Three-loop QCD corrections to quarkonium electroweak decays

arXiv:2207.14259 and arXiv:2208.04302

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Outline

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- Matching the Short Distance Coefficients
- Analytic Anomalous Dimensions
- Phenomenology
- B_c case
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Motivation

Motivation

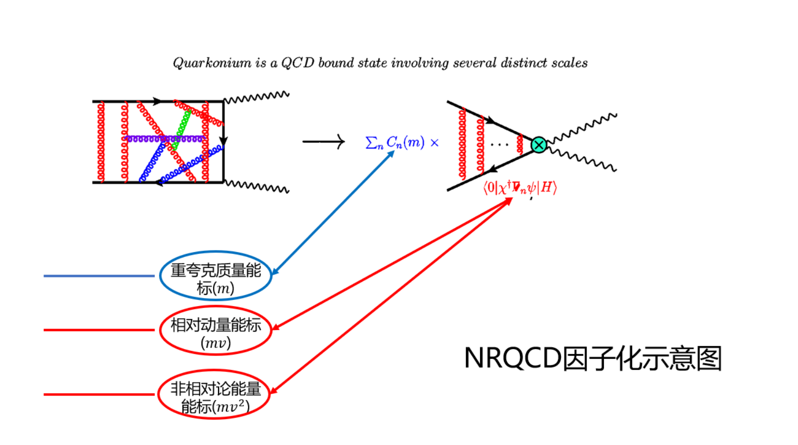
- Quarkonium leptonic decays are fundamental processes in high energy physics experiments.
- Theoretically, It plays an important role of probing the decay constant, which is a basic nonperturbative parameter.
- Previous works: Vector quakonium leptonic decay
  - Tree: Van Royen, Weisskopf, Nuovo Cim, 1967.
  - One loop: Barbieri, R. Gatto, et al., PLB1975; Celmaster, PRD1979.
  - Two loops: Beneke, Signer, Smirnov, PRL1998; Czarnecki, Melniko, PRL1998; Kniehl, Onishchenko, et al., PLB2006; Egner, Fael, et al., PRD2021.
  - Three loops: Marquard, Piclum, et al., NPB2006, PLB2009, PRD2014; Beneke, Kiyo, et al., PRL2014 without singlet and charm mass effect.

  B_c meson:
  - One loop: Braaten, Fleming, PRD1995.
  - Two loops: Onishchenko, Veretin, EPJC2007; Chen, Qiao, PLB2015

In our work, we calculate the complete three-loop correction for the Υ decay constant with massive charm and singlet/indirect contributions, and new three-loop correction to the B_c decay constant.

Short Distance Coefficients (SDCs) calculation

Fractorization of the decay constants



Fractorization of the decay constants

$$\Gamma(V \to l^+ l^-) = \frac{4\pi\alpha^2}{3M_V}|f_V|^2 \quad \alpha \text{ is QED fine structure constant}$$

The leptonic decay constants f_V for vector quarkonia $V = J/\psi, \Upsilon$ are given by

$$\langle 0|\mathcal{J}^\mu_{\mathrm{EM}}|V(\boldsymbol{\epsilon})\rangle = M_V f_V \epsilon^\mu_V, \quad \mathcal{J}^\mu_{\mathrm{EM}} = \sum_f e_f \bar{\Psi}_f \gamma^\mu \Psi_f$$

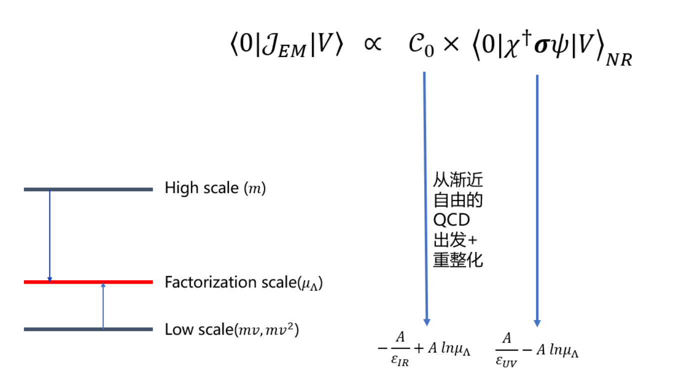
According to the NRQCD factorization,

$$\langle 0|\mathcal{J}^i_{\mathrm{EM}}|V(\epsilon)\rangle = \sqrt{2M_V} e_Q \mathcal{C}_0 \langle 0|\chi^\dagger \sigma^i \psi|V(\epsilon)\rangle_{\mathrm{NR}} + \mathcal{O}\left(v^2\right)$$

