Gluonic evanescent operators in two-loop renormalization

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Based on works with Jin, Ren and Yang
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Outline

- Introduction
- Calculation of bare form factors
- Renormalization in two different schemes
- Results
- Conclusion

Studies of evanescent operator

An operator is said to be evanescent if it vanishes at 4 spacetime dimension via does not vanish in general $d = 4 - 2\epsilon$ spacetime dimensions. Such degrees of freedom should be considered in the dimensional regularization.

In 1990, people began the study of the fermionic evanescent operator. From the two-loop level, evanescent operators should be taken into account in the calculation of the anomalous dimensions.

Buras and Weisz 1990
Bondi, Curci, Paffuti and Rossi, 1990
Dugan and Grinstein 1991
…

Example of evanescent operators: $\bar{\psi}\gamma^{[\mu_1}\ldots\gamma^{\mu_n]}\psi\bar{\psi}\gamma_{[\mu_1}\ldots\gamma_{\mu_n]}\psi\,, \qquad n \geq 5\,.$

Evanescent operator in the Yang-Mills theory

Form factor : $$\int \mathrm{d}^d x e^{-\mathrm{i}q\cdot x}\langle \mathrm{g}_1 \cdots, \mathrm{g}_n|O|0\rangle$$

An operator is called evanescent if its tree form factors vanish. A general delta function can manifest this property, for example,

$$\delta^{\mu_1\mu_2\mu_3\mu_4\mu_5}_{\mu_6\mu_7\mu_8\mu_9\mu_{10}} F_{\mu_1\mu_2}F_{\mu_3\mu_4}F_{\mu_5\mu_6}F_{\mu_7\mu_8}F_{\mu_9\mu_{10}}$$

with

$$\delta^{\mu_1..\mu_n}_{\nu_1...\nu_n} = \det(\delta^{\mu}_{\nu}) = \begin{vmatrix} \delta^{\mu_1}_{\nu_1} & \dots & \delta^{\mu_1}_{\nu_n} \\ \vdots & & \vdots \\ \delta^{\mu_n}_{\nu_1} & \dots & \delta^{\mu_n}_{\nu_n} \end{vmatrix}$$

We initiate the study of gluonic evanescent operators and gave a systematic construction of the higher-dimensional operators in the Yang-Mills theory.

Jin, Ren, Yang, RY, 2022

In this talk, we focus on the two-loop anomalous dimensions of the dimension-10 operators in the planar limit, which is the simplest two-loop calculation of the anomalous dimensions involving gluonic evanescent operators.

The dimension-10 single-trace operator basis

There are length-2, -3, -4 and -5 operators, e.g.

$$O_1 = D^6(F_{\mu_1\mu_2}F_{\mu_1\mu_2}) \qquad O_4 = \frac{D^4(F12F13F23)}{12}$$

$$O_{31} = \frac{1}{8}D_9D_{10}\left[(2\delta^{56789}_{3412(10)} + \delta^{34789}_{5612(10)})F_{12}F_{34}F_{56}F_{78}\right]$$

$$O_{35} = \frac{1}{8}\left[-2\delta^{12347}_{569(10)8}F_{12}F_{56}F_{34}F_{9(10)}F_{78} + \delta^{12349}_{5678(10)}F_{12}F_{56}F_{78}F_{9(10)}F_{34}\right]$$

The basis are classified according to:

1. Physical and evanescent
2. C-parity.
3. Length
4. Helicity sector for physical operators.

30 physical	24 C-even	1 length-2	$O_1 : (-)^2$
		4 length-3	$O_2, O_3 : (-)^3$ $O_4, O_5 : (-)^2+$
		15 length-4	$O_6–O_{10} : (-)^4$ $O_{11}–O_{14} : (-)^3+$ $O_{15}–O_{20} : (-)^2(+)^2$
		4 length-5	$O_{21}, O_{22} : (-)^5$ $O_{23}, O_{24} : (-)^3(+)^2$
	6 C-odd	1 length-3	$O_{25} : (-)^2+$
		5 length-4	$O_{26} : (-)^4$ $O_{27}, O_{28} : (-)^3+$ $O_{29}, O_{30} : (-)^2(+)^2$
6 evanescent	5 C-even	3 length-4	O_{31}, O_{32}, O_{33}
		2 length-5	O_{34}, O_{35}
	1 C-odd	1 length-2	O_{36}

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Calculation of bare form factors

Our calculation is based on the unitarity cut method and the IBP reduction.

