

Recent developments in the CTEQ-TEA  
global analysis:  
CT18CS and CT18As  
and  
Intrinsic Charm in CTEQ-TEA PDFs

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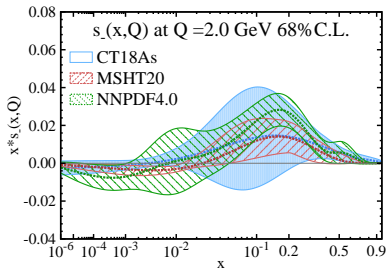
August 23th, 2022



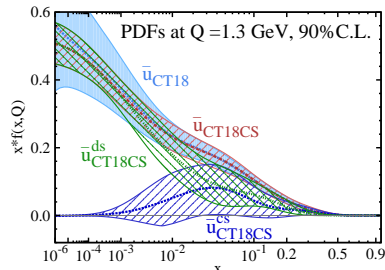
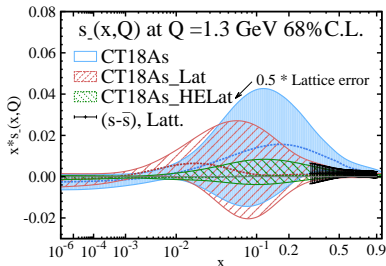
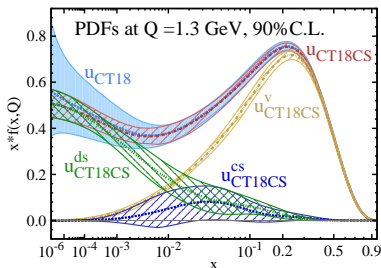
CTEQ

# Global PDF Analysis with Lattice Input

## CT18As



## CT18CS



arXiv:2204.07944

arXiv:2206.02431

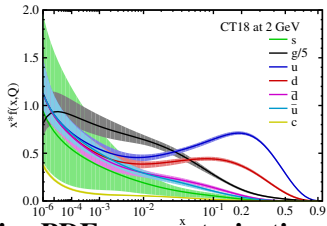
# CT18 in a nutshell

- Start from **CT14-HERAII**:

(PRD95,034003(2017), T.-J. Hou *et al*)

HERAII combined data released after publication of CT14

(PRD93,033006(2016), S. Dulat *et al*).



- Examine a **wide range of non-perturbative PDF parameterizations**.

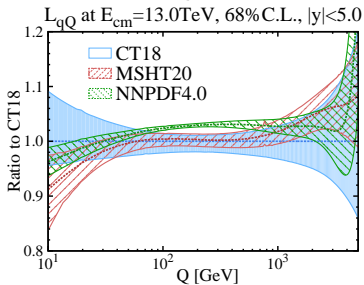
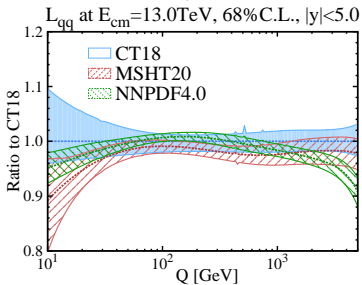
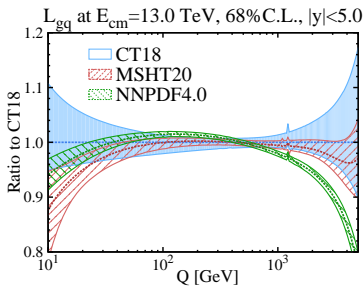
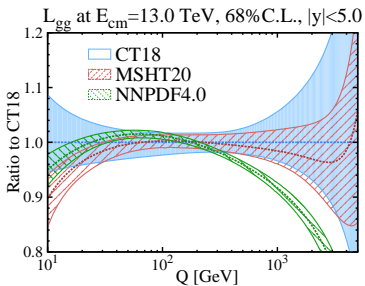
- Use as much relevant **LHC Run II data** as possible; using applgrid/fastNLO interfaces to data sets, with NNLO/NLO K-factors, or fastNNLO tables in the case of top pair (single and double differential) data.

- Implement a **parallelization** of the global PDF fitting to allow for faster turn-around time.

- Use diverse statistical techniques (**PDFSense**, **ePump**, **Gaussian variables**, **Lagrange Multiplier** scans) to examine agreement between experiments.

# PDF Luminosities at 13 TeV LHC

## CT18, MSHT20 and NNPDF4.0



# **Connected and Disconnected Sea Partons from CT18 Parametrization of PDFs**

In Collaborate with Jian Liang, Keh-Fei Liu,  
Mengshi Yan, and C.-P. Yuan  
arXiv:2206.02431

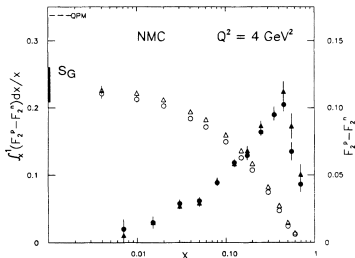
# Gottfried sum rule

Gottfried sum rule (1967) was originally obtained by assuming  $\bar{u}$  and  $\bar{d}$  to be the same, which leads to

$$S_G = \int_0^1 \frac{dx}{x} [F_2^p(x) - F_2^n(x)] = \frac{1}{3}, \quad \text{with } \bar{d}(x) \equiv \bar{u}(x)$$

New Muon Collaboration (NMC) PRL 66, 2712 (1991), PRD 50, R1 (1994),  
 $\mu + p(n) \rightarrow \mu + X$ , obtained

$$S_G = 0.235 \pm 0.026 \quad (Q^2 = 2 \text{ GeV}^2)$$



# Gottfried sum rule

The alternative expression of Gottfried sum rule is,

$$S_G = \frac{1}{3} - \frac{2}{3} \int_0^1 dx (\bar{d}(x) - \bar{u}(x)) + O(\alpha_s^2).$$

Hence, NMC data gives

$$\int_0^1 dx (\bar{d}(x) - \bar{u}(x)) = 0.147 \pm 0.039, \quad \text{at } Q = 2 \text{ GeV}$$

The following experiments like HERMES (PLB387, 419 (1996)) and E866 (PRD64, 052002 (2001)) also shown preference of  $\bar{u}/\bar{d}$  flavor asymmetry.

