QCD Phase Diagram and the Equation of State of Strong-Interaction Matter

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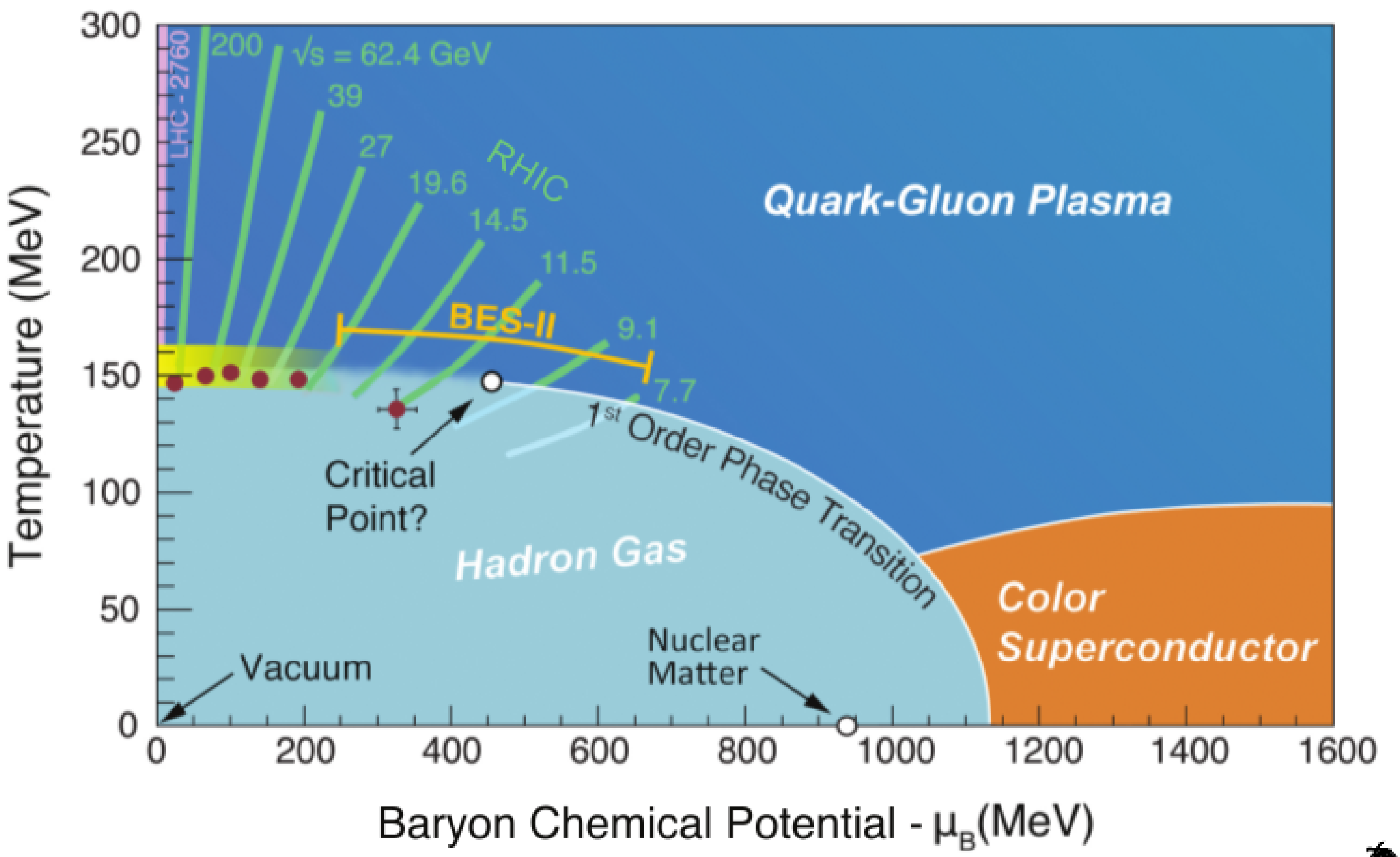
with a lot of input from 'HotQCD,

in particular,
David Clarke
Jishnu Goswami
Anirban Lahiri
Mugdha Sarkar
Sipaz Sharma

- QCD phase diagram – constraining the location of a critical endpoint
- Equation of state in the BES energy range
- Fluctuations and correlations of conserved charges

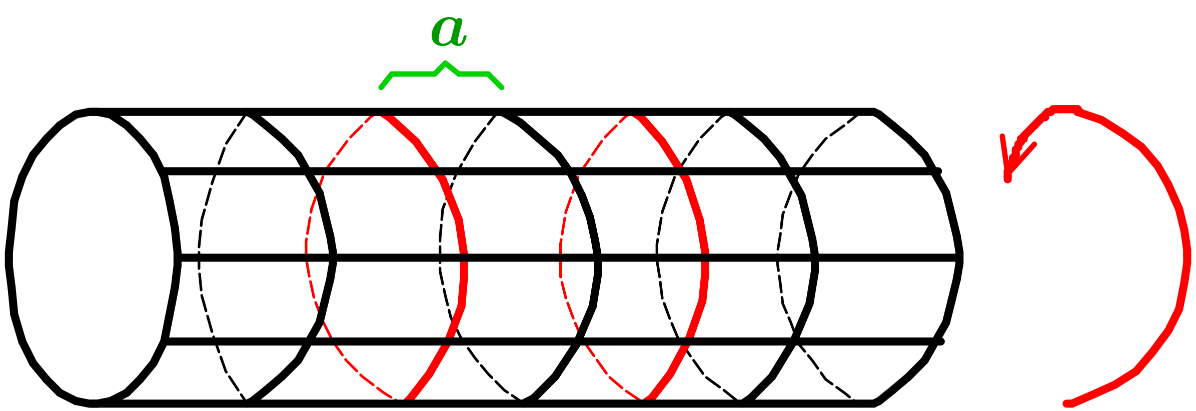
CRC-TR 211 Strong-interaction matter under extreme conditions
DFG Deutsche Forschungsgemeinschaft

Exploring the phase diagram of strong-interaction matter



Where is the critical point?

Simulating strongly interacting matter on a discrete space-time grid (lattice QCD)



$1/T = N_\tau a$

the lattice: $N_\sigma^3 \times N_\tau$

lattice spacing: a

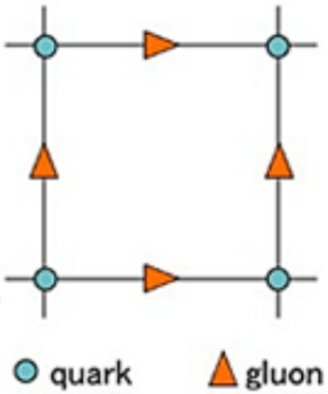
Monte Carlo simulation 1979

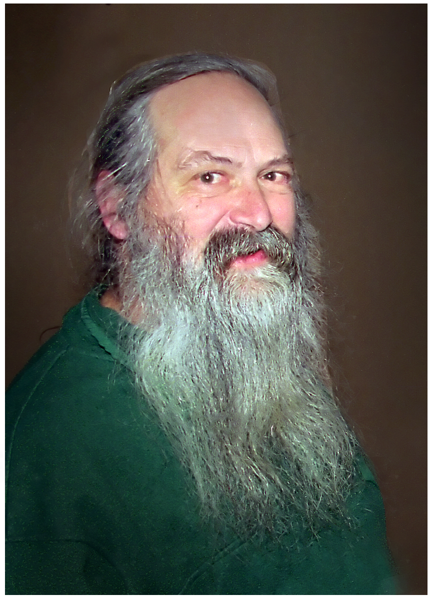
$\longleftarrow$ $V^{1/3} = N_\sigma a$ $\longrightarrow$

partition function:

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{-S_E}$$

$$= \int \mathcal{D}\mathcal{A}\, \det M(\mathcal{A}, m_q)\; e^{-S_G}$$

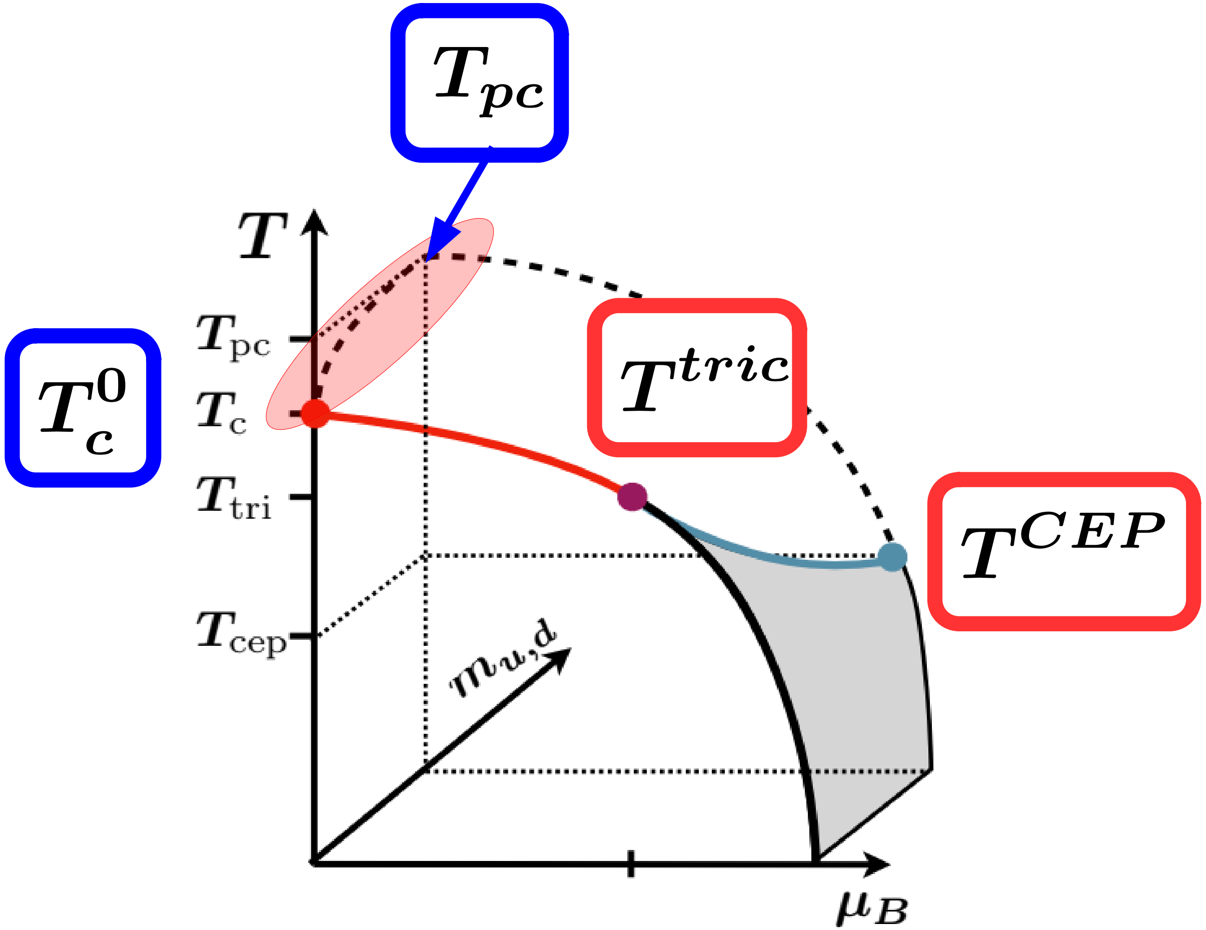


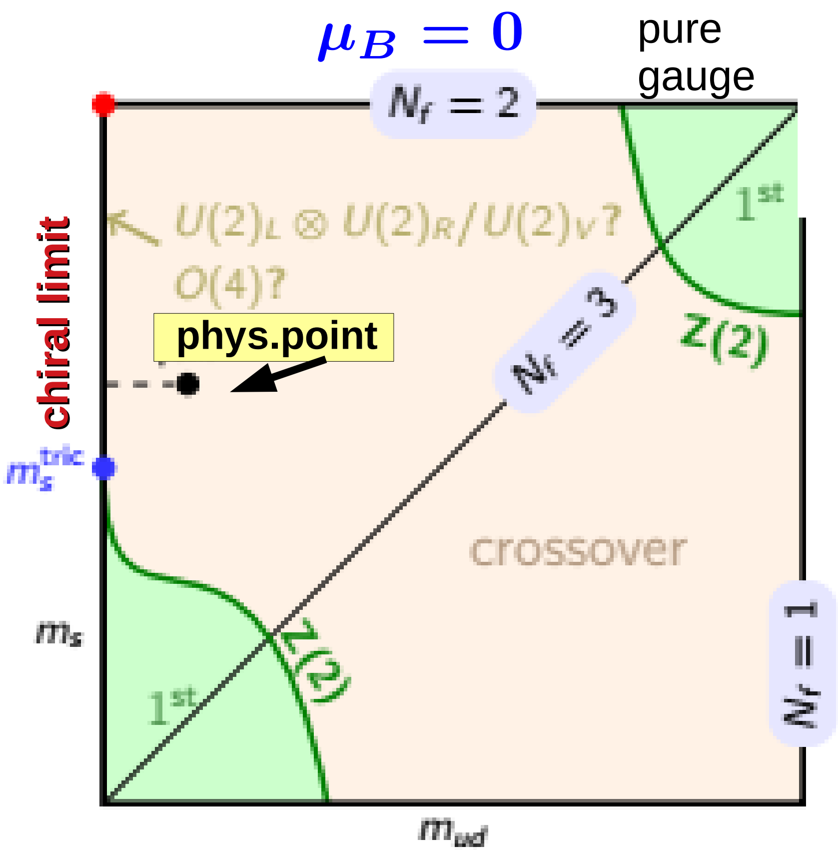


Mike Creutz

Phases of strong-interaction matter

determination of T_c^0 puts an upper limit on T^{CEP}





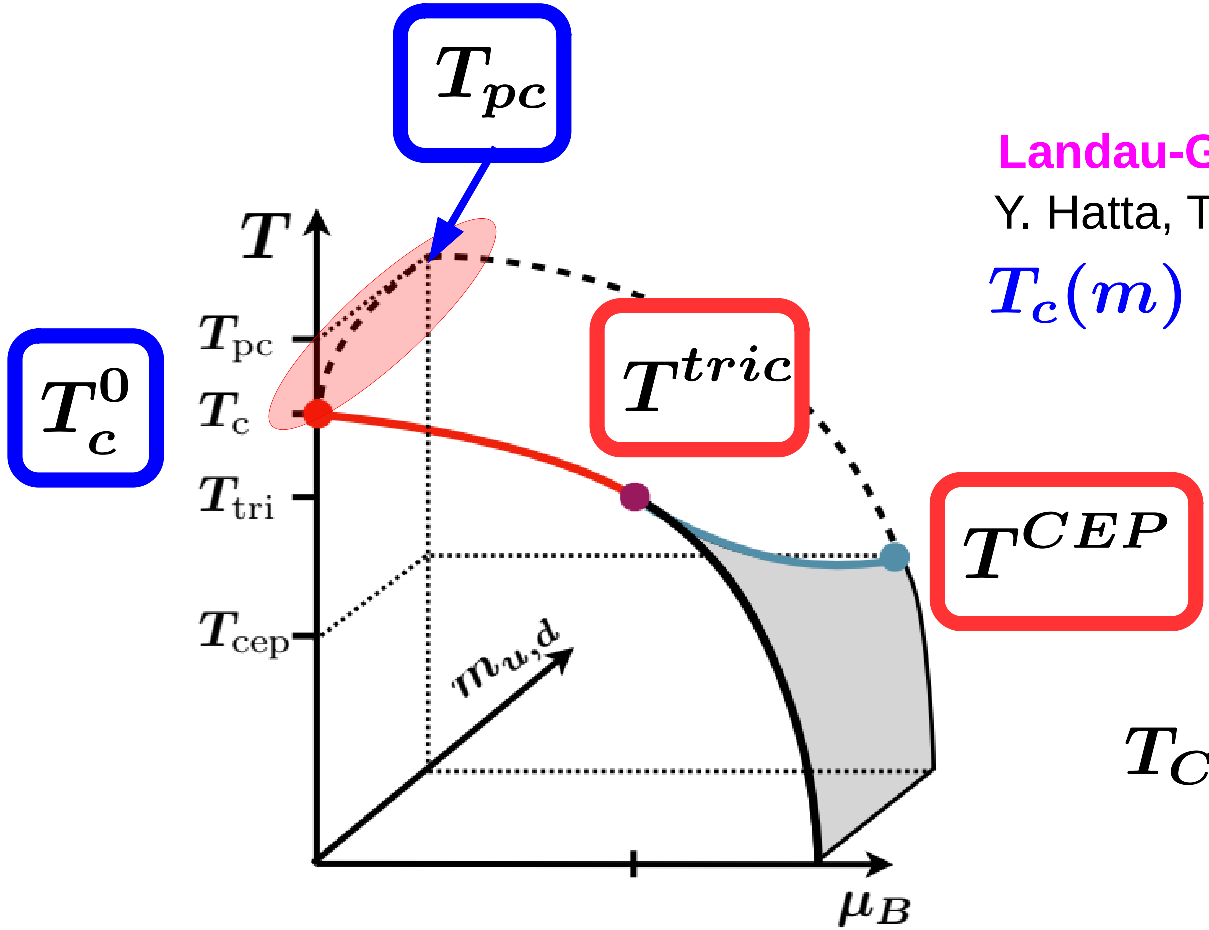
existence of a 1st order region for $m_\ell \rightarrow 0$ increasingly unlikely

F. Cuteri et al, arXiv:2107.12739
Sipaz Sharma et al, arXiv:2111.12599

Random Matrix Model	A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.M. Verbaarschot, Phys. Rev. D58 (1998) 096007
QCD motivated	M. Stephanov, Phys. Rev. D73 (2006) 094508
NJL	M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361

Phases of strong-interaction matter

determination of T_c^0 puts an upper limit on T^{CEP}



Landau-Ginzburg potential:

Y. Hatta, T. Ikeda, Phys. Rev. D67 (2003) 014028

$$T_c(m) = T^{tric} - \mathrm{c}\ m^{2/5}$$

$\mathrm{non-universal,\ but\ c > 0}$

$$T_{CEP} < T^{tric} < T_c^0 < T_{pc}^0$$

Random Matrix Model — A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J.M. Verbaarschot, Phys. Rev. D58 (1998) 096007

