

Hadron Interactions from Lattice QCD - Theory Meets Experiments -



1. Introduction
2. LQCD for hadron interactions
3. LQCD vs. Expt. (RHIC/LHC/J-PARC)
4. Summary

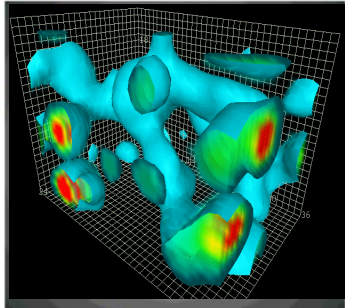
Tetsuo Hatsuda
(RIKEN iTHEMS)

iTHEMS
RIKEN interdisciplinary
Theoretical & Mathematical
Sciences

RHIC-BES Seminar
(Nov.15, 2022)

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} + \bar{q}\gamma^\mu(i\partial_\mu - gt^a A_\mu^a)q - m\bar{q}q$$

QCD vacuum

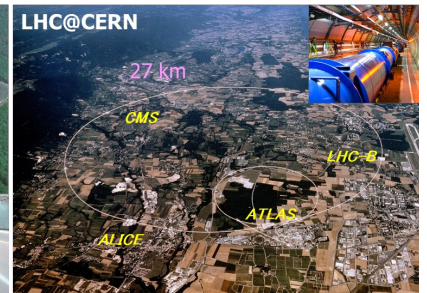
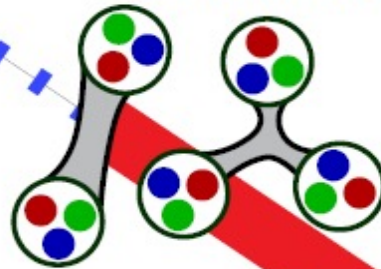


Baryons



Lattice QCD

Interactions

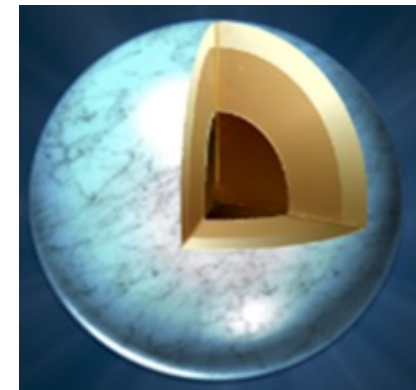


Nuclei



ab-initio nuclear calc.

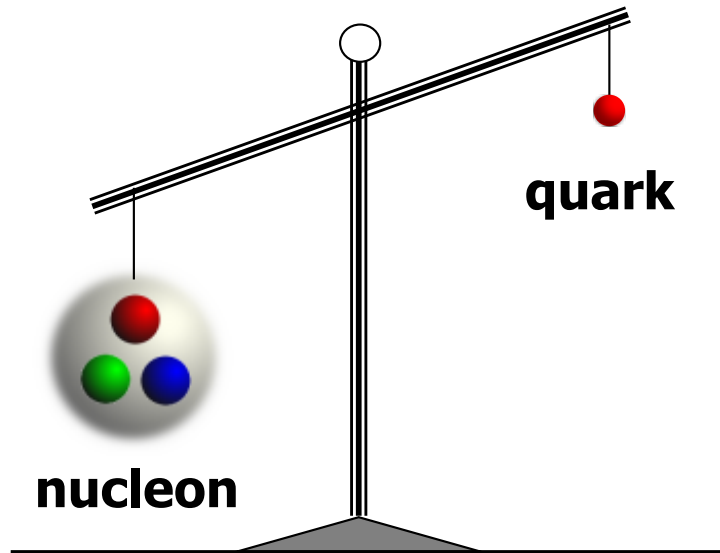
Neutron Stars



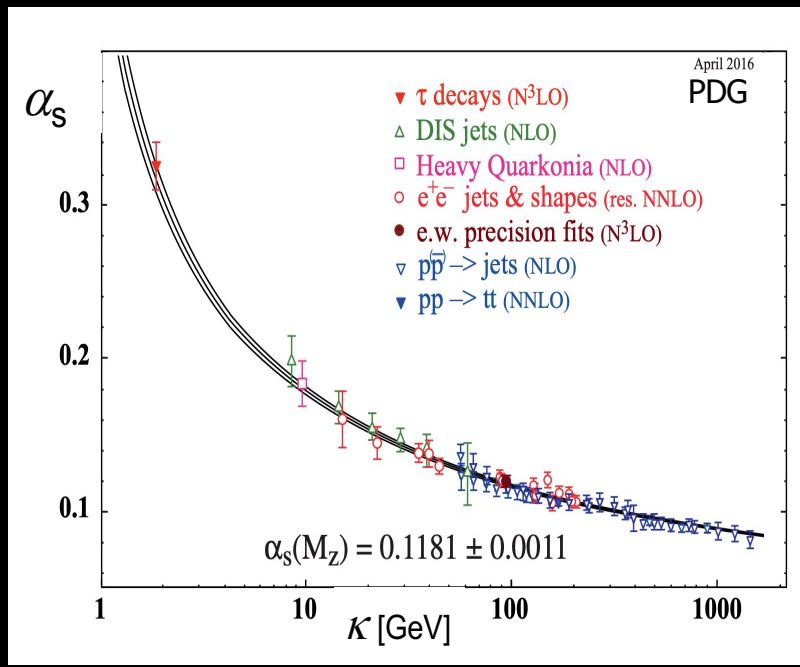
©RIKEN

QCD is non-perturbative

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} + \bar{q}\gamma^\mu(i\partial_\mu - gt^a A_\mu^a)q - m\bar{q}q$$