The SDCs can be obtained by matching the on-shell quark-antiquark vertex function

$$\boxed{Z_2\Gamma = \sqrt{2M}\mathcal{C}_0(\mu_\Lambda)\tilde{Z}_2\tilde{Z}^{-1}(\mu_\Lambda)\tilde{\Gamma} + \mathcal{O}\left(v^2\right)}$$

Fractorization of the decay constants



$$\tilde{Z} \equiv 1 + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \tilde{Z}^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \tilde{Z}^{(3)} + \mathcal{O}\left(\alpha_s^4\right),$$

SDCs

$$\gamma \equiv \frac{\mathrm{d}\ln \tilde{Z}}{\mathrm{d}\ln \mu_\Lambda^2} \equiv \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \gamma^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \gamma^{(3)}(\mu_\Lambda) + \mathcal{O}\left(\alpha_s^4\right).$$

$$\begin{aligned}
&\mathcal{C}_0\left(\frac{\mu_R}{m_Q}, \frac{\mu_\Lambda}{m_Q}, x\right) = \\
&1 + \frac{\alpha_s(\mu_R)}{\pi}\mathcal{C}^{(1)}(x) + \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^2\left[\mathcal{C}^{(1)}\frac{\beta_0}{4}\ln\frac{\mu_R^2}{m_Q^2} + \gamma^{(2)}\ln\frac{\mu_\Lambda^2}{m_Q^2} + \mathcal{C}^{(2)}(x)\right] \\
&+ \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^3\left\{\frac{\mathcal{C}^{(1)}}{16}\beta_0^2\ln^2\frac{\mu_R^2}{m_Q^2} + \left[\frac{\mathcal{C}^{(1)}}{16}\beta_1 + \mathcal{C}^{(2)}(x)\frac{\beta_0}{2}\right]\ln\frac{\mu_R^2}{m_Q^2}\right. \\
&+ \gamma^{(2)}\frac{\beta_0}{2}\ln\frac{\mu_\Lambda^2}{m_Q^2}\ln\frac{\mu_R^2}{m_Q^2} + \frac{1}{4}\left[2\frac{\mathrm{d}\gamma^{(3)}(\mu_\Lambda)}{\mathrm{d}\ln\mu_\Lambda^2} - \beta_0\gamma^{(2)}\right]\ln^2\frac{\mu_\Lambda^2}{m_Q^2} \\
&\left.+ \left[\mathcal{C}^{(1)}\gamma^{(2)} + \gamma^{(3)}(m_Q)\right]\ln\frac{\mu_\Lambda^2}{m_Q^2} + \mathcal{C}^{(3)}(x)\right\} + \mathcal{O}\left(\alpha_s^4\right).
\end{aligned}$$

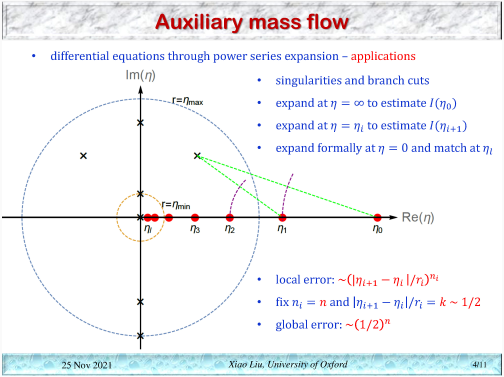
$$x = m_M/m_Q,$$

Tool chain

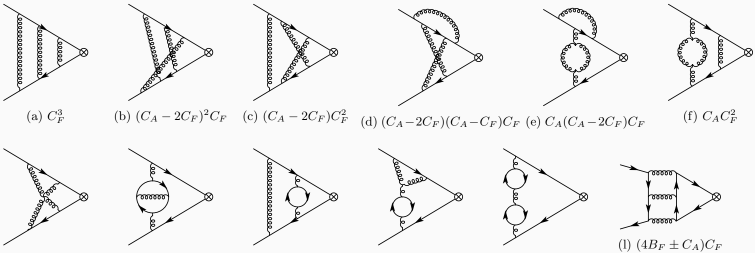
The more than 300 Feynman diagrams are generated by the packages `QGraf`/`FeynArts` for crosscheck. `FormLink`/`FeynCalc` are then utilized to deal with Dirac and color matrices. After applying partial fraction by `Apart` and IBP reduction by `FIRE`, we get roughly 300 master integrals.

The master integrals are evaluated by the auxiliary mass flow method implemented as the package `AMFlow` [arXiv: 1711.09572, 2201.11669].

