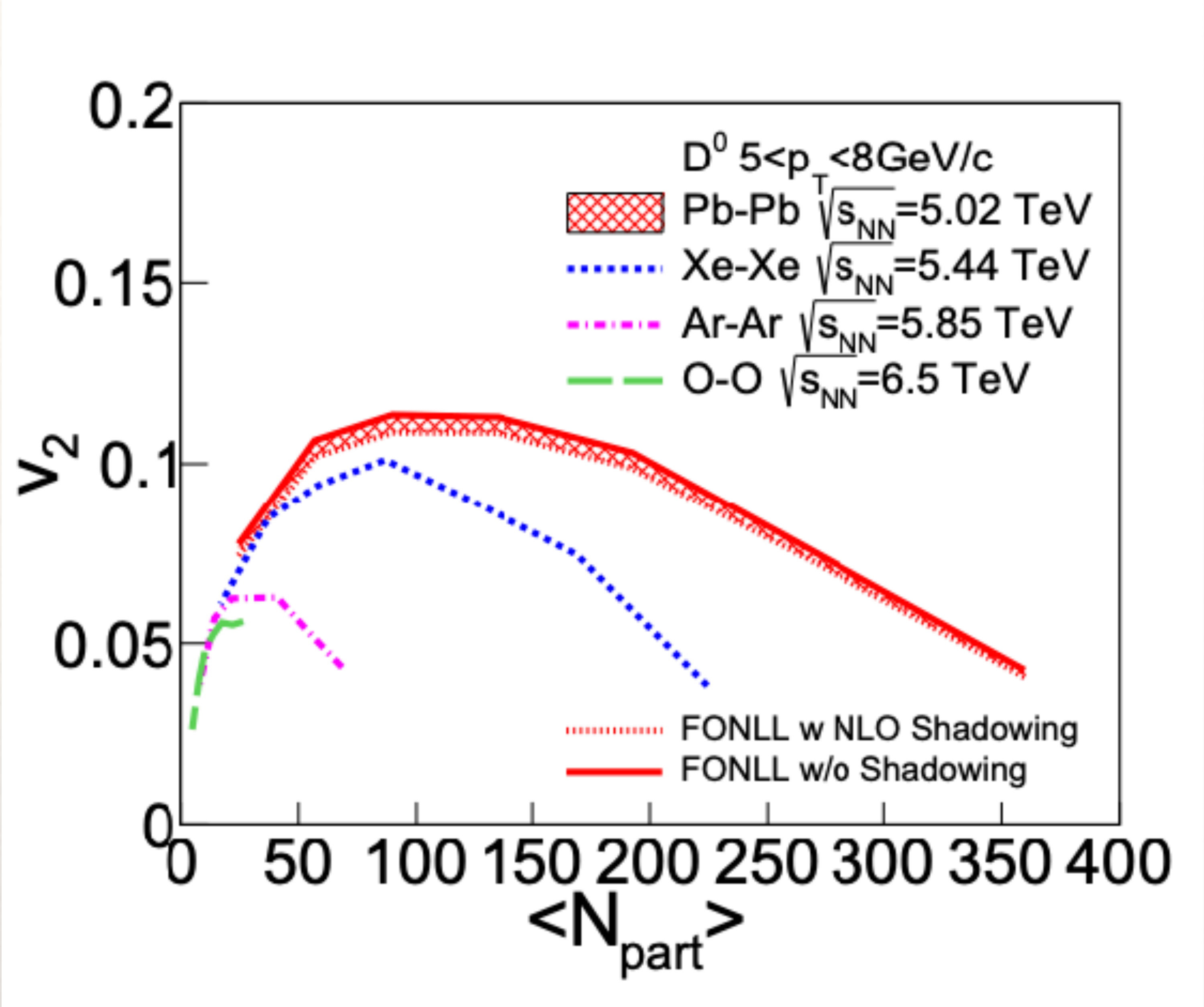


Liu, Xing, Wu, Qin, Cao, Xing, PRC 105 (2022) 4, 044904

- Clear hierarchy of R_{AA} with respect to the system size
- Significant R_{AA} in the small O-O system, existence of QGP
- Scaling of R_{AA} with the system size (quantified by N_{part}) across different collision systems

D meson v_2 in different systems

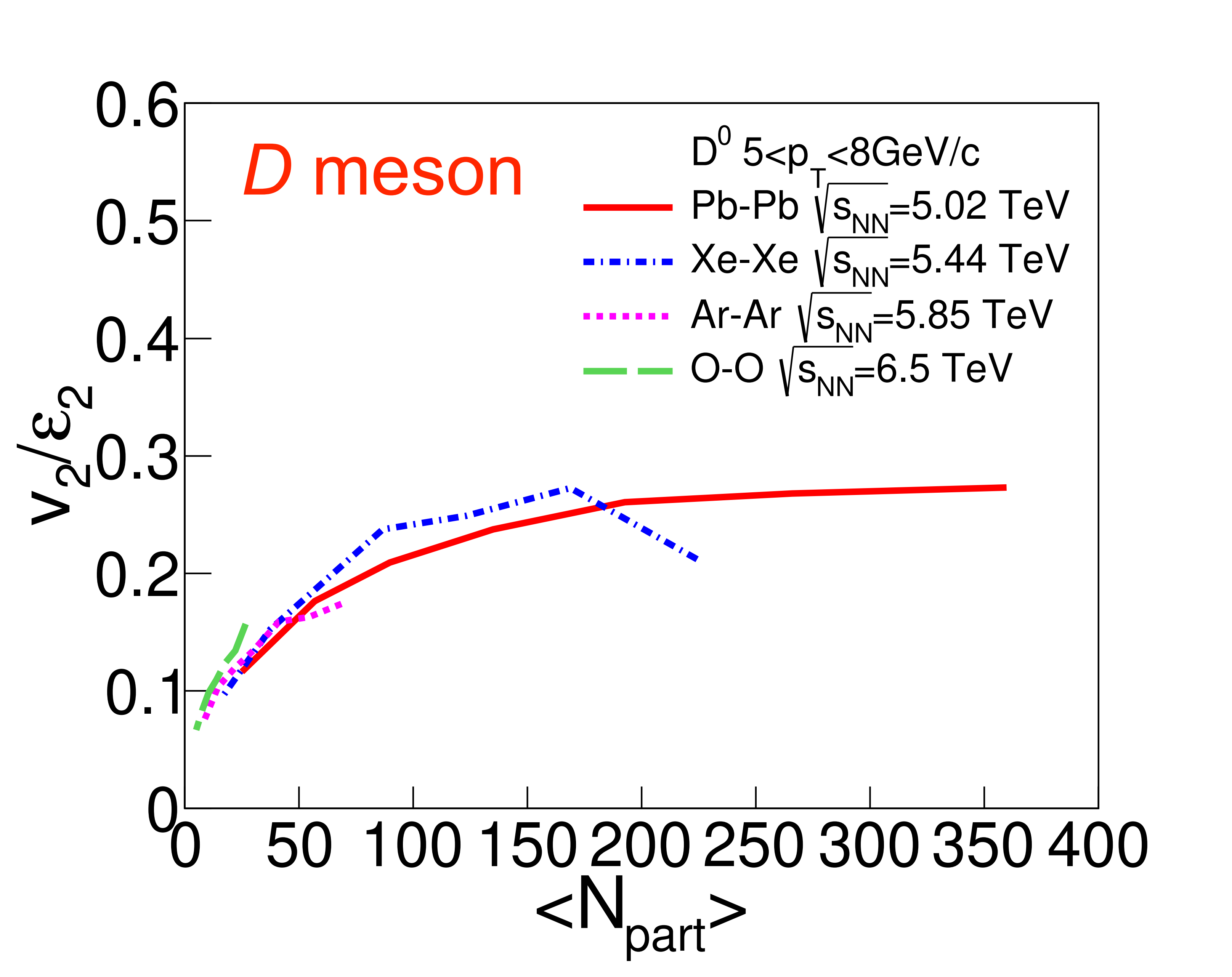


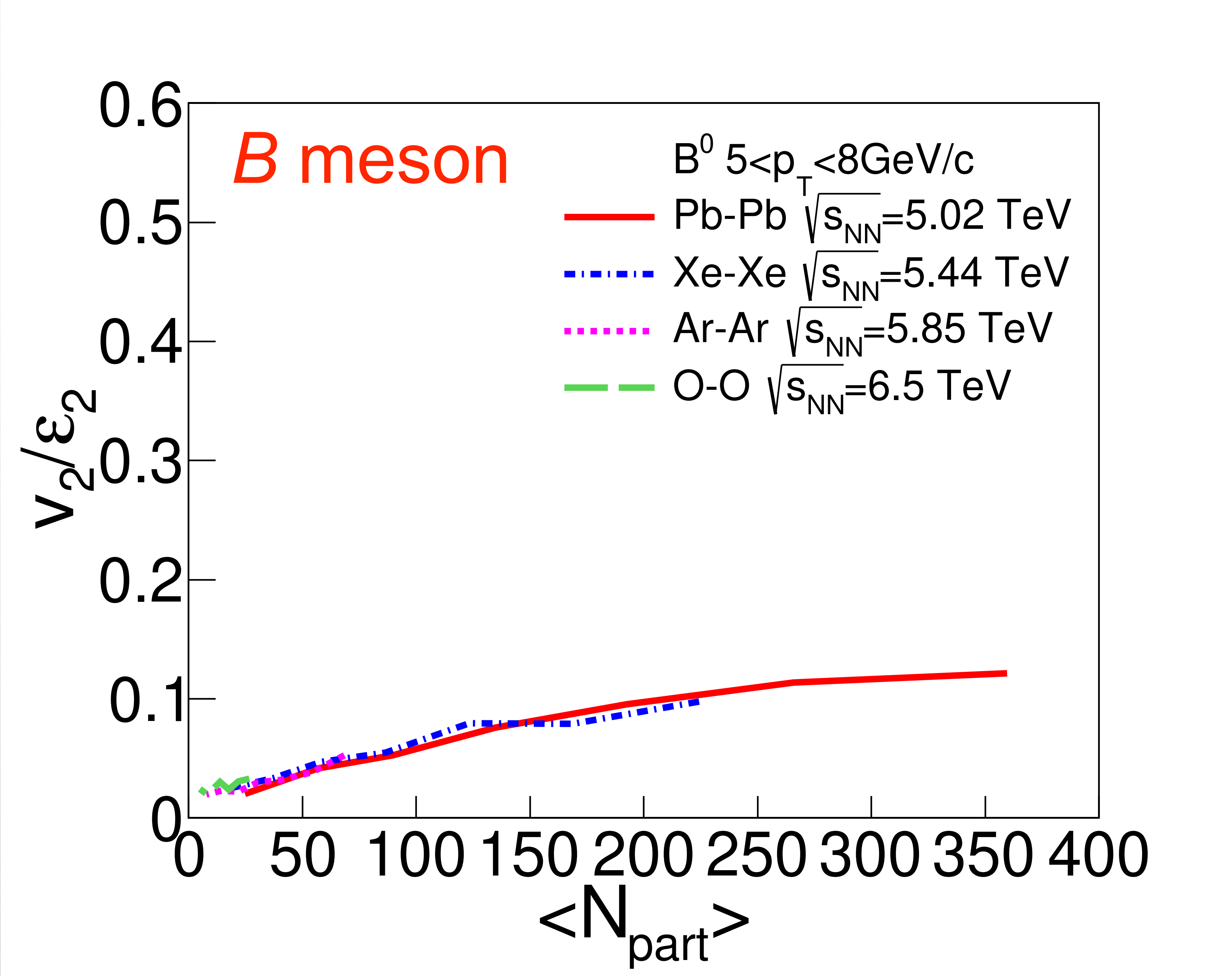


Li, Xing, Wu, Cao, Qin, EPJC 81 (2021) 11, 1035

- Energy loss effect: for a given centrality, v_2 increases with the system size
- Geometry effect: for a given N_{part}, v_2 increases from O-O, Ar-Ar, Xe-Xe to Pb-Pb

Scaling of v_2/ε_2 with respect to N_{part}

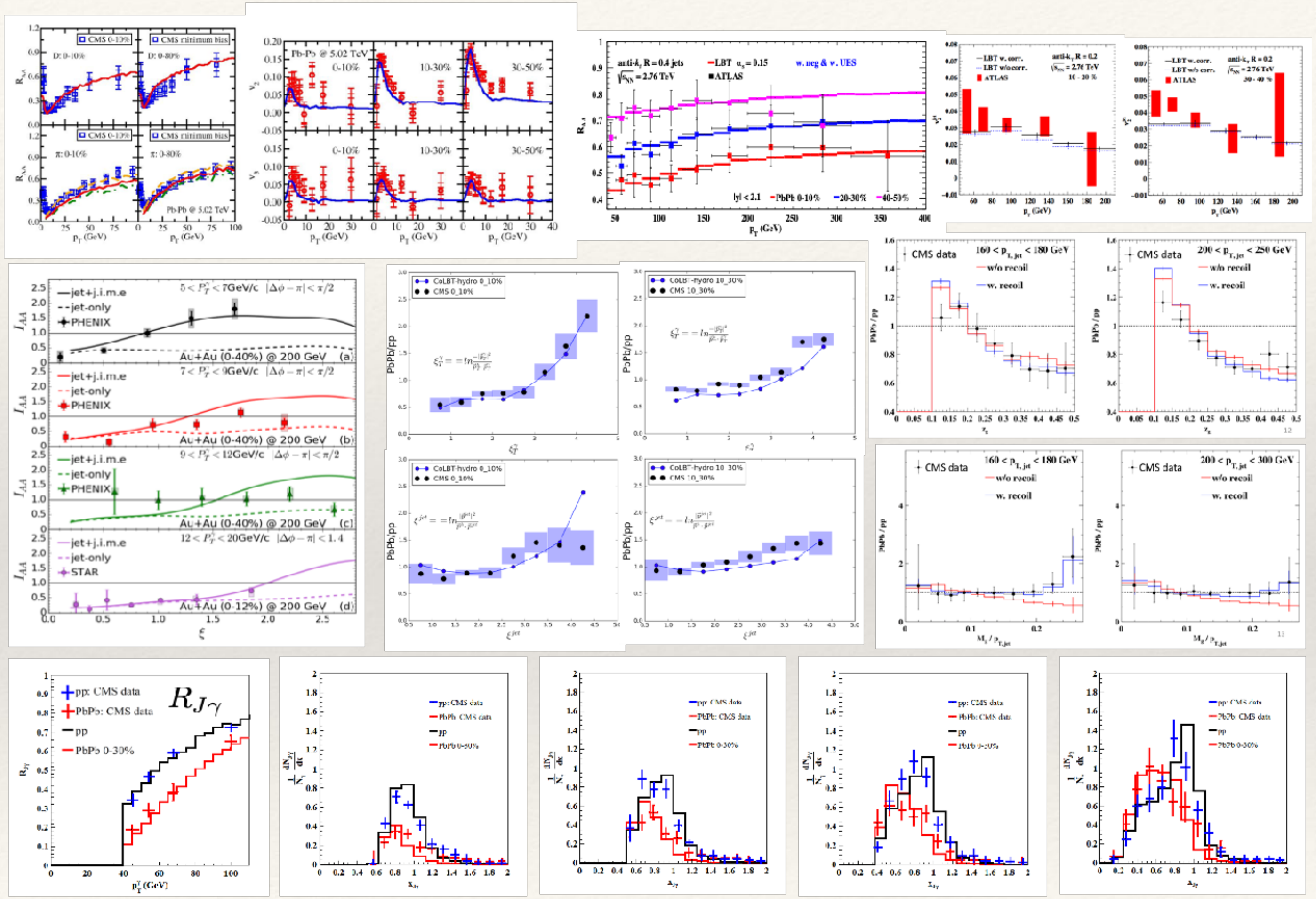




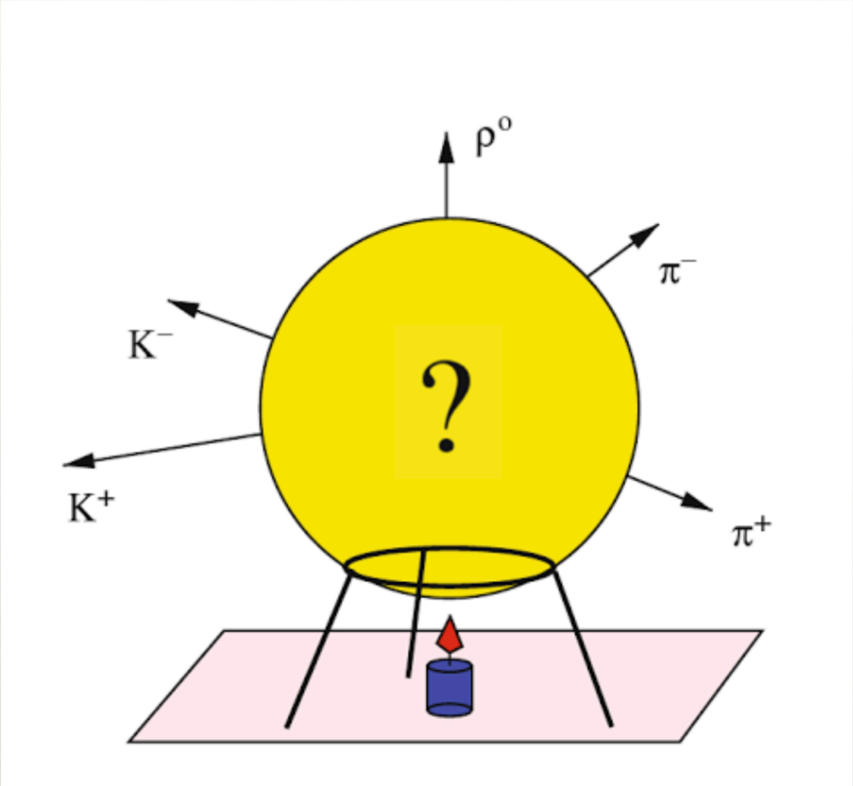
Li, Xing, Wu, Cao, Qin, EPJC 81 (2021) 11, 1035

- Separate energy loss and geometry effects by rescaling heavy quark v_2 with bulk ε_2
- v_2/ε_2 scales with the system size across different collision systems

Extraction of QGP properties from model-to-data comparison

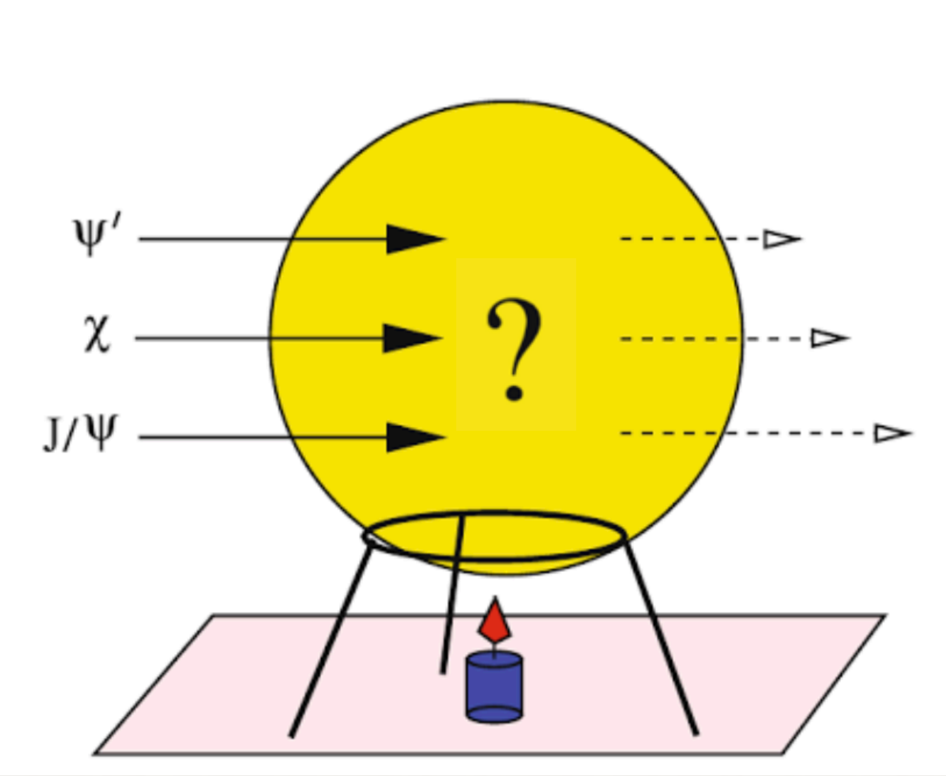


Medium response to jet propagation



Soft hadron emission

+

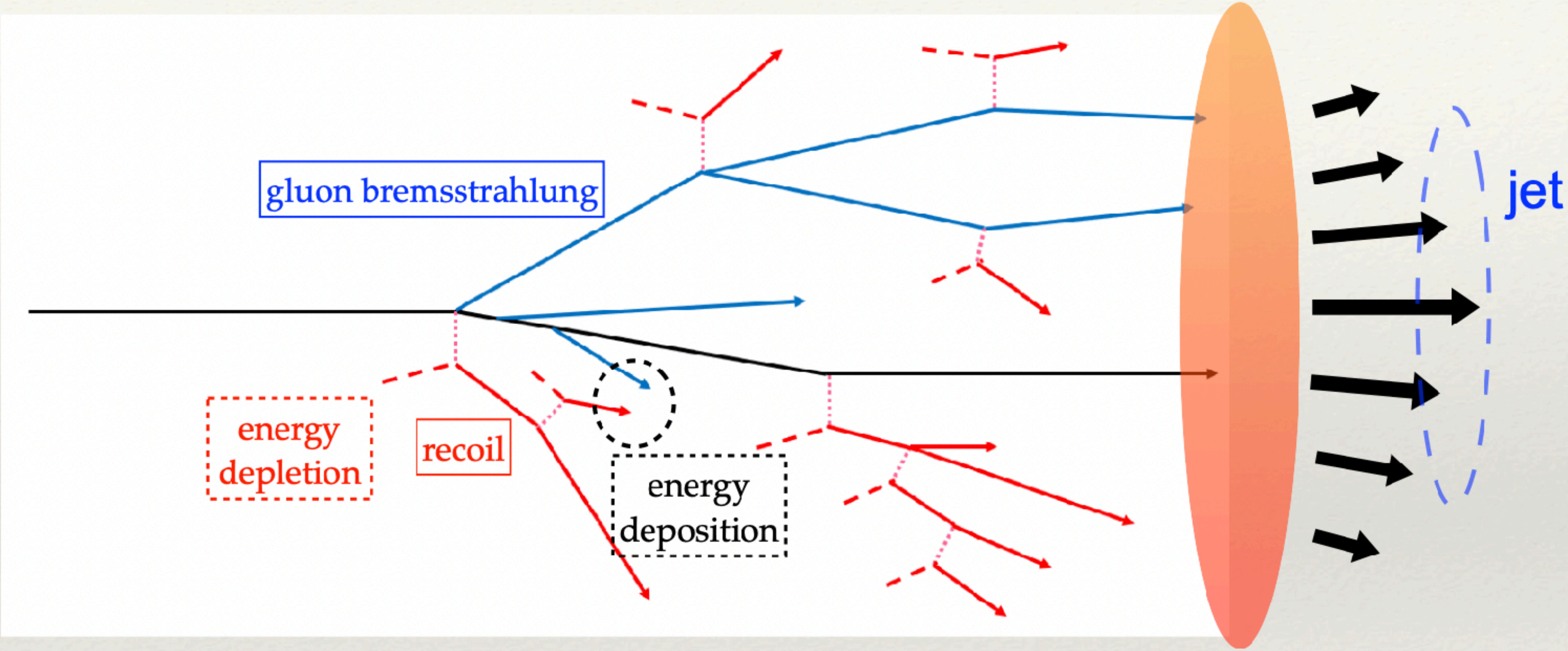


Hard probes

Novel method

Disturb the liquid and study the propagation of the perturbation

What is medium response

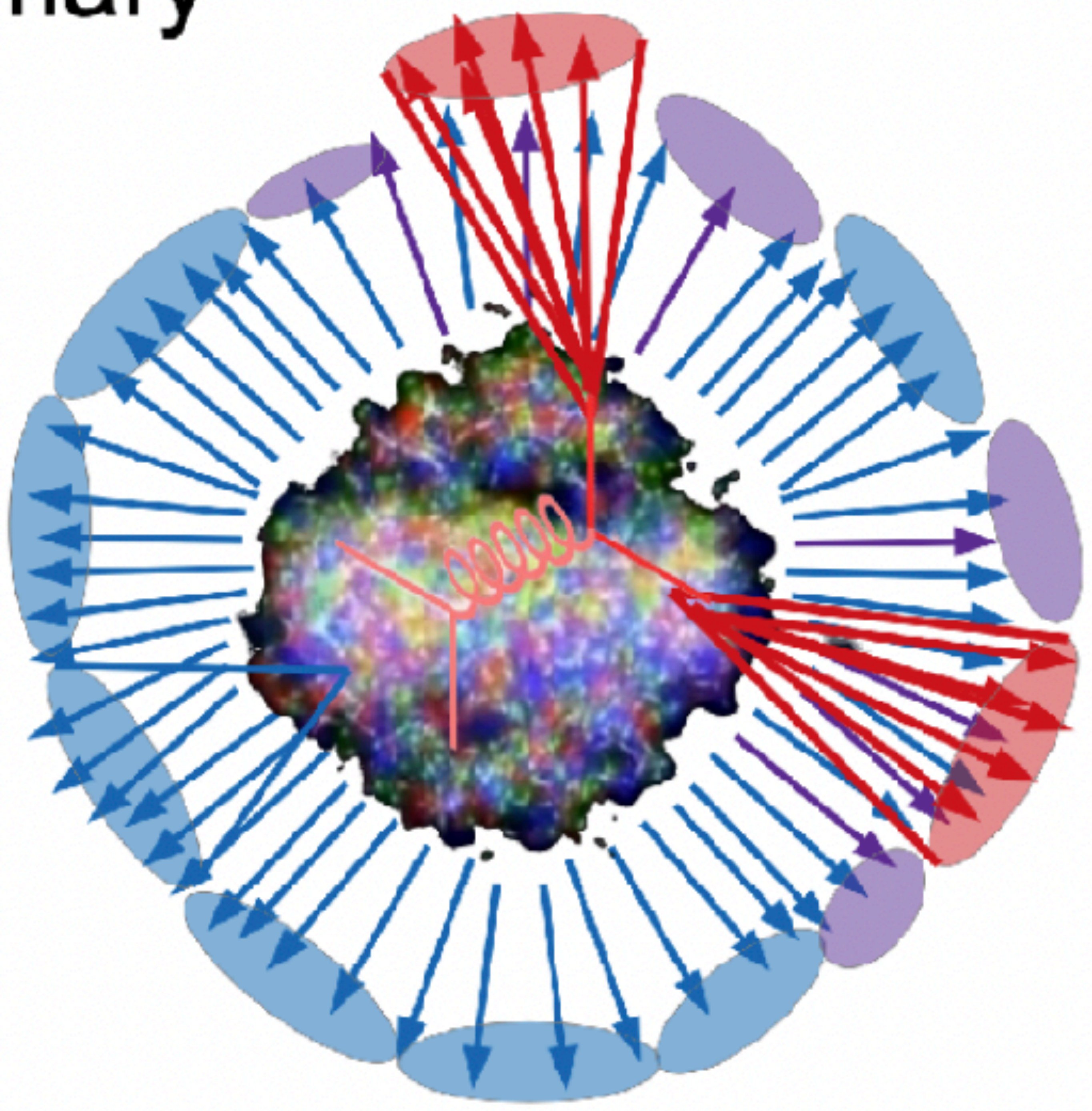


- Energy depletion and energy deposition: jet-induced medium excitation, medium response

