

# New analyses of event shapes and the determination of $\alpha_s$ in $e^+e^-$ annihilation

贵州民族大学

王声权

第二届微扰量子场论研讨会

2022.08.23 杭州

Based on [arXiv:2112.06212](#); [1908.00060](#); [1902.01984](#), in collaboration with  
Stanley J. Brodsky, Xing-Gang Wu, Jian-Ming Shen, and Leonardo Di Giustino

# Outline

- 一. Introduction
- 二. Principle of Maximum Conformality (PMC)
- 三. Event shape observables and a novel method for the determination of  $\alpha_s$  at LEP
- 四. Event shape observables from LEP to CEPC

# 一. Introduction

High precision QCD theoretical calculation is important, it has developed rapidly in recent years.

$$\rho(\mu_R) = r_0 \alpha_s(\mu_R) \left[ 1 + \sum_{k=1}^{\infty} r_k \left( \frac{Q}{\mu_R} \right) \frac{\alpha_s^k(\mu_R)}{\pi^k} \right]$$

为消除红外发散或紫外发散  
引入重整化理论

$$g_0 = Z_g \mu^{\varepsilon/2} g \quad (\varepsilon=4-d)$$

正规化、重整化、能标设定

准确预言具同等重要性

成为当前理论中重要系统误差之一，  
极大地影响微扰论计算精度及预言能力

计算到无穷阶的微扰论预言需与人为引入  
的参数无关

- - 重整化群不变性

物理量

$$\frac{\partial \rho}{\partial \mu_R} \equiv 0; \frac{\partial \rho}{\partial R} \equiv 0$$

# 一. Introduction

## 如何解决能标问题

### Brodsky-Lepage-Mackenzie method (BLM)

引用1214次

PHYSICAL REVIEW D	VOLUME 28, NUMBER 1	1 JULY 1983
On the elimination of scale ambiguities in perturbative quantum chromodynamics		
Stanley J. Brodsky <i>Institute for Advanced Study, Princeton, New Jersey 08540 and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305*</i>		
G. Peter Lepage <i>Institute for Advanced Study, Princeton, New Jersey 08540 and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853*</i>		
Paul B. Mackenzie <i>Fermilab, Batavia, Illinois 60510 (Received 23 November 1982)</i>		

源自QED观察  
引入轻子圈  
问题来自高阶如何处理

### Principle of Minimum Sensitivity (PMS)

引用1200次

PHYSICAL REVIEW D	VOLUME 23, NUMBER 12	15 JUNE 1981
Optimized perturbation theory		
P. M. Stevenson <i>Physics Department, University of Wisconsin-Madison, Madison, Wisconsin 53706 (Received 21 July 1980; revised manuscript received 17 February 1981)</i>		

源自数学处理  
引入驻点  
问题来自物理

### RG-improved effective coupling method (FAC)

引用553次

Volume 95B, number 1	PHYSICS LETTERS	8 September 1980
RENORMALIZATION GROUP IMPROVED PERTURBATIVE QCD		
G. GRUNBERG <sup>1</sup> <i>Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853, USA</i>		

源自实验 - 理论一致性  
引入有效耦合常数  
问题来自与微扰论理念冲突

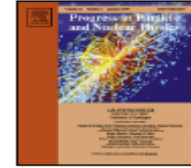
# 一. Introduction



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Progress in Particle and Nuclear Physics

journal homepage: [www.elsevier.com/locate/ppnp](http://www.elsevier.com/locate/ppnp)



Review

The renormalization scale-setting problem in QCD

Xing-Gang Wu<sup>a,\*</sup>, Stanley J. Brodsky<sup>b</sup>, Matin Mojaza<sup>b,c</sup>

<sup>a</sup> Department of Physics, Chongqing University, Chongqing 401331, PR China

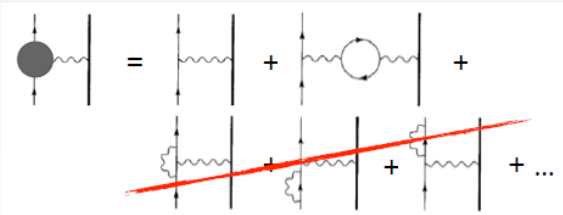
<sup>b</sup> SLAC National Accelerator Laboratory, Stanford University, CA 94039, USA

<sup>c</sup> CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230, Denmark



BLM/FAC/PMS

In the case of QED, the renormalization scale can be set unambiguously by using the Gell-Mann-Low method, which automatically sums all vacuum polarization contributions to the photon propagators to all orders.



BLM=> nf-term  
BLM method reduces in the  
Abelian limit to the  
Gell-Mann-Low method



Quantum Electrodynamics at Small Distances

M. Gell-Mann and F. E. Low

Phys. Rev. **95**, 1300 – Published 1 September 1954

# 二. principle of maximum conformality

PMC首篇正式论文

最初想法是将BLM推到无穷阶

后期发现两者在低阶等价，但PMC理念更基础

PHYSICAL REVIEW D 85, 034038 (2012)

## Scale setting using the extended renormalization group and the principle of maximum conformality: The QCD coupling constant at four loops

Stanley J. Brodsky<sup>1,\*</sup> and Xing-Gang Wu<sup>1,2,†</sup>

<sup>1</sup>*SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA*

<sup>2</sup>*Department of Physics, Chongqing University, Chongqing 401331, China*

(Received 30 November 2011; published 22 February 2012)

PRL 109, 042002 (2012)

PHYSICAL REVIEW LETTERS

week ending  
27 JULY 2012

## Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality

Stanley J. Brodsky<sup>1,\*</sup> and Xing-Gang Wu<sup>1,2,†</sup>

<sup>1</sup>*SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, California 94025, USA*

<sup>2</sup>*Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China*

(Received 29 March 2012; published 23 July 2012)

PRL 110, 192001 (2013)

PHYSICAL REVIEW LETTERS

week ending  
10 MAY 2013



## Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

Matin Mojaza<sup>\*</sup>

*CP3-Origins, Danish Institute for Advanced Studies, University of Southern Denmark, DK-5230 Odense, Denmark  
and SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*

Stanley J. Brodsky<sup>†</sup>

*SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94039, USA*

Xing-Gang Wu<sup>‡</sup>

*Department of Physics, Chongqing University, Chongqing 401331, People's Republic of China  
(Received 13 January 2013; published 10 May 2013)*

## 二. principle of maximum conformality

### PMC基本思想

$$\beta^{\mathcal{R}} = \mu_r^2 \frac{\partial}{\partial \mu_r^2} \left( \frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi} \right) = - \sum_{i=0}^{\infty} \beta_i^{\mathcal{R}} \left( \frac{\alpha_s^{\mathcal{R}}(\mu_r)}{4\pi} \right)^{i+2}$$

基于重整化群方程，利用微扰序列中的非共形 $\beta$ 项确定高能物理过程的有效强耦合常数数值，获得与重整化能标选择无关的理论预言。通过最大程度的逼近共形微扰序列，可同时获得与重整化方案无关的理论预言，符合重整化群不变性要求。

附产品：由于消除具有发散性质的重整化子项，PMC序列将自然地具有更好的微扰收敛性。该收敛性与重整化能标选择无关，因此可以将之认为是高能物理过程的内禀属性。在阿贝尔极限下，将回归QED理论中的GM-L方案。

$$[n! \beta_i^n \alpha_s^n]$$

# 二. principle of maximum conformality

Scale Setting Using the Extended Renormalization Group and the Principle of Maximum Conformality: the QCD Coupling Constant at Four Loops.

[Phys.Rev. D85 \(2012\) 034038.](#)

Eliminating the Renormalization Scale Ambiguity for Top-Pair Production Using the Principle of Maximum Conformality

[Phys.Rev.Lett. 109 \(2012\) 042002.](#)

Self-Consistency Requirements of the Renormalization Group for Setting the Renormalization Scale

[Phys.Rev. D86 \(2012\) 054018.](#)

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

[Phys.Rev.Lett. 110 \(2013\) 192001.](#)

The Renormalization Scale-Setting Problem in QCD

[Prog.Part.Nucl.Phys. 72 \(2013\) 44-98.](#)

Reanalysis of the BFKL Pomeron at the next-to-leading logarithmic accuracy

[JHEP 1310 \(2013\) 117](#)

Systematic Scale-Setting to All Orders: The Principle of Maximum Conformality and Commensurate Scale Relations

[Phys.Rev. D89 \(2014\) 014027.](#)

Renormalization Group Invariance and Optimal QCD

Renormalization Scale-Setting

[Rept.Prog.Phys. 78 \(2015\) 126201.](#)

General Properties on Applying the Principle of Minimum Sensitivity to High-order Perturbative QCD Predictions

[Phys.Rev. D91 \(2015\) , 034006.](#)

Setting the renormalization scale in perturbative QCD: Comparisons of the principle of maximum conformality with the sequential extended Brodsky-Lepage-Mackenzie approach.

