

Extracting CP violation and strong phase in D decays by using quantum correlation in

$$\begin{aligned}\psi'' &\rightarrow D^0 \bar{D}^0 \rightarrow (V_1 V_2)(K\pi) \text{ and} \\ \psi'' &\rightarrow D^0 \bar{D}^0 \rightarrow (V_1 V_2)(V_3 V_4)\end{aligned}$$

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# Outline

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# Introduction

- ▶ In the framework of SM, CP violation in the charm sector is very small, thus any significant amount of CP violation will be a clean signal of NP.
- ▶ Large  $\psi''$  data will be collected at BESIII.  $D \rightarrow VV$  modes have not been extensively explored as  $PP$  and  $VP$  case. Meanwhile, they exhibit rather large branching ratios, of the similar size with respect to  $PP$  or  $VP$  modes, and may provide further new observables, which has not been detailed so far.

$$Br(D^0 \rightarrow \bar{K}^{*0} \rho) = (1.58 \pm 0.35)\%,$$

$$Br(D^0 \rightarrow \rho^0 \rho^0) = (1.82 \pm 0.13) \times 10^{-3}.$$

...

## Various decay modes for correlated double $D$ mesons

One can in principle consider the following different situations :

- ▶  $(PP)(PP), (PP)(VP), (VP)(VP)$ : the only available observable is branching ratio, since the partial waves and helicities are all fixed by angular momentum conservation.
- ▶  $(PP)(VV), (VP)(VV)$ :  $(VV)$  can have three helicity states, and thus there are new angular observables. This will be investigated for  $(PP) = K\pi$  in connection with CKM angle  $\gamma$  measurement.
- ▶  $(VV)(VV)$ : this will be studied with an interest in new observables for CP violation.

# Decay chain

- ▶ antisymmetric coherent state:

$$|(D\bar{D})_{L=1}\rangle = \frac{-|D_1\rangle|D_2\rangle + |D_2\rangle|D_1\rangle}{\sqrt{2}} \quad (1)$$

- ▶ for CKM angle  $\gamma$  measurement

$$\begin{aligned} \psi'' \rightarrow D_1 D_2, \quad D_1 \rightarrow V_1 V_2, \quad D_2 \rightarrow K\pi, \quad (2) \\ V_1 \rightarrow M_1 M'_1, \quad V_2 \rightarrow M_2 M'_2 \end{aligned}$$

- ▶ to investigate CP violation

$$\begin{aligned} \psi'' \rightarrow D_1 D_2, \quad D_1 \rightarrow V_1 V_2, \quad D_2 \rightarrow V_3 V_4, \quad (3) \\ V_1 \rightarrow M_1 M'_1, \quad V_2 \rightarrow M_2 M'_2, \quad V_3 \rightarrow M_3 M'_3, \quad V_4 \rightarrow M_3 M'_2 \end{aligned}$$

In the following we would like to calculate the full angular distribution with helicity amplitudes.

# Helicity amplitudes formula

Amplitudes for decay  $A \rightarrow B + C, B \rightarrow B_1 + B_2, C \rightarrow C_1 + C_2$ :

$$\mathcal{M}(\lambda_{B_1}, \lambda_{B_2}, \lambda_{C_1}, \lambda_{C_2}) = \sum_{\lambda_B, \lambda_C} \mathcal{M}_{\lambda_B, \lambda_C}^{A \rightarrow B+C} \cdot \mathcal{M}_{\lambda_{B_1}, \lambda_{B_2}}^{B \rightarrow B_1+B_2} \cdot \mathcal{M}_{\lambda_{C_1}, \lambda_{C_2}}^{C \rightarrow C_1+C_2} \quad (4)$$

and

$$\mathcal{M}_{\lambda_B, \lambda_C}^{A \rightarrow B+C} = \sqrt{\frac{2J_A + 1}{4\pi}} D_{M_A, \lambda_B - \lambda_C}^{J_A^*}(\phi_A, \theta_A, 0) H_{\lambda_B, \lambda_C}^A, \quad (5a)$$

