TMDWFs and Soft functions at one-loop in LaMET

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based on arXiv: 2207.07280 (Accepted by JHEP)

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第二届微扰量子场论研讨会 杭州

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1. Introduction

- The exploration of underlying structures of hadrons has always been one of the most important frontiers in particle and nuclear physics.
- The TMD wave functions play an important role in theoretical analyses of B meson weak decays[1-4].
- In LaMET, one can construct the directly computable hadron matrix elements with non-local operators, named as quasi-distributions, on the lattice[5-8].

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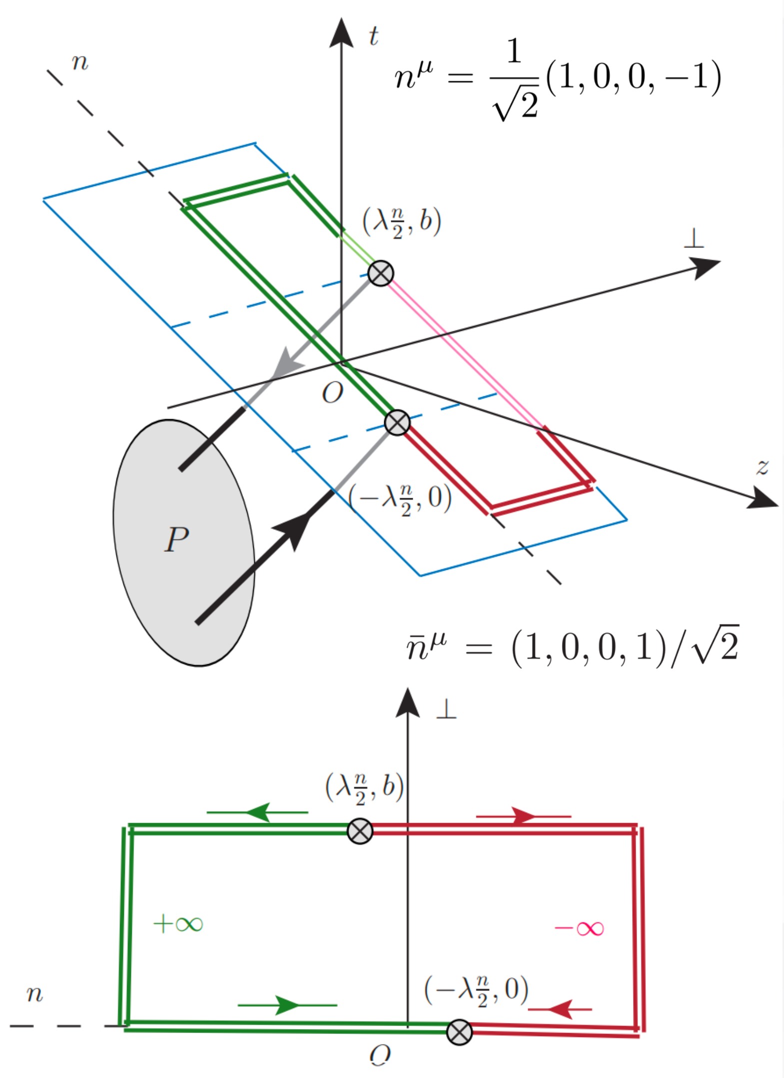
1. Introduction



$$\psi^{\pm}(x, b_\perp, \mu, \delta^-) = \frac{1}{-if_\pi P^+}\int \frac{d(\lambda P^+)}{2\pi} e^{-i(x-\frac{1}{2})P^+\lambda}$$
$$\times \left\langle 0 \left| \overline{\Psi}_n^{\pm}(\lambda n/2 + b)\, \gamma^+\gamma^5 \Psi_n^{\pm}(-\lambda n/2) \right| P \right\rangle |_{\delta^-},$$

$$P^\mu = (P^z, 0, 0, P^z) \qquad b^\mu = (0, \vec{b}_\perp, 0)$$

$$\Psi_n^{\pm}(\xi)|_{\delta^-} = \mathcal{P} e^{ig\int_0^{\pm\infty} ds n\cdot A(\xi + sn) e^{-\frac{\delta^-}{2}|s|}} \psi(\xi)$$

$$\left\langle 0 \left| \overline{\psi}(0)\, \gamma^\mu \gamma^5 \psi(0) \right| \pi \right\rangle = -if_\pi P^\mu$$

1. Introduction

Light-front TMDWFs:

$$\psi^{\pm}\left(x, b_{\perp}, \mu, \delta^{-}\right)=\frac{1}{-i f_{\pi} P^{+}} \int \frac{d\left(\lambda P^{+}\right)}{2 \pi} e^{-i\left(x-\frac{1}{2}\right) P^{+} \lambda} \times\left\langle 0\left|\bar{\Psi}_{n}^{\pm}(\lambda n / 2+b) \gamma^{+} \gamma^{5} \Psi_{n}^{\pm}(-\lambda n / 2)\right| P\right\rangle|_{\delta^{-}},$$

rapidity regulator

quasi TMDWFs:

$$\tilde{\Psi}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right)=\lim_{L \rightarrow \infty} \frac{1}{-i f_{\pi}} \int \frac{d \lambda}{2 \pi} e^{-i\left(x-\frac{1}{2}\right)\left(-P^{z}\right) \lambda} \times \frac{\left\langle 0\left|\bar{\Psi}_{\mp n_{z}}\left(\frac{\lambda n_{z}}{2}+b\right) \gamma^{z} \gamma^{5} \Psi_{\mp n_{z}}\left(-\frac{\lambda n_{z}}{2}\right)\right| P\right\rangle}{\sqrt{Z_{E}\left(2 L, b_{\perp}, \mu\right)}},$$

2. TMDWFs and Soft functions

Since the short-distance coeffifficient is insensitive to the hadrons, in the calculation of TMDWFs one can replace the hadron by the partonic state.