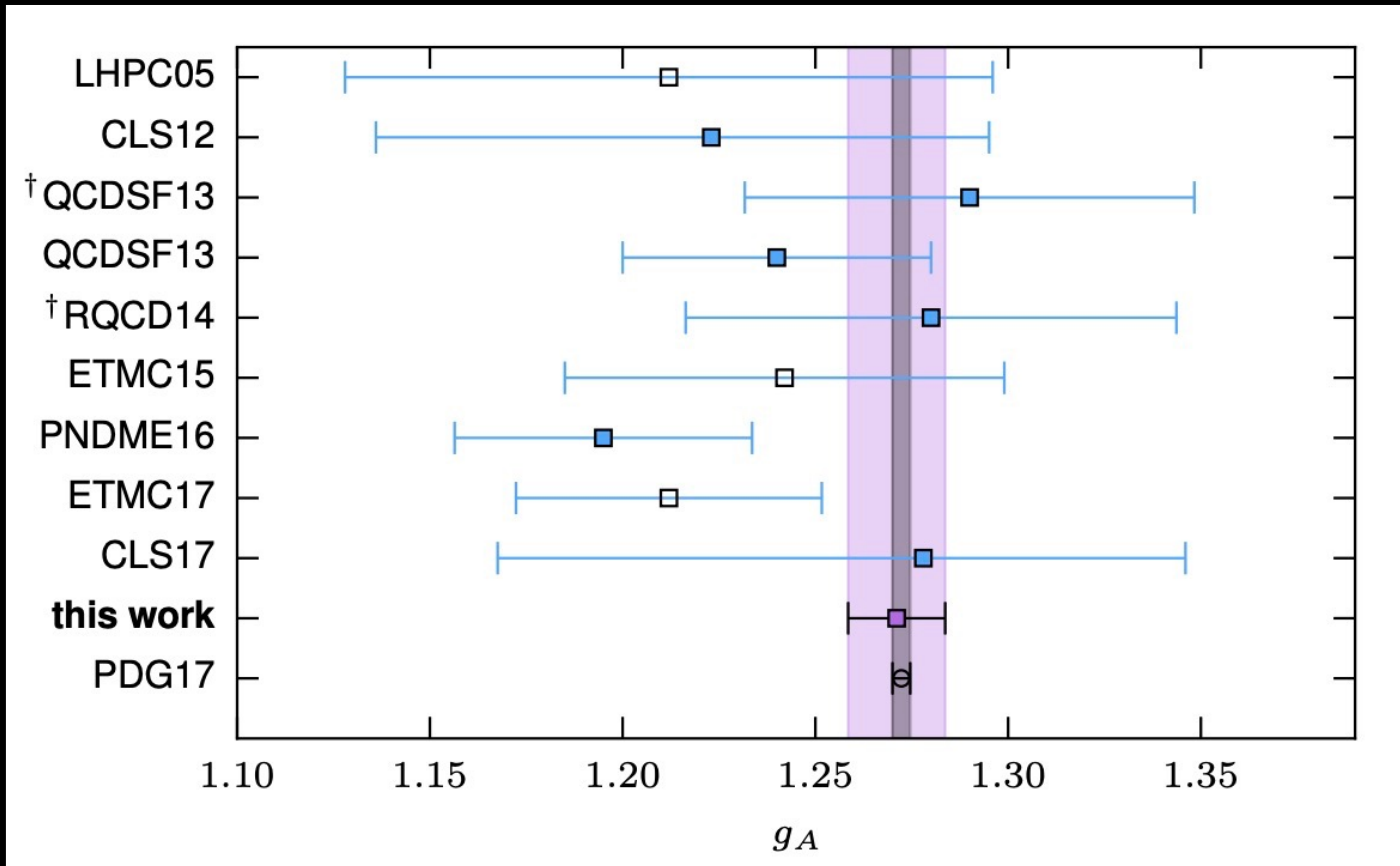
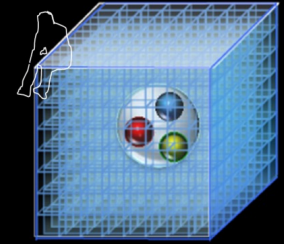
Quark mass ~ 3 MeV
 Nucleon mass ~ 940 MeV



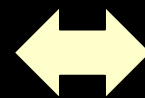
light quarks	MS-bar mass at 2GeV
m_u	2.27(9) MeV
m_d	4.67(9) MeV
m_s	92.0(1.1) MeV

Percent-level determination of g_A from LQCD

Chang (LBNL/iTHEMS)+, Nature 558 (2018) 91

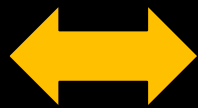
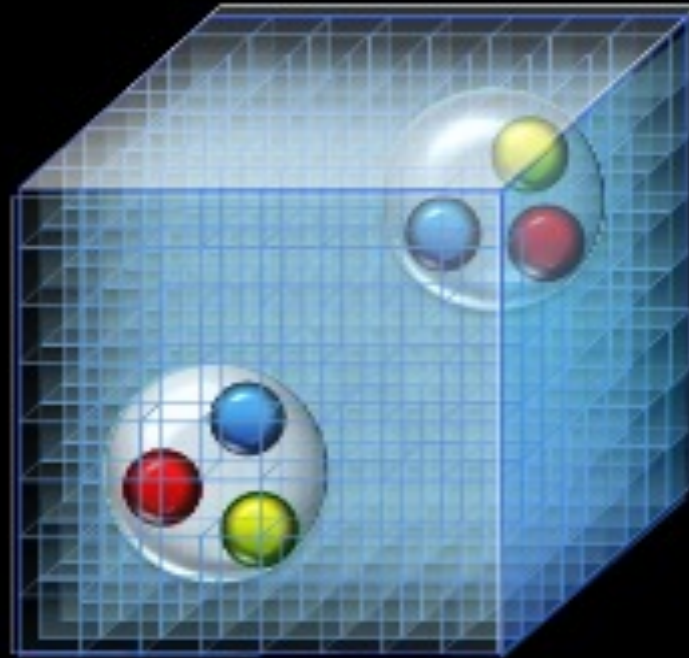


$$(g_A)_{\text{LQCD}} = 1.271(13)$$
$$(\tau_n)_{\text{LQCD}} = 884(15) \text{ s}$$



$$(g_A)_{\text{expt}} = 1.272(2)$$
$$(\tau_n)_{\text{PDG}} = 880.2(1.0) \text{ s}$$

Baryon Interactions



Exotic hadrons

Hypernuclei

Hyperons in neutron star

Interaction between composite particles in QFT

Foundation: Haag, Nishijima, Zimmermann (1957), Borchers (1961)

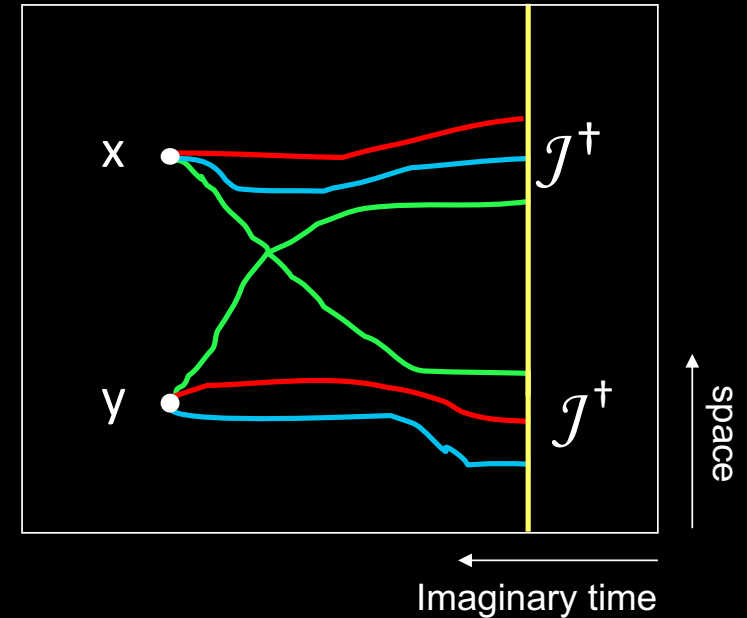
Luscher's Method: Luscher, NPB 354 (1991) 531

HAL QCD Method: Ishii, Aoki, Hatsuda, PRL 99 (2007) 022001
 HAL QCD Coll., PLB 712 (2012) 437