Why to study medium response

Mini-summary

- “Signal” and “background” have different properties, but…
- Always overlap somewhat
- Any procedure to remove “background” will also cut signal

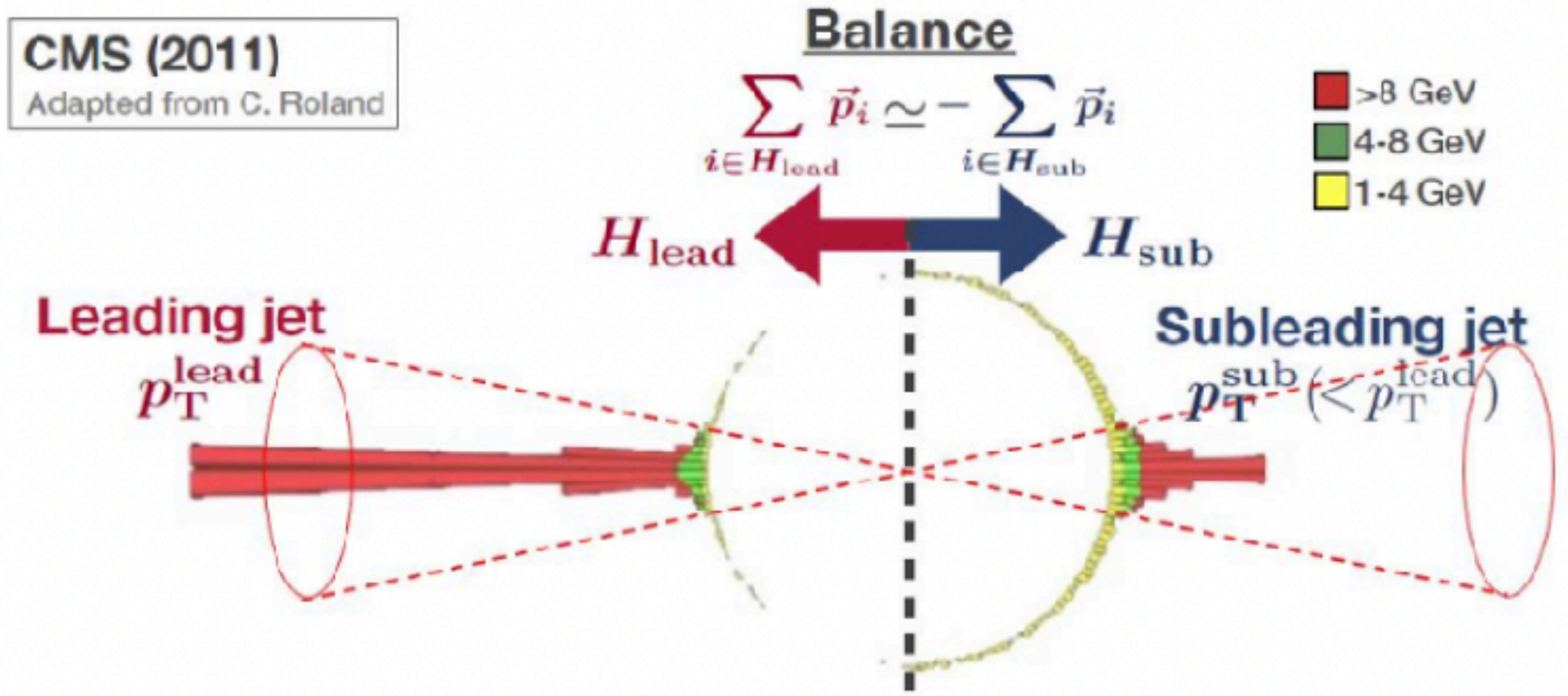


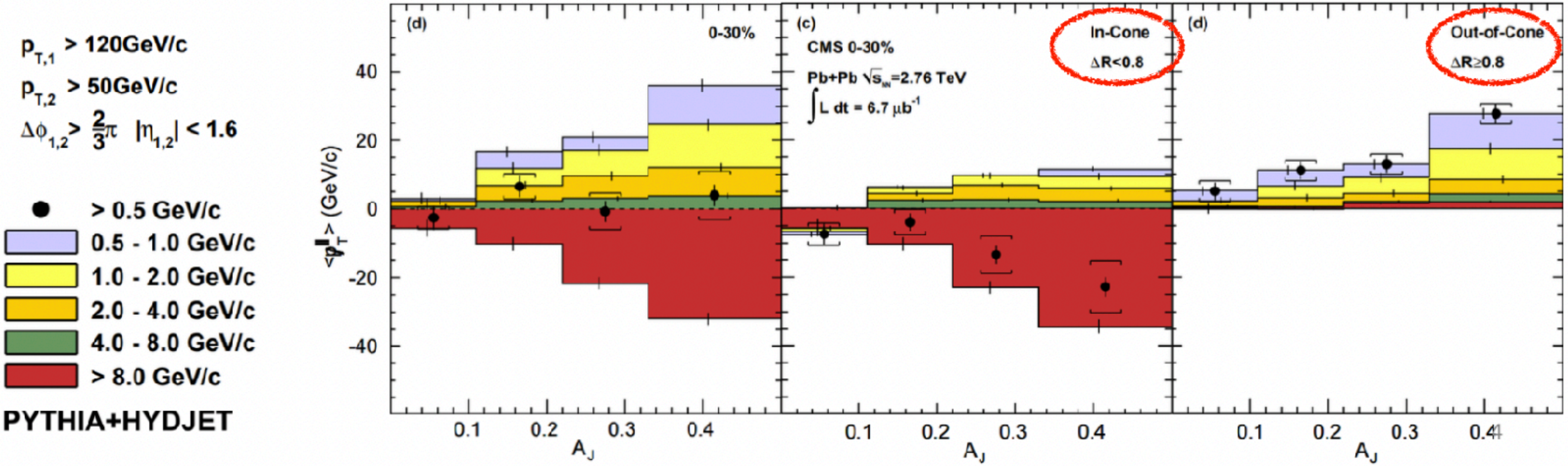
Christine Nattrass (UTK), INT, 28 July 2021 39

- One cannot separate hadrons from QGP emission and jet fragmentation inside jet cone

Why we need medium response

The energy and momentum deposited by the jet shower into the medium appear at large angles away from the jet axis.





Different implementations of medium response

Linear hydrodynamic response

Hydrodynamics with a source: $\partial_\mu T^{\mu\nu}(x) = J^\nu(x).$ $J^\nu(x) \;=\; [dE/d^4x, d\vec{p}/d^4x]$

Linear approximation: $T^{\mu\nu} \approx T_0^{\mu\nu} + \delta T^{\mu\nu};\;\; \partial_\mu T_0^{\mu\nu} = 0,\;\; \partial_\mu \delta T^{\mu\nu} = J^\nu.$

Decomposition: $\delta T^{00} \equiv \delta\epsilon,\;\; \delta T^{0i} \equiv g^i,$ $\delta T^{ij} = \delta^{ij} c_s^2 \delta\epsilon + \frac{3}{4}\Gamma_s(\partial^i g^j + \partial^j g^i + \frac{2}{3}\delta^{ij}\nabla\cdot\vec{g}),$

Fourier transformation of $\partial_\mu \delta T^{\mu\nu} = J^\nu$:

$$J^0 = -i\omega\delta\epsilon + i\vec{k}\cdot\vec{g},$$

$$\vec{J} = -i\omega\vec{g} + i\vec{k}c_s^2\delta\epsilon + \frac{3}{4}\Gamma_s\left[k^2\vec{g} + \frac{\vec{k}}{3}(\vec{k}\cdot\vec{g})\right],$$

$\Rightarrow$

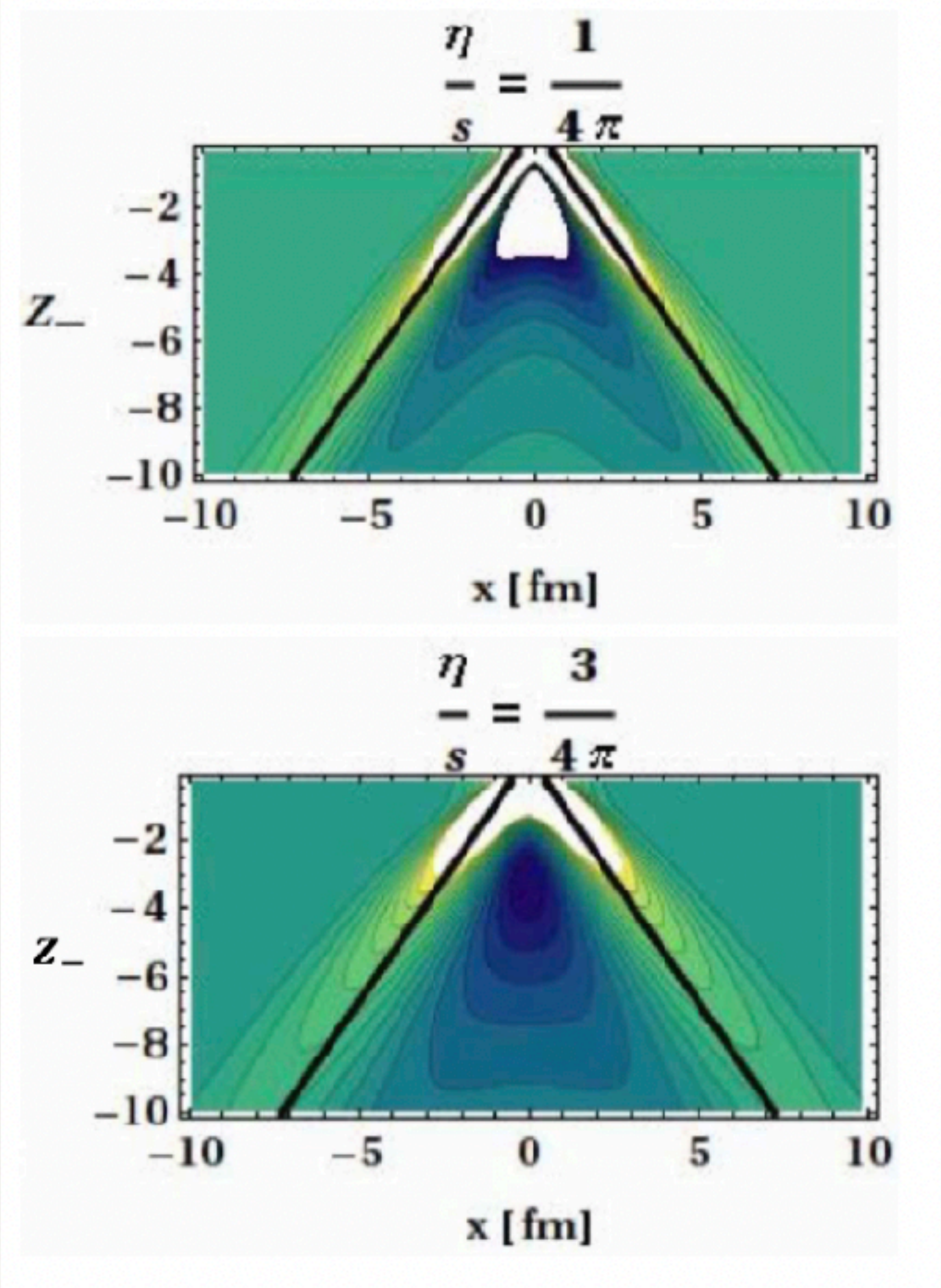
$$\delta\epsilon(\vec{k},\omega) = \frac{(i\omega - \Gamma_s k^2)J^0(\vec{k},\omega) + ikJ_L(\vec{k},\omega)}{\omega^2 - c_s^2k^2 + i\Gamma_s\omega k^2},$$

$$\vec{g}_L(\vec{k},\omega) = \frac{ic_s^2\vec{k}J^0(\vec{k},\omega) + i\omega\hat{k}J_L(\vec{k},\omega)}{\omega^2 - c_s^2k^2 + i\Gamma_s\omega k^2},$$

$$\vec{g}_T(\vec{k},\omega) = \frac{i\vec{J}_T(\vec{k},\omega)}{\omega + \frac{3}{4}i\Gamma_s k^2}.$$

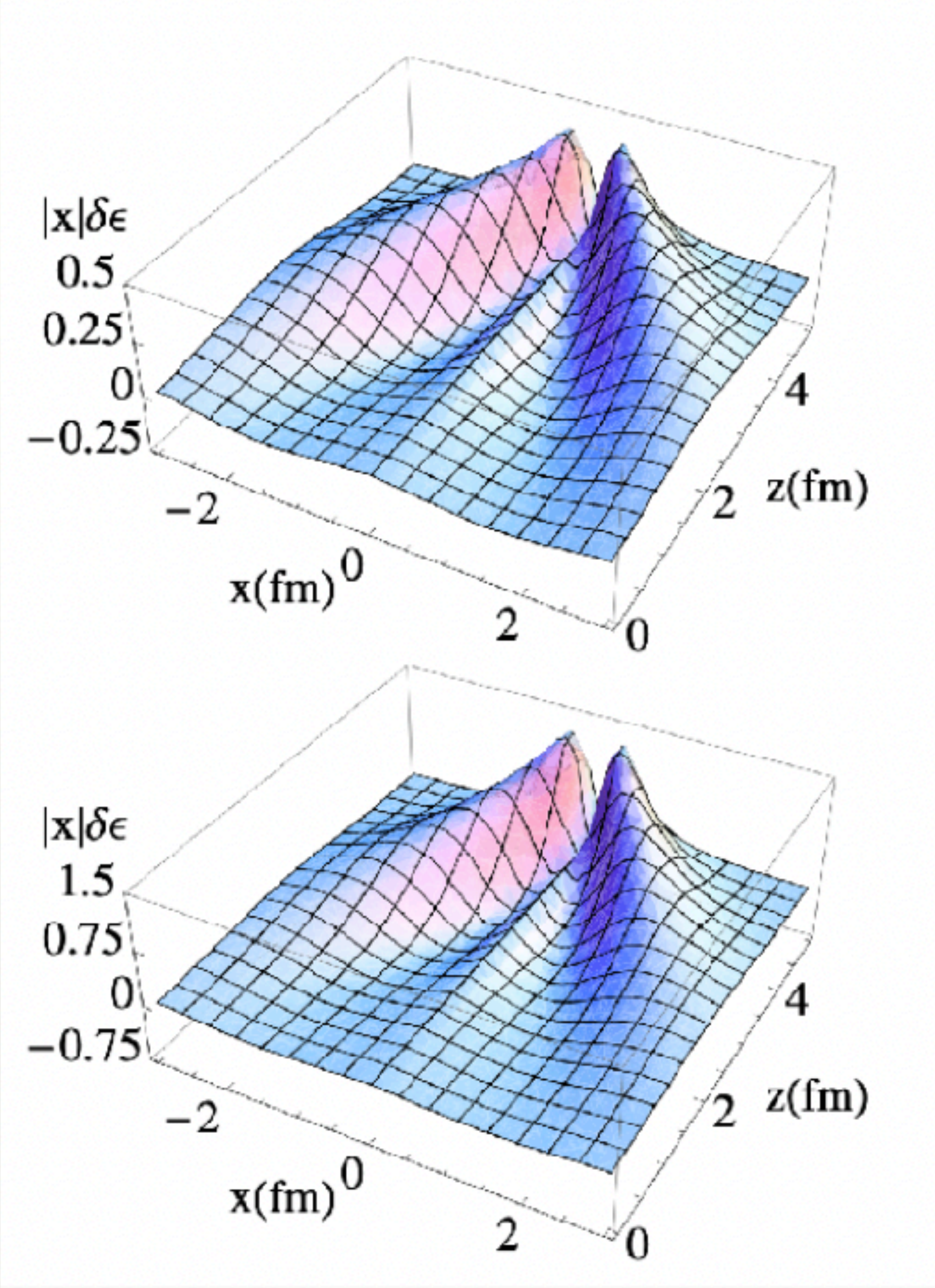
Solution

Linear hydrodynamic response



Neufeld, PRC 79 (2009) 054909

Viscosity dependence of the Mach cone structure



Qin, Majumder, Song, Heinz, PRL 103 (2009) 152303

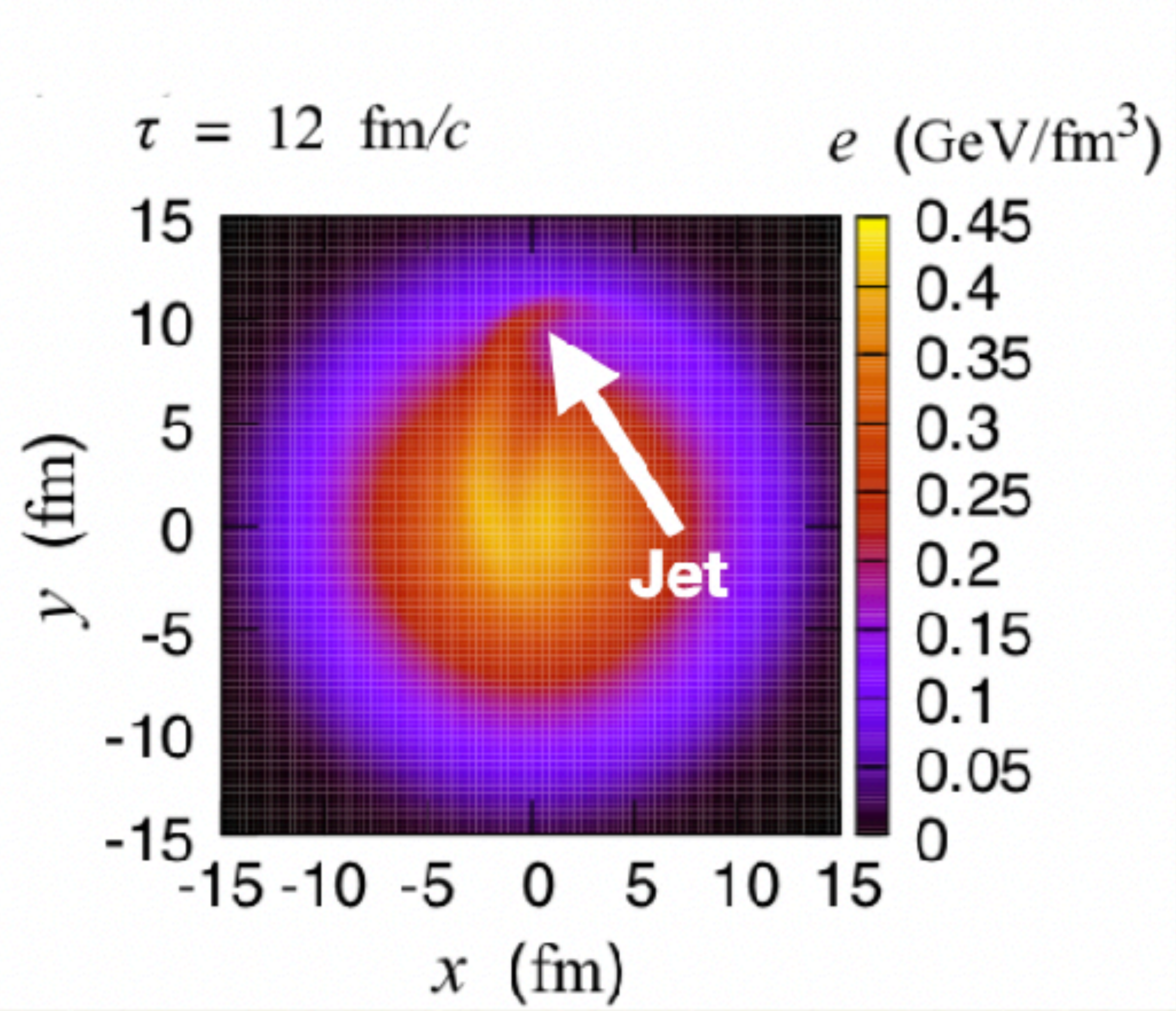
Energy deposition from single quark vs. quark jet shower

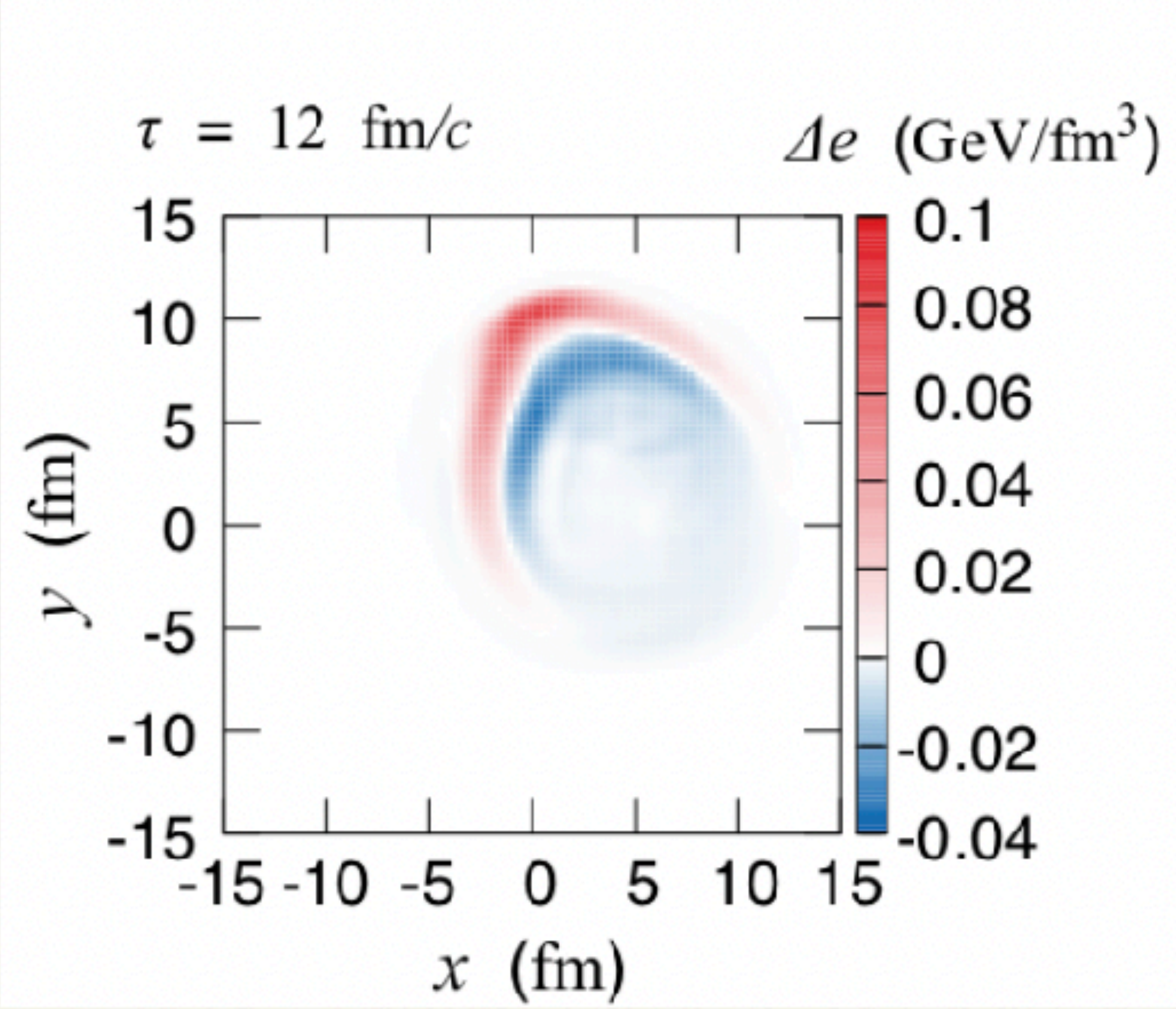
Full hydrodynamic response

When energy deposition is comparable to the unperturbed medium density

Directly solve $\partial_\mu T^{\mu\nu} = J^\nu$

- (1+1)-D hydro Floerchinger, Zapp, EPJC 74 (2014) 12
- (2+1)-D hydro Chaudhuri, Heinz, PRL 97 (2006) 062301
- (3+1)-D hydro Noronha, Torrieri, Gyulassy, Rischke, PRL 105 (2010) 222301; Tachibana, Hirano, PRC 90 (2014) 2, 021902

