Two-loop amplitudes for tW production at hardron colliders

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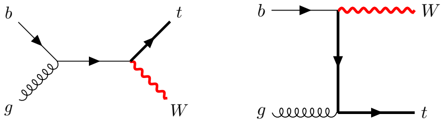
Motivation

The top quark is the heaviest elementary particle in the Standard Model.

Three major modes for single top productions, s-channel, t-channel and tW production.

tW production can be used to probe the the CKM matrix element V_{tb}.



The uncertainty of the measured cross section is about 11 % [A. S. Rodríguez, 2022].

To match experiments, the theoretical predictions must include higher-order corrections.

Motivation

NLO correction [S. Zhu, 2002, Q.-H. Cao, 2008] with top and W decay [J. M. Campbell et al, 2005]

Approximate higher order corrections [N. Kidonakis, 2006, 2010, 2017, 2021]

Effect of the parton shower [S. Frixione et al, 2008, E. Re, 2011, T. Ježo, 2016]

To match experiments, the complete NNLO QCD corrections are important.

Factorization formula

The N-jettiness subtraction is based on the soft-collinear effective theory (SCET).

$$\frac{d\sigma}{d\tau_N} \propto \int H \otimes B_1 \otimes B_2 \otimes S \otimes \left(\prod_{n=1}^{N} J_n \right) . \tag{1}$$

NNLO Beam functions B_i [I.W. Stewart et al, 2010, C.F. Berger et al, 2011, J.R. Gaunt et al, 2014]

NNLO Jet function J [T. Becher et al, 2006, 2011]

NNLO Soft function S [H. T. Li et al, 2016, 2018]

The missing part is NNLO hard function, which demands one-loop squared amplitudes and the interference between two-loop and tree-level amplitudes.

Kinematics and notations

$$g(k_1) + b(k_2) \to W(k_3) + t(k_4),$$
$$k_1^2 = k_2^2 = 0,\ k_3^2 = m_W^2,\ k_4^2 = (k_1 + k_2 - k_3)^2 = m_t^2. \quad (2)$$

The Mandelstam variables

$$s = (k_1 + k_2)^2, t = (k_1 - k_3)^2,\ u = (k_2 - k_3)^2,$$
$$s + t + u = m_W^2 + m_t^2. \quad (3)$$

The polarization summation

$$\sum_i \epsilon_i^{*\mu}(k_3)\epsilon_i^{\nu}(k_3) = -g^{\mu\nu} + \frac{k_3^\mu k_3^\nu}{m_W^2}$$
$$\sum_i \epsilon_i^{\mu}(k_1)\epsilon_i^{*\nu}(k_1) = -g^{\mu\nu} + \frac{k_1^\mu n^\nu + k_1^\nu n^\mu}{k_1 \cdot n} \text{ (can be neglected here)}. \quad (4)$$

Kinematics and notations

The anticommuting γ_5 scheme is implemented.

The tW amplitude can be written as

$$\mathcal{M} = \mathcal{M}^{(0)} + \frac{\alpha_s}{4\pi}\mathcal{M}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{M}^{(2)} + \cdots \tag{5}$$

We do not consider the decay of the top quark and the W boson at the moment.

Do not keep the polarization information, focus on amplitude squared,

$$|\mathcal{M}^{(1)}|^2, \quad |\mathcal{M}^{(0)*}\mathcal{M}^{(2)}|. \tag{6}$$

Then all the Lorentz indices are contracted.

Color structures

According to color structures, we have

$$\mathcal{A}^{(2)} = \sum_{\text{spins}} |\mathcal{M}^{(0)*}\mathcal{M}^{(2)}| = N_c^4 A + N_c^2 B + C + \frac{1}{N_c^2} D + n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l)$$
$$+ n_h (N_c^3 E_h + N_c F_h + \frac{1}{N_c} G_h), \qquad (7)$$

n_l (n_h) is the number of light (heavy) quark flavors. N_c is the color factor.

Leading contribution of the two-loop amplitudes

$$\mathcal{A}^{(2)}_{\mathrm{L.C.+n_l}} \equiv N_c^4 A + n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l). \qquad (8)$$

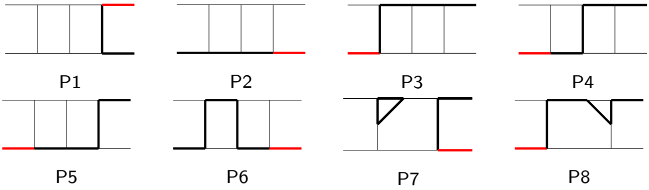
Amplitude calculation

We use FeynArts to generate 199 two-loop diagrams. 73 diagrams contribute to the leading color, 20 diagrams contribute to light fermion loop.