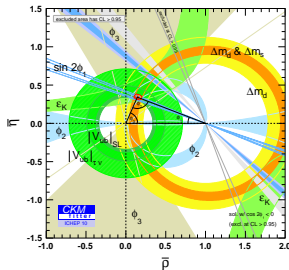
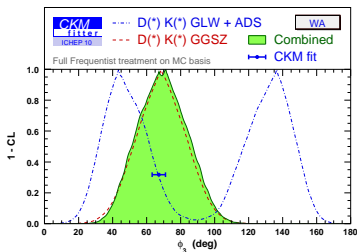
$$\mathcal{M}_{\lambda_{B_1}, \lambda_{B_2}}^{B \rightarrow B_1+B_2} = \sqrt{\frac{2J_B + 1}{4\pi}} D_{\lambda_B, \lambda_{B_1} - \lambda_{B_2}}^{J_B^*}(\phi_B, \theta_B, 0) H_{\lambda_{B_1}, \lambda_{B_2}}^B, \quad (5b)$$

$$\mathcal{M}_{\lambda_{C_1}, \lambda_{C_2}}^{C \rightarrow C_1+C_2} = \sqrt{\frac{2J_C + 1}{4\pi}} D_{-\lambda_C, \lambda_{C_1} - \lambda_{C_2}}^{J_C^*}(\phi_C, \theta_C, 0) H_{\lambda_{C_1}, \lambda_{C_2}}^C, \quad (5c)$$

with rotation matrix:  $D_{m'm}^j(\alpha, \beta, \gamma) = e^{-im'\alpha} d_{m'm}^j(\beta) e^{-im\gamma}$  and  $A_{\parallel} = \frac{1}{\sqrt{2}}(H_{+1} + H_{-1}), \quad A_0 = H_0, \quad A_{\perp} = \frac{1}{\sqrt{2}}(H_{+1} - H_{-1}).$

## Current status of $\gamma$ measurement

current status:  $\gamma = (73_{-25}^{+22})^\circ$ , the least well known of the three angles from  $B$  meson unitarity triangle.



## ADS method

- ▶ In  $B^{+-} \rightarrow D/\bar{D}^0 K$  unitarity triangle, ADS point out CP violation can be greatly enhanced for decays to final states common to both  $D^0$  and  $\bar{D}^0$  are not CP eigenstates.
- ▶ Interference of  $D^0 \rightarrow f$  (doubly Cabibbo suppressed) and  $\bar{D}^0 \rightarrow f$  (Cabibbo allowed) allows the *clean* extraction of  $\gamma$ .
- ▶ ADS method requires determination of the hadronic parameters  $r$  and  $\delta$  in  $D$  decays, which are defined as

$$r \cdot e^{i\delta} = \frac{\langle K^- \pi^+ | \bar{D}_0 \rangle}{\langle K^- \pi^+ | D_0 \rangle}. \quad (6)$$

D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett 78, 3257 (1997).



## $d\Gamma_{2V,K\pi}$ , differential width for $\psi'' \rightarrow 2D \rightarrow (K\pi)(VV)$

Using the helicity formulas for cascade decay, we write

$$\begin{aligned}
 d\Gamma_{2V} &= \frac{9}{4\pi} d(\cos\theta_{V_1})d(\cos\theta_{V_2})d\Phi \times |A^{\psi V_1 V_2}|^2 |A^{D^0 \rightarrow K\pi}|^2 & (7) \\
 &\times \left[ \cos^2\theta_{V_1} \cos^2\theta_{V_2} |A_0^{D^0 \rightarrow V_1 V_2}|^2 (1 + 2r \cos\delta + r^2) \right. \\
 &+ \frac{1}{2} \sin^2\theta_{V_1} \sin^2\theta_{V_2} \cos^2\Phi |A_{\parallel}^{D^0 \rightarrow V_1 V_2}|^2 (1 + 2r \cos\delta + r^2) \\
 &- \frac{\sqrt{2}}{4} \sin 2\theta_{V_1} \sin 2\theta_{V_2} \cos\Phi \operatorname{Re}[A_0^{D^0 \rightarrow V_1 V_2} (A_{\parallel}^{D^0 \rightarrow V_1 V_2})^*] (1 + 2r \cos\delta + r^2) \\
 &+ \frac{1}{2} \sin^2\theta_{V_1} \sin^2\theta_{V_2} \sin^2\Phi |A_{\perp}^{D^0 \rightarrow V_1 V_2}|^2 (1 - 2r \cos\delta + r^2) \\
 &+ \frac{\sqrt{2}}{4} \sin 2\theta_{V_1} \sin 2\theta_{V_2} \sin\Phi \left\{ \operatorname{Re}[A_0^{D^0 \rightarrow V_1 V_2} (A_{\perp}^{D^0 \rightarrow V_1 V_2})^*] (2r \sin\delta) \right. \\
 &\quad \left. + \operatorname{Im}[A_0^{D^0 \rightarrow V_1 V_2} (A_{\perp}^{D^0 \rightarrow V_1 V_2})^*] (1 - r^2) \right\} \\
 &- \sin^2\theta_{V_1} \sin^2\theta_{V_2} \cos\Phi \sin\Phi \left\{ \operatorname{Re}[A_{\parallel}^{D^0 \rightarrow V_1 V_2} (A_{\perp}^{D^0 \rightarrow V_1 V_2})^*] (2r \sin\delta) + \dots \right\} \Big].
 \end{aligned}$$