[Phys.Rev. D91 \(2015\), 094028.](#)

Degeneracy Relations in QCD and the Equivalence of Two Systematic All-Orders Methods for Setting the Renormalization Scale

[Phys.Lett. B748 \(2015\) 13-18.](#)

The Generalized Scheme-Independent Crewther Relation in QCD

[Phys.Lett. B770 \(2017\) 494-499](#)

Novel All-Orders Single-Scale Approach to QCD Renormalization Scale-Setting

[Phys.Rev. D95 \(2017\) , 094006.](#)

Renormalization scheme dependence of high-order perturbative QCD predictions

[Phys.Rev. D97 \(2018\), 036024.](#)

Novel demonstration of the renormalization group invariance of the fixed-order predictions using the principle of maximum conformality and the  $\overline{C}$ -scheme coupling

[Phys.Rev. D97 \(2018\), 094030.](#)

The QCD Renormalization Group Equation and the Elimination of Fixed-Order Scheme-and-Scale Ambiguities Using the Principle of Maximum Conformality

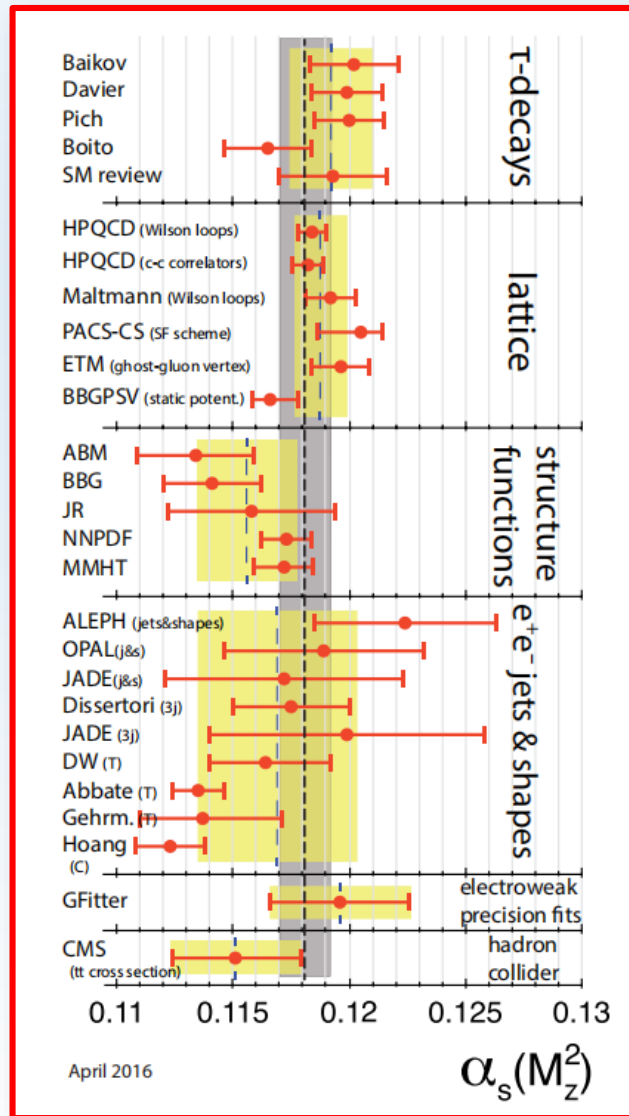
[Prog.Part.Nucl.Phys. 108 \(2019\) 103706](#)

.....

See Yan' s talk



# 三. Event shapes and extracting $\alpha_s$ at LEP



$\alpha_s$  is a free parameter in QCD.

$$\alpha_s(M_Z^2) = 0.1181 \pm 0.0011 ,$$

0.9%

[Particle Data Group],  
Phys. Rev. D98, 030001 (2018)

# 三. Event shapes and extracting $\alpha_s$ at LEP

The classic event shapes: the thrust (T), the heavy jet mass ( $M_H^2/s$ ), the wide and total jet broadenings  $B_W$  and  $B_T$ , the C-parameter (C)

$$T = \max_{\vec{n}} \left( \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \right)$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2},$$

$$\rho \equiv M_H^2/s = \max(M_1^2/s, M_2^2/s)$$

$$M_i^2/s = \frac{1}{E_{\text{vis}}^2} \left( \sum_{k \in H_i} p_k \right)^2$$

$$B_W = \max(B_1, B_2),$$
$$B_T = B_1 + B_2.$$

$$B_i = \frac{\sum_{k \in H_i} |\vec{p}_k \times \vec{n}_T|}{2 \sum_k |\vec{p}_k|}.$$

Currently, the main obstacle for achieving a precise determination of  $\alpha_s(M_Z)$  is not the lack of precise experimental data, especially at  $Z^0$  peak, but the ambiguity of theoretical predictions.

# 三. Event shapes and extracting $\alpha_s$ at LEP

The method for extracting  $\alpha_s(M_Z)$  in  $e^+e^-$  collider:

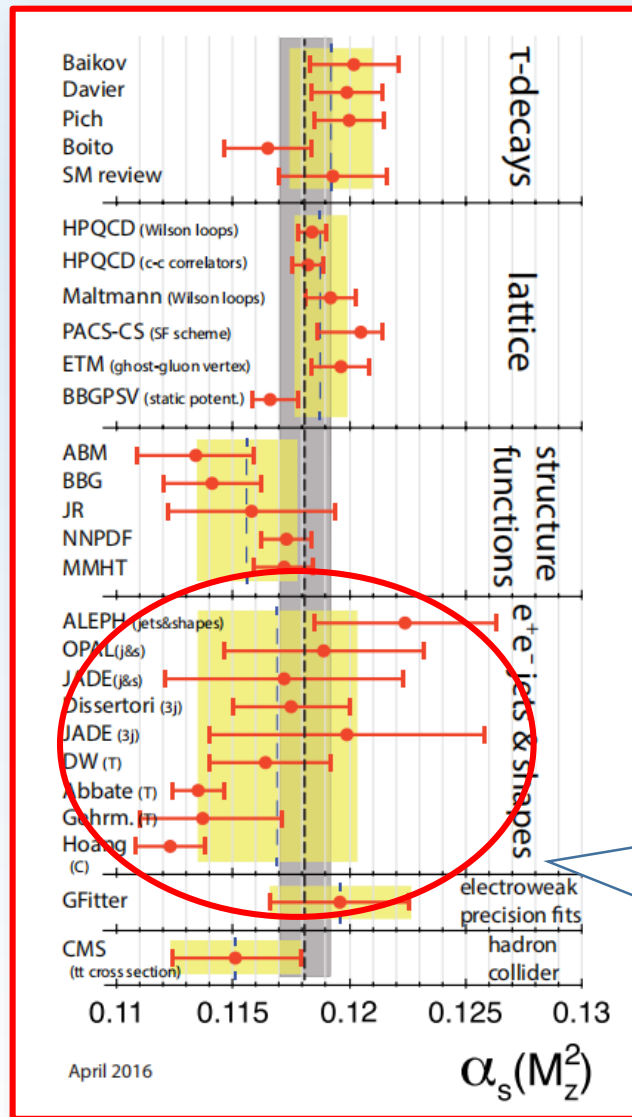
- predictions matched Monte Carlo models to correct for hadronization effects
- based on analytic calculations of non-perturbative and hadronization effects, using methods like power corrections, factorization of soft-collinear effective field theory, dispersive models and low scale QCD effective couplings

We note that there is criticism on both classes of  $\alpha_s$  extractions described above: those based on corrections of non-perturbative hadronization effects using QCD-inspired Monte Carlo generators (since the parton level of a Monte Carlo simulation is not defined in a manner equivalent to that of a fixed-order calculation), as well as studies based on non-perturbative analytic calculations, as their systematics have not yet been fully verified. In particular, quoting rather small overall experimental, hadronization and theoretical uncertainties of only 2, 5 and 9 per-mille, respectively [425,427], seems unrealistic and has neither been met nor supported by other authors or groups.

?

[Particle Data Group],  
Phys. Rev. D98, 030001 (2018)

# 三. Event shapes and extracting $\alpha_s$ at LEP



- 419. G. Dissertori *et al.*, JHEP **0908**, 036 (2009).
- 420. G. Abbiendi *et al.*, Eur. Phys. J. **C71**, 1733 (2011).
- 421. S. Bethke *et al.*, [JADE Collab.], Eur. Phys. J. **C64**, 351 (2009).
- 422. G. Dissertori *et al.*, Phys. Rev. Lett. **104**, 072002 (2010).
- 423. J. Schieck *et al.*, Eur. Phys. J. **C73**, 2332 (2013).
- 424. R.A. Davison and B.R. Webber, Eur. Phys. J. **C59**, 13 (2009).
- 425. R. Abbate *et al.*, Phys. Rev. **D83**, 074021 (2011).
- 426. T. Gehrmann *et al.*, Eur. Phys. J. **C73**, 2265 (2013).
- 427. A.H. Hoang *et al.*, Phys. Rev. **D91**, 094018 (2015).
- 428. R. Frederix *et al.*, JHEP **1011**, 050 (2010).

- The  $\alpha_s(M_Z)$  are plagued by significant **scale uncertainty**
- Some extracted  $\alpha_s(M_Z)$  are deviated from the world average
- non-self-consistent

# 三. Event shapes and extracting $\alpha_s$ at LEP

The differential distribution for a event shape:

$$\frac{1}{\sigma_h} \frac{d\sigma}{d\tau} = \bar{A}(\tau) a_s(Q) + \bar{B}(\tau) a_s^2(Q) + \mathcal{O}(a_s^3).$$

$Q = \sqrt{s}$  using  
conventional method

$$\frac{1}{\sigma_h} \frac{d\sigma}{d\tau} = \bar{A}(\tau) a_s(\mu_r^{\text{pmc}}) + \bar{B}(\tau, \mu_r)_{\text{con}} a_s^2(\mu_r^{\text{pmc}}) + \mathcal{O}(a_s^3)$$

$$\bar{B}(\tau, \mu_r)_{\text{con}} = \frac{11C_A}{4T_R} \bar{B}(\tau, \mu_r)_{n_f} + \bar{B}(\tau, \mu_r)_{\text{in}},$$