Recent review: Aoki & Doi, Front. Phys. 8 (2020) 307

$$F^J(\mathbf{r}, t) = \sum_n a_n^J \psi_n(\mathbf{r}) e^{-E_n t}$$

$t > t^* \sim 1\text{fm}$



Time-dependent HAL QCD Equation

$$(\nabla^2 + \partial_{2t}^2 - m^2) F^J(\mathbf{r}, t) = m \int d^3r U(\mathbf{r}, \mathbf{r}') F^J(\mathbf{r}', t)$$

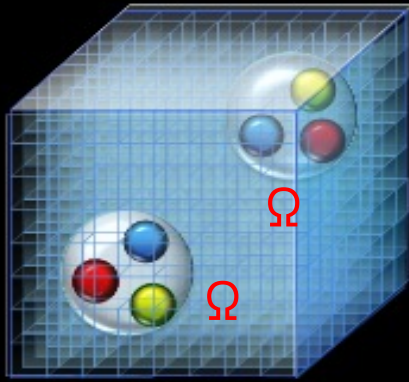
1. Derived from QCD
2. Fully relativistic
3. Faithful to S-matrix
4. Insensitive to lattice volume
5. Applicable to BB, MM, MB, BBB etc
6. Applicable to coupled channel systems

$$U(\mathbf{r}, \mathbf{r}') = V(\mathbf{r}, \mathbf{v}) \delta(\mathbf{r} - \mathbf{r}'),$$

$$V(\mathbf{r}, \mathbf{v}) = \underbrace{V_C(r)}_{\text{LO}} + \underbrace{V_T(r)S_{12}}_{\text{NLO}} + \underbrace{V_{LS}(r)\mathbf{L} \cdot \mathbf{S}}_{\text{N}^2\text{LO}} + \underbrace{O(v^2)}_{\text{N}^2\text{LO}} + \dots$$

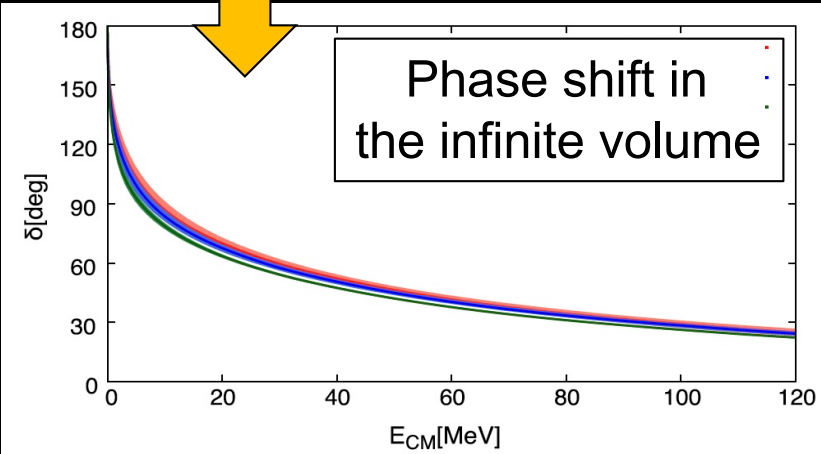
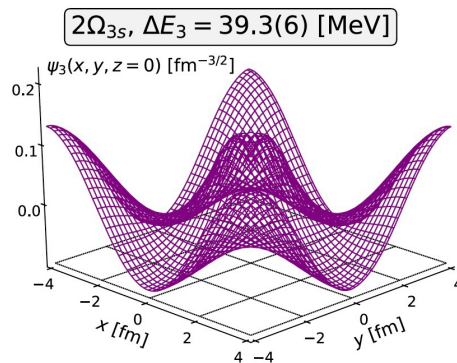
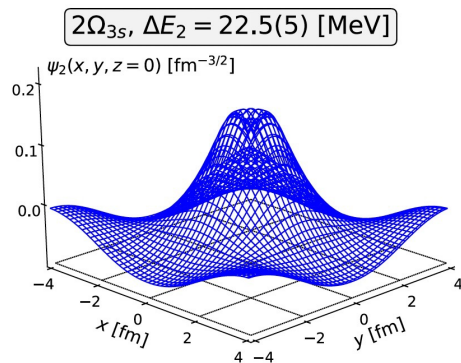
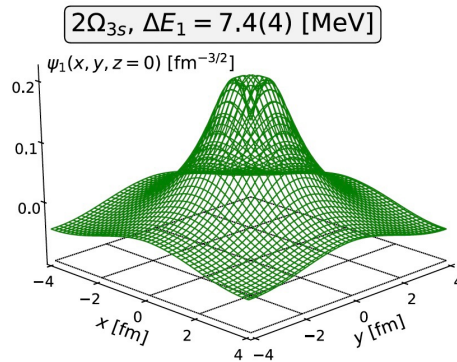
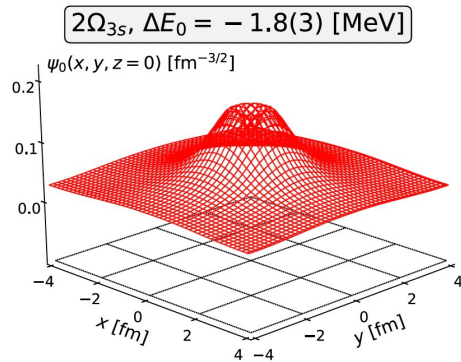
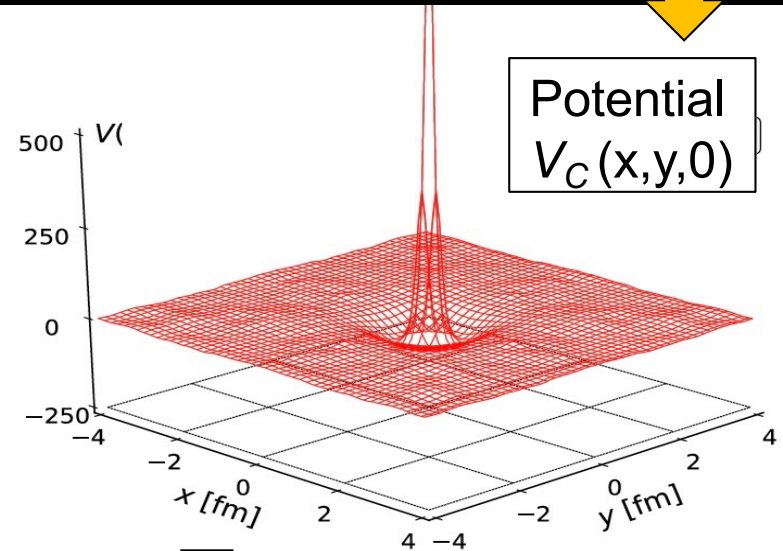
HAL QCD Procedure: an example ($\Omega\Omega$ at $m_\pi=146\text{MeV}$)

