Precision Predictions using e^+e^- Annihilation and Tau Data

FCPPL

Project activity report 2009-2010

LABORATOIRE
DE L'ACCÉLÉRATEUR
LINÉAIRE

IHEP
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WANG Liangliang (co-PhD thesis defended 2009)

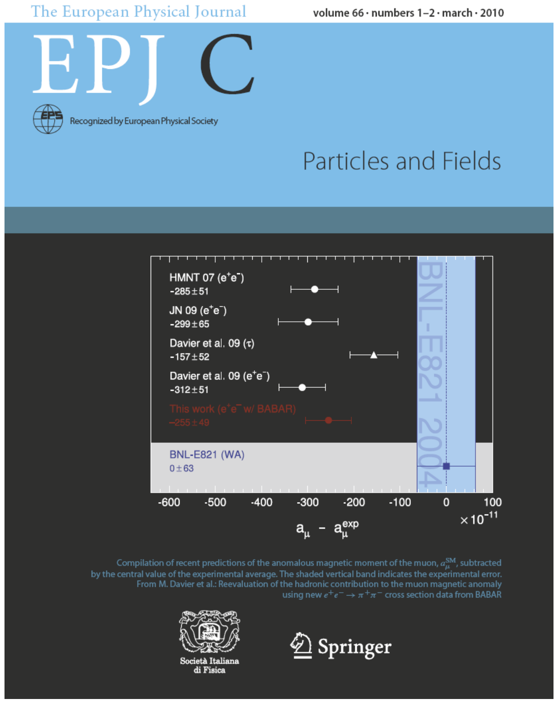
[+A. Höcker (CERN), G. Lopez Castro, G. Toledo (Mexico)]

Main Results Published

1. The discrepancy between tau and e^+e^- spectral functions revisited and the consequences for the muon magnetic anomaly,
   Eur. Phys. J. C66 (2010) 127, arXiv:0906.5443 [hep-ph].

2. Reevaluation of the hadronic contribution to the muon magnetic anomaly using new $e^+e^- \rightarrow \pi^+\pi^-$ cross section data from Babar,
   Eur. Phys. J. C66 (2010) 1, arXiv:0908.4300 [hep-ph].

3. Reevaluation of the hadronic contributions to the muon g-2 and $\alpha(M_Z)$,
   Eur. Phys. J. C71 (2011) 1, arXiv:1010.4180 [hep-ph].


The European Physical Journal
volume 66 · numbers 1–2 · march · 2010
EPJ C
Recognized by European Physical Society
Particles and Fields

Compilation of recent predictions of the anomalous magnetic moment of the muon, a_μ^{SM}, subtracted by the central value of the experimental average. The shaded vertical band indicates the experimental error. From M. Davier et al.: Reevaluation of the hadronic contribution to the muon magnetic anomaly using new $e^+e^- \rightarrow \pi^+\pi^-$ cross section data from BABAR

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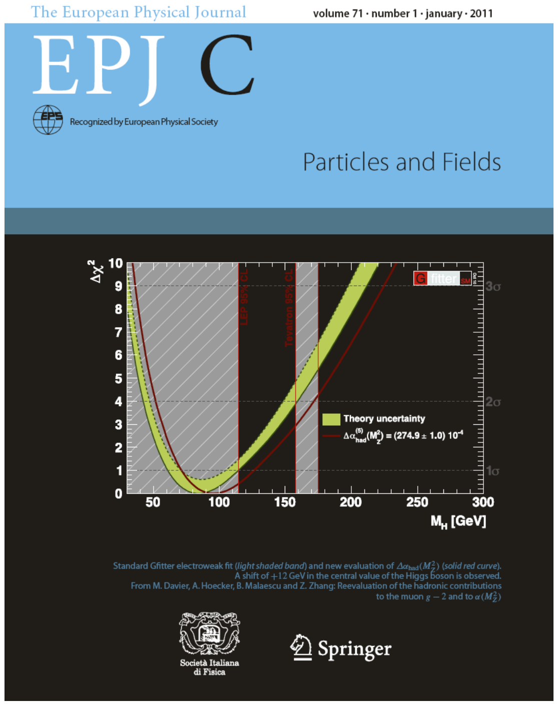
The European Physical Journal
volume 71 · number 1 · january · 2011
EPJ C
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Particles and Fields

Standard Gfitter electroweak fit (*light shaded band*) and new evaluation of $\Delta\alpha_{had}(M_Z^2)$ (*solid red curve*). A shift of +12 GeV in the central value of the Higgs boson is observed. From M. Davier, A. Hoecker, B. Malaescu and Z. Zhang: Reevaluation of the hadronic contributions to the muon $g-2$ and to $\alpha(M_Z^2)$

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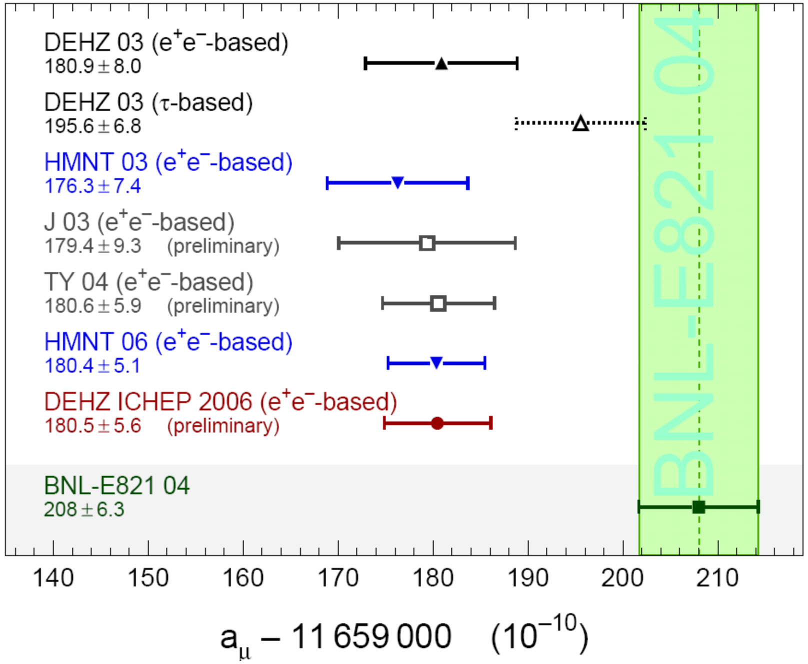
Motivations

- Muon magnetic anomaly a_μ:
  - one of the most precisely measured and prediction quantities in particle physics
  - sensitive to QED, strong and weak sectors of SM
  - data/prediction discrepancy → hint for new physics
  - discrepancy ee & τ-based predictions



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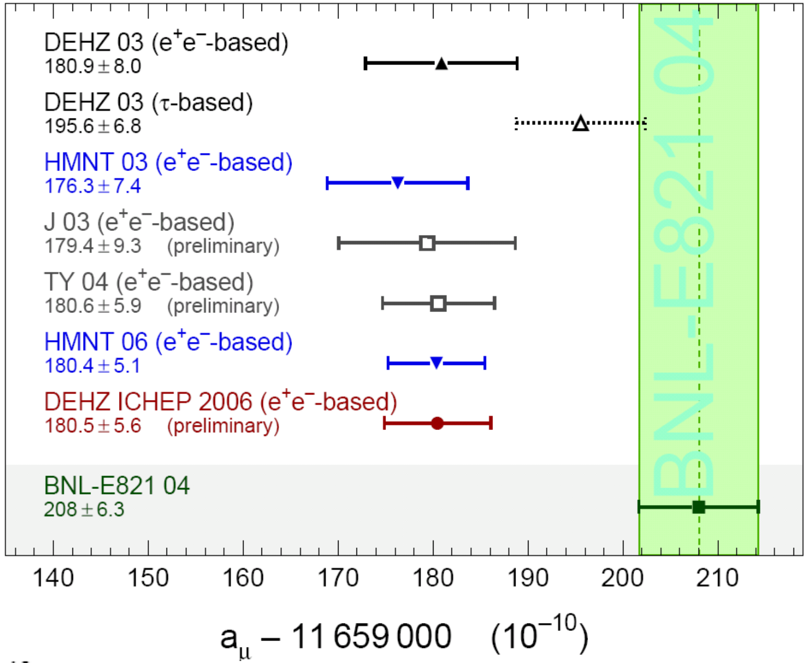
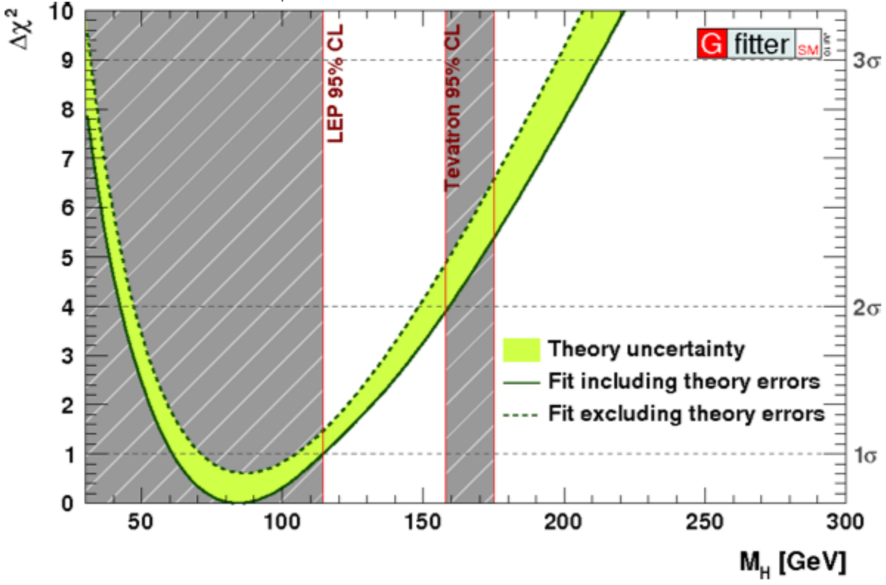


- Running fine-structure constant $\alpha(M_Z)$:

$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha(M_Z)}$$

$$\Delta\alpha(M_Z) = \Delta\alpha_{\text{leptonic}}(M_Z) + \Delta\alpha_{\text{had}}(M_Z)$$

  - one of limiting factors for global fit to EW precision data
  - an example constraint is on Higgs mass

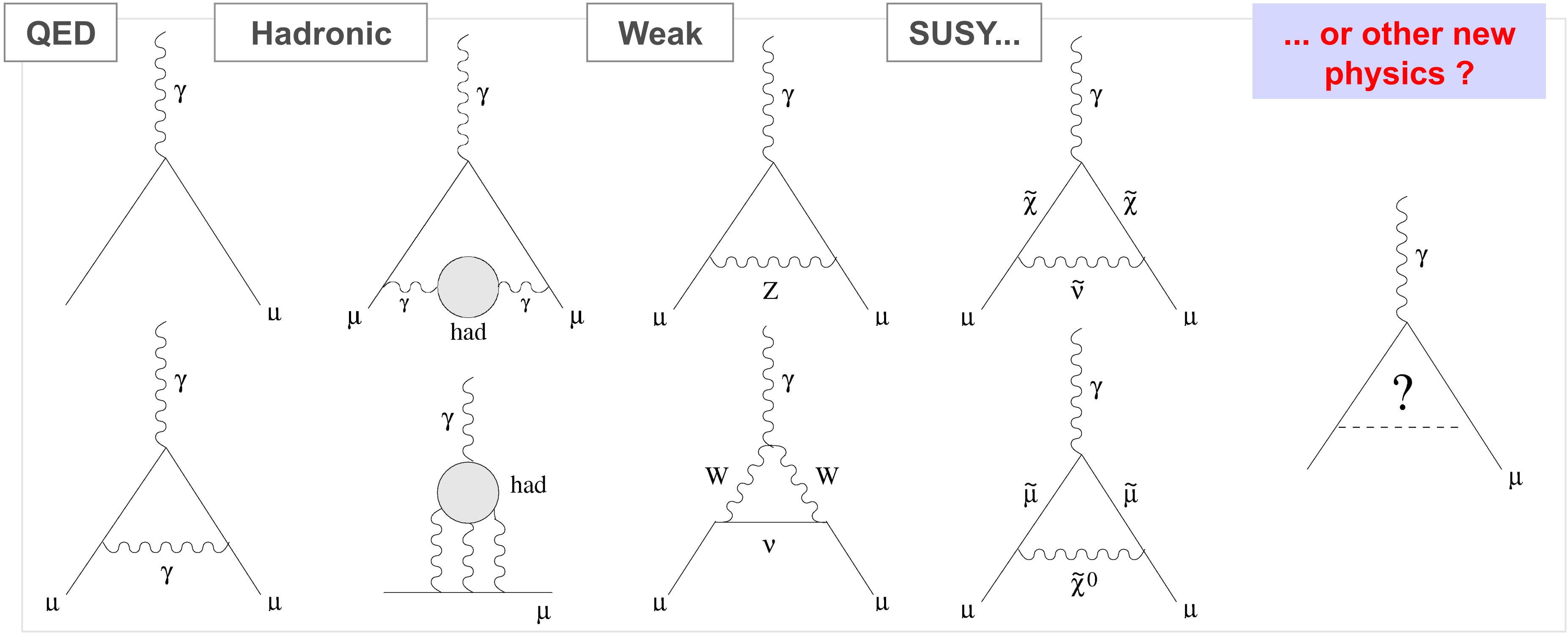


Muon Magnetic Moment Anomaly

$\vec{\mu} = g\frac{\pm e}{2m}\vec{s}$ $g = 2 + \cdots$ ➔ Magnetic Moment anomaly: $a_l = \frac{g-2}{2}$

a_e is better measured but a_μ is more sensitive to new physics effects by $(m_\mu/m_e)^2$~43000)

$$a_\mu^{\text{th}} = a_\mu^{\text{SM}} + a_\mu^{\text{non-SM}}, \qquad a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{Weak}}$$



QED

Hadronic

Weak

SUSY...

... or other new physics ?

Experimental progress on g-2

Miller, de Rafael, Lee Roberts, 2006

Experiment	Beam	Measurement	$\delta a_\mu/a_\mu$	Required th. terms
Columbia-Nevis (57)	μ^+	g=2.00±0.10		g=2
Columbia-Nevis (59)	μ^+	0.001 13(+16)(-12)	12.4%	α/π
CERN 1 (61)	μ^+	0.001 145(22)	1.9%	α/π
CERN 1 (62)	μ^+	0.001 162(5)	0.43%	$(\alpha/\pi)^2$
CERN 2 (68)	μ^+	0.001 166 16(31)	265 ppm	$(\alpha/\pi)^3$
CERN 3 (75)	$\mu^\pm$	0.001 165 895(27)	23 ppm	$(\alpha/\pi)^3$ + had
CERN 3 (79)	$\mu^\pm$	0.001 165 911(11)	7.3 ppm	$(\alpha/\pi)^3$ + had
BNL E821 (00)	μ^+	0.001 165 919 1(59)	5 ppm	$(\alpha/\pi)^3$ + had
BNL E821 (01)	μ^+	0.001 165 920 2(16)	1.3 ppm	$(\alpha/\pi)^4$ + had + weak
BNL E821 (02)	μ^+	0.001 165 920 3(8)	0.7 ppm	$(\alpha/\pi)^4$ + had + weak + ?
BNL E821 (04)	μ^-	0.001 165 921 4(8)(3)	0.7 ppm	$(\alpha/\pi)^4$ + had + weak + ?

➔ Current world average: $\boxed{a_\mu^{exp}=11\ 659\ 208.9 \pm 6.3 \times 10^{-10}}$

Dominated by by BNL-E821: PRD73, 072003 (2006)

SM Predictions: $a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{Weak}}$

$$\begin{array}{rrrl} a_\mu^{\mathrm{QED}} \cdot 10^{10} = \Sigma C_i (\frac{\alpha}{\pi})^i = & 11614097.3 & (1\text{-loop}) & 1 \text{ diagram} \\ + & 41321.8 & (2\text{-loop}) & 9 \\ + & 3014.2 & (3\text{-loop}) & > 100 \\ + & 38.1 & (4\text{-loop}) & > 1000 \\ + & 0.4 & (5\text{-loop}) & > 20000 \end{array}$$

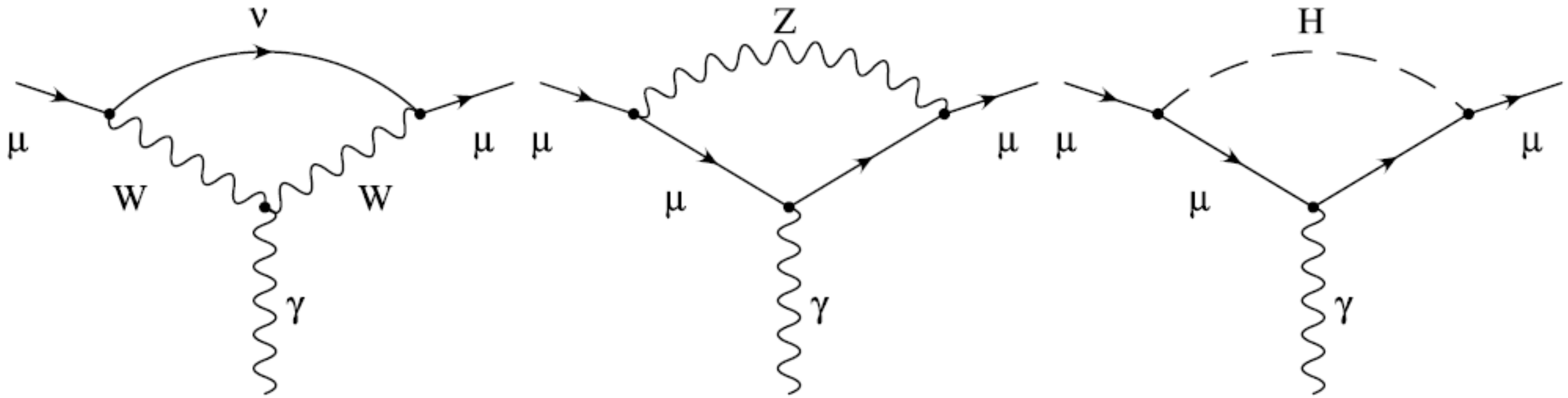
$$\boxed{a_\mu^{\mathrm{QED}} = (11\,658\,471.809 \pm 0.015_{5^{th}\,order \oplus \delta\alpha}) \times 10^{-10}}$$

[PDG'10] i.e. >99%SM

α^3 terms known analytically (S. Laporta, E. Remiddi, 93)
α^4 terms known numerically (T. Kinoshita et al., 03-08)
α^5 terms estimated (T. Kinoshita and M. Nio, 06, A.L. Kataev, 06, K. Chetyrkin et al., 08)
Using latest measurement of α^{-1} = 137.035999084(51) (Hanneke, Fogwell, Gabrielse, 08)

SM Predictions: $a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{Weak}}$

One-loop diagrams:



Order	a_μ^{Weak} (10^{-10})	Ref
1 loop	19.5	Jackiw, Weinberg, 72 Altarelli et al., 72 Bars, Yoshimura, 72
+ 2 loop	$15.4 \pm 0.1_{\mathrm{had}} \pm 0.2_{\mathrm{Mh}}$	Czarnecki et al., 03 Heinemeyer et al., 04 Gribouk, Czarnecki, 05

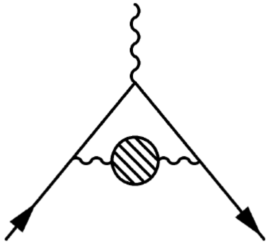
SM Predictions: $a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{Weak}}$

$$a_\mu^{\mathrm{had}} = a_\mu^{\mathrm{had,LO}} + a_\mu^{\mathrm{had,HO}} + a_\mu^{\mathrm{had,LBL}}$$