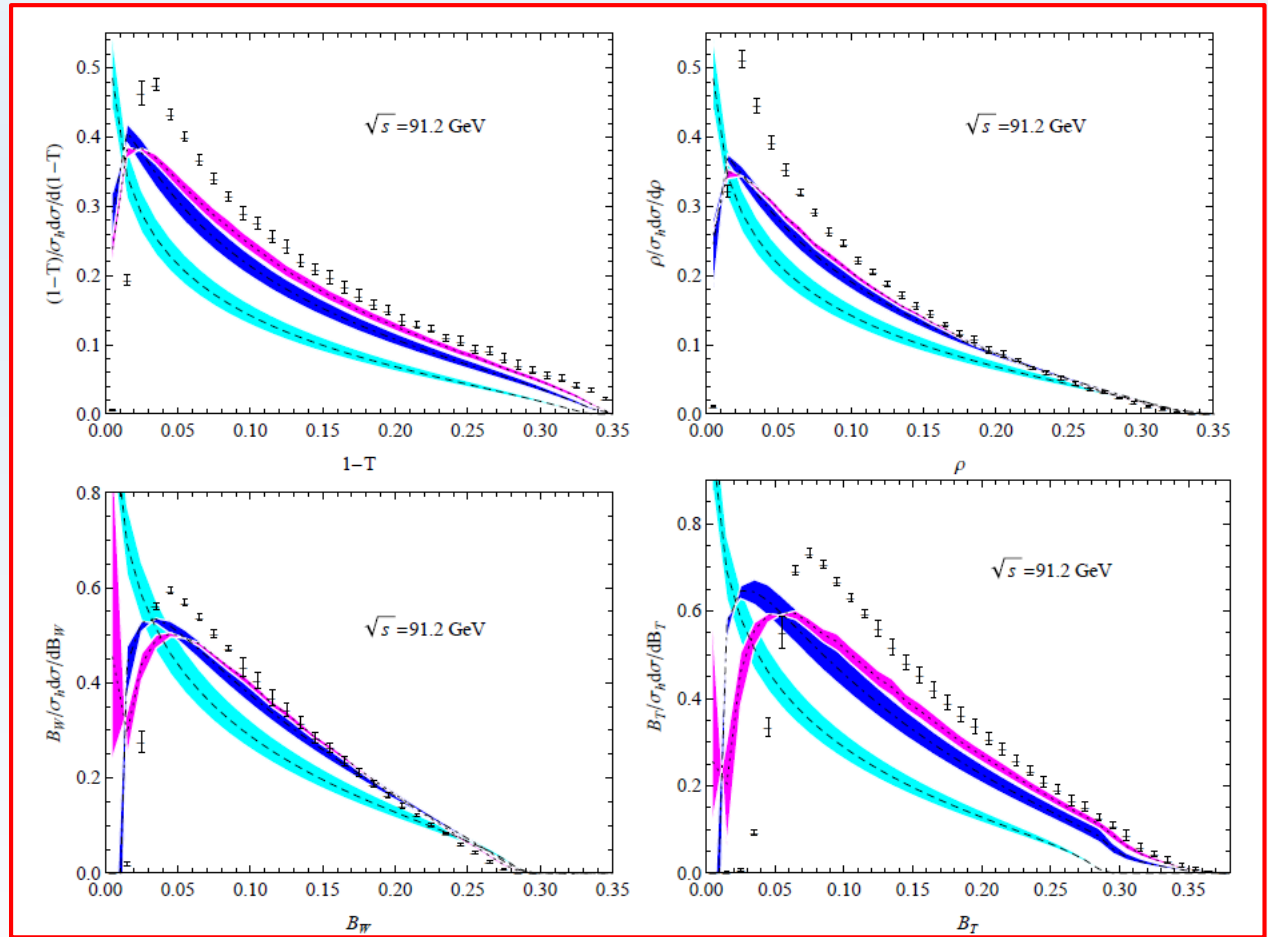
$$\mu_r^{\text{pmc}} = \mu_r \exp \left[ \frac{3\bar{B}(\tau, \mu_r)_{n_f}}{4T_R \bar{A}(\tau)} + \mathcal{O}(a_s) \right].$$

# 三. Event shapes and extracting $\alpha_s$ at LEP

Conventional results  
at 91.2 GeV

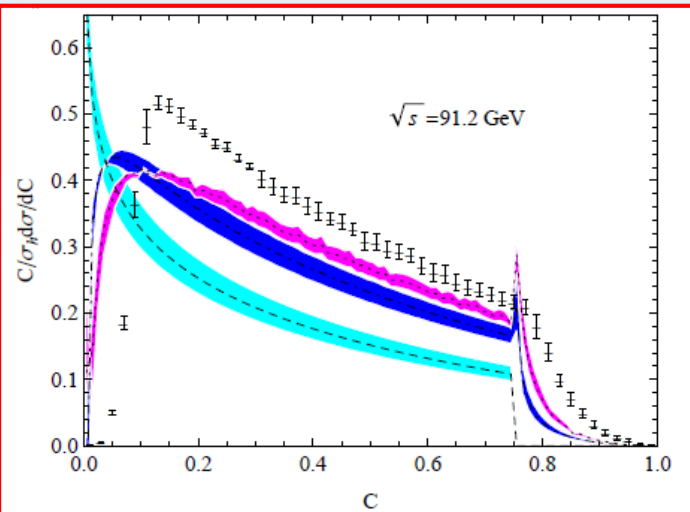
Central values are  $Q =$   
91.2 GeV , the errors  
are  $[Q/2, 2Q]$ .

Phys. Rev. Lett. 99, 132002  
JHEP 0712, 094  
Phys. Rev. Lett. 101, 162001  
JHEP 0906, 041



# 三. Event shapes and extracting $\alpha_s$ at LEP

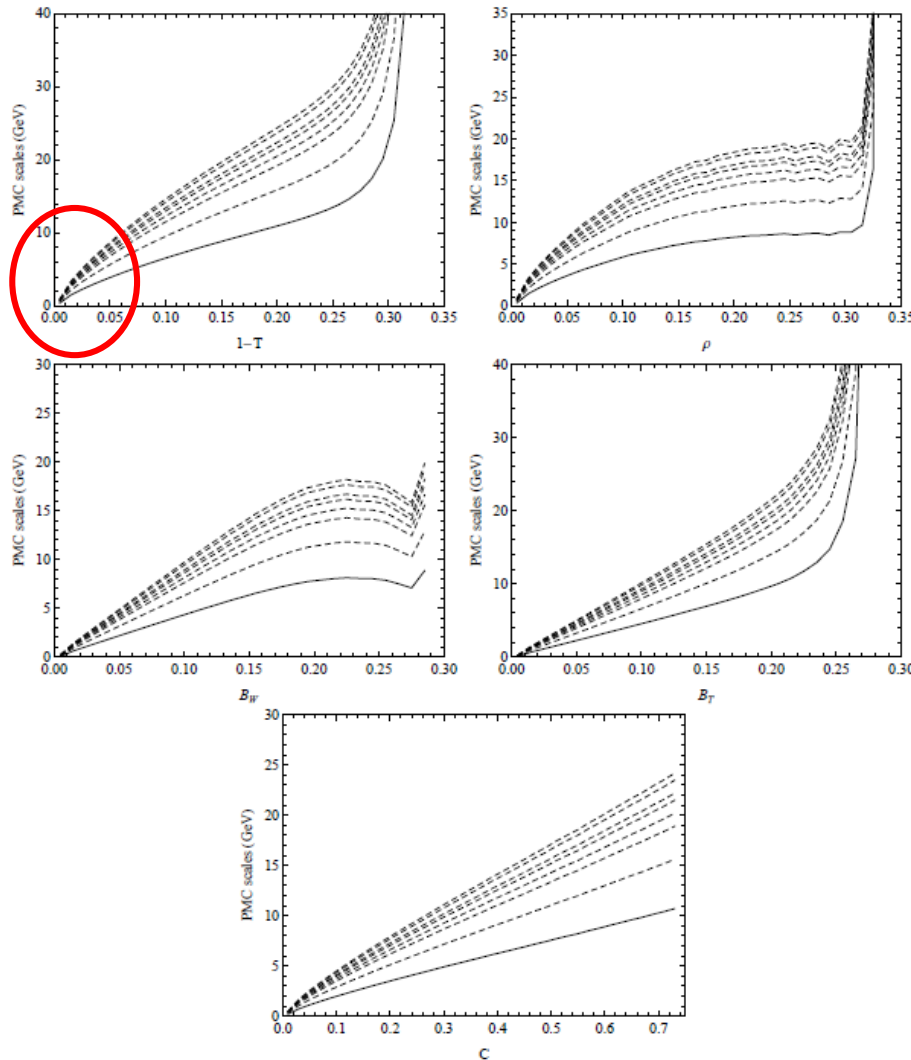
Event shapes using the conventional method:



- The NLO and NNLO are large and the pQCD series shows a slow convergence.
- Estimating the unknown higher order QCD by varying the scale  $[1/2Q, 2Q]$  is unreliable.
- The predictions are plagued by scale uncertainty, and even up to NNLO, the predictions do not match the data.
- The extracted coupling constants are deviated from the world average, and are also plagued by scale uncertainty.

# 三. Event shapes and extracting $\alpha_s$ at LEP

PMC scales:



- ◆ Remarkably, the PMC scales change dynamically with event shapes;
- ◆ The quarks and gluons have soft virtuality near the two-jet region. The PMC scales are very soft in this region, while in the regions away from the two-jet region, the PMC scales are increased, as expected;
- ◆ The PMC scales are small in the wide kinematic regions compared to the conventional method  $\sqrt{s}$ ;
- ◆ The PMC scales increase with the center-of-mass energy;
- ◆ yields the correct physical behavior, and similar behavior are obtained in the SCET theory and other literatures (ZPA 339, 189; EPJC 74, 2896).