QCD motivated — M. Stephanov, Phys. Rev. D73 (2006) 094508

NJL — M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361

Critical behavior in QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a universal scaling function

singular regular

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

$$t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_l}{T}\right)^2$$

$$h \sim \frac{m_l}{T_c}$$

Pseudo-critical temperatures

response functions
2nd order cumulants

- magnetic / mixed / thermal

$$\frac{\partial^2 \ln Z}{\partial h^2} \qquad \frac{\partial^2 \ln Z}{\partial h\, \partial t} \qquad \frac{\partial^2 \ln Z}{\partial t^2}$$

$$\sim \left(\frac{m_l}{T_c}\right)^{1/\delta-1} \qquad \sim \left(\frac{m_l}{T_c}\right)^{(\beta-1)/\beta\delta} \qquad \sim \left(\frac{m_l}{T_c}\right)^{-\alpha/\beta\delta}$$

$$\sim -0.79 \qquad \sim -0.34 \qquad \sim +0.11$$

divergence: strong / moderate / none

O(4) critical exponents
$\alpha = -0.21$
$\beta = 0.38$
$\delta = 4.82$

Fluctuation observables in QCD

– chiral condensate: $\langle\bar{\psi}\psi\rangle_l = \frac{\partial P/T}{\partial m_l/T}$, $\langle\bar{\psi}\psi\rangle_l = (\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d)/2$

– chiral order parameter: $M = \frac{2}{f_K^4}\left[m_s\langle\bar{\psi}\psi\rangle_l - m_l\langle\bar{\psi}\psi\rangle_s\right]$

$$m_l = (m_u + m_d)/2$$

– chiral susceptibility: $\chi_M = m_s\left(\frac{\partial M}{\partial m_u} + \frac{\partial M}{\partial m_d}\right)$ **magnetic**

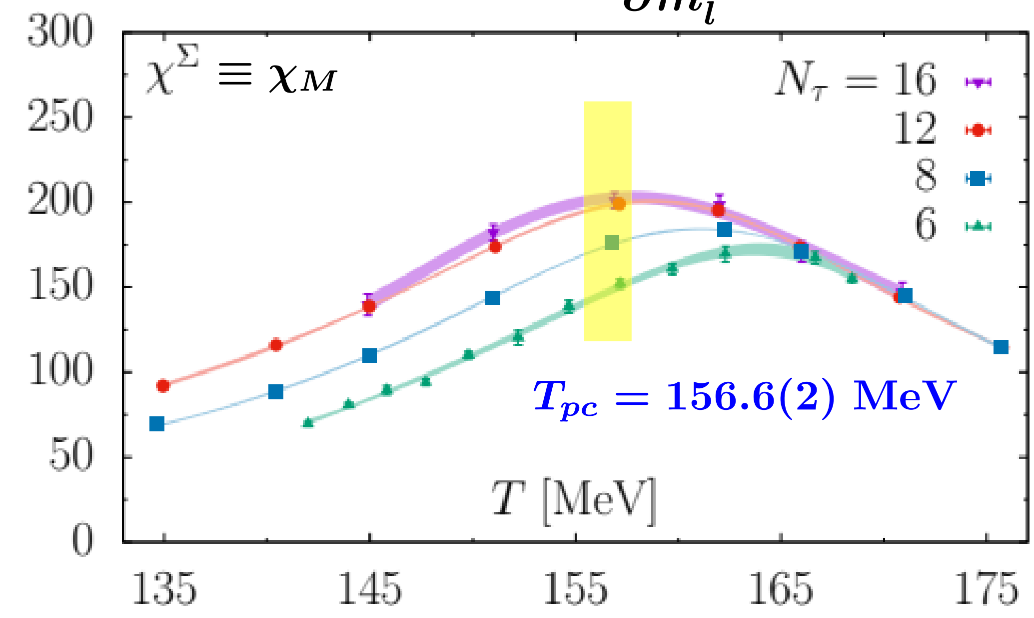
– mixed chiral susceptibility: $\chi_t = T\frac{\partial M}{\partial T}$ **mixed**

– conserved charge fluctuations: $\chi_X = T^4 \frac{\partial^4 P/T^4}{\partial \mu_X^4}\Big|_{\mu_X=0}$ **thermal**

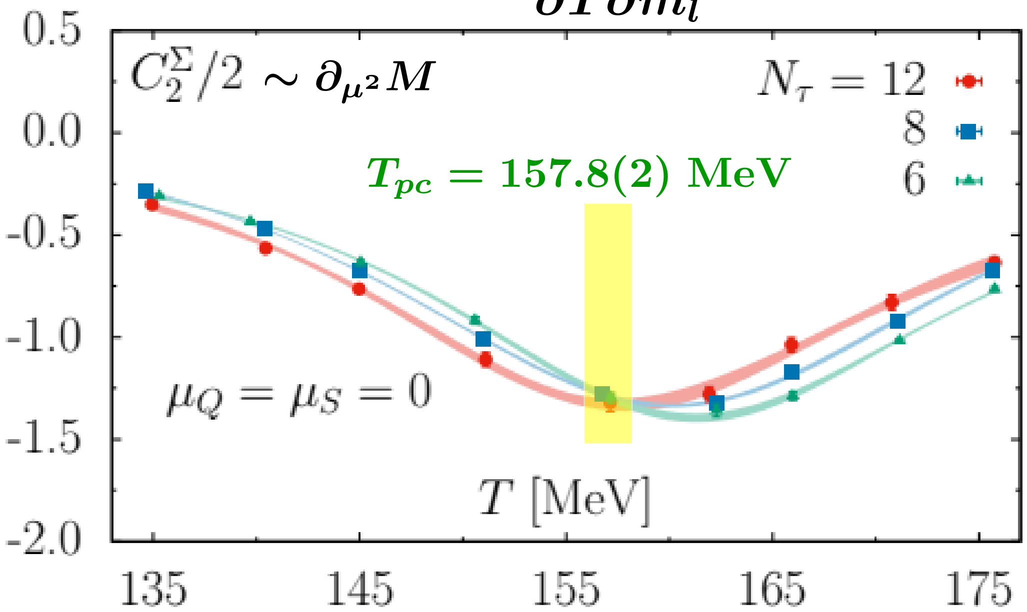
$$X = B,\ S, ...$$

Magnetic, mixed and thermal susceptibilities

magnetic $\sim \frac{\partial^2 \ln Z}{\partial m_l^2}$



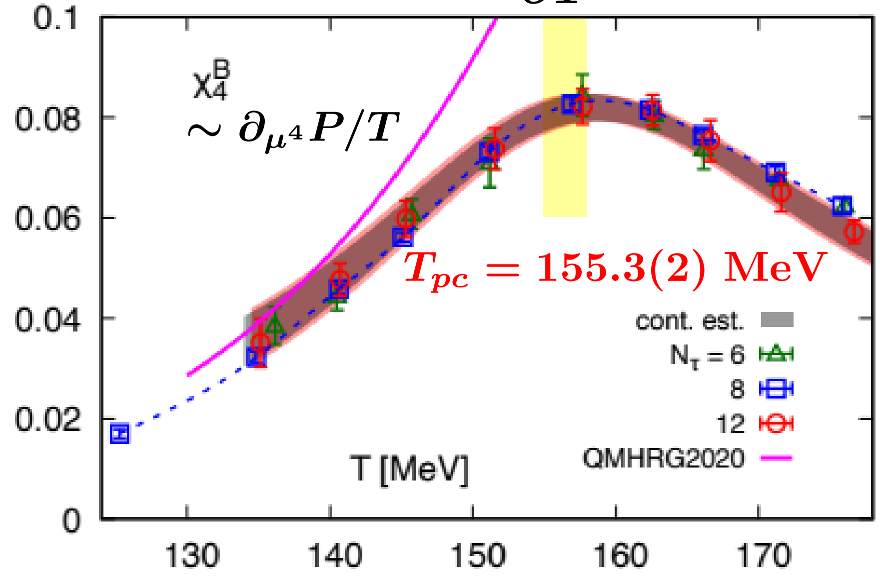
mixed $\sim \frac{\partial^2 \ln Z}{\partial T \partial m_l}$



– T_{pc} is not a unique value

– depends on observable

thermal $\sim \frac{\partial^2 \ln Z}{\partial T^2}$

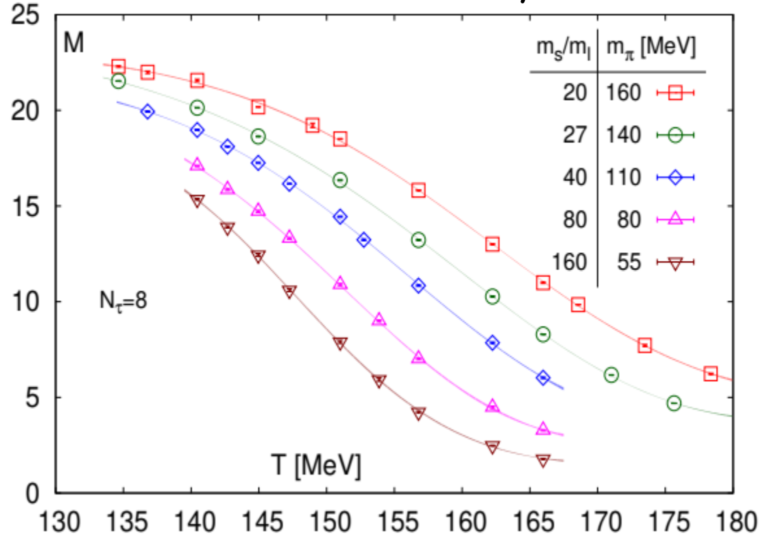


+ 0.5 MeV systematic uncertainty on T-scale setting

The Chiral PHASE TRANSITION in (2+1)-flavor QCD

$m_l \;\Rightarrow\; 0$

$M \sim m_s \partial \ln Z / \partial m_l$



m_s fixed, physical

"magnetic" susceptibility

$$\chi_M \sim \frac{\partial^2 \ln Z}{\partial m_l^2}$$

$$\sim (m_s/m_l)^{0.79}$$

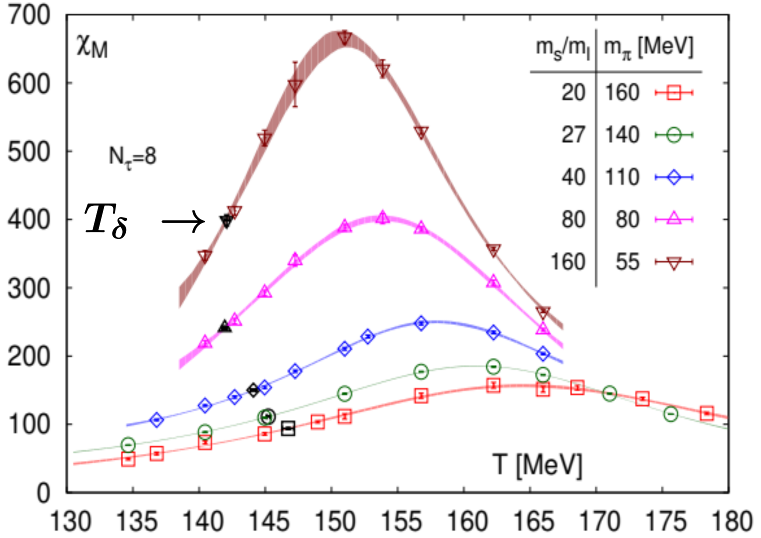
$(160/27)^{0.79} \sim 4$

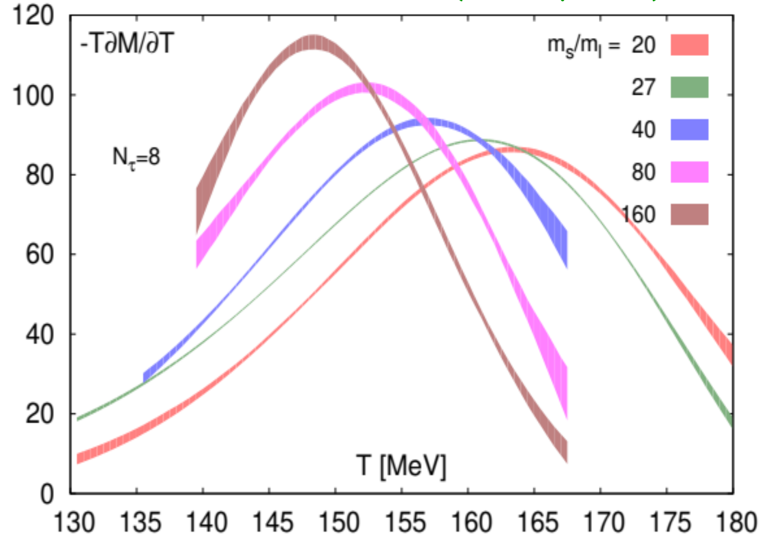
"mixed" susceptibility

$$\chi_t \sim \frac{\partial^2 \ln Z}{\partial T \partial m_l}$$

$$\sim (m_s/m_l)^{0.34}$$

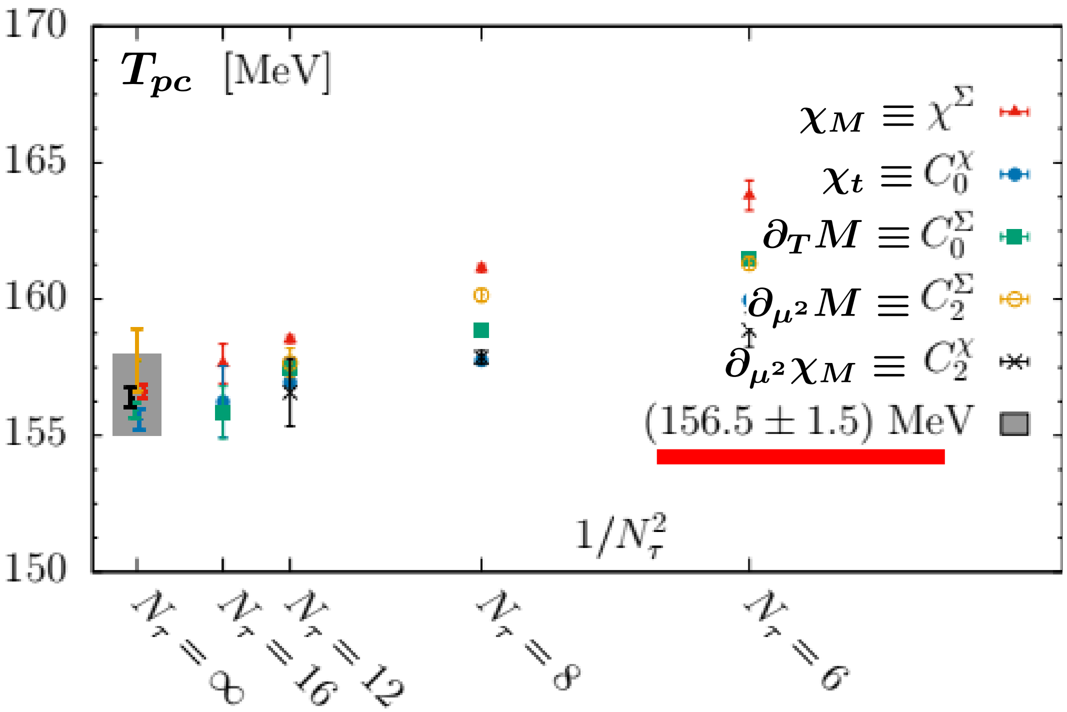
$(160/27)^{0.34} \sim 1.8$





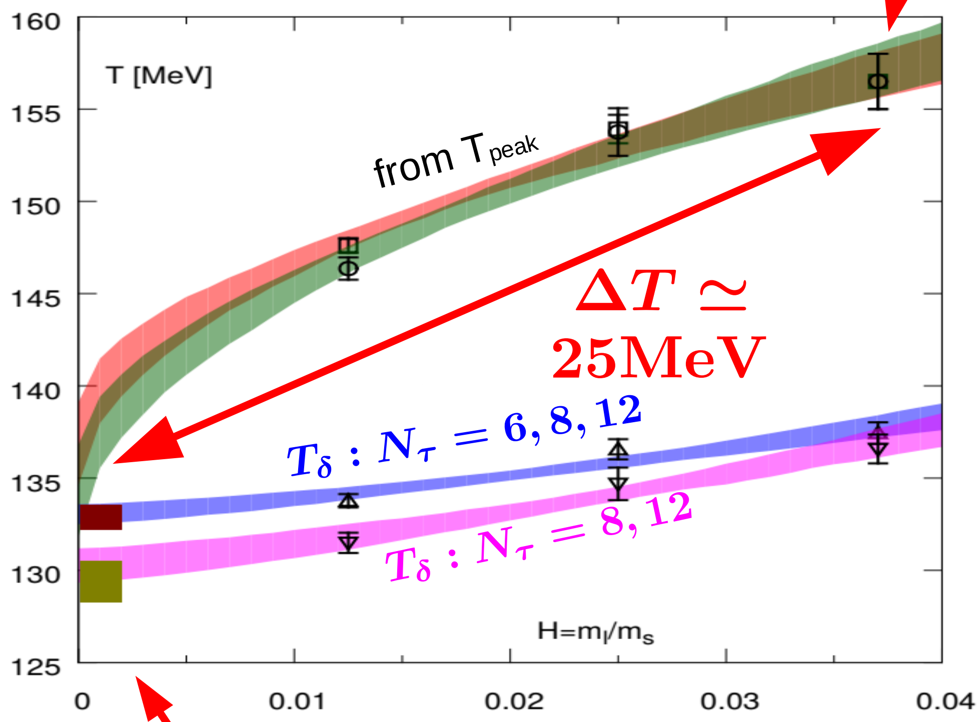
H.T. Ding et al. (HotQCD), PRL 123 (2019) 15, arXiv:1903.04801

Pseudo-critical and critical temperatures



A. Bazavov et al (HotQCD), arXiv:1812.08235

physical masses



H.T Ding et al (HotQCD), arXiv:1903.04801
Anirban Lahiri et al, arXiv:2010:15593

physical masses

$$T_{pc}^{phys} = (156.5 \pm 1.5)\ \mathrm{MeV}$$

chiral limit extrapolations

$$T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$$

also: A. Y. Kotov et al., arXiv: 2105.09842

Curvature of the pseudo-critical line

– μ_B-dependent shift of maxima in susceptibilities

$$\boxed{\frac{\partial \chi_M(T,\mu_B)}{\partial T} = 0 \quad \text{or} \quad \frac{\partial^2 M(T,\mu_B)}{\partial T^2} = 0 \quad \text{or} \quad \frac{\partial^3 M(T,\mu_B)}{\partial T \partial \mu_B^2} = 0}$$

– Taylor series, e.g.