Gongyo+ [HAL QCD], PRL 120 (2018) 212001
Lyu+ [HAL QCD], PRD 105 (2022) 074512

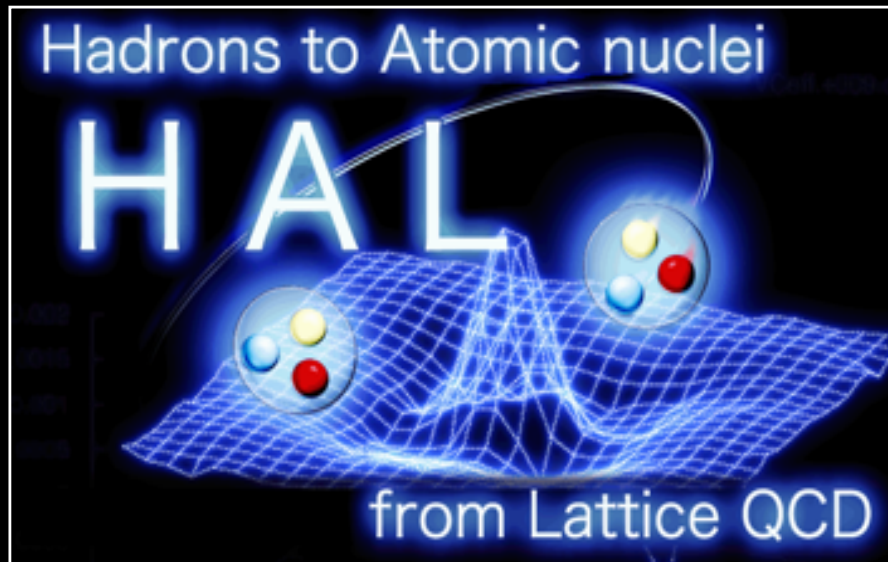


$$F^J(\mathbf{r}, t) = \sum_n a_n^J \psi_n(\mathbf{r}) e^{-E_n t} \rightarrow V_C(r)$$

$\Psi_n(r)$ and E_n in a finite volume



Large scale LQCD simulations for BB interactions



(KEK) T. Aoyama

(RIKEN) T. Doi, T. Hatsuda, T. Sugiura

(Nihon) T. Inoue

(YITP) Y. Akahoshi, S. Aoki, K. Murakami

(RCNP) T. M. Doi, N. Ishii, K. Murano, H. Nemura

(Osaka) Y. Ikeda, K. Sasaki

(Birjand) F. Etminan

(Beijing) Y. Liu, H. Tong

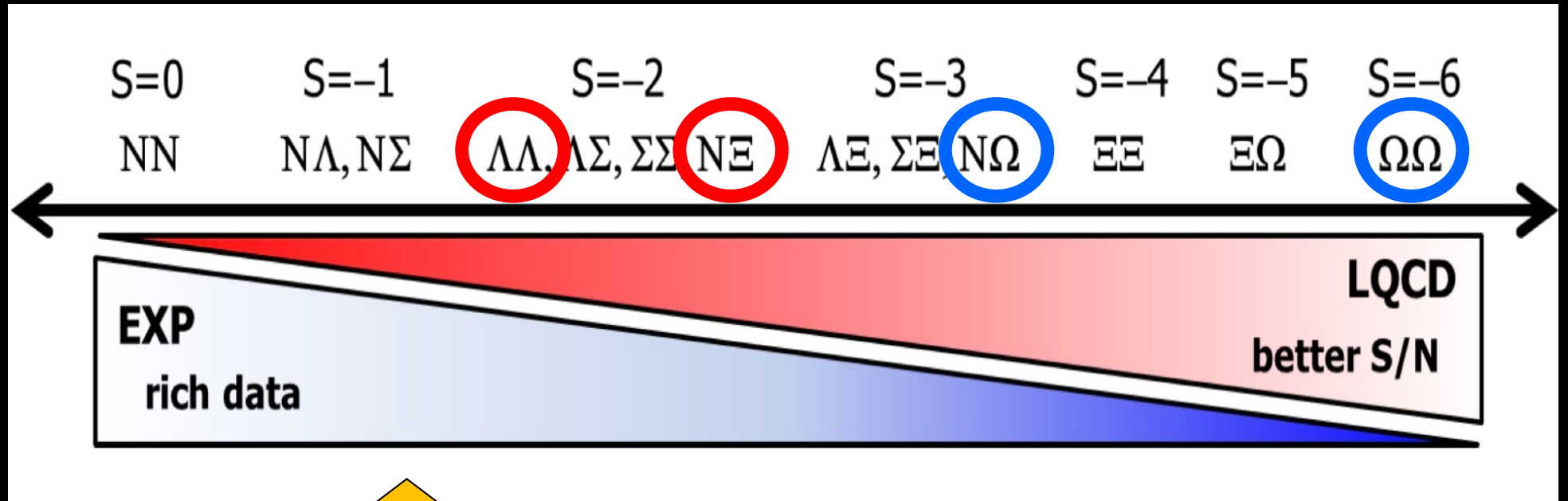
$V=(8.1 \text{ fm})^3$, $m_\pi=146 \text{ MeV}$



$V=(8.1 \text{ fm})^3$, $m_\pi=138 \text{ MeV}$



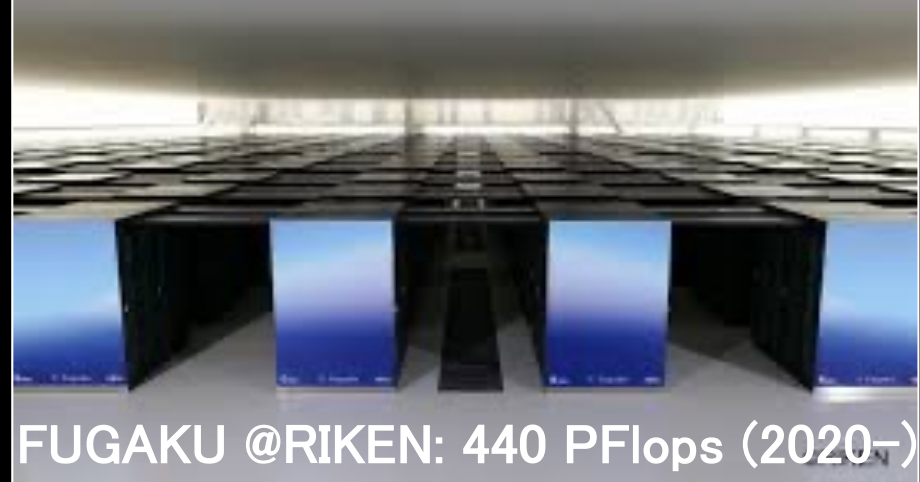
Large scale LQCD simulations for BB interaction