See Yan-Qing Ma's talk



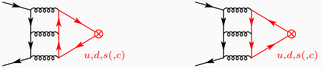
Feynman diagrams



(a) C_F^3 (b) $(C_A - 2C_F)^2 C_F$ (c) $(C_A - 2C_F) C_F^2$ (d) $(C_A - 2C_F)(C_A - C_F) C_F$ (e) $C_A (C_A - 2C_F) C_F$ (f) $C_A C_F^2$

(g) $C_A^2 C_F$ (h) $(C_A - 2C_F) C_F$ (i) $C_A C_F$ (j) C_F^2 (k) C_F (l) $(4B_F \pm C_A) C_F$

Figure 1: Representative diagrams of the direct channel. The color factor B_F is defined as $\sum_{bc} d^{abc} d^{ebc} = 4 B_F \delta^{ae}$ and $B_F = (N_c^2 - 4)/(4N_c)$ for $SU(N_c)$ group.



(a) $(4B_F + C_A)\, C_F$ (b) $(4B_F - C_A)\, C_F$

Figure 2: Representative diagrams of the indirect channel.

$$\langle 0 | \mathcal{J}^i_{\mathrm{EM}} | V(\epsilon) \rangle = \sqrt{2 M_V}\, e_Q \left(\mathcal{C}_{\mathrm{dir}} + \sum_{f \neq Q} \mathcal{C}_{\mathrm{ind,f}} \frac{e_f}{e_Q} \right) \langle 0 | \chi^\dagger \sigma^i \psi | V(\epsilon) \rangle_{\mathrm{NR}} + \mathcal{O}\left(v^2\right)$$

SDCs of Υ decay constants

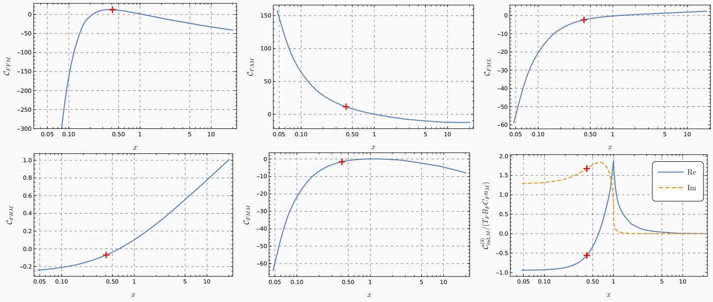
With 4 active flavors in α_s and β_i, terms that are independent of x are given by (Here we only show results from pure gluon diagrams and light-by-light direct terms. For other terms, see arxiv:2207.14259)

$$\begin{aligned}
\mathcal{C}_{\rm dir}^{(3)} =& C_F \big[C_F^2 \mathcal{C}_{FFF} + C_F C_A \mathcal{C}_{FFA} + C_A^2 \mathcal{C}_{FAA} \\
& + T_F n_L \left(C_F \mathcal{C}_{FFL} + C_A \mathcal{C}_{FAL} + T_F n_H \mathcal{C}_{FHL} + T_F n_M \mathcal{C}_{FML}(x) + T_F n_L \mathcal{C}_{FLL} \right) \\
& + T_F n_H \left(C_F \mathcal{C}_{FFH} + C_A \mathcal{C}_{FAH} + T_F n_H \mathcal{C}_{FHH} + T_F n_M \mathcal{C}_{FHM}(x) + B_F \mathcal{C}_{BFH} \right) \\
& + T_F n_M \left(C_F \mathcal{C}_{FFM}(x) + C_A \mathcal{C}_{FAM}(x) + T_F n_M \mathcal{C}_{FMM}(x) \right) \big].
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_{FFF} =& 36.494862458805925376334761898727920316641 81, \\
\mathcal{C}_{FFA} =& -188.07784165988071390579994023278476450389105, \\
\mathcal{C}_{FAA} =& -97.73497326991838634234524500457409843988 7181, \\
\mathcal{C}_{BFH} =& 2.1155782679809064984368222219139443700443356 \\
& + \mathrm{i}\, 0.49421271070067204024121810802016038115522 0487.
\end{aligned}$$

Here we confirm the results (except light by light term) from Egner, Fael, et al. arXiv:2203.11231

Terms that are dependent of x. Red crosses are the physical values with three-loop pole masses of quarks $m_Q \equiv m_b = 4.98$ GeV, $m_M \equiv m_c = 2.04$ GeV(RunDec from $m_c(m_c) = 1.28$GeV, $m_b(m_b) = 4.18$GeV):



Anomalous Dimensions of NRQCD vector current

Renormalization constant of NRQCD operators and anomalous dimensions

Thanks to the extremely high precision of the results generated by `AMFlow`, one can reconstruct the analytic expressions for the non-renormalized poles of the SDCs to infer the renormalization constant of the NRQCD operator and corresponding anomalous dimension.