$$\begin{aligned} \chi_M(T,\mu_B) &= \chi_M(T_{pc},0) \\ &\quad + \frac{\partial \chi_M}{\partial T}(T-T_{pc}) + \frac{1}{2}\frac{\partial^2 \chi_M}{\partial (\mu_B/T)^2}\bigg|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^2 + \ldots \end{aligned}$$

$$\Longrightarrow \quad \kappa_2 \simeq \frac{T^2 \partial^2 \chi_M / \partial \mu_B^2}{2T \partial \chi_M / \partial T}\bigg|_{\mu_B=0}$$

$$\Longrightarrow \quad T_{pc}(\mu_B) = T_{pc}\left(1 - \kappa_2\left(\frac{\mu_B}{T}\right)^2 - \kappa_4\left(\frac{\mu_B}{T}\right)^4 + \ldots\right)$$

Curvature of the pseudo-critical line
– towards the chiral limit –

– universality relations also relate derivatives with respect to T and μ_B

$$\frac{1}{VT^3}\ln Z(V,T,\vec{\mu}) \sim -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta})$$

$$\boxed{h \sim \frac{m_l}{T_c}\ ,\ t \sim \frac{T-T_c}{T_c} + \kappa_2 \left(\frac{\mu_B}{T}\right)^2} \Longleftrightarrow \boxed{\frac{\partial^2}{\partial(\mu_B/T)^2} \simeq \frac{\partial}{\partial T}}$$

FK et al., arXiv:1009.5211

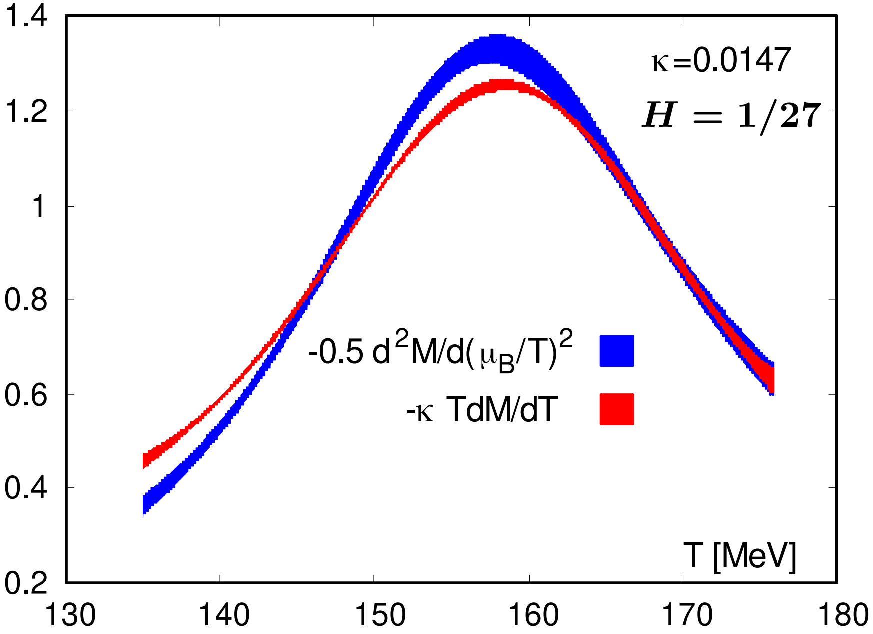
$$\begin{aligned} M(T,\mu_B) &= M(T_c,0) \\ &\quad + \frac{\partial M}{\partial T}(T-T_c) + \frac{1}{2}\frac{\partial^2 M}{\partial(\mu_B/T)^2}\bigg|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^2 + \ldots \end{aligned}$$

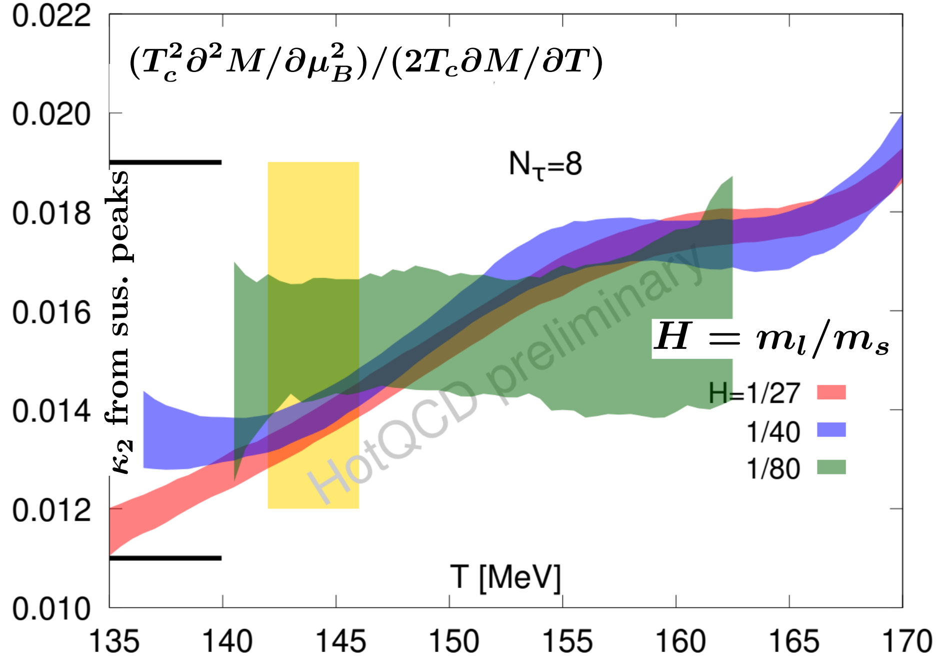
$$\Longrightarrow \qquad \kappa_2 \simeq \frac{T^2\partial^2\chi_M/\partial\mu_B^2}{2T\partial\chi_M/\partial T}\bigg|_{\mu_B=0}$$

Curvature of the pseudo-critical line
– towards the chiral limit –

derivatives of the chiral condensate ⟺ susceptibilities

$$M(T,\mu_B) = M(T_c,0) + \frac{\partial M}{\partial T}(T-T_c) + \frac{1}{2}\frac{\partial^2 M}{\partial(\mu_B/T)^2}\bigg|_{\mu_B=0}\left(\frac{\mu_B}{T}\right)^2 + \ldots$$





Mugdha Sarkar (HotQCD), Lattice 2021
arXiv:2112.15398

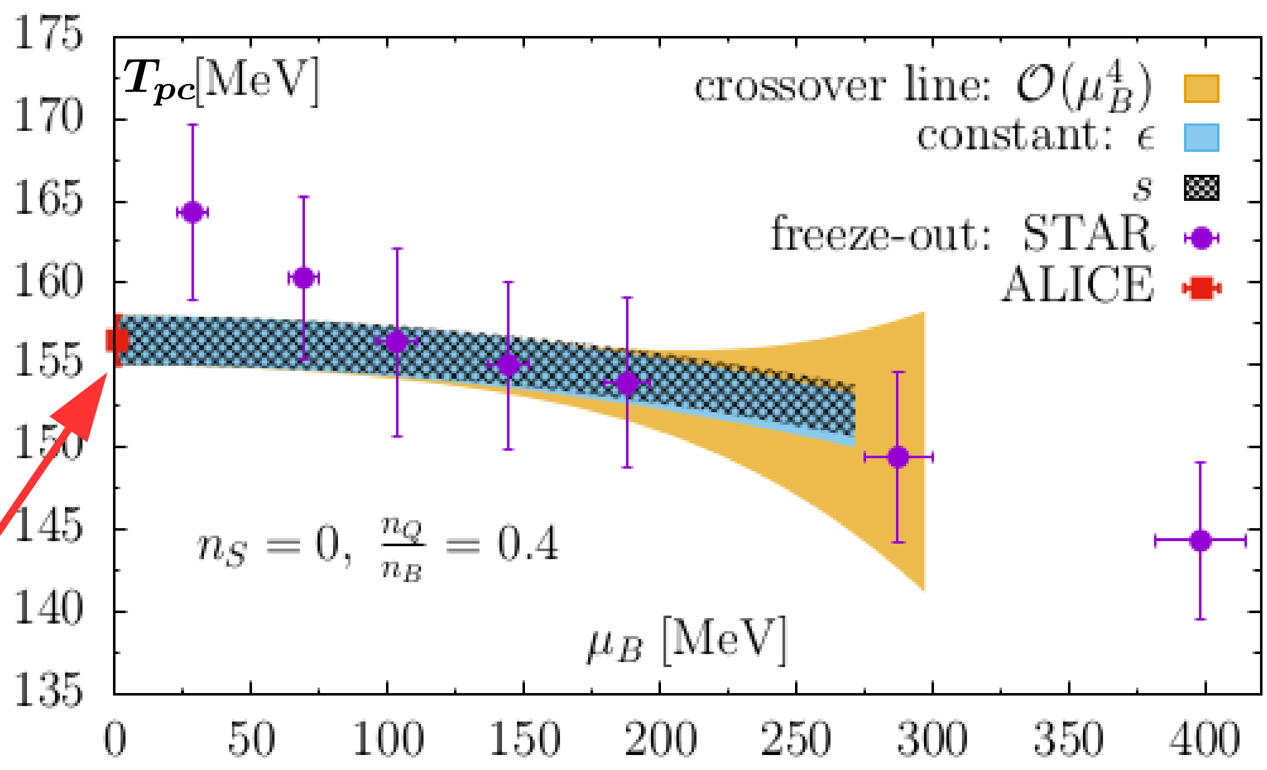
$$t \sim \frac{T-T_c}{T_c} + \kappa_2(H)\left(\frac{\mu_B}{T}\right)^2 , \; H = m_l/m_s$$

curvature of crossover line only mildly dependent on H

Phases of strong-interaction matter

$$T_{pc}(\mu_B) = T_{pc}\left(1 - \kappa_2\left(\frac{\mu_B}{T}\right)^2 - \kappa_4\left(\frac{\mu_B}{T}\right)^4 + \ldots\right)$$

phase diagram at physical values of the quark masses



STAR: arXiv:1701.07065

A. Andronic et al., Nature 561 (2018) 321

$T_{pc} = (156.5 \pm 1.5)$ MeV

$\kappa_2 = 0.012(4)$

$\kappa_4 = 0.000(4)$

A. Bazavov et al. [HotQCD], Phys. Lett. B795 (2019), arXiv:1812.08235

$T_{pc} = (158.0 \pm 0.6)$ MeV

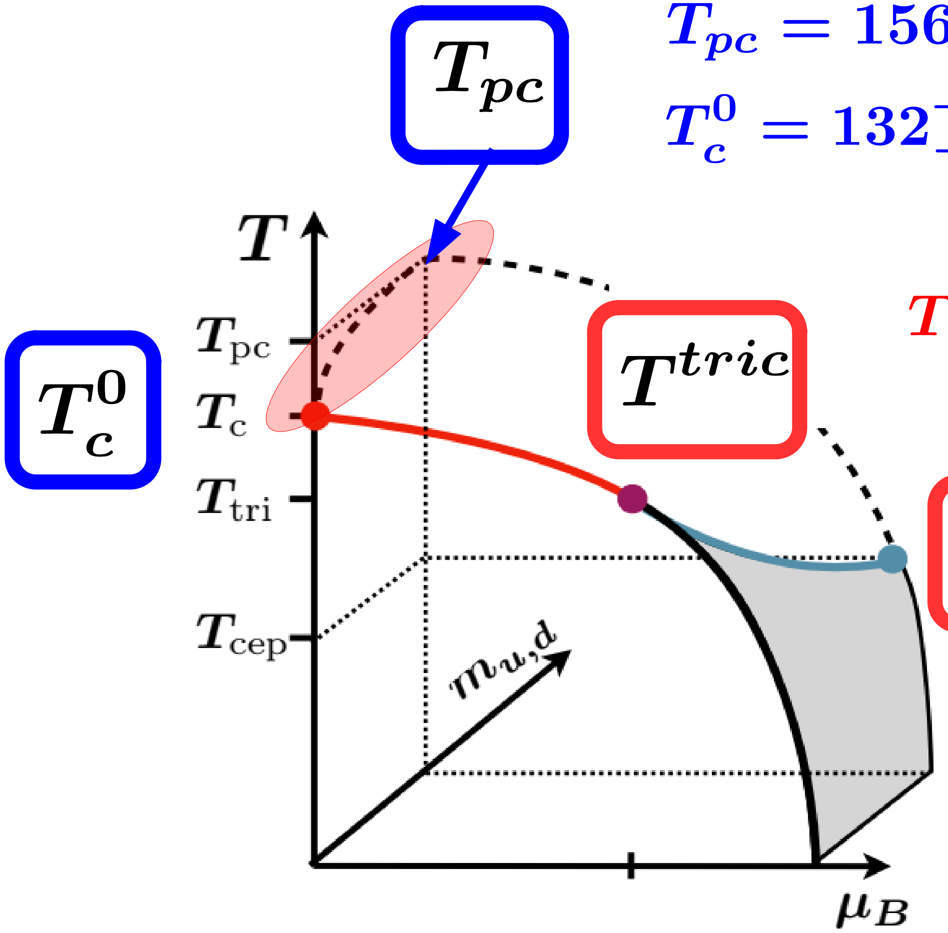
$\kappa_2 = 0.0153(18)$

$\kappa_4 = 0.00032(67)$

S. Borsanyi, et al, PRL 125 (2020) arXiv:2002.02821

Summary: Phases of strong-interaction matter

determination of T_c^0 puts an upper limit on T^{CEP}



$$T_{pc} = 156.5(1.5)\mathrm{MeV}$$

$$T_c^0 = 132^{+3}_{-6}\mathrm{MeV}$$

A. Bazavov et al (HotQCD), arXiv:1812.08235

H.T. Ding et al (HotQCD), arXiv:1903.04801

$$T^{tric} \simeq T_c^0 - 2.2(5)\ \mathrm{MeV} \left(\frac{\mu_B}{T_c}\right)^2_{tric}$$

HotQCD@Lattice2021 (M. Sarkar)
arXiv:2112.15398

constraint on the location of the CEP

$$T^{CEP} < T^{tric} < T_c^0$$

$$\boxed{T^{CEP} < 135\ \mathrm{MeV}}$$

the temperature range below T_{pc} is most important for getting information on a possible CEP

Summary: Phases of strong-interaction matter

determination of T_c^0 puts an upper limit on T^{CEP}

$T_{pc} = 156.5(1.5)\mathrm{MeV}$

$T_c^0 = 132^{+3}_{-6}\mathrm{MeV}$

A. Bazavov et al (HotQCD), arXiv:1812.08235
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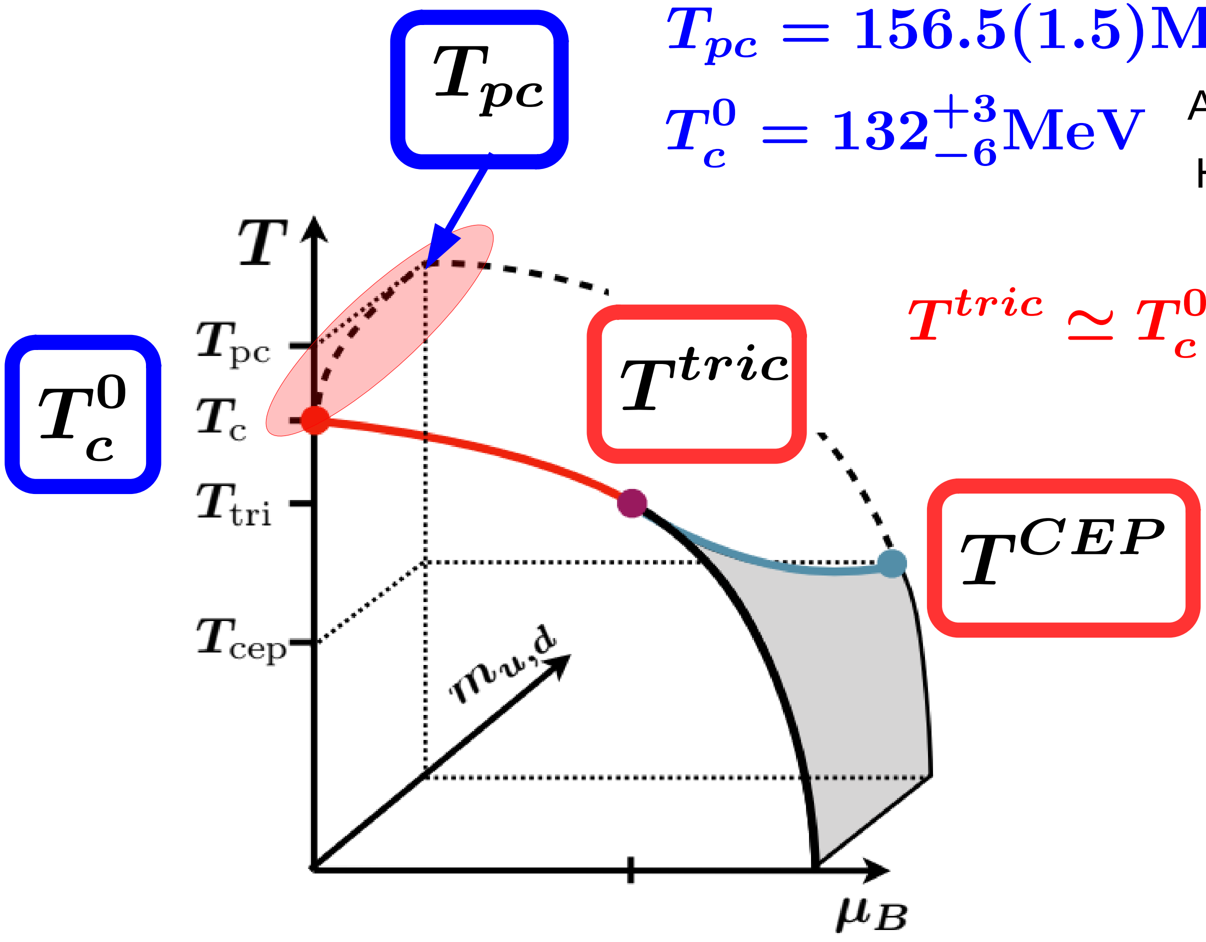

$$T^{tric} \simeq T_c^0 - 2.2(5)\ \mathrm{MeV} \left(\frac{\mu_B}{T_c}\right)^2_{tric}$$

HotQCD@Lattice2021 (M. Sarkar)
arXiv:2112.15398

more likely, if

$(\mu_B/T)_{tric} \geq 2$

$T^{CEP} < T^{tric} < T_c^0$

$\boxed{T^{CEP} < 125\ \mathrm{MeV}}$

the temperature range below T_{pc} is most important for getting information on a possible CEP