## Analysis on $d\Gamma_{2V,K\pi}$

- ▶ The branching ratio only depends on the three amplitude combinations

$$M_0 = A_0(1 + re^{i\delta}), \quad M_{||} = A_{||}(1 + re^{i\delta}), \quad M_{\perp} = A_{\perp}(1 - re^{i\delta}). \quad (8)$$

- ▶ Since  $\delta$  is small, the sensitivity on sine in addition to cosine (PP case) is expected to improve the final results.
- ▶ The above constraint can be improved by exploiting the expected knowledge of polarization of VV modes (single-tag ST). Then the measurement of  $M_i$  in the correlated decay (double-tag DT) may lead to a better result on  $\delta$ .

## Another simplified transversity analysis for low statistics

- ▶ For relatively low statistics, a one-parameter fit to the distribution of the transversity angle yields

$$f_{\perp}^{ST} = \frac{|A_{\perp}|^2}{|A_0|^2 + |A_{||}|^2 + |A_{\perp}|^2}, \quad f_{\perp}^{DT} = \frac{|M_{\perp}|^2}{|M_0|^2 + |M_{||}|^2 + |M_{\perp}|^2} \quad (9)$$

$$\frac{|A_{\perp}|^2}{|A_0|^2 + |A_{||}|^2} = \frac{f_{\perp}^{ST}}{1 - f_{\perp}^{ST}}, \quad \left| \frac{1 + re^{i\delta}}{1 - re^{i\delta}} \right|^2 = \frac{f_{\perp}^{ST}}{1 - f_{\perp}^{ST}} \cdot \frac{1 - f_{\perp}^{DT}}{f_{\perp}^{DT}} \quad (10)$$

which implies a one-dimensional parabolic constraint in the  $(r, \delta)$  plane. It allows to determine  $\delta$  (as function of  $r$ ), however, it gives limited information compared to our proposal.

- ▶ Our main novelty comes from the sensitivity of the completed correlated decay rate to the  $|\sin \delta|$  term.

## Observables for $CP$ violation

If we take the decay chain

$$e^+e^- \rightarrow \psi'' \rightarrow D^0\bar{D}^0 \rightarrow f_a f_b \quad (11)$$

with  $f_a$  and  $f_b$  CP eigenstates of the same CP-parity, we have

$$CP|\psi''\rangle = |\psi''\rangle \quad CP|f_a f_b\rangle = \eta_a \eta_b (-1)^\ell |f_a f_b\rangle = -|f_a f_b\rangle \quad (12)$$

since  $f_a$  and  $f_b$  are in a  $P$  wave.

Therefore, the decay of  $\psi''$  into the states of identical  $CP$  parity is, by itself, a  $CP$  violating observable.

I.I.Y.Bigi and A.I.Sanda, Phys.Lett.B 171, 320 (1986).

## Combined branching ratio.

- ▶ One obtains, neglecting CP-violation in  $D\bar{D}$  mixing, the following combined branching ratio

$$Br((D^0\bar{D}^0)_{C=-1} \rightarrow f_a f_b) = 2Br(D_0 \rightarrow f_a)Br(D_0 \rightarrow f_b) \quad (13)$$

$$(|\rho_a - \rho_b|^2 + r_D |1 - \rho_a \rho_b|^2)$$

$$\rho_f = \frac{A(\bar{D}^0 \rightarrow f)}{A(D^0 \rightarrow f)} \quad r_D = (x^2 + y^2)/2 < 10^{-4} \quad (14)$$

Z.z.Xing, Phys.Rev.D 55, 196 (1997).