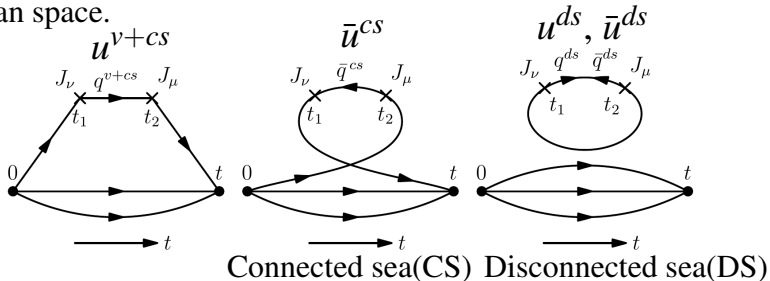
Experiment	$\langle Q^2 \rangle$ (GeV <sup>2</sup> )	$\int_0^1 [\bar{d}(x) - \bar{u}(x)] dx$
NMC/DIS	4.0	$0.147 \pm 0.039$
HERMES/SIDIS	2.3	$0.16 \pm 0.03$
FNAL E866/DY	54.0	$0.118 \pm 0.012$

What is the origin of  $\int dx (\bar{d}(x) - \bar{u}(x)) \neq 0$ ?



# Hadronic tensor in Euclidean path-integral formalism

Motivated by Hadronic tensor in QCD path-integral formalism in Euclidian space.



$$\begin{aligned}
 u &= u^{v+cs} + u^{ds}, & d &= d^{v+cs} + d^{ds} \\
 \bar{u} &= \bar{u}^{cs} + \bar{u}^{ds}, & \bar{d} &= \bar{d}^{cs} + \bar{d}^{ds}
 \end{aligned}$$

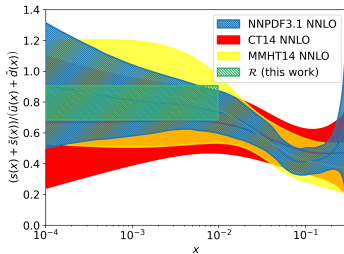
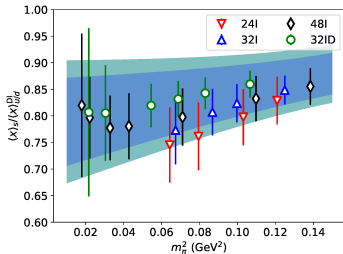
Define  $u^v \equiv u^{v+cs} - \bar{u}^{cs}$ , which is equivalent to defining  $u^{cs} \equiv \bar{u}^{cs}$ .

$$\begin{aligned}
 u - \bar{u} &\equiv (u^{v+cs} + u^{ds}) - (\bar{u}^{cs} + \bar{u}^{ds}) = u^v + (u^{ds} - \bar{u}^{ds}) \\
 &\neq u^v, \quad \text{unless } u^{ds} = \bar{u}^{ds}
 \end{aligned}$$

Similarly,  $d^v \equiv d^{v+cs} - \bar{d}^{cs}$ .

# Lattice input to global fitting of PDFs

Lattice result from overlap on  $N_f = 2 + 1$  DWF on 4 lattices, with one at physical pion mass (J. Liang *et al*,  $\chi$ QCD, PRD, arXiv:1901.07526)



$$\frac{1}{R} = \frac{\langle x \rangle_{s+\bar{s}}}{\langle x \rangle_{\bar{u}+\bar{d}}^{DI}} (\text{at } 1.3 \text{ GeV}) = 0.822(69) \quad (78)$$

With this input from Lattice calculation, we assume

$$u^{ds} = \bar{u}^{ds} = d^{ds} = \bar{d}^{ds} = R = R\bar{s},$$

## Parton degrees of freedom at $Q_0 = 1.3 \text{ GeV}$

If we define  $u^{cs} \equiv \bar{u}^{cs}$  and  $d^{cs} \equiv \bar{d}^{cs}$ , the physical parton degrees of freedom used in CT18CS are then:

	$g = g_{par}$	
	$u^v = u_{par}^v$	
In CT18	$d^v = d_{par}^v$	In CT18CS
	$\bar{u} = \bar{u}^{cs} + \bar{u}^{ds} = \bar{u}_{par} + R s_{par}$	
	$\bar{d} = \bar{d}^{cs} + \bar{d}^{ds} = \bar{d}_{par} + R s_{par}$	
	$s = \bar{s} = s_{par}$	

The non-perturbative PDF functions are further chosen so that

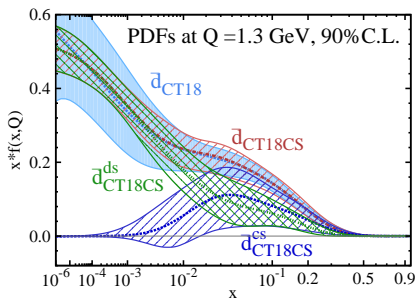
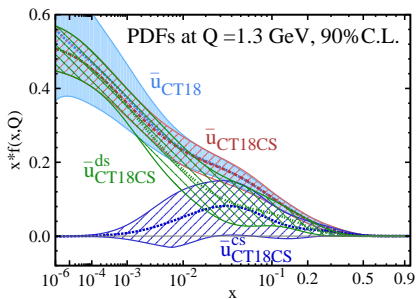
- 1  $\bar{d}/\bar{u} \xrightarrow{x \rightarrow 0} 1$ .  $\Leftarrow$  Isospin symmetry, similar to CT18
- 2  $\bar{u}^{ds}, \bar{d}^{ds}, \bar{s}^{ds} \xrightarrow{x \rightarrow 0} x^{-1}$ .  $\Leftarrow$  Similar to CT18
- 3  $\bar{u}^{cs}, \bar{d}^{cs} \xrightarrow{x \rightarrow 0} u^v, d^v$ .  $\Leftarrow$   $\bar{q}^{cs}$  is in the connected insertion
- 4  $d/u \xrightarrow{x \rightarrow 1} d/u$  of CT18.  $\Leftarrow$  Valence behavior, similar to CT18.
- 5  $\bar{d}/\bar{u} \xrightarrow{x \rightarrow 1} \bar{d}/\bar{u}$  of CT18.  $\Leftarrow$  Describe E866 and E906 data.