$$\tilde{Z} \equiv 1 + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \tilde{Z}^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \tilde{Z}^{(3)} + \mathcal{O}\left(\alpha_s^4\right),$$

$$\gamma \equiv \frac{\mathrm{d} \ln \tilde{Z}}{\mathrm{d} \ln \mu_\Lambda^2} \equiv \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \gamma^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \gamma^{(3)}(\mu_\Lambda) + \mathcal{O}\left(\alpha_s^4\right).$$

We implement the Thiele's interpolation formula to reconstruct rational functions and use PSLQ algorithm to speculate transcendental functions.

Renormalization constant of NRQCD operators and anomalous dimensions

$$
\begin{aligned}
\tilde{Z}_v =& 1 + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \frac{C_F \pi^2}{\epsilon}\left(\frac{1}{12}C_F + \frac{1}{8}C_A\right) + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 C_F\pi^2 \\
& \times \left\{ C_F^2\left[\frac{5}{144\epsilon^2} + \left(\frac{43}{144} - \frac{1}{2}\ln 2 + \frac{5}{48}\ln\frac{\mu_\Lambda^2}{m_Q^2}\right)\frac{1}{\epsilon}\right]\right. \\
& + C_F C_A\left[\frac{1}{864\epsilon^2} + \left(\frac{113}{324} + \frac{1}{4}\ln 2 + \frac{5}{32}\ln\frac{\mu_\Lambda^2}{m_Q^2}\right)\frac{1}{\epsilon}\right] \\
& + C_A^2\left[-\frac{1}{16\epsilon^2} + \left(\frac{2}{27} + \frac{1}{4}\ln 2 + \frac{1}{24}\ln\frac{\mu_\Lambda^2}{m_Q^2}\right)\frac{1}{\epsilon}\right] \\
& + T_F n_L\left[C_F\left(\frac{1}{54\epsilon^2} - \frac{25}{324\epsilon}\right) + C_A\left(\frac{1}{36\epsilon^2} - \frac{37}{432\epsilon}\right)\right] \\
& \left. + T_F n_H \frac{C_F}{60\epsilon} + T_F n_M \frac{1}{\epsilon}\left[\frac{C_F m_Q^2}{60 m_M^2} - \left(\frac{C_F}{18} + \frac{C_A}{12}\right)\ln\frac{\mu_\Lambda^2}{m_M^2}\right]\right\} \\
& + \mathcal{O}\left(\alpha_s^4\right).
\end{aligned}
$$

Frame Title

$$\gamma \equiv \frac{\mathrm{d}\ln \tilde{Z}}{\mathrm{d}\ln \mu_\Lambda^2} \equiv \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \gamma^{(2)} + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \gamma^{(3)}(x, \mu_\Lambda) + \mathcal{O}\left(\alpha_s^4\right)$$

$$x = \frac{m_c}{m_b}$$

$$\begin{aligned}
\gamma_v^{(2)} =& -3\pi^2 C_F \left(\frac{1}{18} C_F + \frac{1}{12} C_A\right), \\
\gamma_v^{(3)}(\mu_\Lambda) =& -3\pi^2 C_F \Bigg\{ \left(\frac{43}{144} - \frac{1}{2}\ln 2\right) C_F^2 + \left(\frac{113}{324} + \frac{1}{4}\ln 2\right) C_F C_A + \left(\frac{2}{27} + \frac{1}{4}\ln 2\right) C_A^2 \\
&+ T_F n_L \left(-\frac{25}{324} C_F - \frac{37}{432} C_A\right) + \frac{1}{60} T_F n_H C_F \\
&+ T_F n_M \left[\frac{C_F}{60x^2} + \left(\frac{1}{18} C_F + \frac{1}{12} C_A\right) \ln x^2\right] + \ln\frac{\mu_\Lambda^2}{m_Q^2} \Bigg[\frac{5}{48} C_F^2 + \frac{5}{32} C_F C_A \\
&+ \frac{1}{24} C_A^2 - T_F n_M \left(\frac{1}{18} C_F + \frac{1}{12} C_A\right)\Bigg]\Bigg\}.
\end{aligned}$$