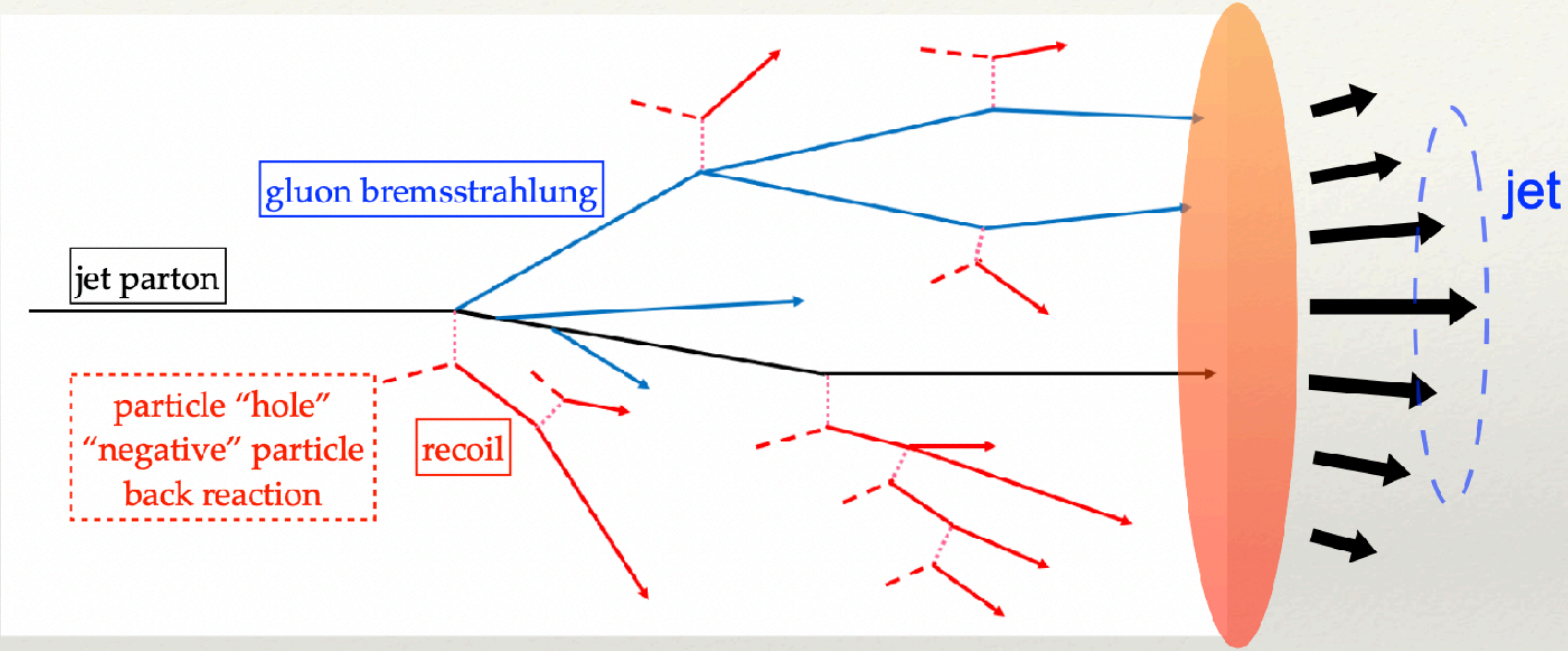




- Need background subtraction
- Distortion of Mach cone by the QGP flow

Tachibana, Chang, Qin, PRC 95 (1017) 4, 044909

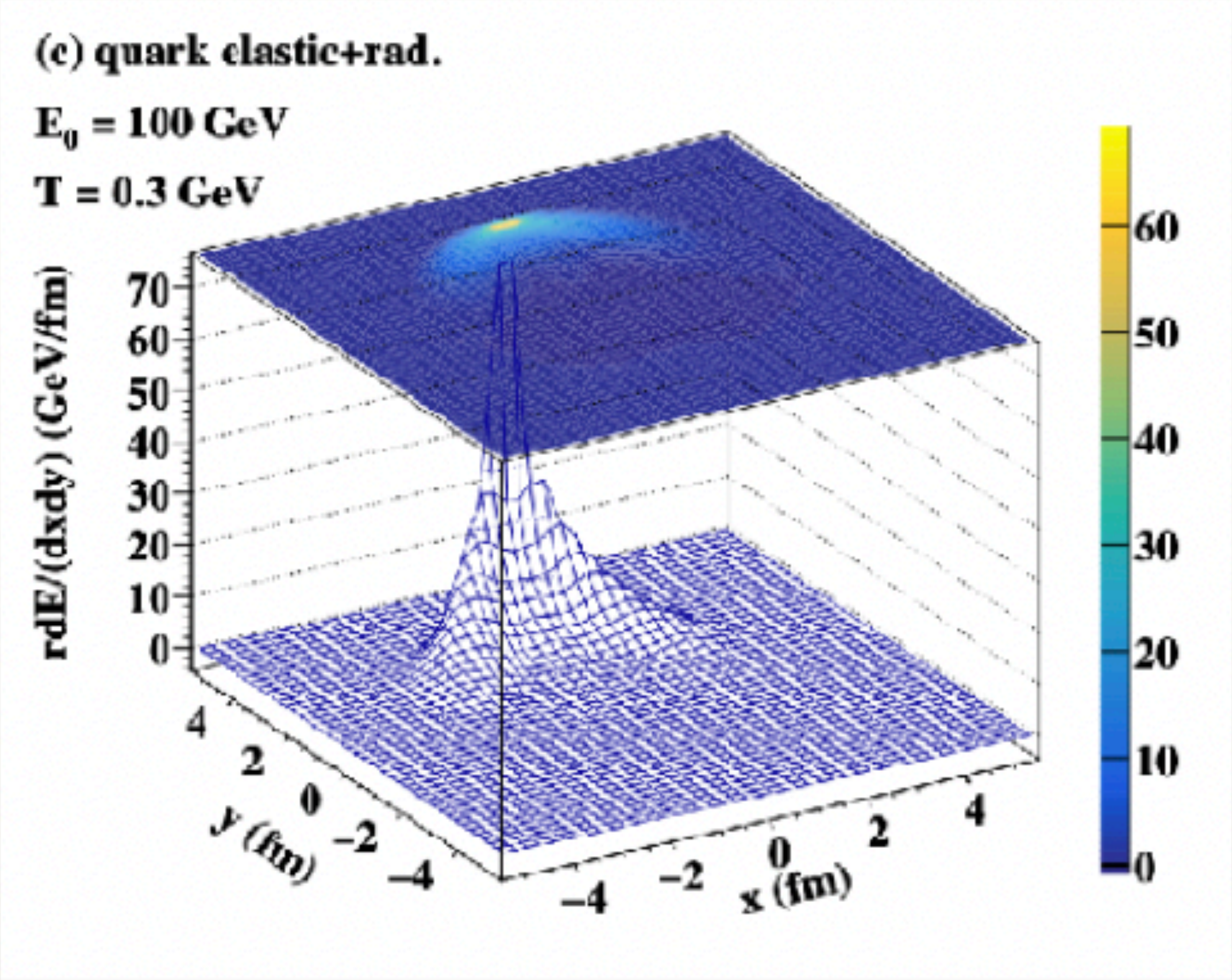
Perturbative approximation of medium response



Describe jet partons, radiated gluons, recoil partons and "negative" partons within the same perturbative transport framework, e.g. linear Boltzmann transport (LBT)

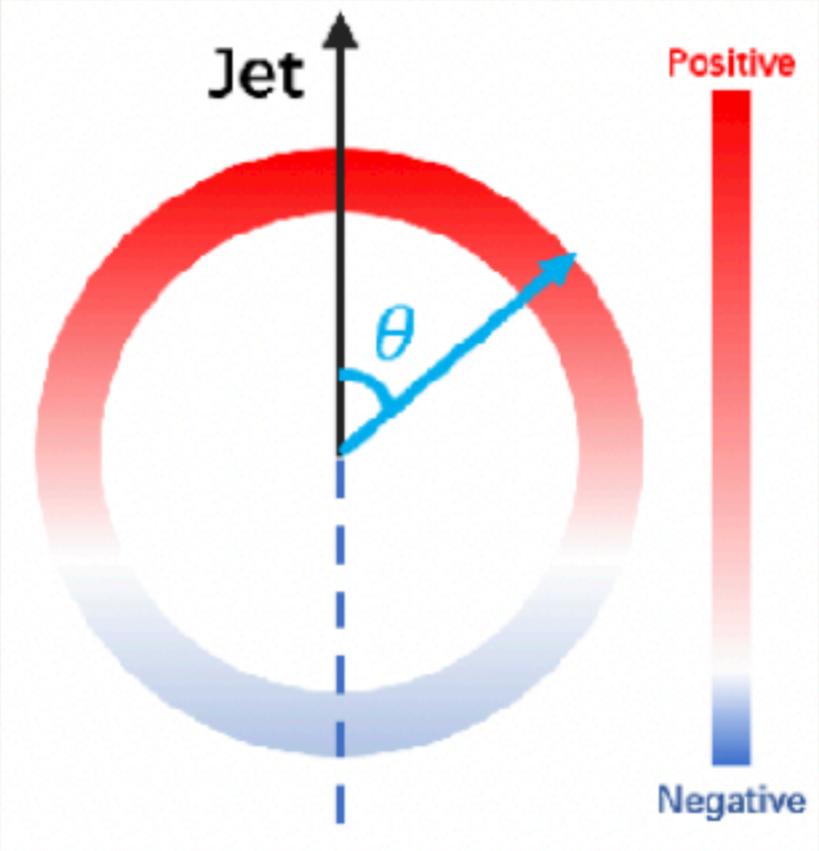
$$p_a \cdot \partial f_a(x_a, p_a) = E_a(\mathscr{C}_a^{\text{el}} + \mathscr{C}_a^{\text{inel}})$$

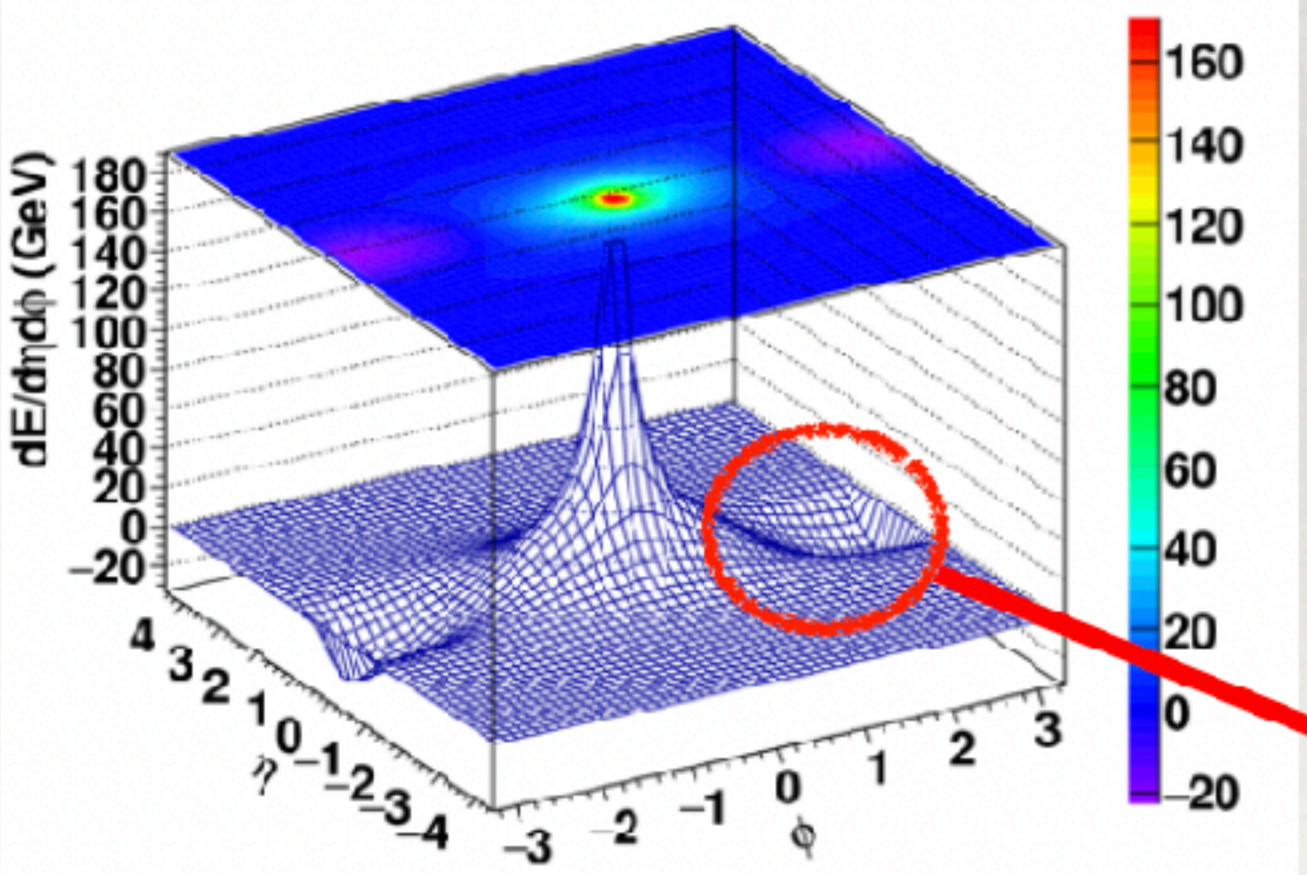
Perturbative approximation of medium response



- Energy (observable) distribution: positive - negative contribution
- Cone structure at t = 4 fm

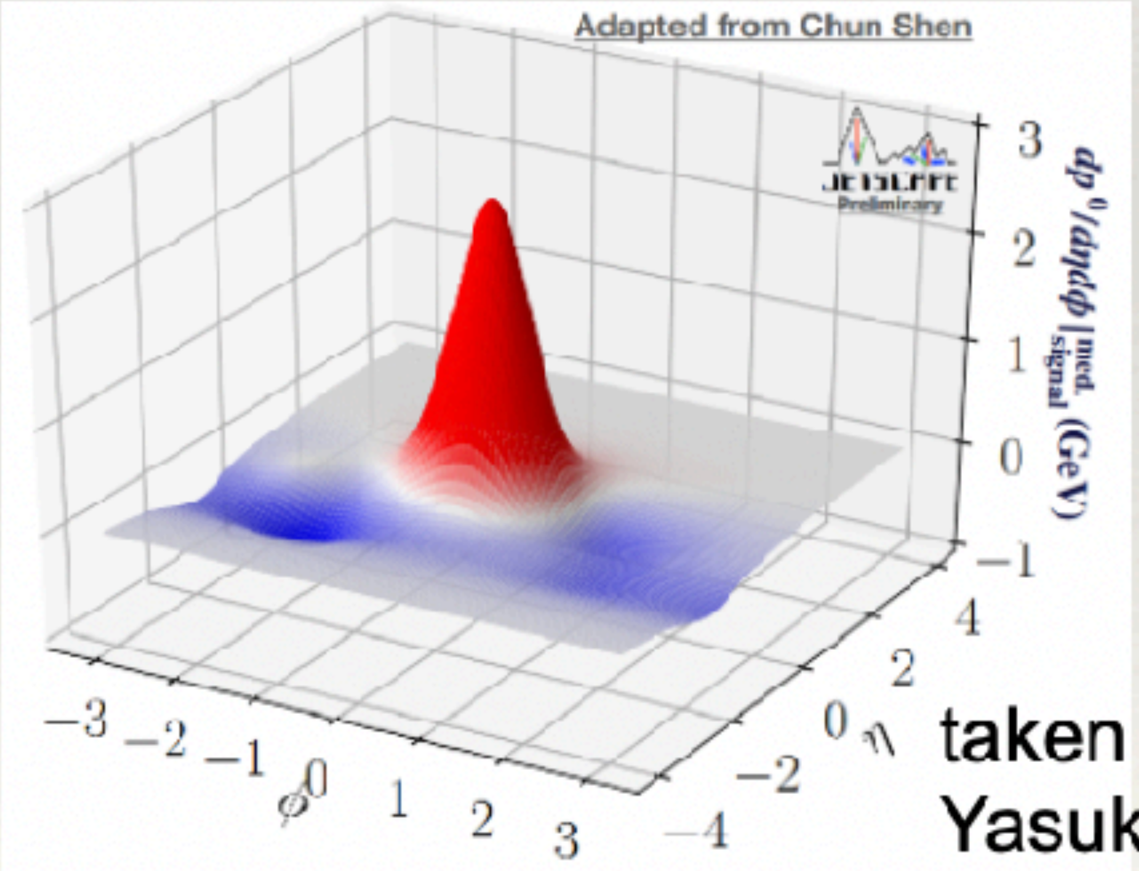
LBT simulation by Tan Luo





Reasonable approximation

diffusion wake



taken from Yasuki Tachibana

LBT with recoil approximation

JETSCAPE, put recoil into hydro evolution

State-of-the-art concurrent simulation of jet+hydro

QGP medium

Hydrodynamics

Going beyond the linear approximation

energetic partons

LBT

$$\partial_\mu T^{\mu\nu} = j^\nu$$

local medium information

$\epsilon, T, \mathbf{u}\ \ldots$

$$\frac{1}{p \cdot u} p^\mu \partial_\mu f = \mathscr{C}$$

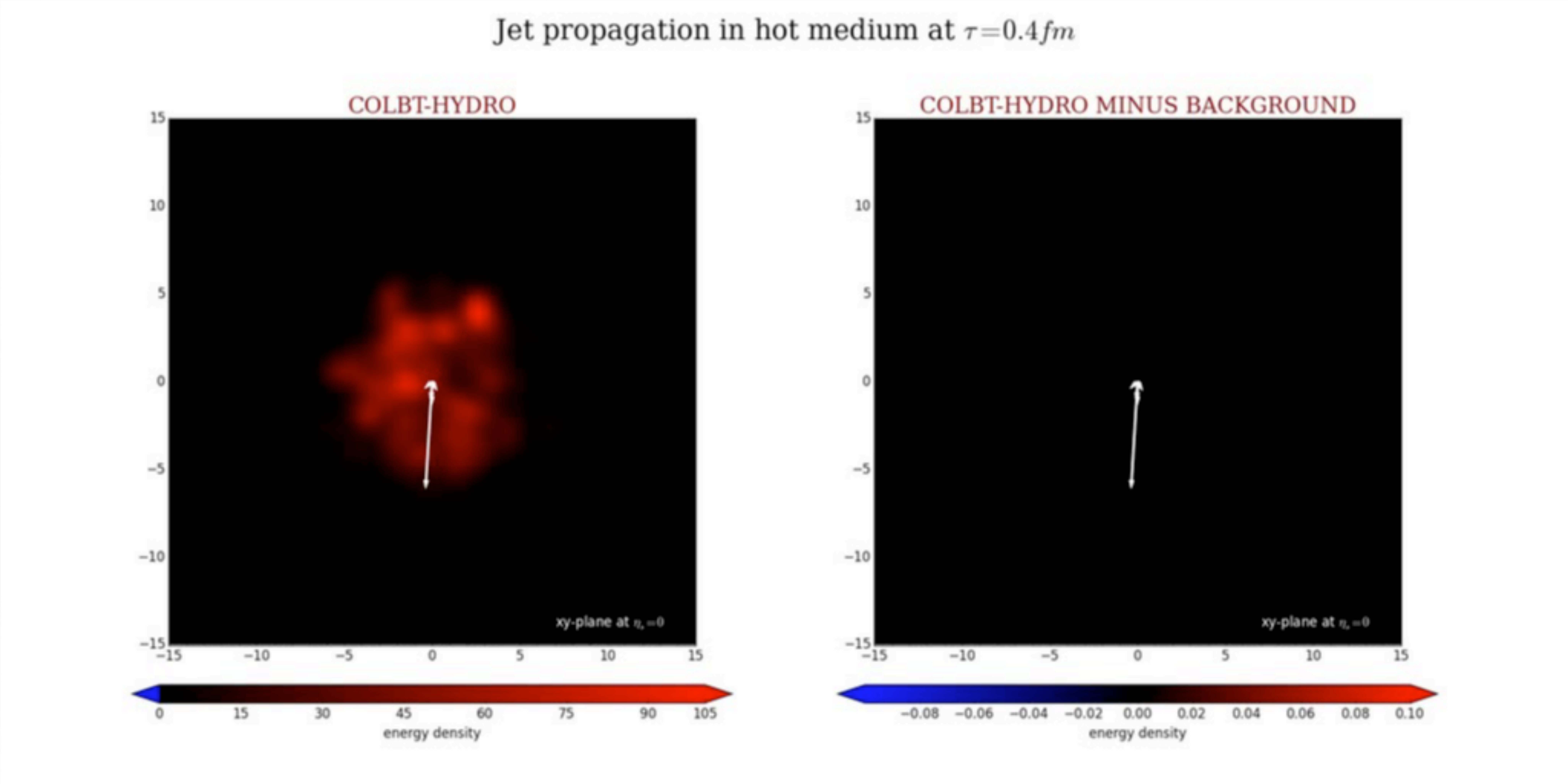
source term

jet and recoil partons with ***E* < *E*$_{cut}$** (comoving frame) [positive] + back reaction [negative]

$$j^\nu = \sum_{i=1}^{n} \frac{dp_i^\nu}{d\tau} \delta^3(\mathbf{x} - \mathbf{x_i})$$

CoLBT-hydro

Concurrent simulation of jet and medium (CoLBT-hydro)



Summary of jet quenching models with medium response

- JEWEL [BDMPS-Z] : recoiled partons transported. (modified parton shower)
- LBT [HT] : recoiled partons transported. (shower + transport)
- MARTINI [AMY] : recoiled partons transported. (shower + transport)

Recoil-medium rescattering

- CoLBT-hydro [HT] : Transport + Hydro parallel simulation. (shower + transport)
- Hybrid [AdS/CFT] : fully thermalized wake. (modified parton shower)
- Coupled Jet-Fluid [HT] : solve Boltzmann equation + Hydro simulation
- EPOS3-HQ : YaJEM + Hydro parallel simulation. (modified parton shower)

Energy momentum deposition into Hydro

Summary of jet quenching models with medium response

modified parton shower + transport

JETSCAPE

- Matter [HT] + LBT [HT] : recoiled partons transported.
- Matter [HT] + MARTINI [AMY] : recoiled partons transported.
- Matter [HT] + ADS/CFT: Hydro simulation.

Recoil-medium rescattering

Energy momentum deposition into Hydro

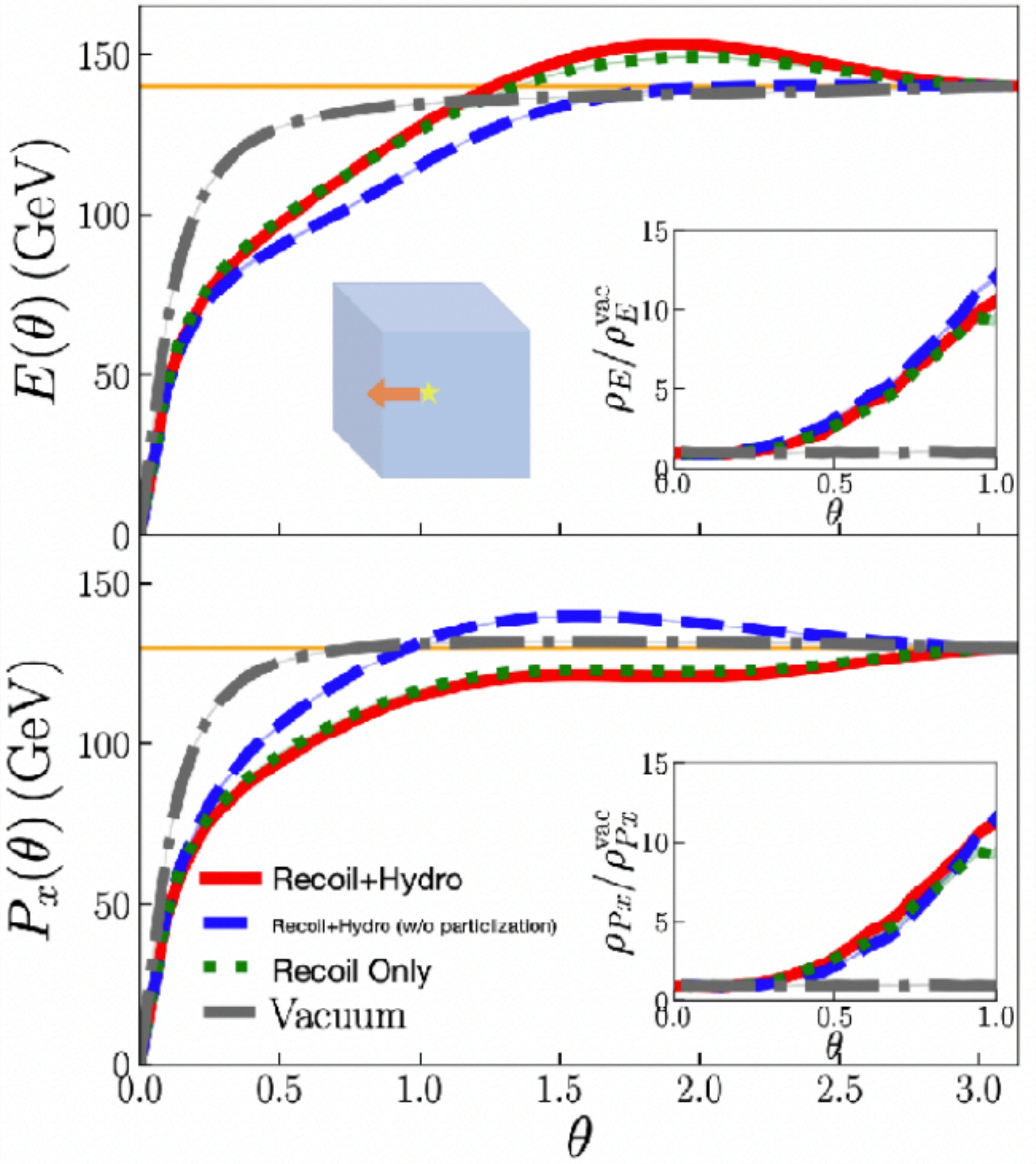
- AMPT
- BAMPS

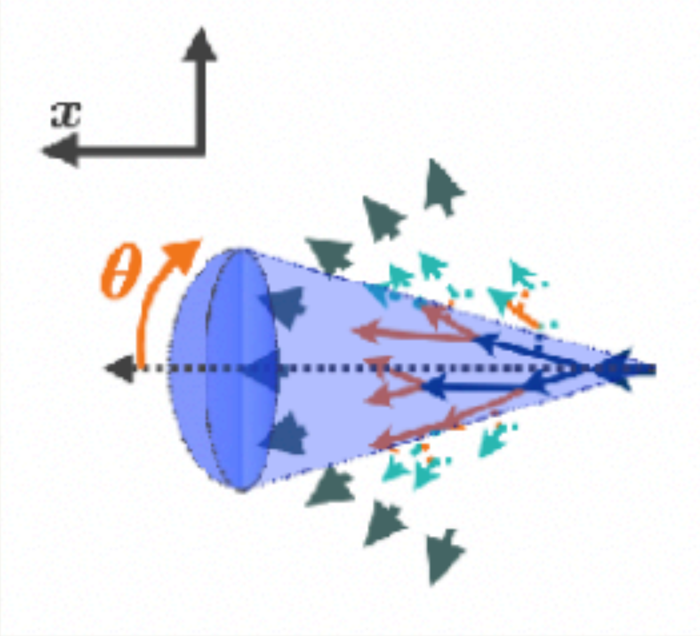
Particle scattering for both medium and jet

- Linearized viscous hydrodynamics with source

Detailed comparison between different implementations

Angular structure of jet in a brick





Recoil only *vs.* recoil+hydro

with vs. w/o particlization in hydro

Integrated and differentiated jet energy (momentum)

$$E(\theta) = \int_0^{\theta} d\theta' \frac{dE}{d\theta'}, \quad \rho_E(\theta) = \frac{1}{E(\theta = 0.4)} \frac{dE}{d\theta}$$

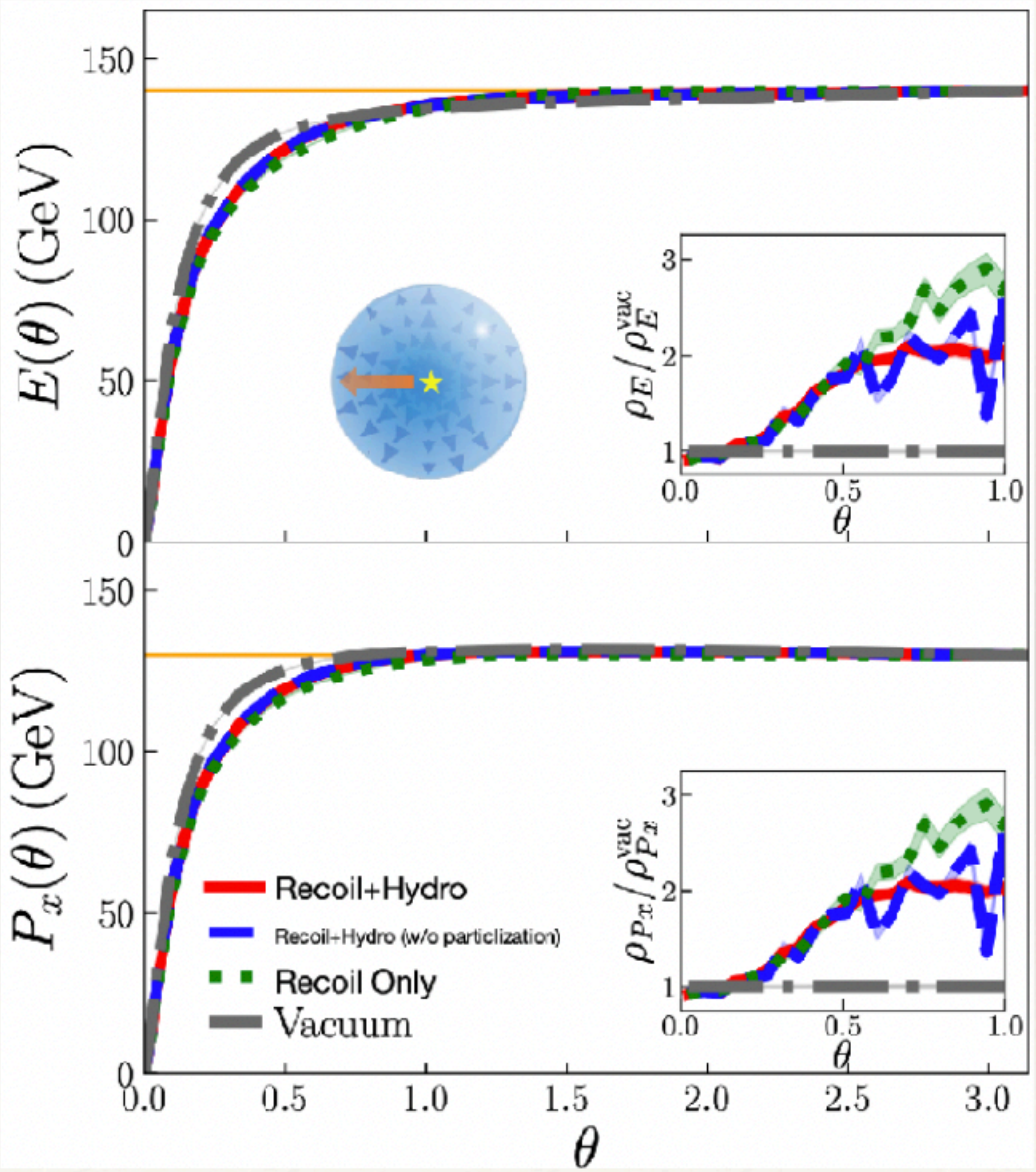
Similar structure between recoil only and recoil+hydro