# CT18CS PDFs

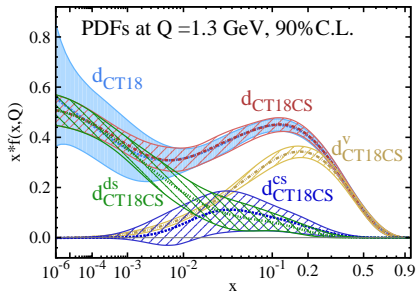
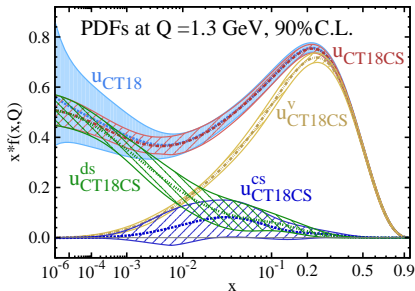
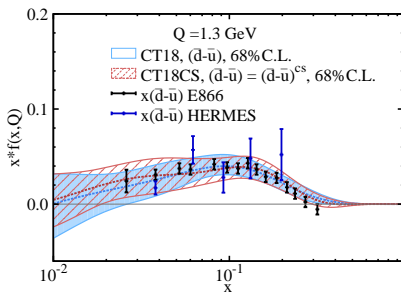
With the input of  $\bar{u}^{ds} = \bar{d}^{ds} = R_s^{ds}$ , and

$$\frac{1}{R} \equiv \frac{\langle x \rangle_{s+\bar{s}}}{\langle x \rangle_{\bar{u}+\bar{d}}(DI)} = 0.822 \text{ at } 1.3 \text{ GeV}$$

from lattice QCD, and considering the ansatz of small- $x$  behavior, we obtain the CT18CS at  $Q_0 = 1.3 \text{ GeV}$  scale.



# CT18CS PDFs



# The $\langle x \rangle$ Moment of CT18CS at 1.3 GeV

The  $\langle x \rangle$  moment of CT18 and CT18CS at 1.3 GeV:

PDF	$\langle x \rangle_{u^v}$	$\langle x \rangle_{d^v}$	$\langle x \rangle_g$	$\langle x \rangle_{\bar{u}}$	$\langle x \rangle_{\bar{d}}$	$\langle x \rangle_s$
CT18	0.325(5)	0.134(4)	0.385(10)	0.0284(22)	0.0361(27)	0.0134(52)
CT18CS	0.323(4)	0.136(3)	0.384(12)	0.0287(25)	0.0364(34)	0.0137(39)

---

PDF	$\langle x \rangle_{u^{v+cs}}$	$\langle x \rangle_{d^{v+cs}}$	$\langle x \rangle_{\bar{u}^{cs}}^*$	$\langle x \rangle_{\bar{d}^{cs}}^*$	$\langle x \rangle_{s^{ds}}^{\S}$
CT18CS	0.335(7)	0.155(8)	0.0120(64)	0.0197(70)	0.0167(49)

More direct comparison between global analysis and lattice calculation can be done for each parton degree of freedom, instead of being limited to  $u - d$  and  $s$ .

	$Q = 2.0$ GeV		$Q = 1.3$ GeV	
	CT18	Lattice	CT18CS	CT18
$\langle x \rangle_{u^+ - d^+}$	0.156(7)	$0.111 - 0.209^{N_f=2+1}$ $0.153 - 0.194^{N_f=2+1+1\dagger}$	0.173(7)	0.175(8)
$\langle x \rangle_{s^+}$	0.033(9)	$0.166 - 0.212^{N_f=2}$ $0.051(26)(5)^{\ddagger}$	0.027(8)	0.027(10)

$\dagger$  Prog. Part. Nucl. Phys., 121:103908, 2021.       $\ddagger$  Phys. Rev. Lett., 121(21):212001, 2018

$$u^+ - d^+ = (u + \bar{u}) - (d + \bar{d}) = (u^{v+cs} + u^{ds} + \bar{u}^{cs} + \bar{u}^{ds}) - (d^{v+cs} + d^{ds} + \bar{d}^{cs} + \bar{d}^{ds})$$

$$\xrightarrow{CT18CS} (u^{v+cs} - d^{v+cs}) + (\bar{u}^{cs} - \bar{d}^{cs})$$

$$s^+ = s + \bar{s} = s^{ds} + \bar{s}^{ds} \xrightarrow{CT18CS} 2s^{ds}$$

# Summary

- PQCD and LQCD are both important methods for studying the structure of hadron. But there were only few physical quantities can be used for comparison in the past.
- With the input from lattice QCD,  $\frac{1}{R} \equiv \frac{\langle x \rangle_{s+\bar{s}}}{\langle x \rangle_{\bar{u}+\bar{d}}(DI)}$ , we consider global analysis with the connected sea parton degrees of freedom taken into account, which are responsible for  $\bar{u} \neq \bar{d}$ , as suggested by data. The result of global analysis, the CT18CS, is found to be compatible with CT18.
- The CT18CS allows to provide direct comparison between lattice calculations and global analysis for each parton degree of freedom.