QCD thermodynamics at non-zero net baryon-density – Taylor expansion –

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\boxed{\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k}$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left.\frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

QCD thermodynamics at non-zero net baryon-density – Taylor expansion –

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$$\boxed{\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k}$$

I) $\mu_S = 0 : \tilde{\chi}_0^{B,n} = \chi_{n00}^{BQS} \equiv \chi_n^B$

II) $n_S = 0 : \hat{\mu}_S \rightarrow \hat{\mu}_S(\hat{\mu}_B) = s_1 \hat{\mu}_B + s_3 \hat{\mu}_B^3 + ...$

$n_Q/n_B = r : \hat{\mu}_Q \rightarrow \hat{\mu}_Q(\hat{\mu}_B) = q_1 \hat{\mu}_B + q_3 \hat{\mu}_B^3 + ...$

isospin symmetric matter ⟺ $\mu_Q = 0 \Leftrightarrow n_Q/n_B = 0.5$

conditions met in heavy ion collisions differ little from (II), e.g. $n_Q/n_B = 0.4$

QCD thermodynamics at non-zero net baryon-density
– Taylor expansion –

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

Constraints on μ_Q & μ_S

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + P_2(T) \left(\frac{\mu_B}{T}\right)^2 + P_4(T) \left(\frac{\mu_B}{T}\right)^4 + \dots$$

$$\frac{n_B(T,\mu_B)}{T^3} = \frac{\mathrm{d}P/T^4}{\mathrm{d}\hat{\mu}_B} = N_1(T) \left(\frac{\mu_B}{T}\right) + N_3(T) \left(\frac{\mu_B}{T}\right)^3 + \dots$$

Taylor expansion to 8th order in μ_B/T

D. Bollweg et al (HotQCD), arXiv:2202.09184 and QM 2022

HotQCD data collection for (2+1)-flavor QCD-EoS

EoS:2017:
arXiv:1701.04325

$N_\tau = 6$				$N_\tau = 8$				$N_\tau = 12$			
β	m_l	T[MeV]	#conf.	β	m_l	T[MeV]	#conf.	β	m_l	T[MeV]	#conf.
5.980	0.00435	135.29	81200	6.245	0.00307	134.64	180320	6.640	0.00196	134.94	5834
6.010	0.00416	139.71	120790	6.285	0.00293	140.45	172110	6.680	0.00187	140.44	5833
6.045	0.00397	145.05	120770	6.315	0.00281	144.95	138150	6.712	0.00181	144.97	13846
6.080	0.00387	150.59	79390	6.354	0.00270	151.00	107510	6.754	0.00173	151.10	14200
6.120	0.00359	157.17	66180	6.390	0.00257	156.78	135730	6.794	0.00167	157.13	15476
6.150	0.00345	162.28	79660	6.423	0.00248	162.25	115850	6.825	0.00161	161.94	16772
6.170	0.00336	165.98	49760	6.445	0.00241	165.98	120270	6.850	0.00157	165.91	19542
6.200	0.00324	171.15	122700	6.474	0.00234	171.02	139980	6.880	0.00153	170.77	21220
6.225	0.00314	175.76	122730	6.500	0.00228	175.64	133070	6.910	0.00148	175.76	12303

added O(10) times new confs.

added O(20) times new confs.

all new

EoS 2022:
arXiv:2202.09184

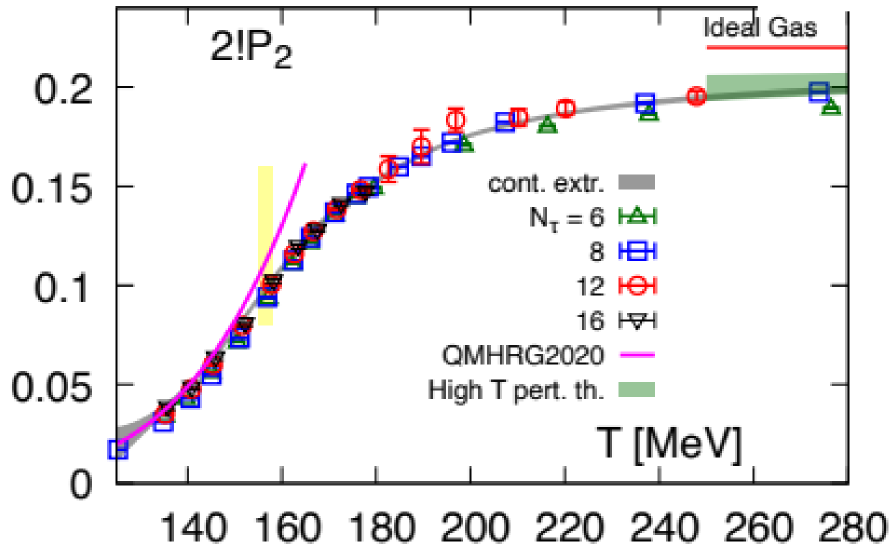
$N_\tau = 8$				$N_\tau = 12$				$N_\tau = 16$			
β	m_l	T[MeV]	#conf.	β	m_l	T[MeV]	#conf.	β	m_l	T[MeV]	#conf.
6.175	0.003307	125.28	2,200,000								
6.245	0.00307	134.84	1,275,380	6.640	0.00196	135.24	330,447	6.935	0.00145	135.80	17671
6.285	0.00293	140.62	1,598,555	6.680	0.00187	140.80	441,115	6.973	0.00139	140.86	23855
6.315	0.00281	145.11	1,559,003	6.712	0.00181	145.40	416,703	7.010	0.00132	145.95	26122
6.354	0.00270	151.14	1,286,603	6.754	0.00173	151.62	323,738	7.054	0.00129	152.19	26965
6.390	0.00257	156.92	1,602,684	6.794	0.00167	157.75	299,029	7.095	0.00124	158.21	21656
6.423	0.00248	162.39	1,437,436	6.825	0.00161	162.65	214,671	7.130	0.00119	163.50	18173
6.445	0.00241	166.14	1,186,523	6.850	0.00157	166.69	156,111	7.156	0.00116	167.53	19926
6.474	0.00234	171.19	373,644	6.880	0.00153	171.65	144,633	7.188	0.00113	172.60	17163
6.500	0.00228	175.84	294,311	6.910	0.00148	176.73	131,248	7.220	0.00110	177.80	3282

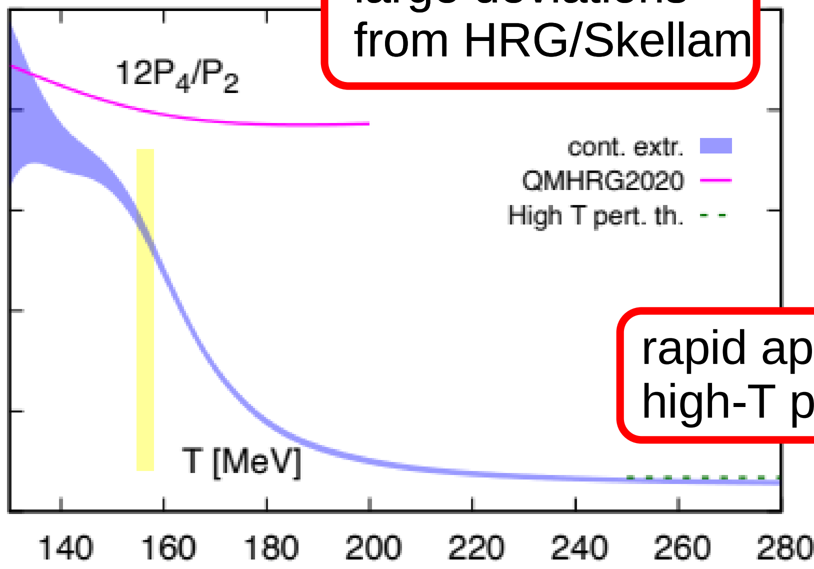
Up to 8th order Taylor expansion for pressure

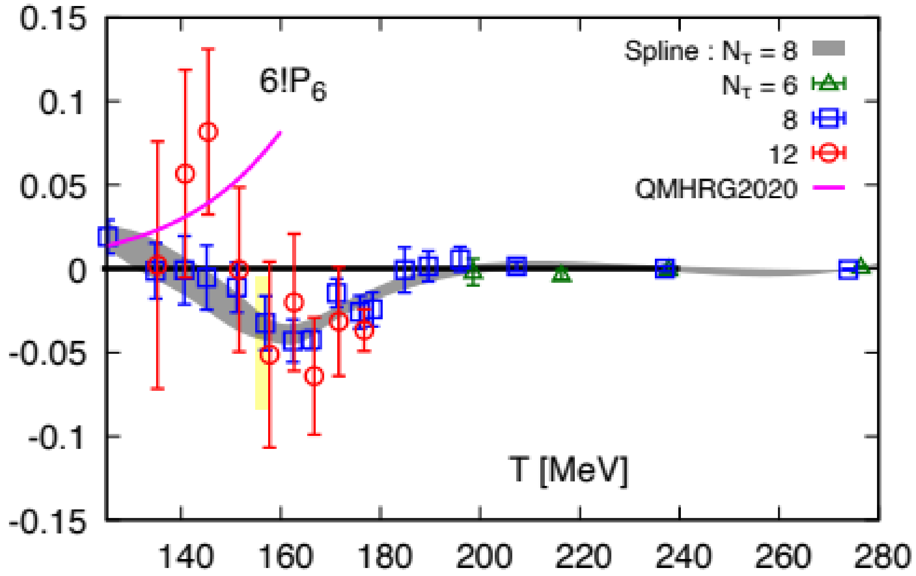
$$P_n \equiv \frac{\tilde{\chi}_n^B}{n!}$$

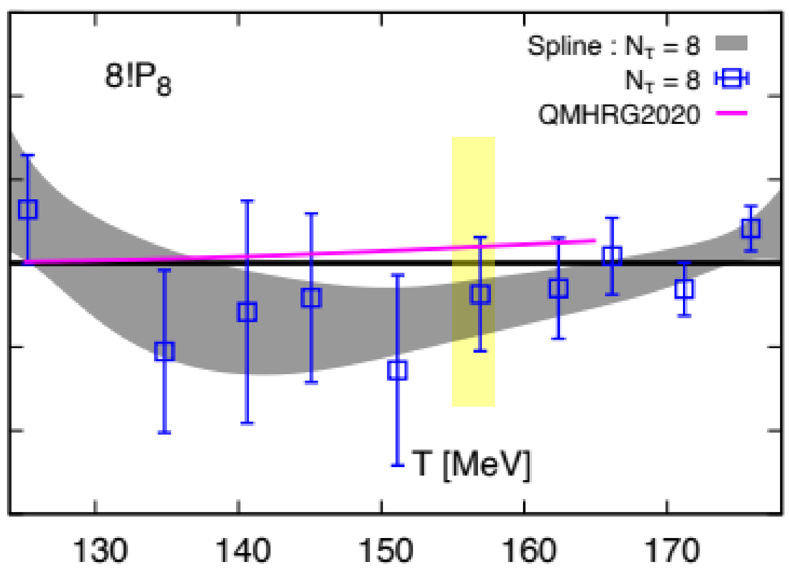
large deviations from HRG/Skellam

rapid approach to high-T pert. theory



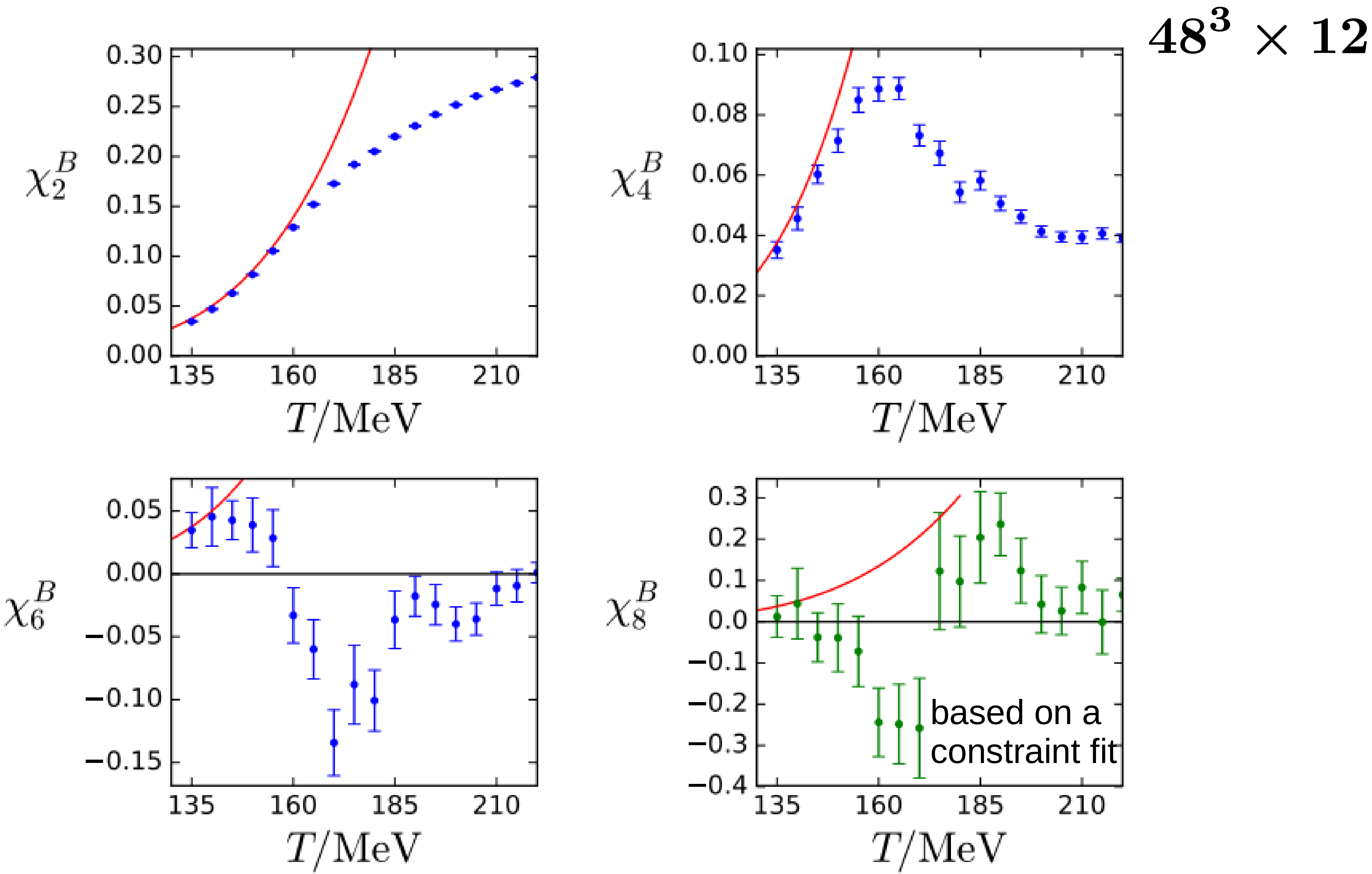






A. Bazavov et al. (HotQCD), Phys. Rev. D 105 (2022) 074511, arXiv:2202.09184 & arXiv:1701.04325

Up to 8th order cumulants are used frequently
– imag. chem. pot. extrapolations –



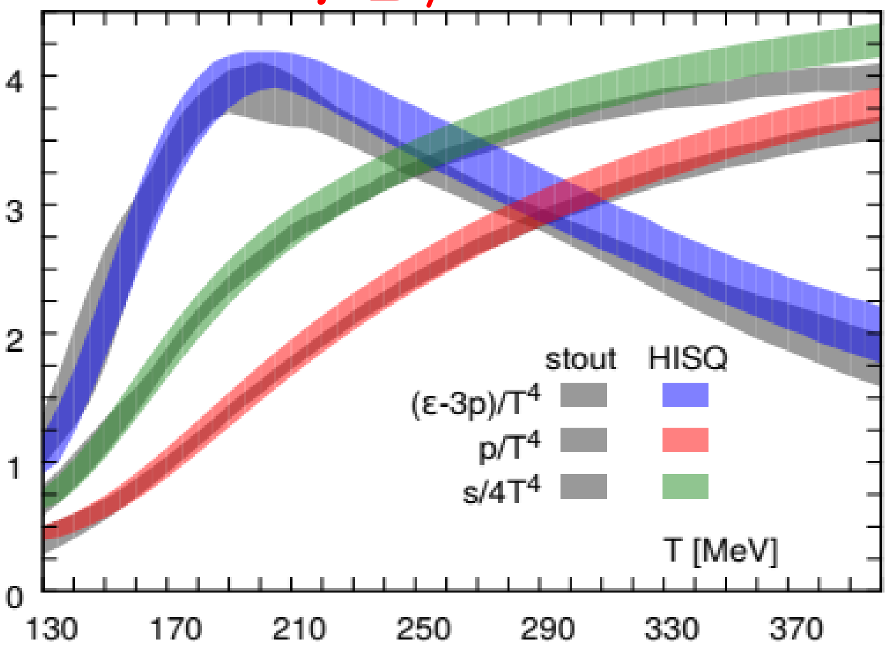
S. Borsanyi et al. , JHEP 10 (2018) 205, arXiv:1805.04445

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

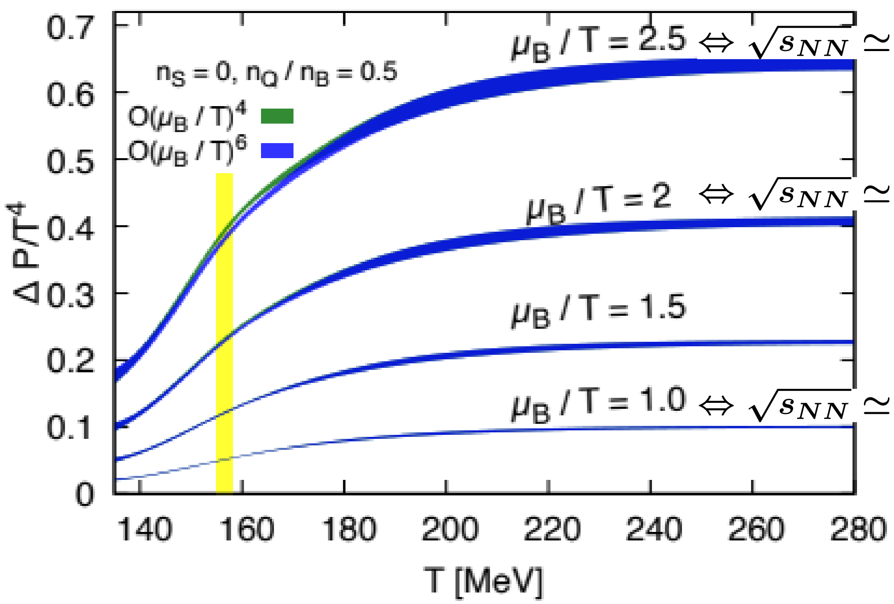
$$\frac{\Delta p(T,\mu_B)}{T^4} = \frac{p(T,\mu_B) - p(T,0)}{T^4} = P_2(T)\left(\frac{\mu_B}{T}\right)^2 + P_4(T)\left(\frac{\mu_B}{T}\right)^4 + P_6(T)\left(\frac{\mu_B}{T}\right)^6 + \ldots$$