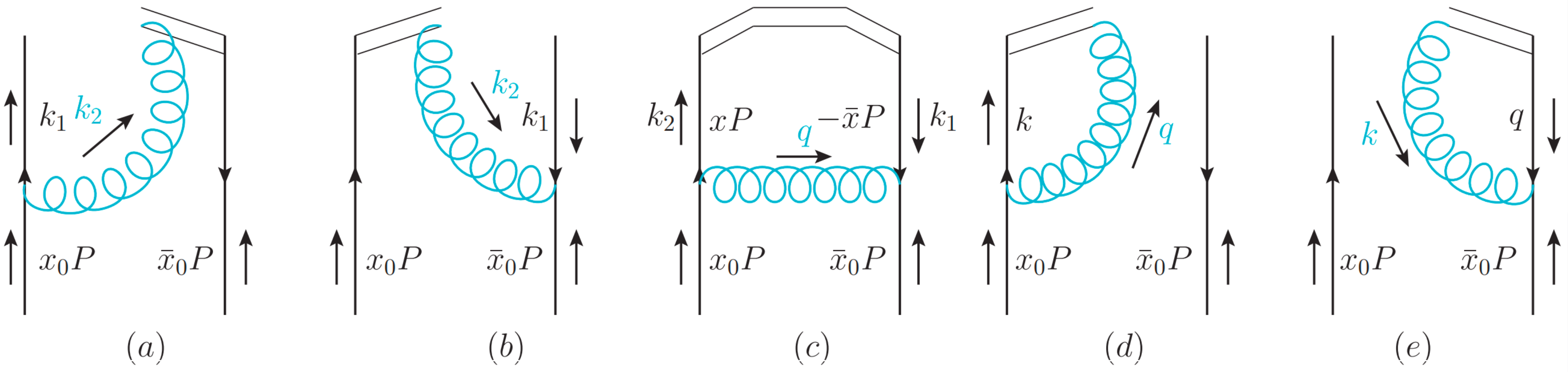
$$\psi^{\pm}_{\bar{q}q}\left(x, b_{\perp}, \mu, \delta^{-}\right) = \frac{1}{2P^{+}} \int \frac{d(\lambda P^{+})}{2\pi} e^{-i(x-\frac{1}{2})P^{+}\lambda} \times \left\langle 0 \left| \overline{\Psi}^{\pm}_{n}\left(\lambda n/2 + b\right) \gamma^{+}\gamma^{5} \Psi^{\pm}_{n}(-\lambda n/2) \right| q\bar{q} \right\rangle |_{\delta^{-}},$$

the quark pair is chosen to have the same J^{PC} with the pion

$$\tilde{\Psi}^{\pm}_{q\bar{q}}\left(x, b_{\perp}, \mu, \zeta^{z}\right) = \lim_{L\to\infty} \int \frac{d\lambda}{4\pi} e^{-i(x-\frac{1}{2})(-P^{z})\lambda} \times \frac{\langle 0 | \overline{\Psi}_{\mp n_z}\left(\frac{\lambda n_z}{2} + b\right) \gamma^{z}\gamma^{5} \Psi_{\mp n_z}\left(-\frac{\lambda n_z}{2}\right) | q\bar{q} \rangle}{\sqrt{Z_E(2L, b_{\perp}, \mu)}}.$$

2. TMDWFs and Soft functions

Light-front TMDWFs:



$$\psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \delta^-) = \delta(x - x_0) + \frac{\alpha_s C_F}{2\pi}\left[f(x, x_0, b_\perp, \mu)\right]_+ + \frac{\alpha_s C_F}{2\pi}\delta(x - x_0)\left[L_b\left(\frac{3}{2} + \ln\frac{-\delta^{-2} \mp i0}{4\bar{x}xP^{+2}}\right) + \frac{1}{2}\right],$$

$$f(x, x_0, b_\perp, \mu) = \left[\left(\frac{x}{x_0(x - x_0)} - \frac{x}{x_0}\right)\left(\frac{1}{\epsilon_{\rm IR}} + L_b\right) + \frac{x}{x_0}\right]\theta(x_0 - x) + \{x \to 1 - x, x_0 \to 1 - x_0\}.$$

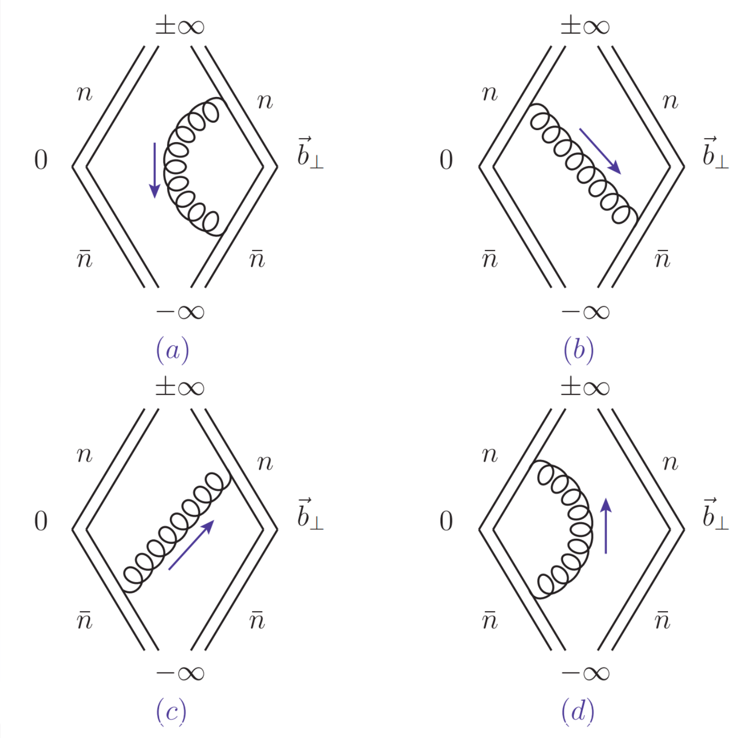
$$L_b = \ln\frac{\mu^2 b_\perp^2}{4e^{-2\gamma_E}}$$

Rapidity divergence

$$\Psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \zeta) = \lim_{\delta^- \to 0}\frac{\psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \delta^-)}{\sqrt{S^{\pm}(b_\perp, \mu, \delta^- e^{2y_n}, \delta^-)}}$$

2. TMDWFs and Soft functions

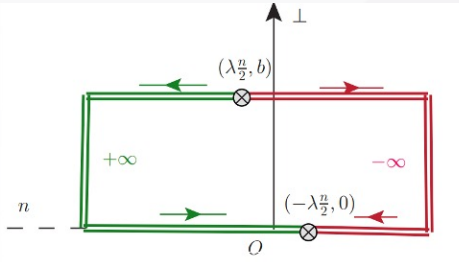
$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-)=\frac{1}{N_c}\mathrm{tr}\langle 0|\mathcal{T}W_{\bar{n}}^{-\dagger}(b_\perp)|_{\delta^+}W_n^{\pm}(b_\perp)|_{\delta^-}\times W_n^{\pm\dagger}(0)|_{\delta^-}W_{\bar{n}}^{-}(0)|_{\delta^+}|0\rangle.$$



$$\begin{aligned}S^{(a)\pm} &= S^{(d)\pm}\\ &= -\mu_0^{2\epsilon}ig^2C_F\int\frac{d^dq}{(2\pi)^d}\frac{\bar{n}^\mu}{q^-+i\frac{\delta^+}{2}}\frac{n_\mu}{q^+\pm i\frac{\delta^-}{2}}\frac{1}{q^2+i\epsilon}\\ &= \frac{\alpha_sC_F}{4\pi}\left[-\frac{2}{\epsilon_{\mathrm{UV}}^2}+\frac{2}{\epsilon_{\mathrm{UV}}}\ln\frac{\mp\delta^-\delta^+-i0}{2\mu^2}\right.\\ &\quad\left.-\ln^2\left(\frac{\mp\delta^-\delta^+-i0}{2\mu^2}\right)-\frac{\pi^2}{2}\right],\end{aligned}$$