A loop form factor can be expanded into a set of IBP master integrals

$$\mathcal{F}^{(l)} = \sum_i c_i I_i^{(l)}$$

$$\mathcal{F}^{(l)}\Big|_{\text{cut}} = \sum_a C_a I_a|_{\text{cut}} \xrightarrow{\text{unitarity-IBP}} \sum_{\text{cut permitted}} c_i M_i \xrightarrow{\text{collect}} \sum_i c_i M_i$$

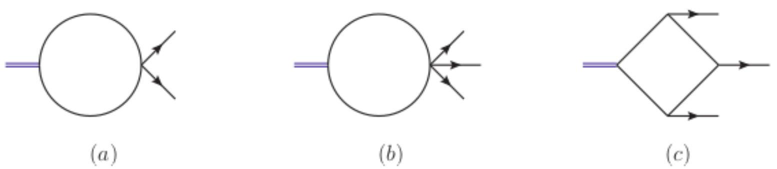
According to generalized unitarity $\mathcal{F}^{(l)}|_{\text{cut}} = \sum_{\text{helicities}} \mathcal{F}^{(0)} \times (\prod_i \mathcal{A}_i^{(0)})$

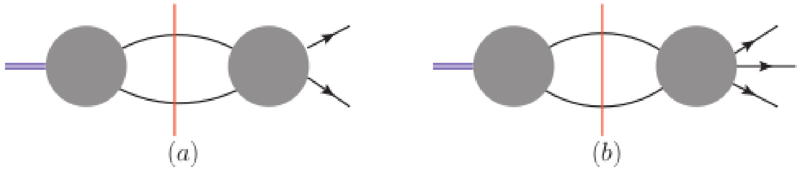
In the conventional dimensional regularization (CDR) scheme, one needs to do the d-dimensional helicity sum

$$\sum_{\text{helicities}} \epsilon_i^{\mu} \epsilon_i^{\nu *} = \eta^{\mu\nu} - \frac{q^{\mu} p_i^{\nu} + p_i^{\mu} q^{\nu}}{p_i \cdot q}$$

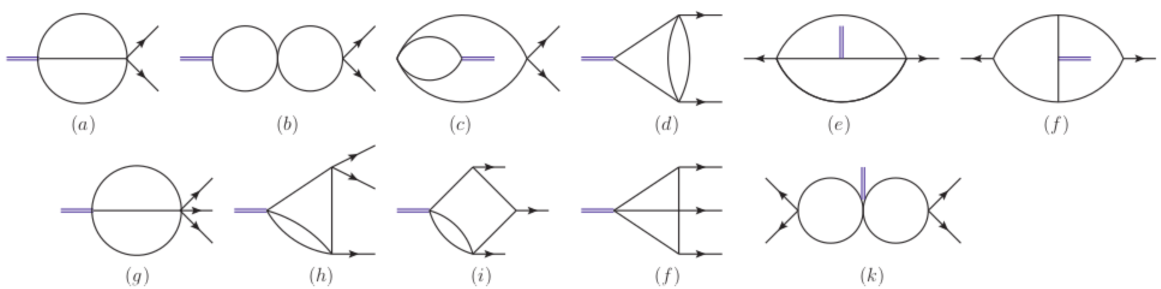
Master and cuts

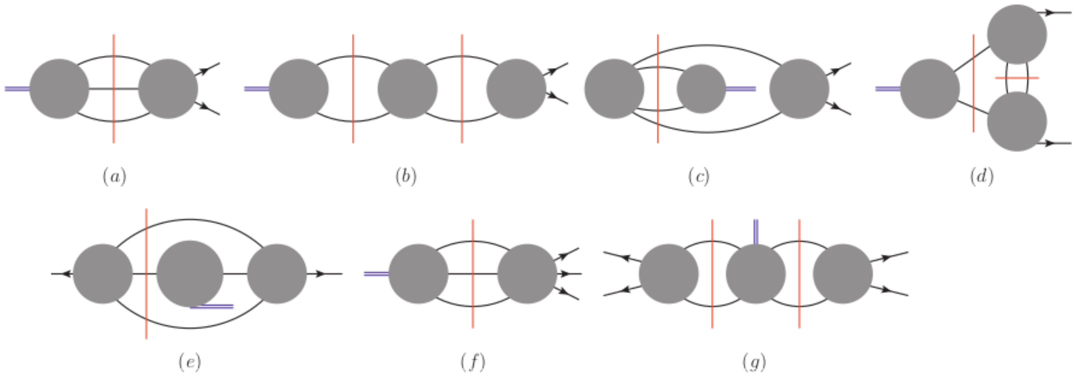
One-loop





Two-loop





Gehrmann and Remiddi, 2001

Tensor reduction

A challenging part of our work is tensor reduction before IBP

$$l_i \cdot e_j \to l_i \cdot p_j, l_i \cdot l_j$$

For 2-point and 3-point form factors, the gauge invariant basis projection is efficient.