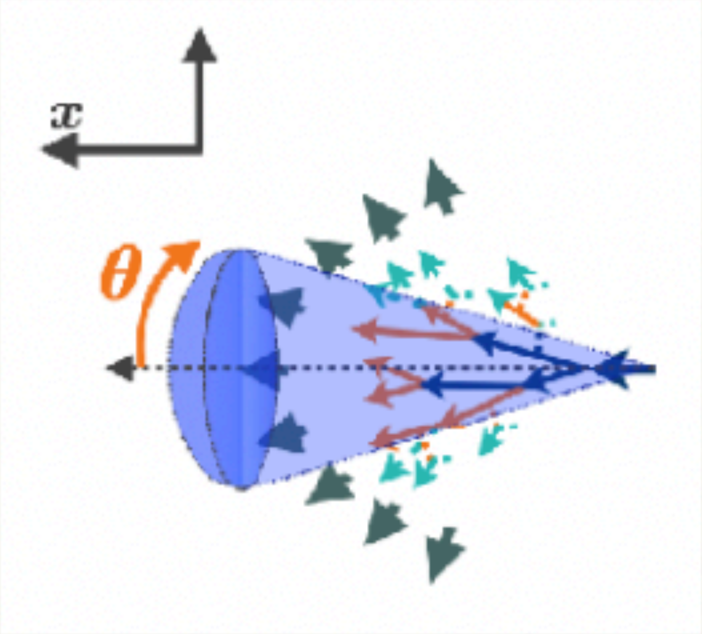
- Backward suppression due to holes (recoil only)
- Backward suppression due to particlization in (recoil+hydro)

Tachibana, Shen, Majumder, arXiv:2001.08321

Detailed comparison between different implementations

Angular structure of jet in an expanding medium





Different flow effects between recoil only and recoil+hydro

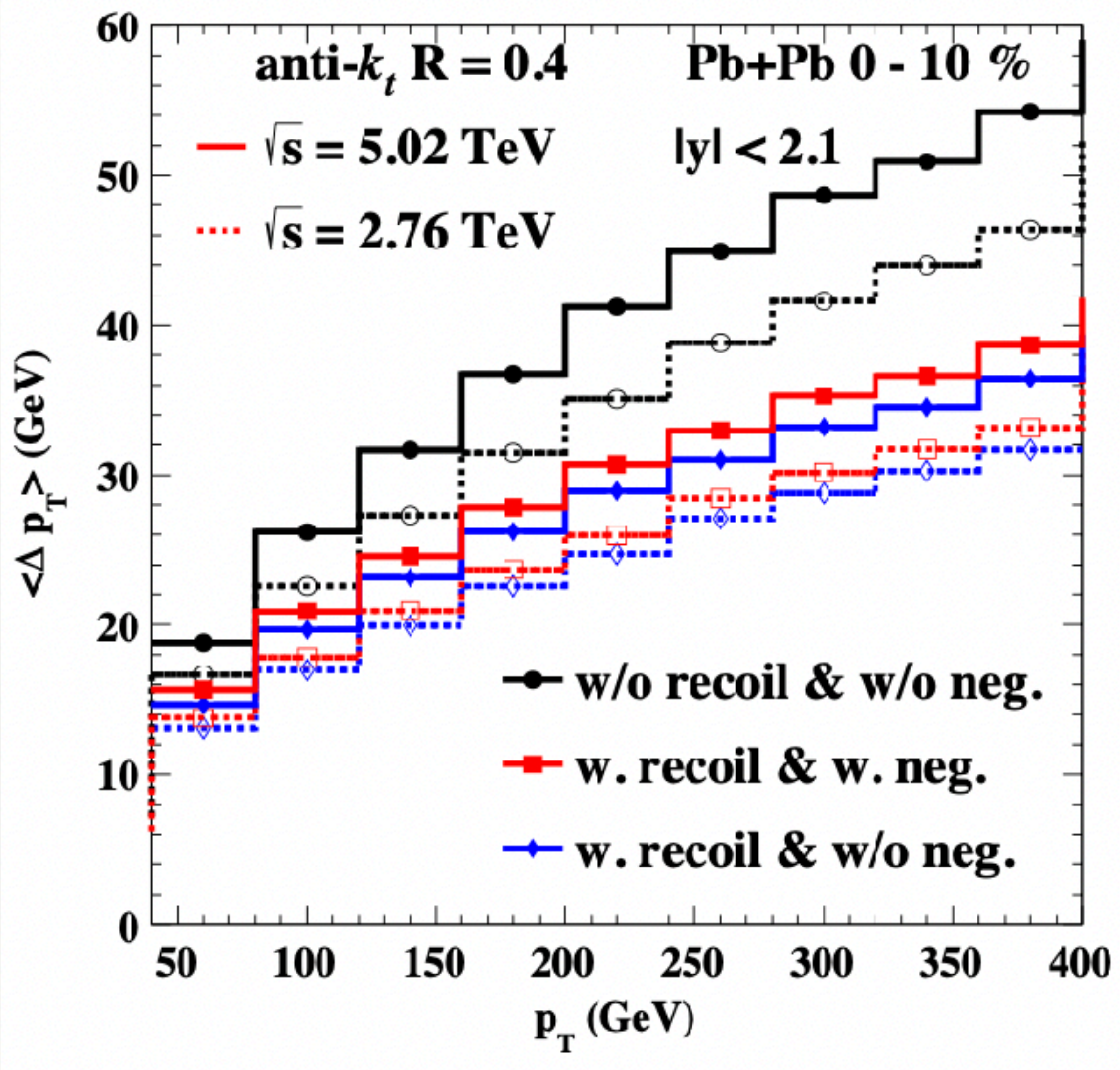
- Collimation due to push by the flow
- No backward suppression due to holes (recoil only)
- No backward suppression due to particlization in (recoil+hydro)
- Different flow effects in hydro response

Tachibana, Shen, Majumder, arXiv:2001.08321

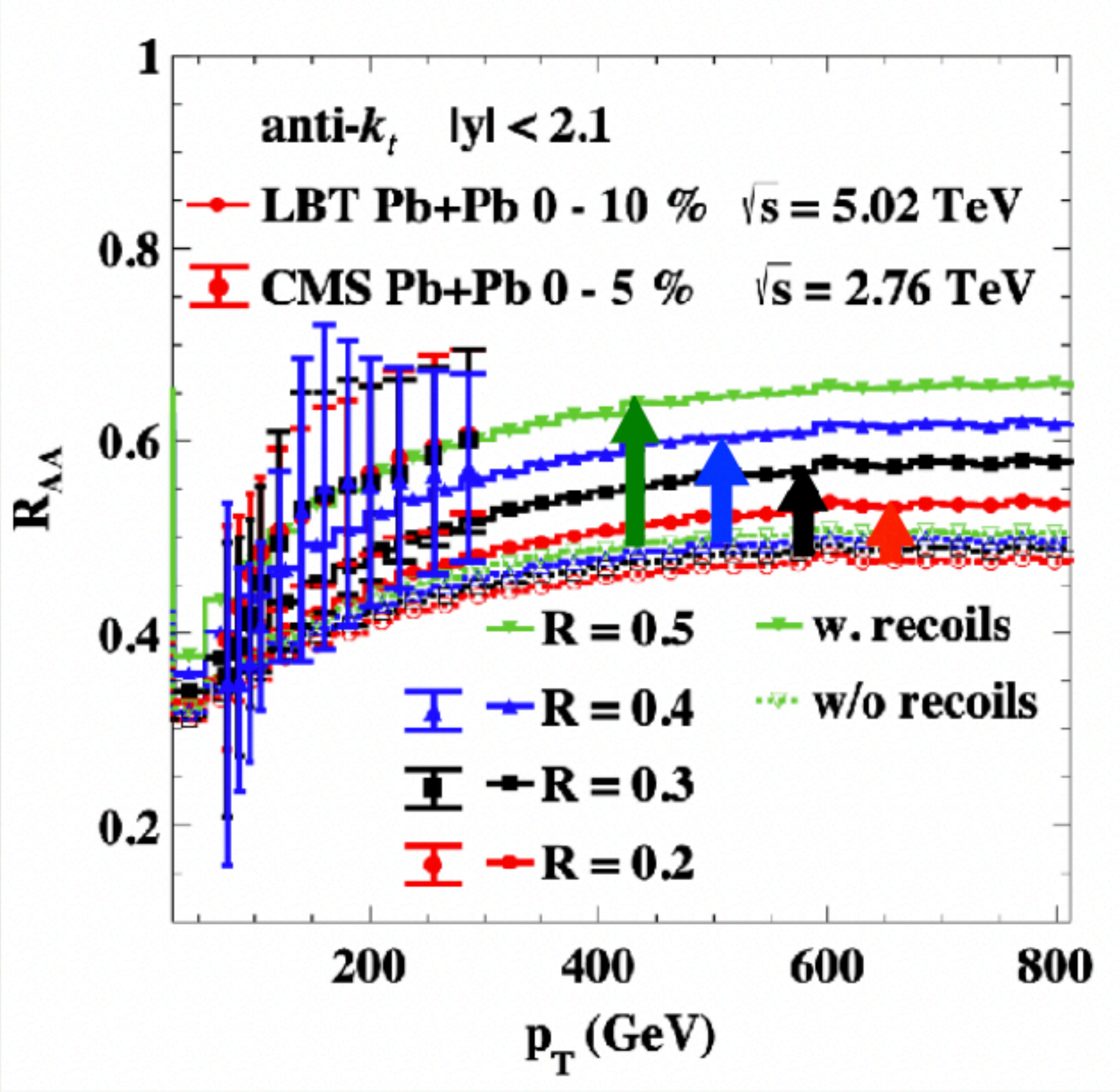
Effects on experimental observables

Jet R_{AA}

Energy loss

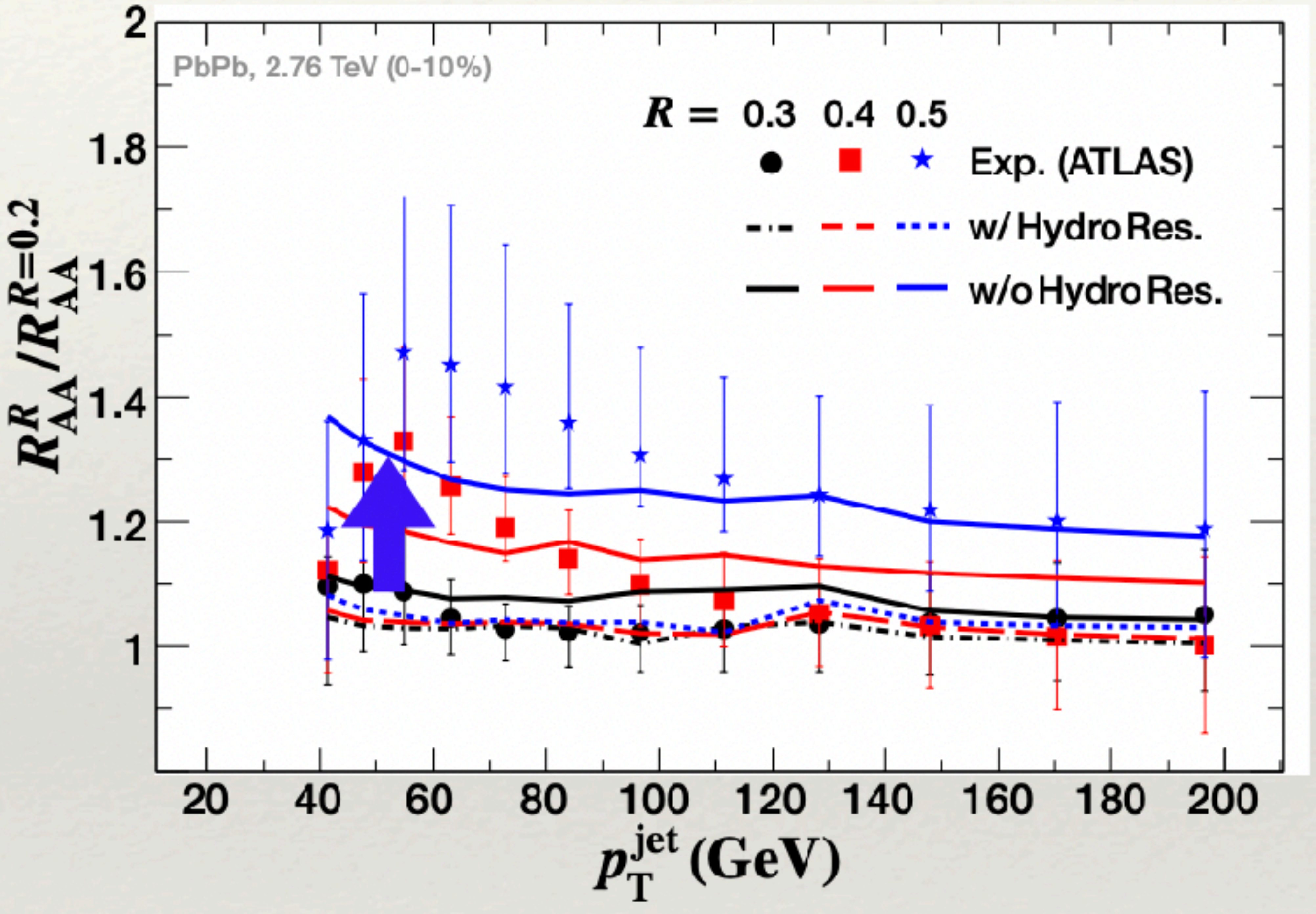


R_{AA}

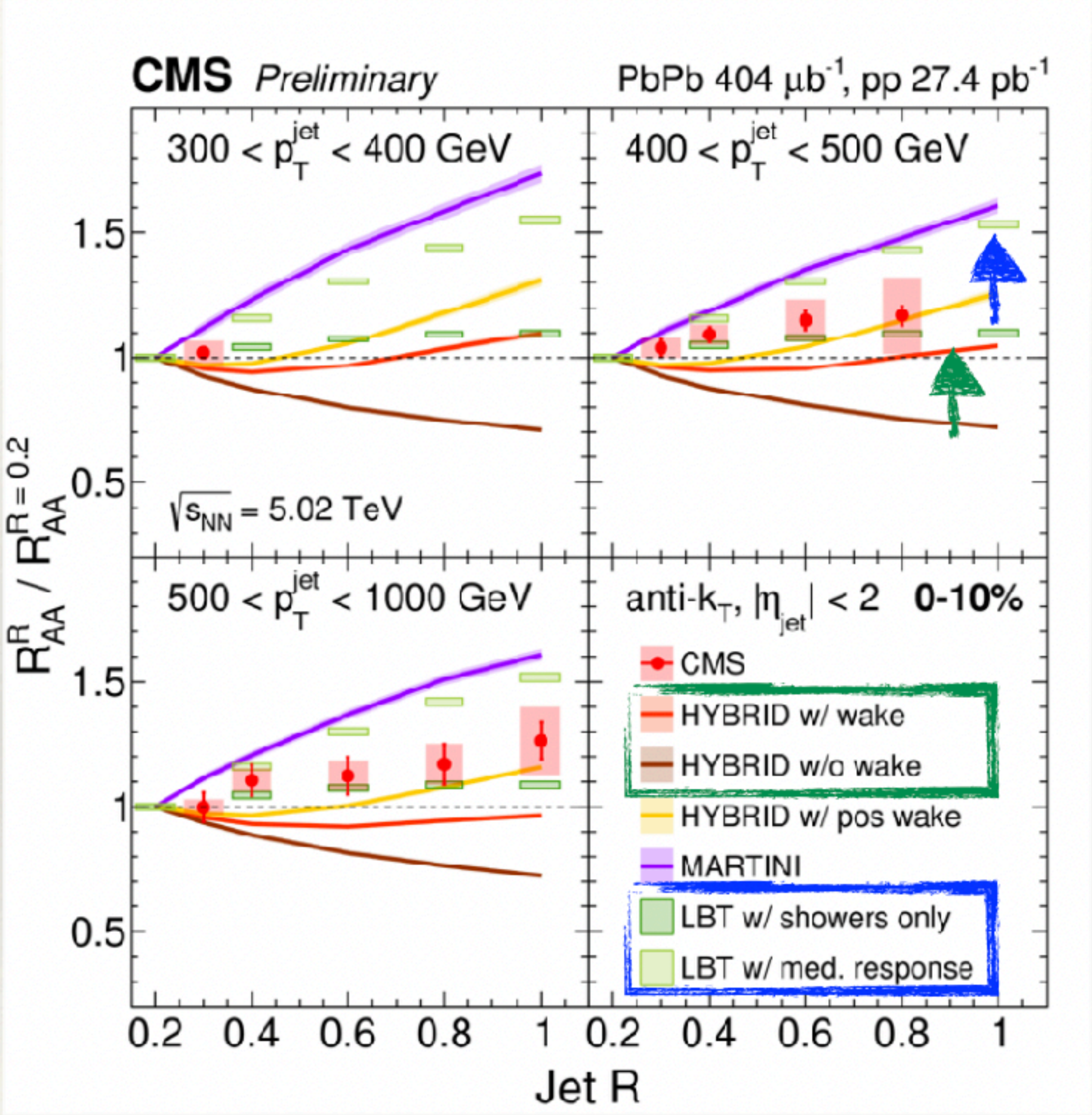


He, Cao, Chen, Luo, Pang, Wang, PRC 99 (2019) 5, 054911

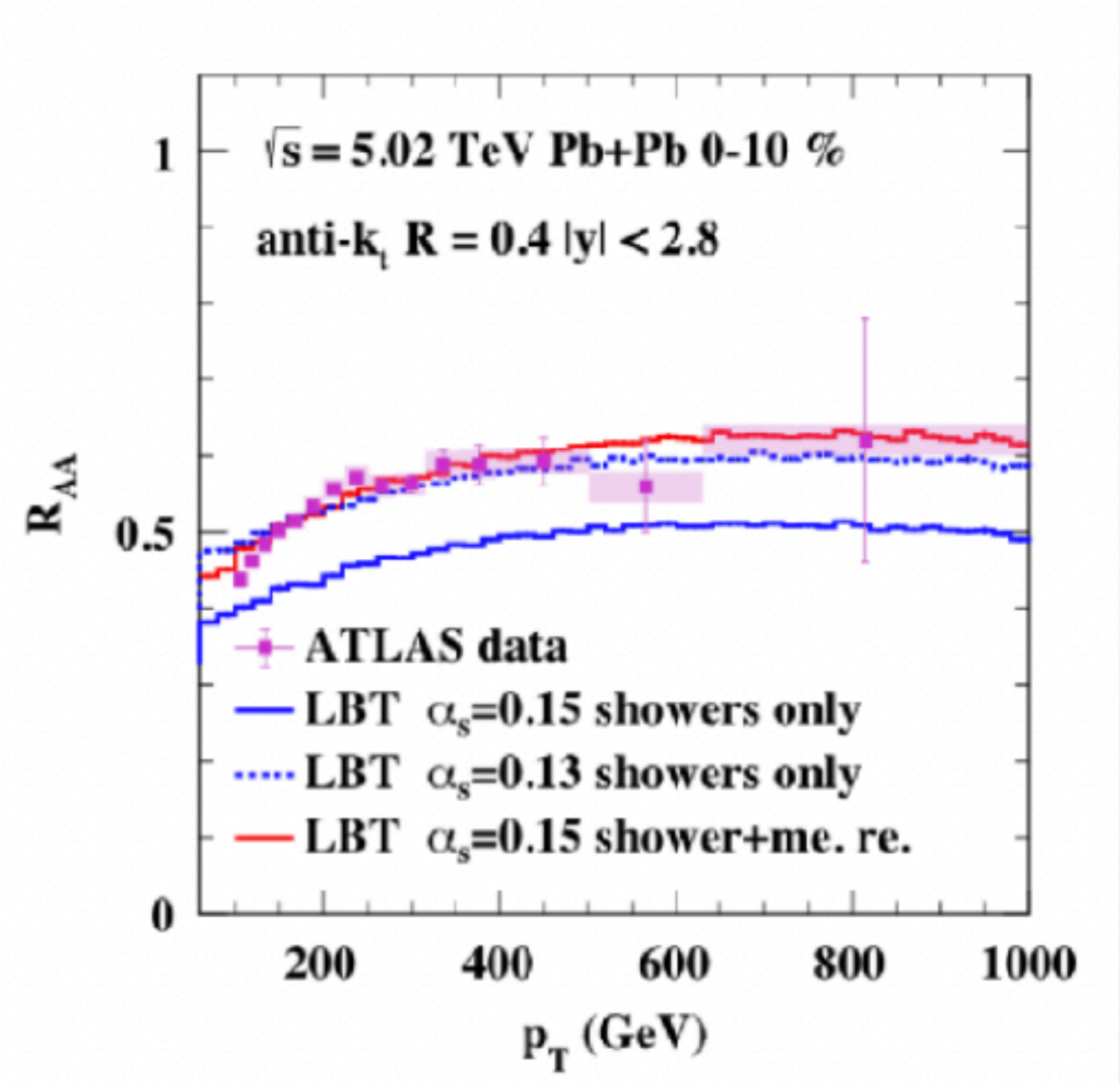
Cone size dependence of jet R_{AA}

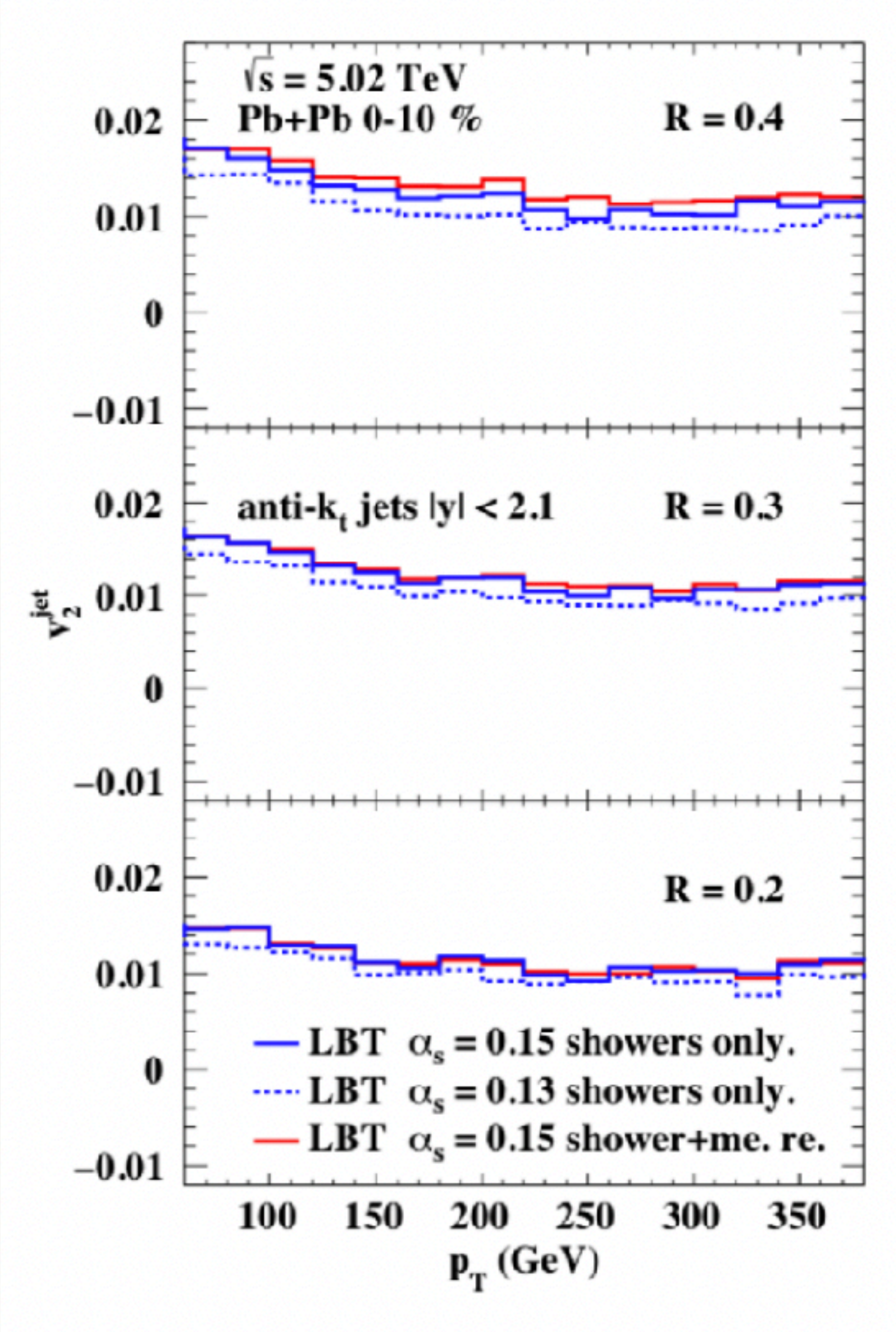


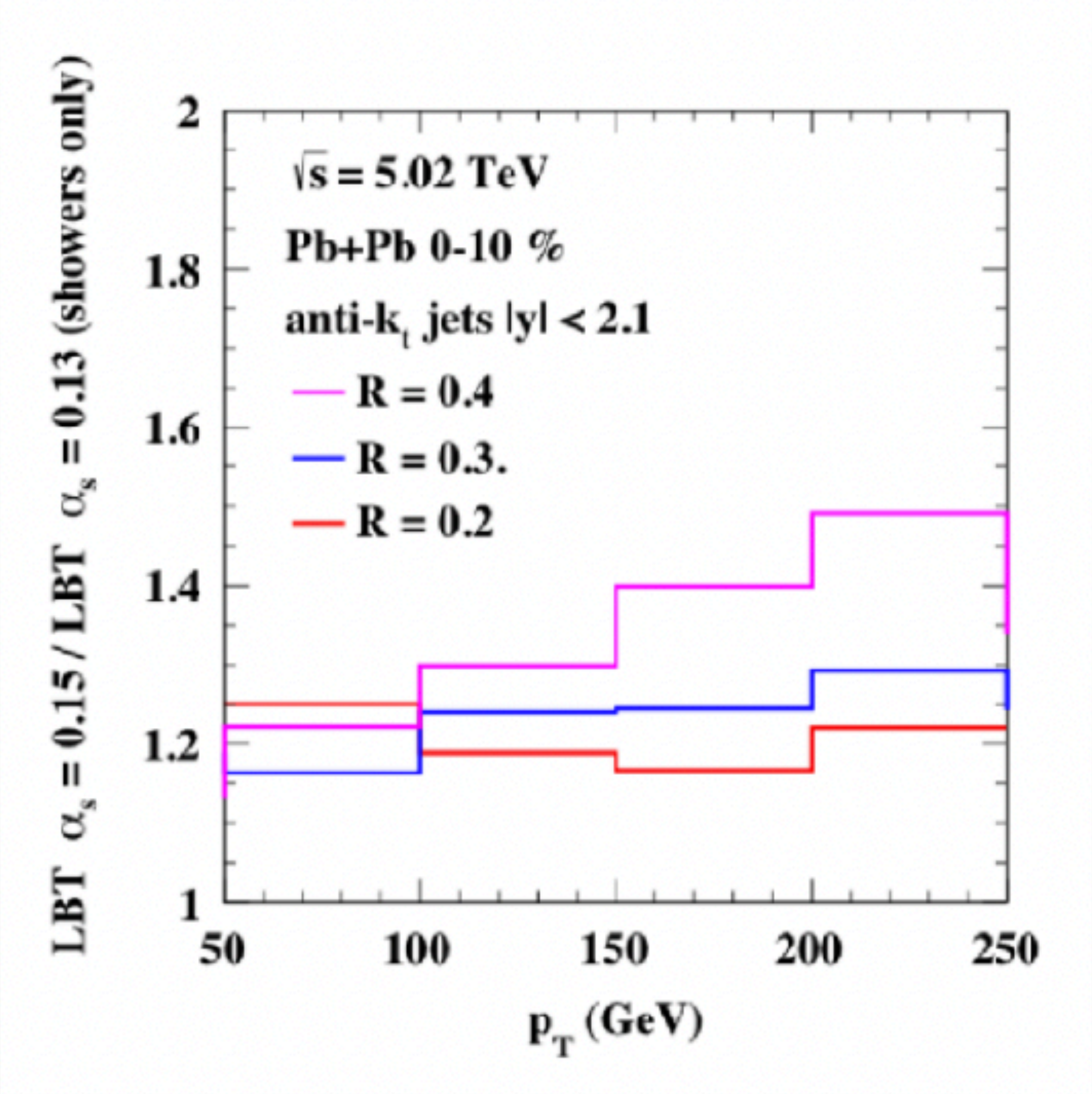
Chang, Tachibana, Qin, PLB 801 (2020) 135181