$$\begin{aligned}S^{(b)\pm} &= S^{(c)\pm}\\ &= \mu_0^{2\epsilon}ig^2C_F\int\frac{d^dq}{(2\pi)^d}\frac{\bar{n}^\mu}{q^-+i\frac{\delta^+}{2}}\frac{n_\mu}{q^+\pm i\frac{\delta^-}{2}}\frac{e^{-iq\cdot b}}{q^2+i\epsilon}\\ &= \frac{\alpha_sC_F}{4\pi}\left[L_b^2+2L_b\ln\frac{\mp\delta^-\delta^+-i0}{2\mu^2}\right.\\ &\quad\left.+\ln^2\left(\frac{\mp\delta^-\delta^+-i0}{2\mu^2}\right)+\frac{2\pi^2}{3}\right].\end{aligned}$$

$$\mu = \mu_0 e^{(\ln(4\pi)-\gamma_E)/2}$$



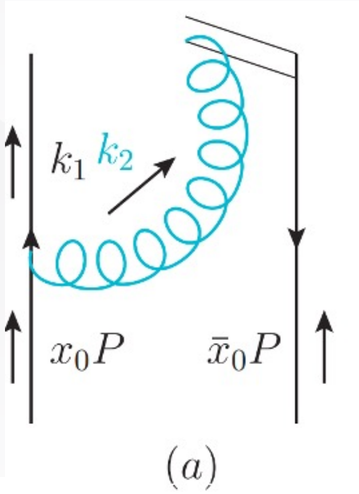
$$S^{\pm}(b_\perp,\mu,\delta^+,\delta^-)=1+\frac{\alpha_sC_F}{2\pi}\left(L_b^2+2L_b\ln\frac{\mp\delta^-\delta^+-i0}{2\mu^2}+\frac{\pi^2}{6}\right)$$

2. TMDWFs and Soft functions

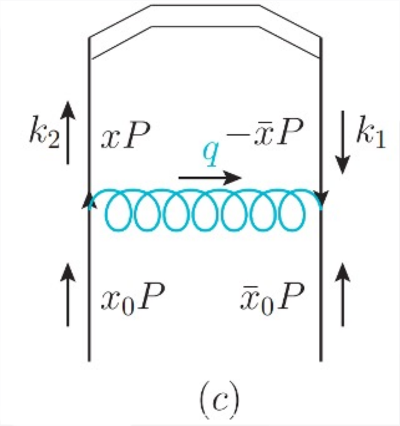
$q^{\mu} \sim (\Lambda, \Lambda, \Lambda)$

$$
\begin{aligned}
\psi_{\bar{q}q}^{(1,a)}|_{\rm soft} &= \mu_0^{2\epsilon}\frac{ig^2C_F}{2}\int\frac{d^dq}{(2\pi)^d}e^{-iq\cdot b}\delta\left[(x-x_0)P^+ + q^+\right] \\
&\times\frac{\bar{v}\gamma^+\gamma^5(x_0\not{P}-\not{q})\not{n}u}{-q^+[(x_0P-q)^2+i\epsilon](q^2+i\epsilon)}|_{soft} \\
&= \delta\left[(x-x_0)P^+\right]\mu_0^{2\epsilon}\frac{ig^2C_F}{2}\int\frac{d^dq}{(2\pi)^d}e^{-iq\cdot b} \\
&\times\frac{\bar{v}\gamma^+\gamma^5x_0P^+\not{\bar{n}}\not{n}u}{-q^+(-2x_0P^+q^-)(q^2+i\epsilon)} \\
&= \delta(x-x_0)\frac{\bar{v}\gamma^+\gamma^5u}{2P^+} \\
&\times\mu_0^{2\epsilon}ig^2C_F\int\frac{d^dq}{(2\pi)^d}e^{-iq\cdot b}\frac{1}{q^+q^-(q^2+i\epsilon)} \\
&= \psi_{\bar{q}q}^{(0)}\times S^{(1,b)}.
\end{aligned}
$$

Soft + Collinear

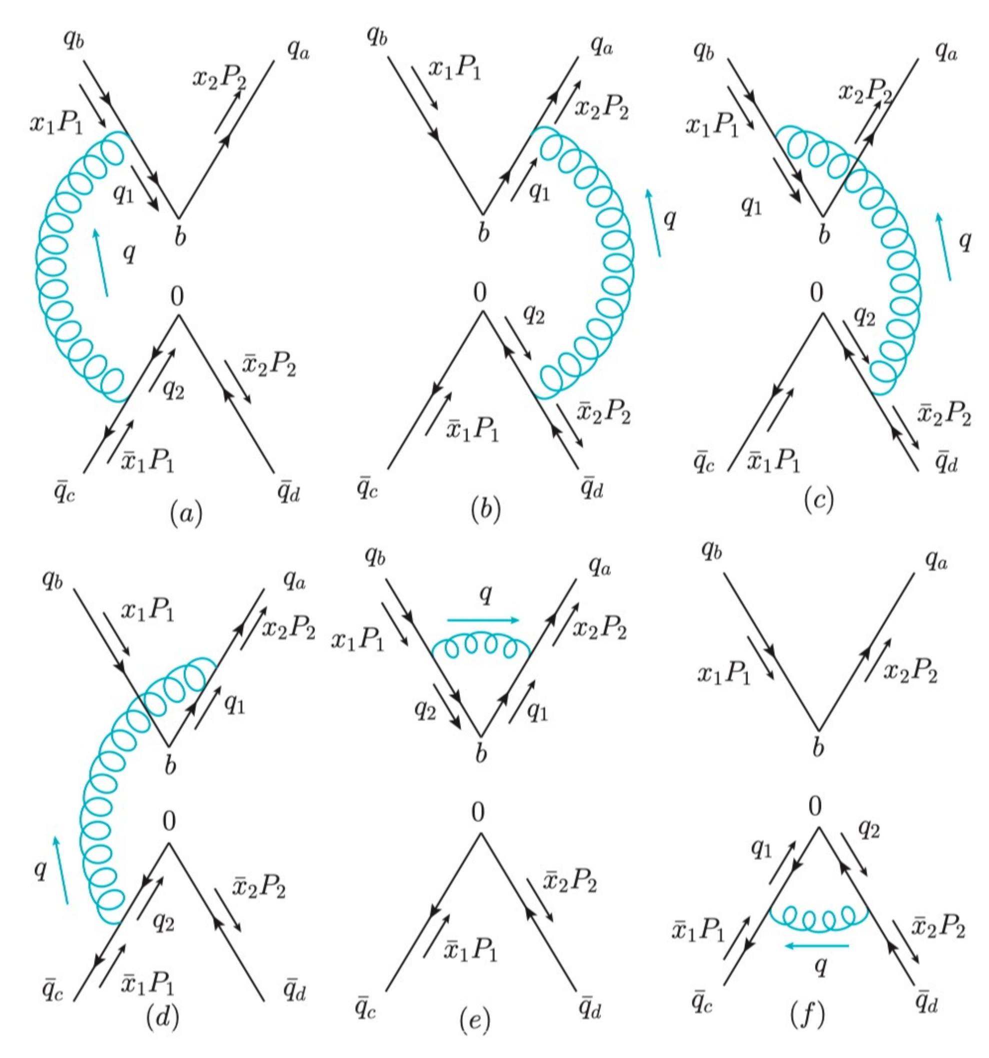


$$\psi_{\bar{q}q}^{(1,a)} = \psi_{\bar{q}q}^{(1,a)}|_{\rm collinear} + \psi_{\bar{q}q}^{(0)} \times S^{(1,b)}$$