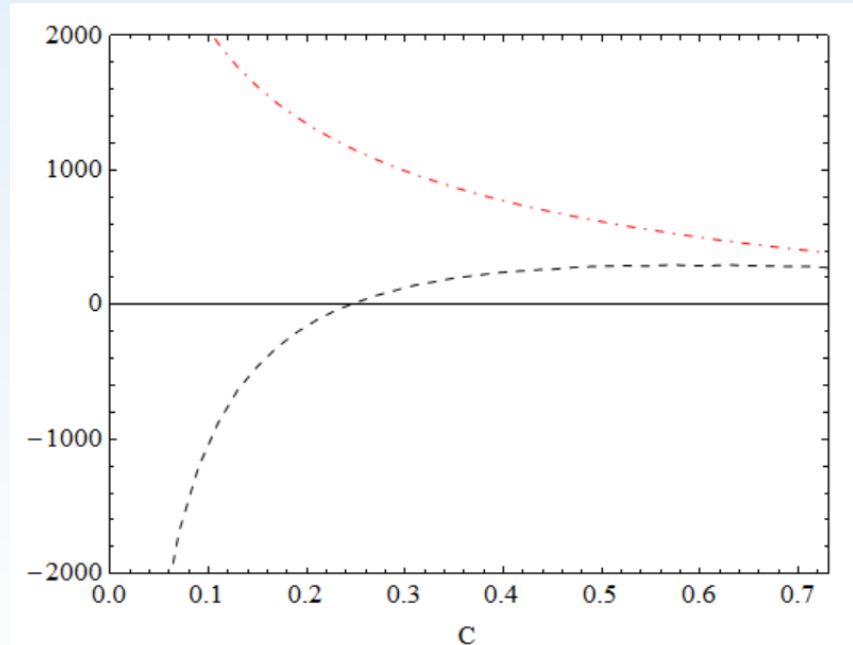
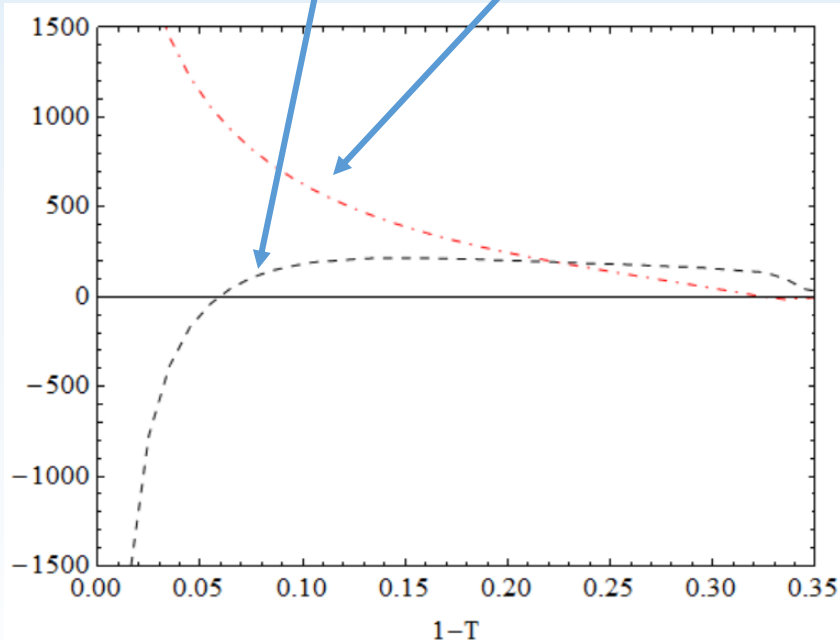


# 三. Event shapes and extracting $\alpha_s$ at LEP

Perturbative coefficients:

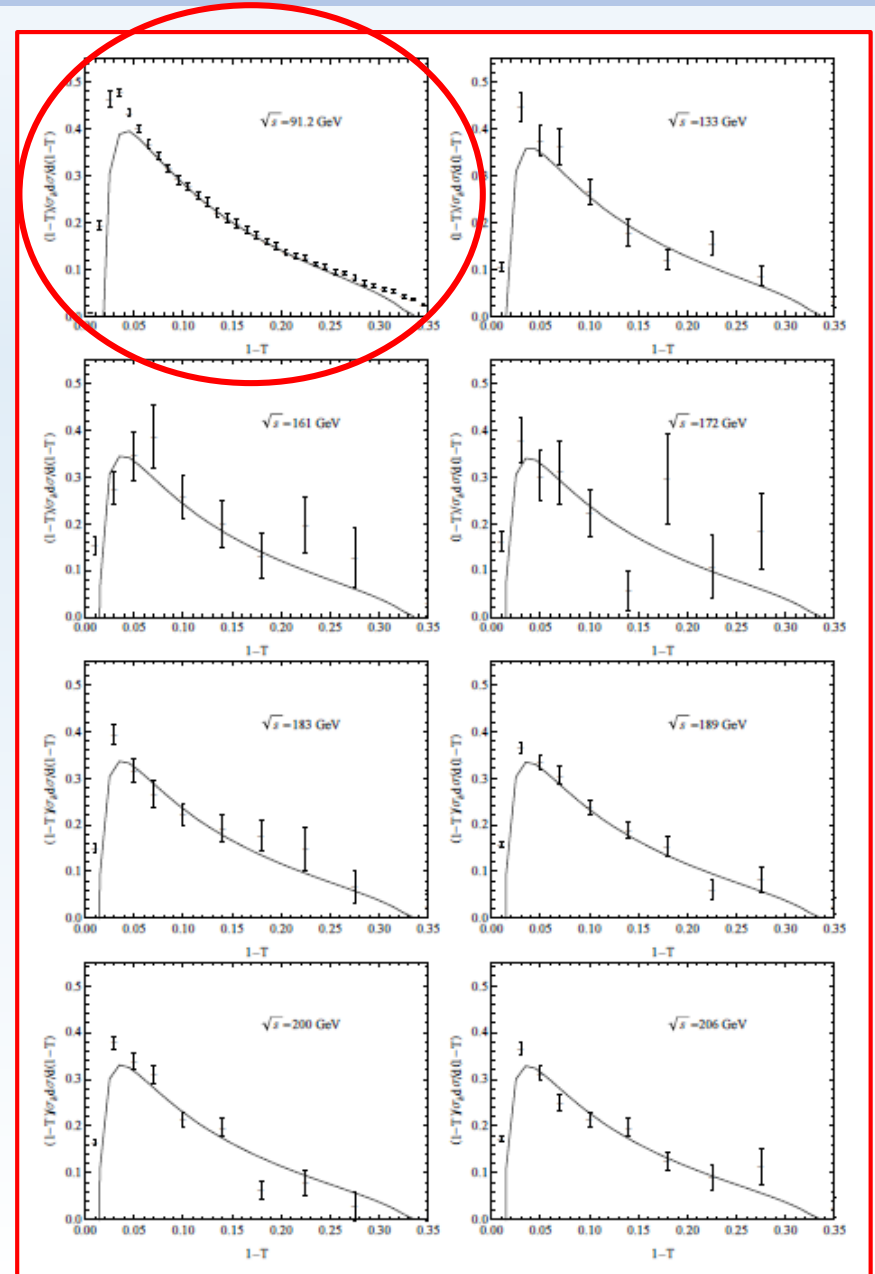
$$\bar{B}(y, \mu_r) = \bar{B}(y, \mu_r)_{\text{con}} + \bar{B}(y, \mu_r)_{\beta_0} \cdot \beta_0,$$

In addition to the PMC scales, the behavior of the PMC conformal coefficients is very different from that of the conventional scale-setting method.

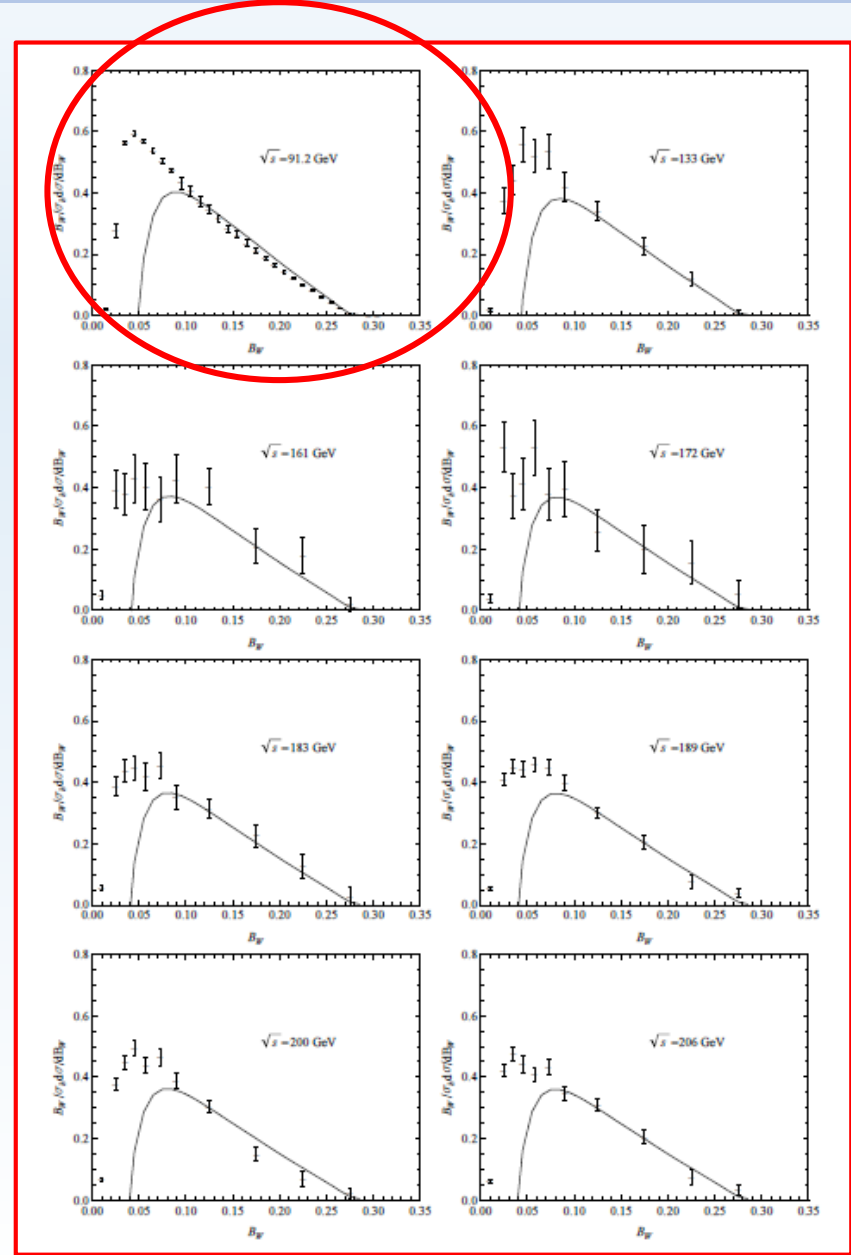
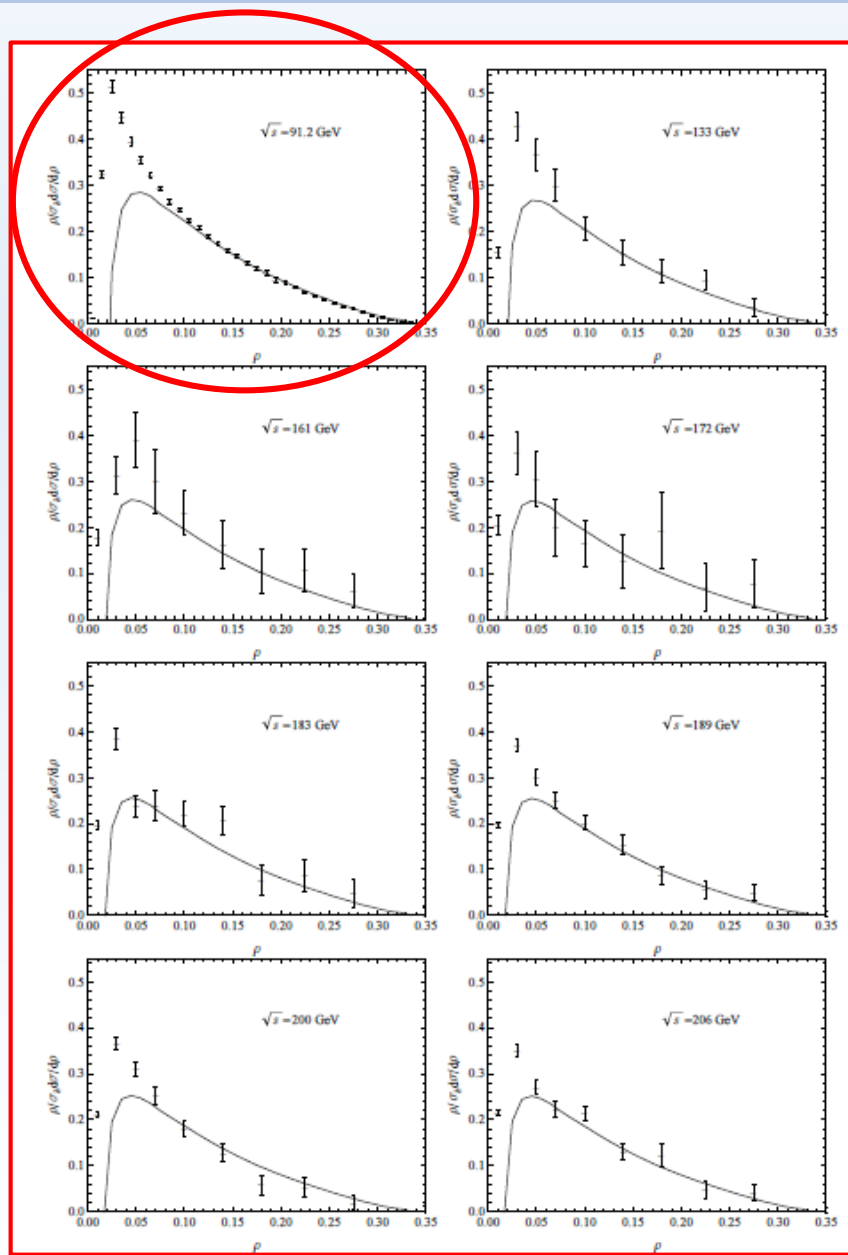