- ▶ CP conservation at the level of the amplitude would require that only two combinations of transversity amplitudes are allowed:  $(0, \perp)$  or  $(\parallel, \perp)$ .
- ▶ CP violating observables :

$$(0, 0) \quad (0, \parallel) \quad (\parallel, 0) \quad (\parallel, \parallel) \quad (\perp, \perp) \quad (15)$$

## CP violating observables.

Exploiting orthogonality relationships for Legendre and Chebyshev polynomials to select specific angular dependence, one can get

$$\begin{aligned} & \int d\Gamma_{4V} \frac{1}{8} (5 \cos^2 \theta_{V_1} - 1)(5 \cos^2 \theta_{V_2} - 1)(5 \cos^2 \theta_{V_3} - 1)(5 \cos^2 \theta_{V_4} - 1) \\ & \quad = |A^{\Psi V_1 V_2 V_3 V_4}|^2 |A_0^{D_0 \rightarrow V_1 V_2}|^2 |A_0^{D_0 \rightarrow V_3 V_4}|^2 \times |\rho_{V_1, V_2}^0 - \rho_{V_3, V_4}^0|^2 \\ & \int d\Gamma_{4V} \frac{1}{32} (5 \cos^2 \theta_{V_1} - 3)(5 \cos^2 \theta_{V_2} - 3)(5 \cos^2 \theta_{V_3} - 3)(5 \cos^2 \theta_{V_4} - 3) \\ & \quad \cdot (4 \cos^2 \Phi - 1)(4 \cos^2 \Psi - 1) \\ & \quad = |A^{\Psi V_1 V_2 V_3 V_4}|^2 |A_{||}^{D_0 \rightarrow V_1 V_2}|^2 |A_{||}^{D_0 \rightarrow V_3 V_4}|^2 \times |\rho_{V_1, V_2}^{||} - \rho_{V_3, V_4}^{||}|^2 \\ & \dots \end{aligned}$$

This projection yields CP violating observables without performing a full angular analysis.

# Decay width for $D^0\bar{D}^0 \rightarrow (f_\eta)(K\pi)$

The process of one  $D$  decaying to  $K^-\pi^+$ , while the other  $D$  decaying to a  $CP$  eigenstate  $f_\eta$  can be described as

$$\Gamma_{K\pi;f_\eta} \equiv \Gamma[(K^-\pi^+)(f_\eta)] \approx A^2 A_{f_\eta}^2 |1 + \eta r e^{-i\delta}|^2 \approx A^2 A_{f_\eta}^2 (1 + 2\eta r \cos\delta), \quad (16)$$

where  $A = |\langle K^-\pi^+ | \mathcal{H} | D^0 \rangle|$  and  $A_{f_\eta} = |\langle f_\eta | \mathcal{H} | D^0 \rangle|$  are the real-valued decay amplitudes,  $\eta = \pm 1$  is CP eigenvalue of eigenstate  $f_\eta$ .

X.D.Cheng, K.L.He, H.B.Li *et al.* Phys.Rev.D 75, 094010 (2007).

## The error on $\cos \delta$

Define an asymmetry

$$\mathcal{A} \equiv \frac{\Gamma_{K\pi;f_+} - \Gamma_{K\pi;f_-}}{\Gamma_{K\pi;f_+} + \Gamma_{K\pi;f_-}}, \quad (17)$$

One can easily get,

$$\mathcal{A} = 2r\cos\delta \quad (18)$$

For a small asymmetry, a general result is that its error  $\Delta\mathcal{A}$  is approximately  $1/\sqrt{N_{K-\pi^+}}$ , where  $N_{K-\pi^+}$  is the total number of events tagged with *CP*-even and *CP*-odd eigenstates, then fixing  $r$  ( $r \approx 0.055$ ) one leads to

$$\Delta(\cos\delta) \approx \frac{1}{2r\sqrt{N_{K-\pi^+}}}. \quad (19)$$



## Numerical values for uncertainty on $\delta$

- ▶ At last, the error on  $\cos \delta$  is given by

$$\Delta(\cos \delta) \approx \frac{1}{2r\sqrt{N_{K^-\pi^+}}} \approx \frac{\pm 284.5}{\sqrt{N(D^0\bar{D}^0)}}. \quad (20)$$

At BESIII, 4 years,  $72 \times 10^6$   $D^0\bar{D}^0$  pairs,  $\Delta(\cos \delta) = \pm 0.03$ .