Phenomenology

Potential model

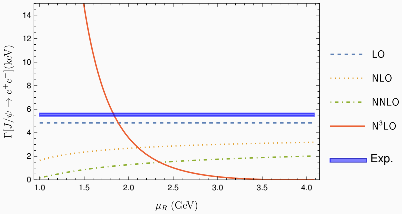
Potential Model	Cornell	Lattice	B-T	Coul. + power
$\lvert R_\Upsilon(0)\rvert^2\,(\mathrm{GeV}^3)$	14.05	5.05613	6.477	$3.909 \sim 9.181$
$\lvert R_{J/\psi}(0)\rvert^2\,(\mathrm{GeV}^3)$	1.454	1.1184	0.810	$0.610 \sim 1.850$
Potential Model	Screened	Power Law	Log	Modified NR
$\lvert R_\Upsilon(0)\rvert^2\,(\mathrm{GeV}^3)$	8.72	4.591	4.916	11.4185
$\lvert R_{J/\psi}(0)\rvert^2\,(\mathrm{GeV}^3)$	1.19	0.999	0.815	1.9767
Potential Model	Bohr	Coulomb	pNRQCD	Semi-relativistic
$\lvert R_\Upsilon(0)\rvert^2\,(\mathrm{GeV}^3)$	$2.1094^{+4.5371}_{-1.2913}$	4.221	3.092	6.143
$\lvert R_{J/\psi}(0)\rvert^2\,(\mathrm{GeV}^3)$	$0.1423^{+0.5007}_{-0.0860}$	0.303	0.421	0.478

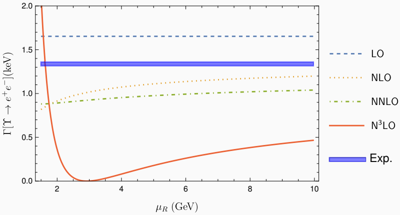
Table 1: The values of $R_V(0)$ estimated in different theoretical approaches.

Leptonic decay width of J/ψ and Υ

$\Gamma(\text{keV})$ \ V	LO	NLO	NNLO	N^{3}LO			PDG
				Direct ($m_M = 0$)	Direct ($m_M \neq 0$)	Total	
Υ	1.6529	$1.1095^{+0.0888}_{-0.2922}$	$0.9750^{+0.0642}_{-0.0942}$	$0.1948^{+1.5900}_{-0.1948}$	$0.1763^{+1.9577}_{-0.1763}$	$0.1764^{+1.9560}_{-0.1764}$	1.340 ± 0.018
J/ψ	4.8392	$2.6999^{+0.4925}_{-1.0391}$	$1.3138^{+0.7094}_{-1.1444}$	$3.2219^{+123.4838}_{-3.2219}$			5.53 ± 0.10

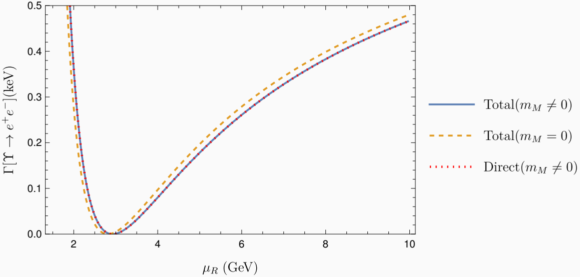
Table 2: Decay width for J/ψ and Υ. The central values of predictions are obtained by setting $\mu_R = m_Q$, while the errors are estimated by varying μ_R from μ_Λ to $2m_Q$.





Charm mass and indirect term effect in Υ decay

Our complete result is ploted in solid line. The dashed line treats charm quark as massless. The dotted line takes out the contribution from the indirect diagrams. It's shown that indirect channel only has invisible effect on the plot, while charm mass leads to visible small correction.



B_c weak decay

Fractorization of the decay constants

$$\Gamma\left(B_c \to l^+\nu_l\right) = \frac{1}{8\pi}|V_{bc}|^2 G_F^2 M_{B_c} m_l^2 \left(1 - \frac{m_l^2}{M_{B_c}^2}\right)^2 f_{B_c}^2$$

The leptonic decay constants f_{B_c} for pseudoscalar quarkonium B_c are given by

$$\langle 0|\bar{b}\gamma^\mu\gamma_5 c|B_c^+(P)\rangle = iP^\mu f_{B_c}$$

According to the NRQCD factorization,

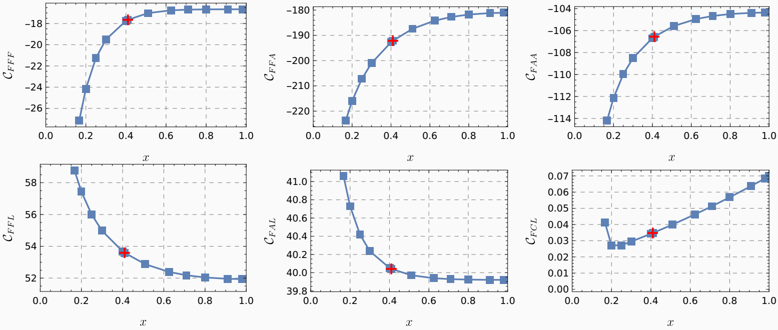
$$\langle 0|\bar{b}\gamma^\mu\gamma_5 c|B_c^+\rangle = \sqrt{2M_{B_c}}\mathcal{C}_0(x)\langle 0|\chi_b^\dagger\psi_c|B_c^+\rangle_{\rm NR} + \mathcal{O}\big(v^2\big) \quad x = \frac{m_c}{m_b}$$