Collinear

No hard mode!



$$F(b_\perp, P_1, P_2, \mu) = \frac{\langle P_2 | (\bar{\psi}_a \Gamma \psi_b)(b)(\bar{\psi}_c \Gamma' \psi_d)(0) | P_1 \rangle}{f_\pi^2 P_1 \cdot P_2}$$

$\Gamma = \Gamma' = I,\ \gamma_5$ or $\gamma_\perp$ and $\gamma_\perp \gamma_5$

$$\langle 0 | \bar{\psi}(0) \gamma^\mu \gamma^5 \psi(0) | P_1 \rangle = -i f_\pi P_1^\mu$$
$$\langle P_2 | \bar{\psi}(0) \gamma_\mu \gamma^5 \psi(0) | 0 \rangle = i f_\pi P_{2\mu}$$

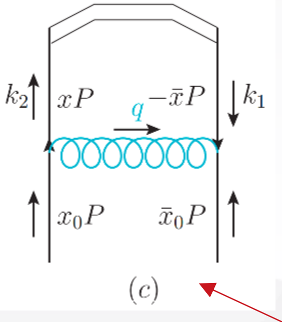
$$\frac{\langle \bar{q}_d(\bar{x}_2 P_2) q_a(x_2 P_2) | (\bar{\psi}_a \Gamma \psi_b)(b)(\bar{\psi}_c \Gamma \psi_d)(0) | q_b(x_1 P_1) \bar{q}_c(\bar{x}_1 P_1) \rangle}{4 P_1 \cdot P_2}$$

$P_1^\mu = (P^z, 0, 0, P^z)$ and $P_2^\mu = (P^z, 0, 0, -P^z)$

2. TMDWFs and Soft functions

$$
\begin{aligned}
F^{(1,a)} = {} & \mu_0^{2\epsilon}\frac{ig^2C_F}{4N_cP_1\cdot P_2}\int\frac{d^dq}{(2\pi)^d}e^{-iq\cdot b}\\
&\times\frac{1}{[(q+x_1P_1)^2+i\epsilon][(q-\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)}\\
&\times c_\Gamma\bar{u}_a(x_2P_2)\gamma_\nu\gamma_5v_d(\bar{x}_2P_2)\\
&\times\bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}-\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(\not{q}+x_1\not{P}_1)\gamma_\mu u_b(x_1P_1)\\
= {} & \mu_0^{2\epsilon}\frac{ig^2C_F}{4P_1\cdot P_2}\int\frac{d^dq}{(2\pi)^d}e^{iq\cdot b}\\
&\times\frac{1}{[(q-x_1P_1)^2+i\epsilon][(q+\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)}\\
&\times(-H_F^{(0)})\bar{u}_a(x_2P_2)\gamma_\nu\gamma_5v_d(\bar{x}_2P_2)\\
&\times\bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}+\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(\not{q}-x_1\not{P}_1)\gamma_\mu u_b(x_1P_1).
\end{aligned}
$$

Fierz transformation



$$
\begin{aligned}
F^{(1,a)} = {} & H_F^{(0)}\mu_0^{2\epsilon}\frac{ig^2C_F}{2P_1^+}\int\frac{d^dq}{(2\pi)^d}e^{iq\cdot b}\\
&\times\frac{1}{[(q-x_1P_1)^2+i\epsilon][(q+\bar{x}_1P_1)^2+i\epsilon](q^2+i\epsilon)}\\
&\times\bar{v}_c(\bar{x}_1P_1)\gamma^\mu(\not{q}+\bar{x}_1\not{P}_1)\gamma^\nu\gamma_5(x_1\not{P}_1-\not{q})\gamma_\mu u_b(x_1P_1)\\
= {} & H_F^{(0)}\times\int dx\psi_{\bar{q}q}^{(1,c)}(x).
\end{aligned}
$$

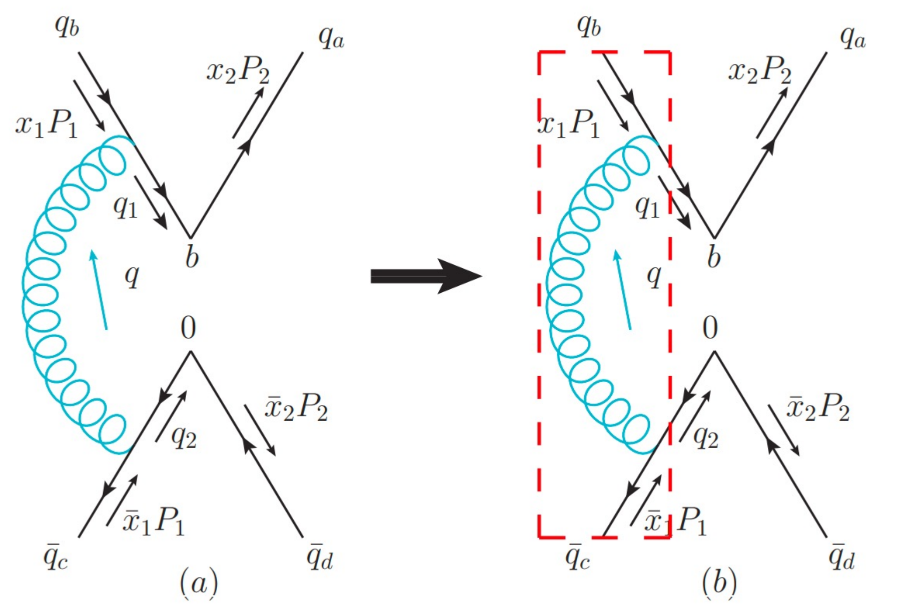


FIG. 6: Factorization of form factor shown in Fig. 5 (a). Only collinear mode contributes in this diagram, while both hard and soft contributions are power suppressed.