EoS 2017:

$\mu_B/T = 0$



$\mu_B/T = 2$: (10-30)% contribution to total pressure at

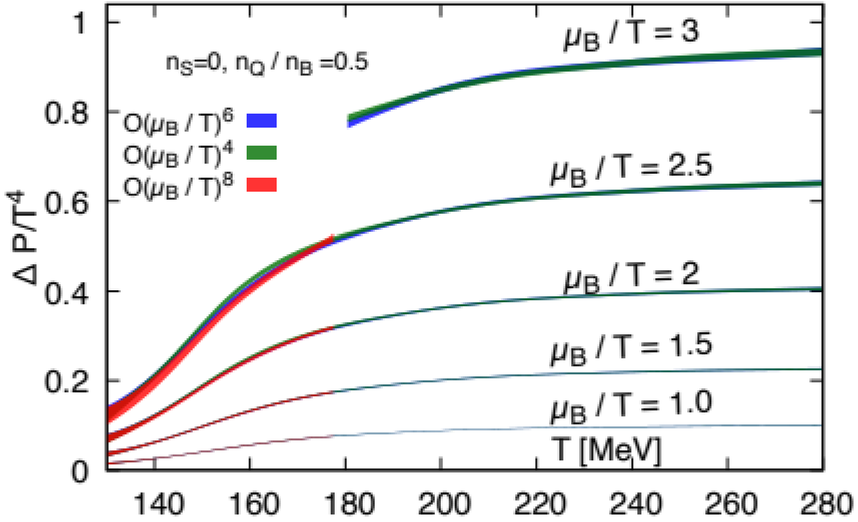


A. Bazavov et al (HotQCD), arXiv:1701.04325

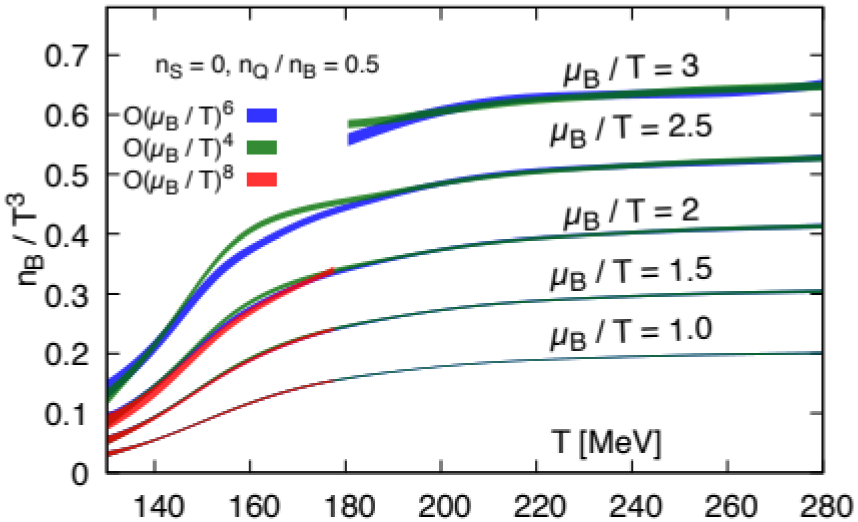
D. Bollweg et al (HotQCD), arXiv:2202.09184

EoS 2022: Taylor series to 8th order

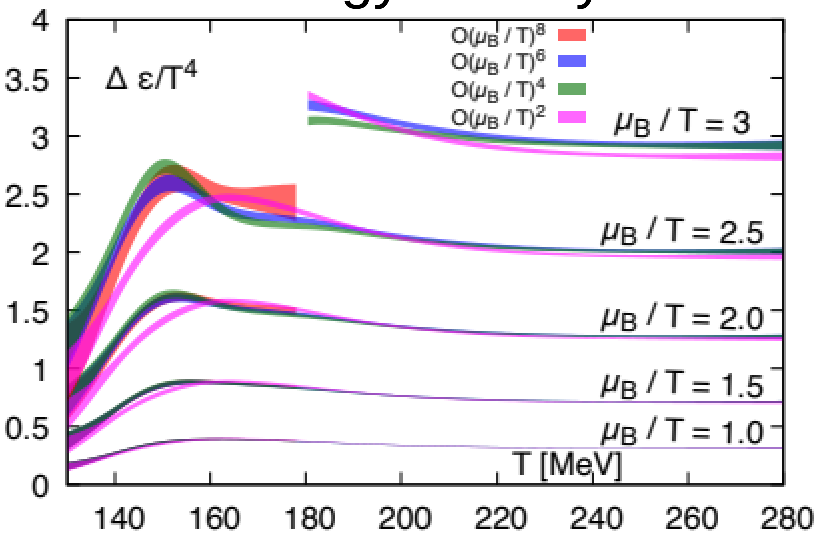
pressure



net baryon-number density



energy density



range of reliability of 8th order results depends on T-region

$$T \geq 200\ \mathrm{MeV} \quad \Rightarrow \quad \mu_B/T \geq 3$$

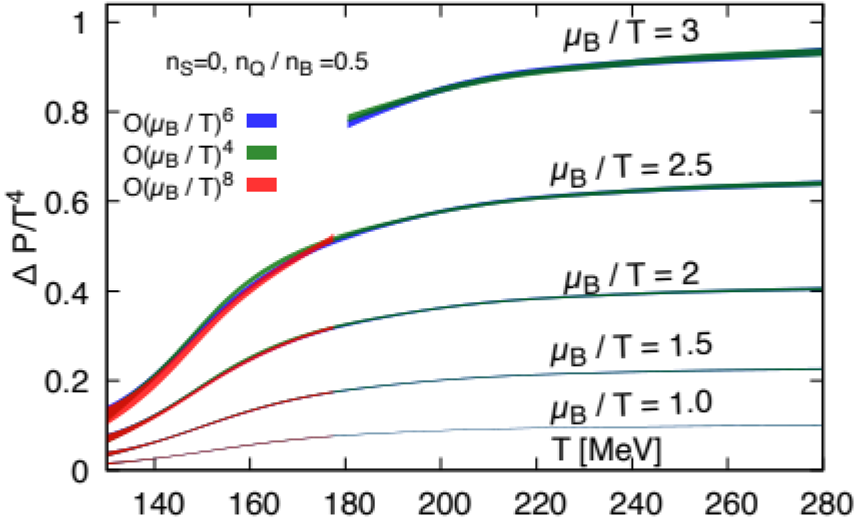
$$T \leq 200\ \mathrm{MeV} \quad \Rightarrow \quad \mu_B/T \simeq 2.5$$

...but limited by statistics

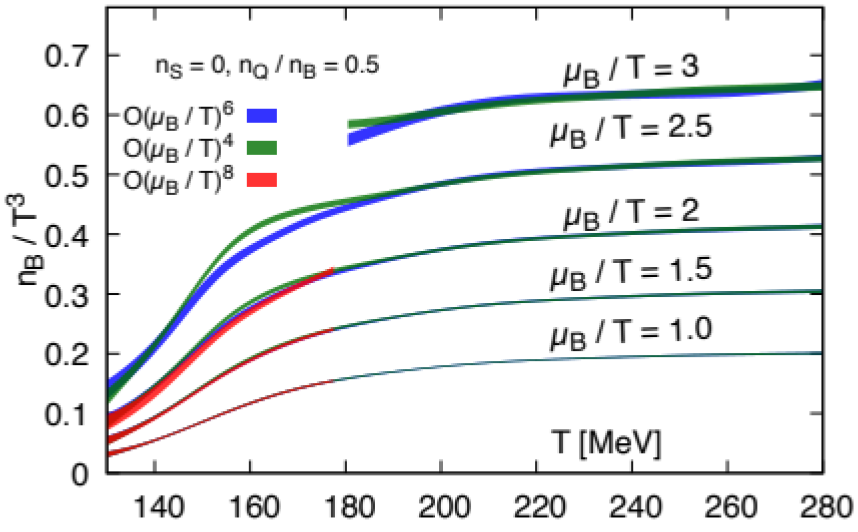
D. Bollweg et al (HotQCD), arXiv:2202.09184
HotQCD in preparation

EoS 2022: Taylor series to 8th order

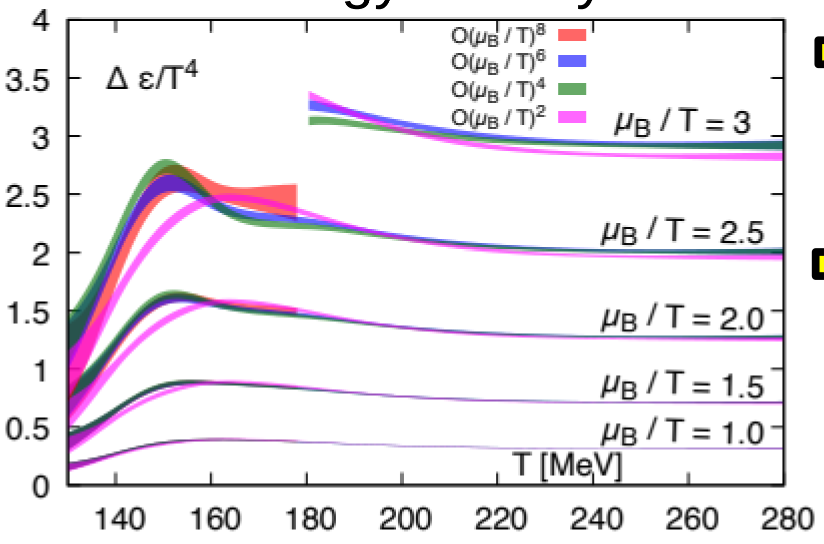
pressure



net baryon-number density



energy density



The EoS (pressure & number density) is well controlled for $\mu_B/T \leq 2$ or equivalently $\sqrt{s_{NN}} \geq 11$ GeV

convergence of expansions for higher order derivatives slower, but same radius of convergence as pressure

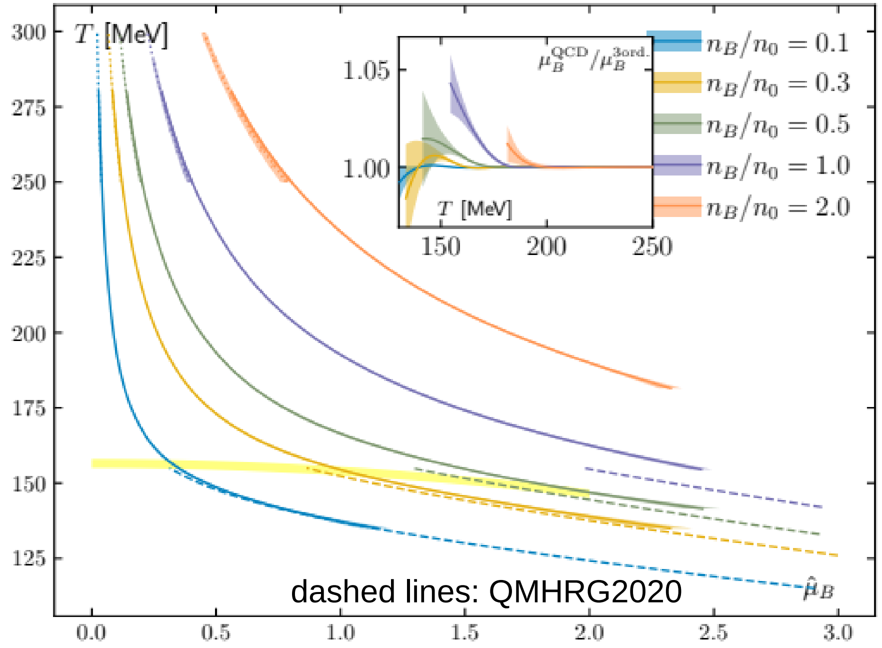
D. Bollweg et al (HotQCD), arXiv:2202.09184
HotQCD in preparation

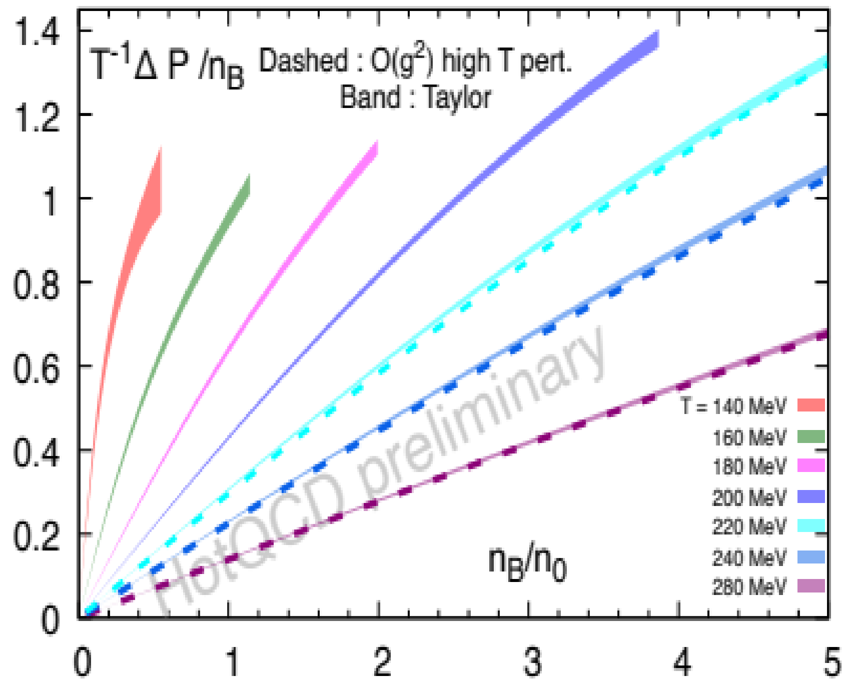
EoS 2022: Taylor series to 8th order

eliminating the chemical potential

$\longrightarrow \quad \Delta p(T, n_B)$

hatched bands:
O(g²) pert.th.





4th order Taylor series is a good approximation in this parameter range

⇨ **closed expression for EoS**

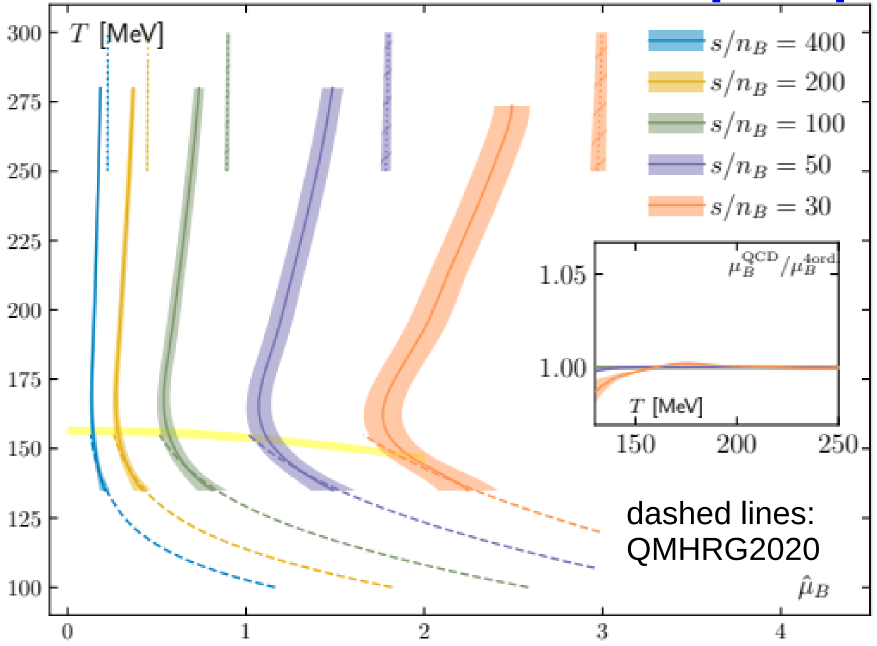
$$\hat{\mu}_B(T, n_B) = y(T, n_B)^{1/3} - \frac{N_1(T)}{3N_3(T)} y(T, n_B)^{-1/3} \, , \qquad \hat{\mu}_B \equiv \mu_B/T$$

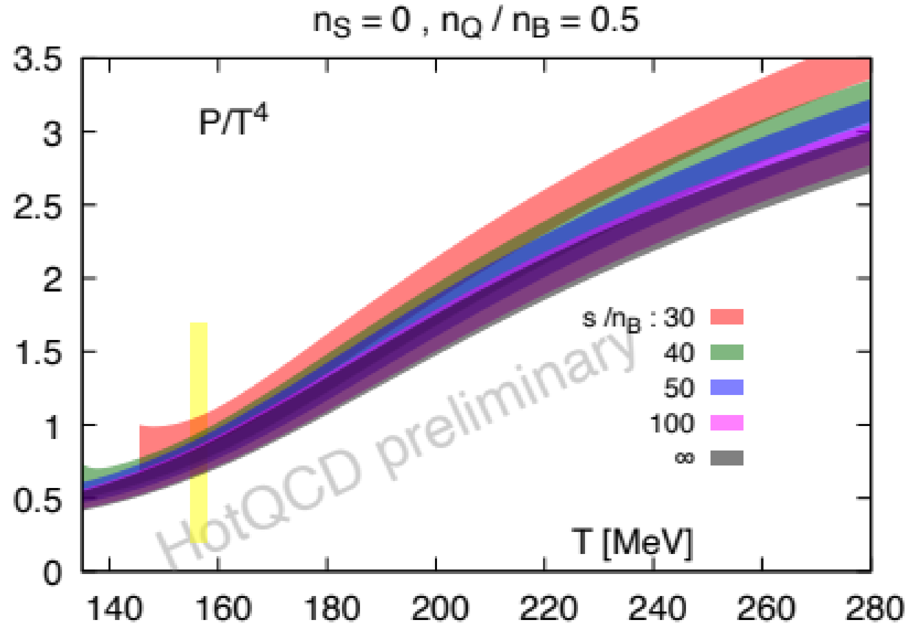
$$y(T, n_B) = \frac{N_1(T)}{N_3(T)} \left(\frac{\hat{n}_B}{2N_1(T)} + \sqrt{\frac{N_1(T)}{27N_3(T)} + \left(\frac{\hat{n}_B}{2N_1(T)}\right)^2} \right) \, , \quad \hat{n}_B \equiv n_B/T^3$$