# **Impact of lattice $s(x) - \bar{s}(x)$ data in the CTEQ-TEA global analysis**

In Collaborate with Huey-Wen Lin, Mengshi Yan, C.-P. Yuan  
arXiv:2204.07944



## CT18 with $s(x) \neq \bar{s}(x)$

In the framework of CT18, 6 d.o.f of partons are parametrized at  $Q_0 = m_c = 1.3 \text{ GeV}$ .

$$g, \quad u^v, \quad d^v, \quad \bar{u}, \quad \bar{d}, \quad s.$$

Where  $\bar{s}(x) \equiv s(x)$  is assumed. The number sum rule for strangeness is satisfied naively. Because the DGLAP equation preserve

$$\int_0^1 [s(x) - \bar{s}(x)] dx = 0,$$

People used to parametrize the strange as  $s_+ = s + \bar{s}$  and  $s_- = s - \bar{s}$ . For example, in CTEQ 6

$$s_+(x, Q_0) = a_0^s x^{a_1-1} (1-x)^{a_2} P_+(x)$$

$$s_-(x, Q_0) = s_+(x, Q_0) \tanh[ax^b(1-x)^c P_-(x)]$$

$$P_-(x) = \left(1 - \frac{x}{x_0}\right) (1 + dx + ex^2 + \dots)$$

## CT18 with $s(x) \neq \bar{s}(x)$

We consider an alternative way on parametrizing the strangeness.  
Consider both  $s$  and  $\bar{s}$  contain an overall factor  $a_0$  :

$$s(x, Q = Q_0) = a_0^s x^{a_1^s - 1} (1-x)^{a_2^s} P^s(x) = a_0^s g^s(x)$$

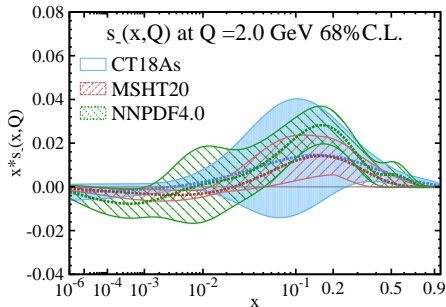
$$\int_0^1 [a_0^s g^s(x) - a_0^{\bar{s}} g^{\bar{s}}(x)] dx = 0$$

By given  $a_0^{\bar{s}}$ , the  $a_0^s$  can be determined by the strange number sum rule.

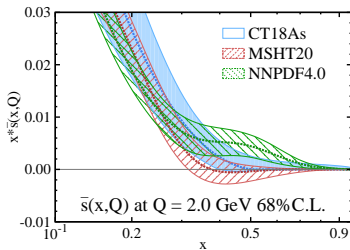
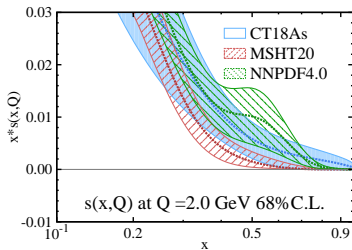
$$a_0^s = \frac{\int a_0^{\bar{s}} g^{\bar{s}}(x) dx}{\int g^s(x) dx}$$

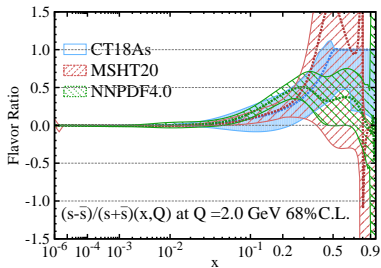
- Different from the root-finding method, there is no presumed requirement on the function form of  $g^s(x)$  and  $g^{\bar{s}}(x)$ .
- But it is relatively hard to control the number of crossing in  $s - \bar{s}$ .

# CT18As NNLO: CT18A with Strangeness Asymmetry

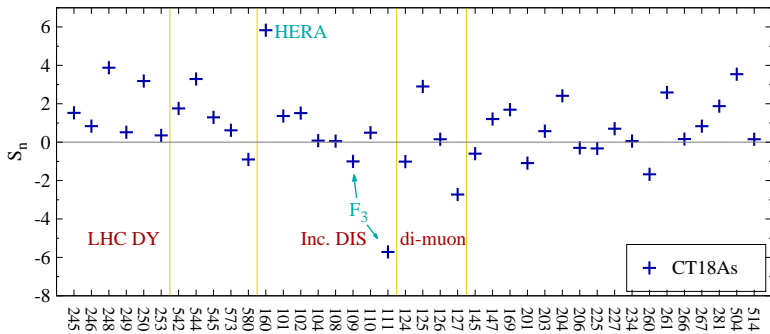
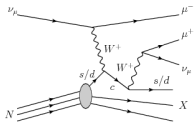


Starting from CT18A, we select the strange asymmetry with single crossing from various trial parametrizations.



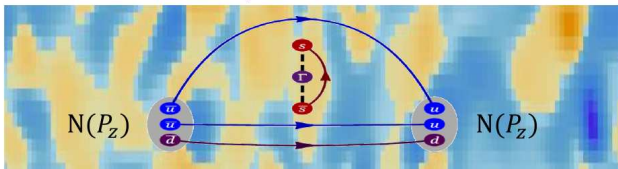


Asymmetry normally reaches  $\sim 50\%$  at  $x \sim 0.25$  in three global fits.  
 $s_- \neq 0$  is preferred by LHC Drell-Yan processes and E866 p/d ratio.