$V=(8.1 \text{ fm})^3$, $m_\pi=146 \text{ MeV}$

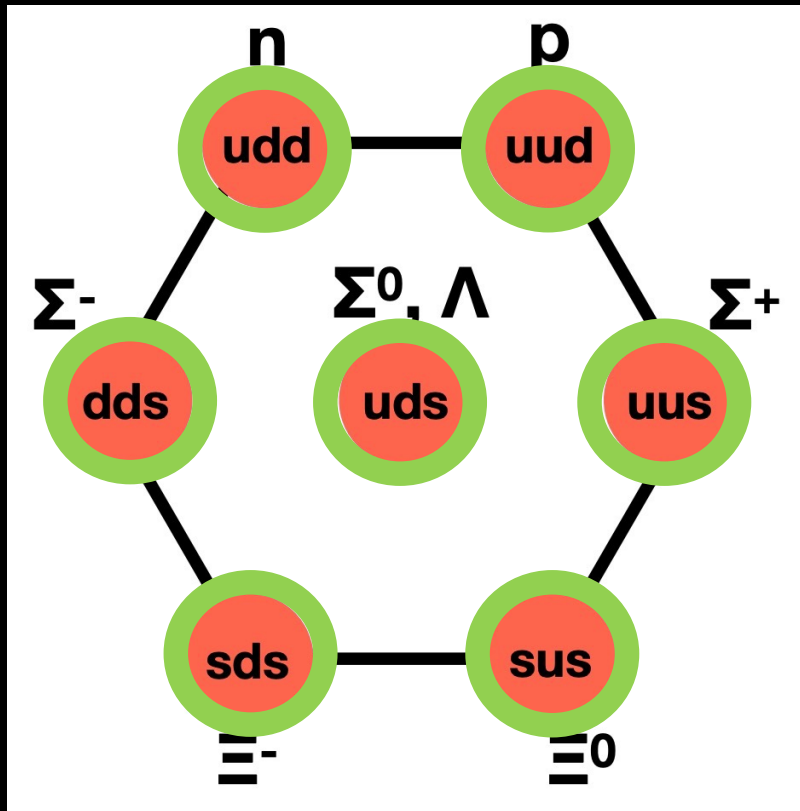


$V=(8.1 \text{ fm})^3$, $m_\pi=138 \text{ MeV}$

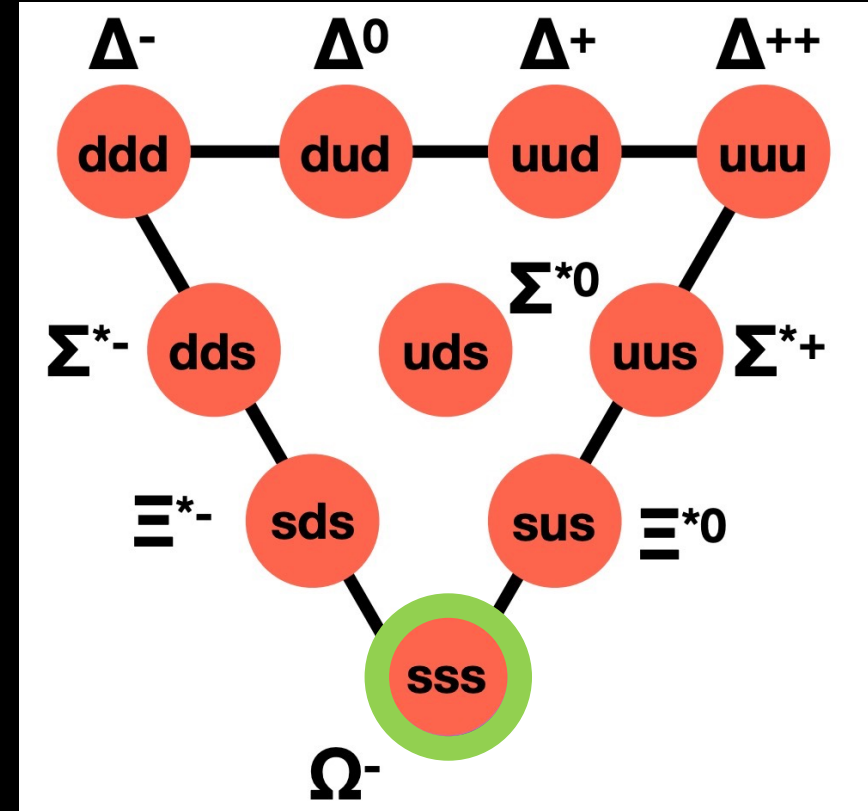


Flavor SU(3) classification

8 (Octet)

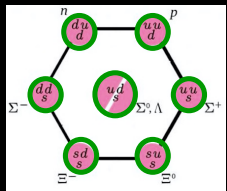


10 (Decuplet)

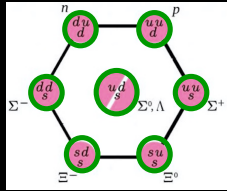


 Stable against strong decays

SU(3)_F classification of BB system



X



$$8 \times 8 = 27 + 8_s + 1 + 10^* + 10 + 8_a$$

NN(¹S₀)

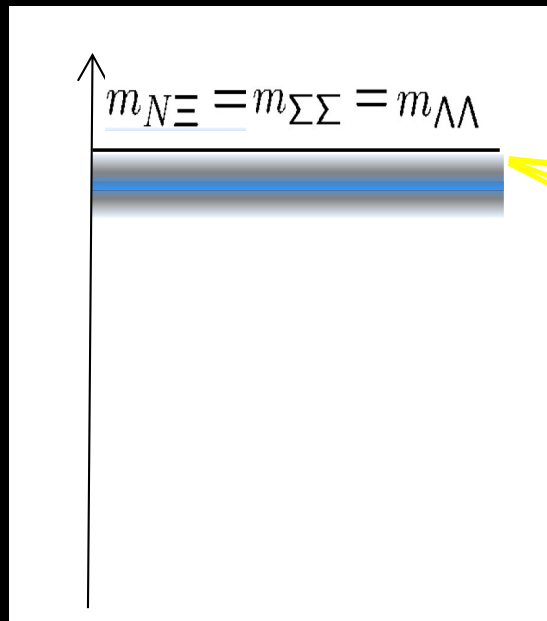
H(¹S₀)

NN(³S₁)

Jaffe (1977)

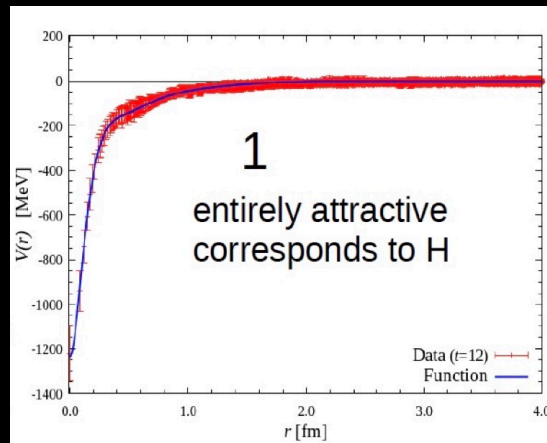
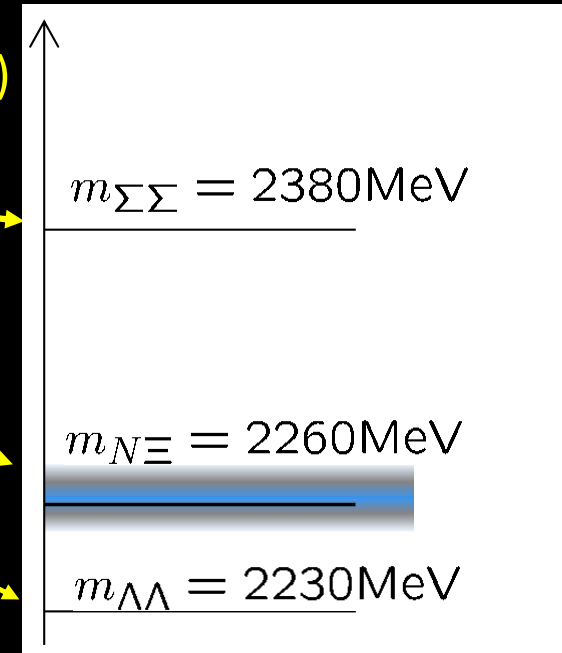
deuteron