- ▶  $\delta = (26.4_{-9.9}^{+9.6})^\circ$  from HFAG.  
 $\Delta(\delta) = \pm 3.9^\circ$  at BES-III;  
 $\Delta(\delta) = 0.4^\circ$  at super- $\tau$ -charm factory with improvement of the luminosity about 100 times.
- ▶ At this stage, the results are pure statistics. The true experimental systematics are required to be studied.
- ▶ One thing to emphasize: size of other terms (e.g.  $\sin \delta$ ) has not been studied yet and may improve the measurement of  $\delta$ .

## CP violating decay rate

- ▶ Parameterize  $\rho_f$  as  $\rho_f = \eta_f(1 + \delta_f)e^{i\alpha_f}$ ,  
 $\delta_f$  CP violation in decay and can be negligible.
- ▶ Illustrative example: the most promising channel  $\rho^0\rho^0/\bar{K}^{*0}\rho^0$   
 (large branching ratio among the CP eigenstates):

$$Br((D^0\bar{D}^0)_{C=-1} \rightarrow \rho^0\rho^0, \bar{K}^{*0}\rho^0) \Big|_{(0,||)}^{CPV} \simeq 8Br^0(D^0 \rightarrow \rho^0\rho^0) \quad (21)$$

$$\cdot Br^{||}(D^0 \rightarrow \bar{K}^{*0}\rho^0) \sin^2 \frac{\alpha_a - \alpha_b}{2}.$$

- ▶ Assuming no CP violating signal events are observed we have the upper limit  $|\alpha_a - \alpha_b| < 4.4^\circ$  at 90%-C.L. at BESIII;  
 $|\alpha_a - \alpha_b| < 0.5^\circ$  at 90%-C.L. at super- $\tau$ -charm factory.
- ▶ We need further study on the systematics.

## Constraints on CP violating branching ratio

We list the upper limits on CP violating branching fraction of some most interesting (VV)(VV) modes at 90%-C.L.

Reaction	Efficiency	Upper limits at BES-III( $\times 10^{-7}$ )
$D^0 \bar{D}^0 \rightarrow (\rho^+ \rho^-)(\bar{K}^{*0} \omega)$	0.13	2.46
$D^0 \bar{D}^0 \rightarrow (\rho^0 \rho^0)(\bar{K}^{*0} \rho^0)$	0.17	1.88
$D^0 \bar{D}^0 \rightarrow (\bar{K}^{*0} \rho^0)(K^{*0} \omega)$	0.10	3.19
$D^0 \bar{D}^0 \rightarrow (\bar{K}^{*0} \rho^0)(\rho^0 \phi)$	0.09	3.55
$D^0 \bar{D}^0 \rightarrow (\bar{K}^{*0} \omega)(\rho^0 \phi)$	0.08	3.99
$D^0 \bar{D}^0 \rightarrow (\rho^0 \rho^0)(\bar{K}^{*0} \omega)$	0.15	2.13
$D^0 \bar{D}^0 \rightarrow (\rho^0 \rho^0)(\rho^0 \Phi)$	0.13	2.46
$D^0 \bar{D}^0 \rightarrow (\rho^+ \rho^-)(\rho^0 \Phi)$	0.11	2.90
$D^0 \bar{D}^0 \rightarrow (\rho^+ \rho^-)(K^{*+} K^{*-})$	0.11	2.90

## Conclusion

- ▶ In the case of  $\psi'' \rightarrow D^0 \bar{D}^0 \rightarrow (V_1 V_2)(V_3 V_4)$ , CP-violating observables can be constructed and phase differences are discussed. We show that **the helicity degree of freedom involved in VV modes creates new observables.**
- ▶ In the case of CP-tagged  $D \rightarrow K\pi$  decays, we expect the determination of the error on  $\delta$  can be improved by taking into account **the dependence of the full angular decay width to the sine of the strong phase.**
- ▶ A further careful study of experimental systematics is required since they presumably dominate the quoted uncertainty.

*Thank you for your attention!*

## Backup slides

- ▶ decay chain  $\Psi \rightarrow D_0 \bar{D}_0 \rightarrow (M_1 M_2)(M_3 M_4)$ .  
 “ $M_1 M_2(M_3 M_4)$ ”: mesons from two-body decay of  $D^0(\bar{D}^0)$
- ▶ CP eigenstates(neglecting CP-violation in  $D\bar{D}$  mixing)

$$|D_1\rangle = \frac{|D_0\rangle + |\bar{D}_0\rangle}{\sqrt{2}}, |D_2\rangle = \frac{|D_0\rangle - |\bar{D}_0\rangle}{\sqrt{2}} \quad (22)$$