Boels and Luo 2017
Boels, Jin and Luo 2018

However, the number of basis blows up for higher-point form factors.

A modified PV reduction

Passarino and Veltman 1979
Kreimer 1992

Given a tensor integral $\int [\mathrm{d}l] \frac{\prod_k l_{i_k}^{\mu_k}}{\prod_j \mathrm{D}_j}$

Momenta in the denominators

Loop momentum decomposition: $l_i^\mu = (x_{i,j} p_j^\mu) + l_{i,\perp}^\mu$

PV for the $l_{i,\perp}^\mu$, The only available tensor: $\eta_\perp^{\mu\nu} = \eta^{\mu\nu} - (G_p)^{-1}_{jk} p_j^\mu p_k^\nu$

$$l_{i_1,\perp}^{\mu_1} \cdots l_{i_m,\perp}^{\mu_m} = \begin{cases} 0\,, & m \text{ odd} \\ \sum_\sigma y_\sigma \eta_\perp^{\mu_{\sigma_1}\mu_{\sigma_2}} \cdots \eta_\perp^{\mu_{\sigma_{m-1}}\mu_{\sigma_m}}\,, & m \text{ even} \end{cases}$$

Finally, do the substitution: $l_{i,\perp} \cdot l_{j,\perp} = l_i \cdot l_j - x_{i,k} x_{j,s} p_k \cdot p_s$

Then we can do the IBP reduction via FIRE6. Smirnov and Chuharev 2019

For two-loop minimal form factors, the calculations are intrinsically 3-point.

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Z matrix and anomalous dimension

Renormalized operator $O_i = Z_i{}^j O_{\mathrm{b},j}$

Dilatation matrix $\mathcal{D} \equiv -\mu \frac{\mathrm{d}Z}{\mathrm{d}\mu} Z^{-1} = \sum_{l=1} \left(\frac{\alpha_s}{4\pi}\right)^l \mathcal{D}^{(l)}$

Together with $Z_i{}^j = \delta_i^j + \sum_{l=1} \left(\frac{\alpha_s}{4\pi}\right)^l Z^{(l)}{}_i{}^j$ and $\mu \frac{\mathrm{d}\alpha_s}{\mathrm{d}\mu} = -2\epsilon\alpha_s - \frac{\beta_0}{2\pi}\alpha_s^2 + \mathcal{O}(\alpha_s^3).$

see e.g. Gehrmann, Jaquier, Glover, Koukoutsakis, 2012

One gets

$$\mathcal{D}^{(1)} = 2\epsilon Z^{(1)}\,,$$

$$\mathcal{D}^{(2)} = 4\epsilon Z^{(2)} - 2\epsilon \left(Z^{(1)}\right)^2 + 2\beta_0 Z^{(1)}$$

Eigenvalues of the dilatation matrix are the anomalous dimensions.

Two renormalization schemes

Evanescent operators can be thought as O(ϵ).

Evanescent operators: $O\left(\epsilon^{-\mathrm{L}}\right) \sim O(\epsilon^{-1})$

Evanescent operators mix to

Physical operators: $O\left(\epsilon^{-(\mathrm{L}-1)}\right) \sim O(\epsilon^{0})$

1. The $\overline{MS}$ scheme

2. Finite renormalization scheme, the finite evanescent-to-physical mixing is also subtracted.

Buras and Weisz 1990
Bondi, Curci, Paffuti and Rossi, 1990

The $\overline{MS}$ scheme

The bare form factor has UV and IR divergence. After renormalization, there would be no UV divergence. The IR divergence takes the universal form

$$\mathcal{F}_i^{(1)} = I^{(1)}(\epsilon)\mathcal{F}_i^{(0)} + \mathcal{F}_i^{(1),\mathrm{fin}} + \mathcal{O}(\epsilon)\,,$$
$$\mathcal{F}_i^{(2)} = I^{(2)}(\epsilon)\mathcal{F}_i^{(0)} + I^{(1)}(\epsilon)\mathcal{F}_i^{(1)} + \mathcal{F}_i^{(2),\mathrm{fin}} + \mathcal{O}(\epsilon)$$

Catani, 1998

where $F_i^{(l)}$ is the lth term of renormalized form factors $Z_i{}^j F_{b,j}$ under the α_s-expansion. Requiring that the divergences are the same on the both side of the equation gives linear equations of the Z matrix.

The efficiency of the calcualtion can be enhanced by using the numerical reconstruction method.