Fate of “H (uuddss)” dibaryon from LQCD

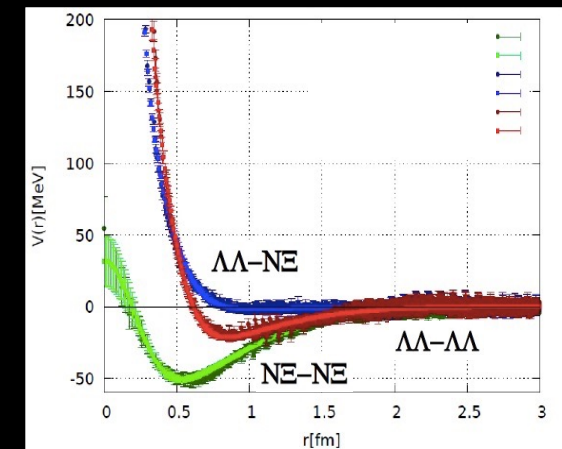


$m_{ud} = m_s$
(~ 100 MeV)

$m_{ud} < m_s$
(~10MeV) (~100MeV)



$$\begin{pmatrix} \langle \Lambda\Lambda | \\ \langle \Sigma\Sigma | \\ \langle N\Xi | \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{27}{40}} & -\sqrt{\frac{8}{40}} & -\sqrt{\frac{5}{40}} \\ -\sqrt{\frac{1}{40}} & -\sqrt{\frac{24}{40}} & \sqrt{\frac{15}{40}} \\ \sqrt{\frac{12}{40}} & \sqrt{\frac{8}{40}} & \sqrt{\frac{20}{40}} \end{pmatrix} \begin{pmatrix} \langle 27 | \\ \langle 8_s | \\ \langle 1 | \end{pmatrix}$$



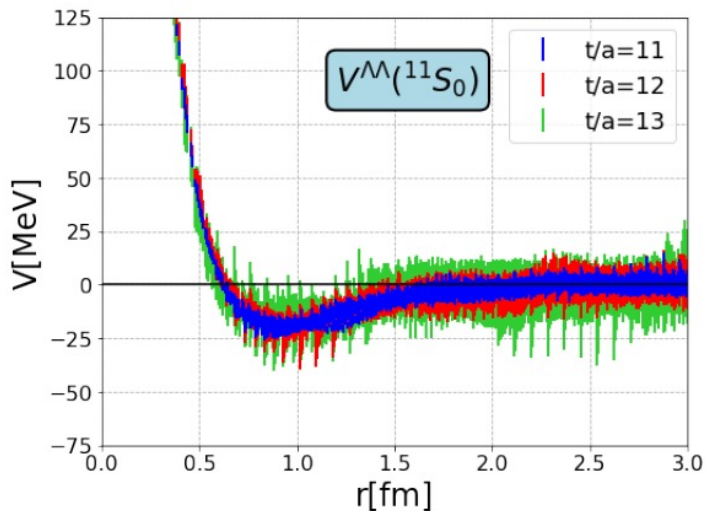
Inoue et al., [HAL QCD Coll.]
Nucl. Phys. A881 (2012) 28

Sasaki et al., [HAL QCD Coll.]
Nucl. Phys. A998 (2020) 121737

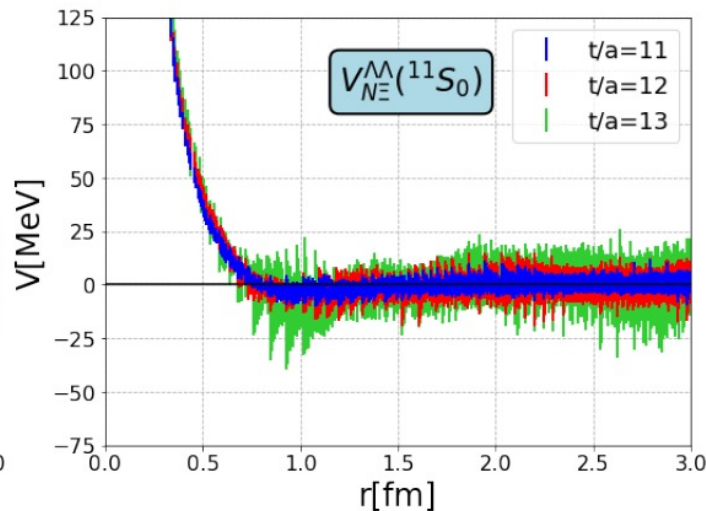
Coupled Channel S=-2 system ($^{11}S_0$)

K. Sasaki+ [HAL QCD Coll.]
Nucl. Phys. **A998** (2020)

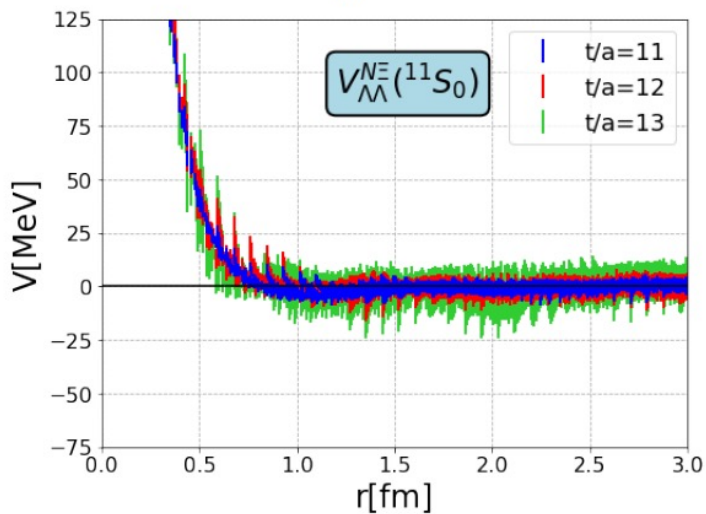
Small $\Lambda\Lambda$
attraction



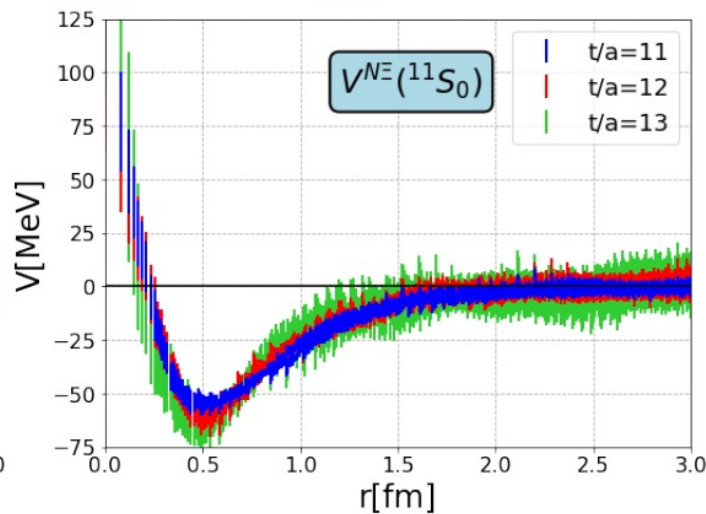
(a)



