Light nuclei production in heavy-ion collisions

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&

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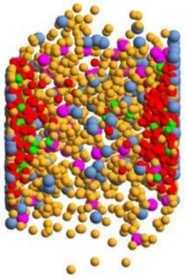
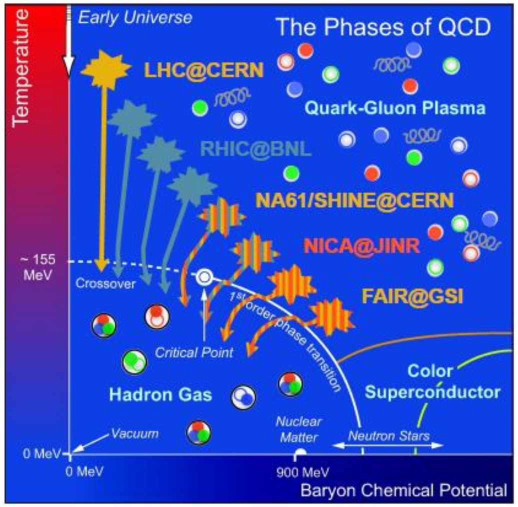
PHQMD

On-line seminar series V
on "RHIC Beam Energy Scan" Fall 2022,
25 October 2022

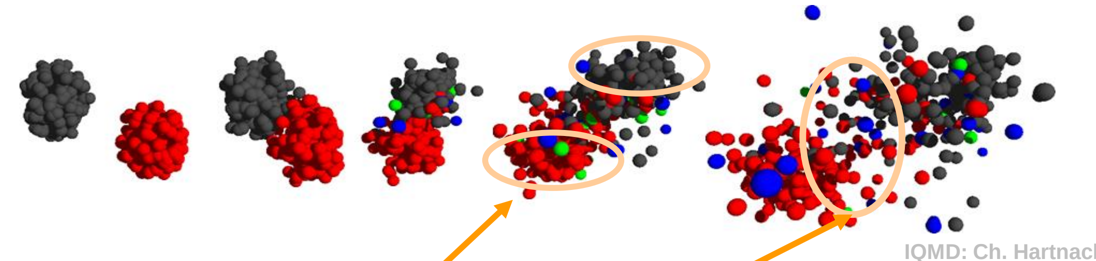


The ‚holy grail‘ of heavy-ion physics:

The phase diagram of QCD



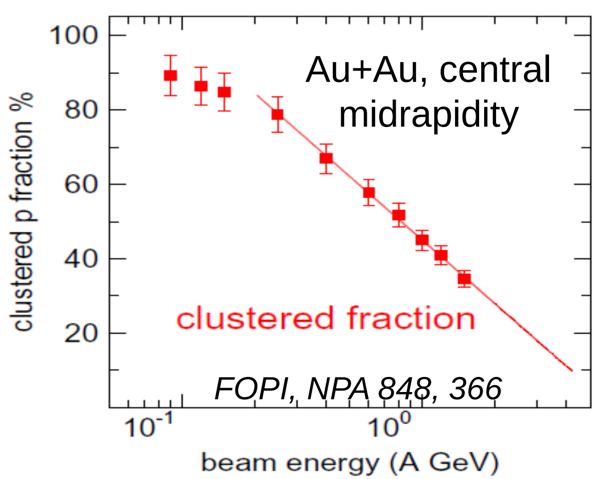
Experimental observables:
... Clusters and (anti-) hypernuclei



- **projectile/target spectators➔ heavy cluster formation**
- **midrapidity➔ light clusters**

! Hyperons are created in participant zone

- **Clusters are very abundant at low energy**

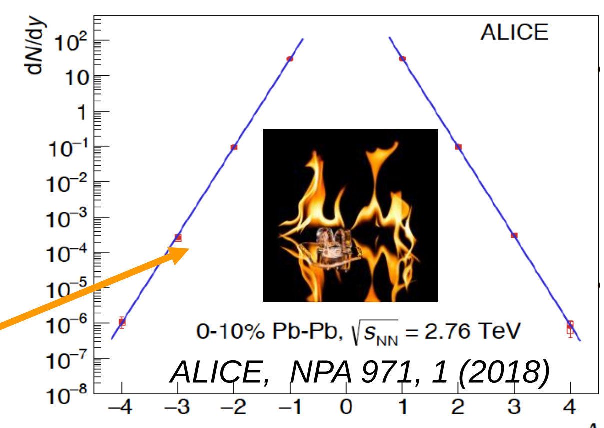


(Anti-) hypernuclei production:

- **at mid-rapidity by coalescence of Λ with nucleons during expansion**
- **at projectile/target rapidity by rescattering/absorption of Λ by spectators**

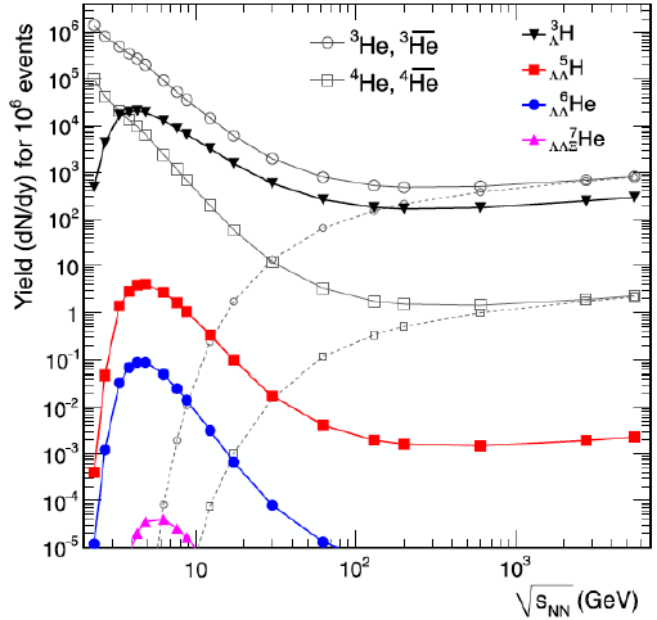
High energy HIC:

‚Ice in a fire‘ puzzle: how the weakly bound objects can be formed and survive in a hot enviroment ?!



Modeling of cluster and hypernuclei formation

A. Andronic et al., PLB 697, 203 (2011)



Existing models for cluster formation:

- **statistical model:**
  - **assumption of thermal equilibrium**
- **coalescence model:**
  - **determination of clusters at a freeze-out time by coalescence radii in coordinate and momentum space**

➔ **don‘t provide information on the dynamical origin of cluster formation**

In order to understand the microscopic origin of cluster formation one needs a realistic model for the dynamical time evolution of the HIC

→ **transport models:**
dynamical modeling of cluster formation based on interactions:

– via potential interaction - potential mechanism

-- by scattering - kinetic mechanism

PHQMD

Cluster formation: QMD vs MF

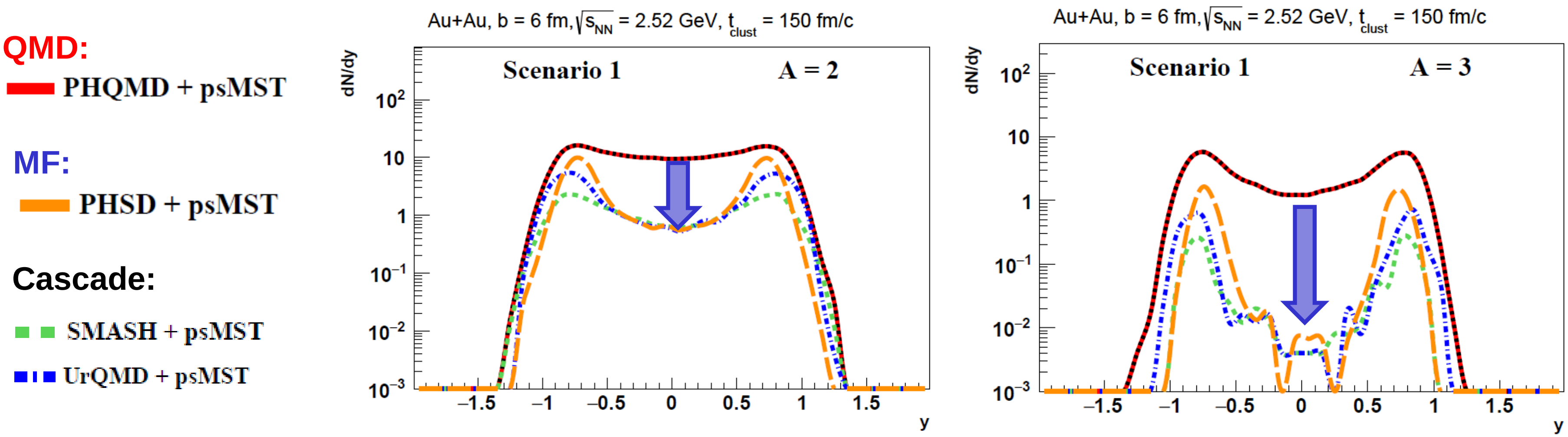
- Cluster formation is sensitive to nucleon dynamics

➔ One needs to keep the nucleon correlations (initial and final) by realistic nucleon-nucleon interactions in transport models:

- QMD (quantum-molecular dynamics) – allows to keep correlations
- MF (mean-field based models) – correlations are smeared out
- Cascade – no correlations by potential interactions

Example: Cluster stability over time:

V. Kireyeu, Phys.Rev.C 103 (2021) 5



➔ n-body QMD dynamics for the description of cluster production

PHQMD

PHQMD: a **unified n-body microscopic transport approach** for the description of heavy-ion collisions and **dynamical cluster formation** from low to ultra-relativistic energies

Realization: combined model **PHQMD = (PHSD & QMD) & (MST/SACA)**

Parton-Hadron-Quantum-Molecular Dynamics

↓

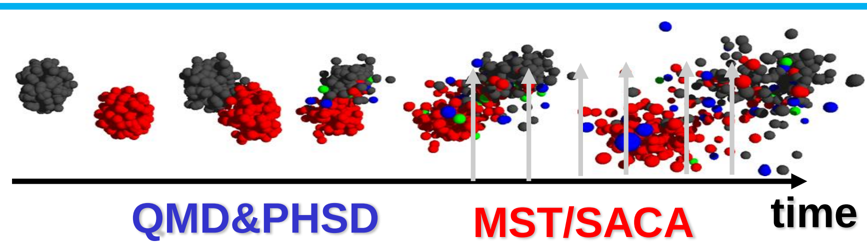
Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

↓

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

↓

Cluster recognition:
SACA (Simulated Annealing Clusterization Algorithm)
or MST (Minimum Spanning Tree)



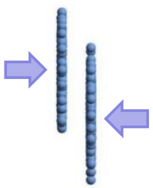
J. Aichelin et al., PRC 101 (2020) 044905;
S. Gläßel et al., PRC 105 (2022) 1

PHQMD Collision Integral → from Parton-Hadron-String-Dynamics

PHSD is a non-equilibrium microscopic transport approach for the description of strongly-interacting hadronic and partonic matter created in heavy-ion collisions

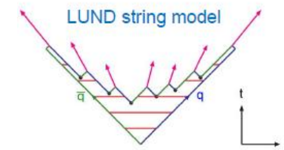
Dynamics: based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory

Initial A+A collision



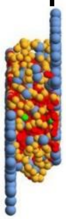
→ PHQMD **Initialization of A-nuclei + QMD propagation of baryons**

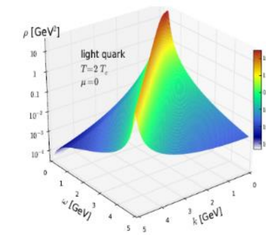
PHSD collision integral → PHQMD



- ❑ **Initial A+A collisions :**
  N+N → string formation → decay to pre-hadrons + leading hadrons

Partonic phase

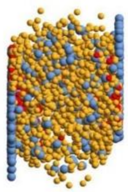


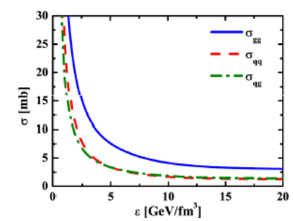


- ❑ **Formation of QGP stage** if local $\varepsilon > \varepsilon_{critical}$:
  dissolution of pre-hadrons → partons

- ❑ **Partonic phase - QGP:**
  QGP is described by the Dynamical QuasiParticle Model (DQPM) matched to reproduce lattice QCD EoS for finite T and μ_B (crossover)

Hadronization

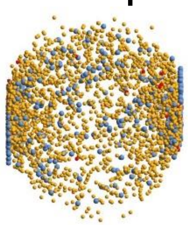


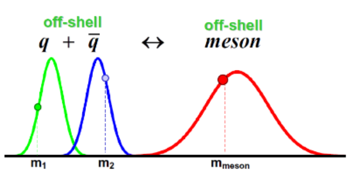


  - Degrees-of-freedom: strongly interacting quasiparticles: massive quarks and gluons (g,q,q_{bar}) with sizeable collisional widths in a self-generated mean-field potential

  - Interactions: (quasi-)elastic and inelastic collisions of partons

Hadronic phase





- ❑ **Hadronization** to colorless **off-shell mesons and baryons**:
  Strict 4-momentum and quantum number conservation

- ❑ **Hadronic phase: hadron-hadron interactions – off-shell HSD**

PHSD: W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168 (2009) 3

QMD propagation

- **Generalized Ritz variational principle:** $\delta \int_{t_1}^{t_2} dt < \psi(t)|i\frac{d}{dt} - H|\psi(t) >= 0.$

Assume that $\psi(t) = \prod_{i=1}^{N} \psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t)$ **for N particles (neglecting antisymmetrization !)**

Ansatz: trial wave function for one particle "*i*" :
Gaussian with width *L* centered at r_{i0}, p_{i0}

[Aichelin Phys. Rept. 202 (1991)]

$$\psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = C\ \mathrm{e}^{-\frac{1}{4L}\left(\mathbf{r}_i - \mathbf{r}_{i0}(t) - \frac{\mathbf{p}_{i0}(t)}{m}t\right)^2} \cdot \mathrm{e}^{i\mathbf{p}_{i0}(t)(\mathbf{r}_i - \mathbf{r}_{i0}(t))} \cdot \mathrm{e}^{-i\frac{\mathbf{p}_{i0}^2(t)}{2m}t}$$

L=4.33 fm^2

- **Equations-of-motion (EoM) for Gaussian centers in coordinate and momentum space:**

$$\dot{r}_{i0} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial \langle H \rangle}{\partial r_{i0}}$$

Hamiltonian: $H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j \neq i} V_{i,j})$

$$V_{i,j} = V(\mathbf{r_i}, \mathbf{r_j}, \mathbf{r_{i0}}, \mathbf{r_{j0}}, t) = V_{\mathrm{Skyrme}} + V_{\mathrm{Coul}}$$

QMD interaction potential and EoS

The expectation value of the Hamiltonian:

$$\langle H \rangle = \langle T \rangle + \langle V \rangle = \sum_i \left(\sqrt{p_{i0}^2 + m^2} - m\right) + \sum_i \langle V_{Skyrme}(\mathbf{r}_{i0}, t) \rangle$$

- **Skyrme potential** **(‘static’) * :**

$$\langle V_{Skyrme}(\mathbf{r}_{i0}, t) \rangle = \alpha \left(\frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0} \right) + \beta \left(\frac{\rho_{int}(\mathbf{r}_{i0}, t)}{\rho_0} \right)^\gamma$$

	α (MeV)	β (MeV)	γ	K [MeV]
S	-390	320	1.14	200
H	-130	59	2.09	380

- **modifed interaction density (with relativistic extension):**

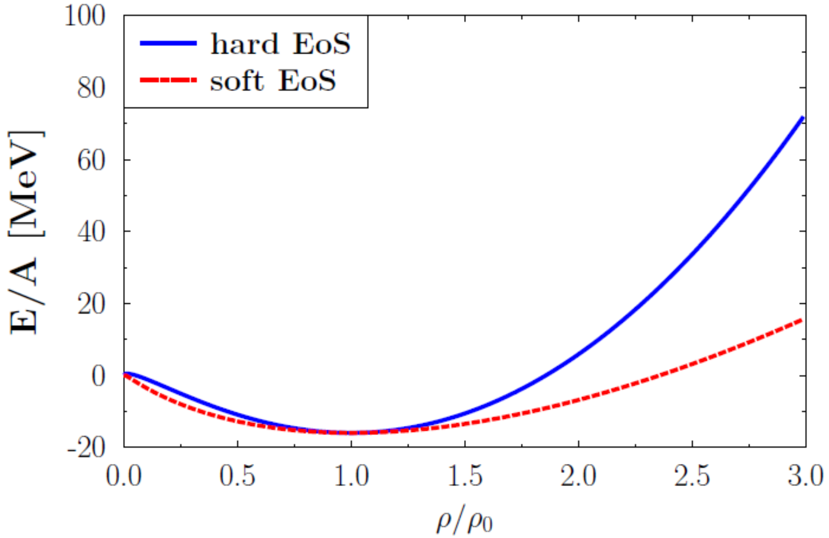
$$\rho_{int}(\mathbf{r}_{i0}, t) \rightarrow C \sum_j \left(\frac{4}{\pi L}\right)^{3/2} e^{-\frac{4}{L}(\mathbf{r}_{i0}^{T}(t) - \mathbf{r}_{j0}^{T}(t))^2} \times e^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r}_{i0}^{L}(t) - \mathbf{r}_{j0}^{L}(t))^2},$$

- **HIC ←→ EoS for infinite matter at rest**

- **compression modulus K of nuclear matter:**

$$K = -V \frac{dP}{dV} = 9\rho^2 \frac{\partial^2 (E/A(\rho))}{(\partial \rho)^2}\Big|_{\rho=\rho_0}.$$