- ▶ Due to the spin of  $\Psi$ , the  $D$  pair is emitted with orbital momentum  $L = 1$ , corresponding to an antisymmetric coherent state:

$$|(D\bar{D})_{L=1}\rangle = \frac{-|D_1\rangle|D_2\rangle + |D_2\rangle|D_1\rangle}{\sqrt{2}} \quad (23)$$

## Useful relations

- ▶  $\lambda_{V_1} = \lambda_{V_2} = \lambda \quad \lambda_{V_3} = \lambda_{V_4} = \kappa$
- ▶ rotation matrix:  $D_{m'm}^j(\alpha, \beta, \gamma) = e^{-im'\alpha} d_{m'm}^j(\beta) e^{-im\gamma}$
- ▶ Wigner d-function:
  - $d_{10}^1(\theta) = -\frac{1}{\sqrt{2}} \sin \theta, \quad d_{00}^1(\theta) = \cos \theta$
  - $d_{-10}^1(\theta) = \frac{1}{\sqrt{2}} \sin \theta, \quad d_{00}^0(\theta) = 1$
- ▶ relevant angle:  $\Phi = \phi_{V_1} - \phi_{V_2}$  and  $\Psi = \phi_{V_3} - \phi_{V_4}$
- ▶ combination of amplitudes

$$H^{\Psi V_1 V_2 V_3 V_4} \equiv H_{D_1 D_2}^{\Psi} H_{M_1 M'_1}^{V_1} H_{M_2 M'_2}^{V_2} H_{M_3 M'_3}^{V_3} H_{M_4 M'_4}^{V_4} \quad (24)$$

$$H_{\lambda}^{D_1 \rightarrow V_1 V_2} \equiv H_{\lambda\lambda}^{D_1 \rightarrow V_1 V_2}, \quad H_{\kappa}^{D_2 \rightarrow V_3 V_4} \equiv H_{\kappa\kappa}^{D_2 \rightarrow V_3 V_4}$$

- ▶ transversity amplitudes

$$A_{\parallel} = \frac{1}{\sqrt{2}}(H_{+1} + H_{-1}), \quad A_0 = H_0, \quad A_{\perp} = \frac{1}{\sqrt{2}}(H_{+1} - H_{-1}).$$

## PP case

The relevant modes for our study can be extracted:

$PP$	$\eta_{CP}(P)\eta_{CP}(P)$	Br (%)	Eff.( $\epsilon$ )
$K^+K^-$	+1	0.39	0.50
$\pi^+\pi^-$	+1	0.14	0.60
$K_S K_S$	+1	0.038	0.30
$\pi^0\pi^0$	+1	0.08	0.24
$K_S\pi^0$	-1	1.22	0.33
$K_S\eta$	-1	0.40	0.26
$K_S a_0(980) \rightarrow K_S(\eta\pi^0)$	+1	0.67	0.18
$K_S a_0(980) \rightarrow K_S(K^+K^-)$	+1	0.31	0.10

- ▶ Only the product of intrinsic parity is given.
- ▶ The reconstruction efficiency is for double tag, it should be  $\sqrt{\epsilon}$  for single D decay.

## PV case

D decays into CP-eigenstates composed of one pseudoscalar and one vector mesons

$PV$	$\eta_{CP}(P)\eta_{CP}(V)$	Br (%)	Eff.( $\epsilon$ )
$\rho^0\pi^0$	-1	0.37	0.29
$\phi\pi^0 \rightarrow (K^+K^-)\pi^0$	-1	0.06	0.10
$K_S\rho^0$	+1	0.77	0.27
$K_S\phi \rightarrow K_S(K^+K^-)$	+1	0.22	0.08
$K_S\omega \rightarrow K_S(\pi^+\pi^-\pi^0)$	+1	0.98	0.20
$\bar{K}^{*0}\eta \rightarrow (K_S\pi^0)(\pi^+\pi^-\pi_0)$	+1	0.03	0.17
$\bar{K}^{*0}\eta \rightarrow (K_S\pi^0)(\gamma\gamma)$	+1	0.06	0.17
$\bar{K}^{*0}\pi_0 \rightarrow (K_S\pi^0)\pi^0$	+1	0.67	0.15