(b)



(c)



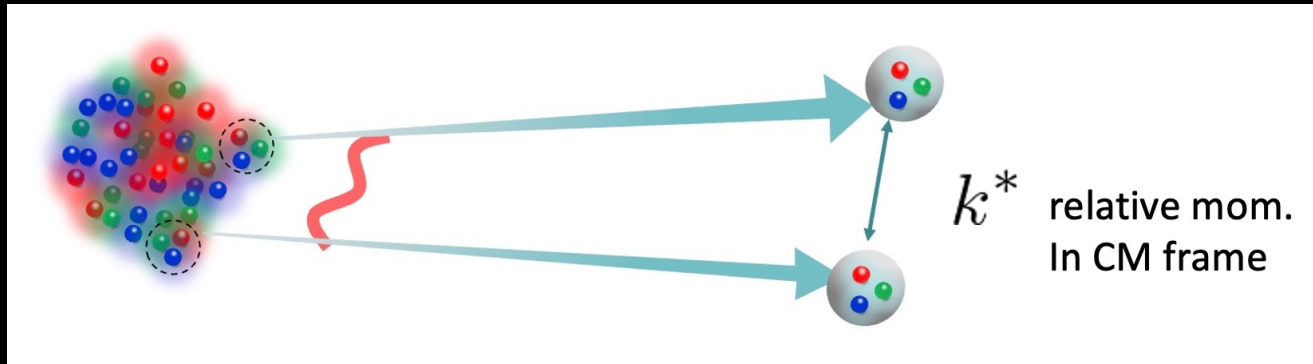
(d)

Short-range
 $N\Xi$ - Λ coupling

Short-range
 $N\Xi$ - Λ coupling
attraction

Large $N\Xi$
attraction

Femtoscscopy : particle correlations in pp, pA and AA



$$C_{\text{expt}}(k^*) = \frac{\xi(k^*) N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)}$$

$$C_{\text{theo}}(k^*)$$

$$= \int d^3r \mathcal{S}(r, k^*) |\Psi(r, k^*)|^2$$

$$= 1 + \int d^3r \mathcal{S}(r, k^*) [|\Psi_0(r, k^*)|^2 - |j_0(k^* r)|^2]$$

Koonin, Phys. Lett. B 70, 43 (1977).

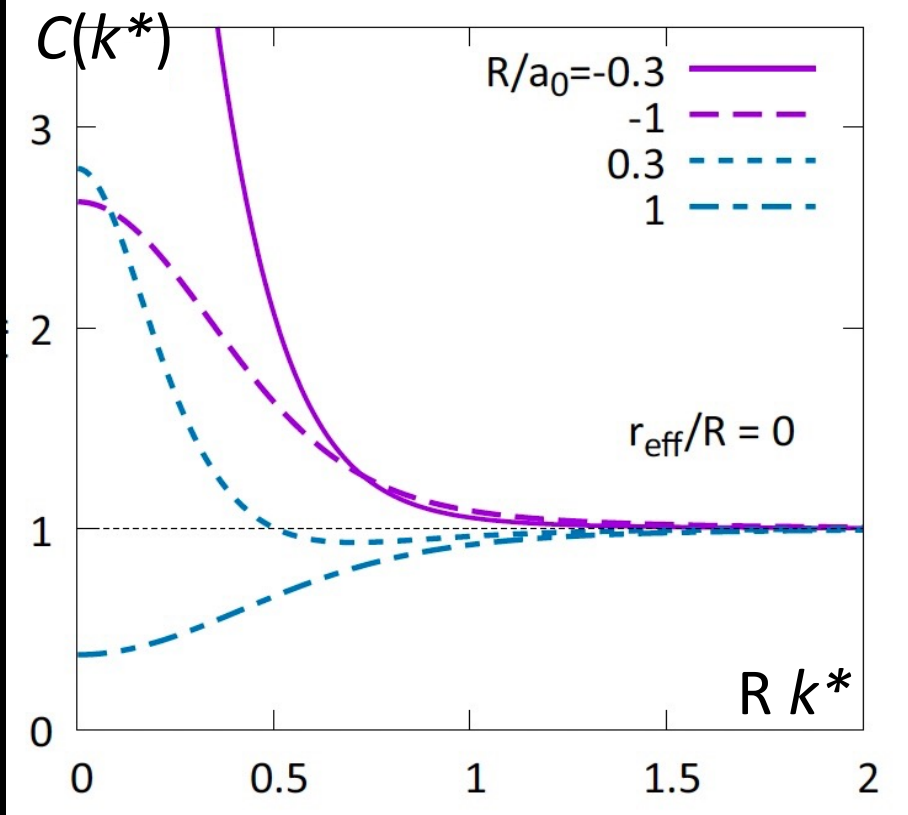
Lednicky and Lyuboshits, Yad. Fiz. 35, 1316 (1981).

Pratt, Phys. Rev. D 33, 1314 (1986).

Anchishkin, Heinz, and Renk, Phys. Rev. C 57, 1428 (1998).

Lednicky, Lyuboshits, and Lyuboshits, Phys. At. Nucl. 61, 2950 (1998).