The finite renormalization scheme

A general dilatation matrix $\begin{pmatrix} \mathcal{D}_{\rm pp} & \mathcal{D}_{\rm pe} \\ \mathcal{D}_{\rm ep} & \mathcal{D}_{\rm ee} \end{pmatrix}$

In this scheme $\begin{pmatrix} \hat{\mathcal{D}}_{\rm pp} & \hat{\mathcal{D}}_{\rm pe} \\ 0 & \hat{\mathcal{D}}_{\rm ee} \end{pmatrix}$

Dugan and Grinstein 1991

This is necessary to involve a finite part in the Z matrix: $Z|_{div} + Z|_{fin}$.

At each loop, the divergent part can be calculated similarly as in the $\overline{MS}$ scheme. The finite part is proportional to the physical operators. To exclude the terms proportional to the evanescent operators, we calculate the bare form factor at 4-dimensional kinematic points. At such points, the terms proportional to the evanescent operators vanish automatically.

Structure of the Z matrix and dilatation matrix

$\overline{MS}$ scheme

$$\begin{pmatrix} Z^{(1)}_{\rm pp} & Z^{(1)}_{\rm pe} \\ 0 & Z^{(1)}_{\rm ee} \end{pmatrix} \quad \begin{pmatrix} Z^{(2)}_{\rm pp} & Z^{(2)}_{\rm pe} \\ Z^{(2)}_{\rm ep} & Z^{(2)}_{\rm ee} \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{D}^{(1)}_{\rm pp} & \mathcal{D}^{(1)}_{\rm pe} \\ 0 & \mathcal{D}^{(1)}_{\rm ee} \end{pmatrix}, \quad \begin{pmatrix} \mathcal{D}^{(2)}_{\rm pp} & \mathcal{D}^{(2)}_{\rm pe} \\ \mathcal{D}^{(2)}_{\rm ep} & \mathcal{D}^{(2)}_{\rm ee} \end{pmatrix}$$

finite scheme

$$\begin{pmatrix} \hat{Z}^{(1)}_{\rm pp} & \hat{Z}^{(1)}_{\rm pe} \\ \hat{Z}^{(1)}_{\rm ep} & \hat{Z}^{(1)}_{\rm ee} \end{pmatrix} \quad \begin{pmatrix} \hat{Z}^{(2)}_{\rm pp} & \hat{Z}^{(2)}_{\rm pe} \\ \hat{Z}^{(2)}_{\rm ep} & \hat{Z}^{(2)}_{\rm ee} \end{pmatrix}$$

$$\begin{pmatrix} \hat{\mathcal{D}}^{(1)}_{\rm pp} & \hat{\mathcal{D}}^{(1)}_{\rm pe} \\ 0 & \hat{\mathcal{D}}^{(1)}_{\rm ee} \end{pmatrix}, \quad \begin{pmatrix} \hat{\mathcal{D}}^{(2)}_{\rm pp} & \hat{\mathcal{D}}^{(2)}_{\rm pe} \\ 0 & \hat{\mathcal{D}}^{(2)}_{\rm ee} \end{pmatrix}$$