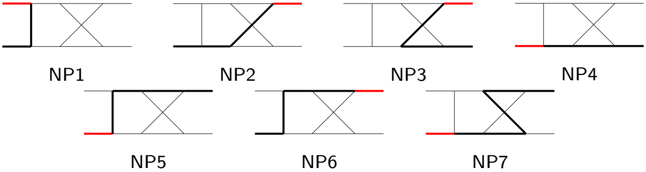
After IBP reduction of FIRE [A. V. Smirnov et al, 2020], $\mathcal{A}^{(2)} = \sum_{\text{spins}} |\mathcal{M}^{(0)*}\mathcal{M}^{(2)}|$ can be reduced to several families of master integrals.

Calculation of master integrals

All master integrals can be expressed in 8 planar and 7 non-planar topologies.



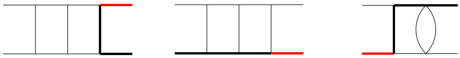
Red lines are W boson, thick lines are top quarks, others are massless particles.



Master integrals of leading contribution

$$\mathcal{A}^{(2)}_{\mathrm{L.C.+n_l}} \equiv N_c^4 A + n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l). \quad (9)$$

Only P1 and P2 and a sub-diagram of P3 relate to leading contribution



P1 P2 sub-diagram of P3

31 and 38 master integrals in P1, P2. Their analytical results have been obtained [L.-B. Chen et.al, 2021, M.-M. Long et.al, 2021] by the differential equations.

We also calculate the P2 topology independently to do the cross check.

Renormalization

Renormalized QCD amplitude is

$$\mathcal{M}_{\text{ren}} = Z_g^{1/2} Z_b^{1/2} Z_t^{1/2} \left(\mathcal{M}_{\text{bare}} \Big|_{\alpha_s^{\text{bare}} \to Z_{\alpha_s} \alpha_s;\ m_{t,\text{bare}} \to Z_m m_t} \right). \quad (10)$$

$Z_{g,b,t}$ is the wave function renormalization factor. α_s and m_t are renormalized by the factor Z_{α_s} and Z_m.

$$\begin{aligned}\mathcal{M}_{\text{ren}} &= \mathcal{M}_{\text{ren}}^{(0)} + \frac{\alpha_s}{4\pi}(\mathcal{M}_{\text{bare}}^{(1)} + \mathcal{M}_{\text{C.T.}}^{(1)}) + \left(\frac{\alpha_s}{4\pi}\right)^2 (\mathcal{M}_{\text{bare}}^{(2)} + \mathcal{M}_{\text{C.T.}}^{(2)}) \\ &= \mathcal{M}_{\text{ren}}^{(0)} + \frac{\alpha_s}{4\pi}\mathcal{M}_{\text{ren}}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{M}_{\text{ren}}^{(2)}. \end{aligned} \quad (11)$$

The on-shell renormalization scheme for wave functions and top-quark mass. α_s is calculated in $\overline{\text{MS}}$ scheme.

IR divergences

IR divergences can be subtracted with a factor $\mathbf{Z}$.

$$\mathcal{M}_{\text{fin}} = \mathbf{Z}^{-1}\mathcal{M}_{\text{ren}} \, . \tag{12}$$

In the framework of SCET, $\mathbf{Z}$ can be investigated through the anomalous-dimensions of the effective operators.

$$\mathbf{Z} = 1 + \frac{\alpha_s}{4\pi}\mathbf{Z}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{Z}^{(2)} \; + \mathcal{O}(\alpha_s^3). \tag{13}$$

For example,

$$\begin{aligned} \mathbf{Z}^{(1)} =& -(C_A + C_F)\frac{\gamma_{\text{cusp}}^{(0)}}{4\epsilon^2} + \frac{\gamma_g^{(0)} + \gamma_b^{(0)} + \gamma_t^{(0)}}{2\epsilon} \\ &+ \frac{\gamma_{\text{cusp}}^{(0)}}{4\epsilon}\left(-C_A \ln\frac{\mu^2}{-s} - C_A \ln\frac{\mu m_t}{m_t^2 - u} + (C_A - 2C_F)\ln\frac{\mu m_t}{m_t^2 - t}\right). \end{aligned} \tag{14}$$

where γ_{cusp}, γ_g, γ_b and γ_t are anomalous dimensions.