Jet v_2







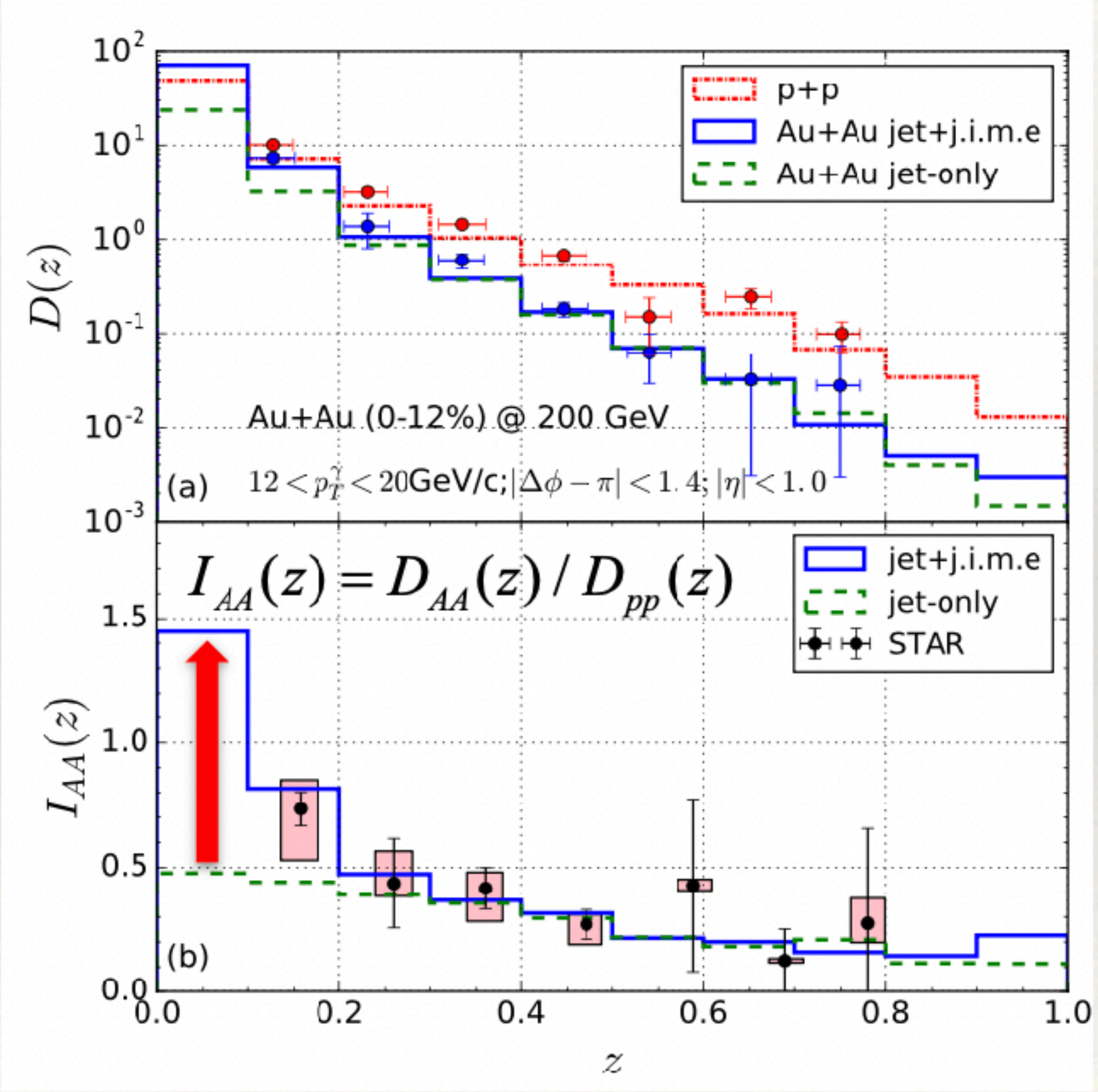
Re-adjust α_s to fit R_{AA} with and without medium response

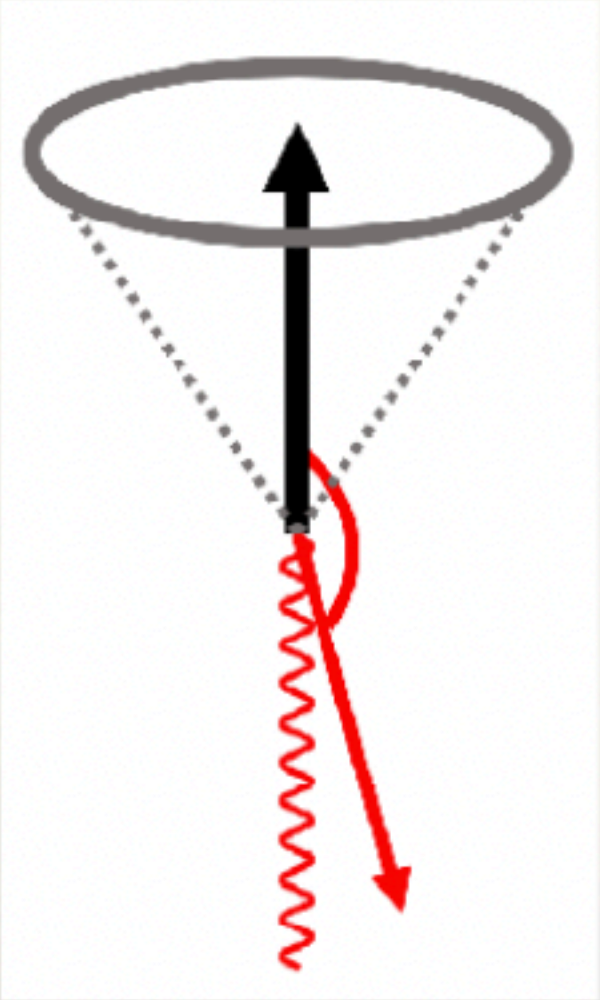
Path-length vs. flow effects

LBT calculation from Yayun He

Medium modification of γ-triggered hadron yield

CoLBT-hydro: $D(z) = \frac{dN_h}{dydz}|_{LBT} + \frac{dN_h}{dydz}|^{w/\ jet}_{hydro} - \frac{dN_h}{dydz}|^{w.o/\ jet}_{hydro}$ $\quad z = p_T^h / p_T^\gamma$





Suppression of leading hadrons at intermediate and large z

LBT:

hard parton energy loss

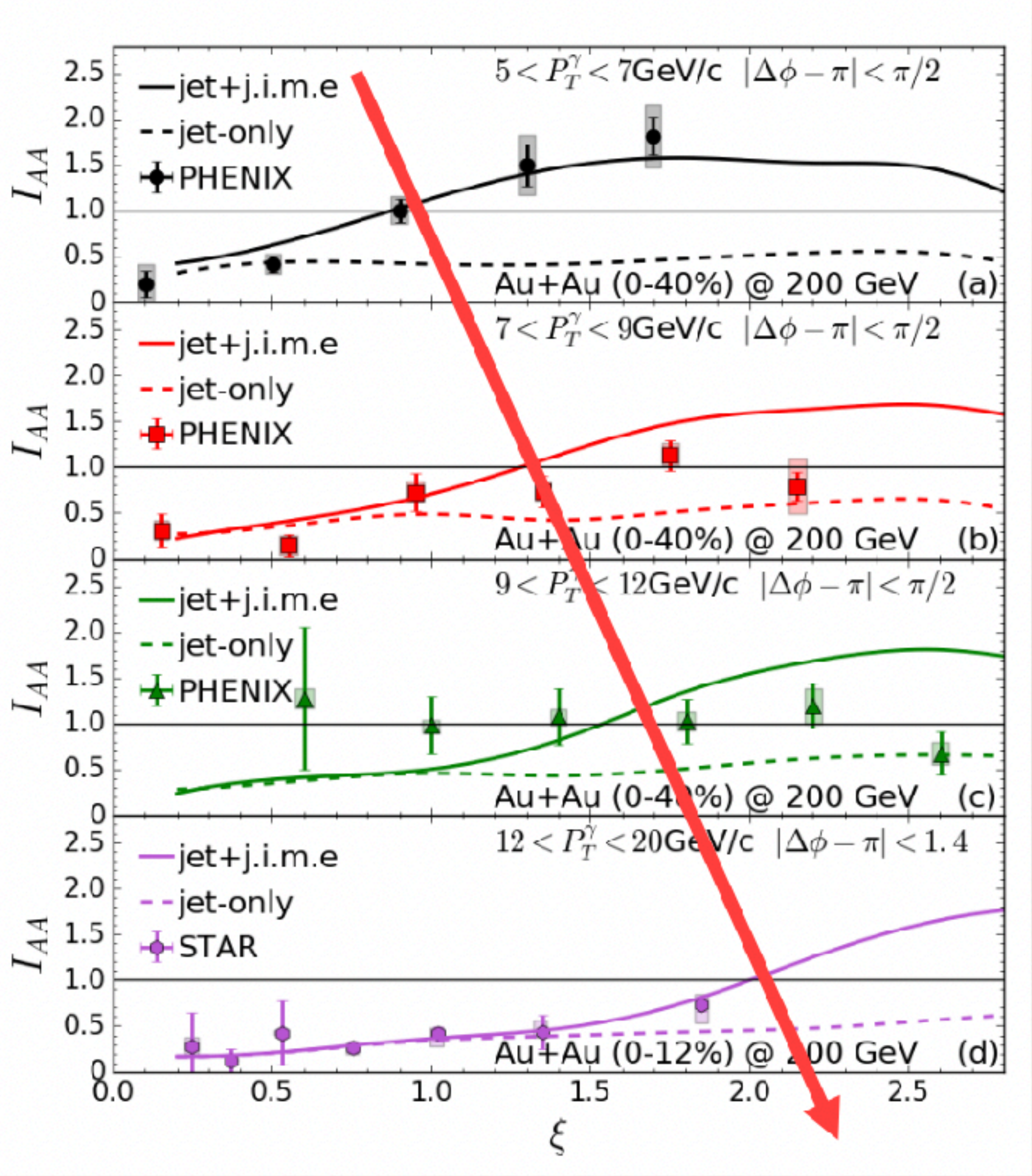
Enhancement of soft hadrons at small z

Hydro:

jet-induced medium excitation (j.i.m.e.)

Chen, Cao, Luo, Pang, Wang, PLB 777 (2018) 86

Medium modification of γ-triggered hadron yield



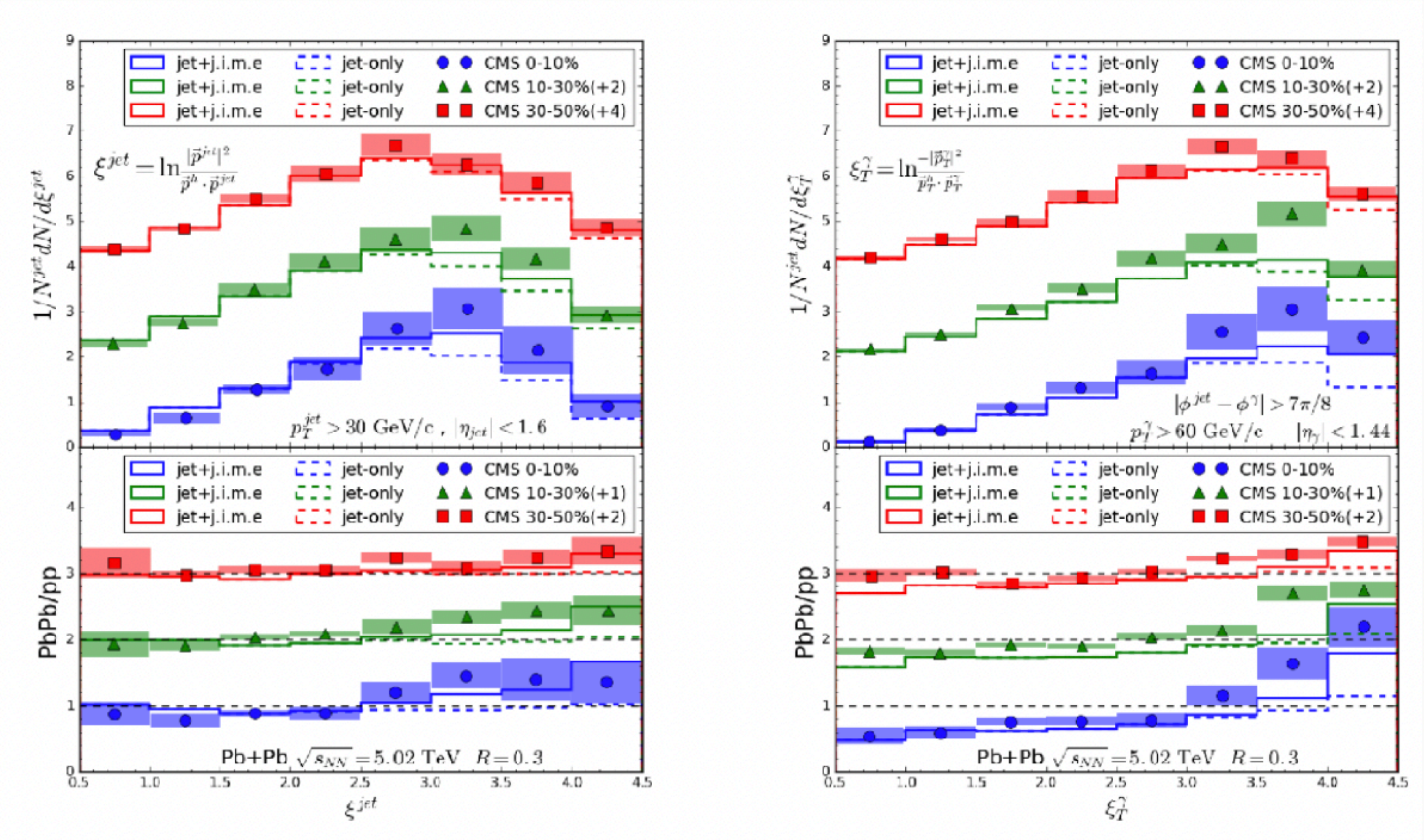
Change of variable: $z = p_T^h \,/\, p_T^\gamma \longrightarrow \xi = \log\frac{1}{z}$

Suppression of hadrons at small ξ (large z), enhancement at large ξ (small z)

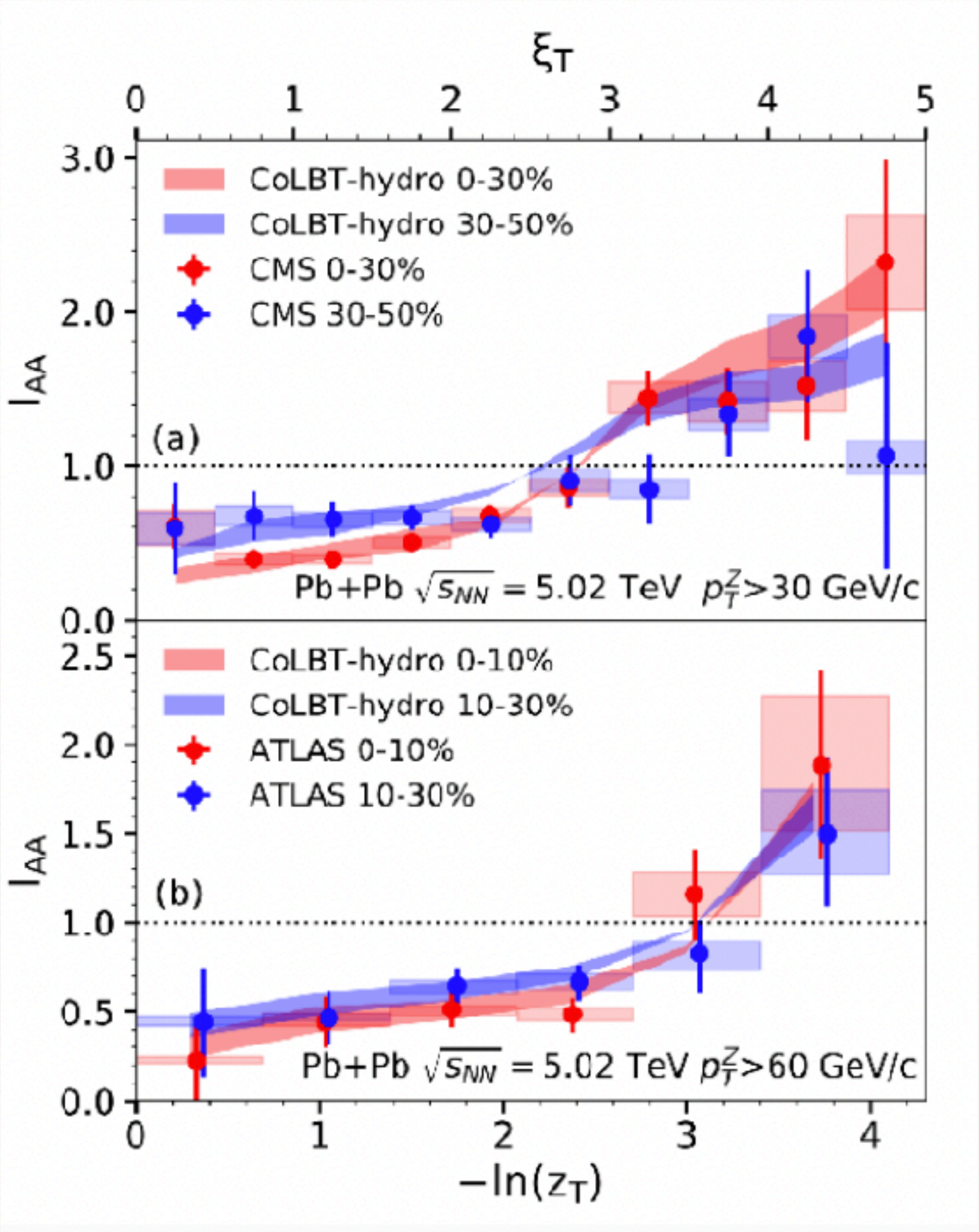
With increasing photon (γ) p_T

- Transition point from suppression to enhancement shifts to larger ξ
- Transition point corresponds to a fixed p_T range of soft hadrons (~ 2GeV)
- Unique feature of j.i.m.e. — thermal property of the medium, independent of the jet energy

γ-jet fragmentation function and Z-triggered hadron yield

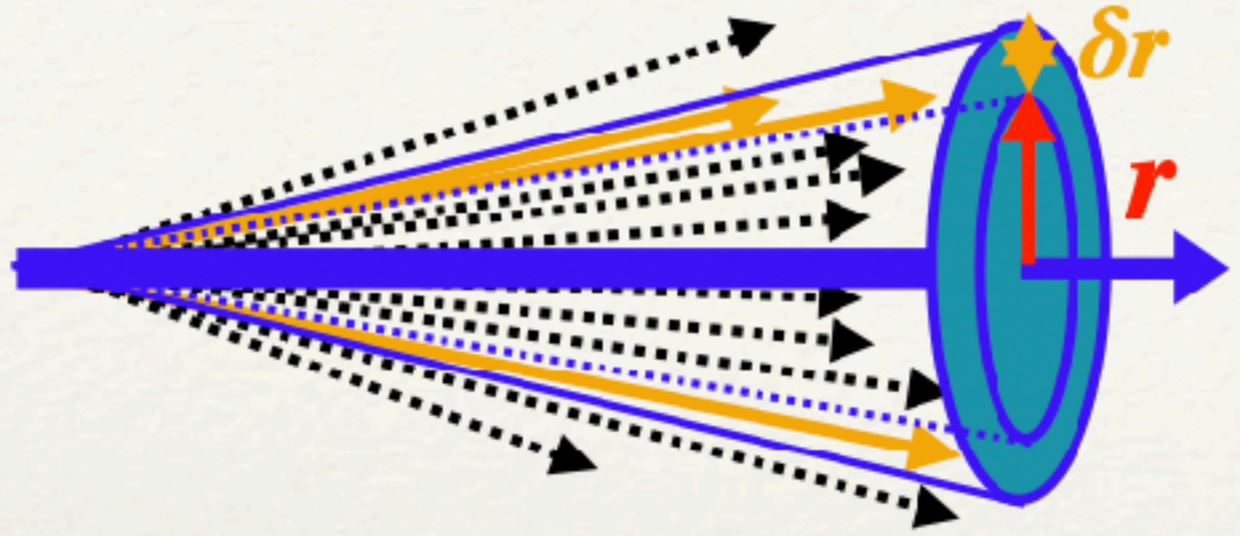


Chen, Cao, Luo, Pang, Wang, PLB 810 (2020) 135783

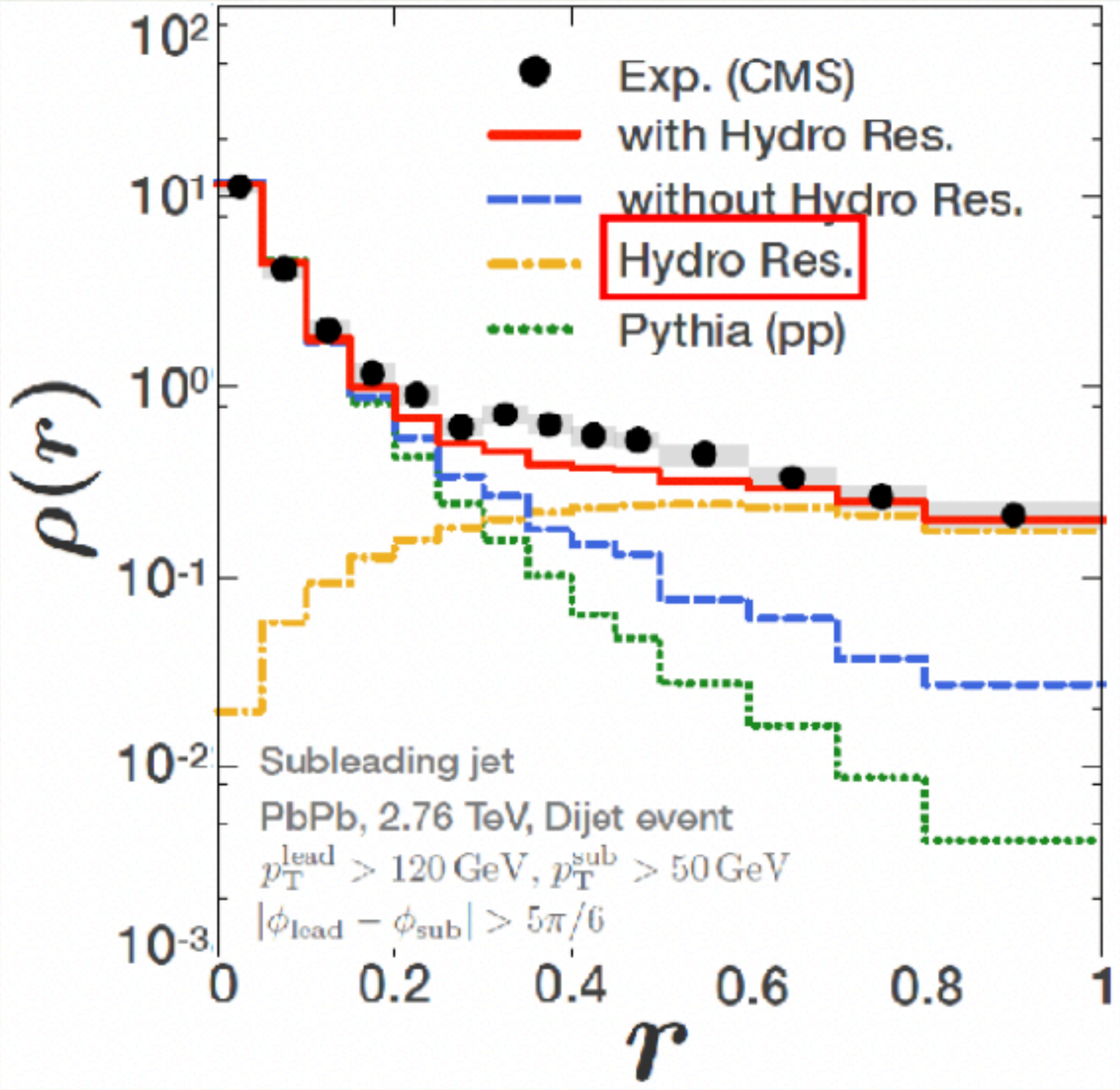


Chen, Yang, He, Ke, Pang, Wang, arXiv:2010.05422

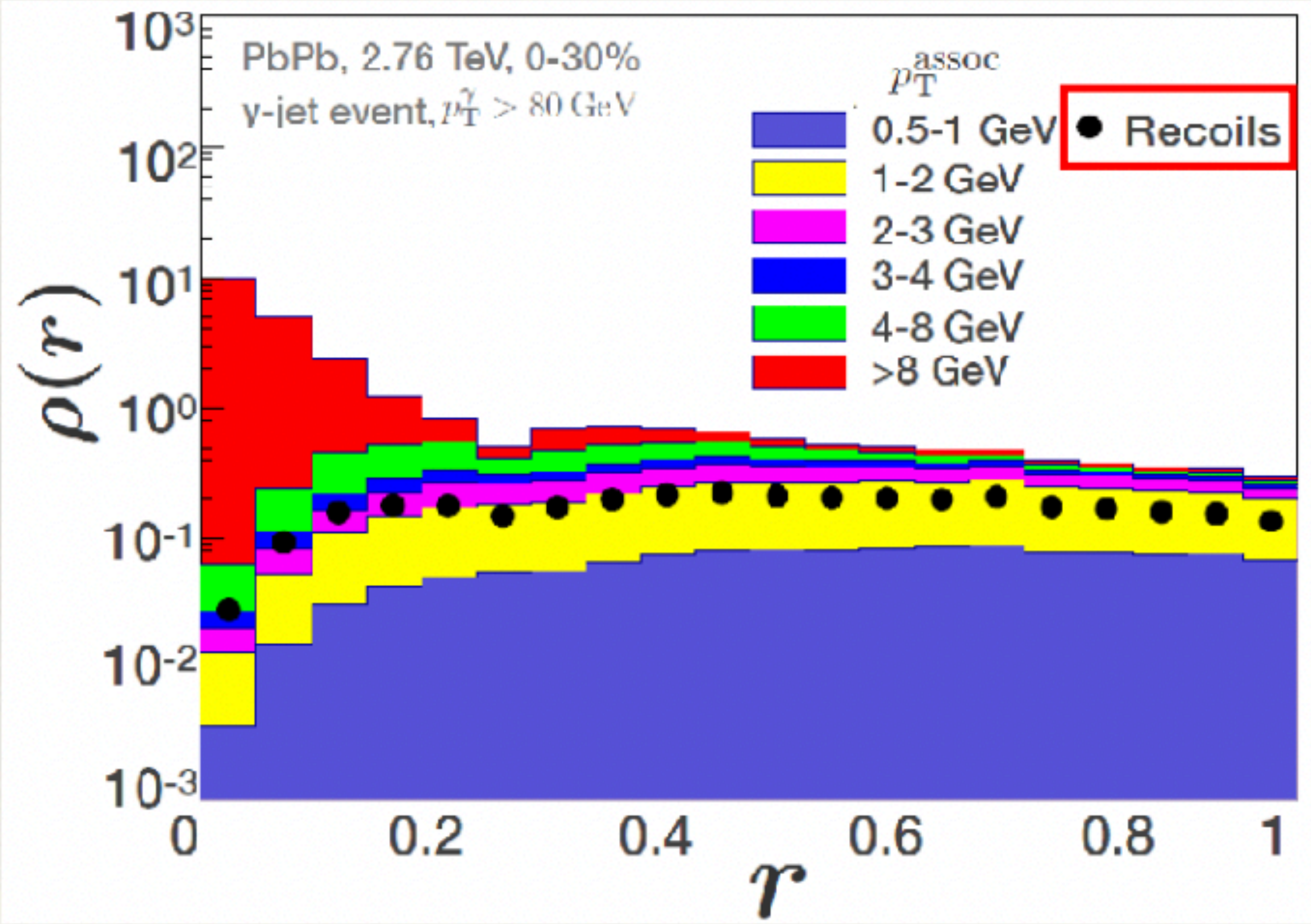
Jet shape



$$\rho(r) = \frac{1}{\delta r}\frac{\sum_{i\in(r-\delta r/2,\, r+\delta r/2)} p_T^i}{\sum_{i\in(0,R)} p_T^i} \qquad \left(r = \sqrt{(\eta_p - \eta^{jet})^2 + (\phi_p - \phi^{jet})^2} \right)$$