# 三. Event shapes and extracting $\alpha_s$ at LEP

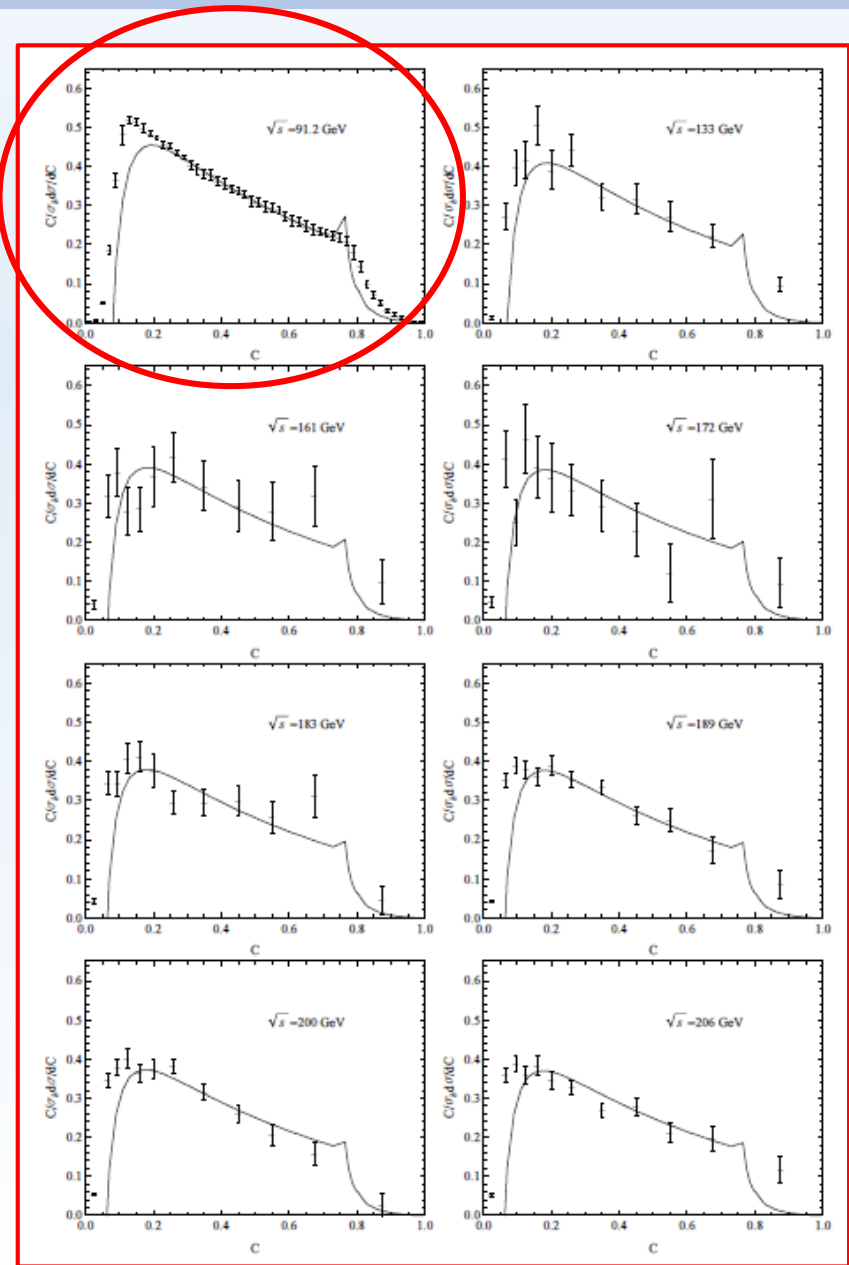
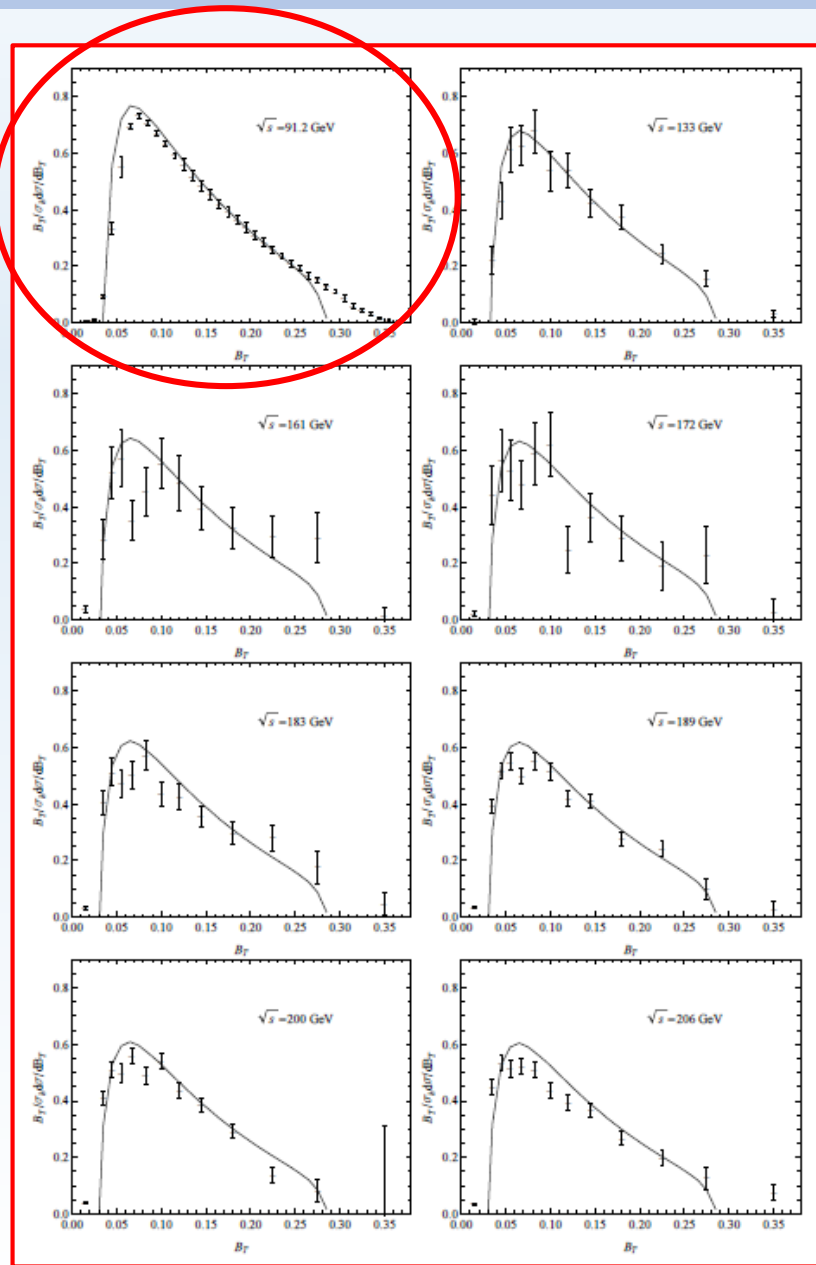
- The resulting PMC predictions are increased in wide kinematic regions compared to the conventional predictions.
- Since the PMC scales are independent of the choice of renormalization scale and the conformal coefficients are also renormalization scale independent, the PMC predictions eliminate the renormalization scale uncertainty.