## VV case

D decays into CP-eigenstates composed of two vector mesons

$VV$	$\eta_{CP}(V)\eta_{CP}(V)$	Br (%)	Eff
$\rho^0\rho^0$	1	0.18	0.24
$\bar{K}^{*0}\rho^0 \rightarrow (K_S\pi_0)(\pi^+\pi^-)$	1	0.27	0.12
$\rho^0\phi$	1	0.14	0.07
$\bar{K}^{*0}\omega \rightarrow (K_S\pi^0)(\pi^+\pi^-\pi^0)$	1	0.33	0.09
$\rho^+\rho^-$	1	[0.62]*	0.18
$\rho^0\omega \rightarrow (\pi^+\pi^-)(\pi^+\pi^-\pi^0)$	1	[ $\approx 0$ ]	0.18
$K^{*+}K^{*-} \rightarrow (K_S\pi^+)(K_S\pi^-)$	1	[0.08]	0.07
$K^{*0}\bar{K}^{*0} \rightarrow (K_S\pi^0)(K_S\pi^0)$	1	0.003	0.09

Note: the values in the brackets have not been measured yet on the experimental side, it's just the estimate from the theory.

## Decay amplitude $M_{12}^m$

$$M_{12}^m = \sum_{\lambda_V} A_{00}^{\Psi \rightarrow D_1 D_2} A_{00}^{V_1 \rightarrow M_1 M'_1} A_{00}^{V_3 \rightarrow M_3 M'_3} A_{00}^{V_2 \rightarrow M_2 M'_2} A_{00}^{V_4 \rightarrow M_4 M'_4} \quad (26)$$

$$A_{\lambda_{V_1} \lambda_{V_2}}^{D_1 \rightarrow V_1 V_2} A_{\lambda_{V_3} \lambda_{V_4}}^{D_2 \rightarrow V_3 V_4}$$

$$= \sqrt{\frac{3}{4\pi}} \frac{9}{(4\pi)^3} \sum_{\lambda_V} D_{m,0}^{1*}(\phi_\Psi, \theta_\Psi, 0) H_{D_1 D_2}^\Psi \quad (27)$$

$$\cdot D_{0, \lambda_{V_1} - \lambda_{V_2}}^{0*}(\phi_{D_1}, \theta_{D_1}, 0) H_{V_1 V_2}^{D_1} D_{\lambda_{V_1}, 0}^{1*}(\phi_{V_1}, \theta_{V_1}, 0) H_{M_1 M'_1}^{V_1}$$

$$\cdot D_{-\lambda_{V_2}, 0}^{1*}(\phi_{V_2}, \theta_{V_2}, 0) H_{M_2 M'_2}^{V_2} D_{0, \lambda_{V_3} - \lambda_{V_4}}^{0*}(\phi_{D_2}, \theta_{D_2}, 0) H_{V_3 V_4}^{D_2}$$

$$\cdot D_{\lambda_{V_3}, 0}^{1*}(\phi_{V_3}, \theta_{V_3}, 0) H_{M_3 M'_3}^{V_3} D_{-\lambda_{V_4}, 0}^{1*}(\phi_{V_4}, \theta_{V_4}, 0) H_{M_4 M'_4}^{V_4}$$

## Amplitude $M_{12}^m$

The probability amplitude becomes

$$\begin{aligned}
 M_{12}^m &= \sqrt{\frac{3}{4\pi}} \frac{9}{(4\pi)^3} e^{im\phi_\Psi} d_{m0}^1(\theta_\Psi) H^{\Psi V_1 V_2 V_3 V_4} \quad (28) \\
 &\times \left[ \cos \theta_{V_1} \cos \theta_{V_2} A_0^{D_1 \rightarrow V_1 V_2} - \frac{1}{\sqrt{2}} \sin \theta_{V_1} \sin \theta_{V_2} \cos \Phi A_{\parallel}^{D_1 \rightarrow V_1 V_2} \right. \\
 &\quad \left. - \frac{i}{\sqrt{2}} \sin \theta_{V_1} \sin \theta_{V_2} \sin \Phi A_{\perp}^{D_1 \rightarrow V_1 V_2} \right] \\
 &\times \left[ \cos \theta_{V_3} \cos \theta_{V_4} A_0^{D_2 \rightarrow V_3 V_4} - \frac{1}{\sqrt{2}} \sin \theta_{V_3} \sin \theta_{V_4} \cos \Psi A_{\parallel}^{D_2 \rightarrow V_3 V_4} \right. \\
 &\quad \left. - \frac{i}{\sqrt{2}} \sin \theta_{V_3} \sin \theta_{V_4} \sin \Psi A_{\perp}^{D_2 \rightarrow V_3 V_4} \right]
 \end{aligned}$$

