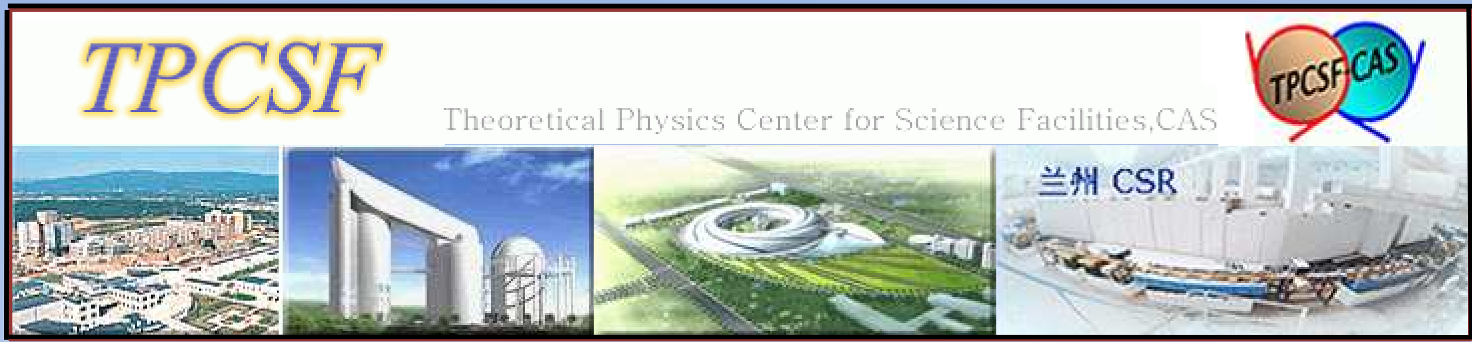

The baryonic form factors in the Heavy Quark Effective Theory

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The Heavy Quark Symmetry (*HQS*) (Isgur, Wise)

Heavy-light mesons $Q\bar{q}$ (as $\bar{B}^0(b\bar{d})$) and baryons Qqq (as Λ_b^0(bud))

- Hadron lenght scale : $R_{had} \sim 1/\Lambda_{QCD}$
- Heavy quark *Q* lenght scale : $\lambda_Q \sim 1/m_Q$

$$m_Q \gg \Lambda_{QCD} \Rightarrow \lambda_Q \ll R_{had}$$

↳ Heavy quark *Q* and light cloud (light $\bar{q}$, *qq*, *g*'s and $q\bar{q}$'s) : **soft gluon** interactions ($q^2 \sim \Lambda_{QCD}^2$)

$\Longrightarrow$ Quantum numbers of *Q* probed by **hard** momentum transfert ($q^2 \gg m_Q^2$, weak bosons, g's)

$\Longrightarrow$ ***HQS*** $m_Q \to \infty$: *Q* **static colour source**

↳ Light cloud insensitive to the **spin** S_Q and to the **flavour** (m_Q) of the heavy quark *Q*

Soft strong interactions preserve:

- Heavy quark **spin** $S_Q = 1/2$
- Total angular momentum of light cloud $j = L \oplus s_\ell$

L : orbital angular momentum

s_ℓ : spin of light cloud

- s_ℓ = 1/2 for mesons
- s_ℓ = 0 or 1 for baryons

↳ Hadronic states classified according to ***spin j*** and ***parity P*** of the light cloud (j^P)

Mesons : $P = (-1)^{L+1}$

Baryons : $P = (-1)^L$

- **Spin** symmetry : only $\mathbf{E}_{chromo}$ contributes $\Longrightarrow$ relativistic effects (colour magnetism)

  vanish in the ***HQS* limit** $m_Q \to \infty$: decoupling of the heavy quark spin S_Q

- **Flavour** symmetry : light cloud does not depend on m_Q

↳ **Spin-flavour** symmetry ***U(2N$_h$)*** of QCD (N_h heavy quark flavours)

HQS $m_Q \to \infty$: Bound-states Qq and Qqq differing by **spin** S_Q and **mass** m_Q of the heavy quark Q

$\Longrightarrow$ have the same light could configuration j^P

$\Longrightarrow$ for $j = L \oplus s_\ell$ fixed : hadrons classified into multiplets with $J = j \oplus S_Q$