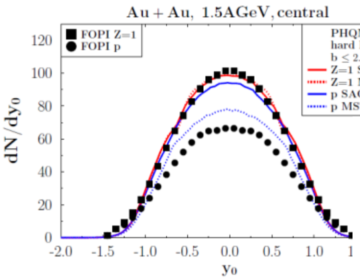
EoS for infinite matter at rest

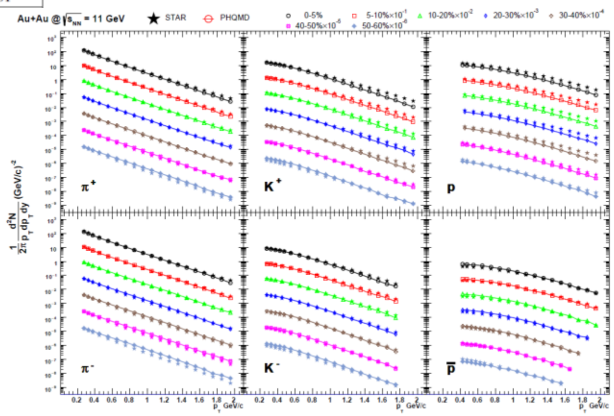


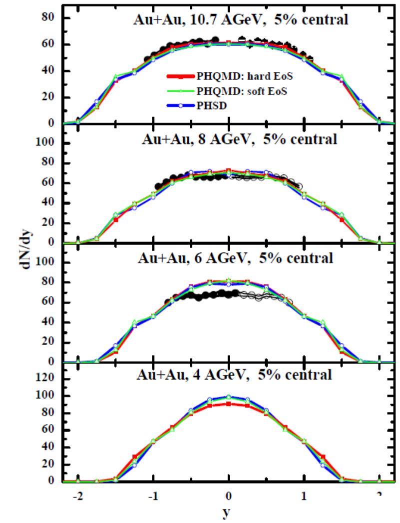
*Work in progress: implementation of momentum-dependent potential (M. Winn)

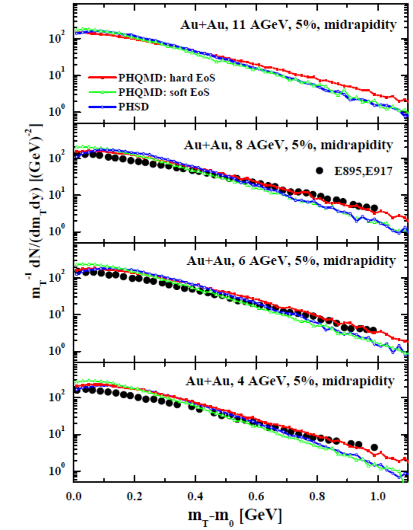
Highlights: PHQMD ‚bulk‘ dynamics from SIS to RHIC

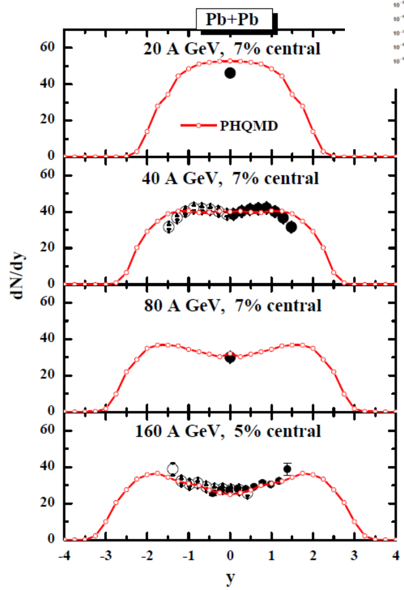
PHQMD: J. Aichelin et al., PRC 101 (2020) 044905

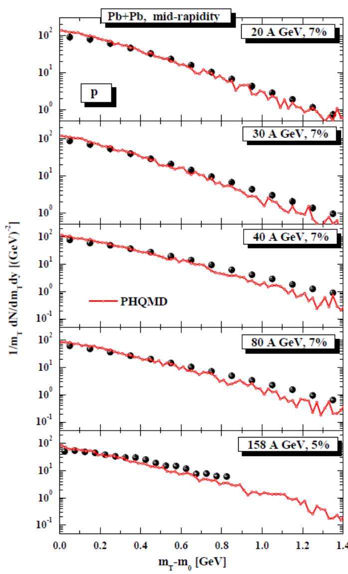


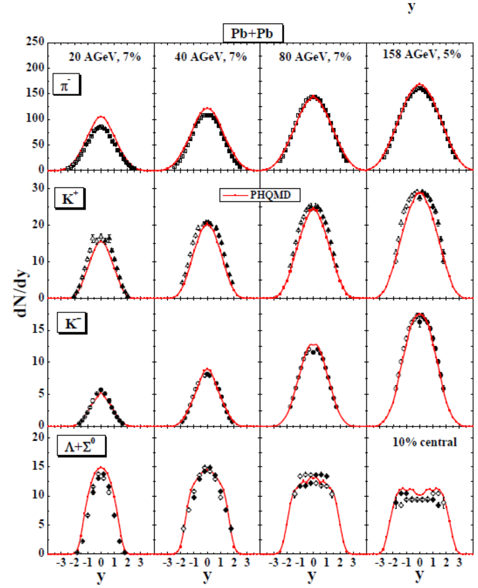


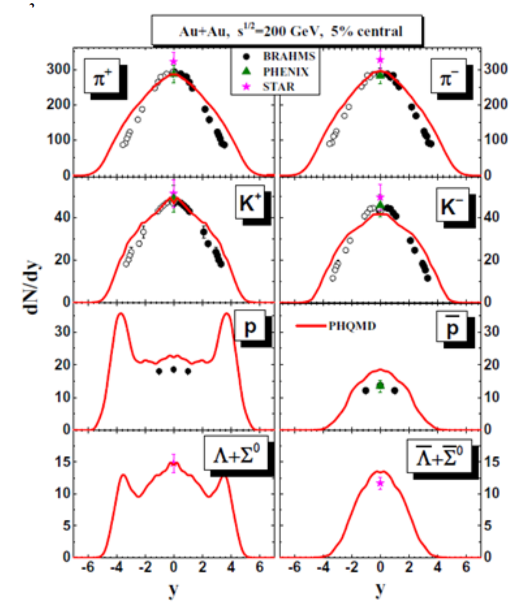










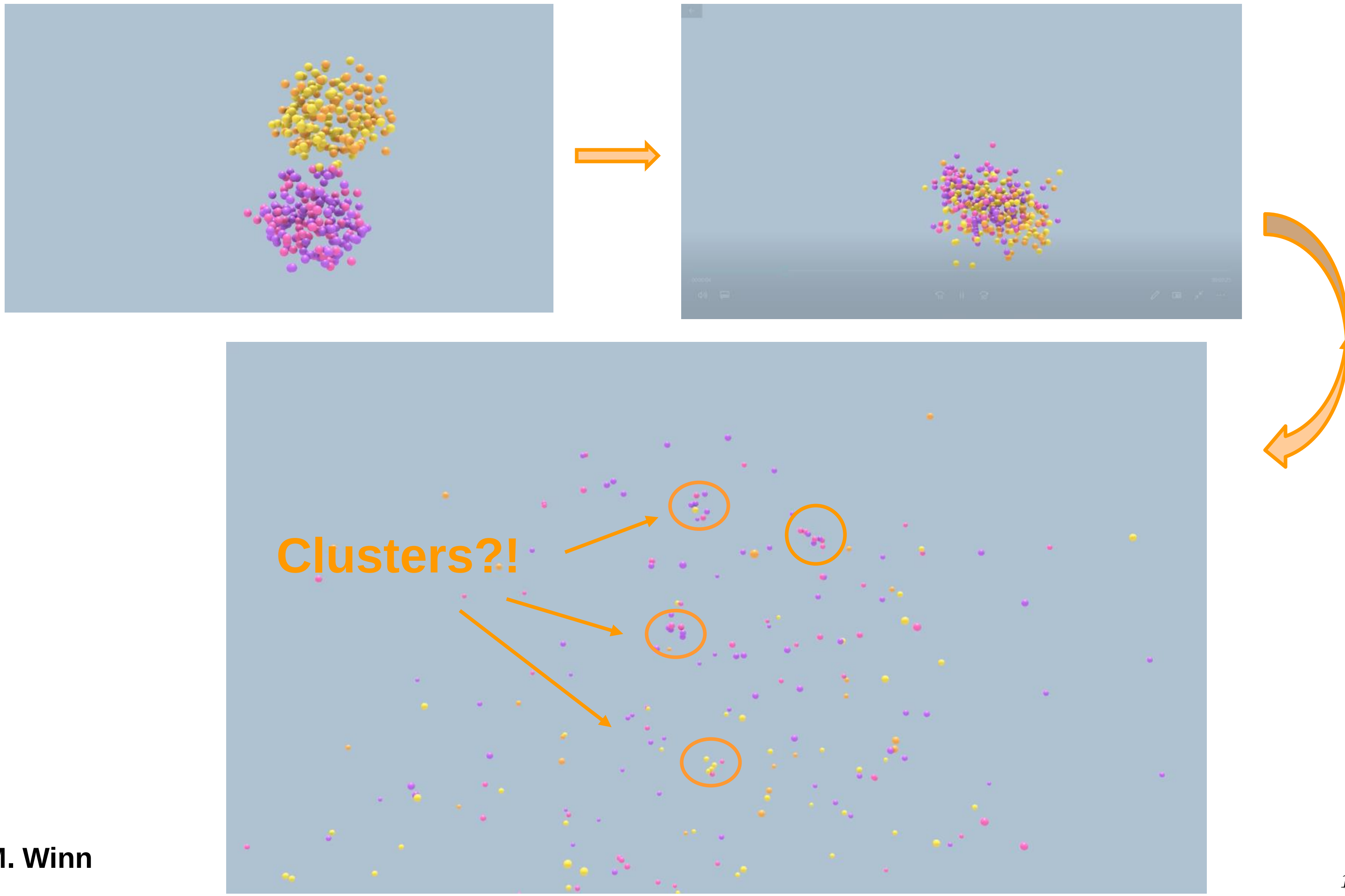


PHQMD provides a good description of hadronic ‘bulk’ observables from SIS to RHIC energies

I. Potential mechanism for cluster production in PHQMD: MST & SACA

PHQMD

Time evolution: Au+Au, b=2 fm, 600 AGeV



By M. Winn

Cluster recognition: Minimum Spanning Tree (MST)

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

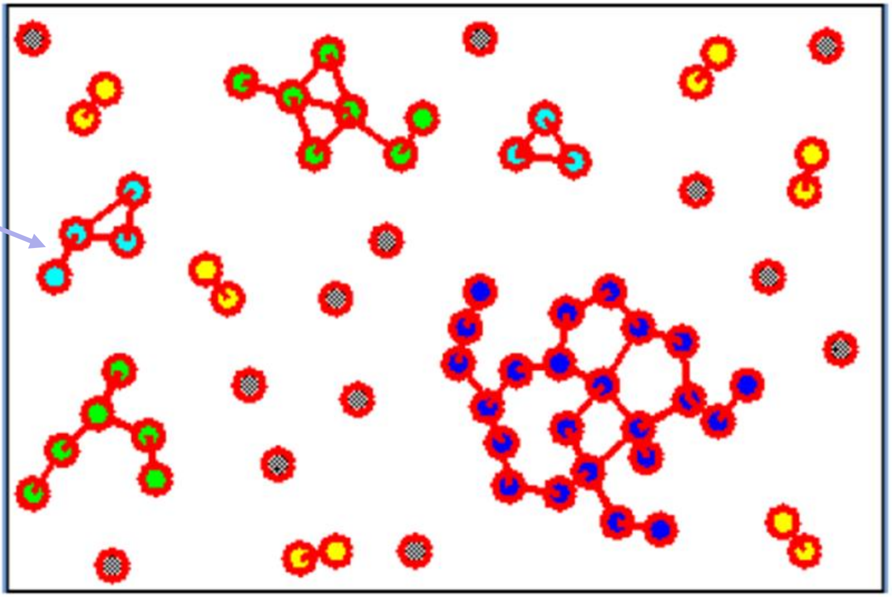
The Minimum Spanning Tree (MST) is a cluster recognition method applicable for the (asymptotic) final states where coordinate space correlations may only survive for bound states.

The MST algorithm searches for accumulations of particles in coordinate space:

1. **Two particles are 'bound' if their distance in the cluster rest frame fulfills**

$$|\vec{r_i} - \vec{r_j}| \leq 4\ \text{fm}$$

2. **Particle is bound to a cluster if it binds with at least one particle of the cluster.**



* Remark:
inclusion of an additional momentum cut (coalescence) leads to small changes: particles with large relative momentum are mostly not at the same position (V. Kireyeu, Phys.Rev.C 103 (2021) 5)

- **MST + extra condition: E_B<0 negative binding energy for identified clusters**

Simulated Annealing Clusterization Algorithm (SACA)

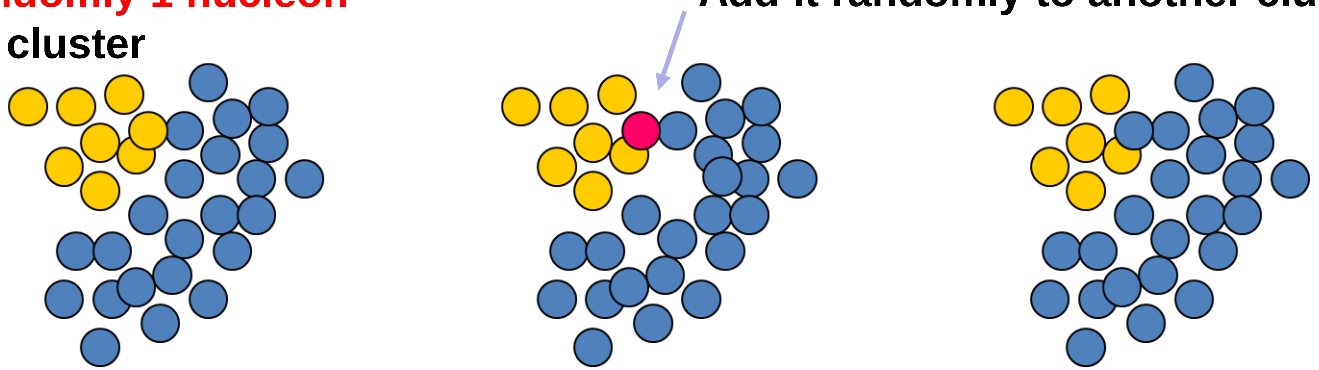
Basic ideas of clusters recognition by SACA:

Based on ideas by Dorso and Randrup (Phys.Lett. B301 (1993) 328)

- **Take the positions and momenta of all nucleons at time t**
- **Combine them in all possible ways into all kinds of clusters or leave them as single nucleons**
- **Neglect the interaction among clusters**
- **Choose that configuration which has the highest binding energy:**

Take randomly 1 nucleon out of a cluster

Add it randomly to another cluster



$E=E^{1}_{kin}+E^{2}_{kin}+V^{1}+V^{2}$

$E'=E^{1'}_{kin}+E^{2'}_{kin}+V^{1'}+V^{2'}$

If E' < E take a new configuration
If E' > E take the old configuration with a probability depending on E'-E
Repeat this procedure many times

→ **Leads automatically to finding of the most bound configurations**

(realized via a Metropolis algorithm)

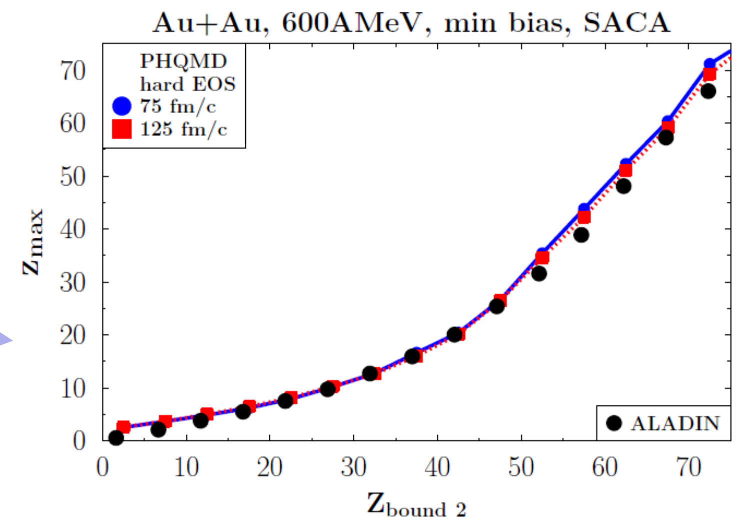
R. K. Puri, J. Aichelin, PLB301 (1993) 328, J.Comput.Phys. 162 (2000) 245-266;
P.B. Gossiaux, R. Puri, Ch. Hartnack, J. Aichelin, Nuclear Physics A 619 (1997) 379-390

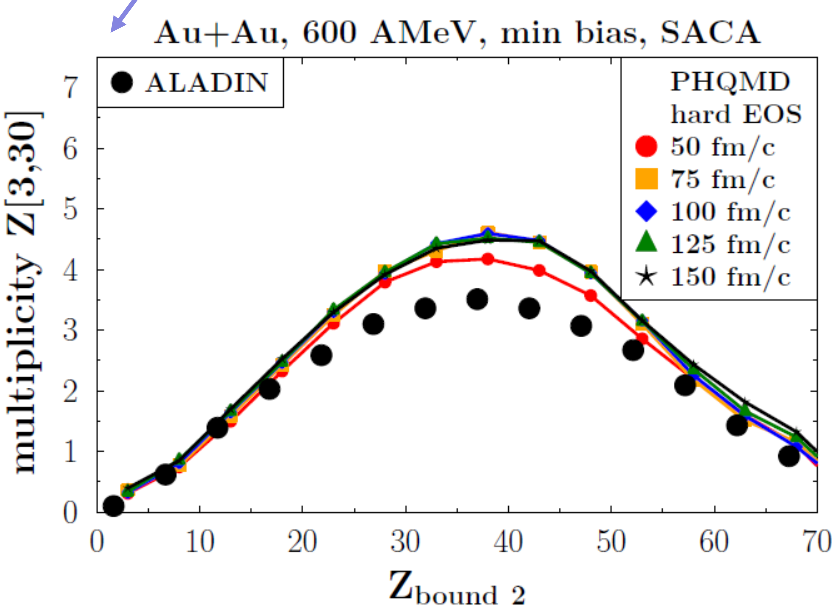
PHQMD: heavy clusters

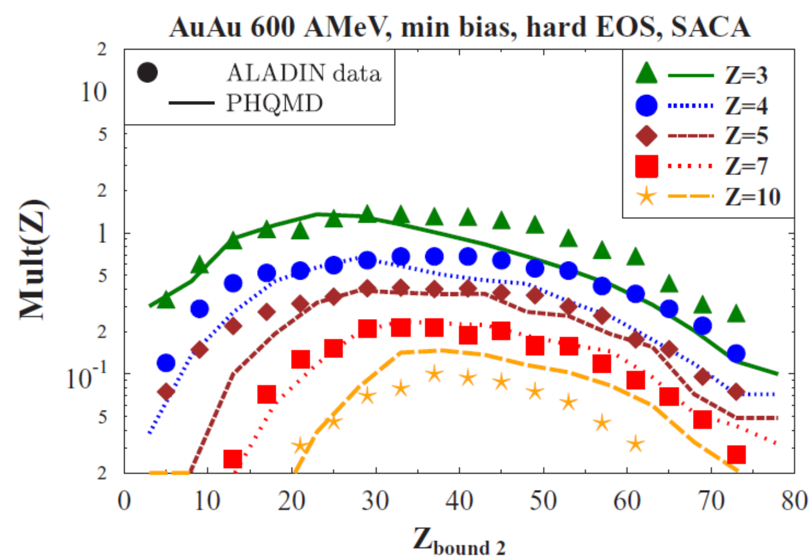
Heavy clusters (spectator fragments): experim. measured up to E_{beam} =1 AGeV (ALADIN Collab.)