## The actual differential cross section

$M_{12}^m$  is actually only one of the two "paths".

$$M^m = (-M_{12}^m + M_{21}^m)/\sqrt{2}, \quad M_{21}^m = M_{12}^m(D_1 \leftrightarrow D_2) \quad (29)$$

differential cross section

$$\begin{aligned}
 d\Gamma_{4V} = & \frac{81}{32\pi^2} d(\cos\theta_{V_1})d(\cos\theta_{V_2})d\Phi d(\cos\theta_{V_3})d(\cos\theta_{V_4})d\Psi \times |A^{\Psi V_1 V_2 V_3 V_4}|^2 \quad (30) \\
 & \times \left[ \cos\theta_{V_1} \cos\theta_{V_2} A_0^{D_0 \rightarrow V_1 V_2} - \frac{1}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \cos\Phi A_{||}^{D_0 \rightarrow V_1 V_2} - \frac{i}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \sin\Phi A_{\perp}^{D_0 \rightarrow V_1 V_2} \right] \\
 & \times \left[ \cos\theta_{V_3} \cos\theta_{V_4} A_0^{\bar{D}_0 \rightarrow V_3 V_4} - \frac{1}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \cos\Psi A_{||}^{\bar{D}_0 \rightarrow V_3 V_4} - \frac{i}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \sin\Psi A_{\perp}^{\bar{D}_0 \rightarrow V_3 V_4} \right] \\
 & - \left[ \cos\theta_{V_1} \cos\theta_{V_2} A_0^{\bar{D}_0 \rightarrow V_1 V_2} - \frac{1}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \cos\Phi A_{||}^{\bar{D}_0 \rightarrow V_1 V_2} - \frac{i}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \sin\Phi A_{\perp}^{\bar{D}_0 \rightarrow V_1 V_2} \right] \\
 & \times \left[ \cos\theta_{V_3} \cos\theta_{V_4} A_0^{D_0 \rightarrow V_3 V_4} - \frac{1}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \cos\Psi A_{||}^{D_0 \rightarrow V_3 V_4} - \frac{i}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \sin\Psi A_{\perp}^{D_0 \rightarrow V_3 V_4} \right] \Big|^2
 \end{aligned}$$

Applied in  $\Psi \rightarrow 2D \rightarrow (K\pi)(VV)$  case.

Introducing

$$r \cdot e^{i\delta} = \frac{\langle K^- \pi^+ | \bar{D}_0 \rangle}{\langle K^+ \pi^- | D_0 \rangle} \quad (31)$$

One obtains:

$$\begin{aligned}
 d\Gamma_{2V} = & \frac{9}{4\pi} d(\cos\theta_{V_1}) d(\cos\theta_{V_2}) d\Phi \times |A^{\Psi V_1 V_2}|^2 |A^{D_0 \rightarrow K\pi}|^2 \quad (32) \\
 & \times \left| \cos\theta_{V_1} \cos\theta_{V_2} (A_0^{\bar{D}_0 \rightarrow V_1 V_2} - r e^{i\delta} A_0^{D_0 \rightarrow V_1 V_2}) \right. \\
 & \quad - \frac{1}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \cos\Phi (A_{\parallel}^{\bar{D}_0 \rightarrow V_1 V_2} - r e^{i\delta} A_{\parallel}^{D_0 \rightarrow V_1 V_2}) \\
 & \quad \left. - \frac{i}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \sin\Phi (A_{\perp}^{\bar{D}_0 \rightarrow V_1 V_2} - r e^{i\delta} A_{\perp}^{D_0 \rightarrow V_1 V_2}) \right|^2
 \end{aligned}$$

## Introducing variables $\rho$

- ▶ introducing two relations

$$A_{0,||,\perp}(\bar{D}_0 \rightarrow V_a V_b) = A_{0,||,\perp}(D_0 \rightarrow V_a V_b) \rho_{V_a, V_b}^{0,||,\perp} \quad (33)$$

In absence of  $CP$  violation, which we will assume in this section, we have:

$$\rho_{V_a, V_b}^{0,||} = -\eta_{CP}(V_a)\eta_{CP}(V_b) = -\rho_{V_a, V_b}^{\perp} \quad (34)$$

- ▶  $\rho_0 = \rho_{||} = -1, \quad \rho_{\perp} = 1.$