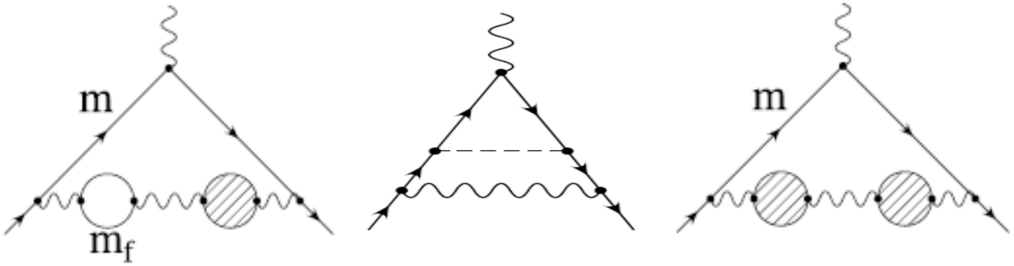
Leading-Order Higher-Order Light-By-Light

$a_\mu^{\mathrm{had,LO}} \sim (\,700 \pm 5\,)\times 10^{-10}$

➔ dominant uncertainty (both e^+e^- and τ based)

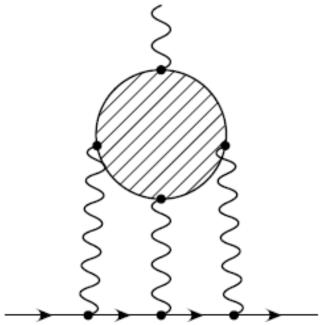


$a_\mu^{\mathrm{had,HO}} = (\,-9.8 \pm 0.1\,)\times 10^{-10}$



$a_\mu^{\mathrm{had,LBL}} \sim (\,12.0 \pm 3.5\,)\times 10^{-10}$

➔ 2nd leading uncertainty

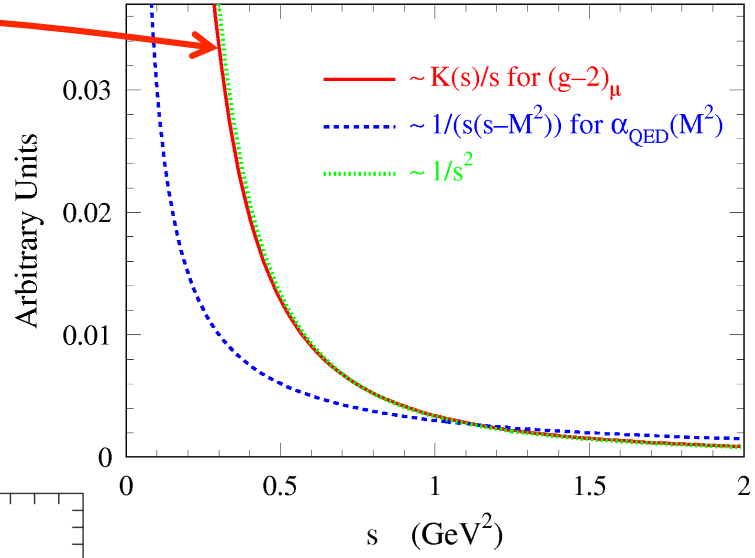


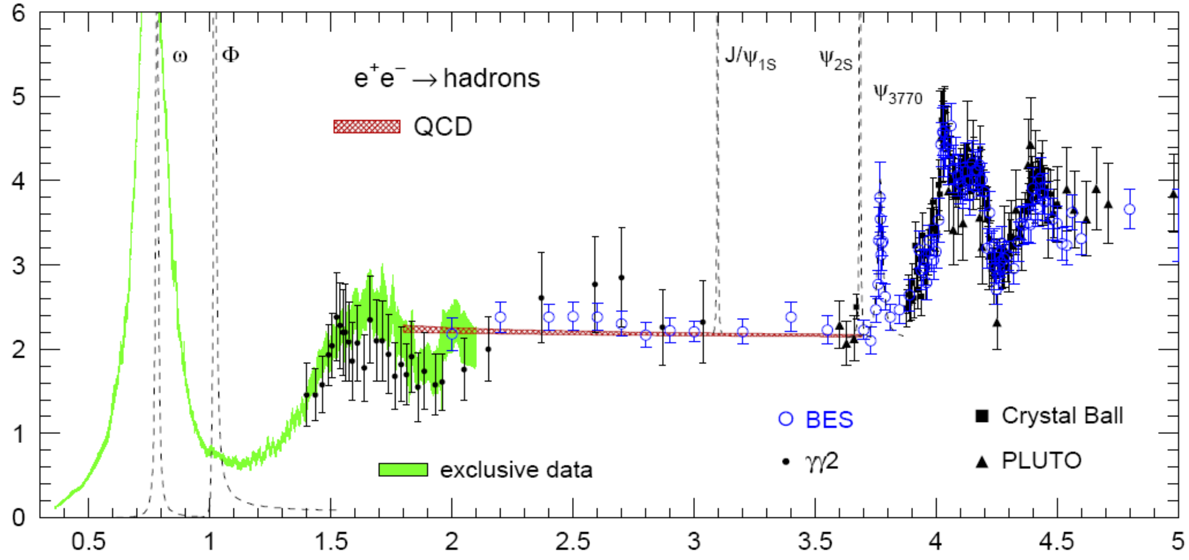
How is the LO Hadronic Contribution Calculated?

Could not predict from 1st principle but can be rigorously calculated using ee annihilation data via Dispersion Relation

$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \, \boxed{\frac{K(s)}{s}} \, R(s)$$

$$R(s) = \frac{\sigma_0(e^+e^- \rightarrow \text{hadrons}\,(\gamma))}{\sigma_{pt}(e^+e^- \rightarrow \mu^+\mu^-)}$$



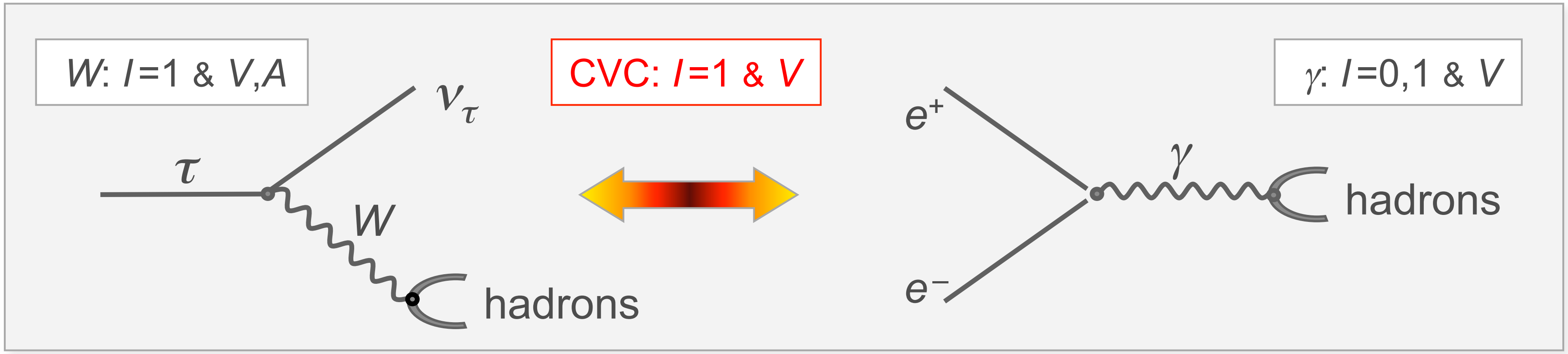


➔ Data driven prediction

➔ Precision is ee data dependent

Connect τ and e^+e^- Data through CVC – SU(2)

R. Alemany, M. Davier, A. Hoecker, Eur. Phys. J. C **2, 123** *(1998)*



Hadronic physics factorizes in Spectral Functions :

Isospin symmetry connects I=1 e^+e^- cross section to vector τ spectral functions:

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\,\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]}\;\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\;\frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

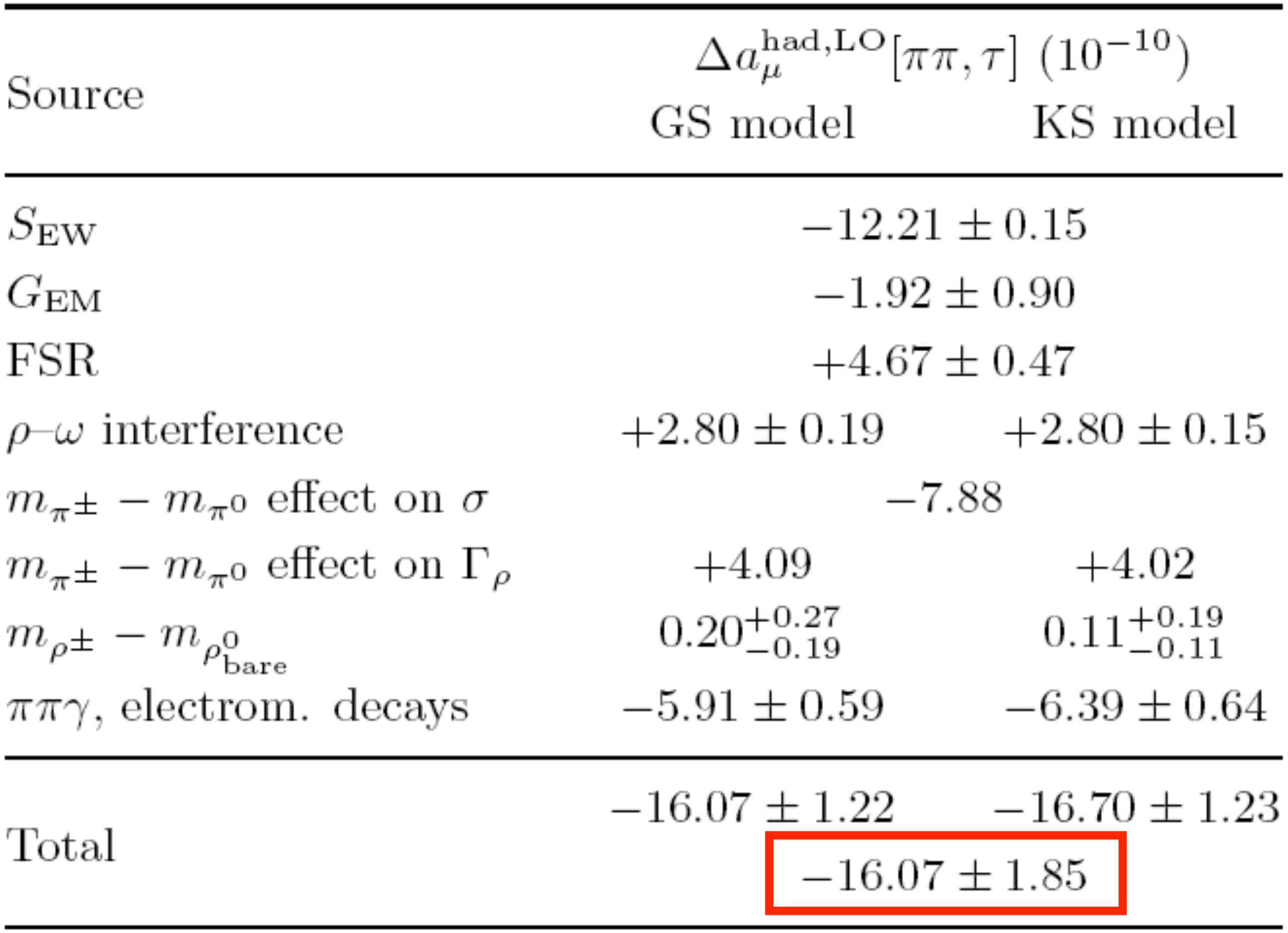
branching fractions

mass spectrum

kinematic factor (PS)

What's new for Tau based prediction?