The SDCs can be obtained by matching the on-shell quark-antiquark vertex function

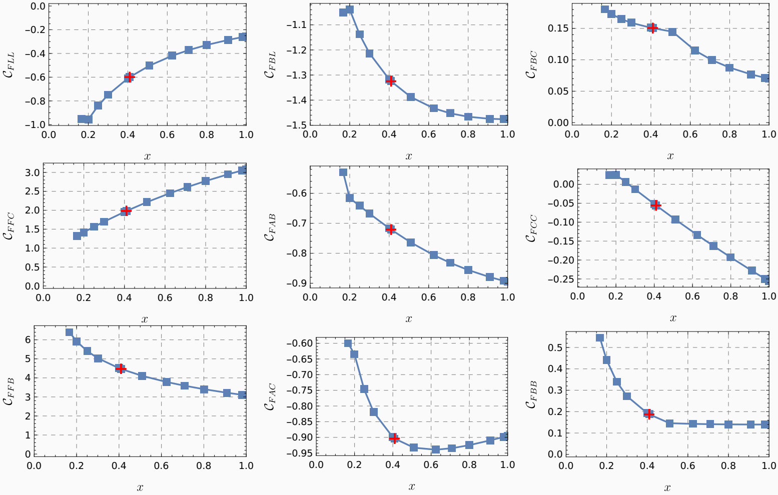
$$\boxed{\sqrt{Z_{2,1}Z_{2,2}}\Gamma = \sqrt{2M}\mathcal{C}_0(\mu_\Lambda)\sqrt{\tilde{Z}_{2,1}\tilde{Z}_{2,2}}\tilde{Z}^{-1}(\mu_\Lambda)\tilde{\Gamma} + \mathcal{O}\left(v^2\right)}$$

SDCs of B_c decay constant

$$\begin{aligned}\mathcal{C}_0(x) =& C_F \Big[C_F^2 \mathcal{C}_{FFF} + C_F C_A \mathcal{C}_{FFA} + C_A^2 \mathcal{C}_{FAA} \\ &+ T_F n_L \left(C_F \mathcal{C}_{FFL} + C_A \mathcal{C}_{FAL} + T_F n_H \mathcal{C}_{FHL} + T_F n_M \mathcal{C}_{FML} + T_F n_L \mathcal{C}_{FLL} \right) \\ &+ T_F n_H \left(C_F \mathcal{C}_{FFH} + C_A \mathcal{C}_{FAH} + T_F n_H \mathcal{C}_{FHH} + T_F n_M \mathcal{C}_{FHM} + B_F \mathcal{C}_{BFH} \right) \\ &+ T_F n_M \left(C_F \mathcal{C}_{FFM} + C_A \mathcal{C}_{FAM} + T_F n_M \mathcal{C}_{FMM} \right) \Big].\end{aligned}$$



SDCs of B_c decay constant



SDCs of B_c decay constant

$m_b = 4.98$ GeV, $m_c = 2.04$ GeV, with 3 active flavors in α_s and β_i.

$\mathcal{C}_{FFF} = -17.6481252546417535 39131,$ $\mathcal{C}_{FFA} = -192.1517982243479087 47121,$

$\mathcal{C}_{FAA} = -106.5570007402788585 9242,$ $\mathcal{C}_{FFL} = 53.5908823803209988 398528,$

$\mathcal{C}_{FAL} = 40.0419439556257077 28391,$ $\mathcal{C}_{FCL} = -0.5995565958860492 0607755,$

$\mathcal{C}_{FBL} = -0.0556736050404740 8860700,$ $\mathcal{C}_{FLL} = -1.3248436752241309 9859707,$

$\mathcal{C}_{FBC} = 0.1504703734097762 0584792,$ $\mathcal{C}_{FFC} = 4.4689270077646697 01991,$

$\mathcal{C}_{FAC} = -0.9039122429495440 874057,$ $\mathcal{C}_{FCC} = 0.1873821757342391 0690057,$