PHQMD with SACA shows an agreement with ALADIN data for very complex cluster observables as

- ❑ **Largest clusters (Z_{bound})**
- ❑ **Multiplicity (Z_{bound})**
- ❑ **Energy independent 'rise and fall'**





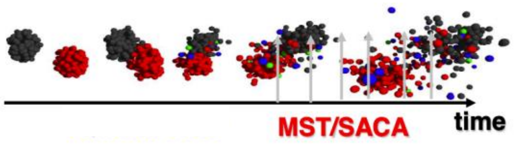


$$Z_{\text{bound 2}} = \sum_i Z_i \, \Theta(Z_i - (1+\epsilon)) \quad (\epsilon < 1)$$

J. Aichelin et al., PRC 101 (2020) 044905

Cluster stability in semi-classical models

Cluster stability problem in semi-classical models (as QMD):



QMD can not describe clusters as 'quantum objects'
➔ the cluster quantum ground state has to respect a minimal average kinetic energy of the nucleons while the semi-classical (QMD) ground state - not!
➔ nucleons may still be emitted from the QMD clusters while in the corresponding quantum system this is not possible
➔ thus, a cluster which is "bound" at time *t* can spontaneously dissolve at *t + Δt*

= QMD clusters are not fully stable over time:
➔ the multiplicity of clusters is time dependent
➔ the form of the final rapidity, p_T distribution and ratio of particles do not change with time

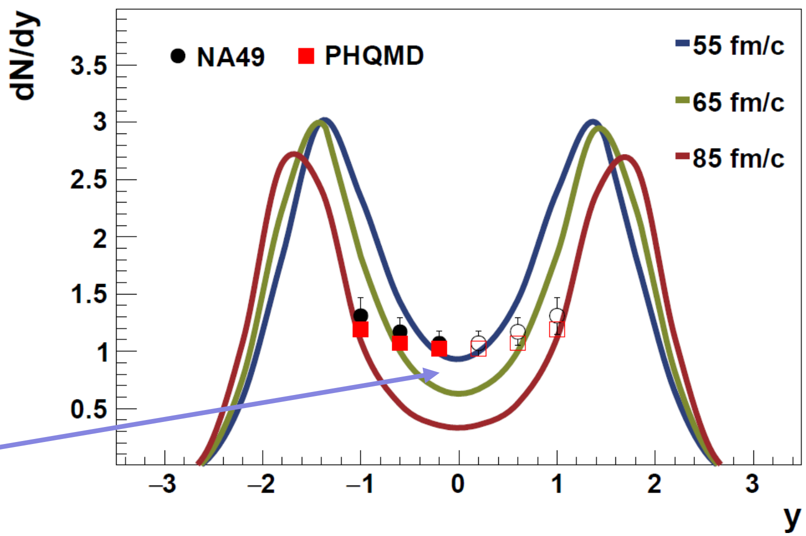
How to stabilize QMD clusters?

Scenario 1: S. Gläßel et al., PRC 105 (2022) 1

PHQMD results are taken at 'physical time' :

$$t = t_0 \cosh(y)$$

where t_0 is the time selected as a best description of the cluster multiplicity at y=0



MST with 'stabilization' procedure

How to stabilize QMD clusters?

Scenario 2:
G. Coci et al., in preparation

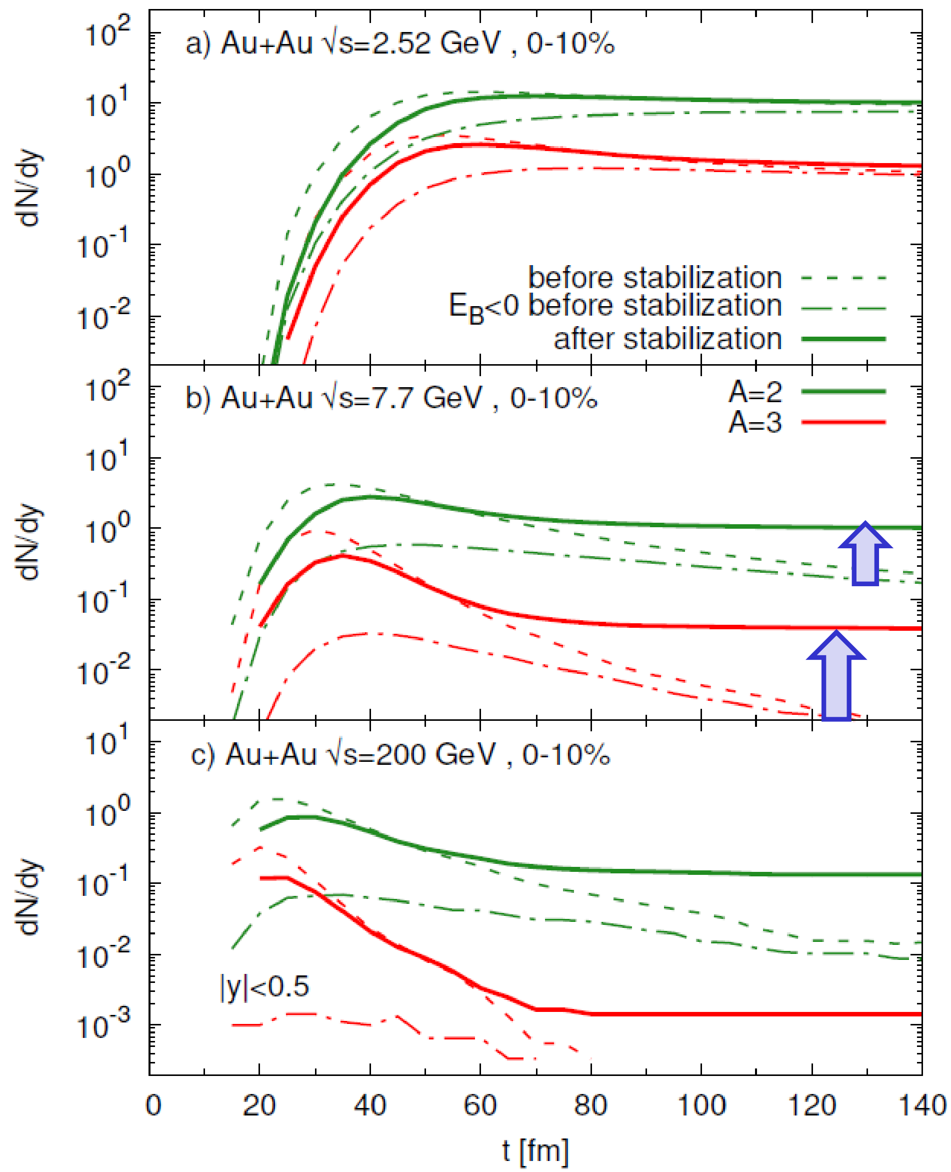
Stabilization Procedure:

- consider asymptotic state: clusters and free nucleons

- For each nucleon in MST track the freezout-time = time at which the last collision occurred

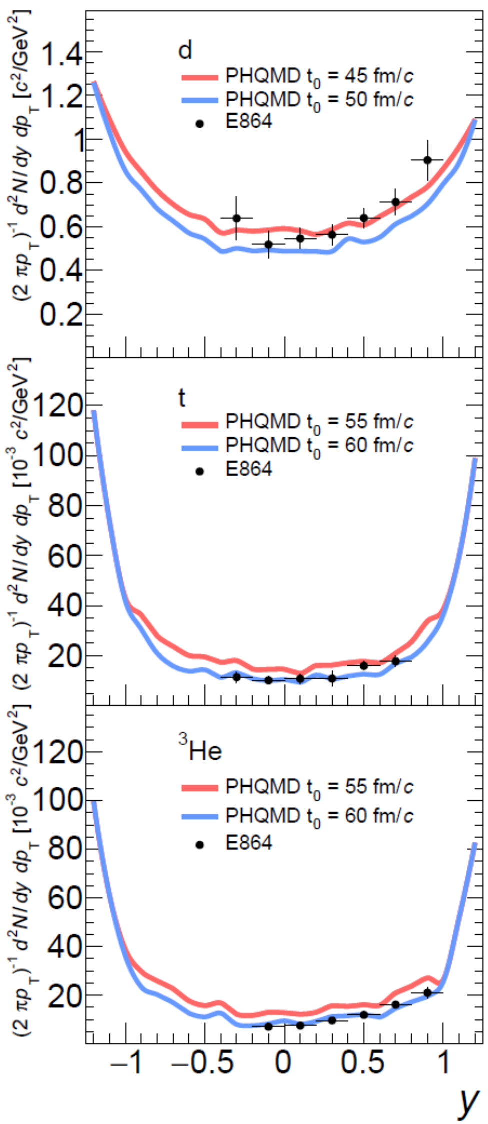
- Recombine nucleons into clusters with $E_B < 0$ if time of cluster disintegration is larger than nucleon freeze-out time

Allows to recover most of "lost" clusters



Cluster production in HICs at AGS energies

y- distributions of d, t, ^{3}He



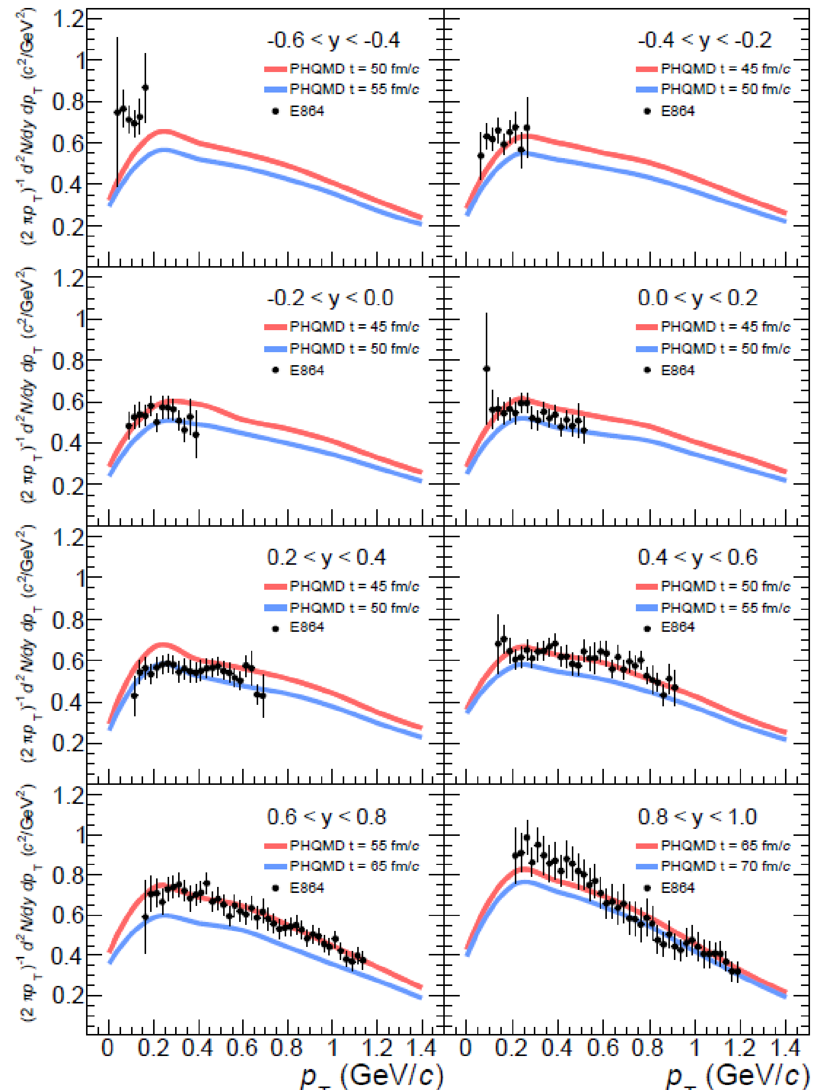
Scenario 1:

The PHQMD results for the y-distribution are taken at 'equal physical time'
$t = t_0 \cosh(y)$,
where t_0 is the time at y=0

Consider t_0 =45 and 50 fm/c

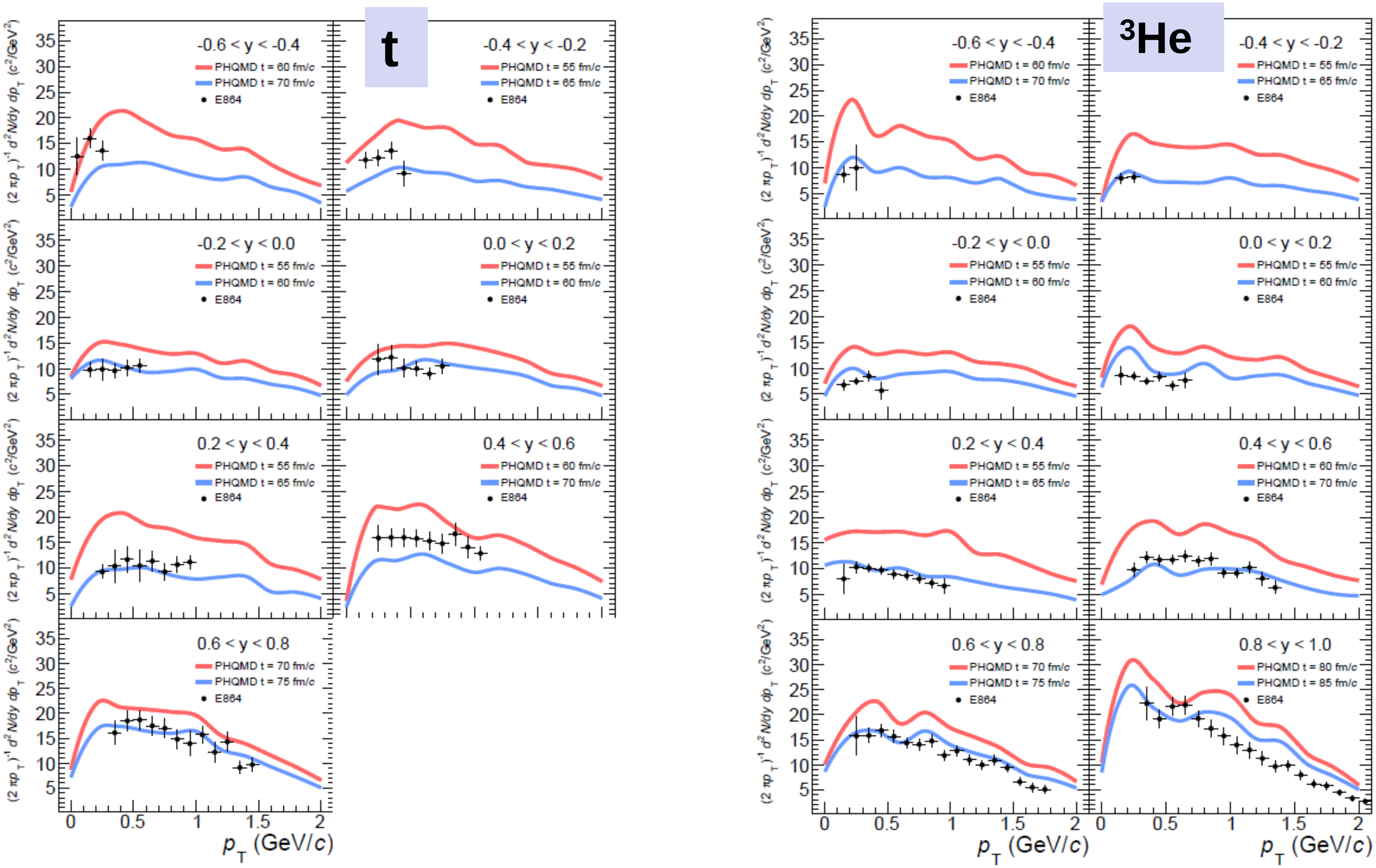
S. Gläßel et al., PRC 105 (2022) 1

p_T - distribution of deuterons



Cluster production in HICs at AGS energies

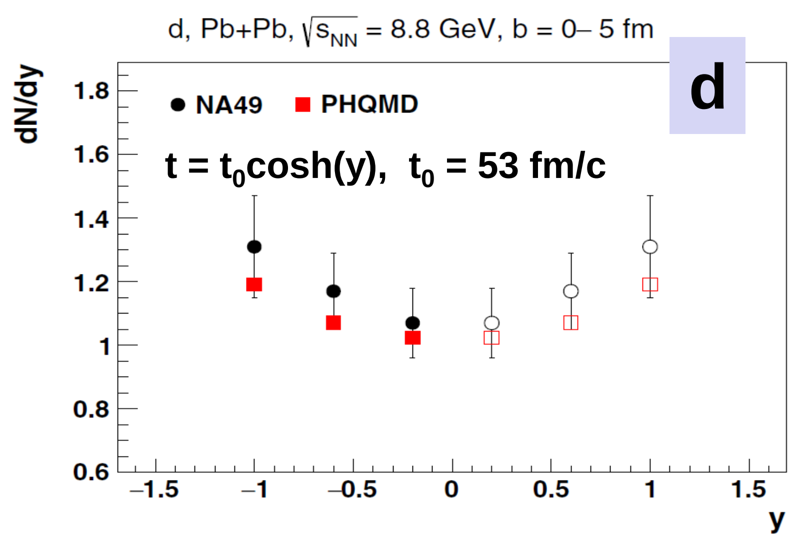
The p_T - distributions of t and ^{3}He from Au+Pb at 10.6 A GeV