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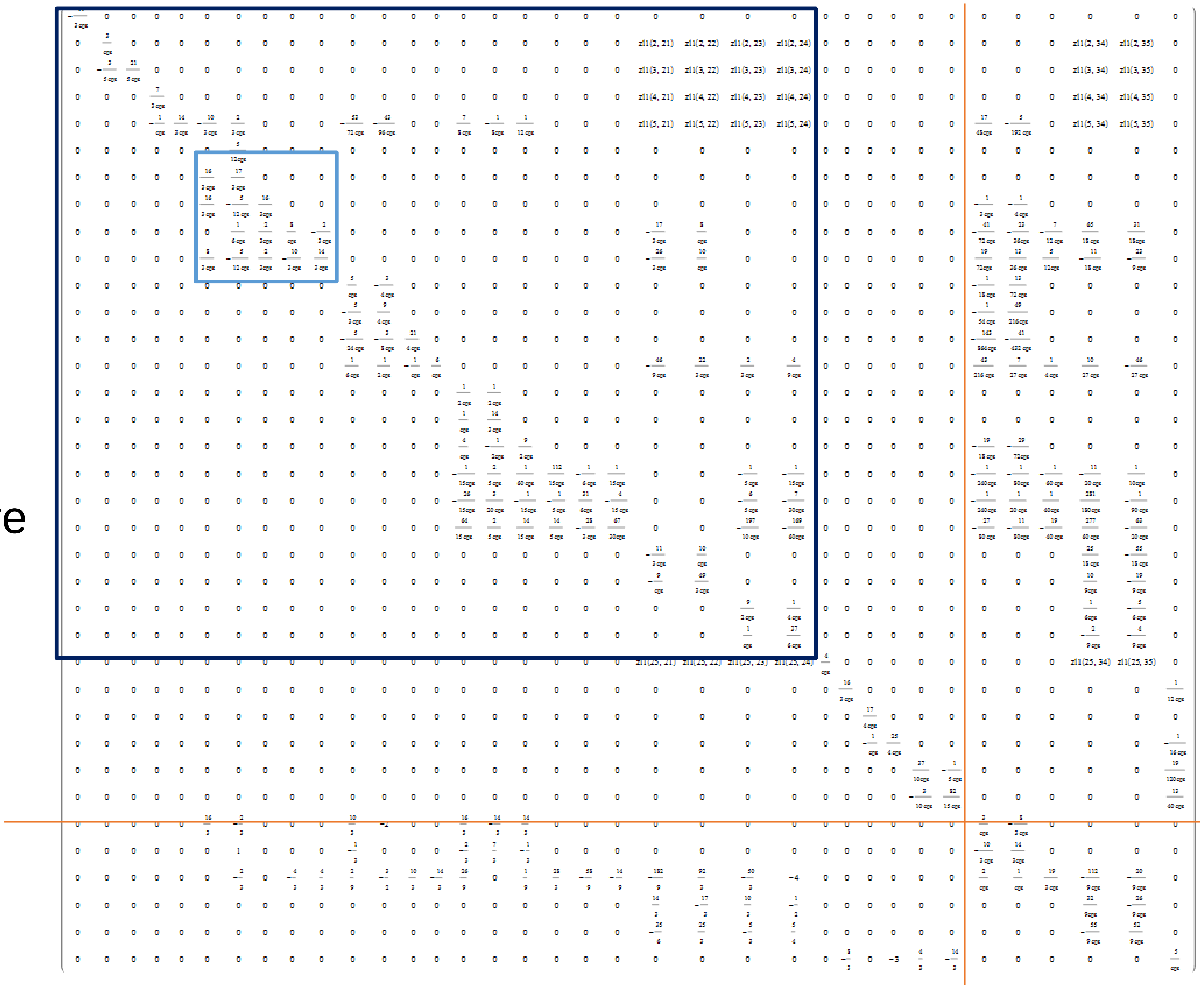
1-loop Z matrix
(The finite scheme)

The blockwise feature:

1. C-parity
2. Length
3. Helicity sector (1-loop)
4. Type of the total derivative

The finite Z_{ep}



The one-loop anomalous dimensions (same in the two schemes)

$$\gamma^{\text{even},(1)}_{\text{p,length-2}} : -\frac{22}{3}\,,$$

$$\gamma^{\text{even},(1)}_{\text{p,length-3}} : \frac{14}{3}, 6, \frac{42}{5}, \frac{28}{3}\,,$$

$$\gamma^{\text{odd},(1)}_{\text{p,length-3}} : \ 8\,,$$

$$\gamma^{\text{even},(1)}_{\text{p,length-4}} : 9, \frac{21}{2}, \frac{32}{3}, 12, \frac{1}{3}\left(17 \pm 3\sqrt{41}\right), \frac{1}{6}\left(31 \pm \sqrt{697}\right), \frac{1}{4}\left(29 \pm \sqrt{201}\right), \frac{2}{3}\left(19 \pm 3\sqrt{5}\right),$$

$$x_1, x_2, x_3\,,$$

$$\gamma^{\text{odd},(1)}_{\text{p,length-4}} : \ \frac{22}{3}, \frac{17}{2}, \frac{32}{3}, 11, \frac{25}{2}\,,$$

$$\gamma^{\text{even},(1)}_{\text{p,length-5}} : \frac{2}{3}\left(19 \pm 3\sqrt{10}\right), \frac{1}{3}\left(32 \pm \sqrt{34}\right)\,,$$

$$\gamma^{\text{even},(1)}_{\text{e,length-4}} = \frac{1}{3}\left(23 \pm \sqrt{345}\right), \frac{38}{3}\,,$$