# First Lattice Strange PDF

§ On the lattice, one needs to calculate the following



2005.12015, Zhang, Lin, Yoon

§ Results by MSULat/quasi-PDF method

☞ Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

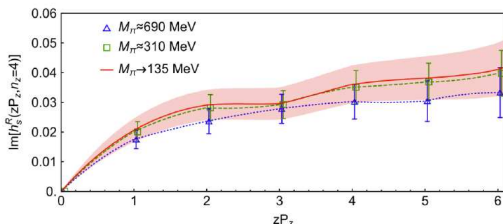
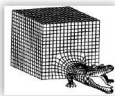
☞ 7,184,000 strange loops

☞ 344,832 nucleon correlators

☞ RI/MOM renormalization

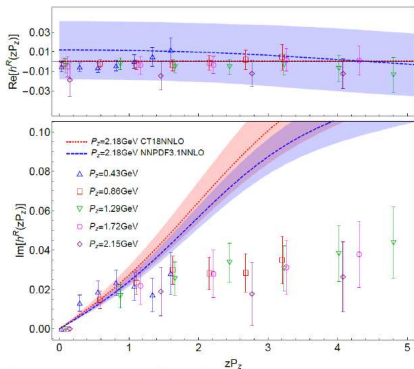
☞ Extrapolated to

$$M_\pi \approx 140 \text{ MeV}$$



# First Lattice Strange PDF

## § Lattice matrix elements



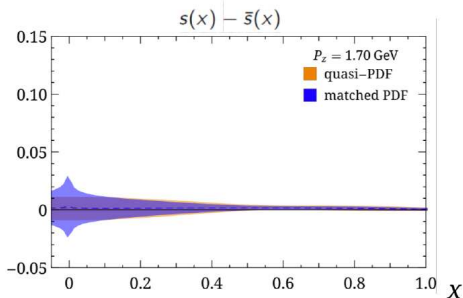
## § From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^1 \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + O\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

## ∞ Strange-antistrange symmetry

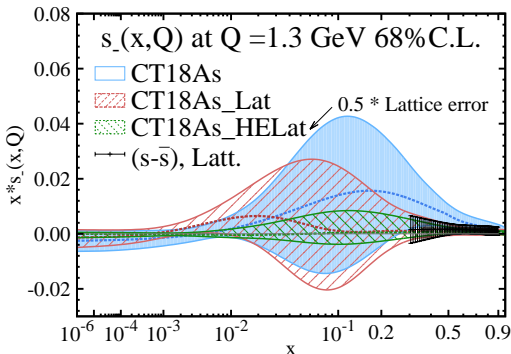
$$\text{Re}[h(z)] \propto \int dx (s(x) - \bar{s}(x)) \cos(xzP_z)$$

$$\text{Im}[h(z)] \propto \int dx (s(x) + \bar{s}(x)) \sin(xzP_z)$$



# CT18As\_Lat:

## Strangeness asymmetry with a lattice QCD constraint



**CT18As:**

CT18A with strange asymmetry.

**CT18As\_Lat:**

PDFs with lattice input.

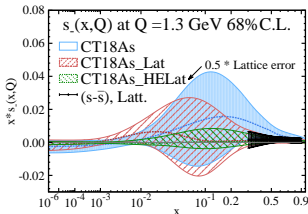
**CT18As\_HELat:**

PDFs if the lattice errors are reduced by 1/2.

- Lattice QCD calculation provide prediction for  $0.3 < x < 0.8$ .
- Lattice QCD prediction disfavors a large  $s_-(x, Q)$  for  $x > 0.3$  region. It cause the reduction in  $s_-(x, Q)/s_+(x, Q)$  for  $x < 0.3$  (depending on the parametrization form).

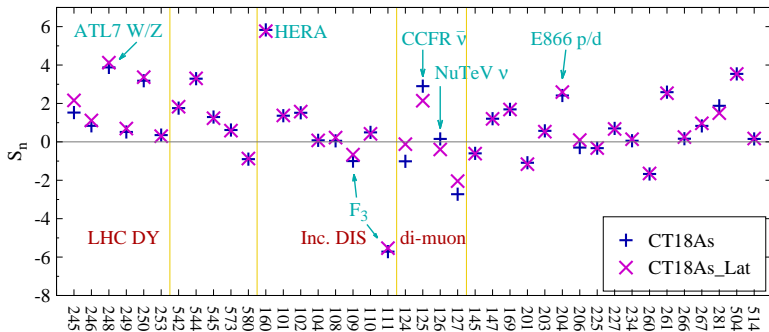
# CT18As\_Lat:

## Strangeness asymmetry with a lattice QCD constraint



Di-muon data provide measurements for  $0.015 < x < 0.336$ .

DIS and dimuon SIDIS show less clear trends.





# Summary

- Lattice QCD calculation disfavors a large  $s_-(x, Q)$  for  $x > 0.3$ .
- By treating lattice QCD calculation as data, we have chance to determine PDFs at  $x \rightarrow 1$ .
- The framework of global analysis provide indirect comparison between lattice calculations and experimental measurements in the overlap region.

# **Intrinsic Charm in CTEQ-TEA PDFs**

## **CT14 IC: answers to important questions**

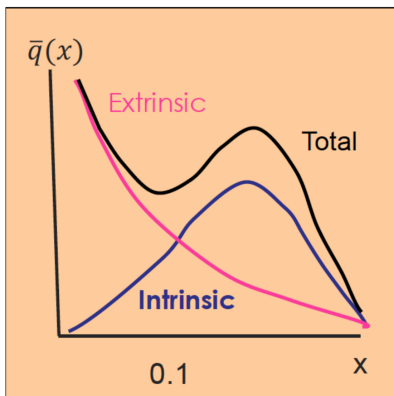
**What are phenomenological constraints on the "intrinsic charm" from the global QCD data?**

⇒ The CT14 charm PDFs allow a "nonperturbative" component carrying a total momentum fraction of  $1 - 2\%$  in DIS at  $Q \sim m_c$ .

**Can we estimate its impact on the LHC predictions?**

Yes, based on the simplest approximation of the "nonperturbative" charm contribution. In most cases, the estimated impact is less than the net CT14 PDF uncertainty.