Revisited isospin corrections *Eur. Phys. J. C66 (2010) 127, arXiv:0906.5443 [hep-ph]*



Source	$\Delta a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10})	
	GS model	KS model
S_{EW}	-12.21 ± 0.15	
G_{EM}	-1.92 ± 0.90	
FSR	$+4.67 \pm 0.47$	
ρ–ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$
$m_{\rho^\pm} - m_{\rho^0_{\mathrm{bare}}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64
Total	-16.07 ± 1.22	-16.70 ± 1.23
	$\boxed{-16.07 \pm 1.85}$	

What's new for Tau based prediction?

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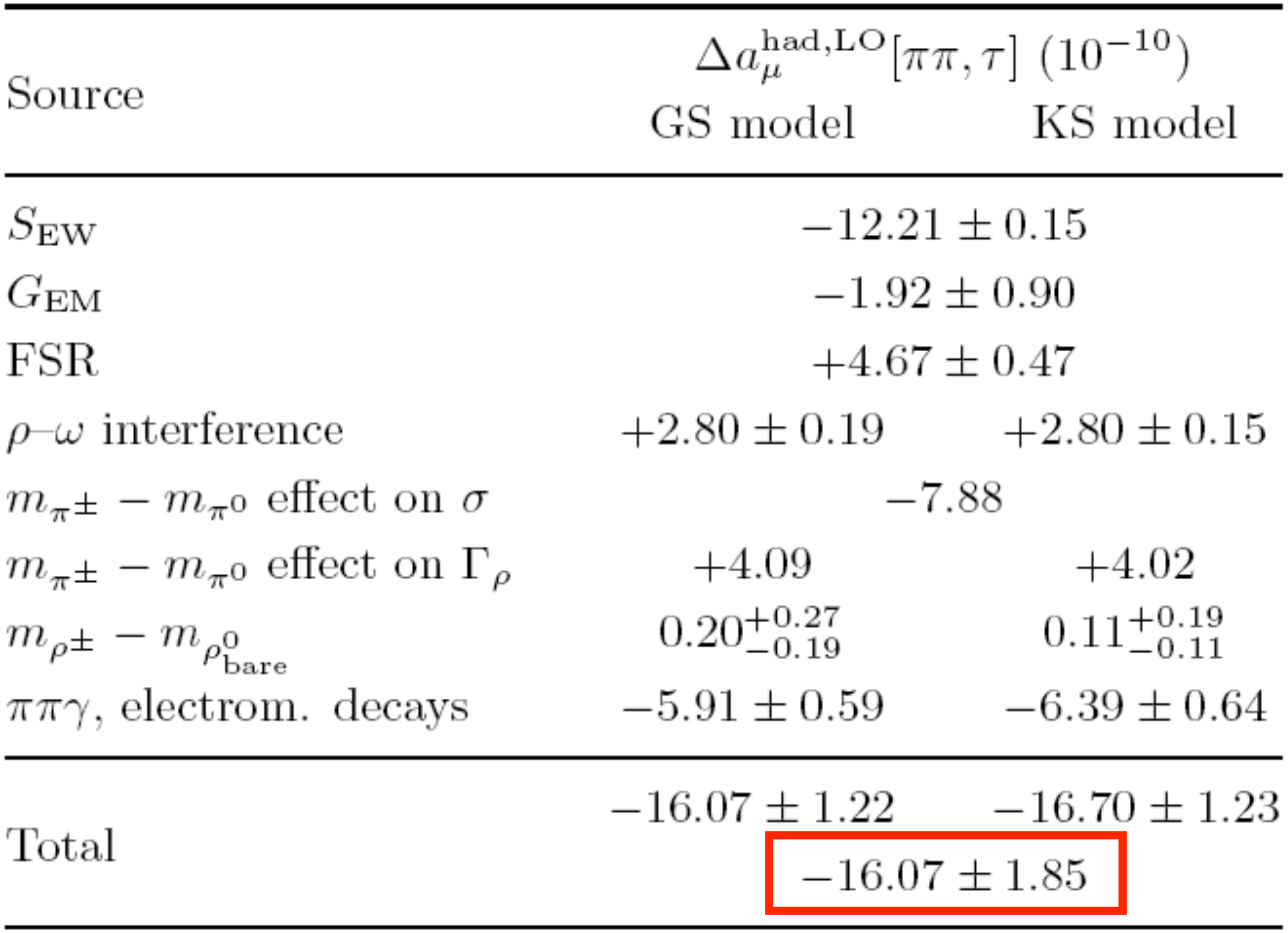


Source	$\Delta a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10})		Old
	GS model	KS model	(DEHZ 03)
S_{EW}	-12.21 ± 0.15		-12.1 ± 0.3
G_{EM}	-1.92 ± 0.90		-1.0
FSR	$+4.67 \pm 0.47$		
ρ–ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$	$+3.5 \pm 0.6$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88		-7.0
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$	$+4.2$
$m_{\rho^\pm} - m_{\rho^0_{\mathrm{bare}}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$	0 ± 2.0
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64	-1.4 ± 1.2
Total	-16.07 ± 1.22	-16.70 ± 1.23	
	$\boxed{-16.07 \pm 1.85}$		-13.8 ± 2.4 (w/o including FSR)

Net change with respect to the previous corrections: -6.9

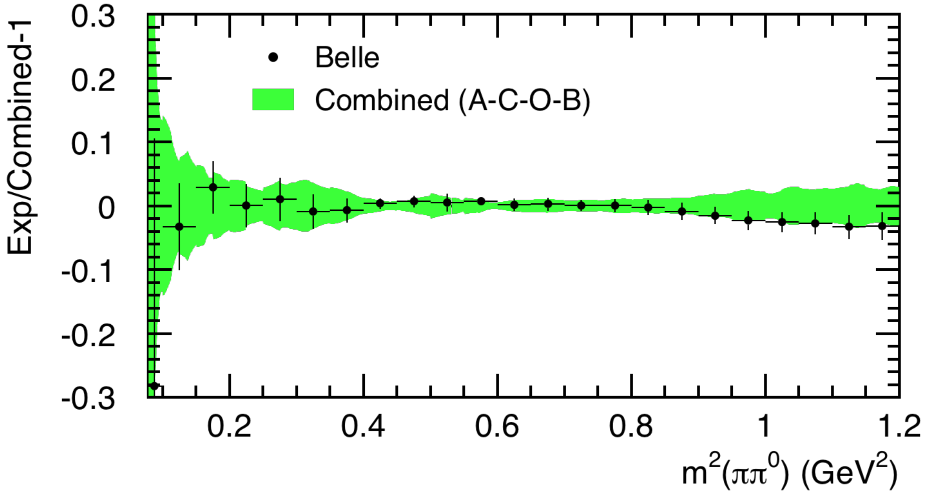
(dominated by em decays)

Other New Items

- ☐ **New tau data from Belle**

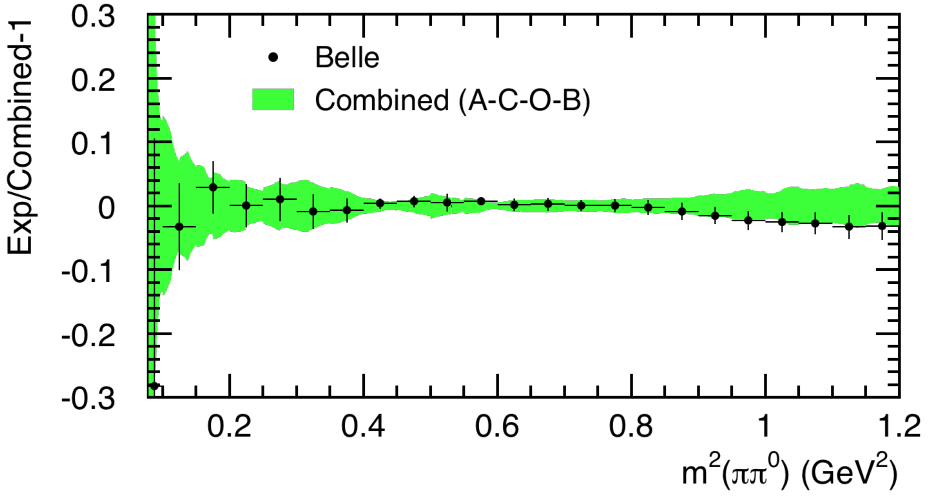
  $\tau \rightarrow h\pi^0\nu_\tau$

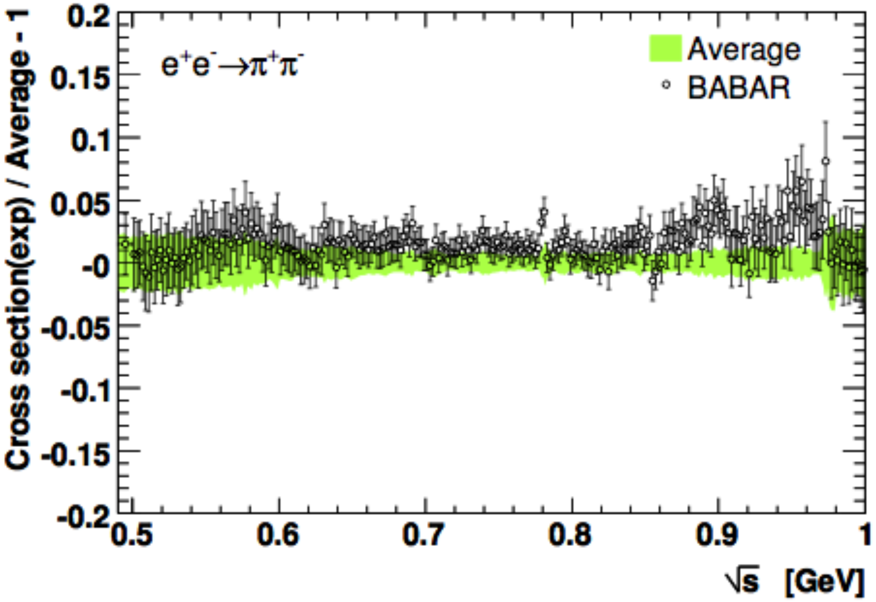
  (5.4×10^6 Belle ←→ 81×10^3 ALEPH)