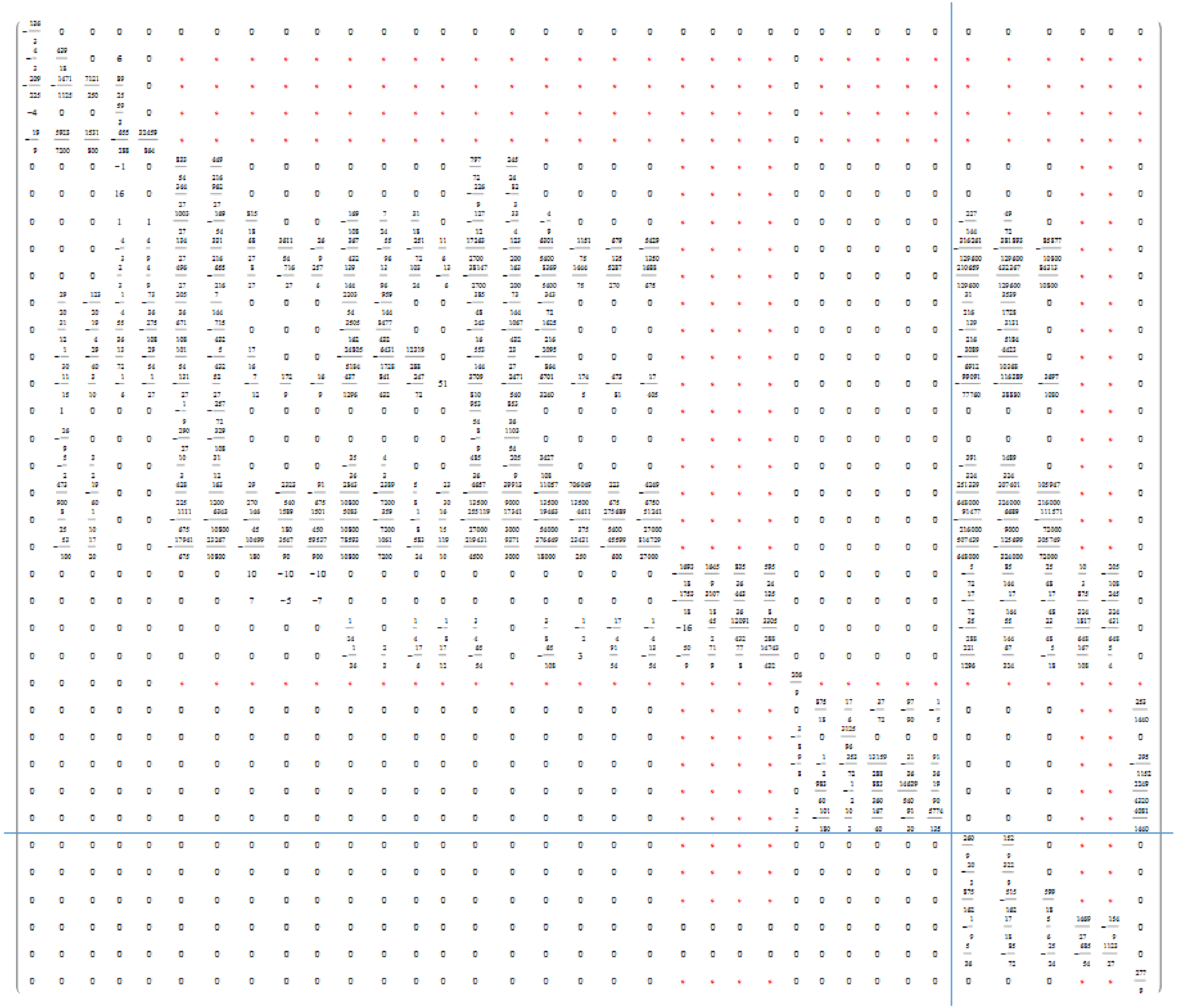
$$\gamma^{\text{even},(1)}_{\text{e,length-5}} = \frac{2}{3}\left(14 \pm \sqrt{170}\right)\,,$$

$$\gamma^{\text{odd},(1)}_{\text{e,length-4}} = 10\,.$$

x_i are the roots of $x^3 - \frac{446x^2}{15} + \frac{769x}{3} - \frac{8014}{15} = 0$

The two-loop dilatation matrix (The finite scheme)

$$\begin{pmatrix} \hat{\mathcal{D}}_{\mathrm{pp}} & \hat{\mathcal{D}}_{\mathrm{pe}} \\ 0 & \hat{\mathcal{D}}_{\mathrm{ee}} \end{pmatrix}$$



The two-loop anomalous dimensions

$\overline{MS}$ scheme

$$\gamma^{\text{even},(2)}_{\text{p,length-2}} : -\frac{136}{3}\,,$$

$$\gamma^{\text{even},(2)}_{\text{p,length-3}} : \frac{59}{3}, \frac{439}{18}, \frac{7121}{250}, \frac{149525}{3996}\,,$$