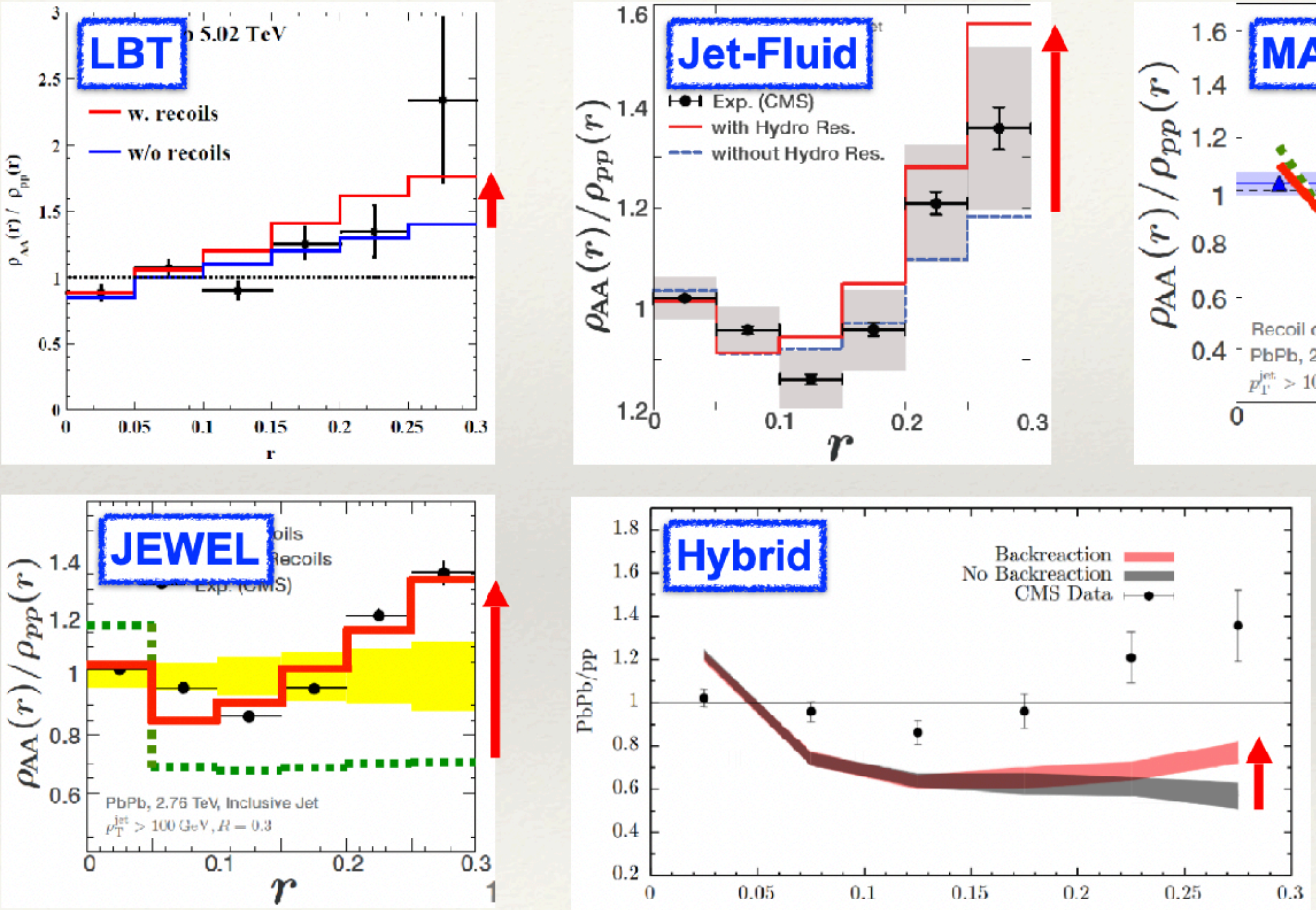


Tachibana, Chang, Qin, PRC 95 (2017) 4, 044909



Luo, Cao, He, Wang, PLB 782 (2018) 707

Medium modification of jet shape



LBT: PLB 782 (2018) 707

Jet-Fluid: PRC 95 (2017) 4, 044909

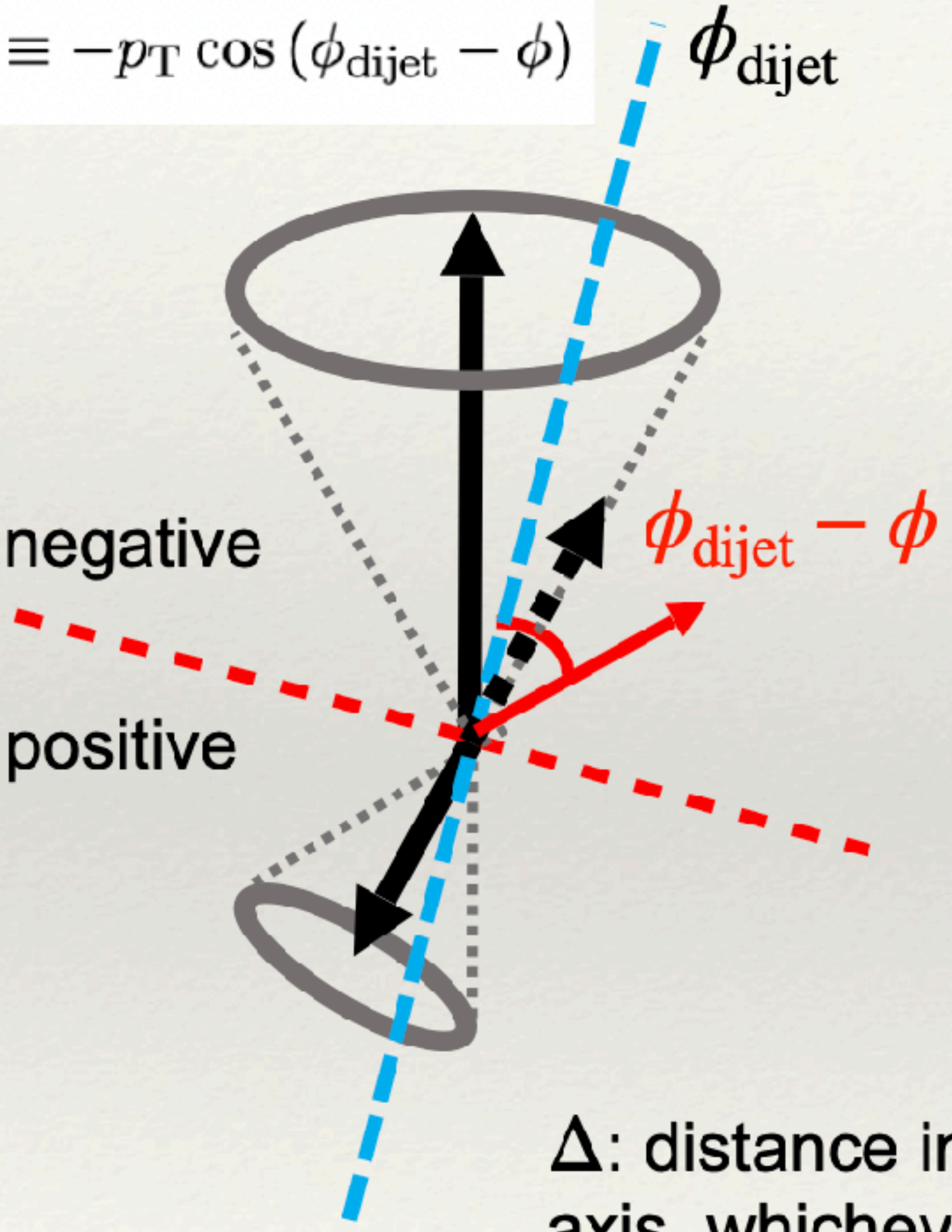
MARTINI: NPA 982 (2019) 643

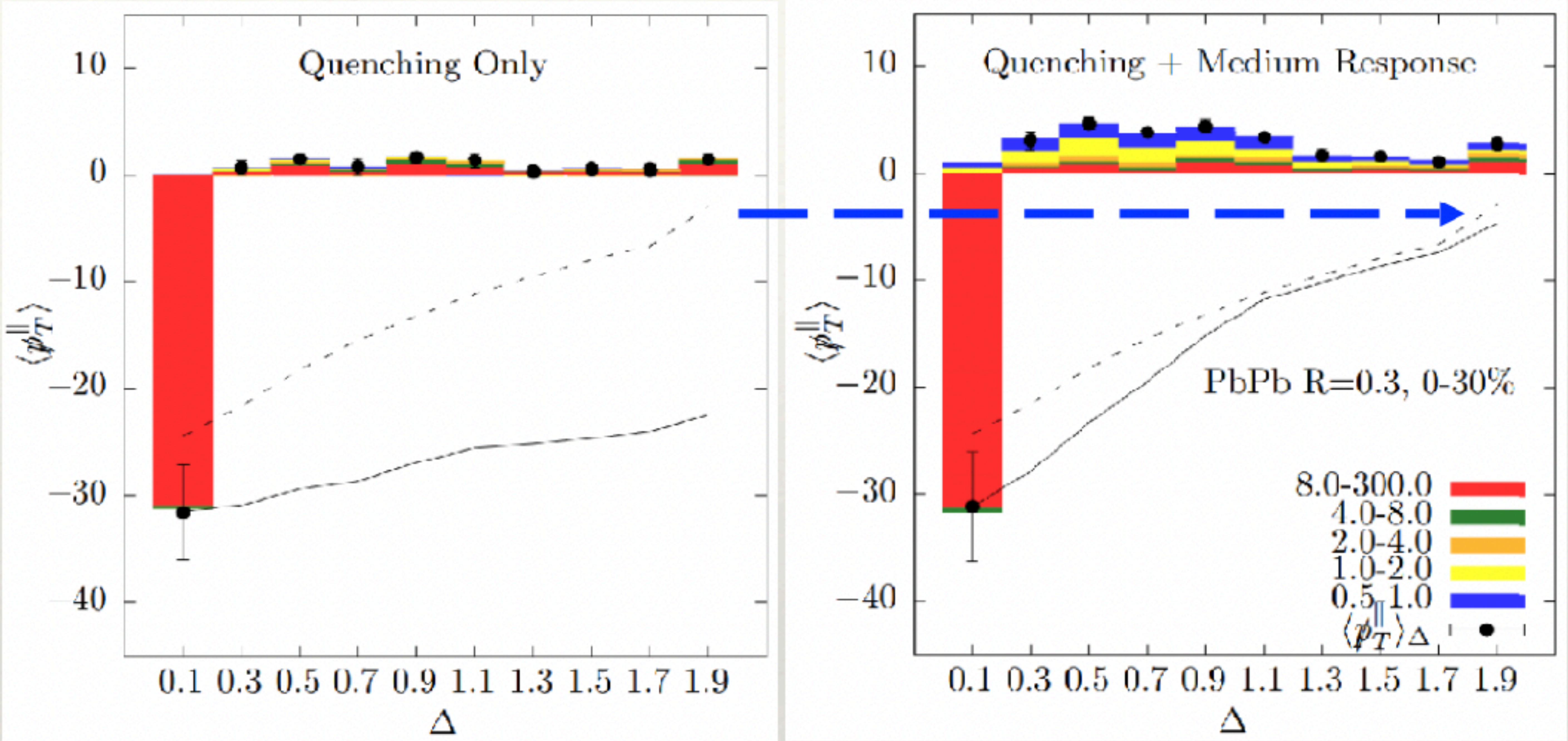
JEWEL: JHEP 07 (2017) 141

Hybrid: JHEP 03 (2017) 135

Recovering missing p_T

$$\not{p}_T^{\parallel} \equiv -p_T \cos\left(\phi_{\mathrm{dijet}} - \phi\right)$$

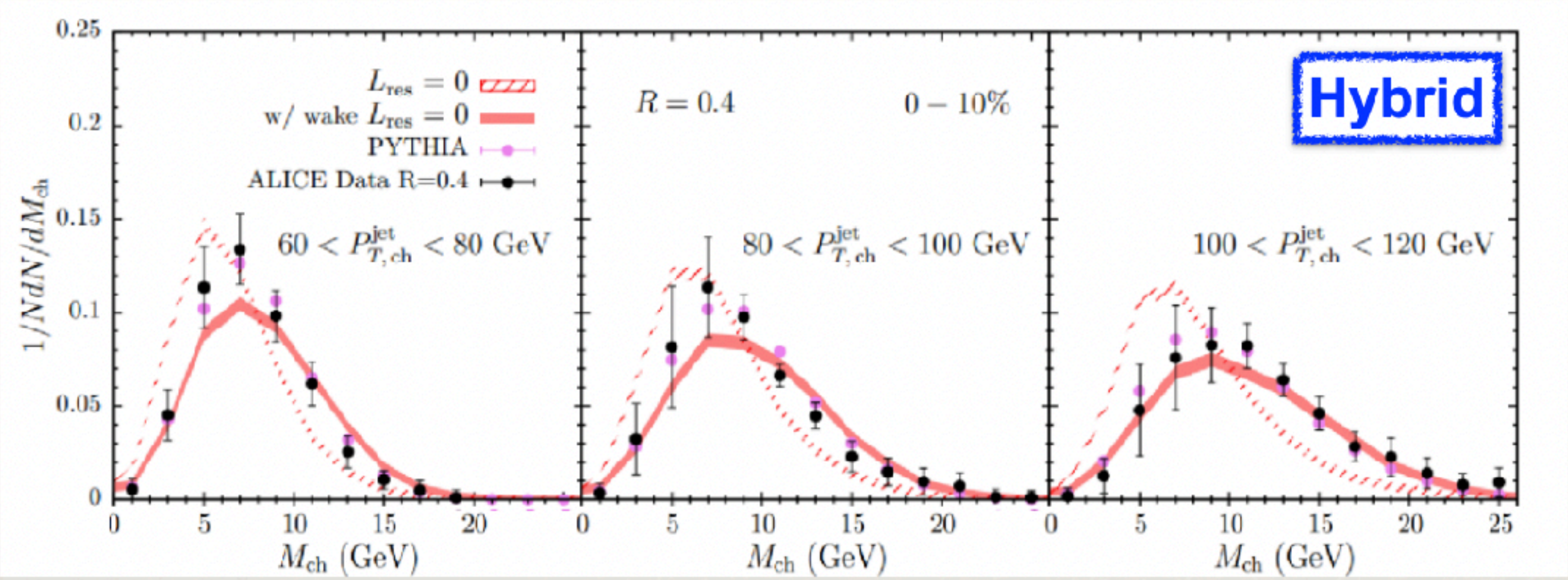




Casalderrey-Solana, Gulhan, Milhano, Pablos, Rajagopal, JHEP 03 (2017) 135

Δ: distance in (η, ϕ) between track and leading or sub-leading axis, whichever closer

Jet mass

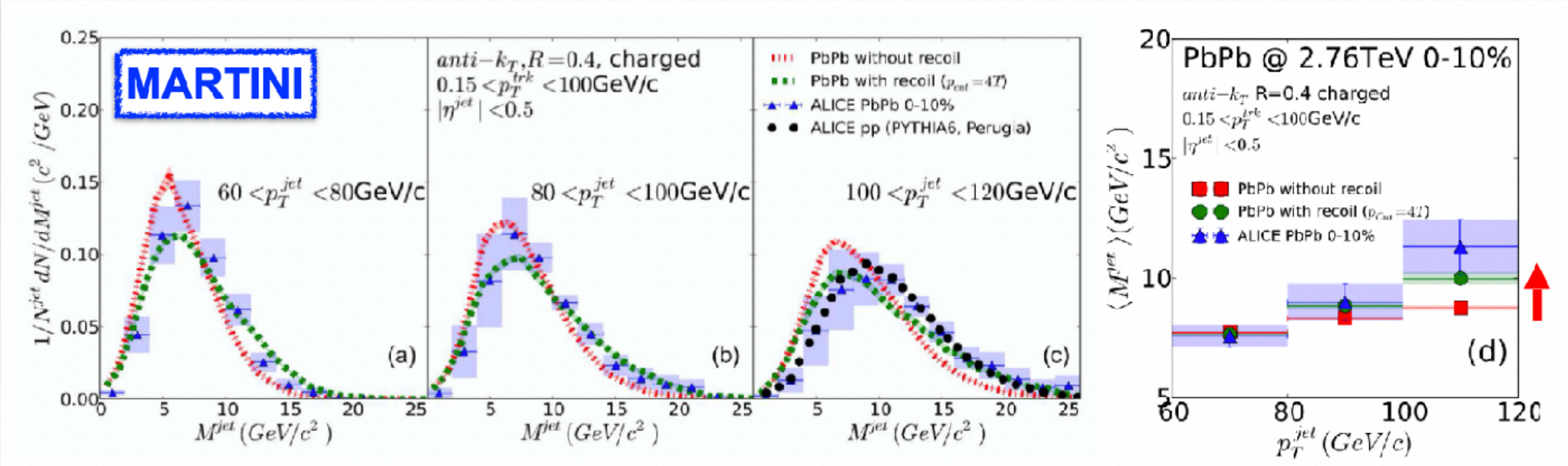


$$M = \sqrt{E^2 - p_T^2 - p_z^2},$$

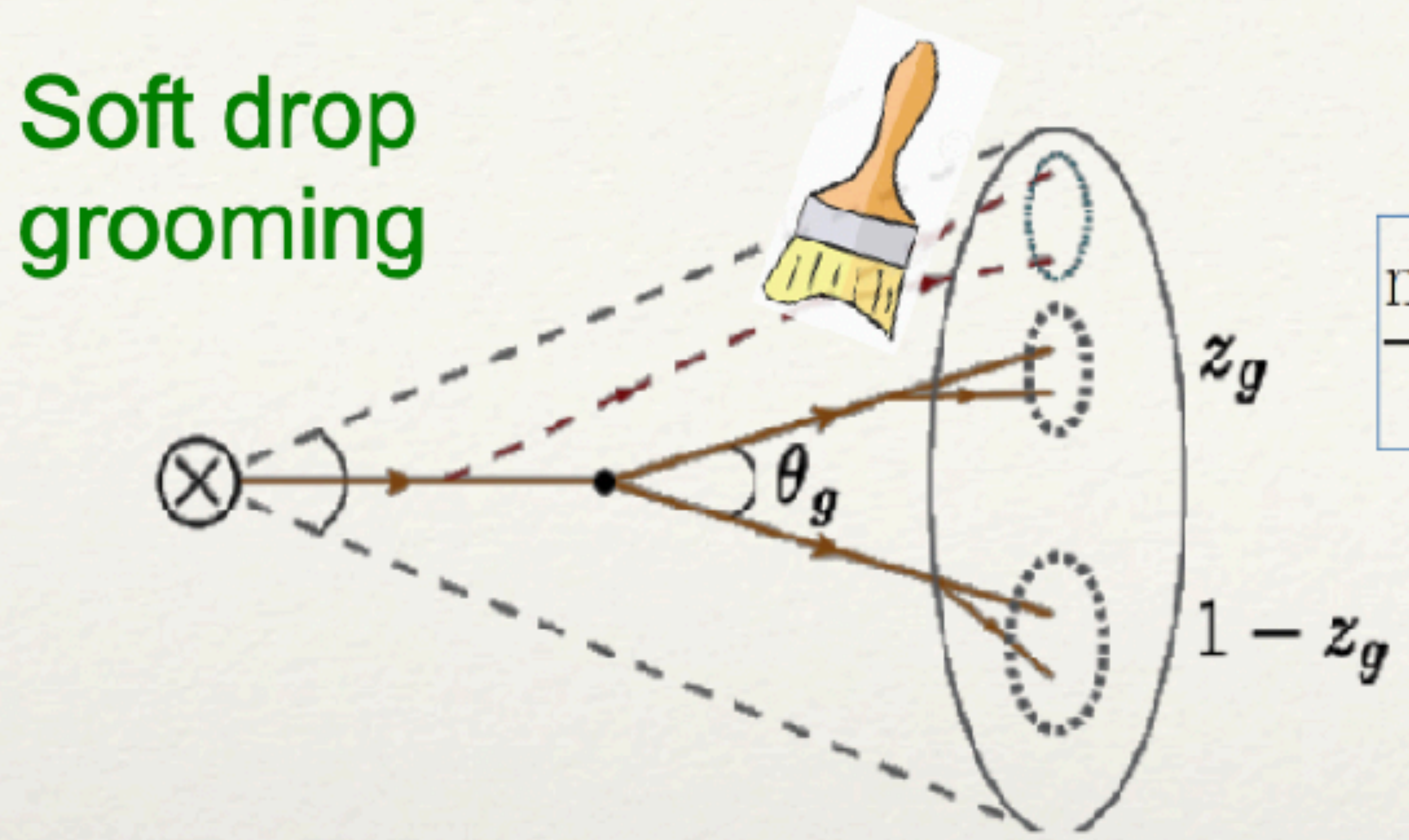
- Jet quenching decreases M
- Medium response increases M