- Meson ground-state doublet (L=0 : $D^{(*)}$) $j^P = \frac{1}{2}^- \ : \begin{pmatrix} J^P = 0^- \ : \ D \\ J^P = 1^- \ : \ D^* \end{pmatrix}$

- Meson excited doublets (L=1 : D^{**}) $j^P = \frac{1}{2}^+ \ : \begin{pmatrix} J^P = 0^+ \ : \ D_0^* \\ J^P = 1^+ \ : \ D_1^* \end{pmatrix}$

  $j^P = \frac{3}{2}^+ \ : \begin{pmatrix} J^P = 1^+ \ : \ D_1 \\ J^P = 2^+ \ : \ D_2^* \end{pmatrix}$

- <u>Baryon ground-state singlet</u> (L=0) $j^P = 0^+ \ : \ \left(J^P = \frac{1}{2}^+ \ : \ \Lambda_c \right)$

- Baryon excited doublet (L=1) $j^P = 1^- \ : \begin{pmatrix} J^P = \frac{1}{2}^- \ : \ \Lambda_{c1}^{1/2} \\ J^P = \frac{3}{2}^- \ : \ \Lambda_{c1}^{3/2} \end{pmatrix}$

The Heavy Quark Effective Theory (*HQET*) (Eichten, Hill, Grinstein, Georgi)

HQS : symmetry of an effective theory of QCD when $m_Q \gg \Lambda_{QCD}$

$\Longrightarrow$ Separation of the large- and short-distance Physics

Heavy *Q* almost on-shell : $p_Q^2 \cong m_Q^2 \Rightarrow p_Q = m_Q v + k$ { residual momentum $k \sim \Lambda_{QCD}$ (dynamic); velocity v as an index (kinematic) }

$\Longrightarrow$ Heavy quark field : $Q(x) = e^{im_Q v\cdot x}[h_v(x) + H_v(x)]$

- h_v annihilates a heavy quark with velocity v
- H_v describes fluctuations around $p_Q^2 = m_Q^2$: cancelled in the effective Lagrangian $\mathcal{L}_{eff}$

$$\mathcal{L}_{eff} = \mathcal{L}_{\mathbf{HQET}} + \frac{1}{2m_Q}\left(\mathcal{L}_1^{(kin)} + \mathcal{L}_1^{(mag)}\right) + O(\frac{1}{m_Q^2})$$

$$= \underbrace{\overline{\mathrm{h}}_\mathrm{v} \mathrm{iv}\cdot \mathrm{Dh}_\mathrm{v}}_{\mathcal{L}_{\mathbf{HQET}}} + \frac{1}{2m_Q}\left(\overline{h}_v(iD_\perp)^2 h_v + \frac{g_s}{2}\overline{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v\right) + O(\frac{1}{m_Q^2})$$

- ***Lagrangian***-type corrections : residual kinetic energy — spin interactions of *Q* with the gluon field

$$\langle H' \mid J_{HQET}(0) \mid H + \delta H\rangle = \langle H' \mid J_{HQET}(0) \mid H\rangle + \frac{1}{2m_Q}\langle H' \mid i\int d^4x T\{J_{HQET}(0), \mathcal{L}_1(x)\} \mid H\rangle$$

$\mid H + \delta H\rangle$: eigenstate of $\mathcal{L}_{eff}$ $\qquad$ $\mid H\rangle$: eigenstate of $\mathcal{L}_{HQET}$

- ***Current***-type corrections : $J = \bar{c}\Gamma b = \underbrace{\overline{\mathrm{h}}^{(c)}\Gamma \mathrm{h}^{(b)}}_{\mathbf{J_{HQET}}} + \frac{1}{2m_b}\overline{h}^{(c)}\Gamma i \not{D} h^{(b)} + \frac{1}{2m_c}\overline{h}^{(c)} i\overleftarrow{\not{D}}\Gamma h^{(b)} + O(\frac{1}{m_Q^2})$

Heavy quark current : $\mathbf{J_{HQET}}$

HQS : form factors ⟹ universal Isgur-Wise (IW) functions (large distance Physics of light cloud)