$$\gamma^{\text{odd},(2)}_{\text{p,length-3}} : \frac{206}{9}\,,$$

$$\gamma^{\text{even},(2)}_{\text{p,length-4}} : \frac{1308521}{35532}, \frac{12319}{288}, \frac{815}{18}, \frac{415}{18}, \frac{37679 \pm 2651\sqrt{41}}{1476}, \frac{2\left(179129 \pm 2352\sqrt{697}\right)}{18819}, \frac{29\left(4108755061\sqrt{201} \pm 115875887553\right)}{112160431296}, \frac{1}{54}\left(3100 \pm 103\sqrt{5}\right), y_1, y_2, y_3\,,$$

$$\gamma^{\text{odd},(2)}_{\text{p,length-4}} : \frac{32885}{1188}, \frac{3125}{96}, \frac{107}{2}, \frac{75421}{1188}, \frac{64211}{1440}\,,$$

$$\gamma^{\text{even},(2)}_{\text{p,length-5}} : \frac{376249 \pm 78535\sqrt{10}}{8604}, \frac{108341113246123\sqrt{34} \pm 4211644375821510}{113297323414176}\,,$$

$$\gamma^{\text{even},(2)}_{\text{e,length-4}} : \frac{\left(6442724032485 \pm 11542242689\sqrt{345}\right)}{213976901880}, \frac{4755559}{75255}\,,$$

$$\gamma^{\text{even},(2)}_{\text{e,length-5}} : \frac{\left(3977690861205 \pm 50021112896\sqrt{170}\right)}{114158809836}\,,$$

$$\gamma^{\text{odd},(2)}_{\text{e,length-4}} : \frac{3079}{540}\,.$$

$$y^3 - \frac{44053970579731y^2}{334691552250} + \frac{4335623758063848120847262203y}{80085267136274439204000} - \frac{12858742227506943574716057437659}{194607199141146887265720000} = 0\,.$$

Finite scheme

$$\hat{\gamma}^{\text{even},(2)}_{\text{p,length-2}} : -\frac{136}{3}\,,$$

$$\hat{\gamma}^{\text{even},(2)}_{\text{p,length-3}} : \frac{59}{3}, \frac{439}{18}, \frac{7121}{250}, \frac{149525}{3996}\,,$$

$$\hat{\gamma}^{\text{odd},(2)}_{\text{p,length-3}} : \frac{206}{9}\,,$$

$$\hat{\gamma}^{\text{even},(2)}_{\text{p,length-4}} : \frac{3427}{108}, \frac{12319}{288}, \frac{815}{18}, \frac{877}{18}, \frac{37679 \pm 2651\sqrt{41}}{1476}, \frac{2\left(179129 \pm 2352\sqrt{697}\right)}{18819}, \frac{129729219 \pm 5049167\sqrt{201}}{4283712}, \frac{1}{270}\left(16160 \pm 251\sqrt{5}\right), y_1, y_2, y_3\,,$$

$$\hat{\gamma}^{\text{odd},(2)}_{\text{p,length-4}} : \frac{8048}{297}, \frac{3125}{96}, \frac{875}{18}, \frac{50825}{1188}, \frac{13159}{288}\,,$$

$$\hat{\gamma}^{\text{even},(2)}_{\text{p,length-5}} : \frac{1}{18}\left(617 \pm 142\sqrt{10}\right), \frac{432664955274 \pm 20543721361\sqrt{34}}{12629285856}\,,$$

$$\hat{\gamma}^{\text{even},(2)}_{\text{e,length-4}} : \frac{97}{3} \mp \frac{59\sqrt{\frac{5}{69}}}{3}, \frac{9098}{261}\,,$$

$$\hat{\gamma}^{\text{even},(2)}_{\text{e,length-5}} : \frac{\left(2513637 \pm 54631\sqrt{170}\right)}{53244}\,,$$

$$\hat{\gamma}^{\text{odd},(2)}_{\text{e,length-4}} : \frac{277}{9}\,.$$

$$y^3 - \frac{250350031847y^2}{1934633250} + \frac{4824966722800230692858971y}{925841238569646696000} - \frac{13849613580264790328390513509}{224979420972424147128000} = 0\,.$$

The physical effect of evanescent operators

1. In the $\overline{MS}$ scheme, the evanescent operators and the physical operators mix together in the dilatation matrix. To get the correct physical anomalous dimensions, it is necessary to renormalize the evanescent operators up to the two-loop order.

2. In the finite scheme, the block D_{ep} vanishes. The anomalous dimensions are just eigenvalues of the block D_{pp}. This does means that the evanescent operators are totally decoupled in the calculation. The term $(-2\epsilon Z_{pe}^{(1)} Z_{ep}^{(1)})$ contributes to the block $D_{pp}^{(2)}$. Therefore, to get the correct two-loop physical anomalous dimensions, we need to renormalize the evanescent operators up to the one-loop order.