Hou et al., JHEP 02(2018)059, arXiv: 1707.00657



Instead of parametrizing the charm as strange in the usual way, we concern the possibility of **valence-like**(intrinsic) and **sea-like**(extrinsic) component of charm.

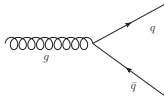
# 1. The Sea-like(extrinsic) component:

- Monotonic in  $x$ , satisfies

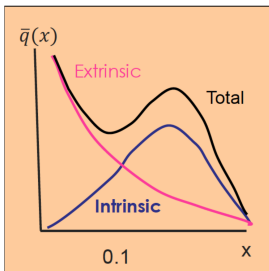
$$q(x) \propto x^{-1}, \text{ for } x \rightarrow 0$$

- May be generate in several ways, e.g.

In PQCD, from gluon splittings



In Lattice QCD, from disconnected diagrams



## 2. Valence-like (intrinsic) component:

- peaks in  $x$ , satisfies

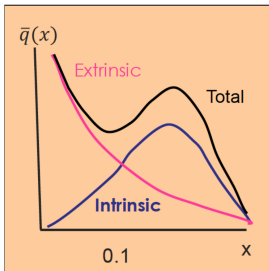
$$q(x) \propto x^{-1/2}, \text{ for } x \rightarrow 0$$

- May be generate in several ways, e.g.

For all flavors, nonperturbatively from a  $|uudQ\bar{Q}\rangle$  Fock state.

(Brodsky, Peterson, Sakai, PRD 1981)

In Lattice QCD, from connected diagrams



# Parametrizations of $c(x, Q_0)$

- ”Valence-like”  $c(x, Q_0)$  according to the BHPS model:  
(Brodsky et al, PLB 1980)

$$c(x) = \bar{c}(x) = \frac{1}{2}A x^2 \left[ \frac{1}{3}(1-x)(1+10x+x^2) - 2x(1+x) \ln(1/x) \right].$$

- ”BHPS3” model: we include intrinsic  $u\bar{u}$ ,  $d\bar{d}$  and  $c\bar{c}$  with **numerical** solutions for the BHPS model.
- ”Sea-like”  $c(x, Q_0)$ :

$$c(x) = \bar{c}(x) = A [\bar{d}(x, Q_0) + \bar{u}(x, Q_0)]$$

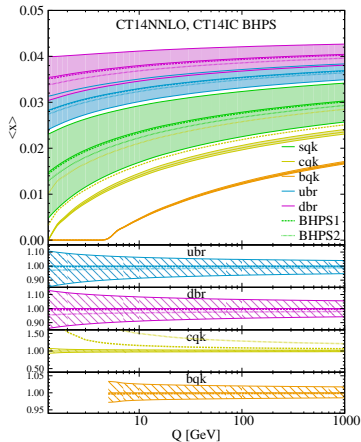
# Charm Momentum Fraction

$$\langle x \rangle_{c+\bar{c}}(Q) = \int_0^1 x [c(x, Q) + \bar{c}(x, Q)] dx$$

Initial scale  $Q_0 \leq m_c$ , intrinsic component only

$$\langle x \rangle_{\text{IC}} = \langle x \rangle_{c+\bar{c}}(Q_0)$$

At  $Q > Q_0$ , growth due to perturbative  $c(x, Q)$



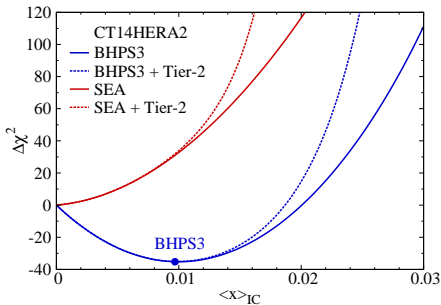
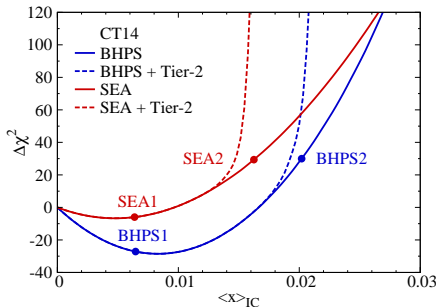
## Setup for the global analysis for CT14 and CT14HERA2:

- $\alpha_s(M_Z) = 0.118$ , compatible with the world average value  $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ ; the default value for recent CT PDF fits. Different value of  $\alpha_s(M_Z)$  yields different PDFs.
- HOPPET - evolution code used to include nonperturbative charm model with NNLO matching, and to evolve the PDF at NNLO.
- Partons are parametrized at the initial energy scale  $Q_0 = 1.295\text{GeV}$ , which is slightly lower than the default charm quark mass  $m_c^{pole} = 1.3\text{GeV}$ .
- Choose experimental data with  $Q^2 > 4\text{GeV}^2$  and  $W^2 > 12.6\text{GeV}^2$  to minimize high-twist, nuclear correction, etc., and focus on perturbative QCD predictions.



## Difference in CT14HERA2 from CT14:

- Combined HERA Run I+II data were used in place of the HERA Run I data in CT14.
- One of the poorly fit NMC data were drop in CT14HERA2.
- Strange quark no longer bound with  $\bar{u}$  and  $\bar{d}$ . Smaller strangeness is prefer than CT14.
- More geneal model BHPS3 use the setup of CT14HERA2.