- $\overline{B} \to D^{(*)} \ell \overline{\nu}_\ell$ (*L*=0)

6 form factors ⟶ the leading ***elastic*** ($1/2^- \to 1/2^-$) meson IW function : $\xi(w)$

$$\langle D(\frac{1}{2}^-)(v') \mid \overline{h}^{(c)}_{v'} \Gamma h^{(b)}_v \mid B(\frac{1}{2}^-)(v)\rangle = -\xi(w) Tr\{\overline{\mathcal{D}}(v')\Gamma \mathcal{B}(v)\}$$

Recoil : $w \equiv v \cdot v' = \dfrac{m^2_{H_i} + m^2_{H_f} - q^2}{2 m_{H_i} m_{H_f}}$ measures the change in velocity of the heavy hadrons

zero recoil : ***w_{min} = 1*** ⇒ H_f is at rest in the rest frame of H_i (max. momentum transfert to leptons)

$\overline{\mathcal{D}}(v'), \mathcal{B}(v)$: 4×4 matrix valued-spin wave functions (Falk, Neubert)

- $\overline{B} \to D^{**} \ell \overline{\nu}_\ell$ (*L*=1)

14 form factors ⟶ ***2 transition*** IW functions ($1/2^- \to 1/2^+$) $\tau_{1/2}(w)$ and ($1/2^- \to 3/2^+$) $\tau_{3/2}(w)$

$$\langle D(\frac{1}{2}^+)(v') \mid \overline{h}^{(c)}_{v'} \Gamma h^{(b)}_v \mid B(\frac{1}{2}^-)(v)\rangle = 2\tau_{1/2}(w) Tr\{\overline{\mathcal{D}}(v')\Gamma \mathcal{B}(v)\}$$

$$\langle D(\frac{3}{2}^+)(v') \mid \overline{h}^{(c)}_{v'} \Gamma h^{(b)}_v \mid B(\frac{1}{2}^-)(v)\rangle = \sqrt{3}\tau_{3/2}(w) Tr\{v_\mu \overline{\mathcal{D}}^\mu(v')\Gamma \mathcal{B}(v)\}$$

- $\Lambda_b \to \Lambda_c \ell \overline{\nu}_\ell$ (*L*=0)

6 form factors ⟶ the leading ***elastic*** ($0^+ \to 0^+$) baryon IW function $\xi_\Lambda(w)$

$$\langle \Lambda_c(v', s') \mid \overline{h}^{(c)}_{v'} \Gamma h^{(b)}_v \mid \Lambda_b(v, s)\rangle = \xi_\Lambda(w) \overline{\mathcal{U}}_{\Lambda_c}(v', s')\Gamma\, \mathcal{U}_{\Lambda_b}(v, s)$$

HQET heavy baryon spinor : $\mathcal{U}_{\Lambda_b}(v,s) = \left(1 + O(\frac{1}{m_b^2})\right) u_{\Lambda_b}(v,s)$: physical spinor ($\overline{u}_\Lambda(v,s) u_\Lambda(v,s) = 2m_\Lambda$)

HQS : model-independent normalization $\xi(1) = \xi_\Lambda(1) = 1$ (CVC)

B, Λ_b semileptonic decays $\longrightarrow$ exclusive determination of $|V_{cb}|$

Bound-state effects of strong interactions parametrized by form factors $\mathcal{F}^{(*)}(w)$,$G_1(w)$

- $$\frac{d\Gamma(\overline{B} \to D^{(*)} \ell \overline{\nu}_\ell)}{dw} = \kappa_M^{(*)} \mid V_{cb} \mid^2 \begin{cases} (w^2-1)^{1/2} \mid \mathcal{F}^*(w) \mid^2 \\ (w^2-1)^{3/2} \mid \mathcal{F}(w) \mid^2 \end{cases}$$

Measurements of $\mid V_{cb} \mid \mathcal{F}^*(w) \Longrightarrow$ Extrapolation at $\mid V_{cb} \mid \mathcal{F}^*(1)$
- experimental imprecision
- hadronic f. f. uncertainty

$$\mathcal{F}^*(1) = \eta_{QCD}\big[\underbrace{\xi(1)}_{=1} + \delta^{(\mathcal{F}^*)}_{1/m_Q^2}\big] = 0.927 \pm 0.024 \quad \Longrightarrow \quad \mid V_{cb} \mid_{excl.} = (38.7 \pm 1.1) \times 10^{-3}$$

Lattice (unquenched)

($\mid V_{cb} \mid_{incl.} = (41.5 \pm 0.7) \times 10^{-3}$ via *OPE*)

Model-independent derivation of $|V_{cb}|$!!