Anomalous dimensions at the WF fixed point

The new Z matrix reads $\tilde{Z} \equiv KZ$

The dilatation matrix under scheme change

see e.g. Vasilev, 2004
Pietro and Stamou, 2018

$$\tilde{\mathcal{D}} = -\frac{\partial K}{\partial \alpha_s}(\mu \frac{d\alpha_s}{d\mu})K^{-1} + K\mathcal{D}K^{-1}$$

The anomalous dimensions do not depend on the scheme at a fixed point.

Require that $\mu \frac{d\alpha_s}{d\mu} = 0$, we get the coupling constant at the WF fixed point:

$$\alpha^* = -\frac{4\pi\epsilon}{\beta_0} - \frac{4\pi\beta_1\epsilon^2}{\beta_0^3} + \mathcal{O}(\epsilon^3)$$

The dilatation matrix: $\mathcal{D}^* = \sum_{i=1} \epsilon^i \mathcal{D}_i^* = \sum_l \left(\frac{\alpha^*}{4\pi}\right)^l \mathcal{D}^{(l)} = (-\frac{\epsilon}{\beta_0} - \frac{\beta_1\epsilon^2}{\beta_0^3})\mathcal{D}^{(1)} + \frac{\epsilon^2}{\beta_0^2}\mathcal{D}^{(2)} + \mathcal{O}(\epsilon^3)$

The anomalous dimensions: $\gamma^* = \sum_{i=1} \epsilon^i \gamma_i^*$

We verity that the anomalous dimensions at the fixed point are the same in the two schemes.

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Conclusion

1. We perfume for the first time the two-loop anomalous dimensions of the dimension-10 operators.

2. Our calculation shows that the evanescent operators do contribute to the two-loop anomalous dimension in dimensional regularization.

3. We check the scheme independence at the WF fixed point.

4. Our strategy is expected to be applied to the two-loop renormalization for higher-dimensional operators in general theories.

Thank you!

Renormalized form factors $Z_i{}^j F_{b,j}$ under the α_s-expansion

$$
\begin{aligned}
\mathcal{F}_i^{(0)} &= \mathcal{F}_{i,\mathrm{b}}^{(0)}\,, \\
\mathcal{F}_i^{(1)} &= S_\epsilon^{-1}\mathcal{F}_{i,\mathrm{b}}^{(1)} + \left(Z^{(1)}{}_i{}^j - \frac{\delta_n}{2}\frac{\beta_0}{\epsilon}\delta_i{}^j\right)\mathcal{F}_{j,\mathrm{b}}^{(0)}\,, \\
\mathcal{F}_i^{(2)} &= S_\epsilon^{-2}\mathcal{F}_{i,\mathrm{b}}^{(2)} + S_\epsilon^{-1}\left(Z^{(1)}{}_i{}^j - \left(1+\frac{\delta_n}{2}\right)\frac{\beta_0}{\epsilon}\delta_i{}^j\right)\mathcal{F}_{j,\mathrm{b}}^{(1)} \\
&\quad + \left(Z^{(2)}{}_i{}^j - \frac{\delta_n}{2}\frac{\beta_0}{\epsilon}Z^{(1)}{}_i{}^j + \frac{\delta_n^2+2\delta_n}{8}\frac{\beta_0^2}{\epsilon^2}\delta_i{}^j - \frac{\delta_n}{4}\frac{\beta_1}{\epsilon}\delta_i{}^j\right)\mathcal{F}_{j,\mathrm{b}}^{(0)}\,.
\end{aligned}
$$

Vanishing of $\widehat{D}_{ep}$

Dugan and Grinstein 1991

The one-loop case is obvious, since the Z_{ep}^{1} is finite. We only keep the finite part of the dilatation matrix, thus D_{ep}^{1} vanishes.

For the two-loop case,

$$Z^{(2)}|_{\frac{1}{\epsilon^2}-\text{part}} = \frac{1}{2}(Z^{(1)})^2 - \frac{\beta_0}{2\epsilon}Z^{(1)}$$ guarantees a finite $D^{(2)}$

For the block $D_{ep}^{(2)}$, which represent the mixing from evanescent to physical operators, the above relation should be one-order higher in the ϵ-expansion.

$$\hat{Z}_{\text{ep}}^{(2)}|_{\frac{1}{\epsilon}-\text{part}} = \frac{1}{2}(\hat{Z}_{\text{ep}}^{(1)}\hat{Z}_{\text{pp}}^{(1)} + \hat{Z}_{\text{ee}}^{(1)}\hat{Z}_{\text{ep}}^{(1)}) - \frac{\beta_0}{2\epsilon}\hat{Z}_{\text{ep}}^{(1)}$$ guarantees a vanishing $D_{ep}^{(2)}$