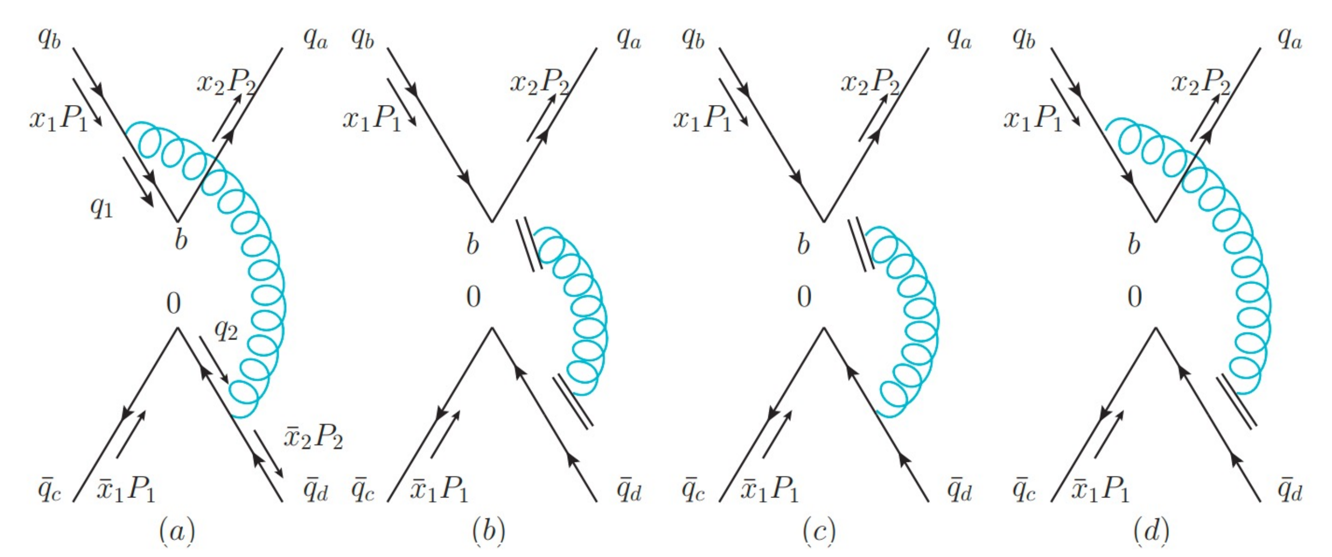
$$F^{(1,a)} = H_F^{(0)}\otimes\psi_{\bar{q}q}^{(1,c)}\otimes(\psi_{\bar{q}q}^{(0)})^\dagger\times\left(\frac{1}{S}\right)^{(0)}$$

Hard　　Collinear　　Soft

$$F^{(1,b)} = H_F^{(0)}\otimes\psi_{\bar{q}q}^{(0)}\otimes(\psi_{\bar{q}q}^{(1,c)})^\dagger\times\left(\frac{1}{S}\right)^{(0)}$$

2. TMDWFs and Soft functions

$$F^{(1,c)}|_{soft} = H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{0})^\dagger \times \left(\frac{1}{S}\right)^{(1,b)}$$



$$F^{(1,c)}|_{collinear} = H_F^{(0)} \otimes \psi_{\bar{q}q}^{(1,a)}|_{\text{collinear}} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)}$$

$q \mathbin{//} P_1$

$$H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(1,a)})^\dagger|_{\text{collinear}} \times \left(\frac{1}{S}\right)^{(0)}$$

$q \mathbin{//} P_2$

2. TMDWFs and Soft functions

$$H_F^{(1,e)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)}$$

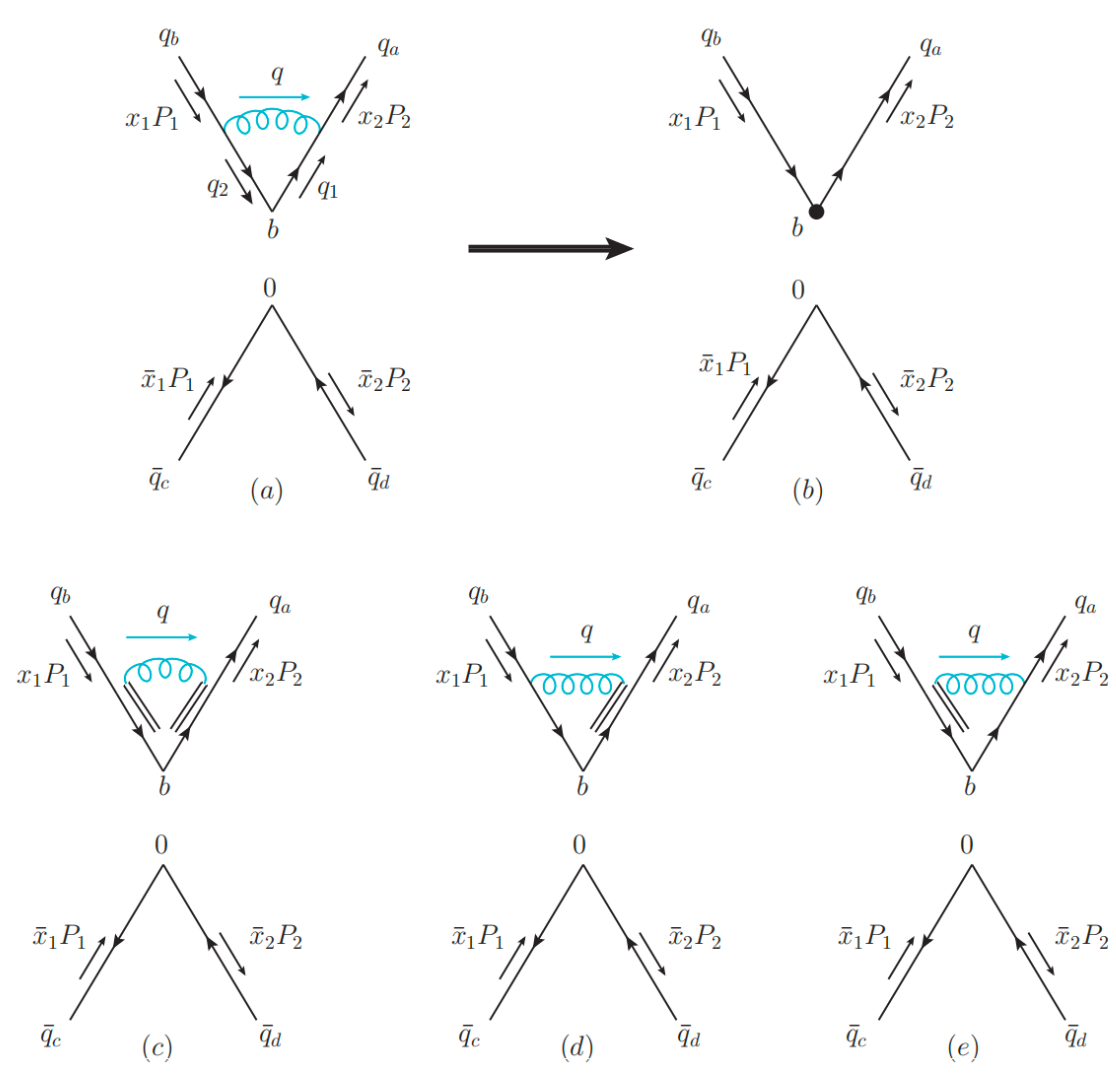
$$+H_F^{(0)} \otimes \psi_{\bar{q}q}^{(1,d)} \otimes (\psi_{\bar{q}q}^{(0)})^\dagger \times \left(\frac{1}{S}\right)^{(0)}$$

$$+H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{(1,d)})^\dagger \times \left(\frac{1}{S}\right)^{(0)}$$