MARTINI: NPA 982 (2019) 643

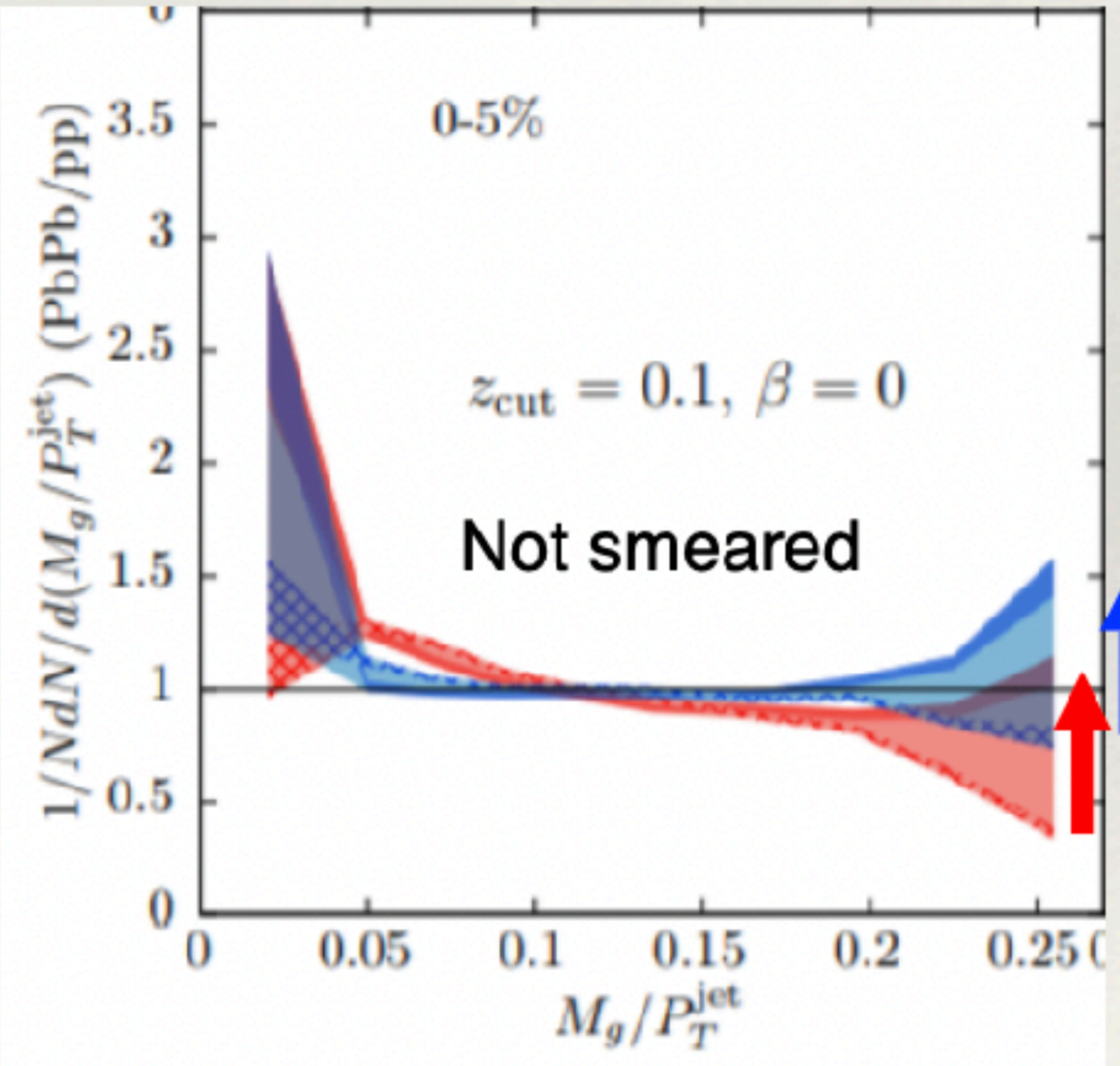
Hybrid: JHEP 01 (2020) 044



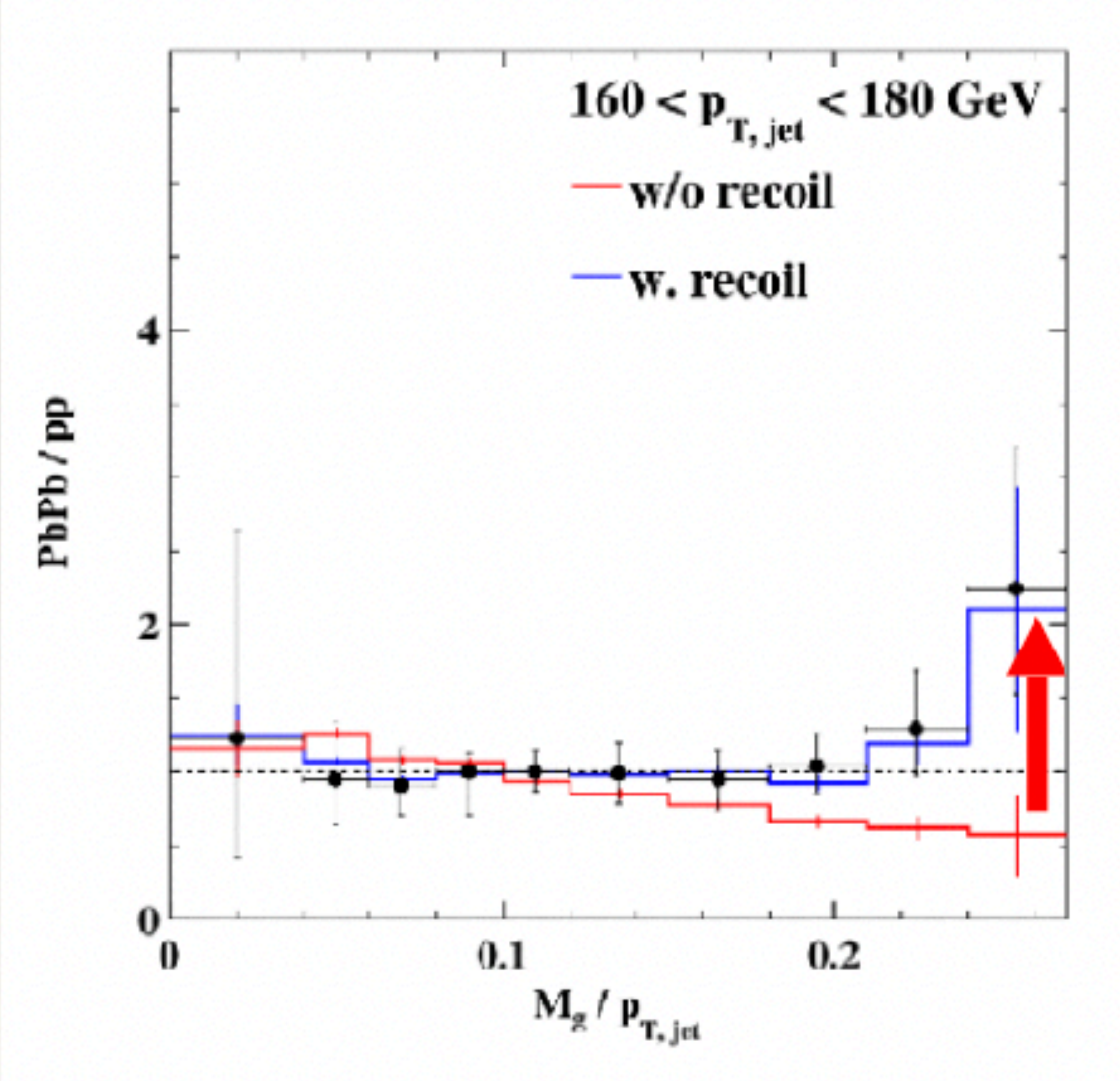
Groomed jet mass



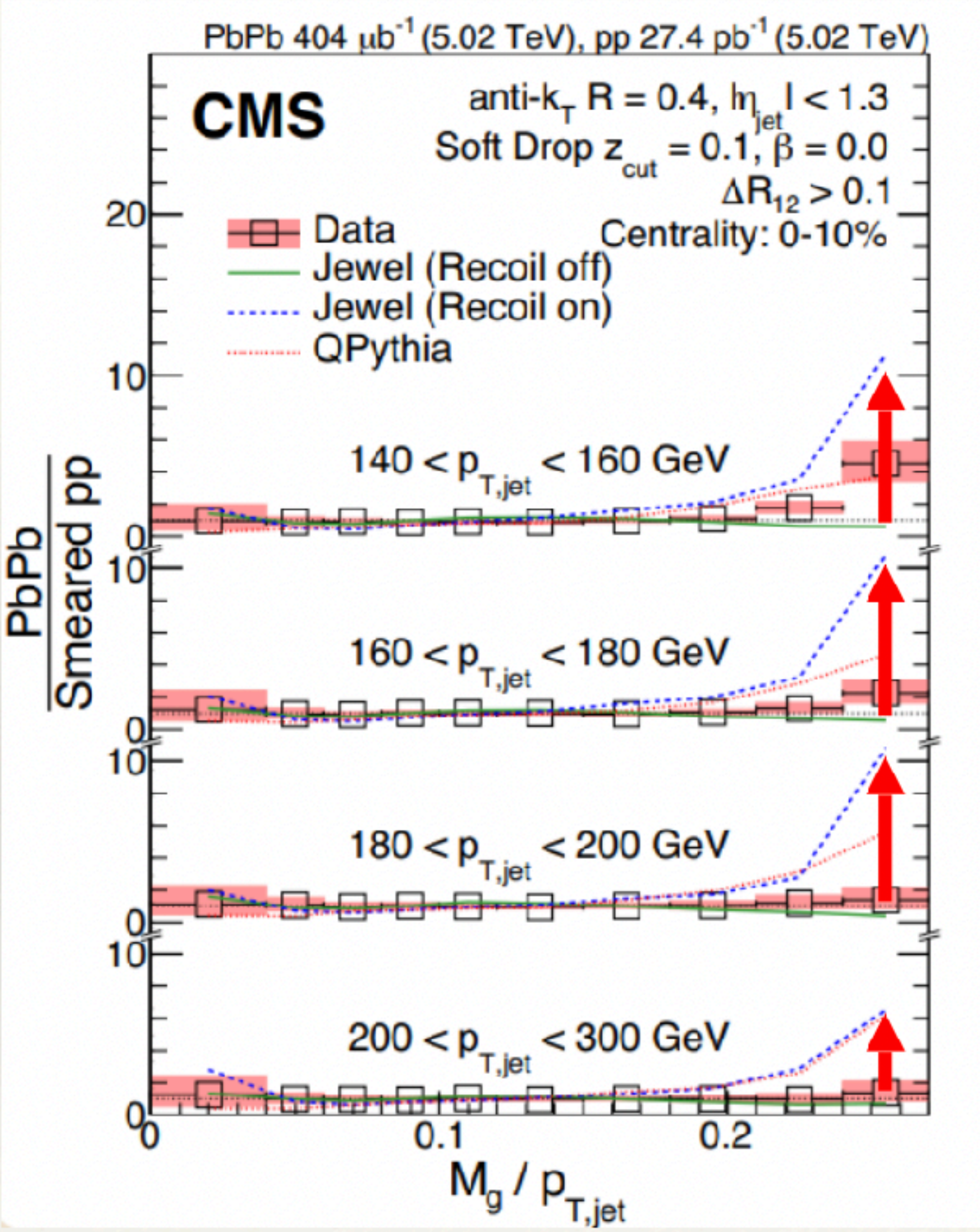
$$\frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} \equiv z_g > z_{\mathrm{cut}} \left(\frac{\Delta R}{R}\right)^{\beta}$$



Casalderrey-Solana, Milhano, Pablos, Rajagopal, JHEP 01 (2020) 044



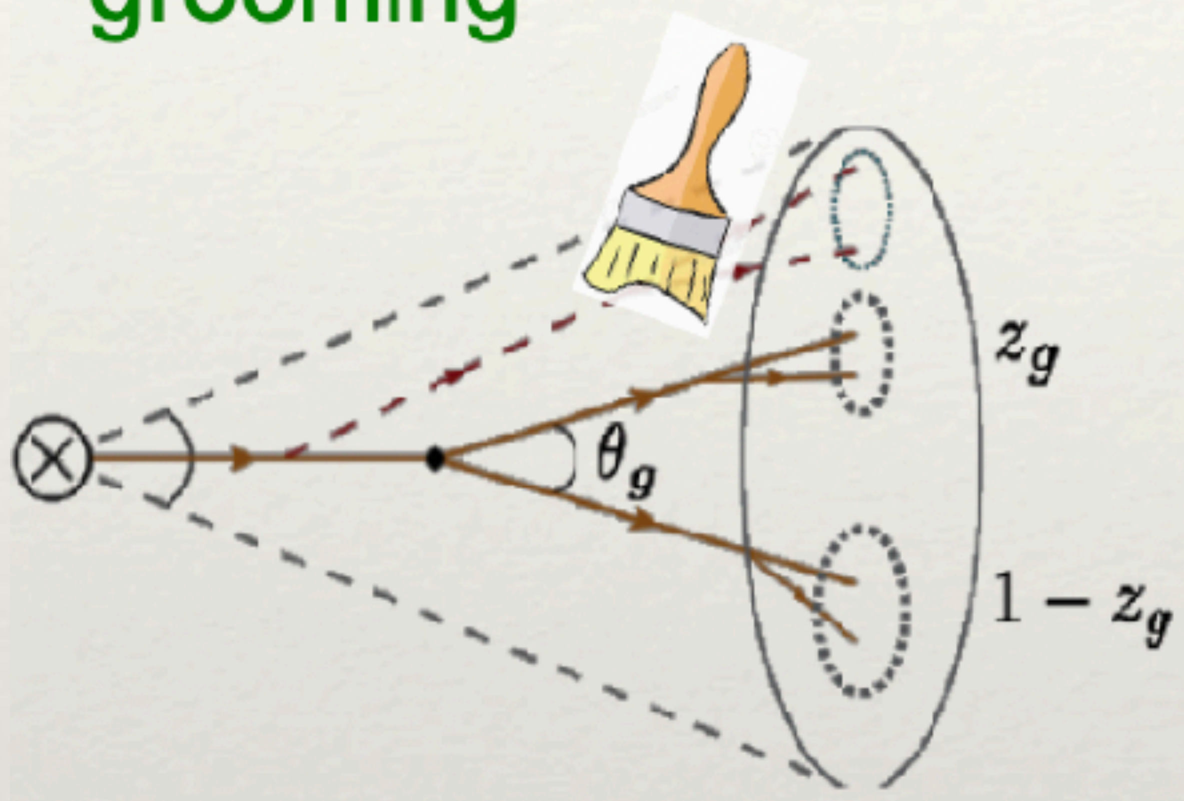
Luo, PoS HardProbes2018 (2019) 036



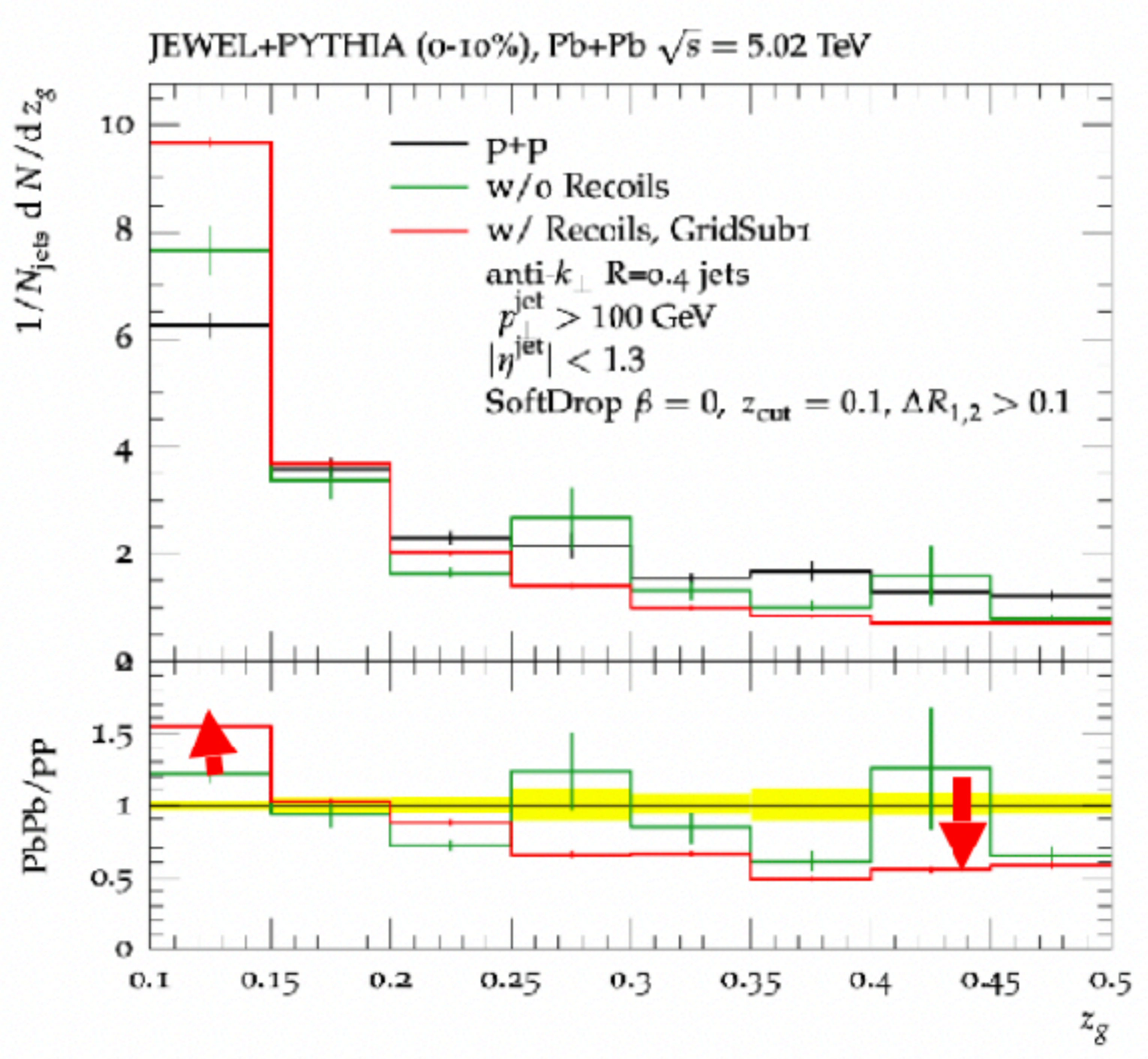
CMS, JHEP 10 (2018) 161

Groomed jet splitting function

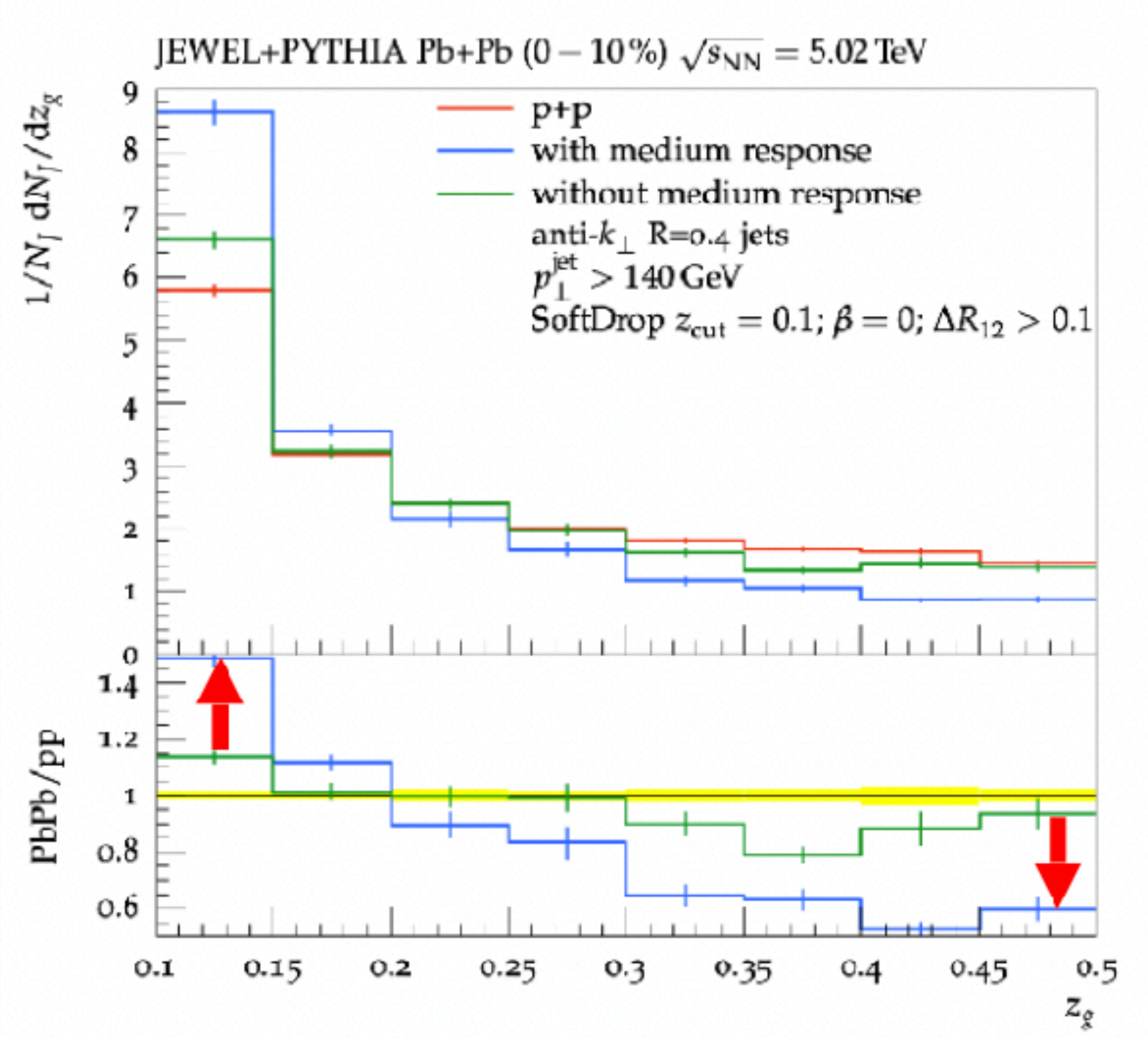
Soft drop grooming



Medium modification of the z_g distribution



Elayavalli, Zapp, JHEP 07 (2017) 141



Mihano, Wiedemann, Zapp, PLB 779 (2018) 409

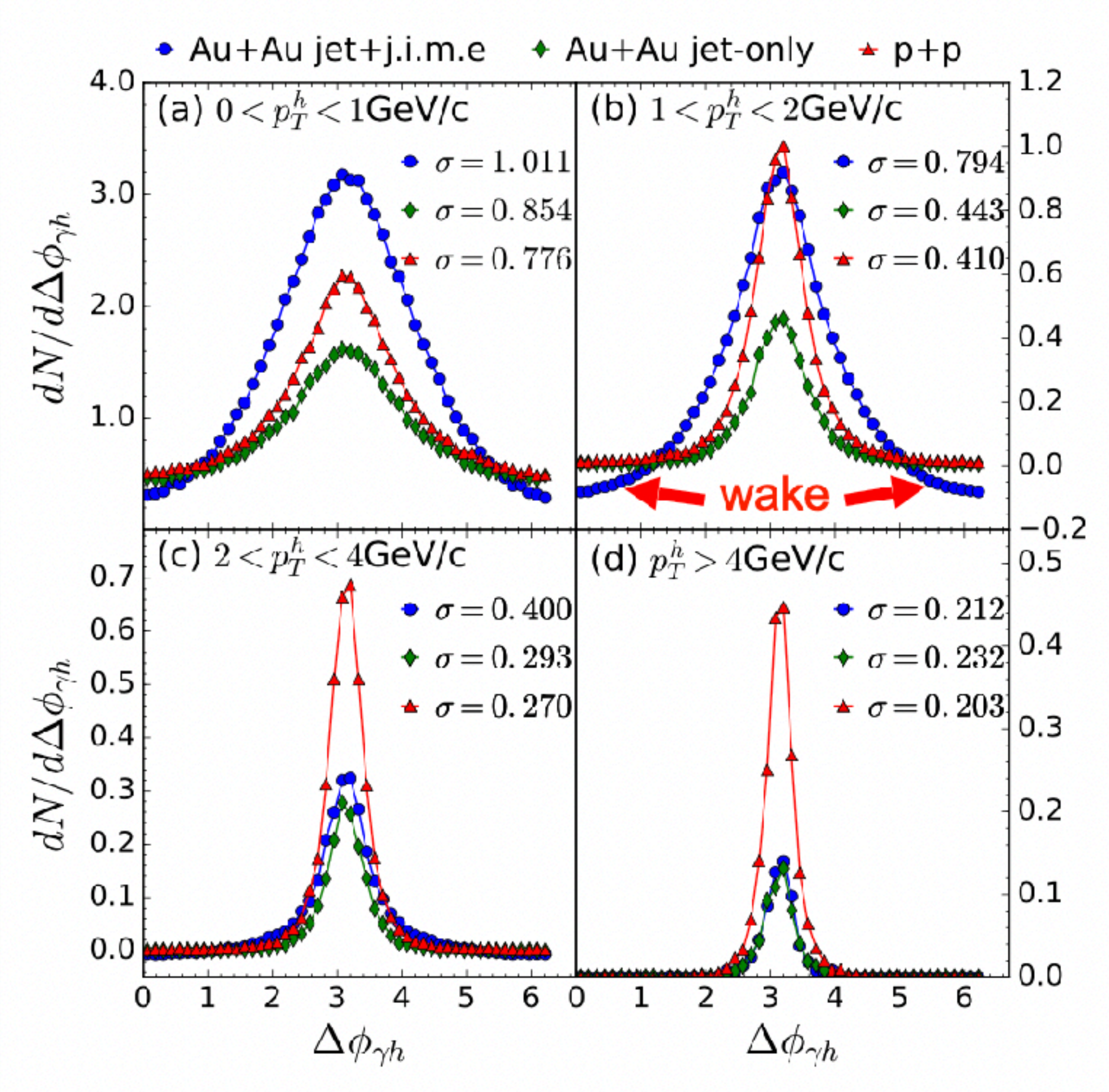
Criticism on medium response study

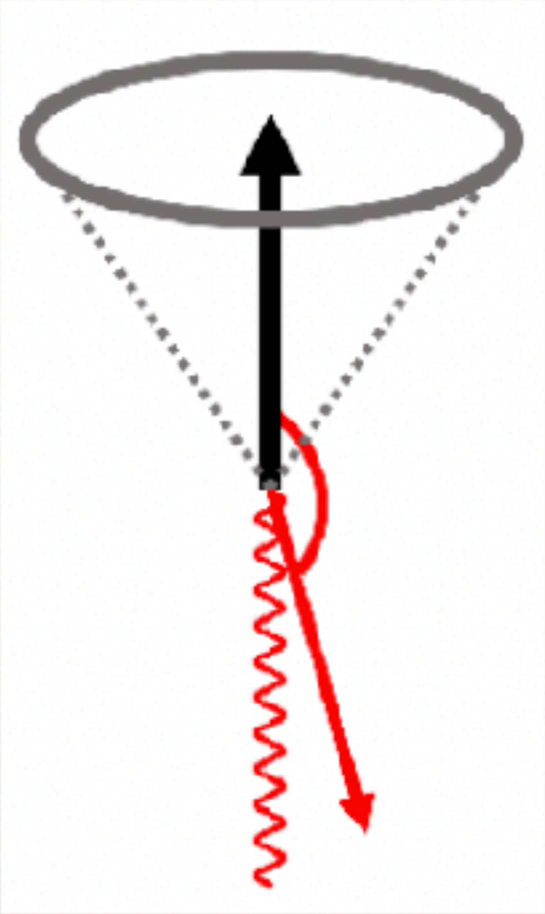
- **Theories without medium response effects can also describe data, why we need medium response?**

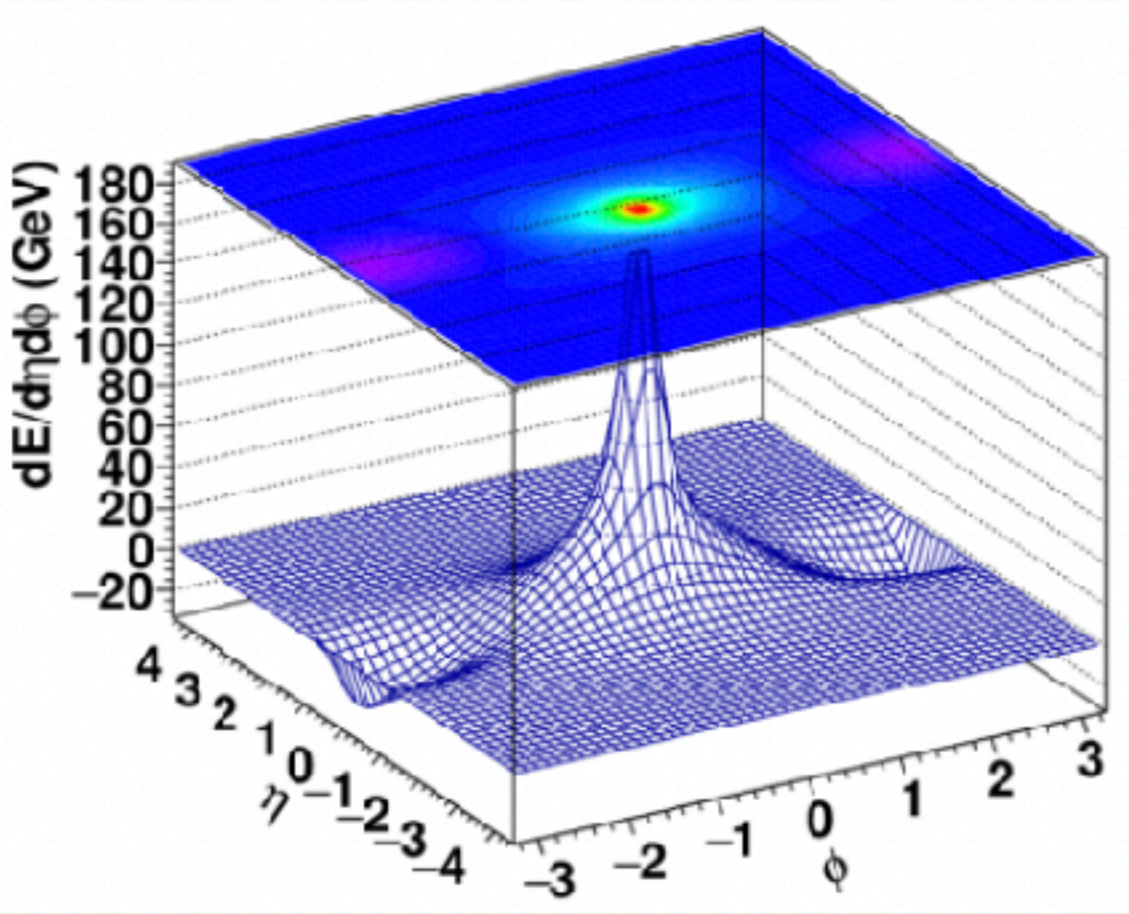
- Short response: one can only judge the effects after studying medium response