$\mathcal{C}_{FFB} = 1.9799127987973044 694123,$ $\mathcal{C}_{FAB} = -0.7210547630289466 943049,$

$\mathcal{C}_{FBB} = 0.0347491174339149 0676344.$

Renormalization constant of the NRQCD current for B_c

$$
\begin{aligned}
\tilde{Z}_p =& 1 + \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^2 \frac{\pi^2 C_F}{\epsilon} \left(\frac{3+z}{8(1+z)} C_F + \frac{1}{8} C_A\right) \\
&+ \left(\frac{\alpha_s(\mu_\Lambda)}{\pi}\right)^3 \pi^2 C_F \left\{ \frac{1}{\epsilon^2} \left(\frac{-1+6z}{72\,(1+z)} C_F^2 - \frac{5}{48\,(1+z)} C_F C_A - \frac{1}{16} C_A^2\right)\right. \\
&+ \frac{1}{\epsilon}\left[C_F^2 \left(\frac{29+38z}{72\,(1+z)} - \frac{7}{12}\ln 2 - \frac{2-3x-22x^2-3x^3+2x^4}{12\,(1-x)\,(1+x)^3}\ln x \right.\right. \\
&\quad \left. + \frac{1}{12}\ln(1+z) + \frac{-1+6z}{24\,(1+z)}\ln\frac{\mu_\Lambda^2}{m_b m_c}\right) + C_F C_A \left(\frac{93+52z}{216\,(1+z)} + \frac{1}{8}\ln 2\right. \\
&\quad \left. - \frac{5+2x+5x^2}{48\,(1-x)\,(1+x)}\ln x + \frac{1}{8}\ln(1+z) + \frac{18+11z}{48\,(1+z)}\ln\frac{\mu_\Lambda^2}{m_b m_c}\right) \\
&\quad \left. + C_A^2 \left(\frac{2}{27} + \frac{5}{24}\ln 2 + \frac{1}{24}\ln(1+z) + \frac{1}{24}\ln\frac{\mu_\Lambda^2}{m_b m_c}\right)\right] \\
&+ T_f n_L \left[\left(\frac{3+z}{36\,(1+z)}\frac{1}{\epsilon^2} - \frac{15+7z}{108\,(1+z)}\frac{1}{\epsilon}\right) C_F + \left(\frac{1}{36}\frac{1}{\epsilon^2} - \frac{37}{432}\frac{1}{\epsilon}\right) C_A\right] \\
&\left. + T_f n_{H,b} \left(\frac{1}{15\,(1+1/x)^2}\frac{1}{\epsilon}\right) C_F + T_f n_{H,c}\left(\frac{1}{15\,(1+x)^2}\frac{1}{\epsilon}\right) C_F \right\}
\end{aligned}
$$

where $x = \frac{m_c}{m_b}$, $z = \frac{1}{2}\left(x + \frac{1}{x}\right)$. For corresponding anomalous dimension.

Anomalous dimension of the NRQCD current for B_c

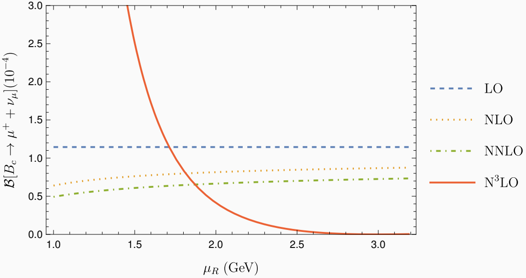
$$\gamma^{(2)}(x) = -\pi^2 C_F \left(\frac{3+z}{4\,(1+z)} C_F + \frac{1}{4} C_A \right),$$

$$\begin{aligned}
\gamma^{(3)}\left(x, \frac{\mu_\Lambda^2}{m_b m_c}\right) = -\pi^2 C_F \Bigg[& \left(\frac{29+38z}{24\,(1+z)} - \frac{7}{4}\ln 2 - \frac{2-3x-22x^2-3x^3+2x^4}{4\,(1-x)\,(1+x)^3}\ln x + \frac{1}{4}\ln(1+z) \right. \\
& \left. + \frac{-1+6z}{8\,(1+z)} \ln \frac{\mu_\Lambda^2}{m_b m_c} \right) C_F^2 + \left(\frac{93+52z}{72\,(1+z)} + \frac{3}{8}\ln 2 - \frac{5+2x+5x^2}{16\,(1-x)\,(1+x)}\ln x + \frac{3}{8}\ln(1+z) \right. \\
& \left. + \frac{18+11z}{16\,(1+z)} \ln \frac{\mu_\Lambda^2}{m_b m_c} \right) C_F C_A + \left(\frac{2}{9} + \frac{5}{8}\ln 2 + \frac{1}{8}\ln(1+z) + \frac{1}{8}\ln \frac{\mu_\Lambda^2}{m_b m_c} \right) C_A^2 \\
& - T_F n_l \left(\frac{15+7z}{36\,(1+z)} C_F + \frac{37}{144} C_A \right) + T_F n_b \frac{1}{5\,(1+1/x)^2} C_F + T_F n_c \frac{1}{5\,(1+x)^2} C_F \Bigg].
\end{aligned}$$

where $x = \frac{m_c}{m_b}$, $z = \frac{1}{2}\left(x + \frac{1}{x}\right)$.