# 三. Event shapes and extracting $\alpha_s$ at LEP

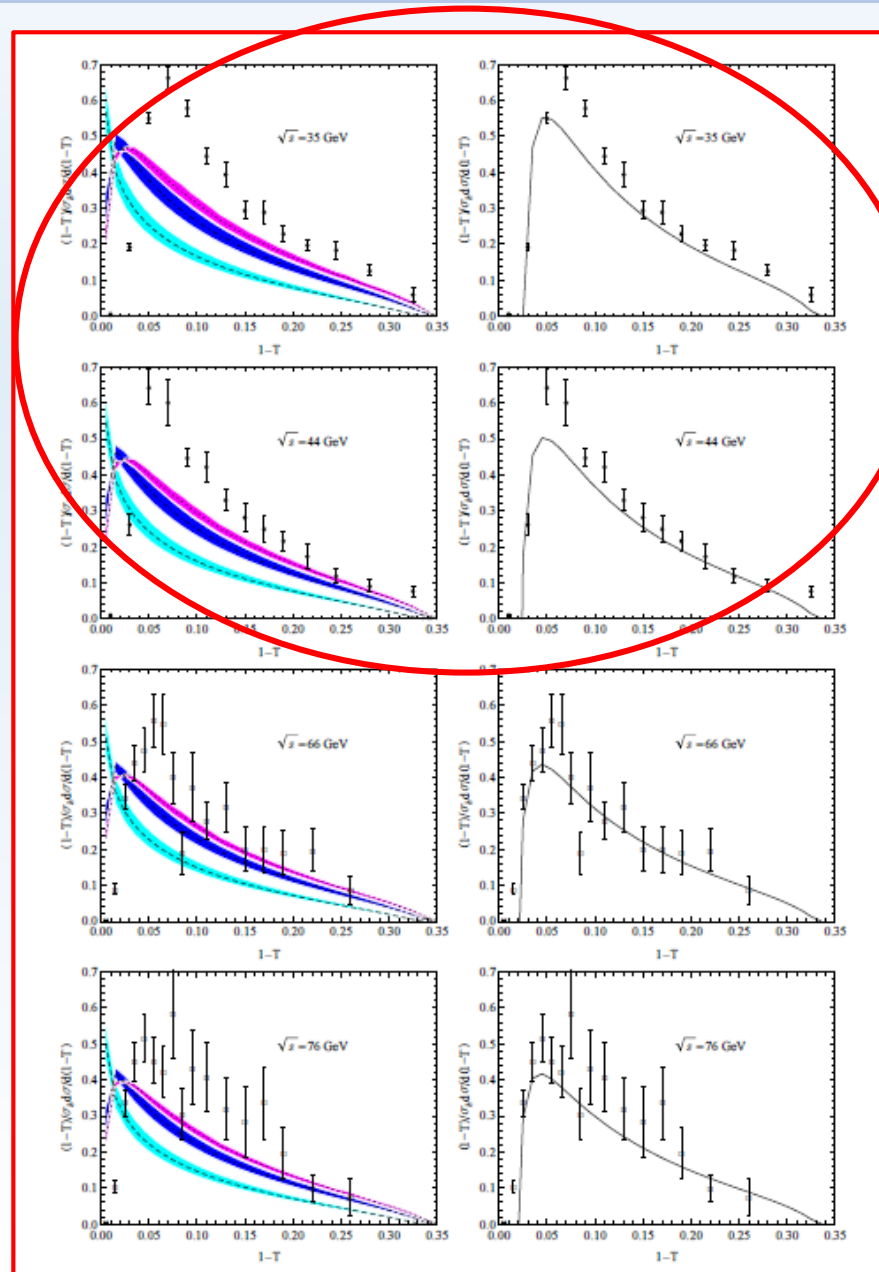


# 三. Event shapes and extracting $\alpha_s$ at LEP

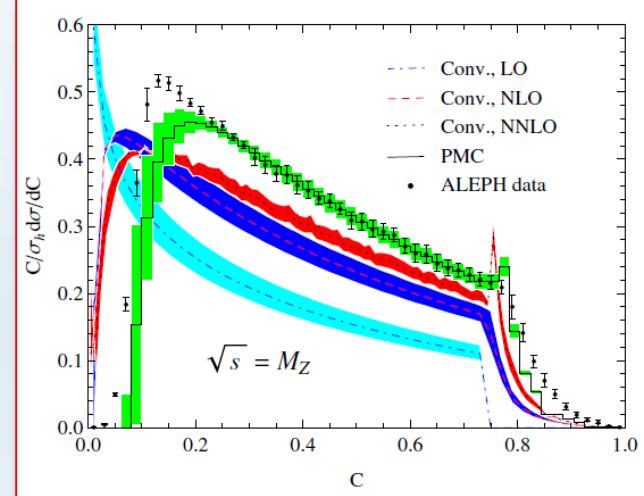
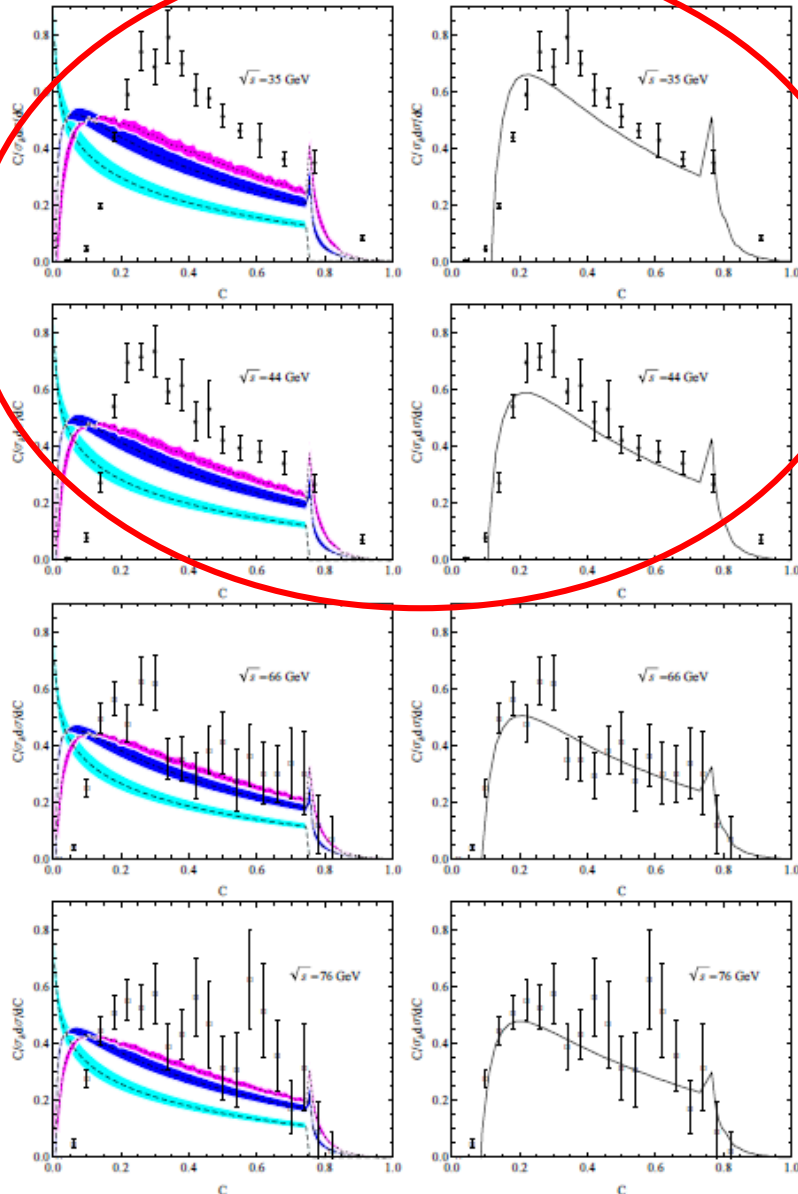


# 三. Event shapes and extracting $\alpha_s$ at LEP

Event shape distributions  
below  $Z^0$  peak

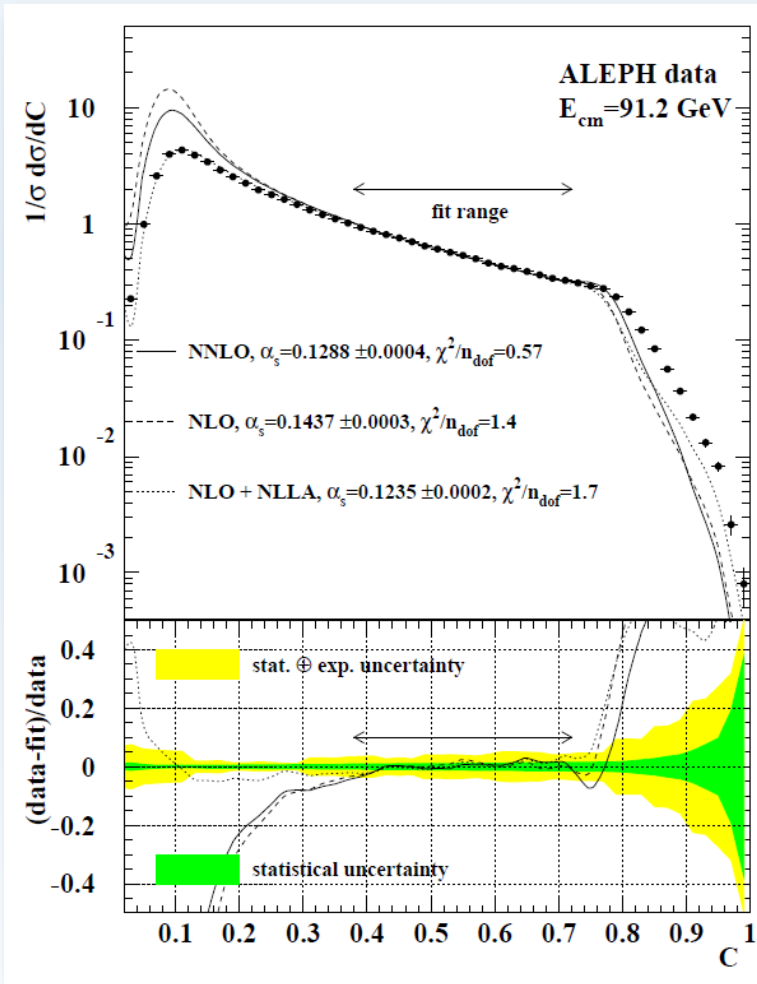


# 三. Event shapes and extracting $\alpha_s$ at LEP



- The PMC predictions are greatly increased in wide kinematic regions, which leads PMC results to be closer to the experimental data.
- There are some deviations near the two-jet and multijet regions, since there are large logarithms that spoil the perturbative regime of the QCD. The resummation of large logarithms is thus required for the PMC results especially near the two-jet regions.