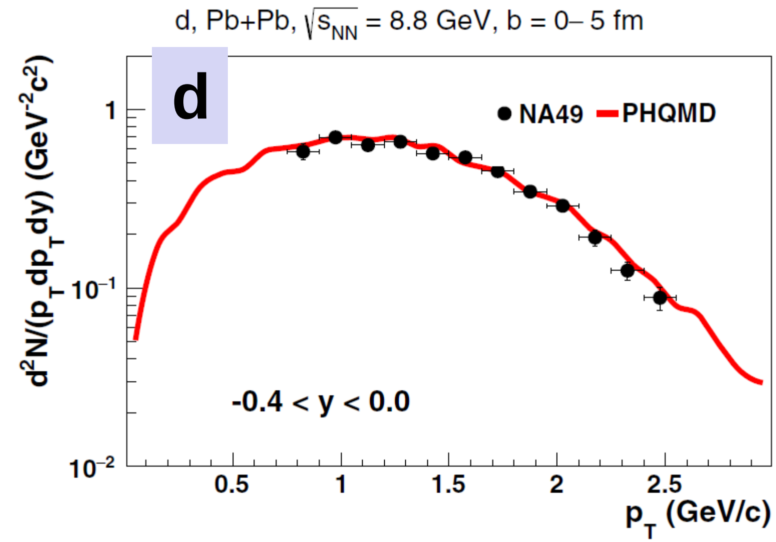


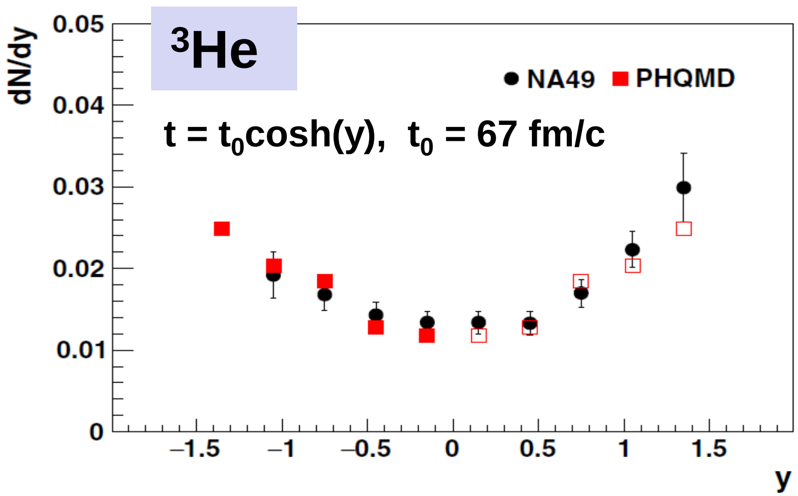
S. Gläßel et al., Phys. Rev. C 105 (2022) 1

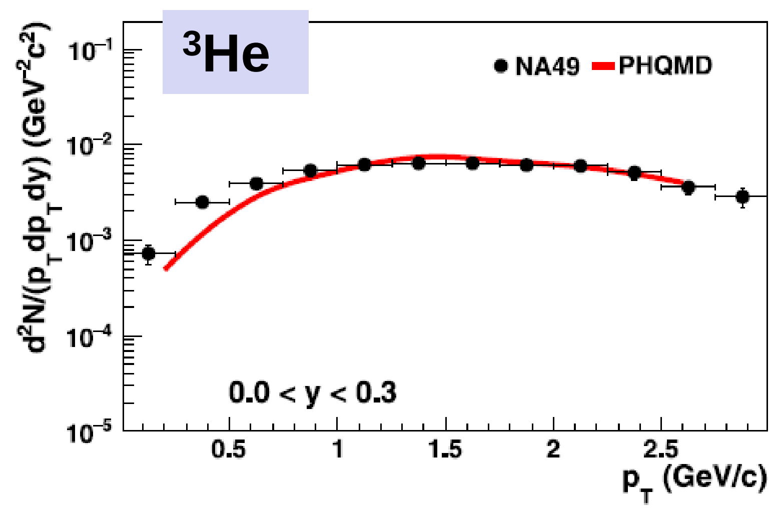
Cluster production in HICs at SPS energies

The rapidity and p_T-distributions of d and ^{3}He from Pb+Pb at 30 A GeV





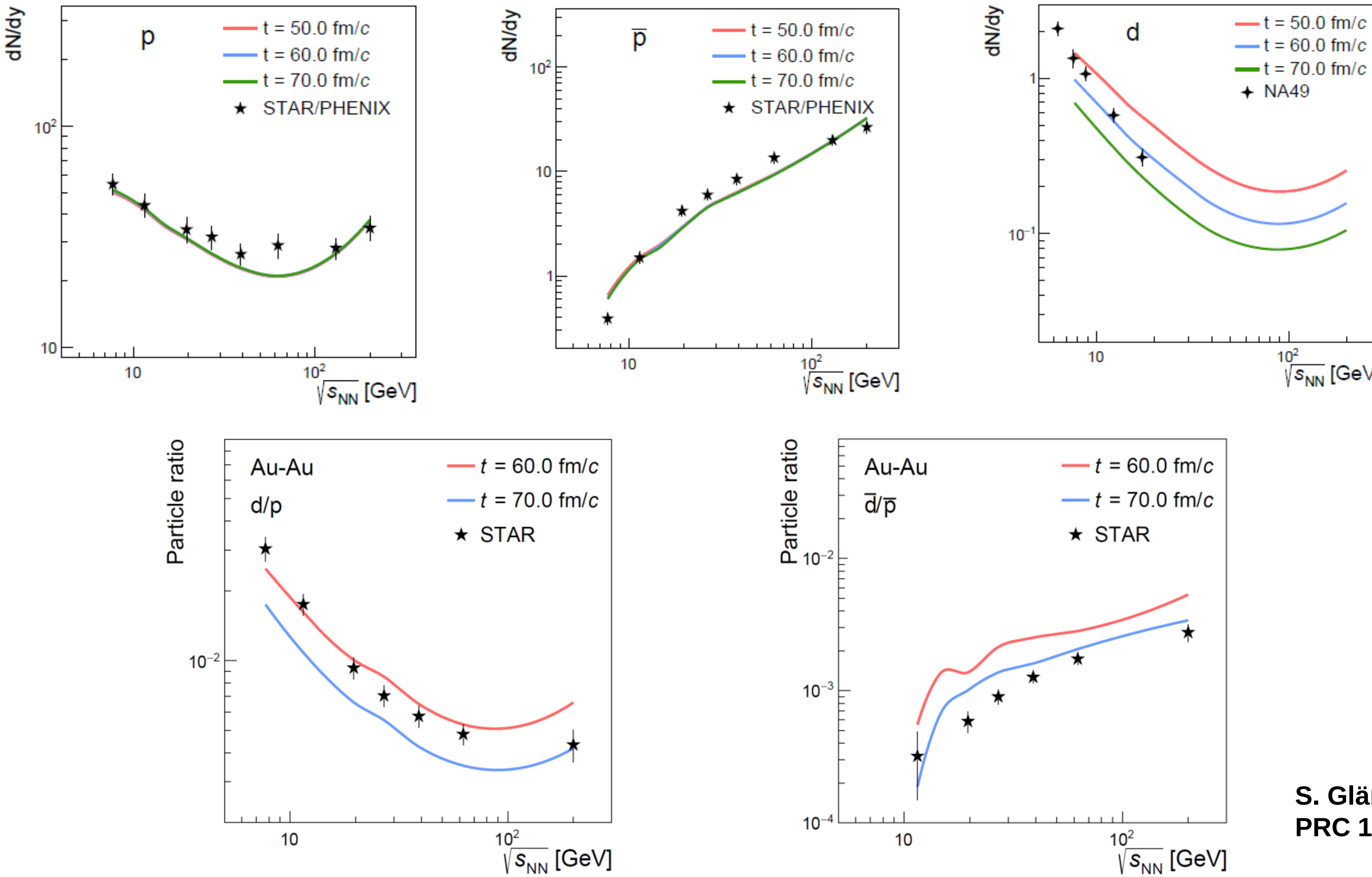




The PHQMD results for d and ^{3}He agree with NA49 data

S. Gläßel et al., Phys. Rev. C 105 (2022) 1

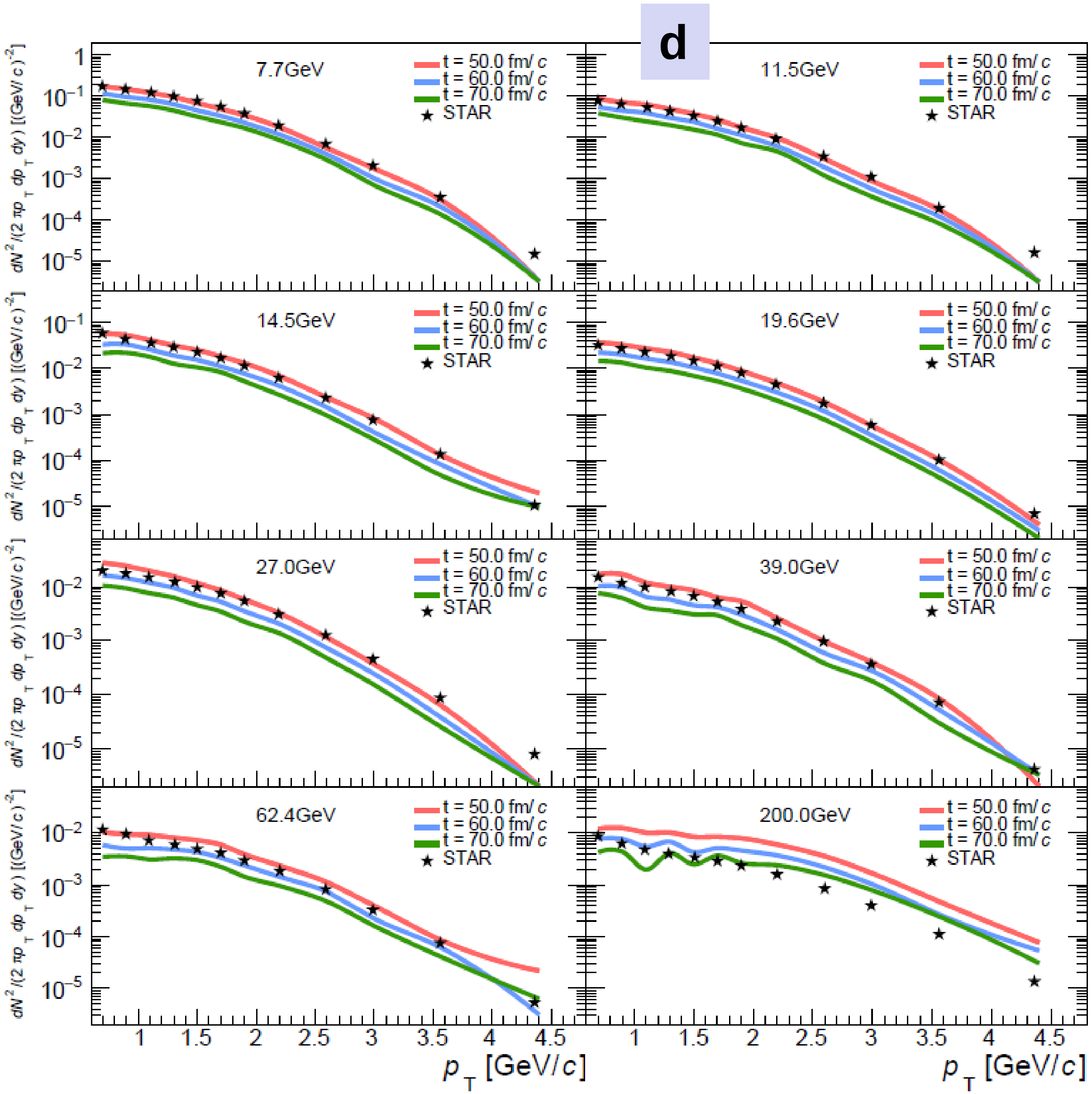
Excitation function of multiplicity of $p, \bar{p}, d, \bar{d}$



S. Gläßel et al., PRC 105 (2022) 1

The $p, \bar{p}$ yields at y~0 are stable, the $d, \bar{d}$ yields are better described at *t= 60-70* fm/c

Deuteron p_T spectra from 7.7GeV to 200 GeV



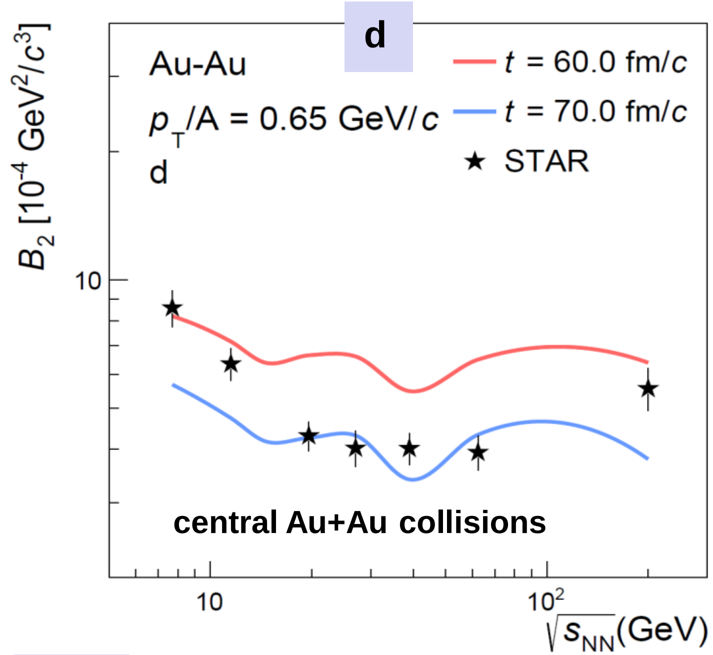
Comparison of the PHQMD results for the deuteron p_T-spectra at midrapidity with STAR data

S. Gläßel et al., Phys. Rev. C 105 (2022) 1

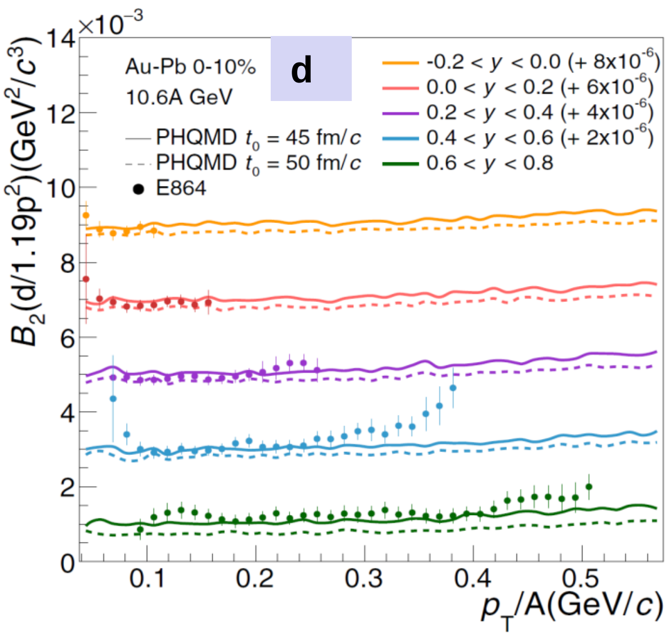
Coalescence parameter B_2 for d and ^{3}He

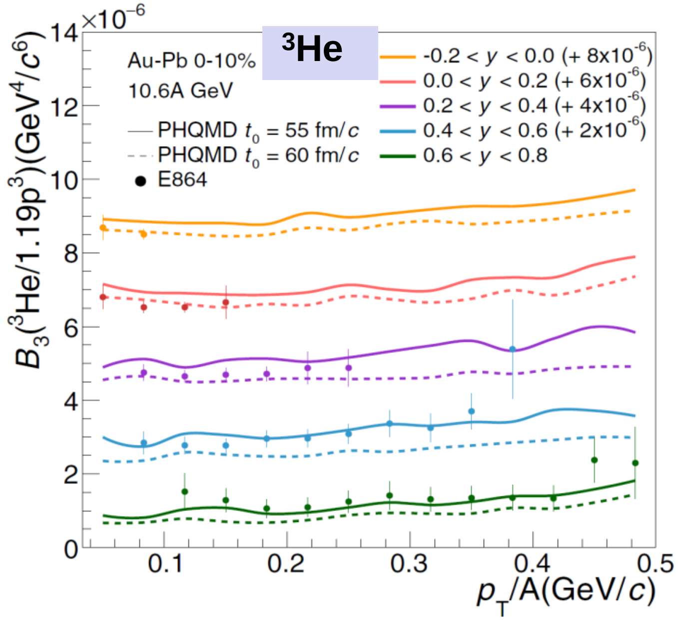
Coalescence parameter B_2:

$$B_2 = \frac{E_d \frac{d^3 N_d}{d^3 P_d}}{\left(E_p \frac{d^3 N_p}{d^3 p_p} \Big|_{p_p = P_d/2} \right)^2}$$



S. Gläßel et al., Phys. Rev. C 105 (2022) 1



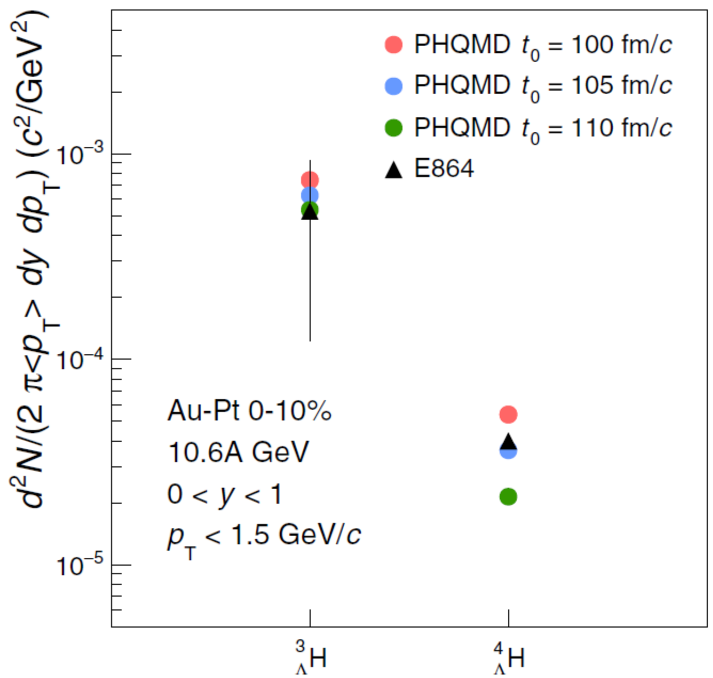


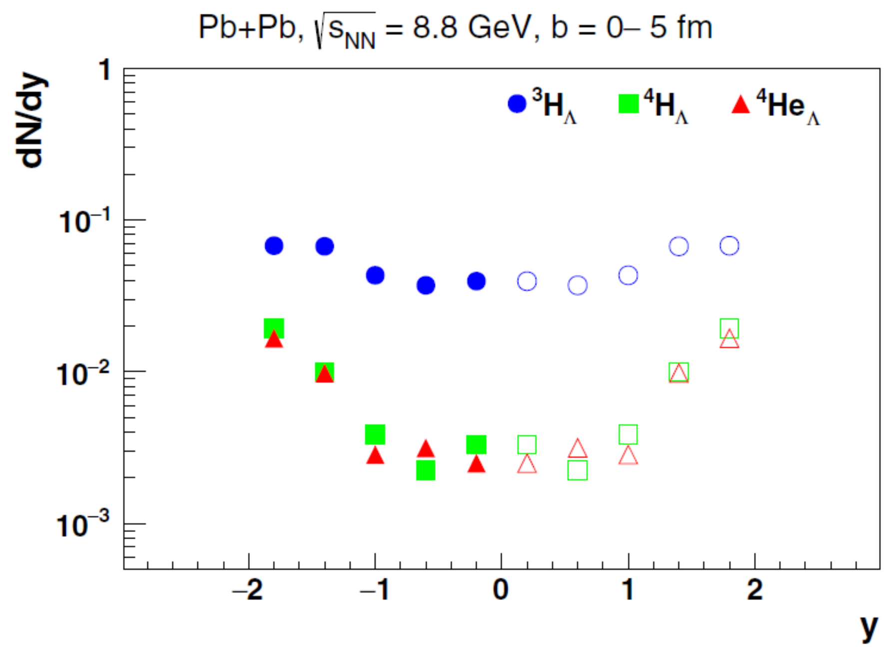
$$t = t_0 \cosh(y)$$

Hypernuclei production at 10.6 and 30 AGeV

The PHQMD results for hypernuclei production in Au+Pt central collisions at 10.6 A GeV