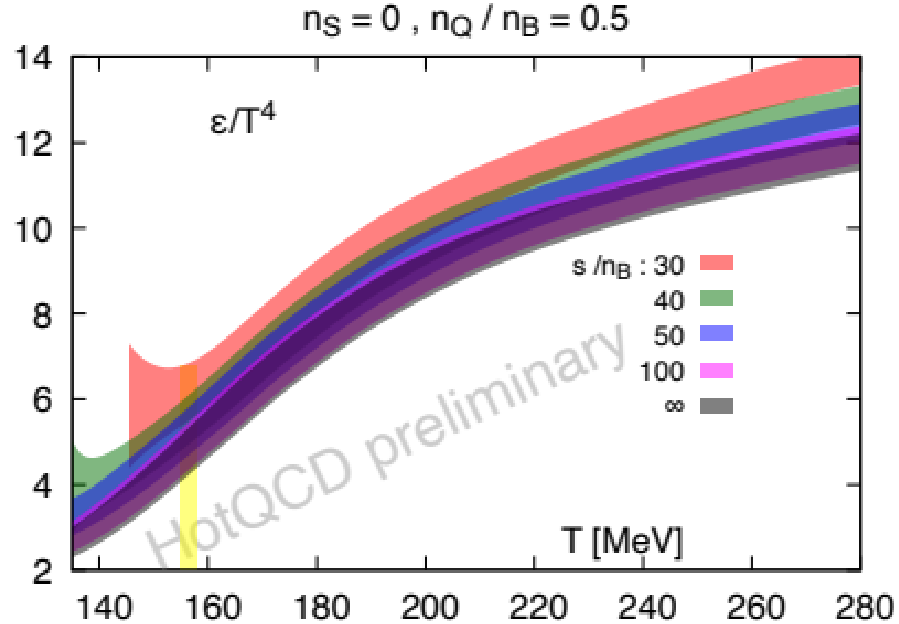
Pressure and energy density on
lines of constant s/n$_B$

$\sqrt{s_{NN}} \simeq$

200 39 27 11.5 [GeV]







– 4th order Taylor series suffices to describe EoS in almost the entire parameter range covered by BES (collider mode)

HotQCD, in preparation

Comparing Taylor series and Pade resummation

Taylor series

$$\frac{\Delta p}{T^4} \equiv \frac{p(T,\mu_B)}{T^4} - \frac{p(T,0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T)\left(\frac{\mu_B}{T}\right)^{2k}$$

$$= \frac{P_2^2}{P_4}\left(\bar{x}^2 + \bar{x}^4 + c_{6,2}\bar{x}^6 + c_{8,2}\bar{x}^8 + \ldots\right)$$

use $P_2 > 0,\ P_4 > 0$

$$\bar{x} \equiv \sqrt{P_4/P_2}\,(\mu_B/T)$$

$$c_{6,2} = \frac{P_6 P_2}{P_4^2}\ ,\quad c_{8,2} = \frac{P_8 P_2^2}{P_4^3}$$

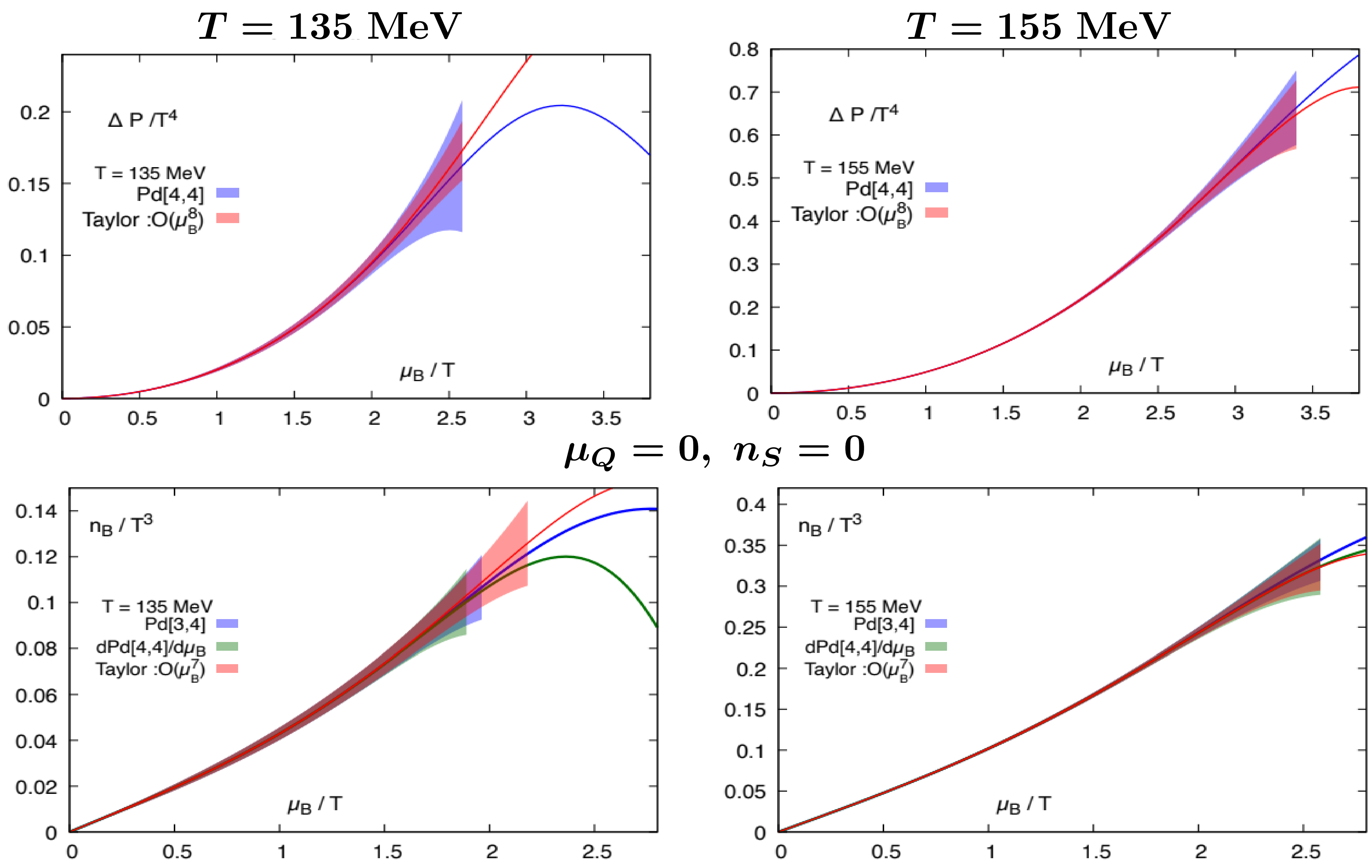
Pade approximation

$$P_{[4,4]} = \frac{(1-c_{6,2})\bar{x}^2 + (1-2c_{6,2}+c_{8,2})\,\bar{x}^4}{(1-c_{6,2}) + (c_{8,2}-c_{6,2})\bar{x}^2 + (c_{6,2}^2 - c_{8,2})\bar{x}^4}$$

pressure: $\left.\frac{p(T,\mu_B)}{T^4}\right|_{[nm]} = \frac{P_2^2}{P_4} P_{[nm]}$ (similar for n_B/T^3, e/T^4.....) OR .....

energy density: $\left(\frac{\Delta\epsilon}{T^4}\right)_{[nm]} = 3\left(\frac{\Delta p}{T^4}\right)_{[nm]} + T\frac{\mathrm{d}}{\mathrm{d}T}\left(\frac{P_2^2}{P_4}P_{[nm]}\right)$

Comparing Taylor series and Pade resummation

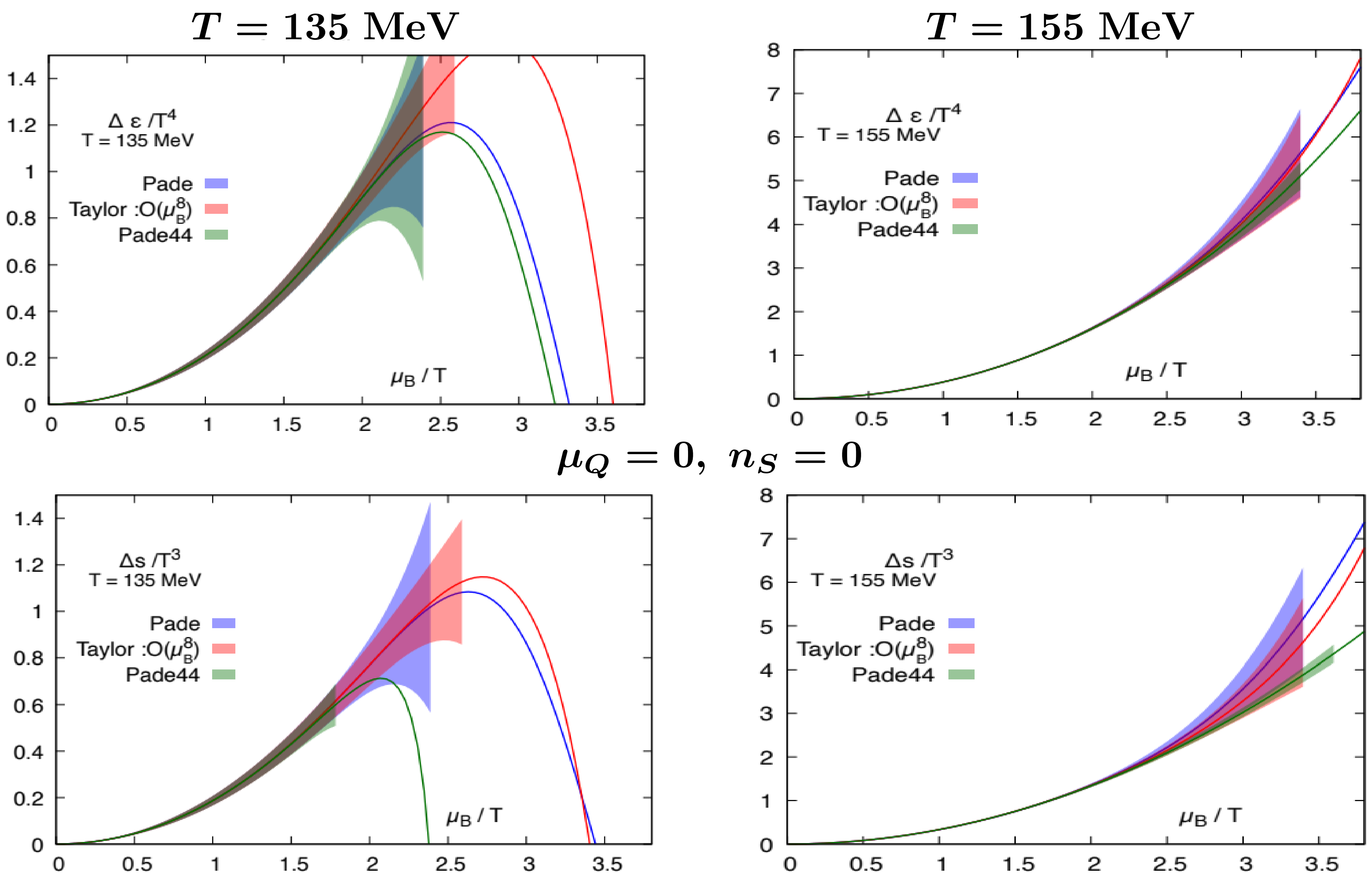


agreement between Taylor series and Pade approximants in a larger μ_B range at higher temperature; qualitatively similar μ_B dependence

HotQCD in preparation

& arXiv:2202.09184

Comparing Taylor series and Pade resummation

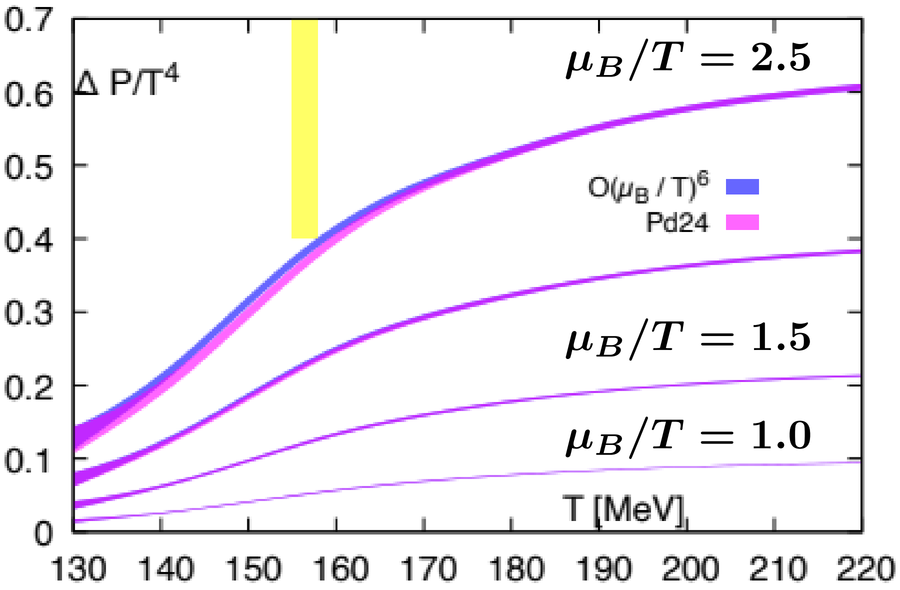


agreement between Taylor series and Pade approximants in a larger μ_B range at higher temperature; qualitatively similar μ_B dependence

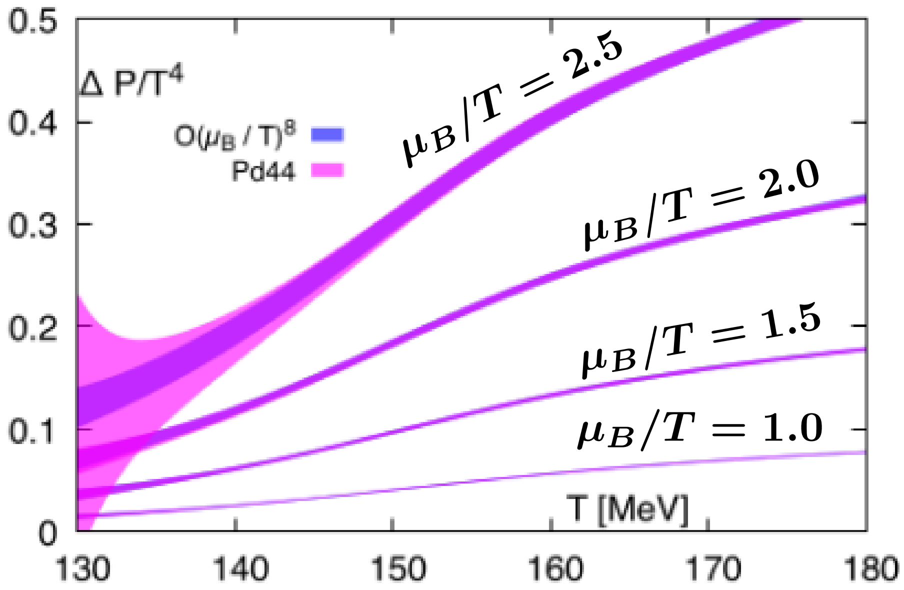
HotQCD in preparation
& arXiv:2202.09184

EoS 2022: Taylor series versus Pade

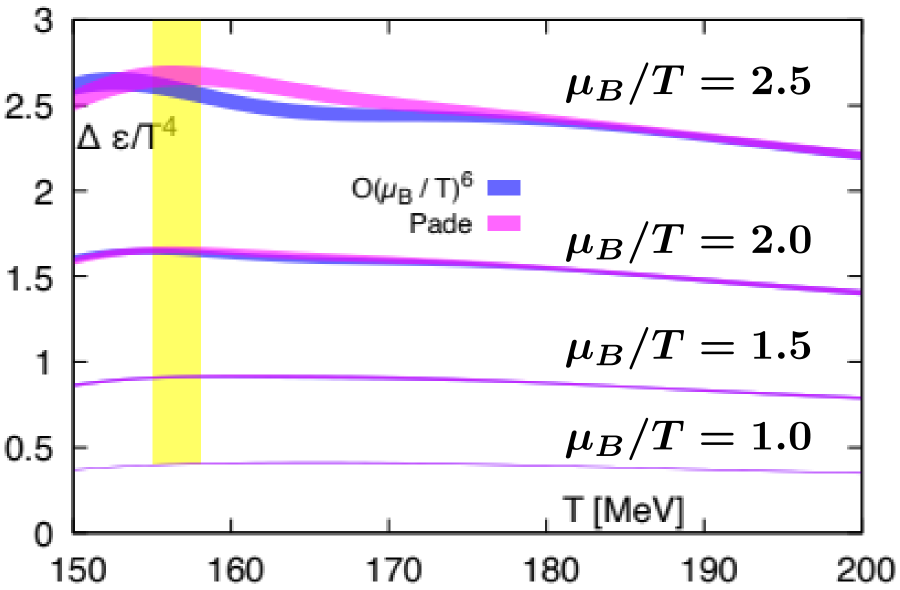
pressure 6th order and [2,4] Pade



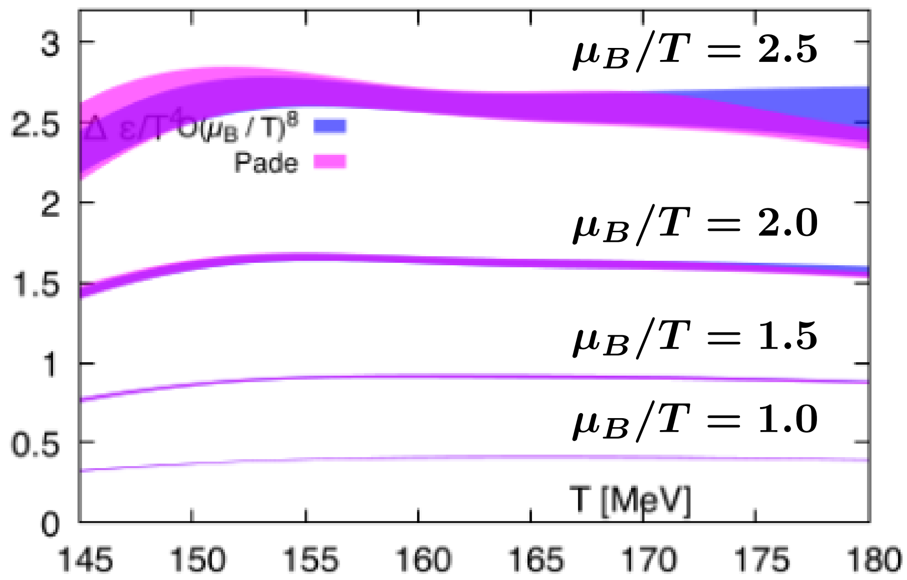
pressure 8th order and [4,4] Pade



energy 6th order and Pade-deriv.



energy 8th order and Pade-deriv.