# ≡. Event shapes and extracting $\alpha_s$ at LEP



$$Q = \sqrt{s} = M_Z$$

Conv.

- ✓ One value  $\alpha_s$  at scale  $M_Z$  is extracted ( $\alpha_s(M_Z)$ ).
- ✓ the fit range of T (C) distribution is narrow.
- ✓ the fit range is arbitrary, different fit range leads to different  $\alpha_s$ .

# 三. Event shapes and extracting $\alpha_s$ at LEP

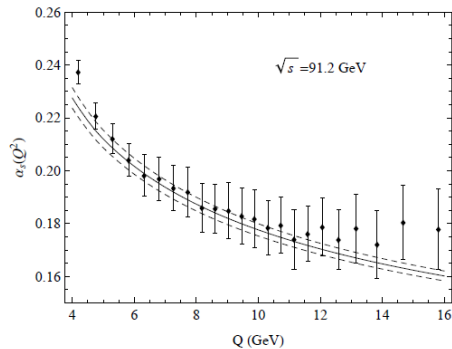


Figure 12. The extracted running coupling  $\alpha_s(Q^2)$  comparing the PMC predictions with the ALEPH data at  $\sqrt{s} = 91.2$  GeV. As a comparison, the solid line is the world average and two dashed lines represent its uncertainty.

$$Q = \sqrt{s} = M_Z$$

$$4 < Q < 16 \text{ GeV}$$

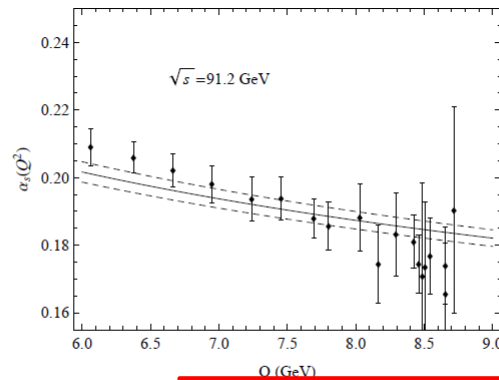


Figure 13. Similar to Fig. (12), but  $\alpha_s$

$$6 < Q < 9 \text{ GeV}$$

$$4 < Q < 7 \text{ GeV}$$

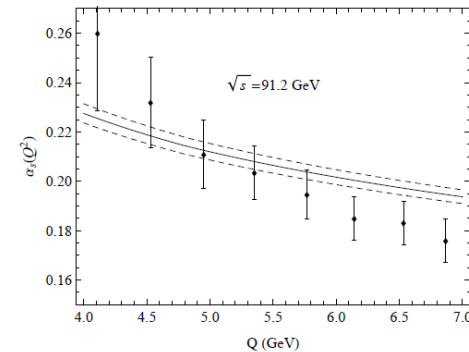


Figure 14. Similar to Fig. (12), but  $\alpha_s(Q^2)$  extracted from the wide jet broadening ( $B_W$ ) distribution.



# 三. Event shapes and extracting $\alpha_s$ at LEP

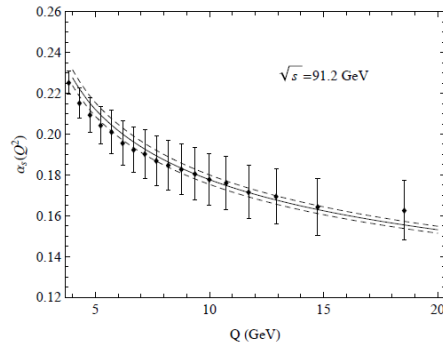


Figure 15. Similar to Fig. (12), but  $\alpha_s(Q^2)$  extracted from the total jet broadening ( $B_T$ ) distribution.

$4 < Q < 19 \text{ GeV}$

- ✓ The extracted  $\alpha_s$  are in agreement with the world average in wide range of  $Q$ .
- ✓ The extracted  $\alpha_s$  are not plagued by scale uncertainty.
- ✓ Since PMC scale varies with event shapes, we can extract the strong coupling at a wide scale range using the experimental data at single center-of mass-energy.

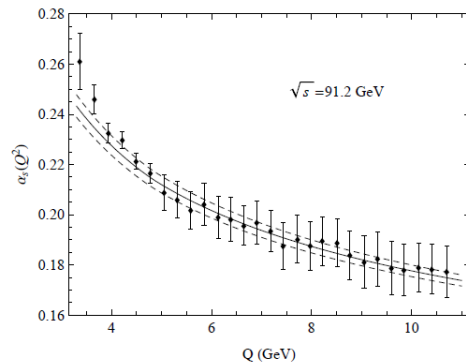


Figure 16. Similar to Fig. (12), but  $\alpha_s(Q^2)$  extracted from the C-parameter ( $C$ ) distribution.

$3 < Q < 11 \text{ GeV}$

In QED, the running of the QED coupling at a wide scale range can be determined from events at a single energy  
e.g., (OPAL Collaboration), EPJC 45, 1 (2006)

# ≡. Event shapes and extracting $\alpha_s$ at LEP

the mean value of event shapes,

$$\langle y \rangle = \int_0^{y_0} \frac{y}{\sigma_h} \frac{d\sigma}{dy} dy,$$

- ✓ it involves an integration over the full phase space.
- ✓ it provides an important complement to the differential distributions and to determinate  $\alpha_s$

$$\mu_r^{\text{pmc}}|_{\langle 1-T \rangle} = 0.0695\sqrt{s}, \text{ and } \mu_r^{\text{pmc}}|_{\langle C \rangle} = 0.0656\sqrt{s},$$

$\mu_r^{\text{pmc}} \ll \sqrt{s}$  is also suggested by

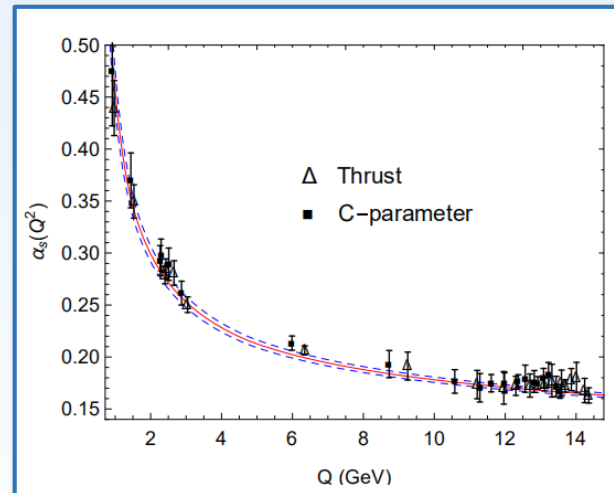
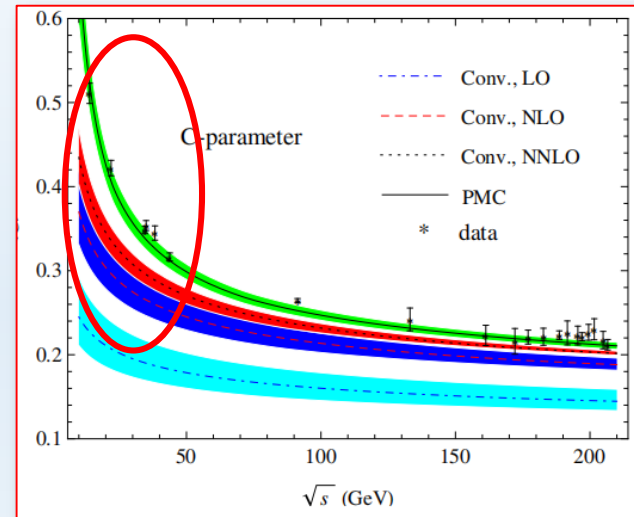
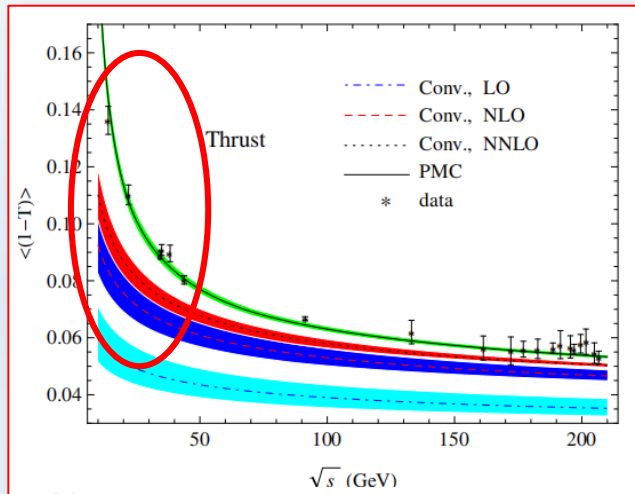
- ✓ PMC scales of differential distribution are also very small.
- ✓ the average of the PMC scale for differential distribution is close to the scale of mean value. **self-consistent.**

**Studies of QCD at  $e^+e^-$  centre-of-mass energies between 91 and 209 GeV**

The ALEPH Collaboration

**Eur. Phys. J. C 35, 457 – 486 (2004)**

# 三. Event shapes and extracting $\alpha_s$ at LEP



# ≡. Event shapes and extracting $\alpha_s$ at LEP

$$\begin{aligned}\alpha_s(M_Z^2) &= 0.1185 \pm 0.0011(\text{Exp.}) \pm 0.0005(\text{Theo.}) \\ &= 0.1185 \pm 0.0012,\end{aligned}\quad (3)$$

T

$$\begin{aligned}\alpha_s(M_Z^2) &= 0.1193^{+0.0009}_{-0.0010}(\text{Exp.})^{+0.0019}_{-0.0016}(\text{Theo.}) \\ &= 0.1193^{+0.0021}_{-0.0019},\end{aligned}\quad (4)$$

C

Cited by LHeC and FCC group and PDG

[Particle Data Group], Prog.  
Theor. Exp. Phys. 2020 (2020),  
083C01.