The PHQMD predictions for dN/dy of $^3H_\Lambda$, $^4H_\Lambda$ and $^4He_\Lambda$ from central Pb+Pb collisions at 30 A GeV ($s^{1/2}$ = 8.8 GeV)





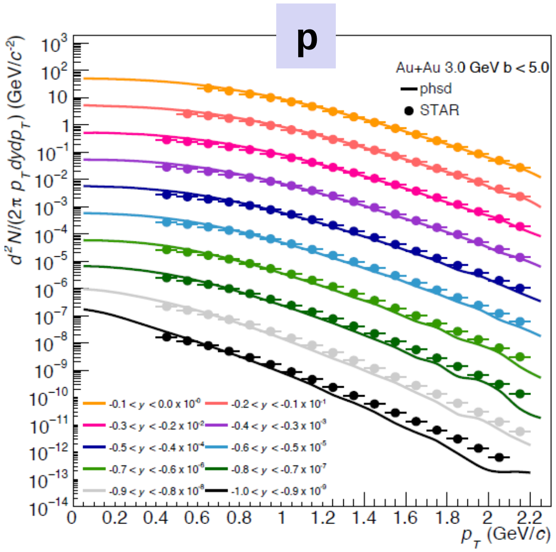
- **Assumption on nucleon-hyperon potential: $V_{N\Lambda}$ = 2/3 V_{NN}**

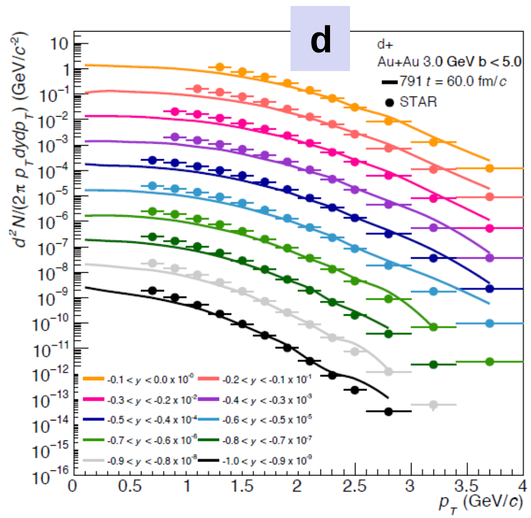
S. Gläßel et al., Phys. Rev. C 105 (2022) 1

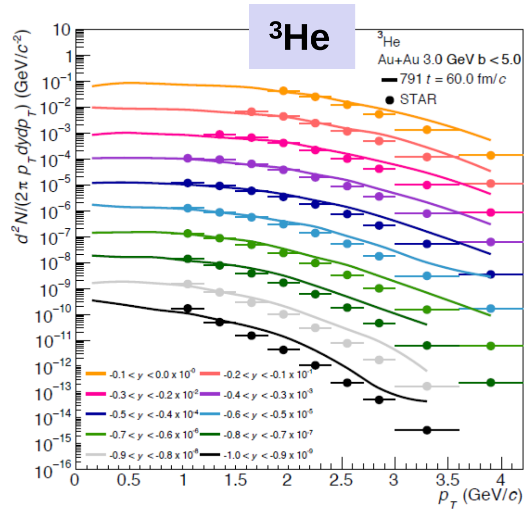
Light cluster production at $s^{1/2}$ = 3 GeV

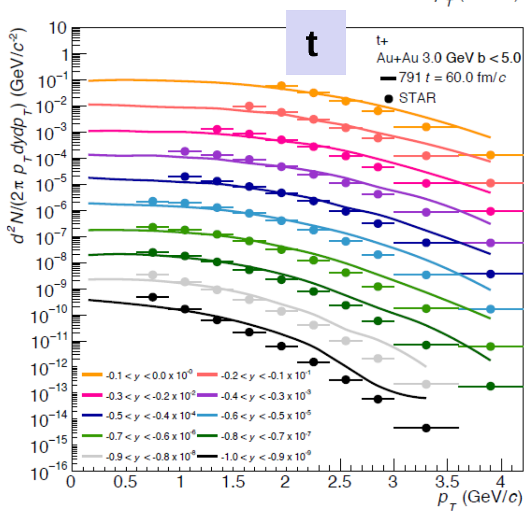
The PHQMD comparison with recent STAR fixed target p_T distribution of p, d, t, ^{3}H, ^{4}H from Au+Au central collisions at $\sqrt{s}$ =3 GeV

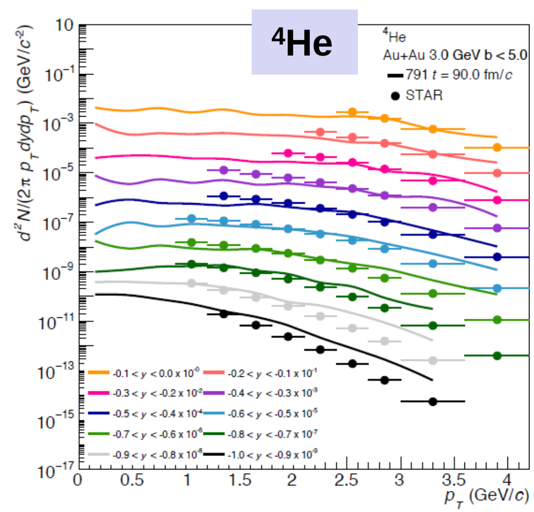
PHQMD: t = 60 fm/c











(preliminary) STAR data – talk by Hui Liu at QM'2022

➔ Good description of cluster production

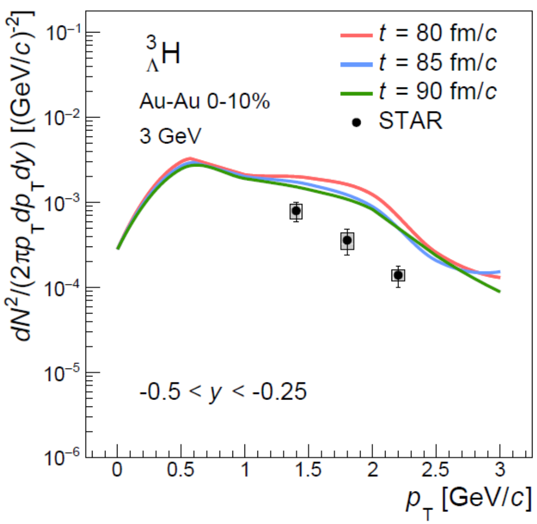
S. Gläßel, PHQMD calculations: arXiv:2208.11802

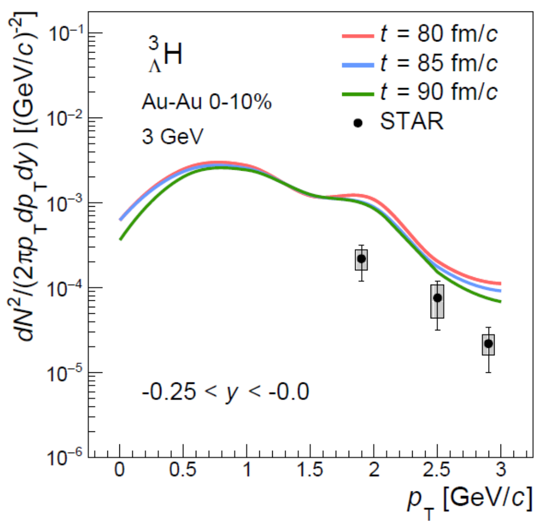
Hypernuclei production at $s^{1/2}$ = 3 GeV

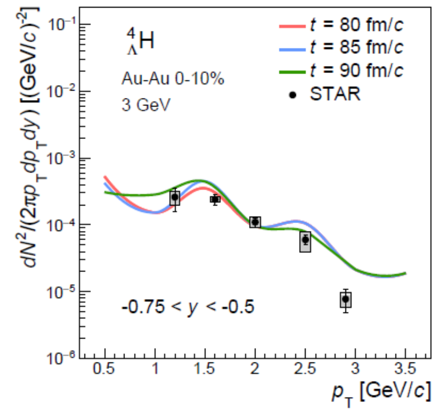
The PHQMD comparison with recent STAR fixed target p_T distribution of $^3H_\Lambda$, $^4H_\Lambda$ from Au+Au central collisions at $\sqrt{s}$ =3 GeV

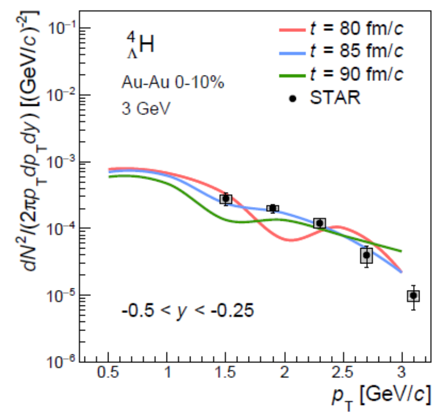
STAR: Phys. Rev. Lett. 128, 202301 (2022)

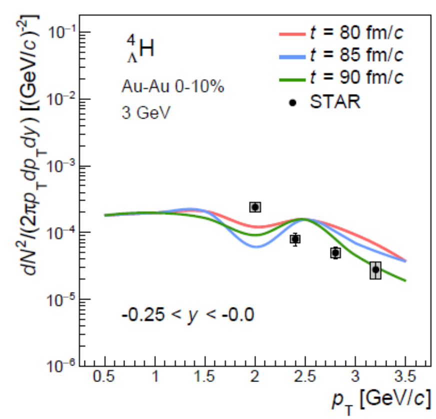
- **Assumption for nucleon-hyperon potential: $V_{N\Lambda}$ = 2/3 V_{NN}**











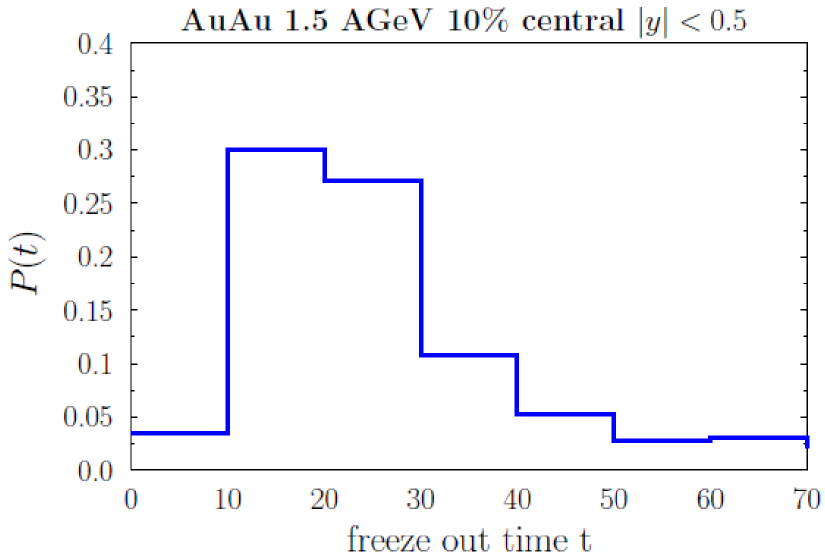
➔ Reasonable description of hypernuclei production at $\sqrt{s}$ =3 GeV

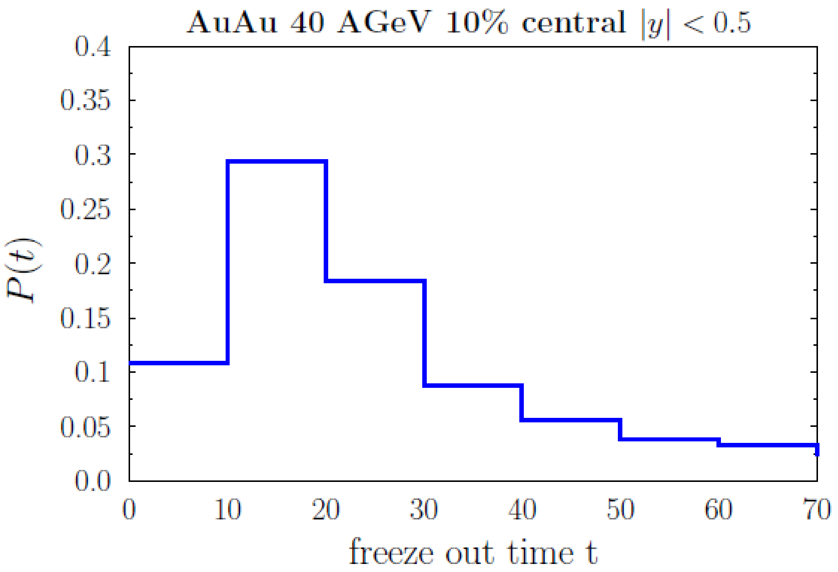
PHQMD: S. Gläßel et al., Phys. Rev. C 105 (2022) 1

When does the system freeze out?

- The normalized distribution of the freeze-out time of baryons (nucleons and hyperons) which are finally observed at mid-rapidity |y|<0.5

 * Here freeze-out time as defined by the last elastic or inelastic collision, after that only potential interaction between baryons occurs





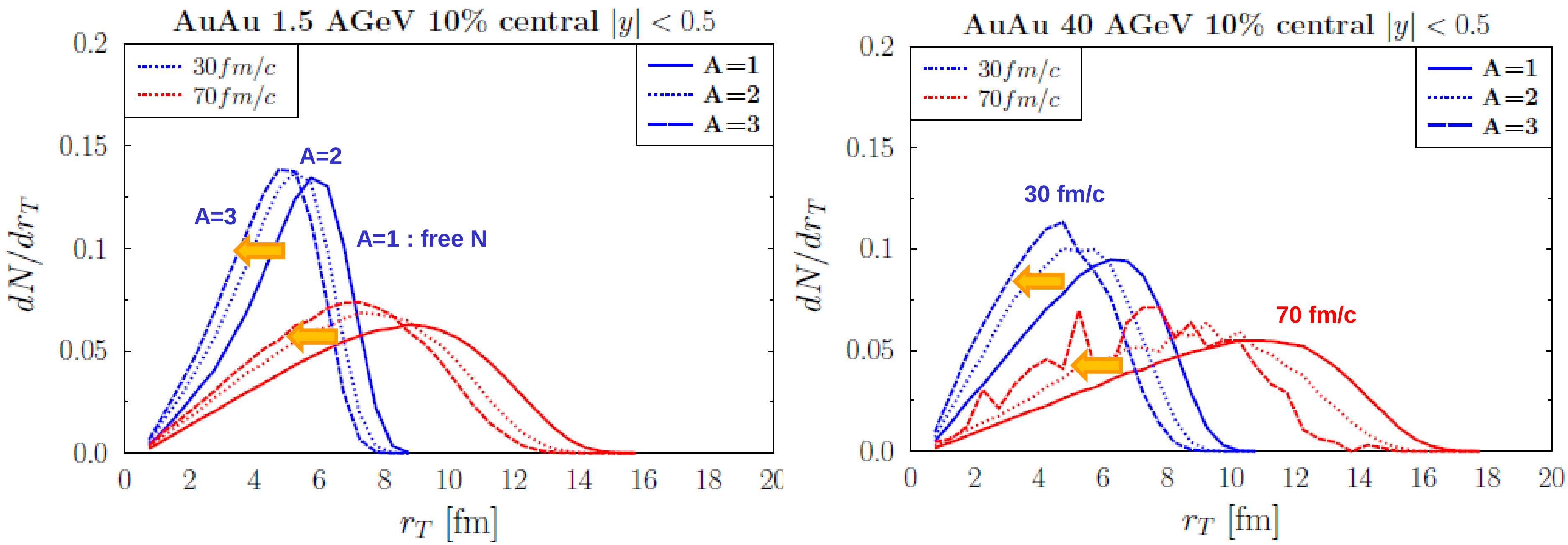
→ **Freeze-out time of baryons in Au+Au at 1.5 AGeV and 40 AGeV:**

- **similar profile since expansion velocity of mid-rapidity fireball is roughly independent of the beam energy**

S. Gläßel et al., Phys. Rev. C 105 (2022) 1

Where are the clusters formed?