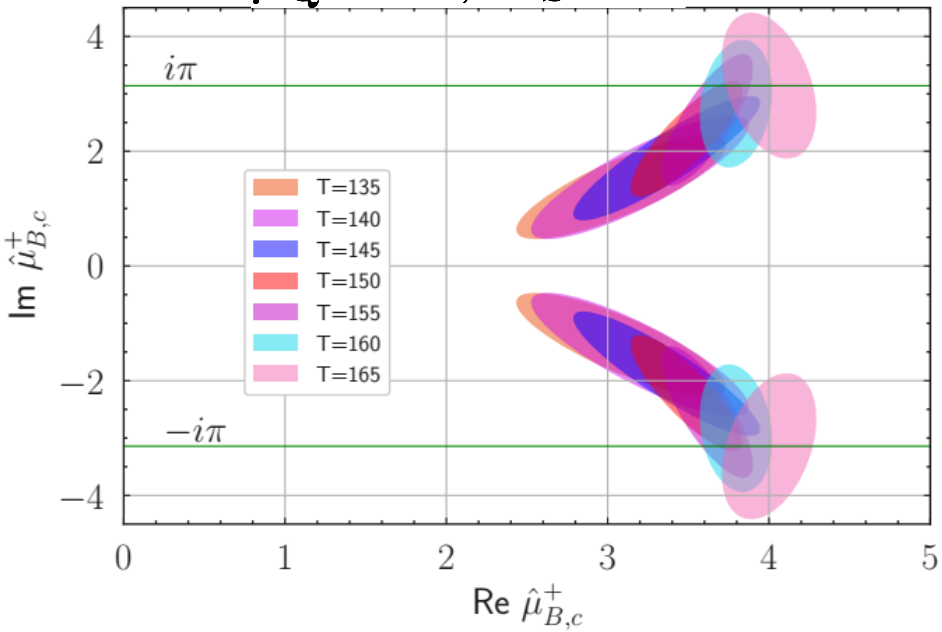
Poles of [n,n] Pade approximants in QCD

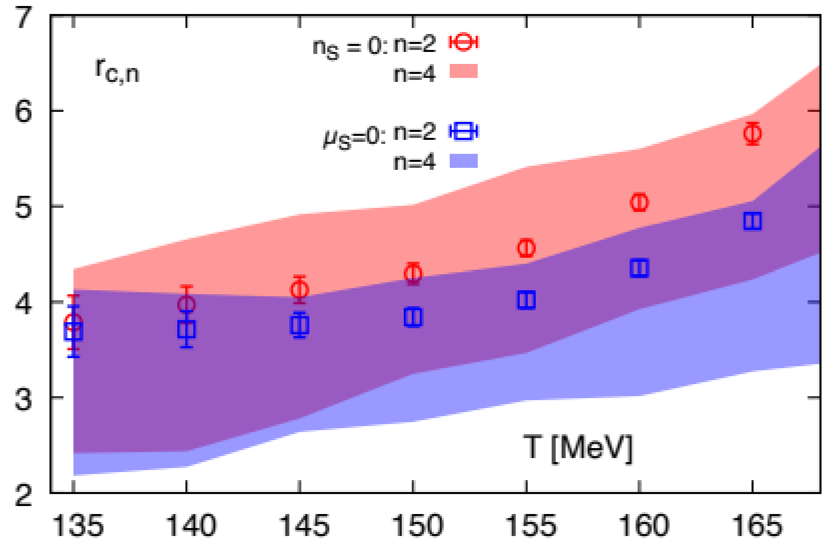
$$\hat{\mu}^{\pm}_{B,c} = \pm r_{c,4} e^{\pm i\Theta_{c,4}} \ , \ r_{c,4} = \sqrt{\frac{12\tilde{\chi}_0^{B,2}}{\tilde{\chi}_0^{B,4}}} \left| \frac{1-c_{6,2}}{c_{6,2}^2 - c_{8,2}} \right|^{1/4} \ , \ c_{2k,2} = \frac{2\tilde{\chi}_0^{B,2}}{(2k)!\tilde{\chi}_0^{B,2k}} \left(\frac{12\tilde{\chi}_0^{B,2}}{\tilde{\chi}_0^{B,4}} \right)^{k-1}$$

complex poles move to real axis as temperature decreases

distance of complex poles from the origin is given by the **Mercer-Roberts estimator** for the radius of convergence

$\mu_Q = 0 \ , \ n_S = 0$





HotQCD, arXiv:2202.09184

within current errors poles on the real axis (critical point) are possible only for $T \leq 135\mathrm{MeV} \ , \ \mu_B/T > 2.5$

higher statistics will sharpen the constraint

Higher order cumulants
– Taylor expansion of QCD EoS and the HRG –

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

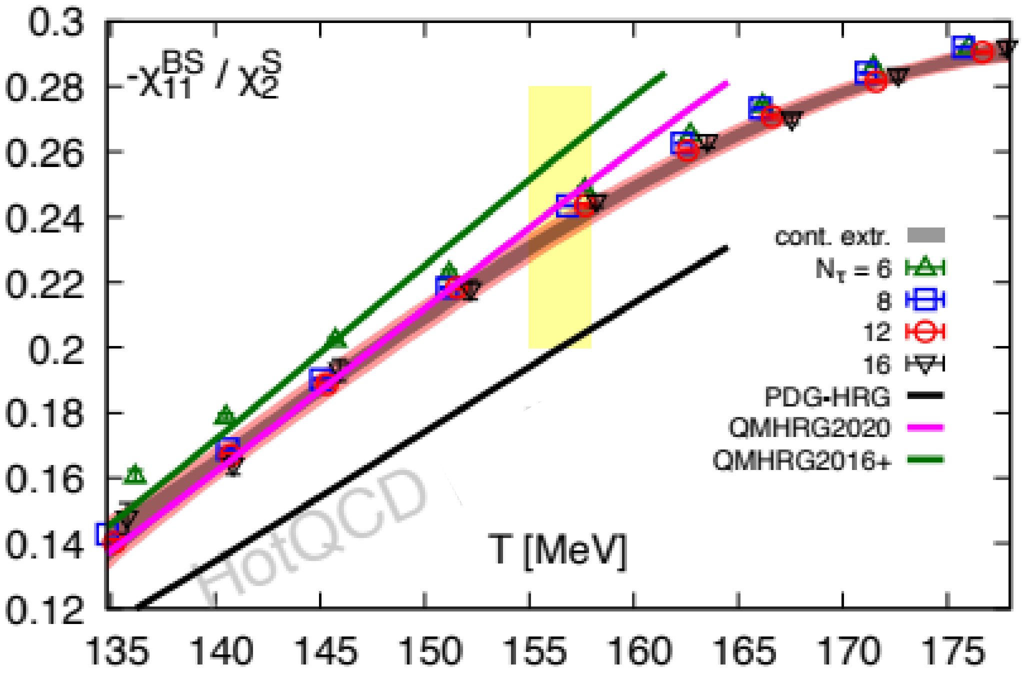
cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left.\frac{\partial^{i+j+k} P/T^4}{\partial\hat{\mu}_B^i \partial\hat{\mu}_Q^j \partial\hat{\mu}_S^k}\right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

cumulants at vanishing chemical potential provide information on the equation of state as well as freeze-out conditions at small non-zero chemical potential

Ratio of baryon number – strangeness correlation and net strangeness fluctuations

$$\frac{\mu_S}{\mu_B} \equiv -\frac{\chi_{11}^{BS}}{\chi_2^S} - q_1 \frac{\chi_{11}^{QS}}{\chi_2^S} + \mathcal{O}(\mu_B^2)$$



- evidence for experimentally not yet observed strange baryons?
- evidence for strong flavor correlations

$$-\frac{\chi_{11}^{BS}}{\chi_2^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{11}^{us}}{\chi_2^s}$$

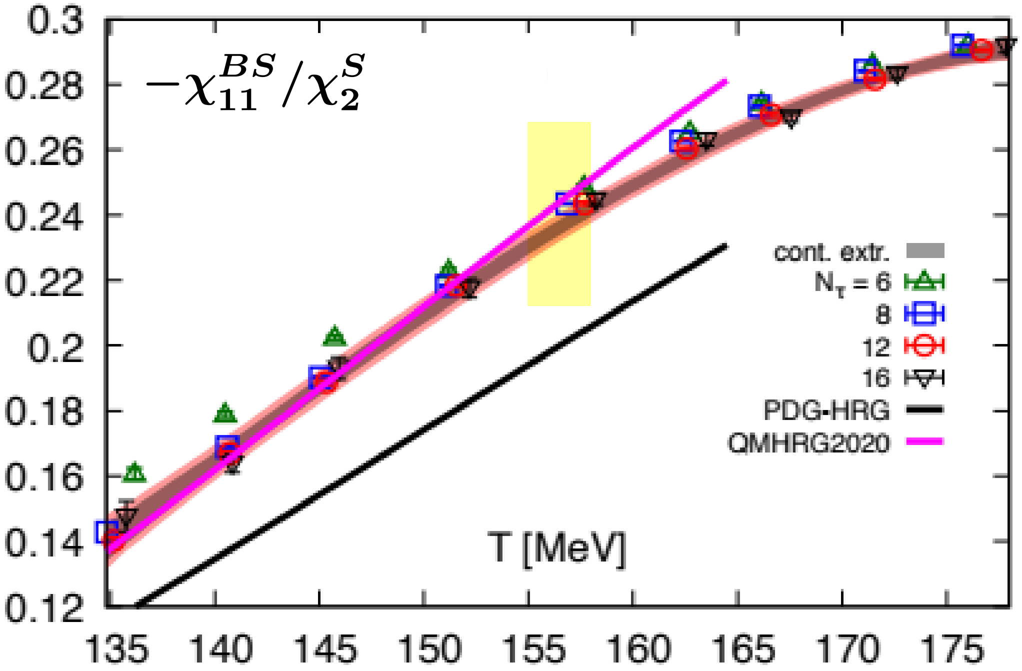
D. Bollweg et al. (HotQCD), arXiv:2107.10011

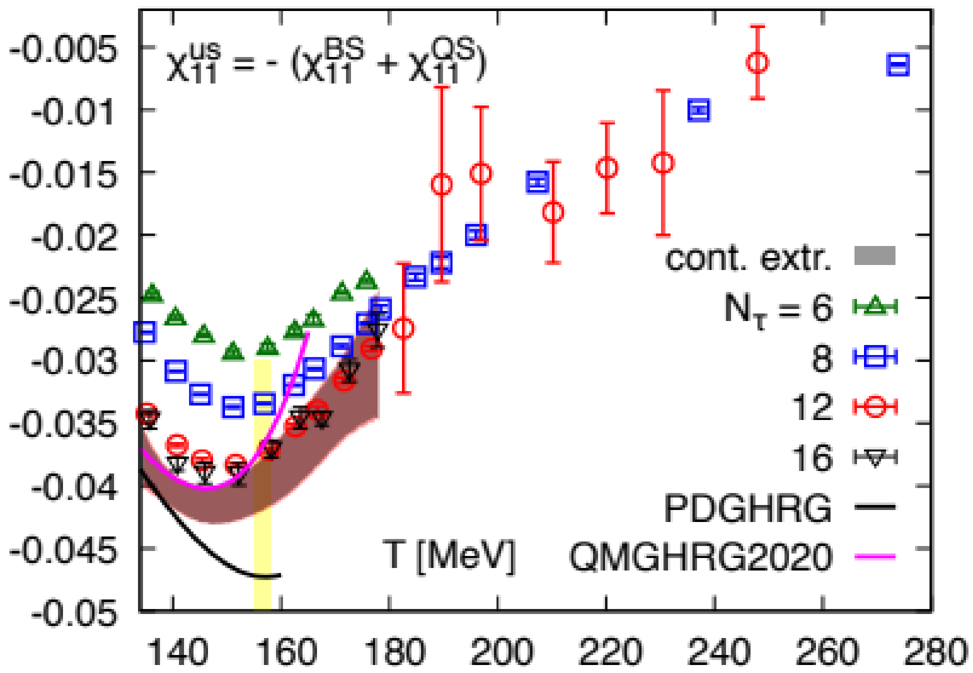
PDG-HRG: uses experimentally known hadron spectrum listed by the Particle Data Group
QM-HRG: uses additional hadrons predicted to exist in Quark Model calculations

Ratio of baryon number – strangeness correlation and net strangeness fluctuations

BS ratios probe flavor-correlations

$$-\frac{\chi_{11}^{BS}}{\chi_2^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{11}^{us}}{\chi_2^s}$$





conserved charge ⟺ quark number fluctuations:

$$\chi_{11}^{BS} = -\tfrac{1}{3}\chi_{11}^{us} - \tfrac{1}{3}\chi_{11}^{ds} - \tfrac{1}{3}\chi_2^s$$

$$\mathcal{O}(g^6 \ln g^2)$$

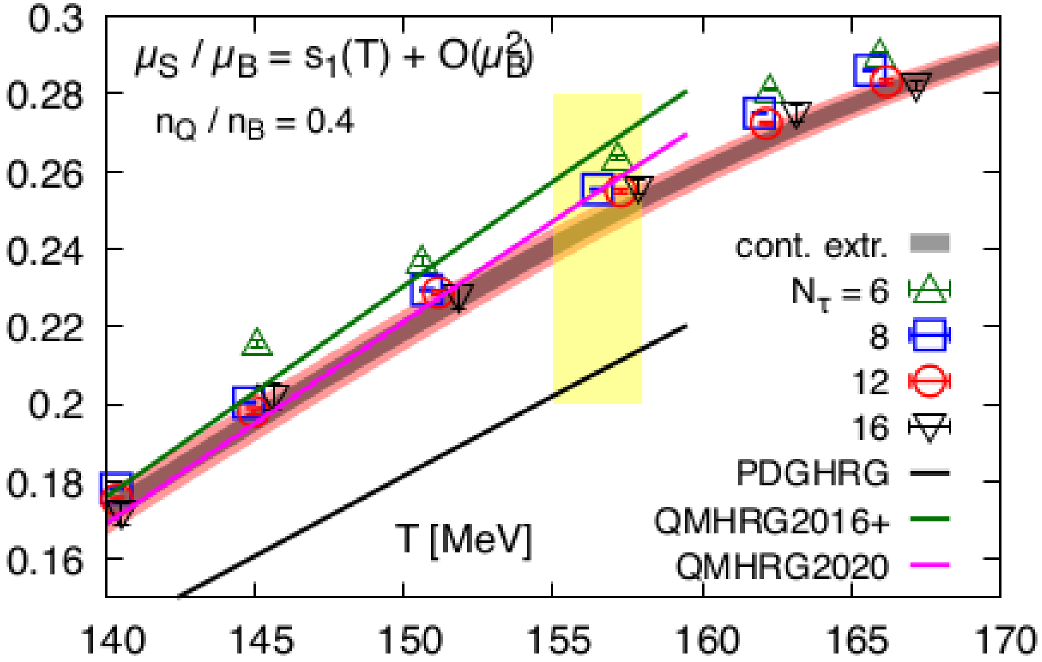
J.-P. Blaizot, E. Iancu, A. Rebhan, Phys. Lett. B 523 (2001) 143

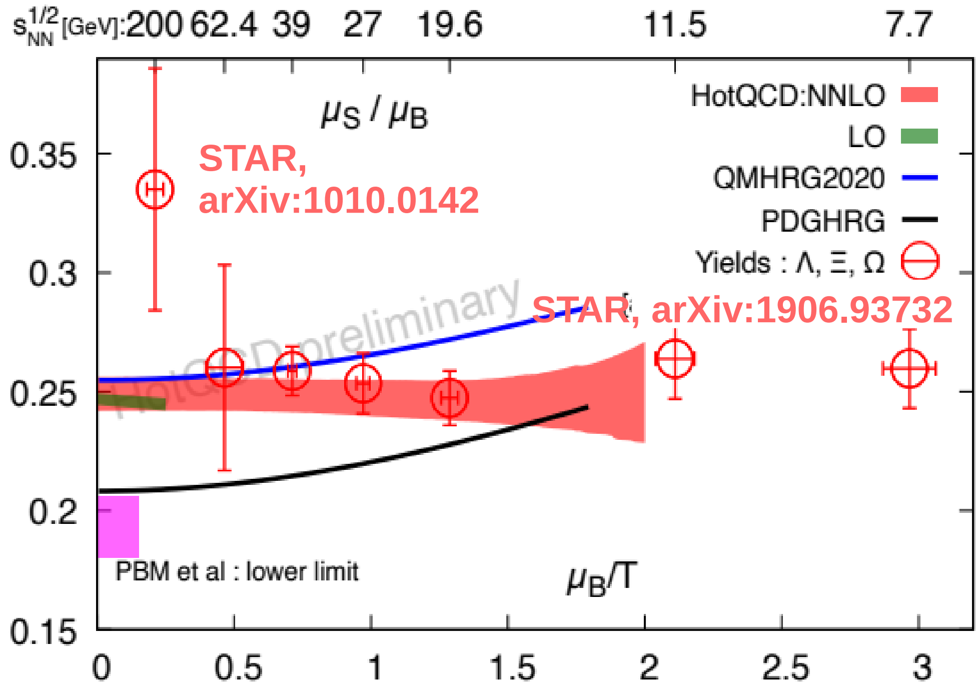
O(10) stronger flavor correlations at T_{pc} than at $2T_{pc}$

Baryon number – strangeness chemical potentials at freeze-out from strange baryon yields

BS ratios probe strangeness content in an HRG

$$\frac{\mu_S}{\mu_B} \equiv -\frac{\chi_{11}^{BS}}{\chi_2^S} - q_1 \frac{\chi_{11}^{QS}}{\chi_2^S} + \mathcal{O}(\mu_B^2)$$