	LO	NLO	NNLO	$\mathrm{N^3LO}$
leptonic width($\times 10^{-7}$ eV)	1.4776	$0.97314^{+0.15579}_{-0.14772}$	$0.77476^{+0.17194}_{-0.13966}$	$3.9847^{+32.3584}_{-3.9796}$
$\mathcal{B}(B_c \to \mu^+ + \nu_\mu)$ ($\times 10^{-4}$)	1.1448	$0.75395^{+0.12070}_{-0.11445}$	$0.60025^{+0.13321}_{-0.10820}$	$3.0872^{+25.070}_{-3.0832}$

Table 3: Branching ratio of the B_c leptonic decay. The uncertainties are given by varying renormalization scale from factorization scale to $m_q = \sqrt{m_b m_c}$. Central value is chosen at $m_r = \frac{m_b m_c}{m_b + m_c}$.



Conclusions

Conclusions

- We confirm the known three loop results of SDCs of Υ, J/ψ electromagnetic leptonic decays. Furthermore, our results of indirect and charm mass effect to the SDC of Υ decay is new. While the former is invisible, the latter is obvious.
- A new term in NRQCD anomalous dimension of vector NQRCD current $\chi^\dagger \boldsymbol{\sigma} \psi$ corresponding to the intermediate flavor of quark is first obtained.
- For B_c weak decay, the three loop SDC and anomalous dimension of pseudoscalar NQRCD current $\chi_b^\dagger \psi_c$ are totally new. Different color index coefficients varies quite differently.

Conclusions

- Our three-loop results for Υ and B_c share similar patterns that N^3LO corrections heavily depend on the renormalization scale and can grow very large and spoil the convergence of the perturbative expansion. This phenomenon may draw forth a new puzzle which deserves further research. Our treatment of the phenomenology is different from Beneke, Kiyo, et al., PRL2014.

Thank you for your attention

Backup

Decoupling relation:

$$\frac{\alpha_s^{(n_L+n_M+n_H)}(\mu_R)}{\pi} = \frac{\alpha_s^{(n_L+n_M)}(\mu_R)}{\pi}$$
$$+ \left(\frac{\alpha_s^{(n_L+n_M)}(\mu_R)}{\pi}\right)^2 T_F n_H \left[\frac{1}{3}\ln\frac{\mu_\Lambda^2}{m_Q^2} + \left(\frac{1}{6}\ln^2\frac{\mu_\Lambda}{m_Q} + \frac{1}{36}\pi^2\right)\epsilon + \mathcal{O}\left(\epsilon^2\right)\right]$$
$$+ \left(\frac{\alpha_s^{(n_L+n_M)}(\mu_R)}{\pi}\right)^3 T_F n_H \Bigg[\left(\frac{1}{4}\ln\frac{\mu_\Lambda^2}{m_Q^2} + \frac{15}{16}\right) C_F + \left(\frac{5}{12}\ln\frac{\mu_\Lambda^2}{m_Q^2} - \frac{2}{9}\right) C_A$$
$$+ \frac{1}{9} T_F n_H \ln^2\frac{\mu_\Lambda}{m_Q} + \mathcal{O}(\epsilon)\Bigg] + \mathcal{O}\left(\alpha_s^4\right)$$

Backup

Phenomenology of Beneke, Kiyo, et al., PRL2014:

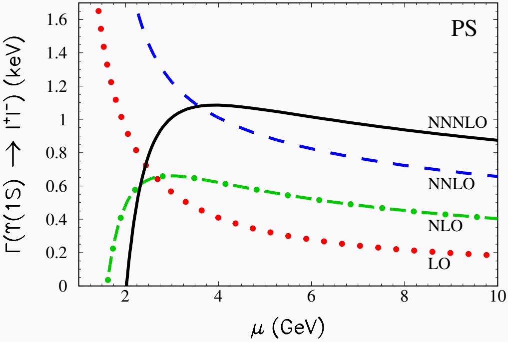


Figure 1: Caption