- $$\frac{d\Gamma(\Lambda_b \to \Lambda_c \ell \overline{\nu}_\ell)}{dw} = \kappa_B \mid V_{cb} \mid^2 (w^2-1)^{1/2} \mid G_1(w) \mid^2$$

$\Gamma(\Lambda_b^0 \to \Lambda_c^+ \ell^- \overline{\nu}_\ell) \simeq 5\%$ (@ Tevatron)

$\Gamma(\Lambda_b^0 \to \Lambda_c^+ \ell^- \overline{\nu}_\ell + anything) \simeq 10\%$

LHC$_b$ will study dΓ/dw in detail

$$G_1(w) = \xi_\Lambda(w) + \left(\frac{1}{2m_b} + \frac{1}{2m_c}\right)\left[\frac{w-1}{w+1}\overline{\Lambda}\xi_\Lambda(w) + A(w)\right] + O\left(\frac{1}{m_Q^2}\right) \quad \Longrightarrow \quad \boxed{G_1(1) = 1 + \delta^{(G_1)}_{1/m_Q^2}}$$

Current-type correction
(instead of 2 subleading meson f.f. !)

Lagrangian-type correction $A(1) = 0$
(instead of 3 subleading meson f.f. !)

$\overline{\Lambda} = m_{\Lambda_Q} - m_Q + O(\frac{1}{m_Q})$: light cloud energy

$$\xi_\Lambda(w) = 1 - \rho_\Lambda^2(w-1) + \frac{\sigma_\Lambda^2}{2}(w-1)^2 + \ldots$$

- ρ_Λ^2 : slope
- σ_Λ^2 : curvature

HQET Sum Rules (SR) and the leading constraints (Bjorken, Oliver et al.)

$\Lambda_b(v_i) \to \Lambda_c^{(n)}(v') \to \Lambda_b(v_f)$ **non-forward** transition $v_i \neq v_f$ (Ulratsev) $\Longrightarrow$ general SRs :

T-product of 2 heavy-heavy currents ($J_{i,f} = \overline{c}\,\Gamma_{i,f}\, b$) : $i \int d^4x e^{-iq\cdot x} \langle \Lambda_b(v_f) \mid T\{J_f(0)\,,\, J_i(x)^\dagger\} \mid \Lambda_b(v_i)\rangle$

- ***intermediate charmed*** $\Lambda_c^{(n)}$ ***states***

  $J = j \pm 1/2$ with $\begin{cases} j^P = L^P \\ P = (-1)^L \end{cases}$

- ***Operator Product Expansion (OPE)***

HQET SRs :

$$L_{Hadrons}(w_i, w_f, w_{if}) = R_{OPE}(w_i, w_f, w_{if})$$

$$(w_i = v_i \cdot v' \;,\; w_f = v_f \cdot v' \;,\; w_{if} = v_i \cdot v_f)$$

(in a certain domain, e.g. $w_{i,f} \geq 1$)

$\Longrightarrow$ vector ($\Gamma_{i,f} = \not{v}'$) HQET SR : ($w_i = w_f = w$)

$$\xi_\Lambda(w_{if}) = \sum_{L\geq 0}\sum_{n\geq 0}\left[\tau_L^{(n)}(w)\right]^2 \sum_{0\geq k\geq \frac{L}{2}} (-1)^k \frac{(L!)^2}{(2L)!}\frac{(2L-2k)!}{k!(L-k)!(L-2k)!}(w^2-1)^{2k}(w^2-w_{if})^{L-2k}$$