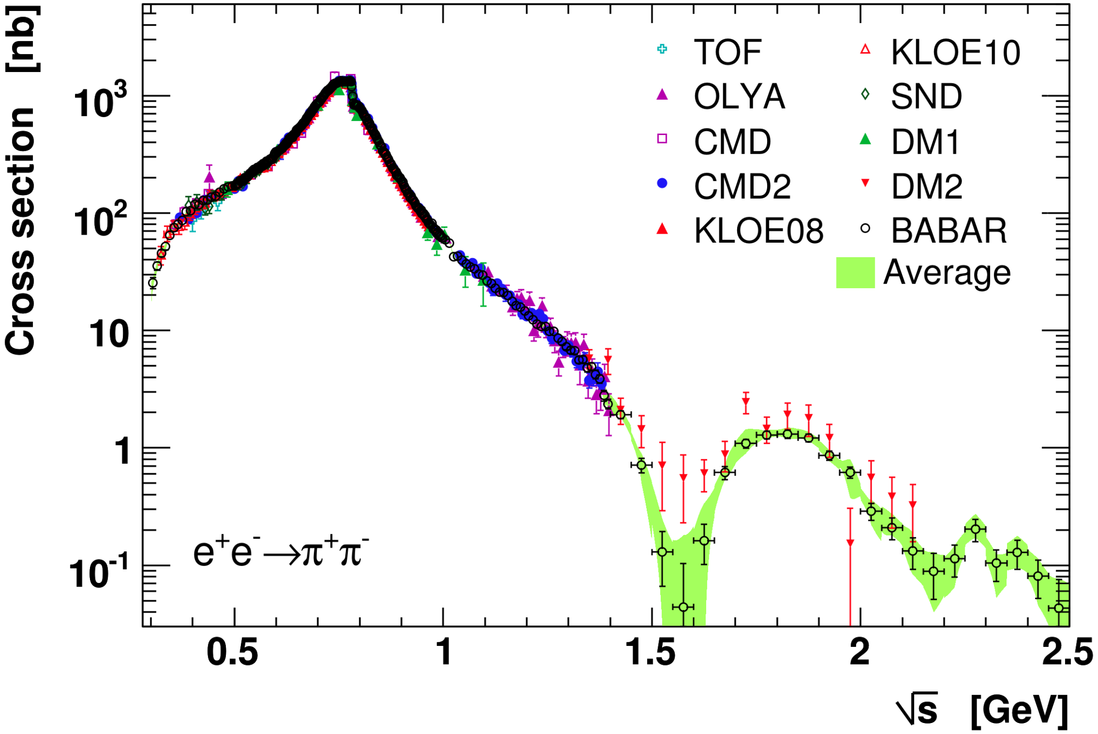
Other New Items

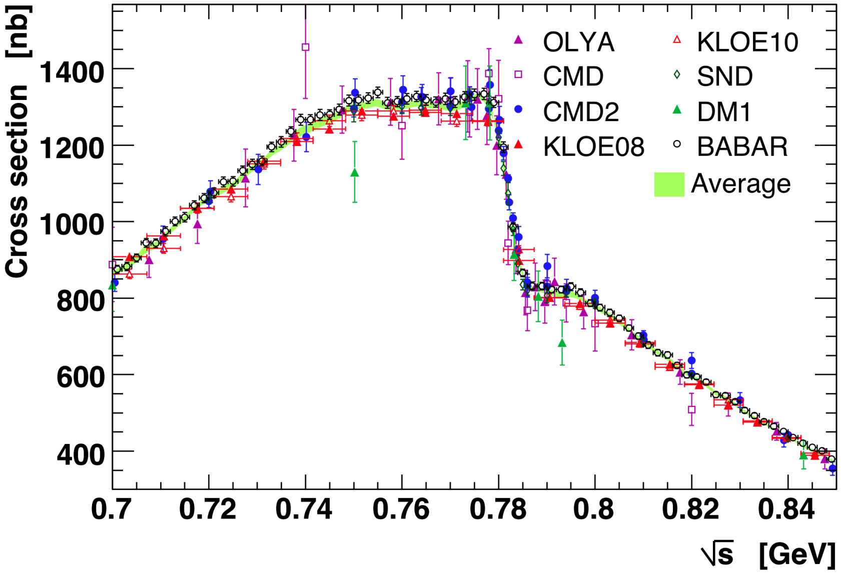
- **New tau data from Belle**
  - $\tau \to h\pi^0\nu_\tau$
  - (5.4×10^6 Belle ↔ 81×10^3 ALEPH)
- **New e+e- annihilation data**
  - $\pi^+\pi^-$
  - $\pi^+\pi^-\pi^0$, $2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$
  - other multi-hadron channel
- **Include unmeasured channels through isospin relations**
  - 5, 6π channels
  - $K\bar{K}[n\pi]$ channels
- **New HVPTools package**
  - data combination
  - data interpolation
  - handling inter-exp, inter-channel correlations