NNLO hard function

$$H^{(2)} = \mathcal{M}^{(2)}_{\text{fin}}\mathcal{M}^{(0)*}_{\text{fin}} + \mathcal{M}^{(0)}_{\text{fin}}\mathcal{M}^{(2)*}_{\text{fin}} + \left|\mathcal{M}^{(1)}_{\text{fin}}\right|^2 \quad (15)$$

According to color structures

$$\begin{aligned} H^{(2)} =& N_c^4 H_A + N_c^2 H_B + H_C + \frac{1}{N_c^2} H_D + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right) \\ &+ n_h \left(N_c^3 H_{Eh} + N_c H_{Fh} + \frac{1}{N_c} H_{Gh} \right) . \end{aligned} \quad (16)$$

Leading contribution of hard function

$$H^{(2)}_{\mathrm{L.C.+n_l}} \equiv N_c^4 H_A + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right) . \quad (17)$$

Numerical results

$\beta_t = \sqrt{1 - m_t^2/E_t^2}$ measures the velocity of the top quark and θ is the angle between gluon and top quark.

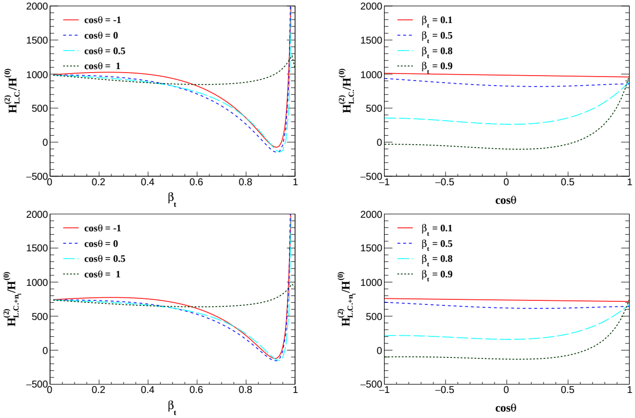
The divergence of ϵ^{-4} and ϵ^{-3} have been canceled analytically.

Other divergence have been checked numerically in high precision.

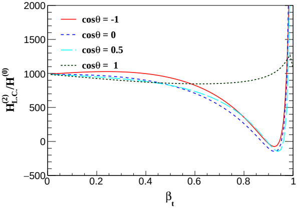
Numerical results

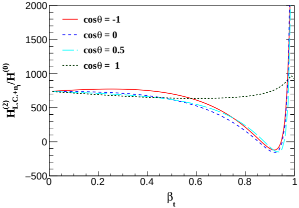
$$H = H^{(0)} + \frac{\alpha_s}{4\pi} H^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 H^{(2)}$$

$$H^{(2)}_{\mathrm{L.C.+n_l}} \equiv N_c^4 H_A + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right)$$



Numerical results





Divergence in the limit of $\beta_t \to 1$.

Fermion-loop diagrams provide negative contributions, decrease L.C. result by about 30%.

Integrate over all phase space, the L.C. and nl dependent NNLO hard function contribute 5.4% and -1.4% to the LO cross section at the 13 TeV LHC.

m_W expansion

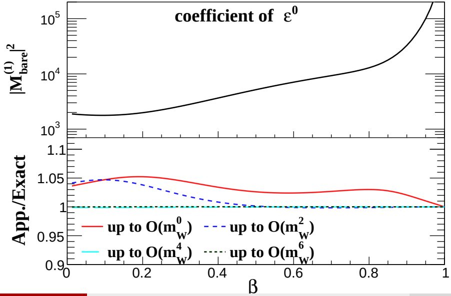
$m_W = 0$ does not bring new IR divergences due to the massive top quark propagator.

$$I^i_{n_1,n_2,n_3,n_4}\left(s,u,m_W^2,m_t^2\right) = \sum_{n=0}^{\infty} \frac{(m_W^2)^n}{n!} \left.\frac{\partial^n I^i_{n_1,n_2,n_3,n_4}}{\partial (m_W^2)^n}\right|_{m_W^2=0} . \tag{18}$$

1.The number of these master integrals is less.

2.The analytic computation is easier to perform.

m_W expansion in one-loop squared.