- The snapshot (taken at time 30 and 70 fm/c) of the normalized distribution of the transverse distance r_T of the nucleons to the center of the fireball.
- It is shown for A=1 (free nucleons) and for the nucleons in A=2 and A=3 clusters



➔ **Transverse distance profile of free nucleons and clusters are different!**

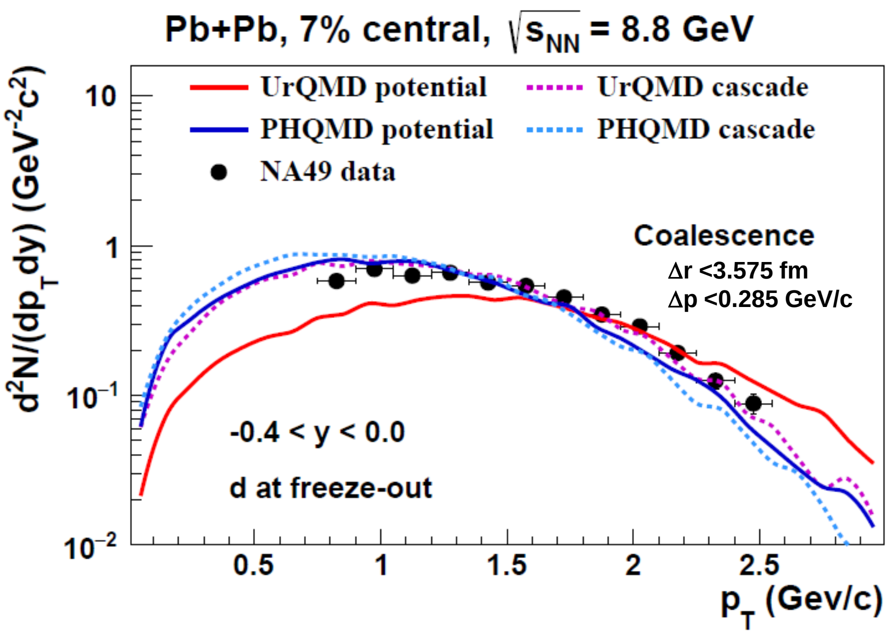
➔ **Clusters are mainly formed behind the 'front' of free nucleons of expanding fireball**

➔ **'ice' is behind the 'fire' ➔ cluster can survive**

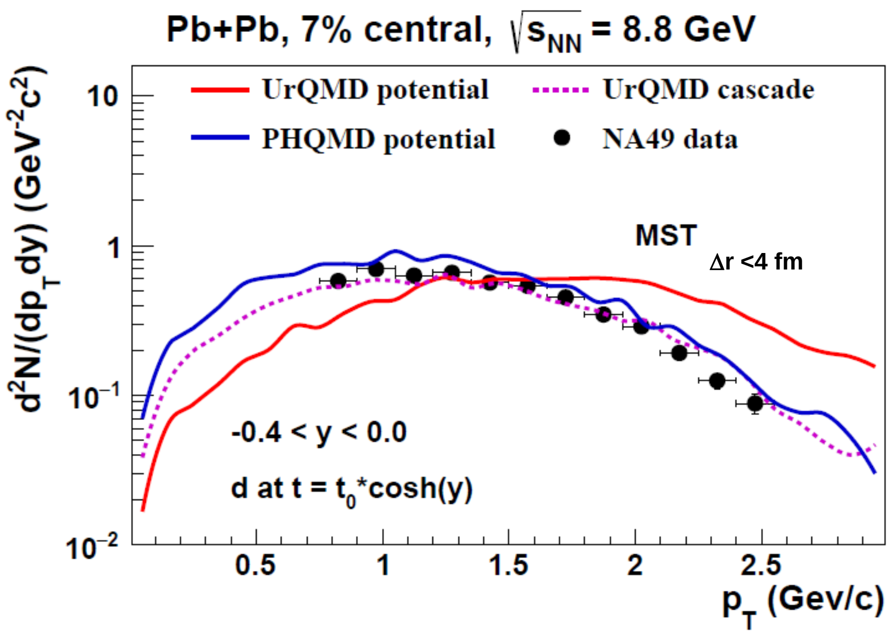
S. Gläßel et al., Phys. Rev. C 105 (2022) 1

Comparison of the coalescence and MST for d

Coalescence



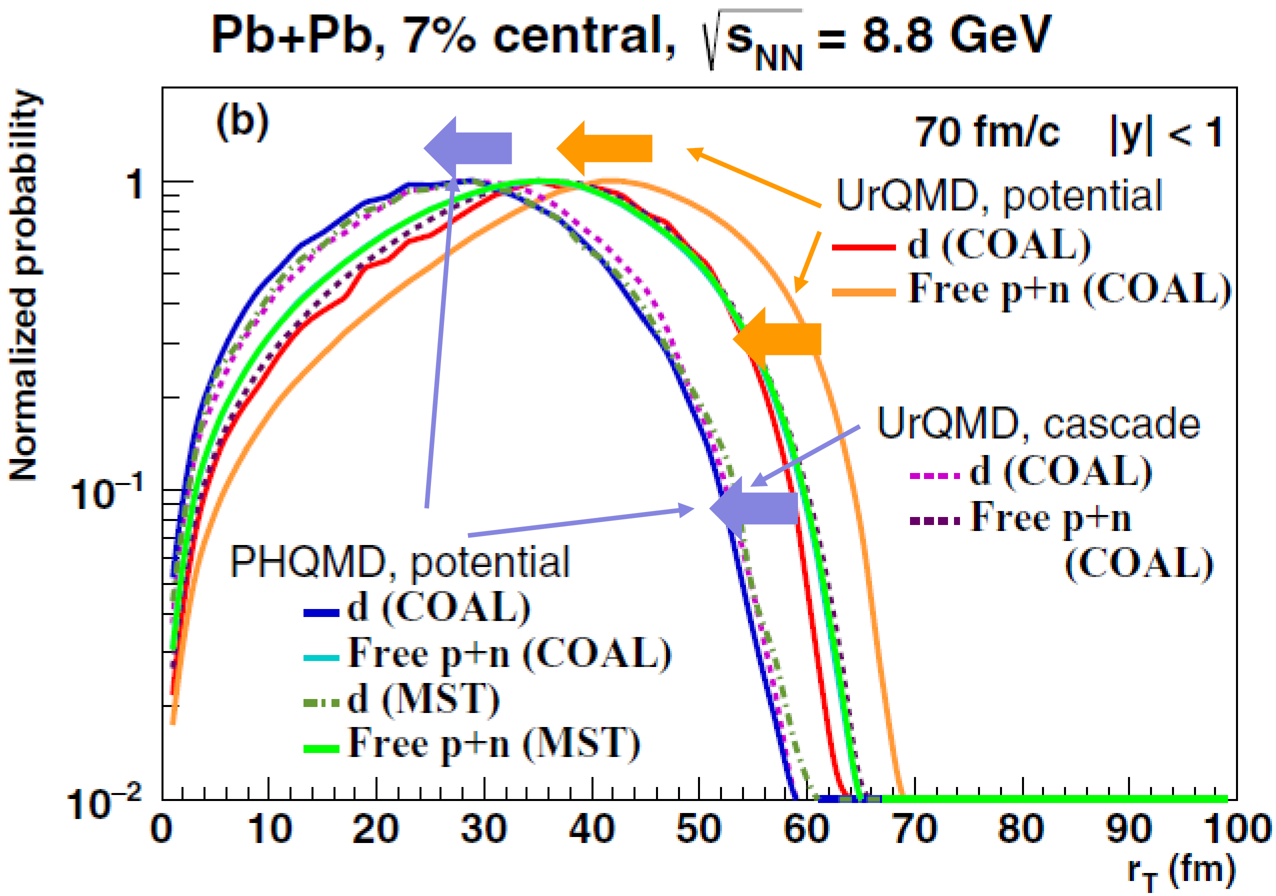
MST



- ➔ **Coalescence and MST give very similar multiplicities and y- and p_T –distributions**
- ➔ **PHQMD and UrQMD results in the cascade mode are very similar**
- ➔ **Deuteron production is sensitive to the realization of potential in transport approaches**

V. Kireyeu, J. Steinheimer, M. Bleicher, J. Aichelin, E.B., Phys. Rev. C 105 (2022) 044909

Comparison of the coalescence and MST for d



➔ **Coalescence as well as the MST procedure show that the deuterons remain in transverse direction closer to the center of the heavy-ion collision than free nucleons**

➔ **deuterons are behind the fast nucleons (and pion wind)**

V. Kireyeu, J. Steinheimer, M. Bleicher, J. Aichelin, E.B., Phys. Rev. C 105 (2022) 044909

II. Kinetic mechanism for deuteron production in PHQMD

Gabriele Coci et al., in preparation

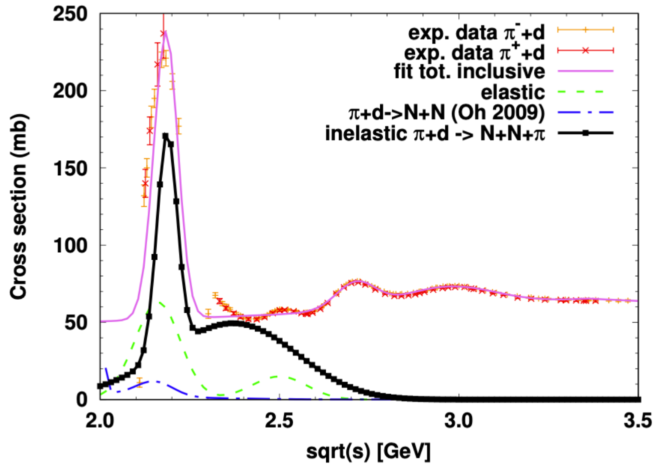
Deuteron production by hadronic reactions

"Kinetic mechanism"

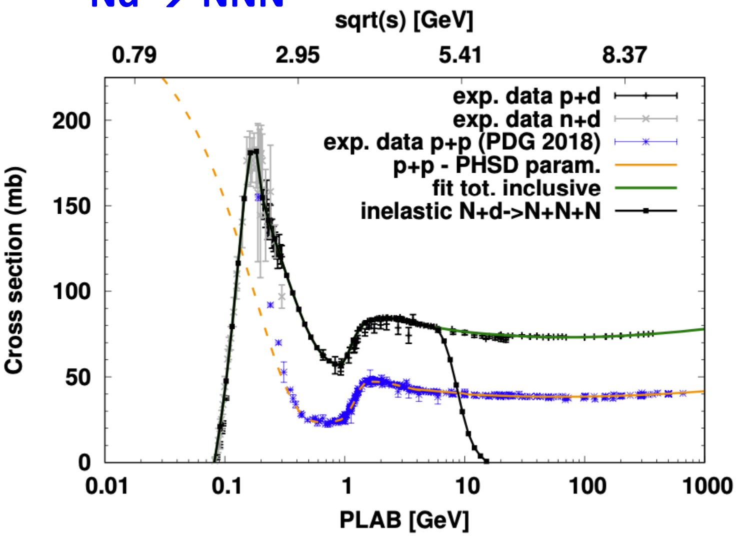
1) hadronic inelastic reactions NN ⟷ dπ , πNN ⟷ dπ , NNN ⟷ dN
2) hadronic elastic π+d, N+d reactions

- Hadronic reactions for d+π and d+N scattering have very large cross sections $\sigma_{peak} \approx 200$ mb

πd → πNN



Nd → NNN

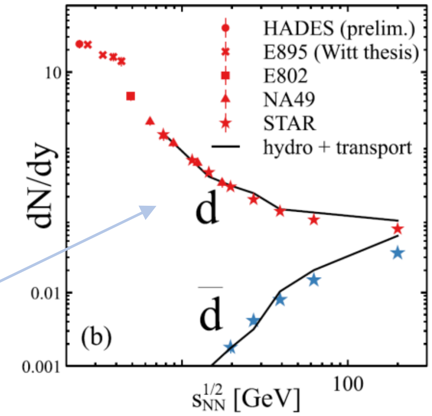


- the rates for the inverse processes pNN → pd, NNN → dN in hadronic matter are large due to the time-reversal symmetry

* Kinetic production by inverse reaction N + p + n → N + d first studied in HICs at $E_{Lab} \backsim 1$ AGeV by P.J. Siemens, J. Kapusta PRL 43 (1979) 1486

Models for deuteron production by hadronic reactions

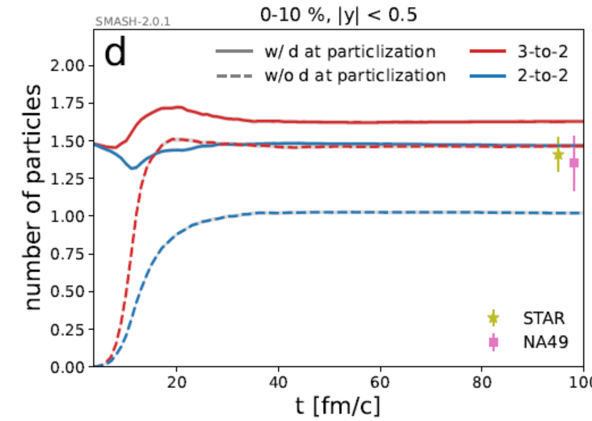
- **SMASH, AMPT:** Inverse reactions X+N+N → X+d (X=π,N with X catalyzer) important for d formation in HICs
- at RHIC and LHC energies: large π abundance
  → deuterons form by π-catalysis: π+p+n → π+d
  at SIS energies: large N abundance
  → deuterons form by N-catalysis: N+p+n → N+d



- **SMASH (hydro + kinetic):** $\pi NN \leftrightarrow d\pi$, $NNN \leftrightarrow dN$ are realized

1) via fictitious dibaryon resonance d' as two-step processes of N+N → d' and π + d' → π + d
   D. Oliinychenko PRC 99 (2019) 4, 044907

2) via 3 ⟷ 2 transition rates
   J. Staudenmaier et al., PRC 104 (2021) 3, 034908



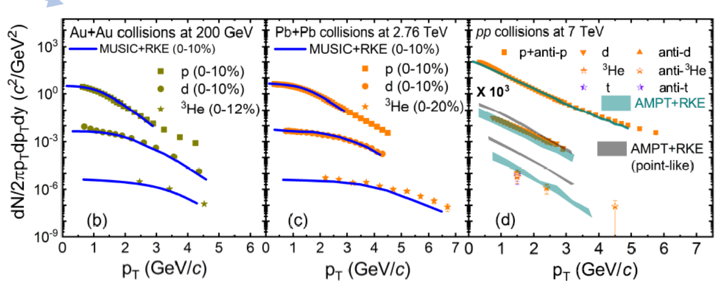
- **AMPT:** $\pi NN \leftrightarrow d\pi$ via impulse approximation:

$$\mathcal{M}_{\pi d\to\pi NN} \rightarrow \langle\tilde{p}_N|\phi_d\rangle\mathcal{M}_{\pi N\to\pi N}$$

+ accounting of the finite size of deuterons via Wigner function

$$|\phi_d(\tilde{\mathbf{p}})|^2 = \int d^3\mathbf{r}\gamma_d W_d = (4\pi\sigma_d^2)^{3/2}e^{-\tilde{\mathbf{p}}^2\sigma_d^2}$$

leads to the suppression of d production in pp



K.-J. Sun, R. Wang, C.-M. Ko et al., 2106.12742

Collision Integral: covariant rate for n←→m reactions

PHQMD

- In Boltzmann Equation the Collision Integral accounts for all dissipative processes (hadronic reactions ...)

W. Cassing NPA 700 (2002) 618

$$p_{1,\mu}\partial^\mu_x f_i(x,p_1) = I^i_{coll} = \sum_n \sum_m I^i_{coll}[n \leftrightarrow m]$$

$$I^i_{coll}[n \leftrightarrow m] = \frac{1}{2}\frac{1}{N_{id}!}\sum_\nu\sum_\lambda \left(\frac{1}{(2\pi)^3}\right)^{n+m-1} \left(\prod_{j=2}^{n}\int\frac{d^3\vec{p}_j}{2E_j}\right)\left(\prod_{k=n+1}^{n+m}\int\frac{d^3\vec{p}_k}{2E_k}\right)$$
$$\times(2\pi)^4\delta^4(p_1^\mu + \sum_{j=2}^{n} p_j^\mu - \sum_{k=1}^{n+m} p_k^\mu)\, W_{n,m}(p_1,p_j;i,\nu \mid p_k;\lambda)$$
$$\times\left[\prod_{k=n+1}^{n+m} f_k(x,p_k) - f_i(x,p_1)\prod_{j=2}^{n} f_j(x,p_j)\right]$$

(n-1) initial + m final integrations

Transition amplitude squared

Gain - Loss

- Collision rate for hadron "*i*" is the number of reactions in the covariant volume $d^4x = dt*dV$

$$\frac{dN_{coll}[n(i)\to m]}{dtdV} \propto \int\frac{d^3p_1}{2E_1}f_i(x,p_1)\int\left(\prod_{j=2}^{n}\frac{d^3p_j}{2E_j}f_j(x,p_j)\right)\int\left(\prod_{k=n+1}^{n+m}\frac{d^3p_k}{2E_k}\right)$$
$$\times(2\pi)^4\delta^4(\sum_{j=1}^{n} p_j^\mu - \sum_{k=n+1}^{n+m} p_k^\mu)\, W_{n,m}(p_j;\tau(i),\nu \mid p_k;\lambda)$$

... similar for m→n(i)

Collision Integral: covariant rate formalism

PHQMD

- **With n=2 initial particles , the covariant rate can be expressed in terms of the reaction cross section**

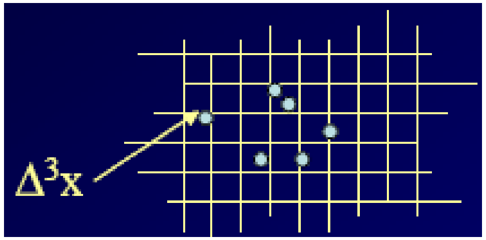
W. Cassing NPA 700 (2002) 618

$$\frac{dN_{coll}[1(d)+2\to 3+4]}{dtdV} \propto \frac{1}{(2\pi)^6}\int \frac{d^3p_1}{2E_1} f_1(x,p_1) \int \frac{d^3p_2}{2E_2} f_2(x,p_2) \times$$

$$\int \frac{d^3p_3}{(2\pi)^3 2E_3}\int \frac{d^3p_4}{(2\pi)^3 2E_4} W_{2,2}(p_1,p_2;p_3,p_4)(2\pi)^4\,\delta^4(p_1+p_2-p_3-p_4) \;\rightarrow\; 4E_1E_2 v_{rel}\sigma_{2,2}(\sqrt{s})$$

- **Using test-particle ansatz for *f(x,p)* the collision integral is numerically solved dividing the coordinate space in cells of volume ΔV_{cell} where the reaction rate at each time step Δt are sampled stochastically with probability:**

$$\frac{\Delta N_{coll}[1(d)+2\to 3+4]}{\Delta N_1 \Delta N_2} = P_{2,2}(\sqrt{s}) = v_{rel}\sigma_{2,2}(\sqrt{s})\frac{\Delta t}{\Delta V_{cell}}$$



Similarly…

$$\frac{\Delta N_{coll}[1(d)+2\to 3+4+5]}{\Delta N_1 \Delta N_2} = P_{2,3}(\sqrt{s}) = v_{rel}\sigma_{2,3}(\sqrt{s})\frac{\Delta t}{\Delta V_{cell}}$$

Lang, Babovsky, Cassing, Mosel, Reusch and Weber, J. Comp. Phys., vol. 106, no. 2, (1993)
Used by BAMPS - Xu and Greiner PRC v. 71, (2005)