$\tau_L^{(n)}(w)$: $0^+ \to L^P$ baryon transition IW functions

- the derivatives of $\xi_\Lambda(w)$: $(-1)^p \xi_\Lambda^{(p)}(1) \geq 0$ is an ***alternate series*** in powers of (*w*-1) (Oliver et al.)
- the slope of $\xi_\Lambda(w)$: $\rho_\Lambda^2 \geq 0$
- the curvature of $\xi_\Lambda(w)$: $\sigma_\Lambda^2 \geq \frac{3}{5}[\rho_\Lambda^2 + (\rho_\Lambda^2)^2]$
- $\rho_\Lambda{}^2 \to 0$ implies $\sigma_\Lambda{}^2 \to 0$

Lagrangian perturbations of the semileptonic decay of the Λ_b (preliminary)

$$\lim_{w\to 1}\frac{1}{\sqrt{w^2-1}}\frac{d\Gamma(\Lambda_b\to\Lambda_c\ell^-\overline{\nu}_\ell)}{dw}=\frac{G_F^2\mid V_{cb}\mid^2}{4\pi^3}m_{\Lambda_c}^2(m_{\Lambda_b}-m_{\Lambda_c})^2\mid G_1(1)\mid^2$$ (@ LHC$_b$)

HQET : axial-vector form factor at *w*=1 : $\mid G_1(1)\mid^2=1+\delta^{(G_1)}_{1/m_Q^2}$

$$\mid G_1(1)\mid^2=1+\left[\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)^2+\frac{8}{3}\frac{1}{2m_c}\frac{1}{2m_b}\right]\lambda-\sum_{\Lambda_c^{(n)}}\frac{\mid\langle\Lambda_c^{(n)}\mid\vec{A}\mid\Lambda_b\rangle\mid^2}{4m_{\Lambda_c^{(n)}}m_{\Lambda_c}}$$ ← intermediate states $j^P = 0^+, 1^+$

a) residual kinetic energy of *Q* : $\lambda=\frac{\langle\Lambda_Q(v)\mid\mathcal{L}_1^{(kin)(Q)}(0)\mid\Lambda_Q(v)\rangle}{2m_{\Lambda_Q}}=\frac{\langle\Lambda_Q(v)\mid\overline{h}_v^{(Q)}(iD_\perp)^2h_v^{(Q)}(0)\mid\Lambda_Q(v)\rangle}{2m_{\Lambda_Q}}$

(*Q* almost on-shell)

b) kinetic part $\mathcal{L}_1^{(kin)}$ of the Lagrangian correction-type subleading axial form factor *A(w)*

$$A(w)\overline{\mathcal{U}}_{\Lambda_c}(v',s')\Gamma\,\mathcal{U}_{\Lambda_b}(v,s)=\langle\Lambda_c(v',s')\mid i\int d^4xT\{\overline{h}_{v'}^{(c)}\Gamma h_v^{(b)}(0)\,,\,\mathcal{L}_1^{(kin)}(x)\}\mid\Lambda_b(v,s)\rangle$$

$$A(w)=\frac{1}{2}\sum_{n\neq 0}\frac{\xi^{(n)}(w)}{m_{\Lambda_c^{(n)}}-m_{\Lambda_c}}\frac{\langle\Lambda_c^{(n)}(v,s)\mid\mathcal{L}_1^{(kin)(c)}(0)\mid\Lambda_c(v,s)\rangle}{\sqrt{4m_{\Lambda_c^{(n)}}m_{\Lambda_c}}}$$: intermediate states $\Lambda_c^{(n)}$ ($j^P = 0^+$ because $\mathcal{L}_1^{(kin)}$)

($0^+ \to 0^+$) IW function $\xi_\Lambda^{(n=0)}(1)=1$ but $\xi_\Lambda^{(n\neq 0)}(1)=0$ $\Longrightarrow$ **A(1)=0**

$$-\delta^{(G_1)}_{1/m_Q^2}\geq-\frac{1}{2}\left[\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)^2+\frac{8}{3}\frac{1}{2m_c}\frac{1}{2m_b}\right]\lambda+\frac{12}{5}\left[\left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)\right]^2\frac{[A'(1)]^2}{\sigma_\Lambda^2-\frac{3}{2}[\rho_\Lambda^2+(\rho_\Lambda^2)^2]}$$

$\rho_\Lambda^2 \to 0$ implies $A'(1) \to 0$

strong correlation between the ***leading f.f.*** $\xi_\Lambda(w)$ shape and the ***subleading f.f.*** *A(w)*