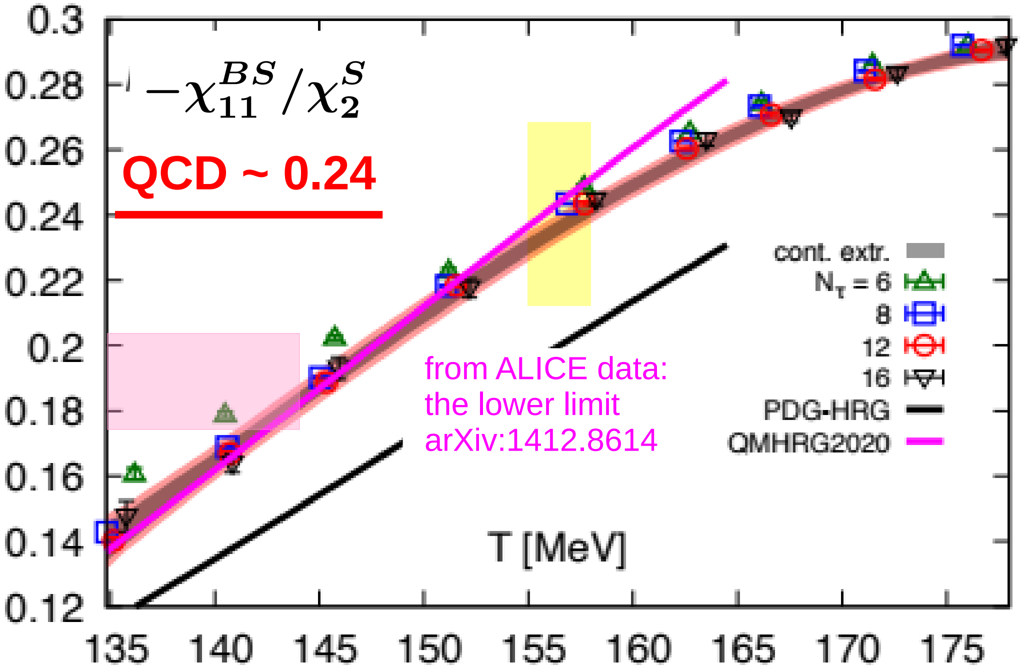


STAR multi-strange baryon yields are consistent with freeze-out at T_{pc} and a μ_S/μ_B that reflects contributions from additional strange baryons

at T_{pc} QCD: $\frac{\mu_S}{\mu_B} \simeq 0.24$

PDG-HRG: $\frac{\mu_S}{\mu_B} \simeq 0.21$

baryon number – strangeness correlation from HIC



for Skellam distributions mean values and second order cumulants as well as correlations are related:

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(\langle N_q\rangle + \langle N_{-q}\rangle)$$

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}\sum_{n=1}^{|q|} n^2(\langle N_n\rangle + \langle N_{-n}\rangle)$$

$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3}\sum_{n=-q_N}^{q_N}\sum_{m=-q_M}^{q_M} nm\langle N_{n,m}\rangle$$

$$|q| = 1\ (B),\ 2\ (Q),\ 3\ (S)$$

testing assumption of Skellam distribution:

$$-\frac{\chi_{BS}}{T^2} > \frac{1}{VT^3}[2\langle\Lambda+\Sigma^0\rangle + 4\langle\Sigma^+\rangle + 8\langle\Xi\rangle + 6\langle\Omega^-\rangle] = 97.4 \pm 5.8.$$

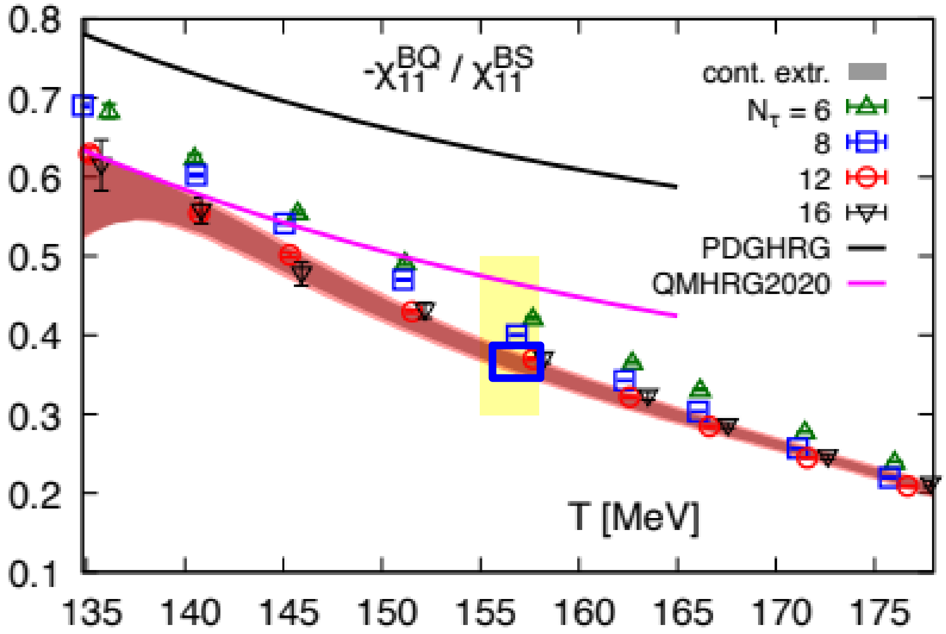
$$\frac{\chi_S}{T^2} \simeq \frac{1}{VT^3}[(\langle K^+\rangle + \langle K^0\rangle + \langle\Lambda+\Sigma^0\rangle + \langle\Sigma^+\rangle + \langle\Sigma^-\rangle + 4\langle\Xi^-\rangle + 4\langle\Xi^0\rangle + 9\langle\Omega^-\rangle + \text{antiparticles}) - (\Gamma_{\phi\to K^+} + \Gamma_{\phi\to K^-} + \Gamma_{\phi\to K^0} + \Gamma_{\phi\to\bar{K}^0})\langle\phi\rangle] = (504 \pm 24).$$

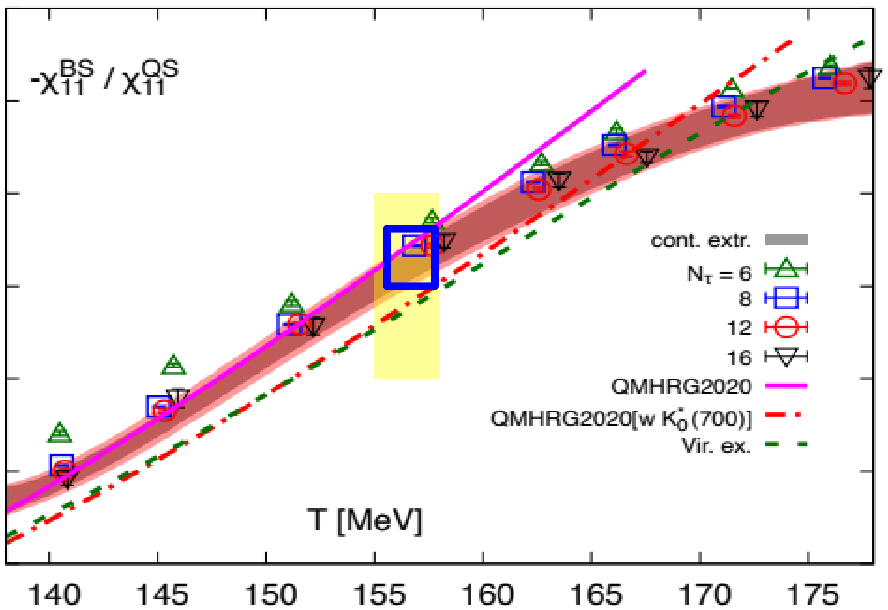
getting control over B rather than P fluctuations is important!!!

$$-\frac{\chi_{BS}}{\chi_S} \geq 0.193(22)$$

P. Braun-Munzinger, A. Kalweit, K. Redlich, J. Stachel, Phys. Lett. B747 (2015) 292, arXiv:1412.8614

Ratios of second order cumulants at $\mu_B = 0$
– observables at the LHC ? –





$$\chi_{11}^{BQ}/\chi_{11}^{BS} = -0.368(24)$$

$$\chi_{11}^{BS}/\chi_{11}^{QS} = 0.623(40)$$

$$\chi_{11}^{BS}/\chi_{2}^{S} = -0.236(11)$$

at ALICE freeze-out temperature
== QCD crossover temperature

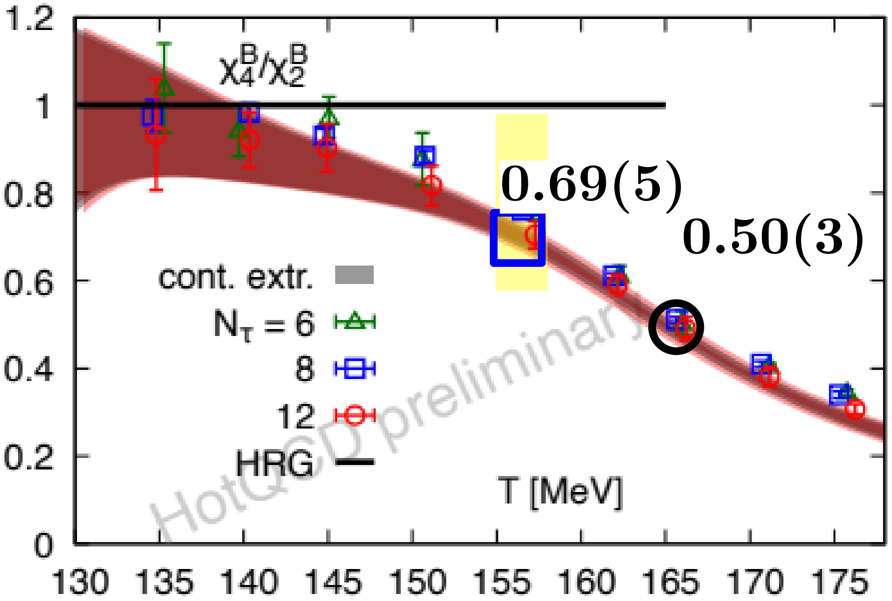
$$T_{fo} = 156.5(1.5)\mathrm{MeV}$$

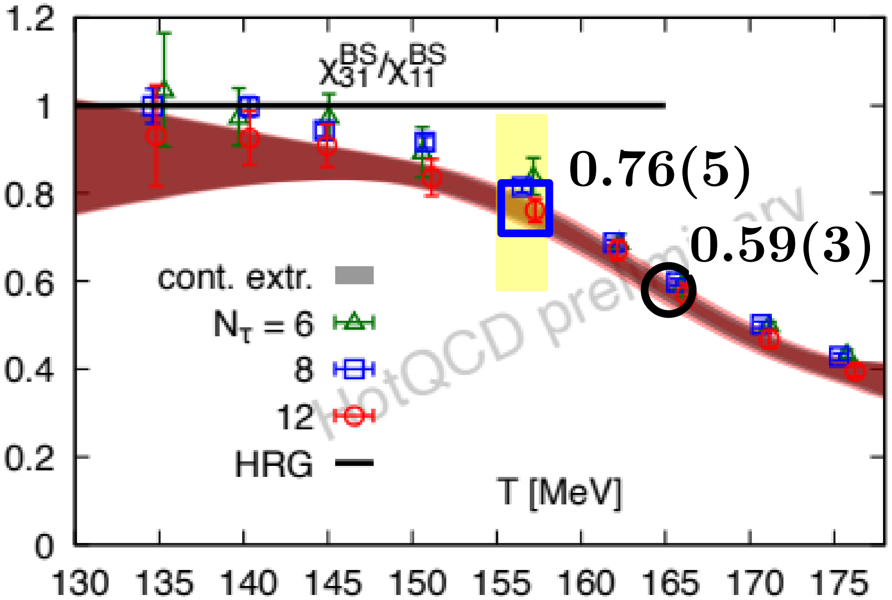
large deviations from PDGHRG
still 25% deviations from QMHRG2020

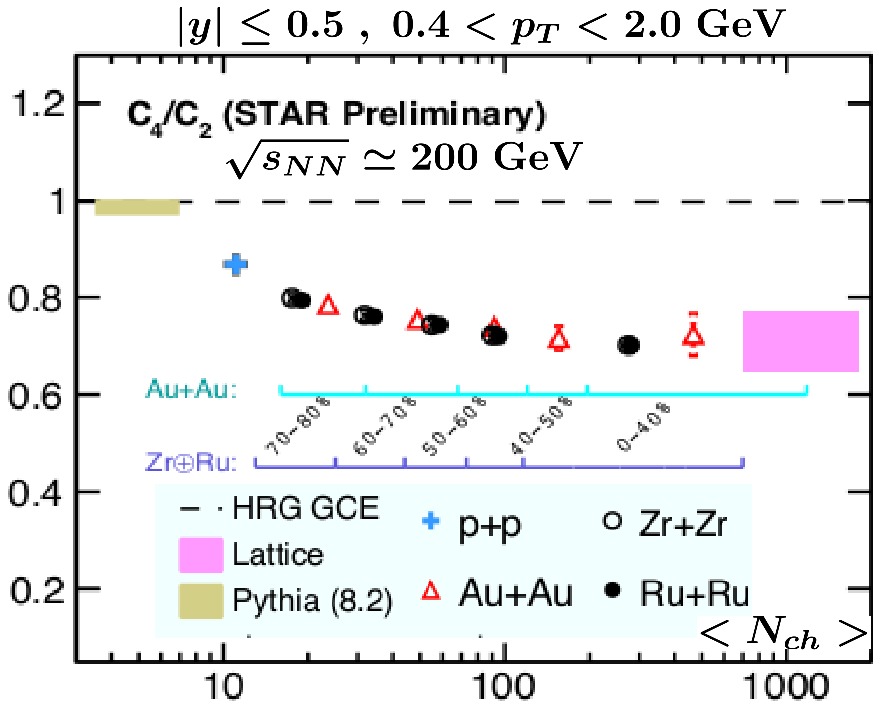
sensitive test of QCD

Ratios of 4th and 2nd order cumulants

– large deviations from Skellam –







Ho-San Ko @ QM22

– ratios of 4th and 2nd order cumulants differ from non-inter. HRG for T>145 MeV

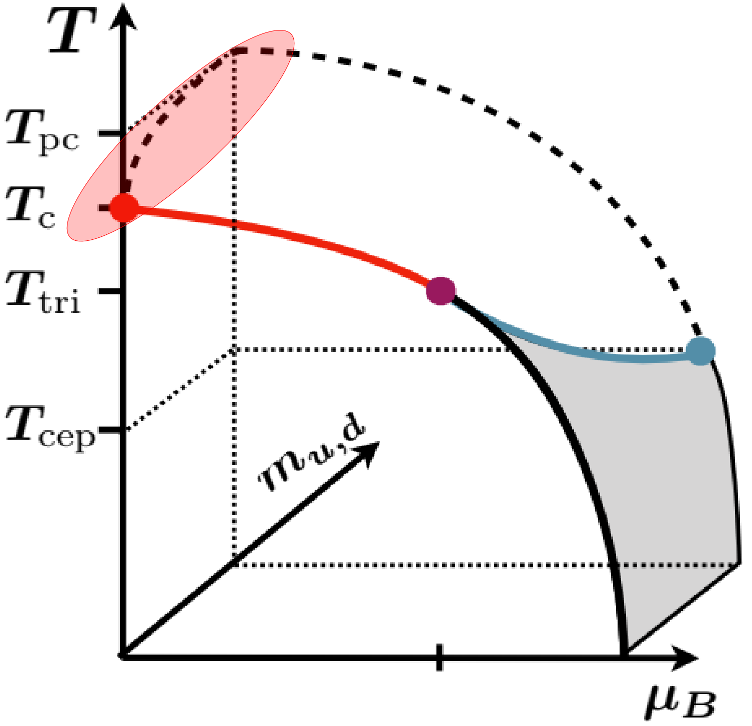
– they change by ~(20-40)% in the crossover region

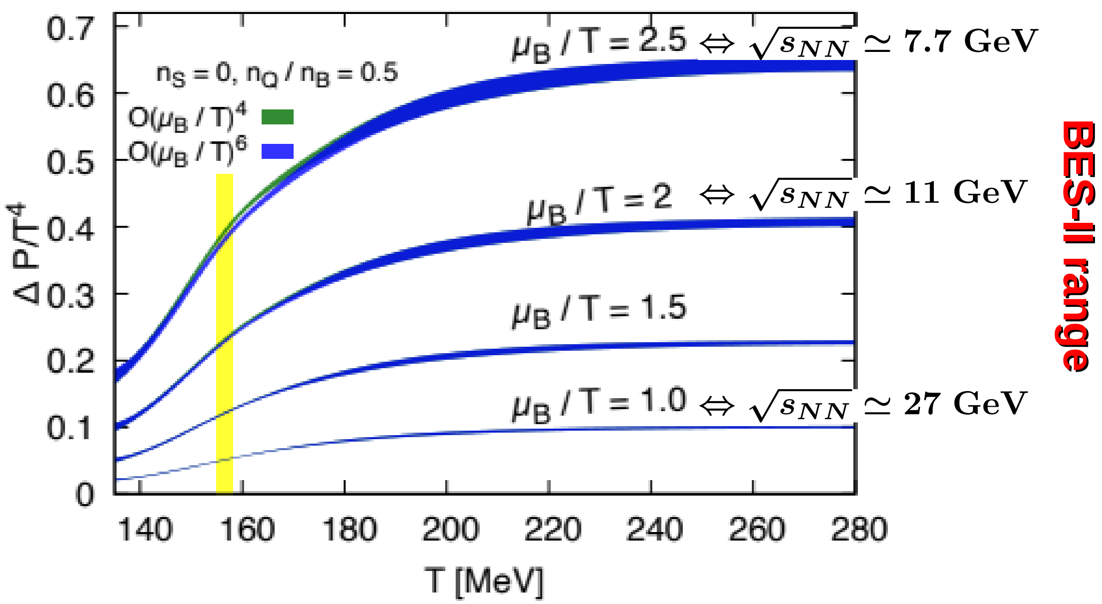
sensitive probes for freeze-out

at ALICE freeze-out temperature $T_{fo} = 156.5(1.5)\mathrm{MeV}$

STAR at 200 GeV: $T_{fo} = 165(3)\mathrm{MeV}$

Conclusions





What we learned so far about the CEP in QCD from lattice QCD calculations:

I) the critical temperature is below $T_c = 132^{+3}_{-6}$ MeV

II) the corresponding critical chemical potential is likely to be above 400 MeV

→ Taylor expansions need to be resummed in order to reach CEP

– no CEP for $\mu_B/T \leq 2.5$

– CEP not in the BES-II range (in collider mode)

– EoS (pressure & number density) well controlled for

$$\mu_B/T \leq 2.0 \; \forall T > 135 \text{ MeV}$$

(larger range for higher T)

– reliable μ_B - range is smaller for higher order cumulants, given only an 8th order Taylor series for the pressure