- **$\Delta t \to 0$, $\Delta v_{cell} \to 0$ convergence to exact solution**

Collision Integral: covariant rate formalism

- **With n > 2 initial particles, the covariant rate cannot be expressed in terms of the reaction cross section**

W. Cassing NPA 700 (2002) 618

$$\frac{dN_{coll}[3+4+5\to 1(d)+2]}{dtdV} = \int \left(\prod_{k=3}^{5} \frac{d^3p_k}{(2\pi)^3 2E_k} f_k(x,p_k) \right) \times$$
$$\int \frac{d^3p_1}{(2\pi)^3 2E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} W_{3,2}(p_3,p_4,p_5;p_1,p_2)(2\pi)^4\, \delta(p_1+p_2-p_3-p_4-p_5)$$

- **With the assumption for the TRANSITION AMPLITUDE: $W(\sqrt{s})$ + detailed balance the covariant collision rate can be still expressed in terms of the reaction probability. With test particle ansatz the transition rate for 3→2 reactions:**

$$\frac{\Delta N_{coll}[3+4+5\to 1(d)+2]}{\Delta N_3 \Delta N_4 \Delta N_5} = P_{3,2}(\sqrt{s})$$

$$P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3E_4E_5} \frac{R_2(\sqrt{s},m_1,m_2)}{R_3(\sqrt{s},m_3,m_4,m_5)} \frac{1}{\Delta V_{cell}}$$

Energy and momentum of final particles

2,3-body phase space integrals [Byckling, Kajantie]

PHQMD: deuteron reactions in the box

π+p+n⟷ d+π , p+n+N ⟷ d+N , N+N ⟷ d+π , d+X elastic

- **2 ⟶ 2 and 2 ⟶ 3 either by geometric criterium or stochastic method**

Kodama et al. Phys. Rev. C 29 (1984)

$$d_T < \sqrt{\frac{\sigma_{tot}^{2,3}(\sqrt{s})}{\pi}}$$

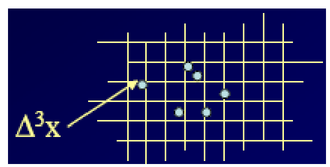
W. Cassing NPA 700 (2002) 618

$$P_{2,3}\left(\sqrt{s}\right) = \sigma_{tot}^{2,3}(\sqrt{s})v_{rel}\frac{\Delta t}{\Delta V_{cell}}$$

- **3 ⟶ 2 realized via covariant rate formalism by stochastic method**

W. Cassing NPA 700 (2002) 618

- **Numerically tested in "static" box**



- **PHQMD provides a good agreement with analytic solutions from rate equations**

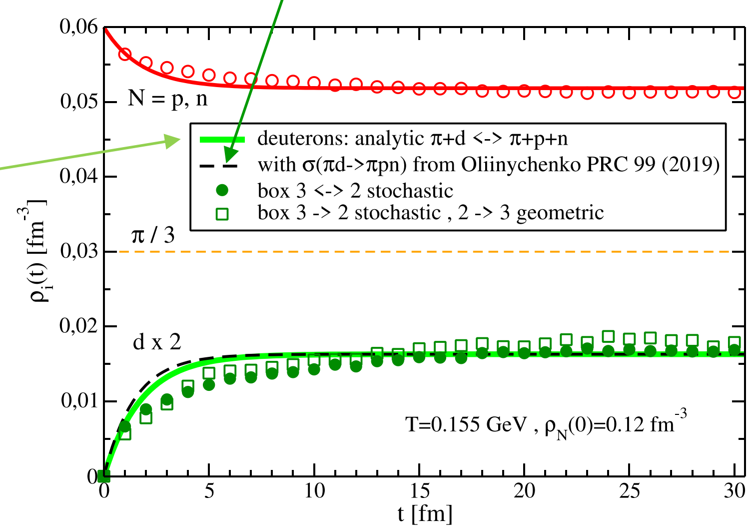
$$\begin{cases} \dot{\lambda}_d = \sum < v_{rel}\sigma_{\pi d} > \left(\frac{g_d g_\pi}{g_N^2 g_\pi}\lambda_N^2 - \lambda_d\right) n_\pi^{eq}\lambda_\pi \\ \dot{\lambda}_N = -\sum < v_{rel}\sigma_{\pi d} > \left(\frac{g_d g_\pi}{g_N^2 g_\pi}\lambda_N^2 - \lambda_d\right) n_\pi^{eq}\lambda_\pi \\ \dot{\lambda}_\pi = 0 \end{cases}$$

+ initial conditions

[Y. Pan S. Pratt, PRC 89 (2014), 044911]

Comparison to SMASH cross sections: F_{iso} = 1

J. Staudenmaier et al., PRC 104 (2021) 3, 034908



Density inside the box at temperature T: $\rho_i = n^{eq}(T)*\lambda_i(t)$

Isospin deuteron reactions in the box

π+N+N⟷ d+π , d+N ⟷ p+n+N , N+N ⟷ d+π , d+X elastic

$$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$$
$$\pi^- + p + p \leftrightarrow \pi^0 + d$$
$$\pi^+ + n + n \leftrightarrow \pi^0 + d$$
$$\pi^0 + p + p \leftrightarrow \pi^+ + d$$
$$\pi^0 + n + n \leftrightarrow \pi^- + d$$

Novel aspects in PHQMD:

N+N+π inclusion of all possible channels allowed by total isospin T conservation:

$$P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3E_4E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

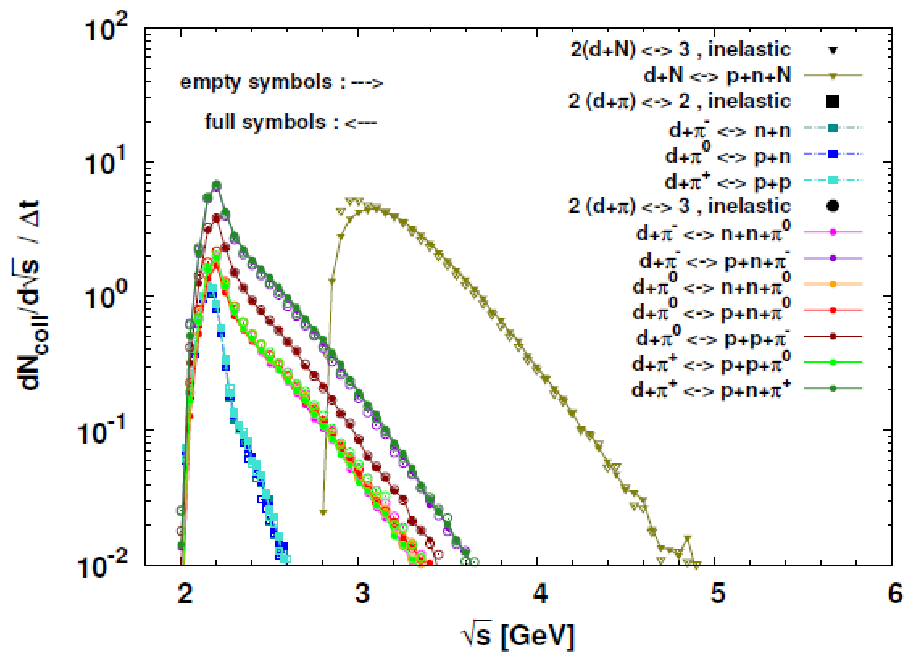
- **NNπ expanded as superposition of eigenstates of total isospin T**

$$|N, N, \pi\rangle = \sum_T \sum_{T_3=-T}^{-T} \langle T, T_3 | N, N, \pi\rangle \, |T, T_3\rangle$$

- **Fourier coefficient of eigenstate of total isospin 1 (= T(d π)=T(π))**

$$F_{iso} = |\langle N, N, \pi | T(d+\pi) = 1, T_3\rangle|^2$$

➔ **For the realistic description of HICs: Important to account for all possible isospin channels !**

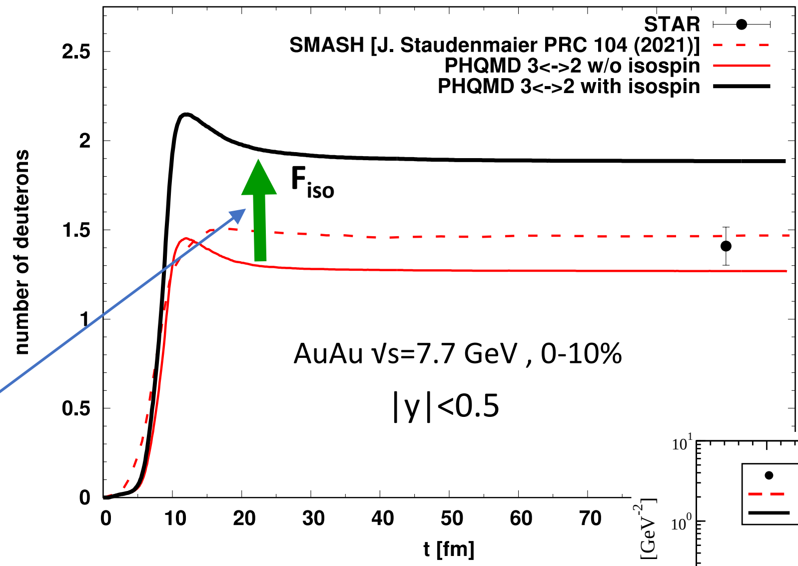


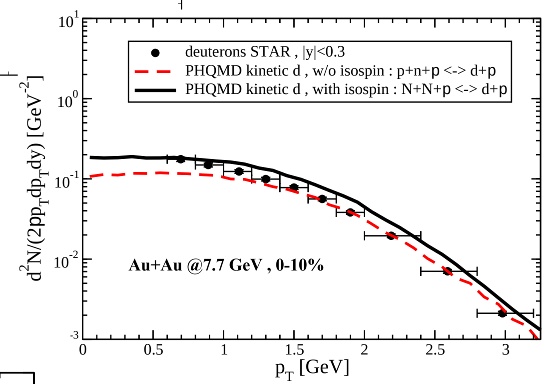
→ **Detailed balance condition fulfilled**

Kinetic deuterons in PHQMD – isospin effects

<ins>RHIC BES energy √s = 7.7 GeV:</ins>

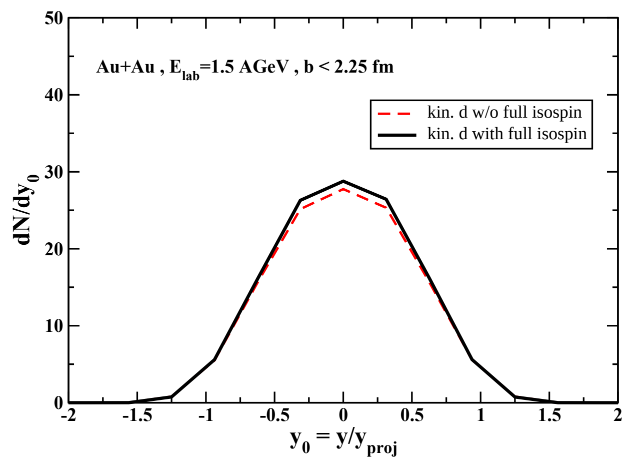
- **Hierarchy due to large π abundance**
  π+N+N→π+d >> N+p+n→N+d
- **Inclusion of all isospin channels enhances deuteron yield ~ 50%.**
- **p_T slope is not affected**





<ins>GSI SIS energy √s < 3GeV :</ins>

- **Baryon dominated matter**
- **Enhancement due to inclusion of isospin π+N+N channels is negligible**



Modelling finite-size effects in kinetic mechanism

How to account for the quantum nature of deuteron, i.e. for

1) **the finite-size of *d* in coordinate space (*d* is not a point-like particle) – for in-medium d production**
2) **the momentum correlations of *p* and *n* inside *d***

Realization 1) assume that a deuteron can not be formed in a high density region, i.e. if there are other particles (hadrons or partons) inside the 'excluded volume':

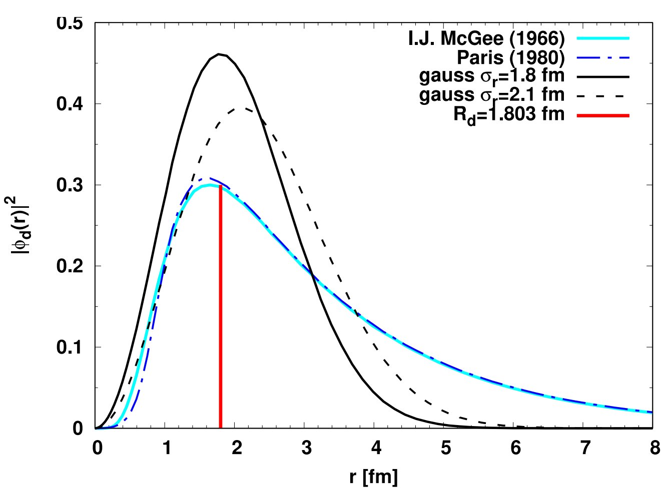
Excluded-Volume Condition:

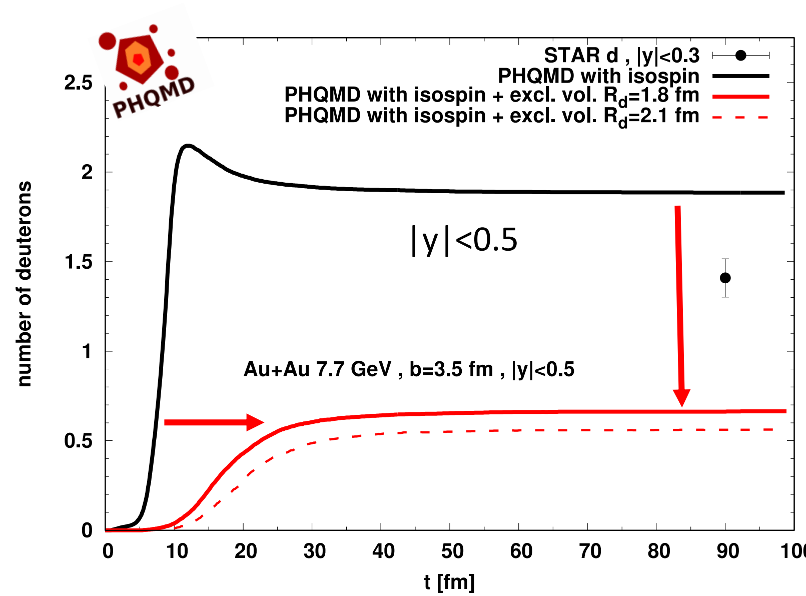
$$|\vec{r}(i)^{*} - \vec{r}(d)^{*}| < R_d$$

"i" is any particle not participating in $\pi NN \rightarrow \pi d$, $NNN \rightarrow Nd$, $NN \rightarrow d\pi$
*** means that positions are in the cms of pre-calculated "candidate" deuteron**

The exclusion parameter R_d is tuned to the physical radius

$$\langle r_d^2 \rangle = \int_0^\infty r^2 |\phi_d(r)|^2 dr \sim (1.8\, fm)^2$$

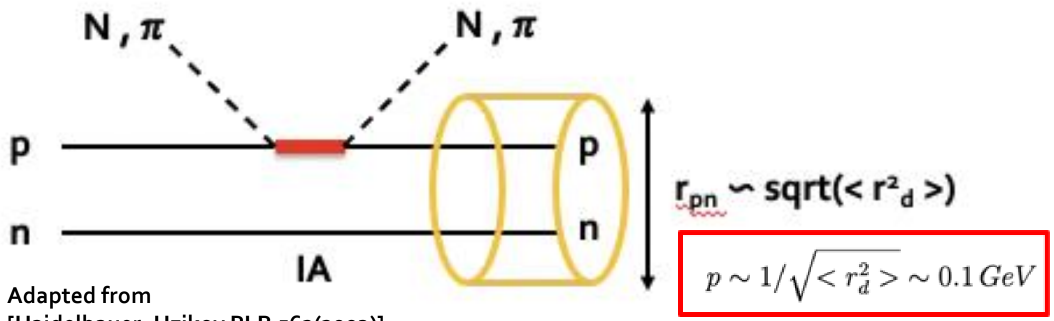




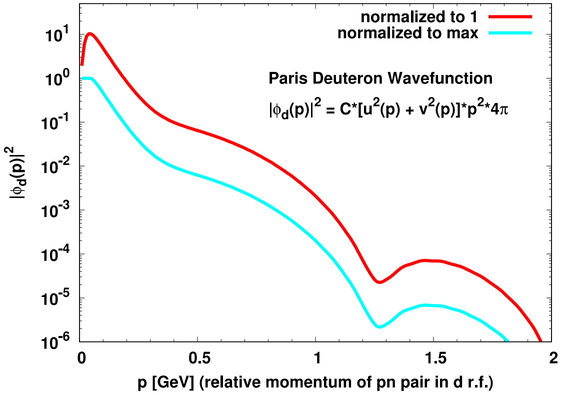
- ❑ **Strong reduction of d production!**
- ❑ **p_T slope is not affected by excluded volume condition**

Modelling finite-size effects in kinetic mechanism

2) QM properties of deuteron must be also in momentum space
→ momentum correlations of pn-pair

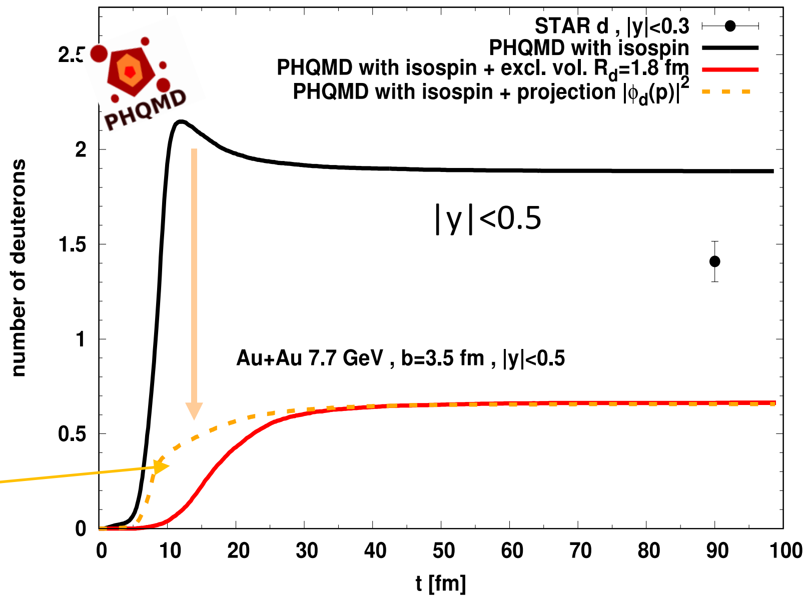


Adapted from
[Haidelbauer, Uzikov PLB 562(2003)]
[Hoftiezer et al. PRC23 (1981)]
Same spirit as AMPT [K.-J. Sun, R. Wang, C.-M. Ko et al., 2106.12742]