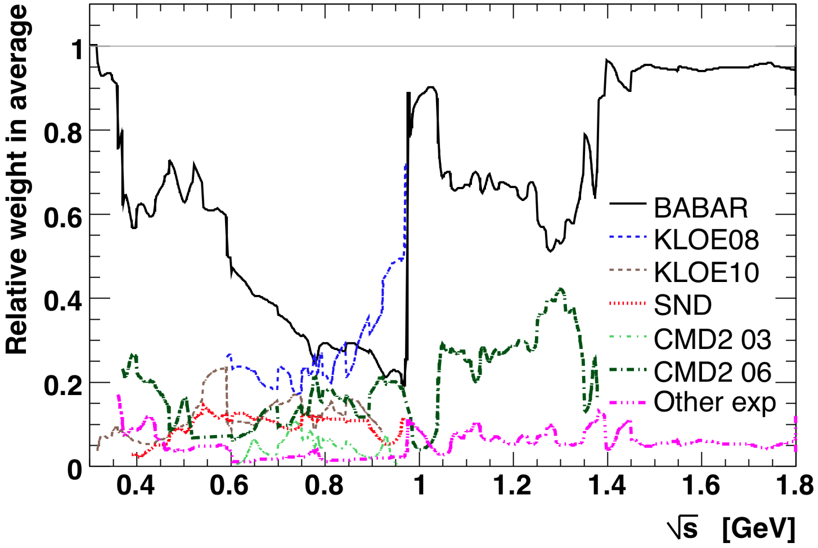


2π Channel

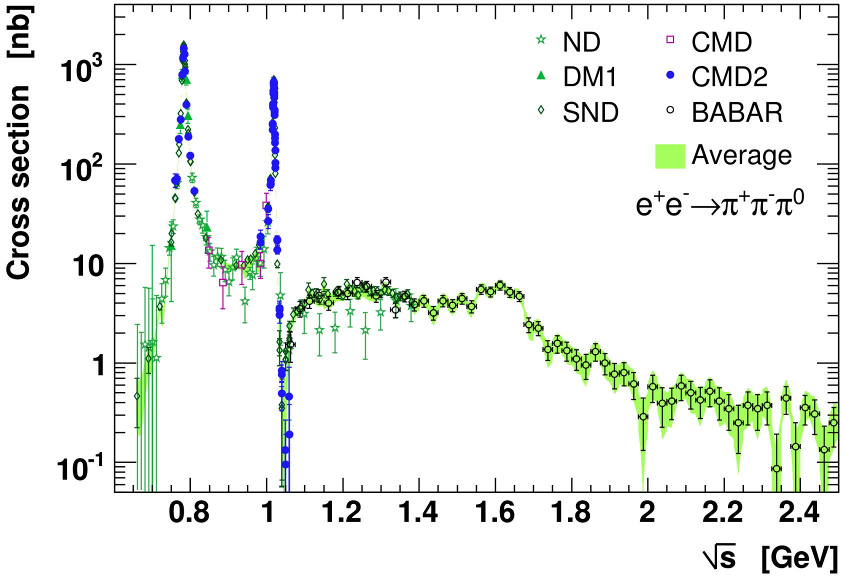


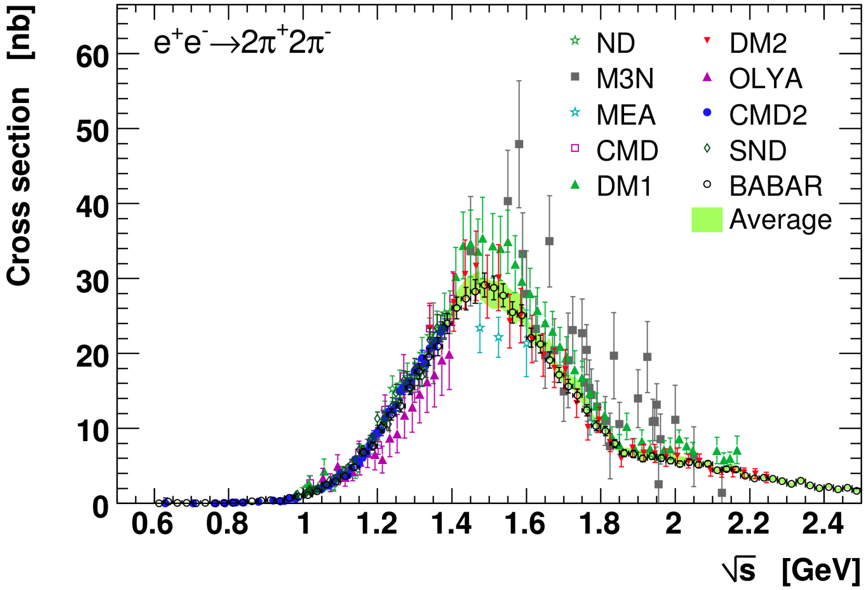


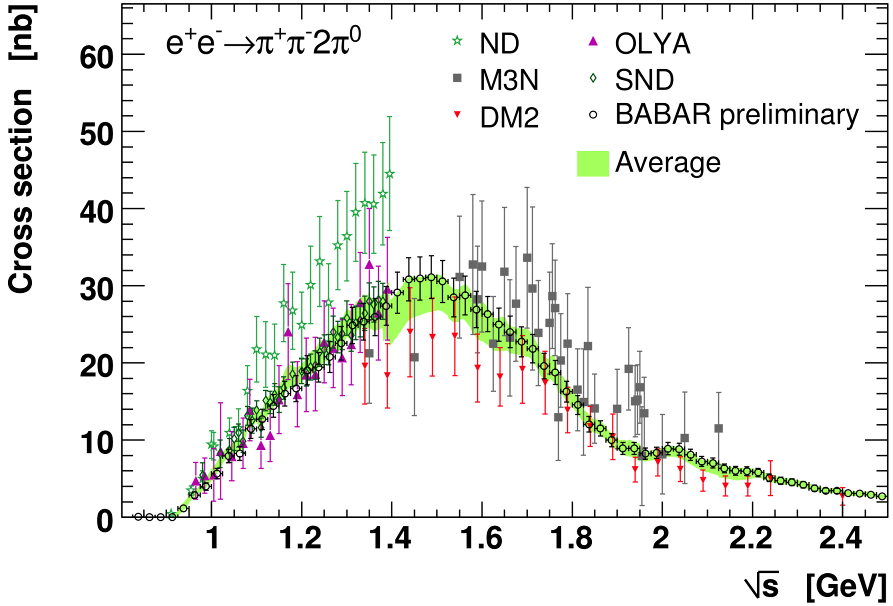
- Measured by many experiments
- BABAR dominates over almost all energy region
- Discrepancy between BABAR and KLOE



$\pi^+\pi^-\pi^0$, $2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$ Channels

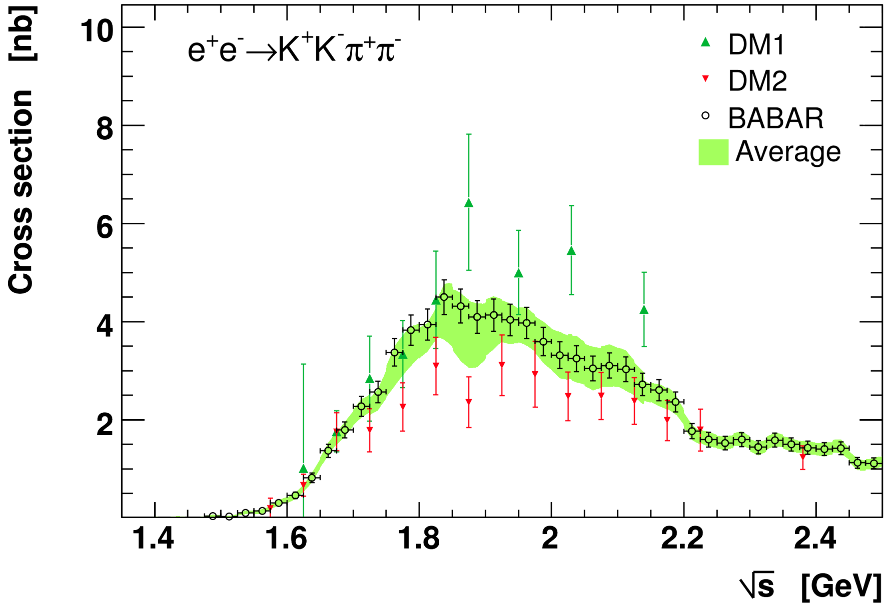


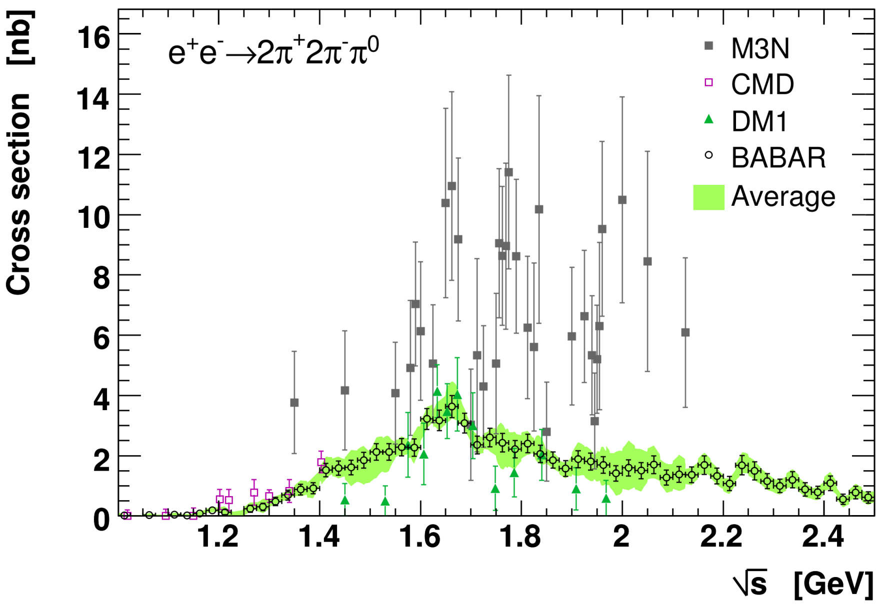


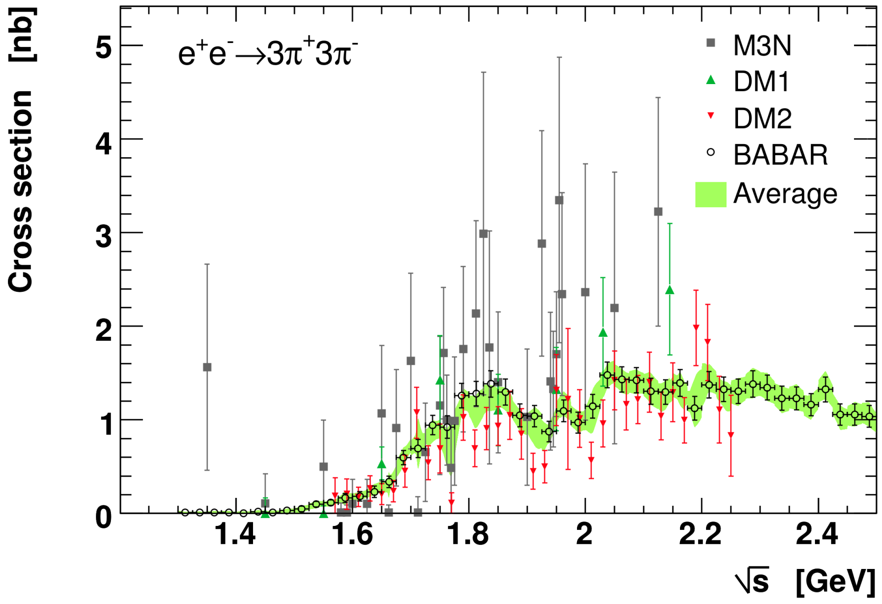


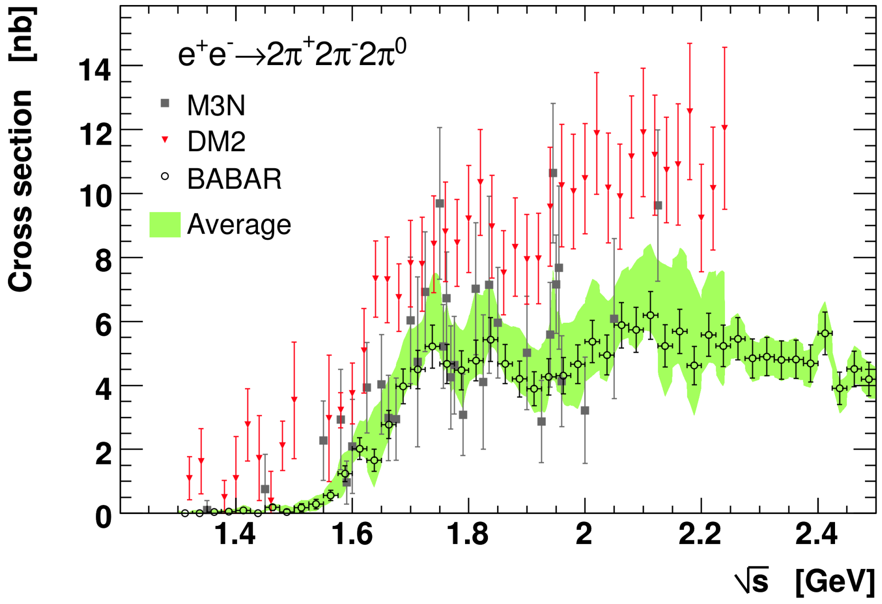
Again BABAR dominates over other experiments though data for $\pi^+\pi^-2\pi^0$ channel still preliminary