$$+H_F^{(0)} \otimes \psi_{\bar{q}q}^{(0)} \otimes (\psi_{\bar{q}q}^{0)})^\dagger \times \left(\frac{1}{S}\right)^{(1,d)}$$

$$H_F(Q^2, \bar{Q}^2) = H^{Sud}(-Q^2) H^{Sud}(-\bar{Q}^2)$$

arXiv:1705.07167



2. TMDWFs and Soft functions

$$F(b_\perp, P_1, P_2, \mu) = \int dx_1 dx_2 H_F(Q^2, \bar{Q}^2, \mu^2)$$
$$\times \left[\frac{\psi^{\pm}_{\bar{q}q}(x_2, b_\perp, \mu, \delta'^+)}{\sqrt{S^{\pm}(b_\perp, \mu, \delta'^+, \delta^-)}}\right]^\dagger \left[\frac{\psi^{\pm}_{\bar{q}q}(x_1, b_\perp, \mu, \delta'^-)}{\sqrt{S^{\pm}(b_\perp, \mu, \delta^+, \delta'^-)}}\right]$$
$$\times \frac{S^{\pm}(b_\perp, \mu, \delta^+, \delta^-)}{\sqrt{S^{\pm}(b_\perp, \mu, \delta'^+, \delta^-) S^{\pm}(b_\perp, \mu, \delta^+, \delta'^-)}}$$

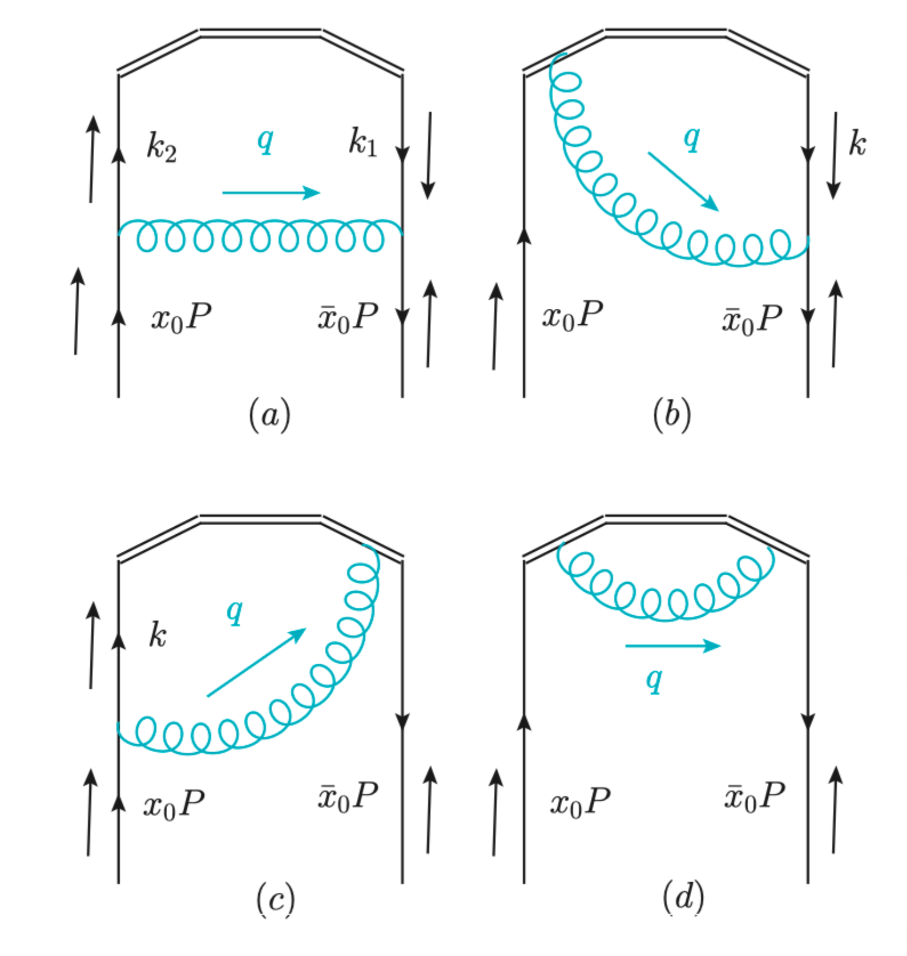
$$H(x_1, x_2) = \frac{H_F(Q^2, \bar{Q}^2, \mu^2)}{\left[H_1^{\pm}(\zeta_2^z, \bar{\zeta}_2^z, \mu)\right]^\dagger \left[H_1^{\pm}(\zeta_1^z, \bar{\zeta}_1^z, \mu)\right]}.$$

$$F(b_\perp, P_1, P_2, \mu) = \int dx_1 dx_2 H(x_1, x_2) S_r(b_\perp, \mu)$$
$$\times \tilde{\Psi}^\dagger_{q\bar{q}}(x_2, b_\perp, \mu, \zeta_2^z) \tilde{\Psi}_{q\bar{q}}(x_1, b_\perp, \mu, \zeta_1^z)$$

$$\tilde{\Psi}^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \zeta^z)\, S_r^{\frac{1}{2}}(b_\perp, \mu) = H_1^{\pm}(\zeta^z, \bar{\zeta}^z, \mu)$$
$$\times e^{\frac{1}{2}\ln\frac{\mp\zeta^z + i0}{\zeta} K_1(b_\perp, \mu)} \Psi^{\pm}_{\bar{q}q}(x, b_\perp, \mu, \zeta)$$

Matching

quasi TMDWFs:

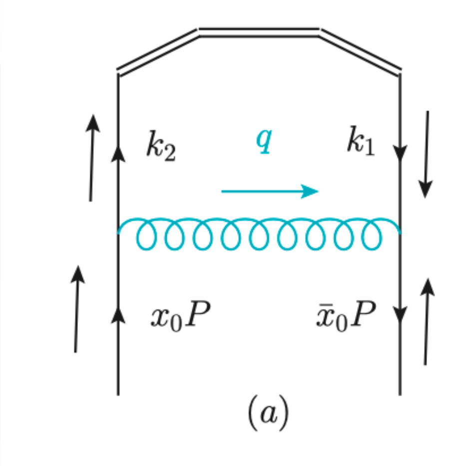


- ✓ **1-loop diagrams**
- ✓ **Self-energies of external lines are not shown**

$$\bar{u}\gamma^z\gamma_5 \ldots \sim \bar{u}(\gamma^+ - \gamma^-)\gamma_5 \sim \bar{u}\gamma^+\gamma_5$$

$$\bar{u}\gamma^t\gamma_5 \ldots \sim \bar{u}(\gamma^+ + \gamma^-)\gamma_5 \sim \bar{u}\gamma^+\gamma_5$$