Summary and Overlook

We obtain the analytical one-loop square and two-loop leading contribution amplitudes for tW production by using differential equations.

The renormalized amplitude squared has up to ϵ^{-4} poles, which have been checked against the general infrared structures predicted by anomalous dimensions.

The finite part gives rise to about a few percent corrections compared to the corresponding LO results.

We investigate how to obtain approximated results by the expansion in m_W.

We will calculate complete two-loop amplitude in the future.

Backup

In the differential equations of P1, there is square root

$$r_1 = \sqrt{(s-(m_t+m_W)^2)(s-(m_t-m_W)^2)},$$
$$s = m_t^2 \frac{(x+z)(1+x\,z)}{x}, \quad m_W^2 = m_t^2\, z^2. \tag{19}$$

In the differential equations of P2, besides r_1, there is another square root

$$r_2 = \sqrt{s(m_t^2-u)(m_t^2\,(-4m_W^2+s+4u+4)-4m_t^4-s\,u)} \tag{20}$$

The problem is to rationalized more than one square roots simultaneously.

One choice is to use package **RationalizeRoots**. [M. Besier et.al, 2020]. For example,

$$m_W^2 = \frac{b_1 b_2 m_t^2}{1-b_2}, \quad s = -\frac{(b_1+1)m_t^2}{b_2-1}, \quad u = \frac{b_2(b_3(b_1+1)(b_3b_2-2)+4b_1)m_t^2}{b_2(b_3(b_1+1)(b_3b_2-2)-4)+4}, \tag{21}$$

Backup

By constructing the canonical basis, the differential equations can be transforming to ϵ form [J. M. Henn, 2013].

$$d\,\mathbf{F}(s,t,m_W^2;\epsilon) = \epsilon\,(d\,\tilde{A})\,\mathbf{F}(s,t,m_W^2;\epsilon), \tag{22}$$

The square roots may appear. Combining the boundary conditions, the solutions can be expressed in multiple polylogarithms (MPLs) [A. B. Goncharov, 1998] or GPLs.

$$G_{a_1,a_2,\ldots,a_n}(x) \equiv \int_0^x \frac{\mathrm{d}t}{t-a_1} G_{a_2,\ldots,a_n}(t)\,, \tag{23}$$

$$G_{\vec{0}_n}(x) \equiv \frac{1}{n!}\ln^n x\,. \tag{24}$$

Use PolyLogTools [D. Claude, 2019] to obtain numerical results of GPLs.

Backup

$$
\begin{aligned}
\mathcal{M}_{\text{fin}} &= \mathcal{M}_{\text{fin}}^{(0)} + \frac{\alpha_s}{4\pi}\mathcal{M}_{\text{fin}}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathcal{M}_{\text{fin}}^{(2)}, \\
\mathcal{M}_{\text{fin}}^{(0)} &= \mathcal{M}_{\text{ren}}^{(0)}\,, \\
\mathcal{M}_{\text{fin}}^{(1)} &= \mathcal{M}_{\text{ren}}^{(1)} - \mathbf{Z}^{(1)}\mathcal{M}_{\text{ren}}^{(0)}\,, \\
\mathcal{M}_{\text{fin}}^{(2)} &= \mathcal{M}_{\text{ren}}^{(2)} + ((\mathbf{Z}^{(1)})^2 - \mathbf{Z}^{(2)})\mathcal{M}_{\text{ren}}^{(0)} - \mathbf{Z}^{(1)}\mathcal{M}_{\text{ren}}^{(1)}\,.
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
H &= H^{(0)} + \frac{\alpha_s}{4\pi}H^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 H^{(2)}\,, \\
H^{(0)} &= \left|\mathcal{M}_{\text{fin}}^{(0)}\right|^2\,, \\
H^{(1)} &= \mathcal{M}_{\text{fin}}^{(1)}\mathcal{M}_{\text{fin}}^{(0)*} + \mathcal{M}_{\text{fin}}^{(0)}\mathcal{M}_{\text{fin}}^{(1)*}\,, \\
H^{(2)} &= \mathcal{M}_{\text{fin}}^{(2)}\mathcal{M}_{\text{fin}}^{(0)*} + \mathcal{M}_{\text{fin}}^{(0)}\mathcal{M}_{\text{fin}}^{(2)*} + \left|\mathcal{M}_{\text{fin}}^{(1)}\right|^2\,.
\end{aligned} \tag{26}
$$