Other Multi-hadron Channels

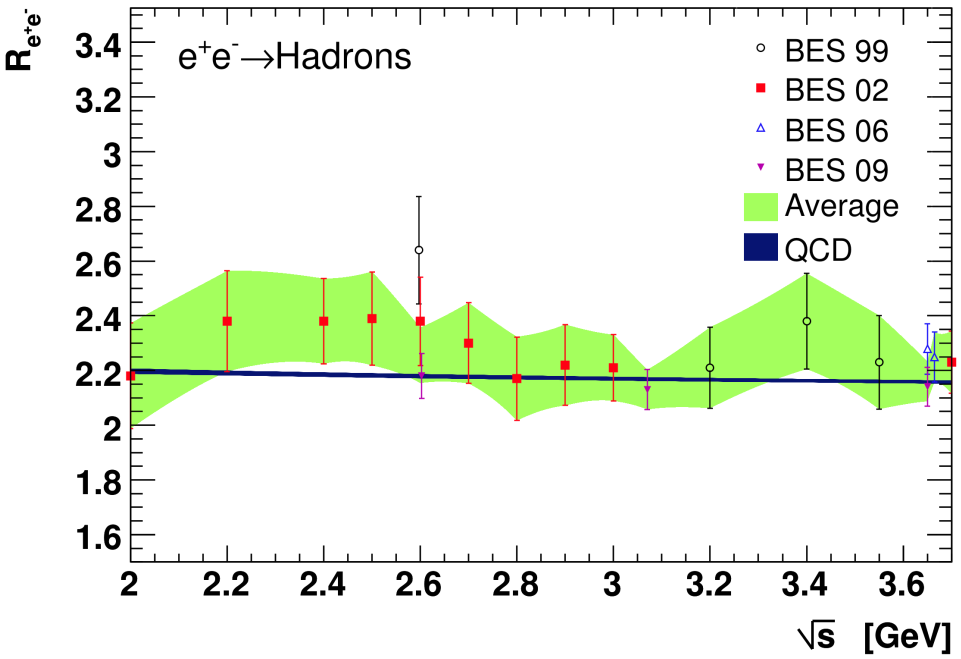


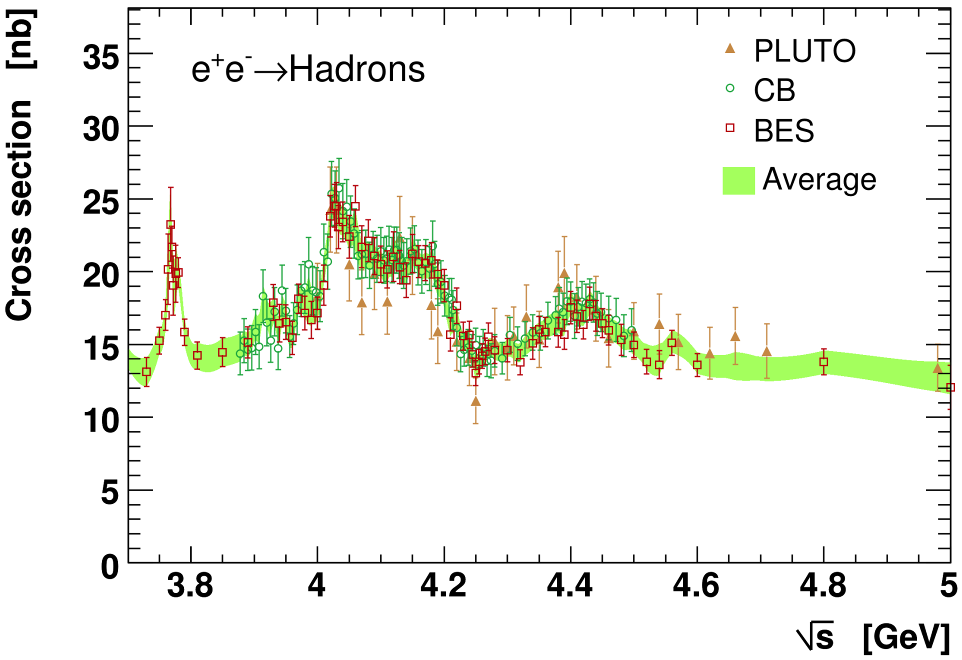






Regions below and above DDbar



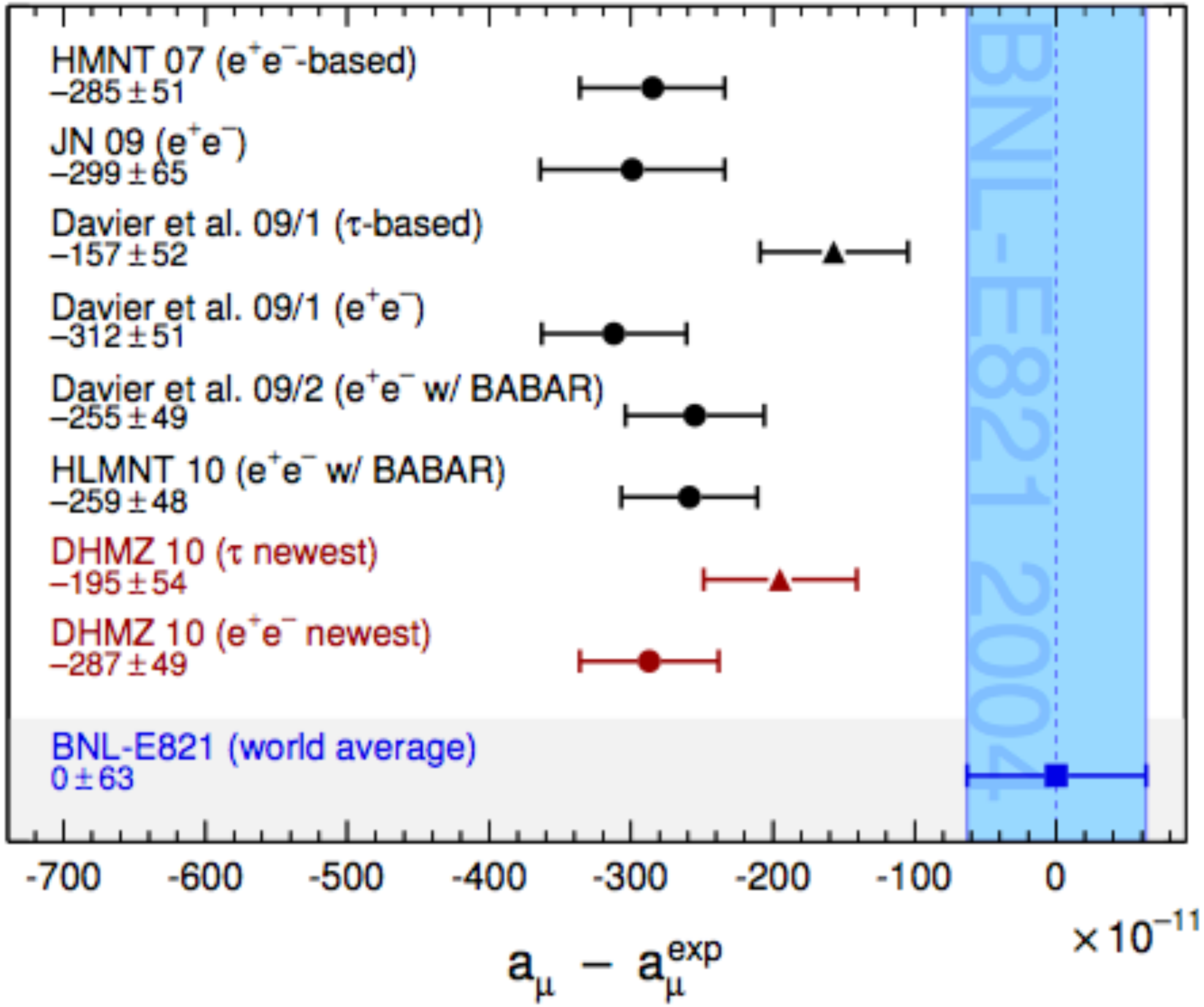


pQCD calculation in good agreement with the direct measurements in non-resonance regions and are applied down to 1.8GeV

BES data precision steadily improving

$R_{\rm QCD}$	$[1.8 - 3.7\ {\rm GeV}]_{uds}$
$R_{\rm QCD}$	$[5.0 - 9.3\ {\rm GeV}]_{udsc}$
$R_{\rm QCD}$	$[9.3 - 12.0\ {\rm GeV}]_{udscb}$
$R_{\rm QCD}$	$[12.0 - 40.0\ {\rm GeV}]_{udscb}$
$R_{\rm QCD}$	$[> 40.0\ {\rm GeV}]_{udscb}$
$R_{\rm QCD}$	$[> 40.0\ {\rm GeV}]_{t}$