[Lacombe et al. PL (1981)]

- For a "candidate" deuteron calculate the relative momentum p of the interacting pn-pair in the deuteron rest frame
- The probability of the pn-pair to bind into a final deuteron with momentum p is given by the DWF $|\phi_d(p)|^2$
- Bound pn-pairs are selected by projection on DWF $|\phi_d(p)|^2$

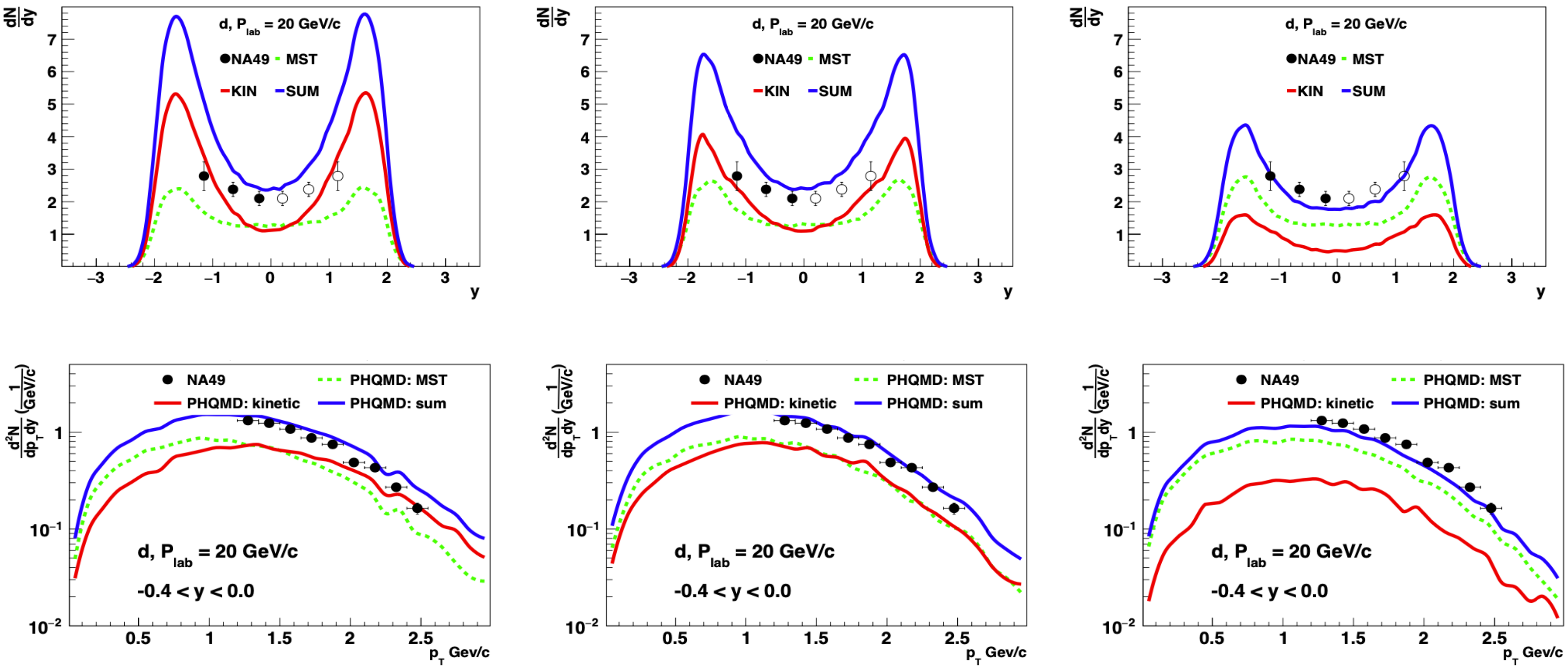


❑ Strong reduction of d production by projection on DWF $|\phi_d(p)|^2$

Kinetic vs. potential deuteron production

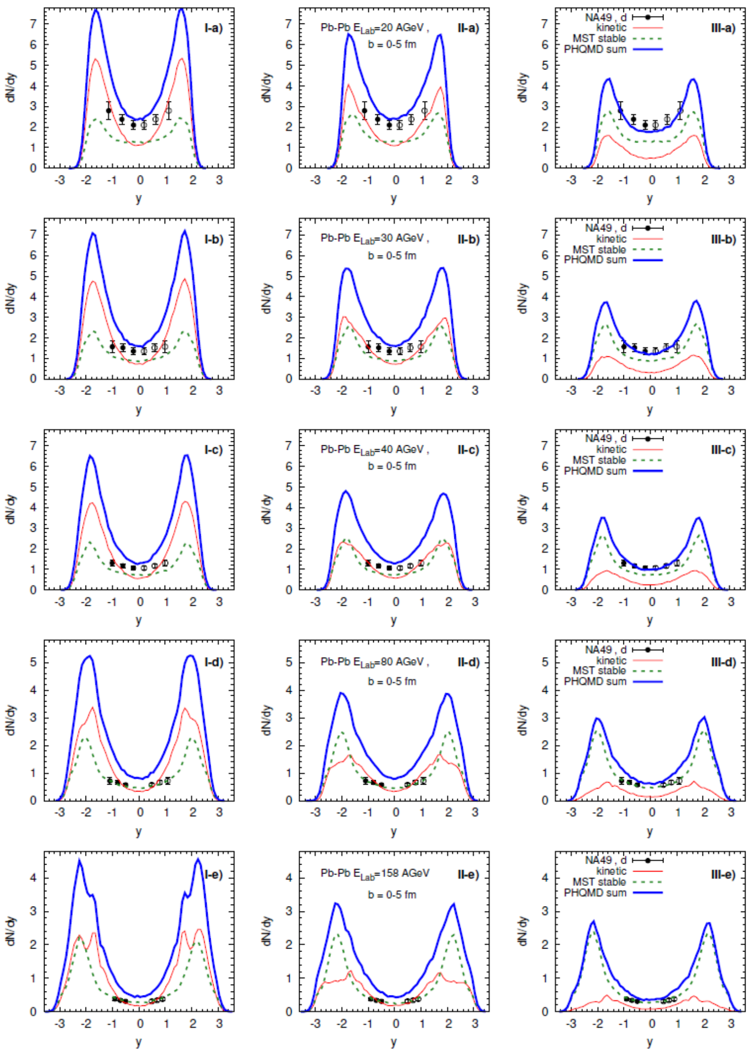
Total deuteron production = Kinetic mechanism with finite-size effects
+ MST (with stabilization) identification of deuterons (“stable” bound (E_B<0) A=2, Z=1 clusters)

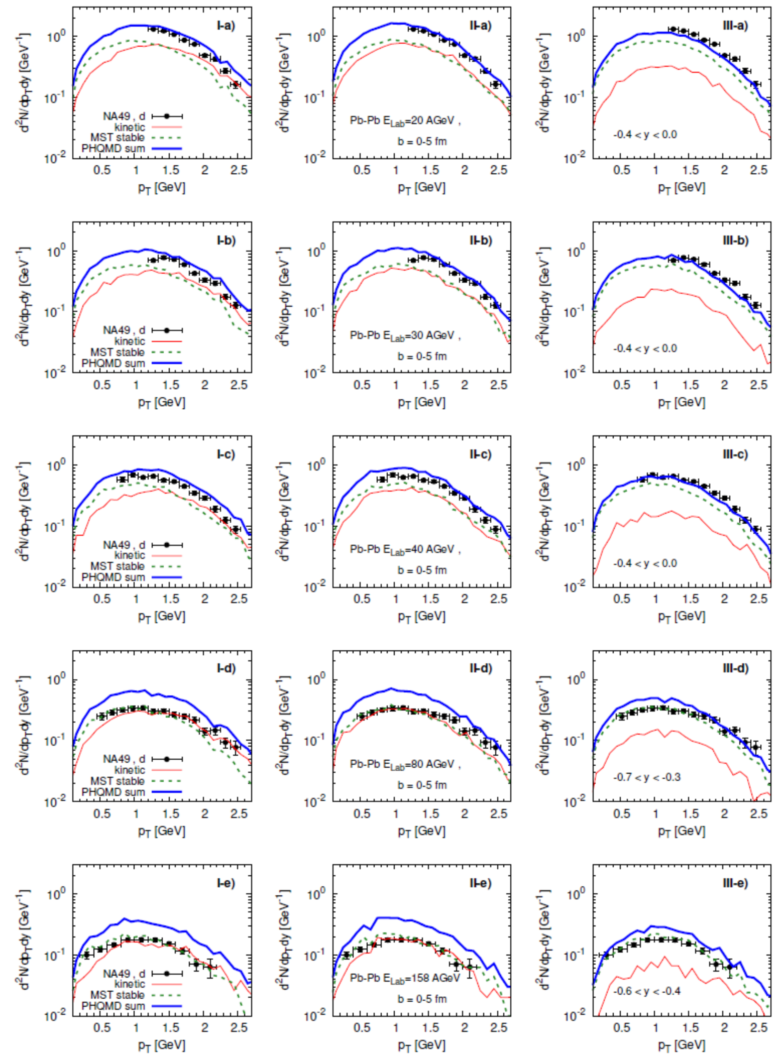
1) excluded-volume **2) Momentum projection** **3) both effects**



- **Good description of mid-rapidity NA49 data** [PRC 94 (2016) 04490699]

Kinetic vs. potential deuteron production



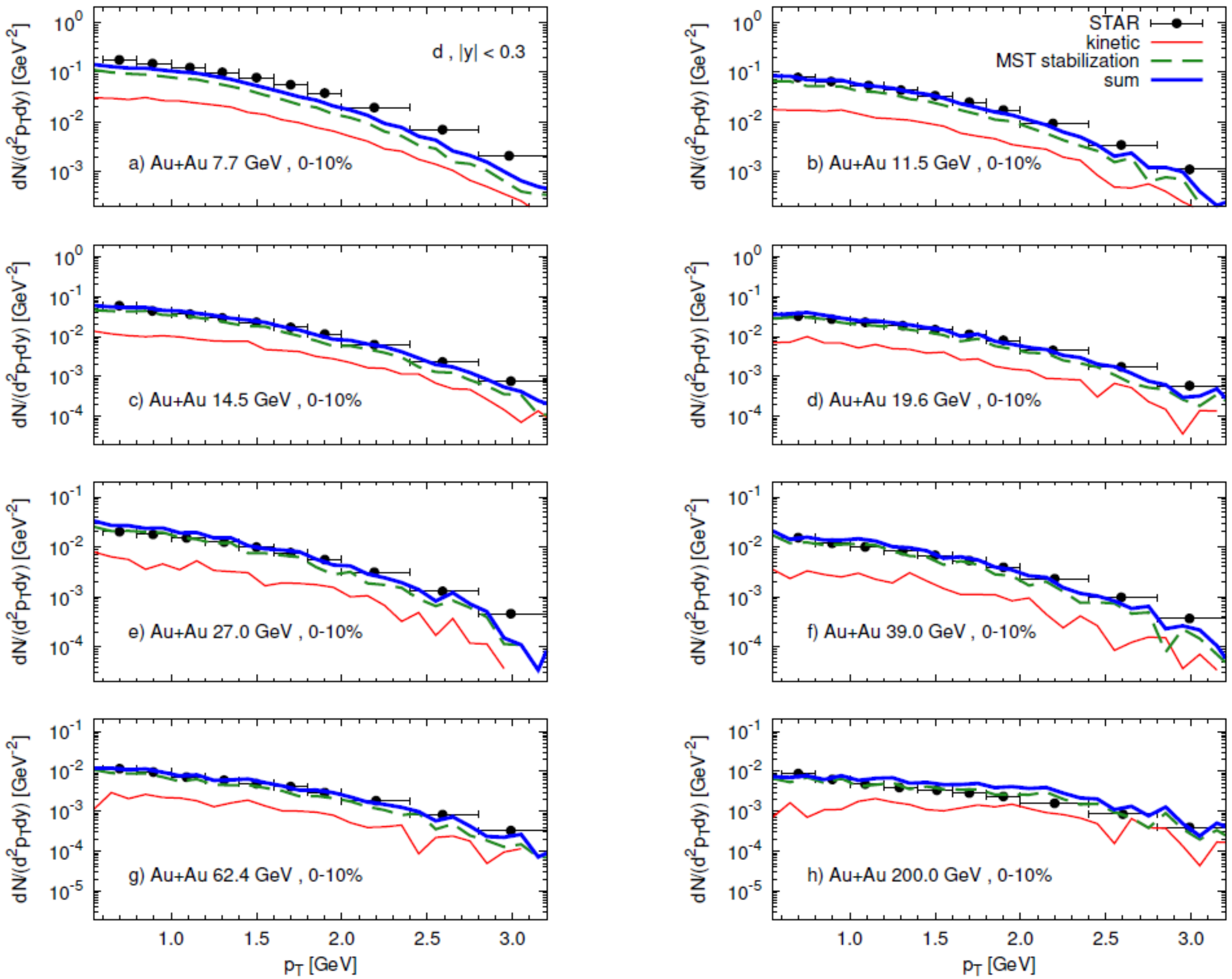


Total deuteron production = Kinetic mechanism with finite-size effects

+ MST (with stabilization) identification of deuterons (“stable” bound (E_B<0) A=2 , Z=1 clusters)

Kinetic vs. potential deuteron production

Total d = Kinetic mechanism with finite-size effects + MST (with stabilization) identification of d

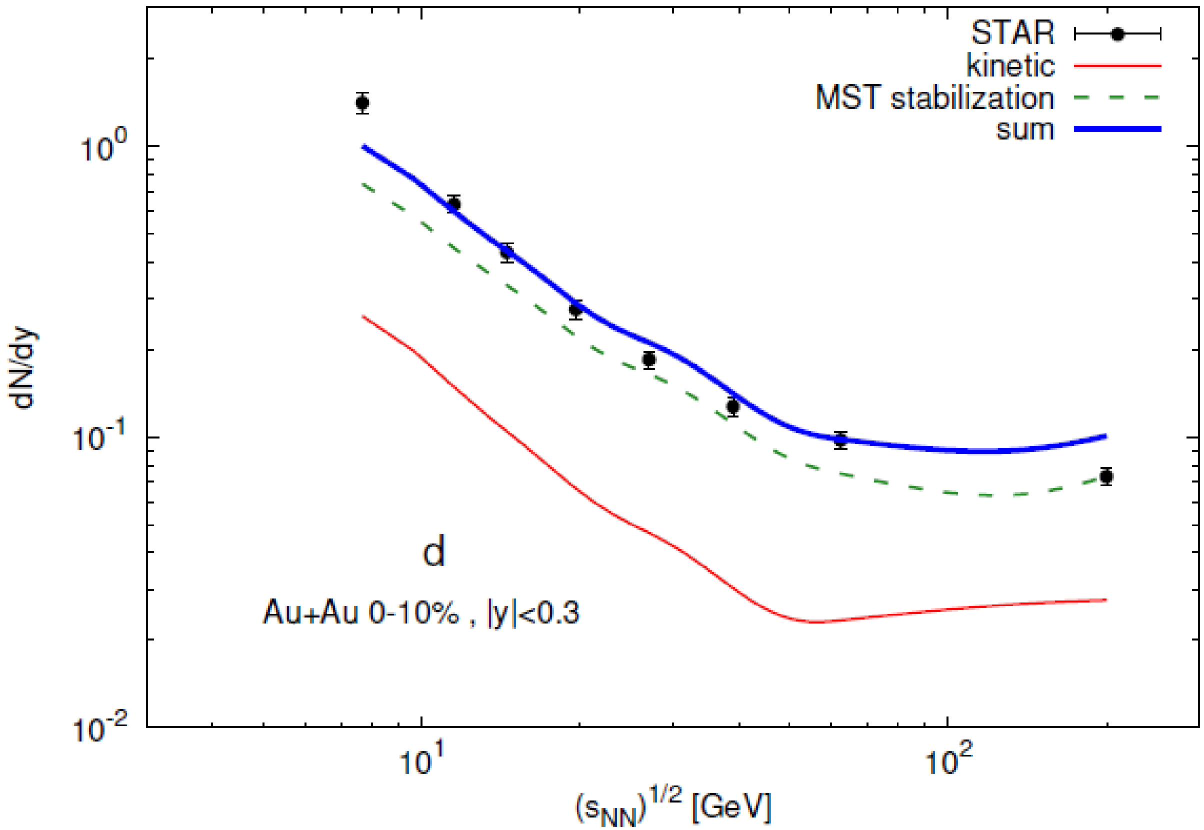


- **Good description of mid-rapidity STAR data** [PRC 99, (2019)]

PHQMD

Kinetic vs. potential deuteron production

Excitation function dN/dy of deuterons at midrapidity



- **PHQMD provides a good description of STAR data**
- **The potential mechanism is dominant for d production at all energies!**

Summary

- The PHQMD is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster and hypernuclei formation

combined model PHQMD = (PHSD & QMD) & (MST | SACA)

- Clusters are formed dynamically by potential interactions among nucleons and hyperons and identified by Minimum Spanning Tree model

- Kinetic mechanism for deuteron production is implemented in the PHQMD with inclusion of full isospin decomposition for hadronic reactions which enhances d production
- However, accounting for the quantum properties of the deuteron, modelled by the finite-size excluded volume effect in coordinate space and projection of relative momentum of the interacting pair of nucleons on the deuteron wave-function in momentum space, leads to a strong reduction of d production, especially at target/projectile rapidities

- The PHQMD reproduces cluster and hypernuclei data on dN/dy and dN/dp$_T$ as well as ratios d/p and $\bar{d}/\bar{p}$ for heavy-ion collisions from AGS to top RHIC energies.

A detailed analysis reveals that stable clusters are formed

- shortly after elastic and inelastic collisions have ceased
- behind the front of the expanding energetic hadrons
- since the 'fire' is not at the same place as the 'ice', cluster can survive

- Coalescence and MST give very similar deuteron distributions within the PHQMD and UrQMD transport approaches

Outlook

PHQMD:

- LHC energies → numerous computational efforts
- Momentum-dependent potential - important for low energies of SIS, FAIR
- Realistic description of hyperon-nucleon potential – important for hypernuclei dynamics
- Kinetic formation of light clusters like t, He ?
- Extended study of collective observables for clusters

New experimental data are needed!

y-distributions → mechanisms for cluster formation at large y

Collective observables v_1, v_2, …