- Motivation: we should search for unambiguous signatures of medium response

Diffusion wake





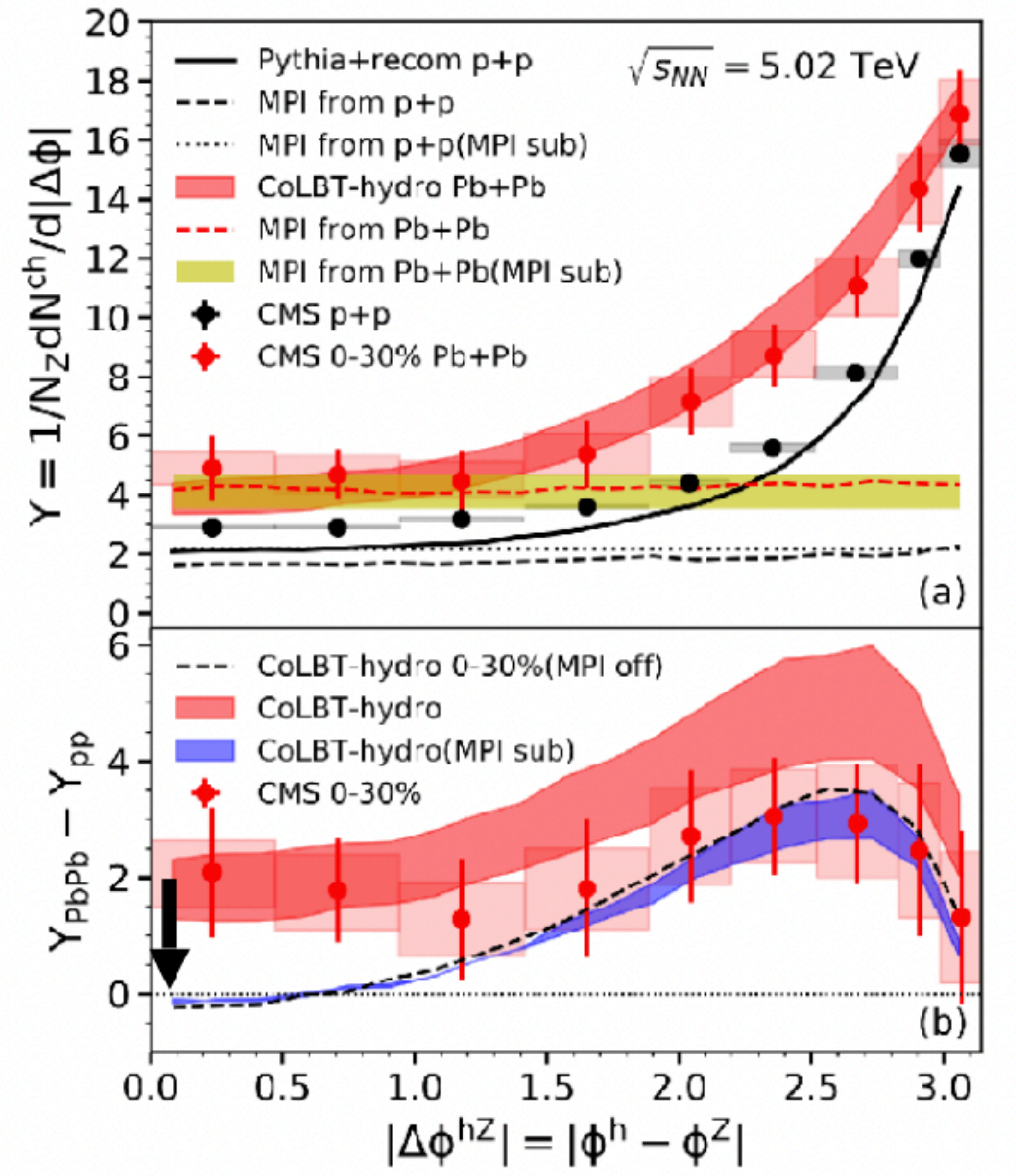


- High p_T, suppression, no broadening
- Low p_T, enhancement, broadening
- Diffusion wake predicted in $1 < p_T < 2$ GeV bin

Chen, Cao, Luo, Pang, Wang, PLB 777 (2018) 86

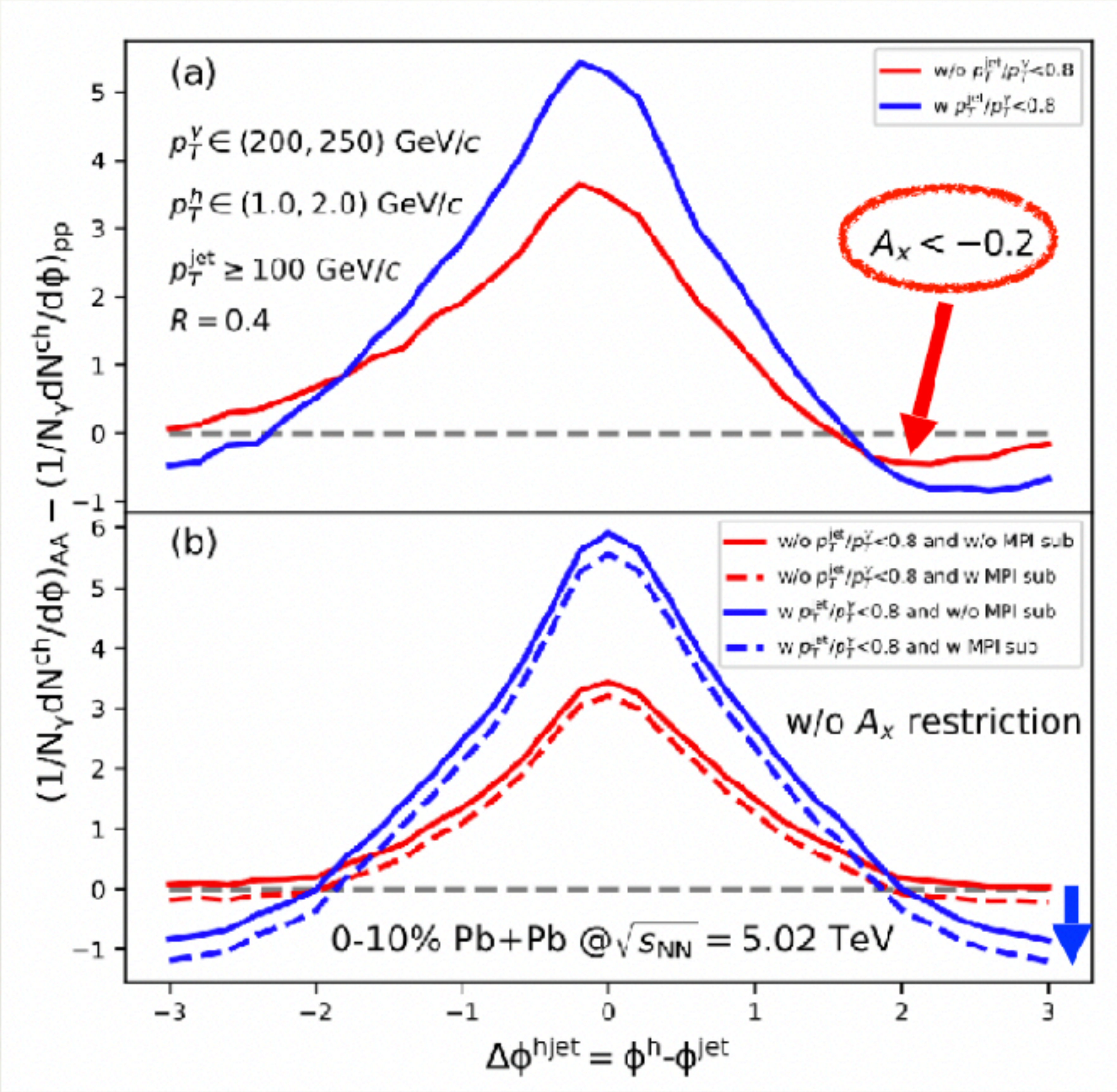
Enhancing diffusion wake effects

Subtract MPI in Z-hadron correlation



$$\frac{dN^{hZ}_{\rm MPI}}{d\phi} \approx \frac{dN^{hZ}_{\rm mix}}{d\phi} - \int_1^\pi \frac{d\phi}{\pi}\left(\frac{dN^{hZ}}{d\phi} - \frac{dN^{hZ}}{d\phi}|_{\phi=1}\right)$$

Apply 2D jet tomography



$$A_{\vec{n}} = \frac{\int d\phi[(dN^h/d\phi)_{\phi-\phi_n>0} - (dN^h/d\phi)_{\phi-\phi_n<0}]}{\int d\phi dN^h/d\phi}$$

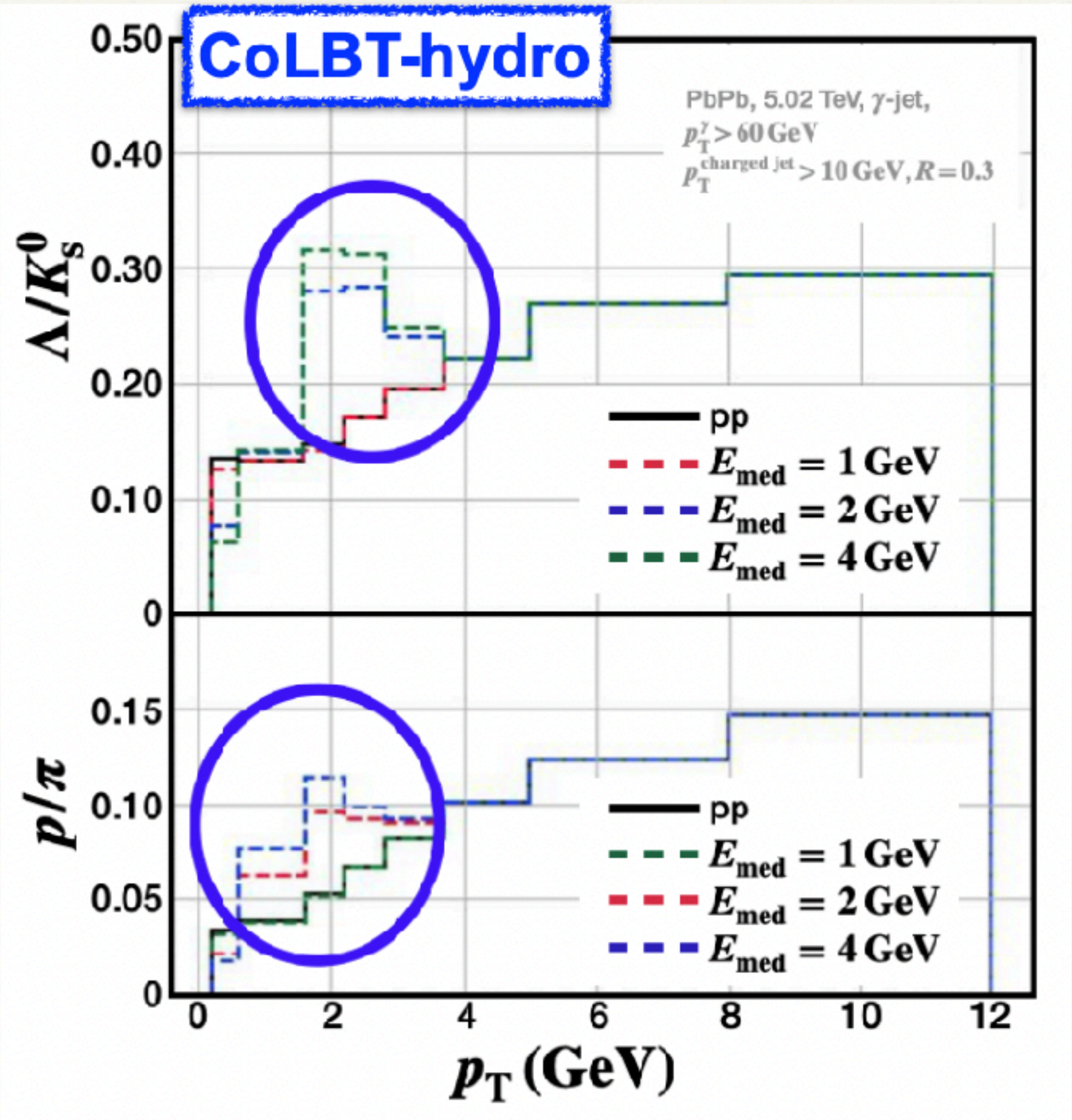
azimuthal asymmetry of soft hadron dist. w.r.t. $\hat{n}$

Direction and path-length bias with:

- A_x cut
- $p_T^{\rm jet}/p_T^{Z/\gamma}$ cut

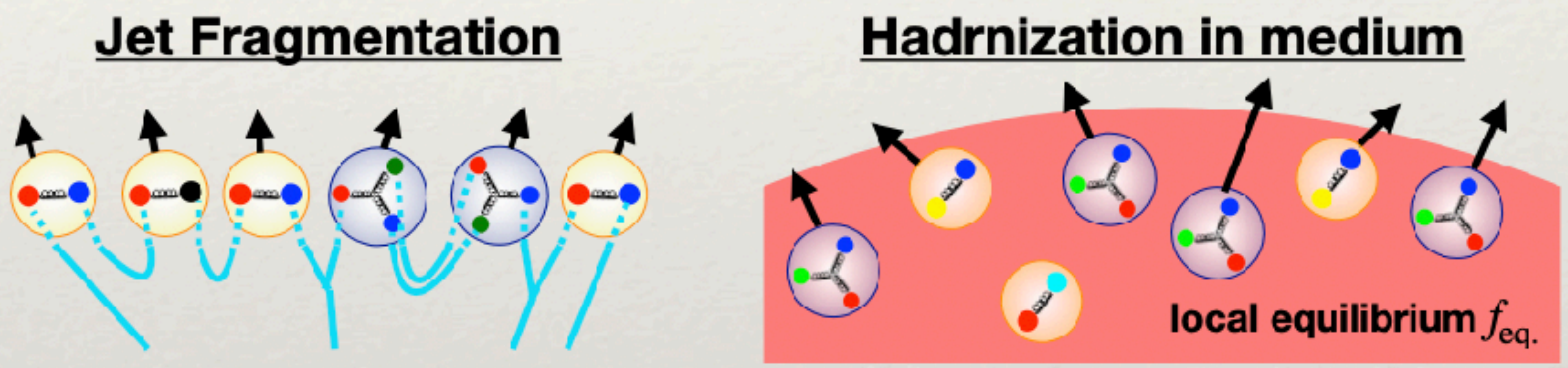
Chen, Yang, He, Ke, Pang, Wang, arXiv:2101.05422

Hadron chemistry inside jets



CoLBT-hydro calculation from Wei Chen (QM 2019)

- Different mechanism between showering partons and medium
- More baryons form medium



Baryon meson ratio in jet

- Enhancement around 2 GeV
- Sensitive to E_{med}

Monte-Carlo method

A view of Monte Carlo, Monaco

Why Monte-Carlo

Limits of semi-analytical calculation

Splitting function: $P^{\mathrm{vac}}_{q\to qg}(x) = \frac{4}{3}\frac{1+(1-x)^2}{x}$

Number of emitted gluon: $N_g = \int_{x_{\mathrm{cut}}}^{1} P(x)dx$

Energy taken away from the quark: $\Delta E = \int_{x_{\mathrm{cut}}}^{1} xE \cdot P(x)dx$

- *E* on the RHS does not change with parton evolution
- Do not naturally include energy fluctuation, average energy loss only
- Hard to include other fluctuations with the presence of medium

Monte-Carlo method

$$P^{\mathrm{vac}}_{q\to qg}(x) = \frac{4}{3}\frac{1+(1-x)^2}{x} \qquad N_g = \int_{x_{\mathrm{cut}}}^{1} P(x)dx \qquad \Delta E = \int_{x_{\mathrm{cut}}}^{1} xE \cdot P(x)dx$$

Monte Carlo methods are computational algorithms that rely on repeated ***random sampling*** based on probability distributions to obtain numerical results – naturally include quantum fluctuations in realistic physics processes and connect theories and experiments.

- Use N_g (if N_g << 1, otherwise e^{-N_g}) as the probability of splitting
- If a splittings happen, sample x using $P(x)$ and take energy xE away from parent quark
- Repeat the process to obtain numerical results of multiple splittings

MC sampling: hit-or-miss method

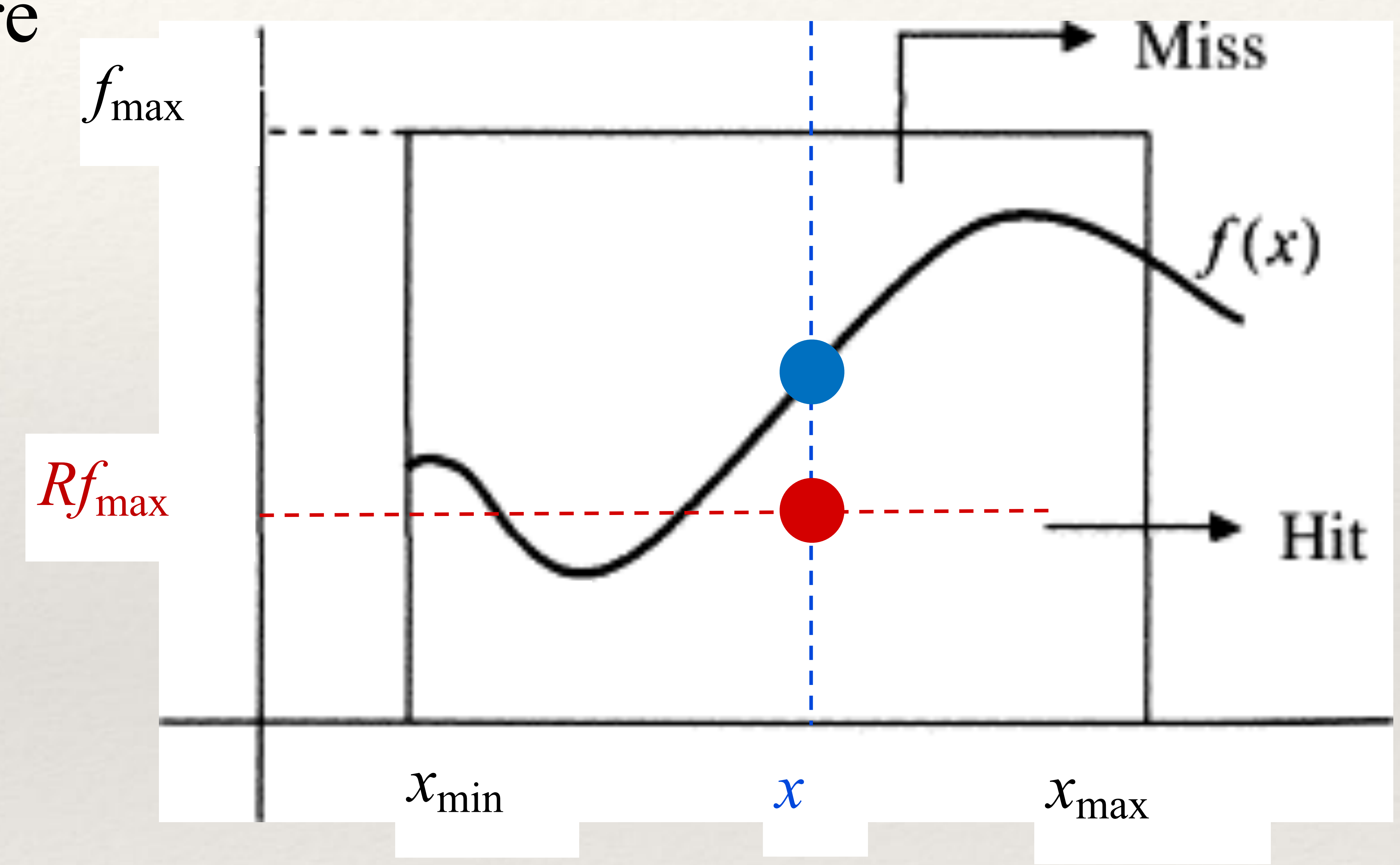
If the maximum of $f(x) \leq f_{max}$ in the range where we want to sample x:

1. Select an x with even probability in the allowed range: $x = x_{min}+R(x_{max}-x_{min})$;

2. Compare a (new) Rf_{max} with f(x);

   if $Rf_{max} \leq f(x)$, accept x;

3. Otherwise repeat 1 and 2 until x is found.



Hit-or-miss: example

Example: sample medium-induced gluon spectrum (x and $k_\perp$)

$$f(x,k_\perp) = \frac{dN_g}{dx dk_\perp^2 dt} = \frac{2\alpha_s C_A P(x)}{\pi k_\perp^4} \hat{q} \left(\frac{k_\perp^2}{k_\perp^2 + x^2 M^2} \right)^4 \sin^2\left(\frac{t - t_i}{2\tau_f} \right)$$

1. Calculate N_g within Δt at given E, T and t (record $f_{\max}$ when evaluating integral);

$$\langle N_g \rangle (E, T, t, \Delta t) = \Delta t \int dx dk_\perp^2 \frac{dN_g}{dx dk_\perp^2 dt}$$

2. If emission happens sample x and $k_\perp$ independently within physical ranges;

3. Compare a new $Rf_{\max}$ with $f(x,k_\perp)$, accept x and $k_\perp$ if $Rf_{\max} \leq f(x,k_\perp)$;

4. Otherwise repeat 2 and 3 until x and $k_\perp$ are found.

MC sampling: inverse-primitive-function method

1. Find the primitive function of $f(x)$:

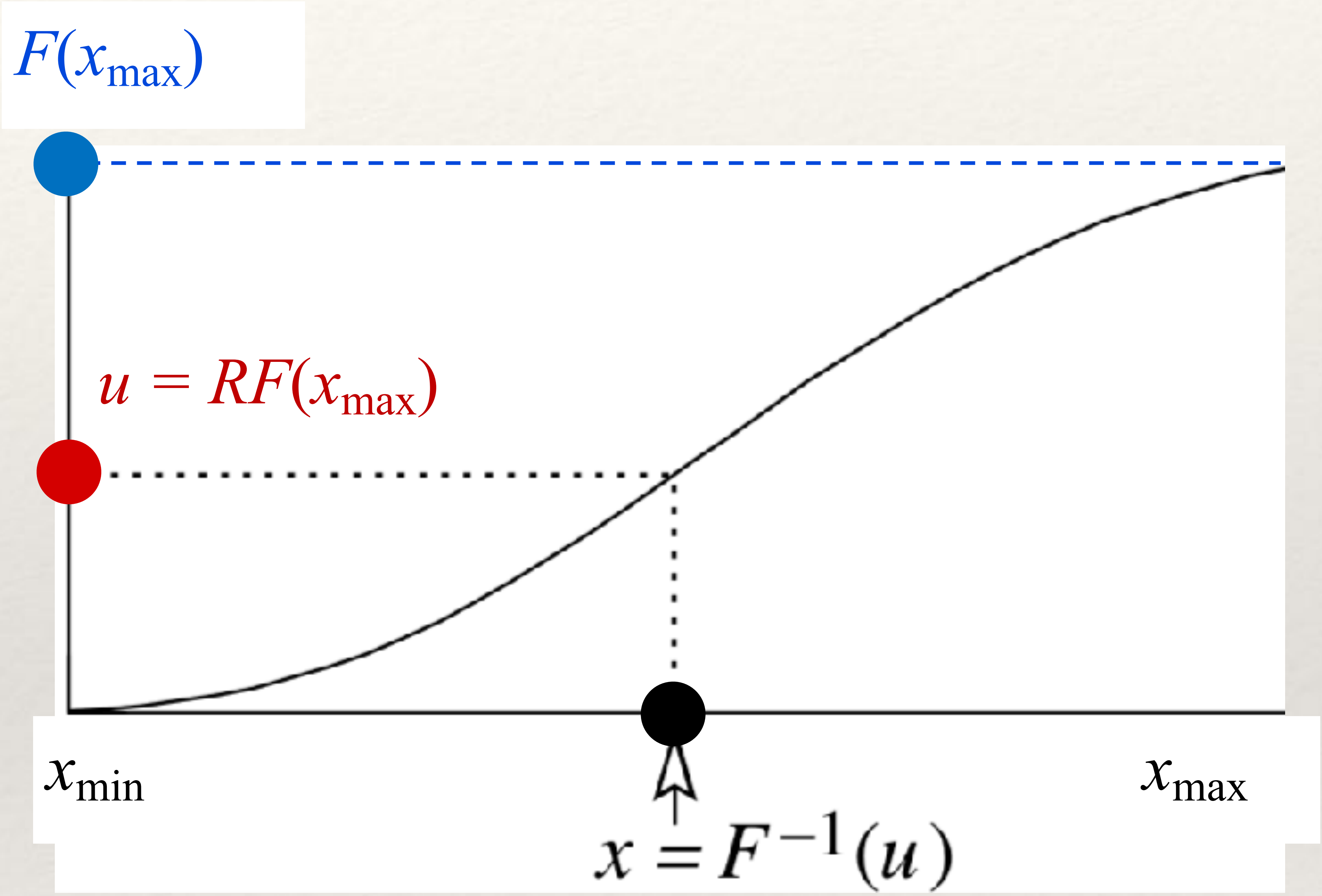
$$F(x) = \int_{x_{\min}}^{x} f(x)dx;$$

2. Sample a random number u in $[0, F(x_{max})]$ with $RF(x_{max})$;

3. Solve x with the inverse function of $F(x)$:

$$x = F^{-1}(u).$$

Efficient when the analytical solution of the inverse function exists. Still doable even it does not exist.



Inverse-primitive-function: example

Example: sample x from splitting function $P(x)$ [used in MATTER]

1. Calculate the integrated splitting function (primitive function);

$$F(x) = \int_{z_{\mathrm{cut}}}^{x} dz P(z) \qquad [x_{\max} = 1 - z_{\mathrm{cut}}]$$

2. Sample a random number u in $[0, F(x_{max})]$ with $RF(x_{max})$;

3. Solve $F(x) = u$ numerically (bisection method) --

(a) set $x_{mid} = (x_{min}+x_{max})/2$;

(b) if $F(x_{mid}) < u$, set $x_{min} = x_{mid}$; if $F(x_{mid}) > u$, set $x_{max} = x_{mid}$;

(c) Repeat (a) and (b) until $|F(x_{mid})-u| < \varepsilon$, and then $x = x_{mid}$.

MC sampling: combined method

Assume the existence of a function $g(x)$, with $f(x) \leq g(x)$ over the x range of interest. Here $g(x)$ is picked to be a “simple” function, such that the primitive function $G(x)$ and its inverse $G^{-1}(x)$ are known.

1. Select an x according $g(x)$ using the inverse primitive function method;
2. Compare a (new) $Rg(x)$ with the ratio $f(x)$; if $Rg(x) \leq f(x)$ accept x;
3. Otherwise repeat 1 and 2 until x is found.