Last a_μ Predictions Comparing Measurement



Measurement/predictions discrepancy:

e^+e^-: 3.6σ, τ: 2.4σ

New Results on $\alpha(M_Z)$ & Constraint on M_H

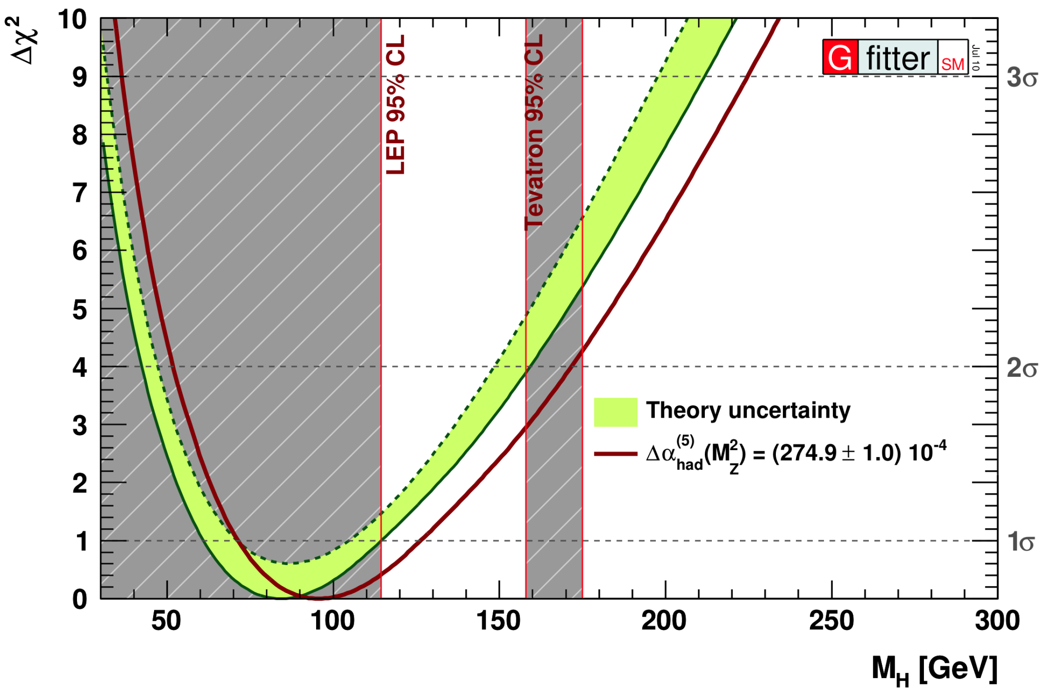
$$\Delta\alpha_{had}(M_Z) = 274.21 \pm 0.17_{stat} \pm 0.78_{uncor\text{-}syst} \pm 0.41_{cor\text{-}syst} \pm 0.18_{\psi} \pm 0.52_{QCD}\ (\times 10^{-4})$$

$$= 274.21 \pm 1.04_{total}\ (\times 10^{-4})$$

➔ $\Delta\alpha_{had}^{(5)}(M_Z) = 274.9 \pm 1.0\ (\times 10^{-4})$, $\alpha^{-1}(M_Z) = 128.962 \pm 0.015$

To be compared with

HMNT (06): $\Delta\alpha_{had}^{(5)}(M_Z) = 276.8 \pm 2.2\ (\times 10^{-4})$, $\alpha^{-1}(M_Z) = 128.937 \pm 0.030$



The fitted (Gfitter) Higgs mass shifted from 84^{+30}_{-23} GeV to 96^{+31}_{-24} GeV

The new upper limits are:

170 GeV @90% CL

201 GeV @95% CL

Summary and Prospects

- Active & fruitful collaboration
- Providing the most precise predictions on a_μ and $\alpha(M_Z)$
  - → Discrepancy measurement/prediction on a_μ could be a 1st hint of new physics
  - → New $\alpha(M_Z)$ favors a lager Higgs mass in better agreement with direct LEP search

- Making connection with direct searches at LHC
  - → exciting prospect
- Looking forward to new e+e- and τ data from BES3
  - → wish to continue the project

New Project: Moving from BNL to Fermilab

K. Lynch, tau workshop 2008

Systematic uncertainty (ppm)	1998	1999	2000	2001	New Goal
Magnetic field – ω_p	0.5	0.4	0.24	0.17	0.07
Anomalous precession – ω_a	0.8	0.3	0.31	0.21	0.07

Combined syst: 0.28ppm

New syst: 0.10ppm

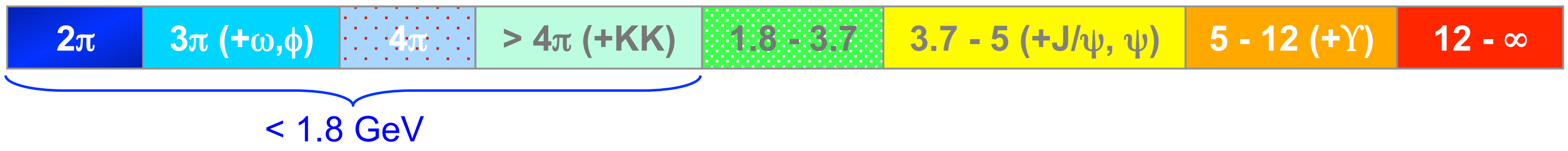
Combined stat: 0.46ppm

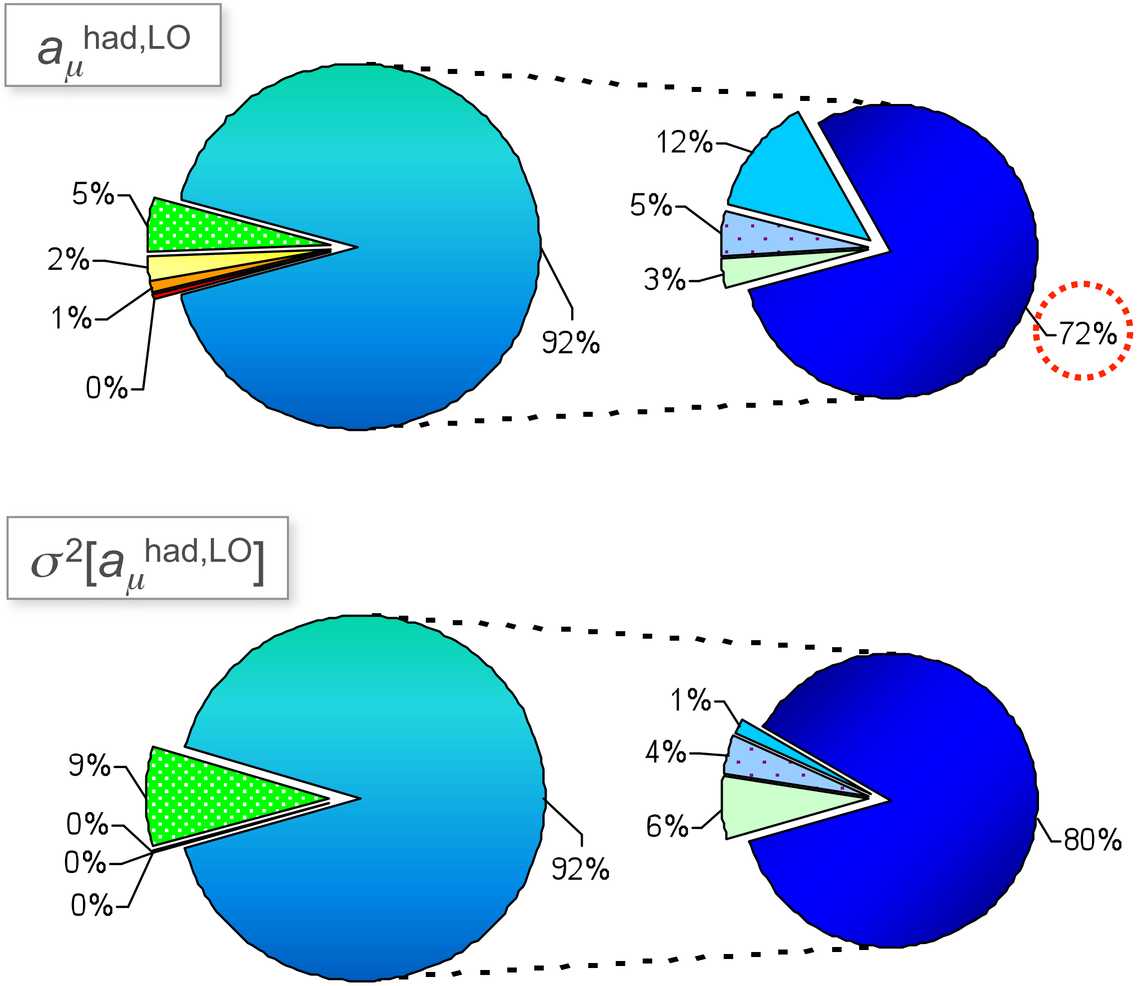
Statistical uncertainty (ppm)	4.9	1.3	0.62	0.66	0.10
Total Uncertainty (ppm)	5.0	1.3	0.73	0.72	0.14

Combined total: 0.54ppm

➔ To improve δa_μ by a factor of 4 from 6.5×10^{-10} to 1.6×10^{-10}

Relative Contribution of Input Data vs Energy





- ➔ 2π channel contributes more than 70% !
- ➔ The e+e- data precision (was) limited

- ➔ Use (complement with) tau data

  [Alemany-Davier-Höcker,
  EPJ C2(98)123]

Isospin Breaking (IB) Corrections Revisited

Corrections for SU(2) breaking applied to τ data for dominant $\pi^-\pi^+$ contribution:

- Electroweak radiative corrections:
  - dominant contribution from short distance correction S_{EW}
  - subleading corrections (small)
  - long distance radiative correction $G_{EM}(s)$

Marciano-Sirlin' 88

Braaten-Li' 90

Cirigliano-Ecker-Neufeld' 02
Lopez Castro et al.' 06

- Charged/neutral mass splitting:
  - $m_{\pi^-} \neq m_{\pi 0}$ leads to phase space (cross sec.) and width (FF) corrections
  - ρ -ω mixing (EM $\omega \rightarrow \pi^-\pi^+$ decay) corrected using FF model
  - $m_{\rho -} \neq m_{\rho 0}$ and $\Gamma_{\rho -} \neq \Gamma_{\rho 0}$

Alemany-Davier-Höcker' 97, Czyż-Kühn' 01

Flores-Baez-Lopez Castro' 08
Davier et al.'09

- Electromagnetic decays: $\rho \rightarrow \pi\pi\gamma$, $\rho \rightarrow \pi\gamma$, $\rho \rightarrow \eta\gamma$, $\rho \rightarrow l^+l^-$
- Quark mass difference $m_u \neq m_d$ (negligible)

Isospin Breaking (IB) Corrections Revisited

$$\Delta^{\mathrm{IB}} a_\mu^{\mathrm{LO,had}}[\pi\pi,\tau] = \frac{\alpha^2 m_\tau^2}{6\,|V_{ud}|^2\pi^2}\,\frac{\mathcal{B}_{\pi^-\pi^0}}{\mathcal{B}_{e^-\overline{\nu}_e\nu_\tau}}\int_{4m_\pi^2}^{m_\tau^2} ds\,\frac{K(s)}{s}\,\frac{dN_X}{N_X\,ds}\left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\left[\frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}}-1\right]$$

$S_{EW}=1.0235\pm0.0003$

$$R_{\mathrm{IB}}(s) = \frac{\mathrm{FSR}(s)}{G_{\mathrm{EM}}(s)}\,\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2$$

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