3. Expansion by regions

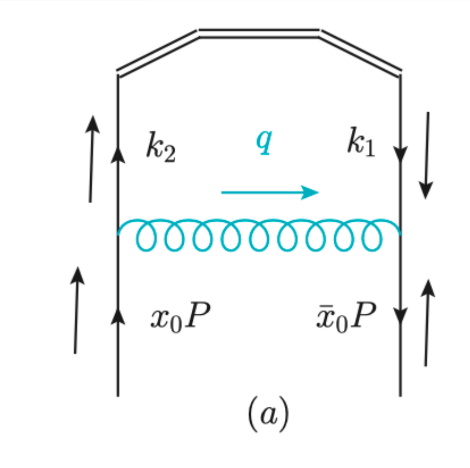


Expansion by regions:

- ✓ **Hard:** $q^\mu \sim (Q, Q, Q)$
- ✓ **Collinear:** $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ **Soft:** $q^\mu \sim (\Lambda, \Lambda, \Lambda)$

$$q^\mu = (q^+, q^-, q_\perp)$$

$$\tilde{\psi}_{\bar{q}q}^{\pm(1,a)} = \mu_0^{2\epsilon} i \frac{g^2 C_F}{2} (\bar{u}\gamma^l\gamma^5 v) \int \frac{d^d q}{(2\pi)^4} \frac{\frac{D-2}{P^z}[(\bar{x}_0P+q)^2 q^l - (x_0P-q)^2 q^l - P^l q^2]}{[(\bar{x}_0P+q)^2+i\epsilon][(x_0P-q)^2+i\epsilon](q^2+i\epsilon)} e^{-iq\cdot b_\perp} \delta\Big[(x-x_0)P^z + q^z\Big].$$



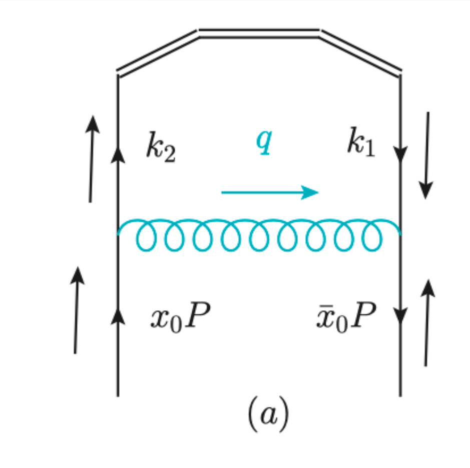
Expansion by regions:

- ✓ **Hard: $q^\mu \sim (Q, Q, Q)$**
- ✓ Collinear: $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ Soft: $q^\mu \sim (\Lambda, \Lambda, \Lambda)$

$$\tilde{\psi}_{\bar{q}q}^{\pm(1,a)} = \mu_0^{2\epsilon} i \frac{g^2 C_F}{2} (\bar{u}\gamma^l \gamma^5 v) \int \frac{d^d q}{(2\pi)^4} \frac{\frac{D-2}{P^z}[(\bar{x}_0 P + q)^2 q^l - (x_0 P - q)^2 q^l - P^l q^2]}{[(\bar{x}_0 P + q)^2 + i\epsilon][(x_0 P - q)^2 + i\epsilon](q^2 + i\epsilon)} e^{-iq\cdot b_\perp} \delta\left[(x - x_0)P^z + q^z\right].$$

$$\Lambda_{\mathrm{QCD}} \ll 1/b_\perp \ll P^z$$

Highly Oscillation

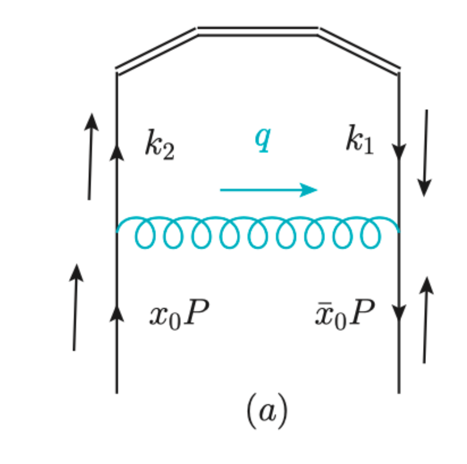


Expansion by regions:

- ✓ Hard: $q^\mu \sim (Q, Q, Q)$
- ✓ Collinear: $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$
- ✓ Soft: $q^\mu \sim (\Lambda, \Lambda, \Lambda)$

$$\tilde{\psi}_{\bar{q}q}^{\pm(1,a)} = \mu_0^{2\epsilon} i \frac{g^2 C_F}{2} (\bar{u}\gamma^l\gamma^5 v) \int \frac{d^dq}{(2\pi)^4} \frac{\frac{D-2}{P^z}[(\bar{x}_0P+q)^2 q^l - (x_0P-q)^2 q^l - P^l q^2]}{[(\bar{x}_0P+q)^2+i\epsilon][(x_0P-q)^2+i\epsilon](q^2+i\epsilon)} e^{-iq\cdot b_\perp} \delta\left[(x-x_0)P^z + q^z\right].$$

$$\frac{\Lambda^4 * \Lambda^2}{\Lambda * \Lambda * \Lambda^2} \sim \Lambda^2$$, Power suppress!



Expansion by regions:

- ✓ Hard: $q^\mu \sim (Q, Q, Q)$
- ✓ Collinear: $q^\mu \sim (Q, \Lambda^2/Q, \Lambda)$ $\frac{\Lambda^4 * \Lambda^2}{\Lambda^2 * \Lambda^2 * \Lambda^2} \sim 1$
- ✓ Soft: $q^\mu \sim (\Lambda, \Lambda, \Lambda)$

Leading power!

$$\tilde{\psi}^{\pm(1,a)}_{\bar{q}q} = \mu_0^{2\epsilon} i \frac{g^2 C_F}{2} (\bar{u}\gamma^l \gamma^5 v) \int \frac{d^d q}{(2\pi)^4} \frac{\frac{D-2}{P^z}[(\bar{x}_0 P + q)^2 q^l - (x_0 P - q)^2 q^l - P^l q^2]}{[(\bar{x}_0 P + q)^2 + i\epsilon][(x_0 P - q)^2 + i\epsilon](q^2 + i\epsilon)} e^{-iq\cdot b_\perp} \delta\left[(x - x_0)P^z + q^z\right].$$