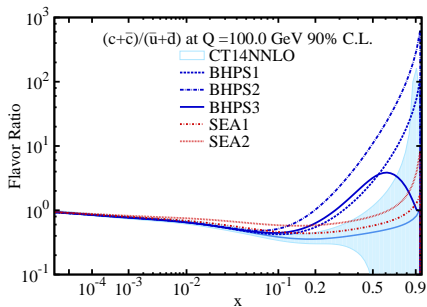
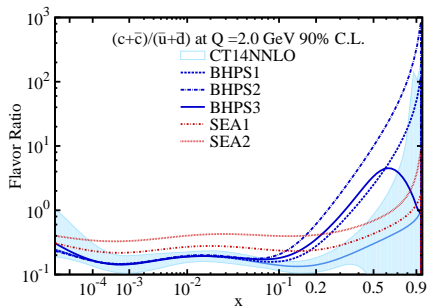
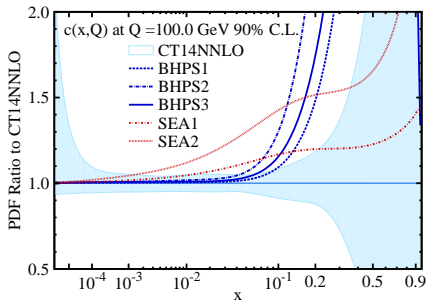
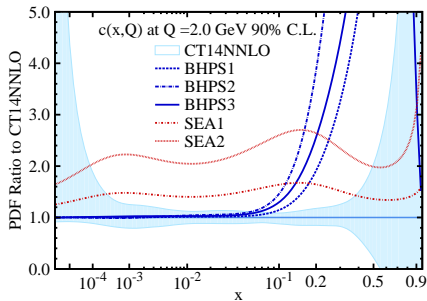
For 90% C.L.,

$\langle x \rangle_{IC} \lesssim 0.021$  for CT14 BHPS,

$\langle x \rangle_{IC} \lesssim 0.024$  for CT14HERA2 BHPS,

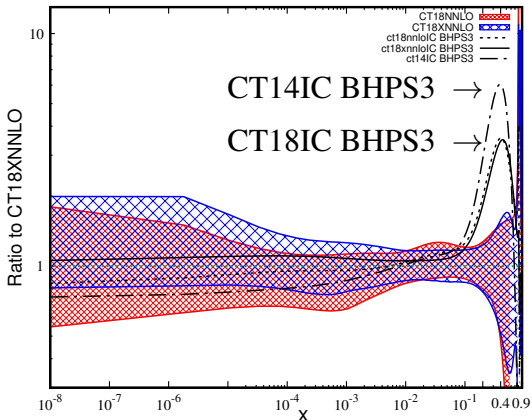
$\langle x \rangle_{IC} \lesssim 0.016$  for CT14 and CT14HERA2 SEA.

# Impact of IC on the PDFs and their ratios



# CT18IC PDFs

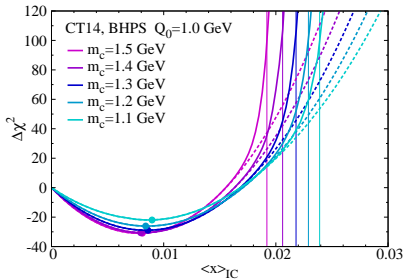
Charm quark fraction  $(c+cb)/(ub+db)$   $Q = 2 \text{ GeV}$



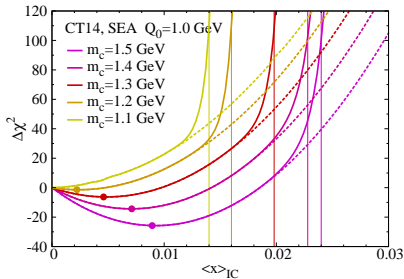
With the inclusion of LHC RUN II data, the CT18IC BHPS 3 shows lower Brodsky peak.

# Dependence of Fit on the Charm-Quark Mass

The combined HERA charm production and inclusive DIS data play an important role in the description of the goodness of fit.  $m_c$  is a key input scale.



BHPS model: the position of the  $\chi^2$  minimum is relatively stable as  $m_c$  is varied, while the upper limit on the amount of IC decreases to 1.7%. BHPS model is not dramatically affected by variations of  $m_c$

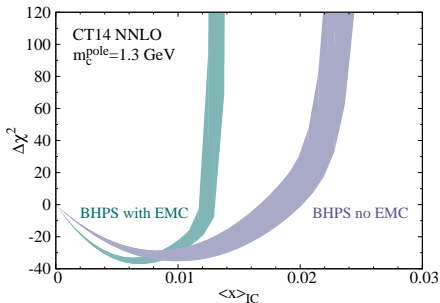


SEA model: limits on the amount of IC allowable are shifted towards higher values.  $u_{bar}$  and  $d_{bar}$  are well constrained by data (vector boson production in  $pp$  and  $p_{bar}p$ ) in the intermediate/small  $x$  region, and cannot change too much

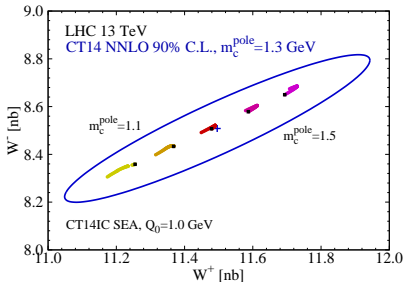
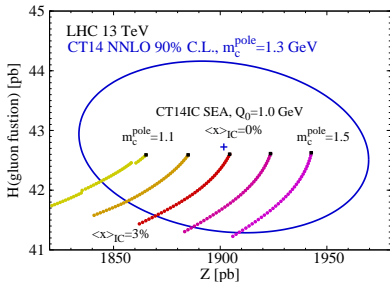
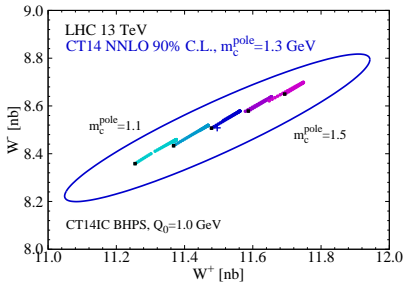
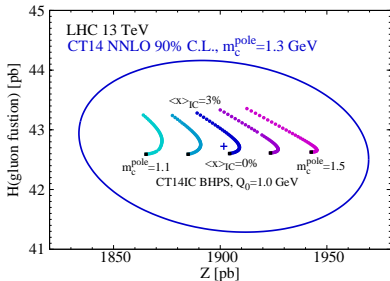
# European Muon Collaboration (EMC)

Semi-inclusive dimuon and trimuon production in DIS on an iron target

- Excess in a few high- $x$  bins of the  $F_{2c}(x, Q)$
- No systematic error.
- Analysis done at the leading order of QCD.
- Tension with various inclusive and semi-inclusive DIS data.