PDG

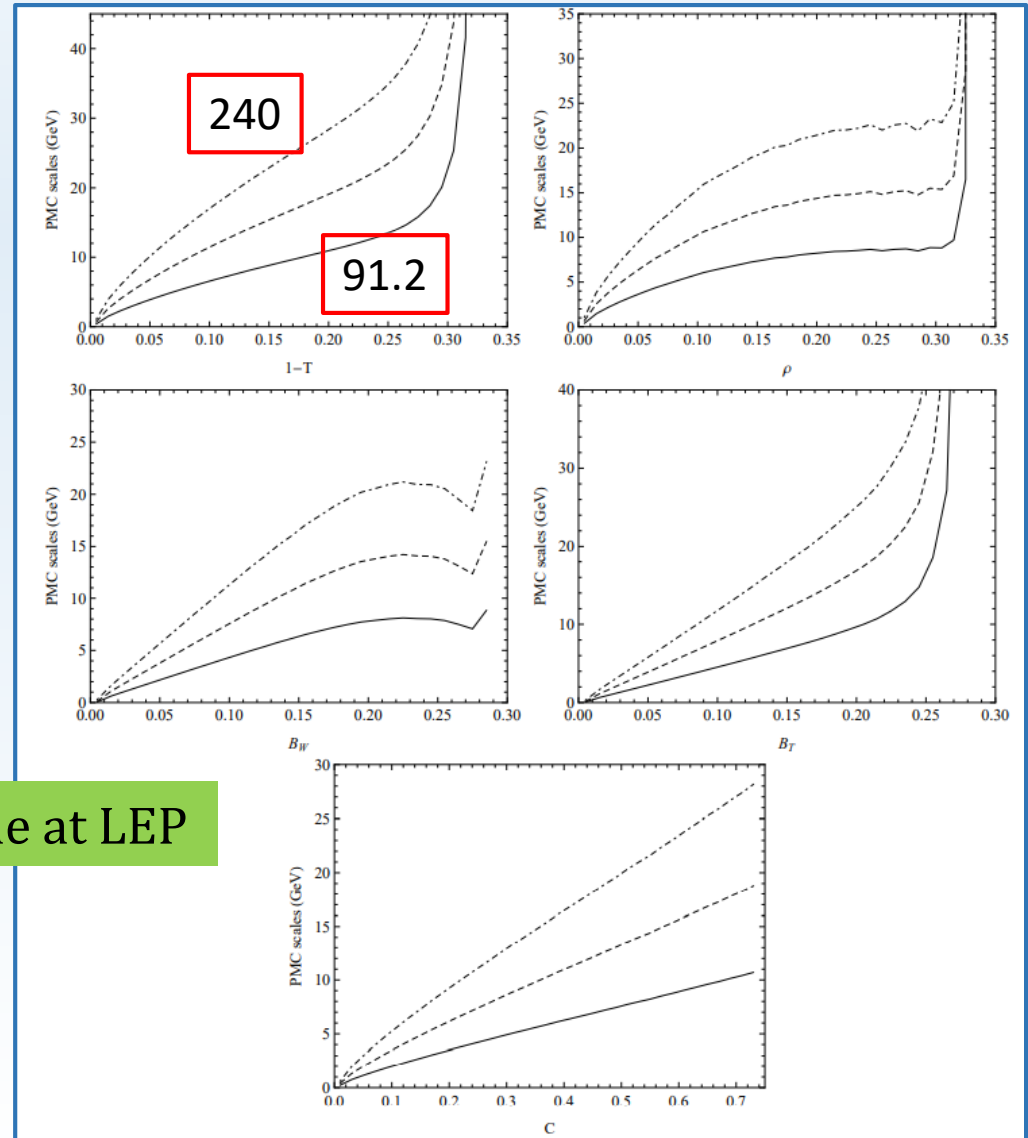
**The Large Hadron-Electron Collider at the HL-LHC**  
LHeC Collaboration and FCC-he Study Group (P. Agostini (Santiago  
CERN-ACC-Note-2020-0002, JLAB-ACP-20-3180  
e-Print: [arXiv:2007.14491](https://arxiv.org/abs/2007.14491) [hep-ex] | [PDF](#)

mean value for other event  
shapes, EEC,  $\rho$ ,  $B_W$ ,  $B_T$ ...

# 四. Event shapes from LEP to CEPC

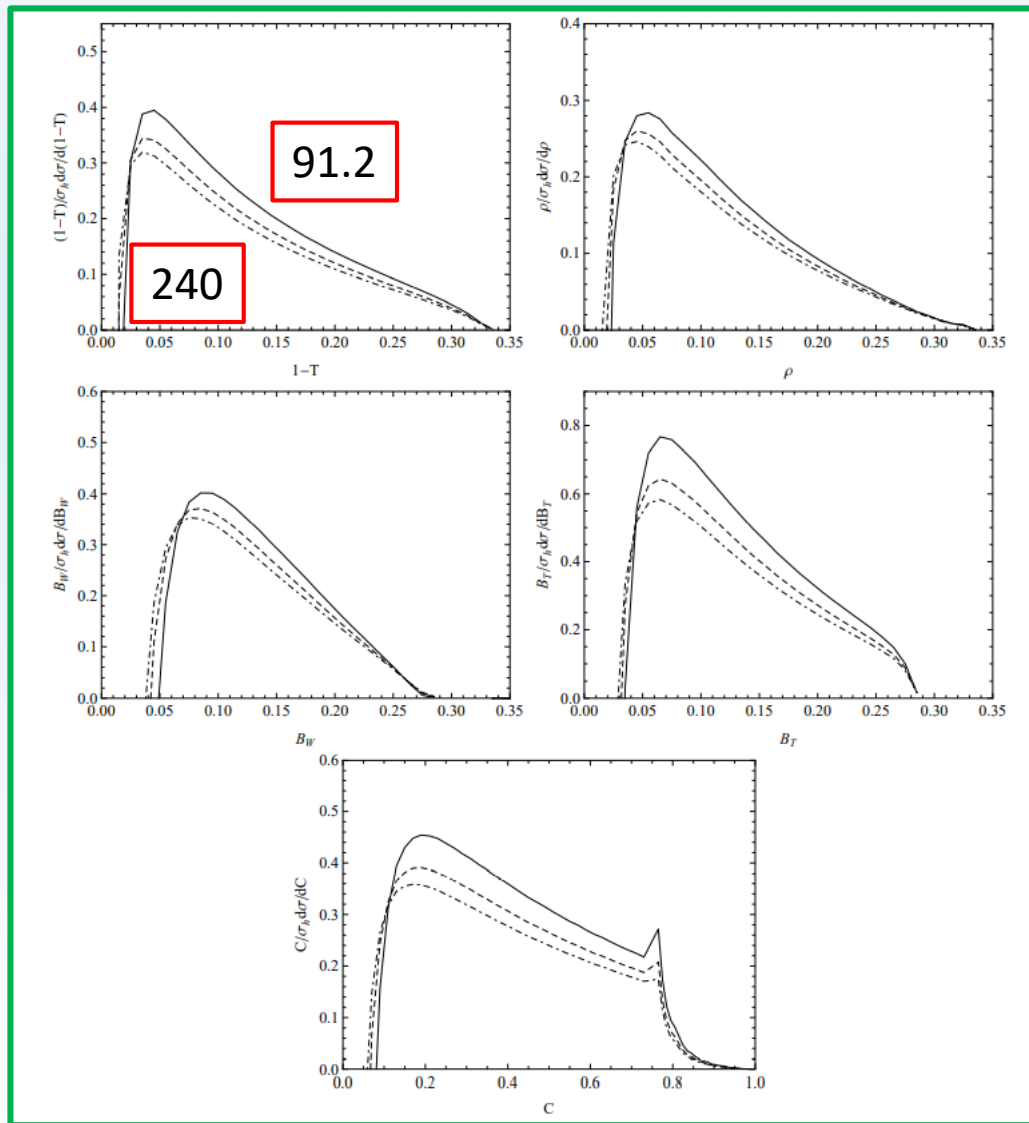
We calculate the classical event shapes at the CEPC at 91.2, 160 and 240 GeV.

PMC scales for event shape observables at CEPC



Similar to the case of the PMC scale at LEP

# 四. Event shape observables at CEPC



Our precise and scale-independent predictions for event shape observables, and a novel way to verify the running of  $\alpha_s(Q^2)$  call for the precise measurements at CEPC.

## 四. Summary

基于-重整化群方程以及基本重整化群不变性--提供具可系统设定高能物理过程“正确动量流动”的方案--从而解决传统方案下的重整化能标和重整化方案依赖问题

### PMC能标设定方案

- 1) 可自然改变微扰收敛性
- 2) 可更快地逼近物理量的真实值
- 3) 微扰低阶下就可与重整化能标选择无关, 获得每一阶准确值
- 4) 采用与方案无关的共形序列, 得到微扰展开收敛性的固有属性, 可用于估算未知高阶项贡献

粲夸克偶素?

### 传统能标设定方案

- 1) 收敛慢 (重整化子项发散)
- 2) 计算到任意高阶也无法获得每一阶准确值
- 3) 足够高阶时才能获得与重整化能标无关的物理量的真实值
- 4) 因每阶的数值都不准确, 不能很好判断微扰展开的收敛性, 无法给出令人信服的未知高阶项估算值



thanks