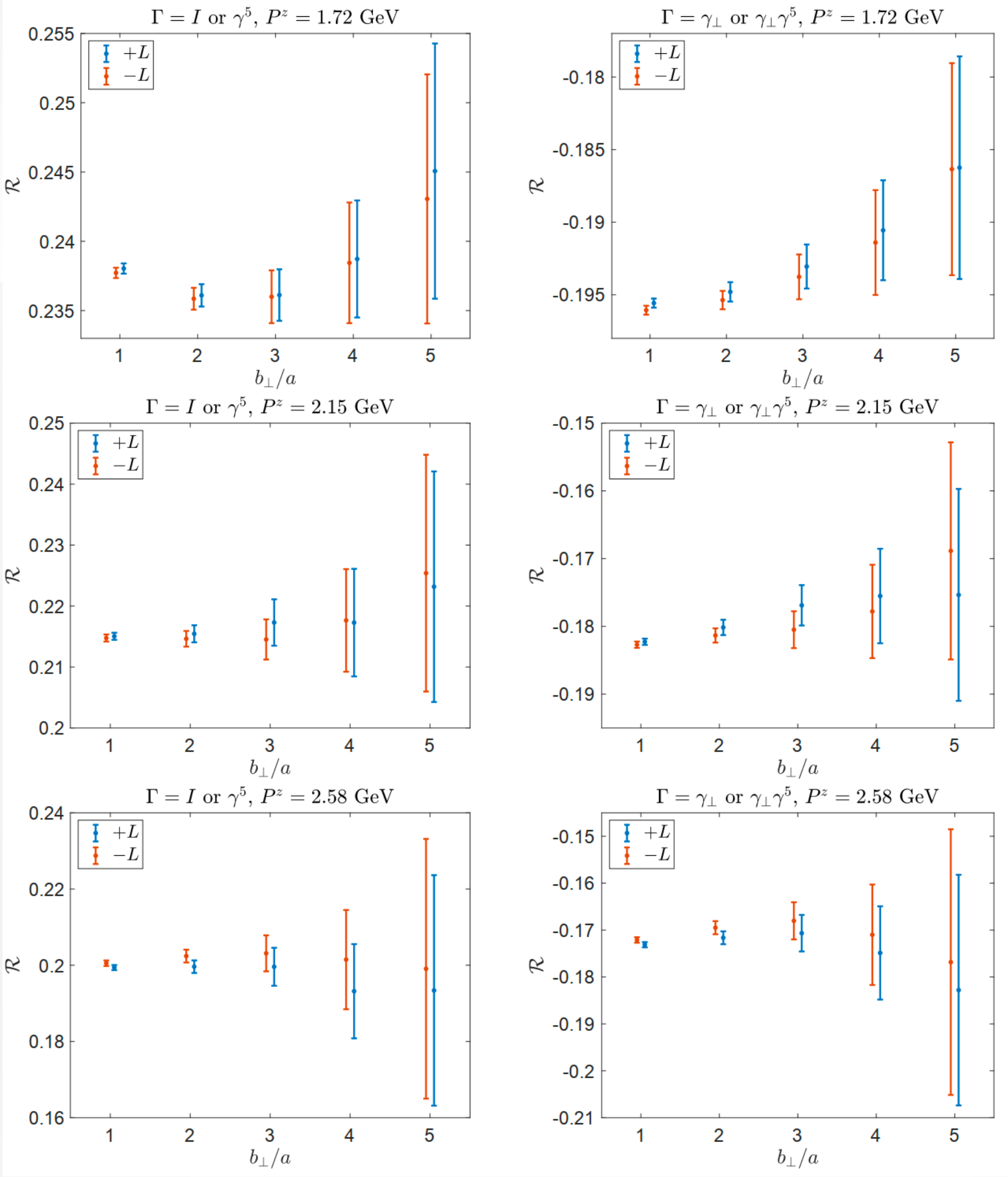
↓

$$\mu_0^{2\epsilon} i \frac{g^2 C_F}{2} (\bar{u}\gamma^l \gamma^5 v) \int \frac{d^d q}{(2\pi)^d} \frac{D-2}{P^z} \frac{-P^l q_\perp^2}{[(\bar{x}_0 P + q)^2 + i\epsilon][(x_0 P - q)^2 + i\epsilon](q^2 + i\epsilon)} e^{-iq\cdot b} \sqrt{2}\delta\left[(x - x_0)P^+ + q^+\right]$$

Light-front result !

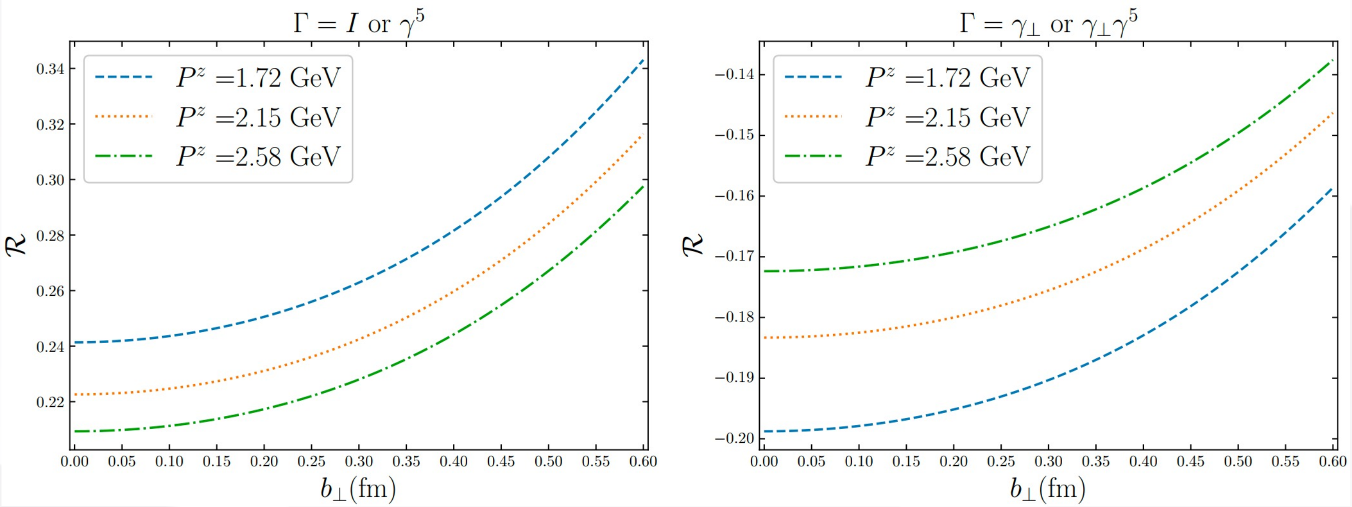
Collinear mode in quasi TMDWFs = TMDWFs

4. Lattice results



The lattice data on quasi-TMDWFs from LPC.

arXiv: hep-lat/2204.00200



Phenomenological model for quasi-TMDWFs.

arXiv: hep-ph/0702085

$$\mathcal{R} = \frac{\mathcal{H}_1 - \mathcal{H}_0}{\mathcal{H}_0}$$

$$S_r(b_\perp, \mu) = \frac{F(b_\perp, P_1, P_2, \mu)}{\mathcal{H}}$$

$$\mathcal{H} = \int dx_1 dx_2 H(x_1, x_2) \times \tilde{\Psi}^\dagger(x_2, b_\perp, P^z, \zeta_2^z)\tilde{\Psi}(x_1, b_\perp, P^z, \zeta_1^z)$$

5.Summary

1. In LaMET, the TMDWFs and soft functions can be extracted from the simulation of a four-quark form factor.

2. The one-loop TMD factorization of the form factor can be proofed in expansion by regions approach.

3. The perturbative corrections of soft functions depend on the operator to define the form factor, but are less sensitive to the transverse separation.

4. These results will be helpful to precisely extract the soft functions and TMD wave functions from the first-principle in future.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感谢聆听