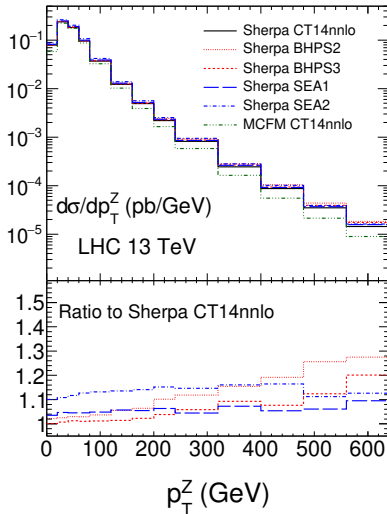
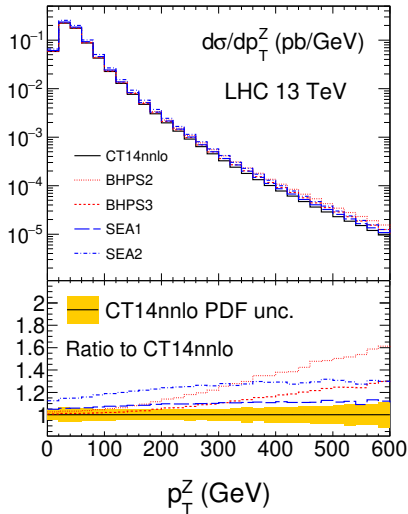


# LHC: NNLO Total inclusive electroweak boson production cross sections $\sigma_{tot}(pp \rightarrow VX)$



# LHC Searches for Intrinsic Charm

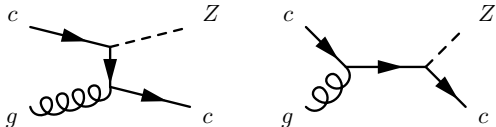
Z+c NLO computation with various models, without (left) and with parton shower (right)



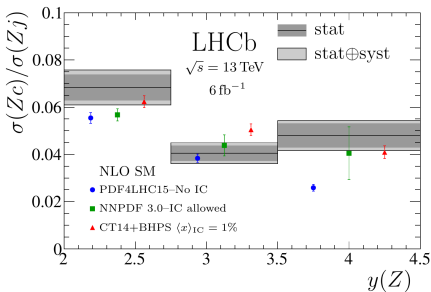
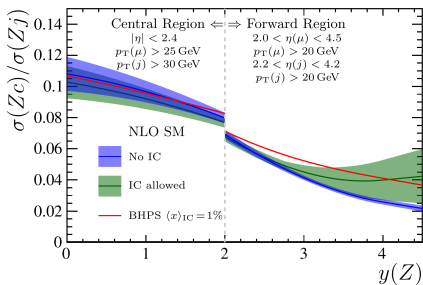


# LHCb Z+c

LHCb provide measurement of Z+c at 13 TeV with  $6 \text{ fb}^{-1}$  for  $2.0 < y(Z) < 4.5$ . (Phys.Rev.Lett. 128 (2022) 8, 082001 [arXiv: 2109.08084])



CT14IC BHPS3 with  $\langle x \rangle_{IC} = 1\%$  can explain the excess in the high  $y(Z)$  bin in LHCb Z+c data.



# EIC, charm production

Orders-of-magnitude more events for some IC models

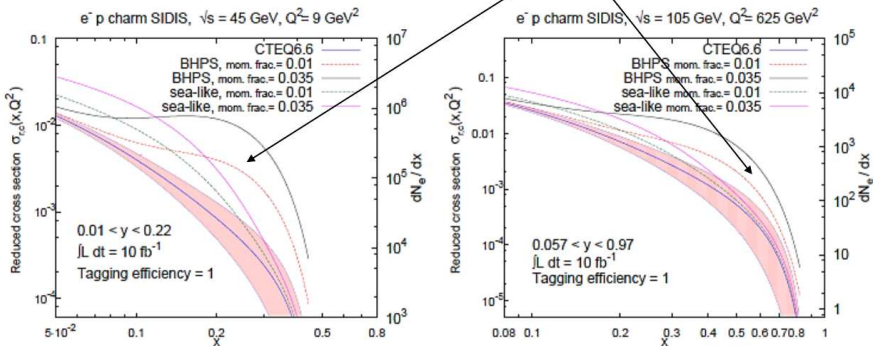


Figure 1.20. Charm contribution to the reduced NC  $e^-p$  DIS cross section at  $\sqrt{s} = 45$  and 105 GeV. For each IC model, curves for charm momentum fractions of 1% and 3.5% are shown. For comparison we display the number of events  $dN_e/dx$  for  $10 \text{ fb}^{-1}$ , assuming perfect charm tagging efficiency.

[Guzzi, Nadolsky, Olness, in arXiv:1108.1713;  
see also T. Hobbs, arXiv:1707.06711]

# Summary

- Intrinsic charm was estimated in the framework of CT14 and CT18 by using valence-like BHPS model and sea-like model.
- Best fit of CT14IC with  $\langle x \rangle_{IC} \sim 1\%$ , the CT14IC BHPS 3 PDF, was presented which is not sensitive to the choice of  $m_c$ .
- The LHCb  $Z+c$  data has good agreement with CT14IC in high  $y(z)$  bin.

Thank you  
for your attention