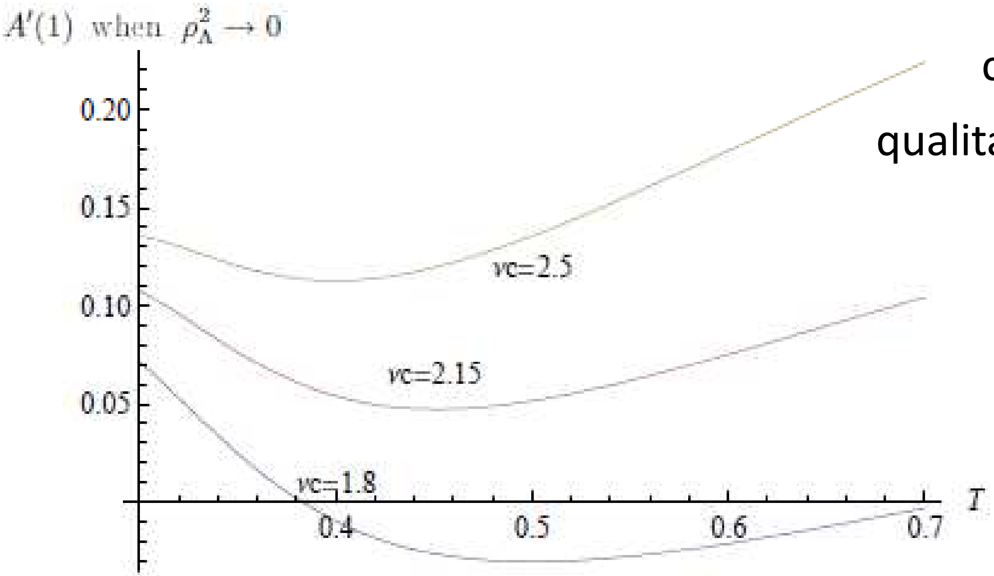
The kinetic Lagrangian-type subleading axial form factor A(w) from QCD Sum Rules

(Grozin et al., Dai et al., Wang et al., Korner et al.)

$$A(w) = -\frac{e^{2\overline{\Lambda}/T}}{4f^2}\left[J(w) - \xi_\Lambda(w)J(1)\right] \quad \Longrightarrow \quad A(1) = 0 \text{ imposed in QCDSR } (\, \xi_\Lambda(1) = 1 \,)$$

$$J(w) = \left(\frac{1}{2\pi}\frac{1}{w+1}\right)^4 w\int_0^{\nu_c} d\nu\nu^6 e^{-\nu/T} + \frac{m_0^2\langle\overline{q}q\rangle^2}{6T}w\left[3+\frac{m_0^2}{4T^2}(w^2-1)\right]e^{-\frac{m_0^2}{4T^2}(w+1)} + \frac{\langle\alpha_s G^2\rangle}{3}\left(\frac{1}{2\pi}\frac{T}{w+1}\right)^3(2w^3+8w^2+4w+5)$$

Decay constant : $f^2 e^{-2\overline{\Lambda}/T} = \left(\frac{1}{2\pi}\right)^4 \frac{1}{80}\int_0^{\nu_c} d\nu\nu^5 e^{-\nu/T} + \frac{\langle\overline{q}q\rangle^2}{6}e^{-\frac{m_0^2}{2T^2}} + \frac{T^2}{(2\pi)^3}\frac{\langle\alpha_s G^2\rangle}{16}$



continuum threshold 1.8 < ν_c < 2.5 GeV

qualitative Borel stability window T ≅ 0.3 - 0.6 GeV

$$\langle\overline{q}q\rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3$$

$$\langle\frac{\alpha_s}{\pi}G^2\rangle = (0.012 \pm 0.004)^4 \text{ GeV}^4$$

$$m_0^2 = 0.8 \text{ GeV}^2$$

$$\Longrightarrow \boxed{A'(1) \underset{\rho_\Lambda^2 \to 0}{=} 0.05 \pm 0.05 \text{ GeV}}$$

- ***compatible*** with model-independent HQET result
- ***HQET constraint*** must be imposed in the QCDSR calculations