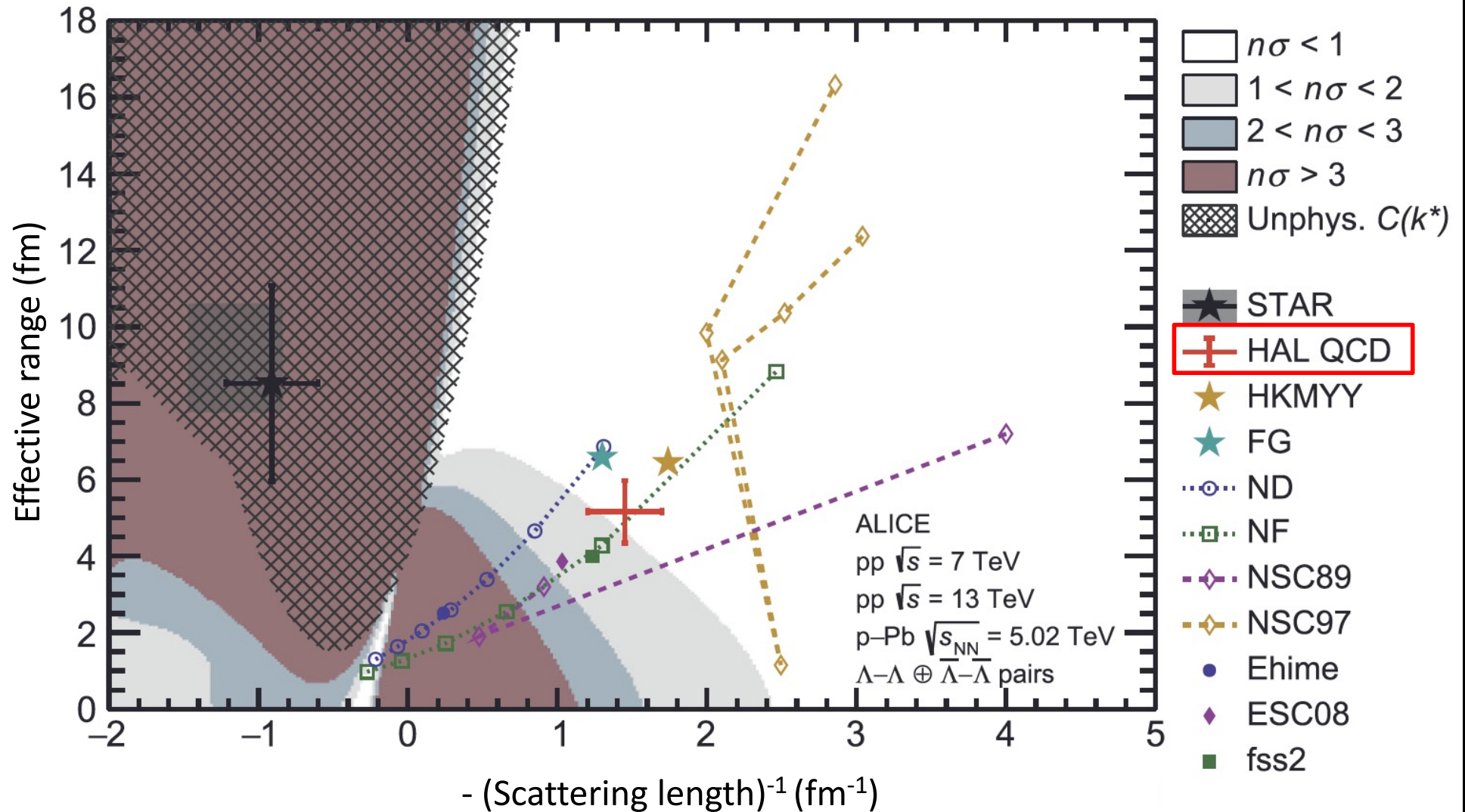
Haidenbauer, Nucl. Phys. A 981, 1 (2019).



Morita et al., Phys. Rev. C 101 (2020) 015201

$\Lambda\Lambda$ correlation in pp and pA

ALICE Coll., Phys. Lett. B797 (2019)

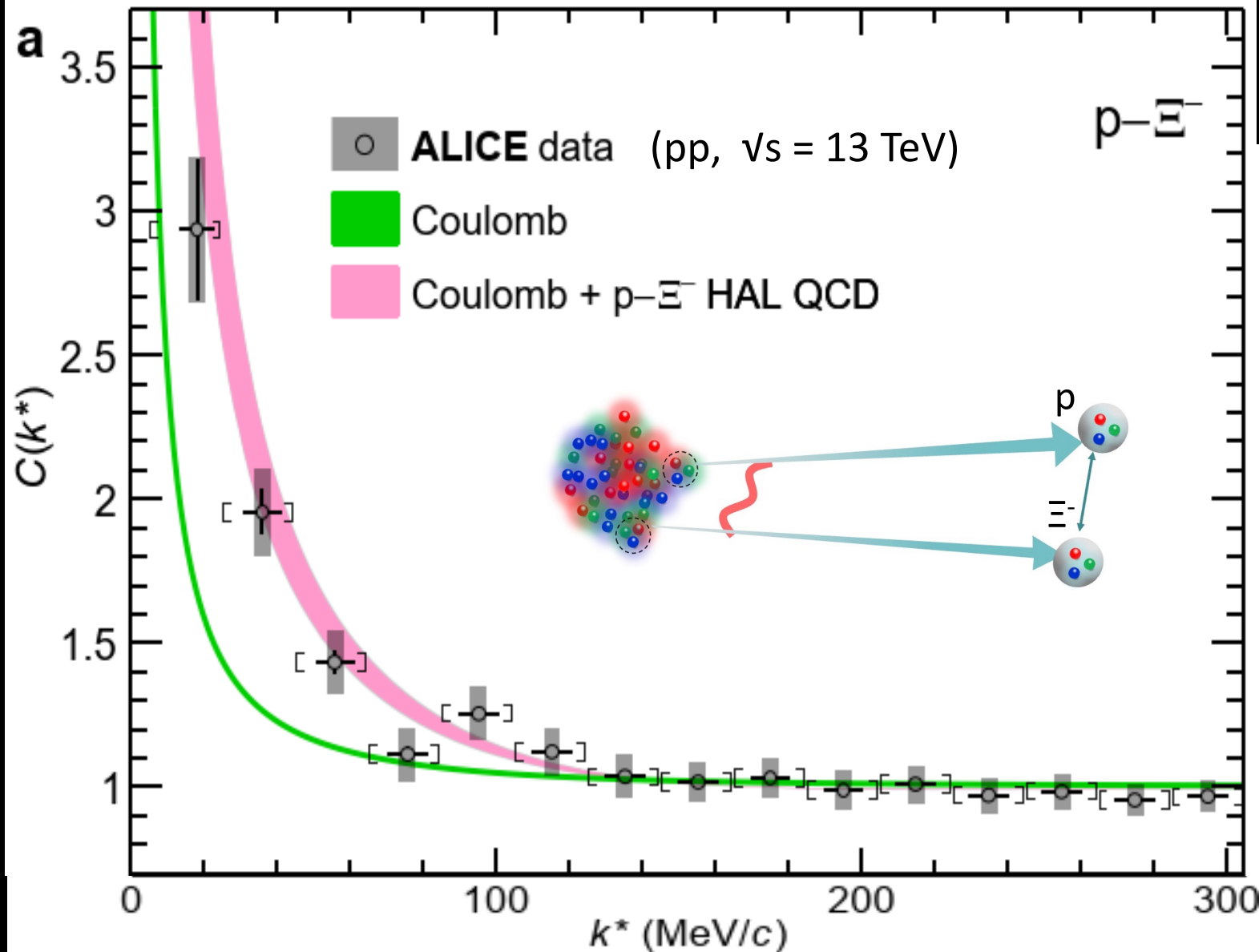
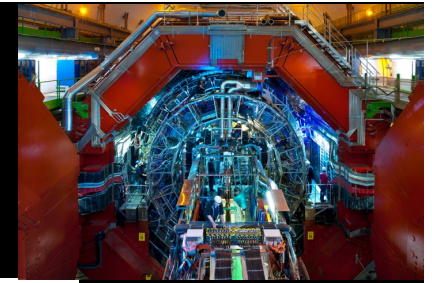


K. Sasaki+ [HAL QCD Coll.]
Nucl. Phys. **A998** (2020)

$$a_0^{(\Lambda\Lambda)} = -0.81 \pm 0.23_{-0.13}^{+0.00} \text{ [fm]}, \quad r_{\text{eff}}^{(\Lambda\Lambda)} = 5.47 \pm 0.78_{-0.55}^{+0.09} \text{ [fm]},$$

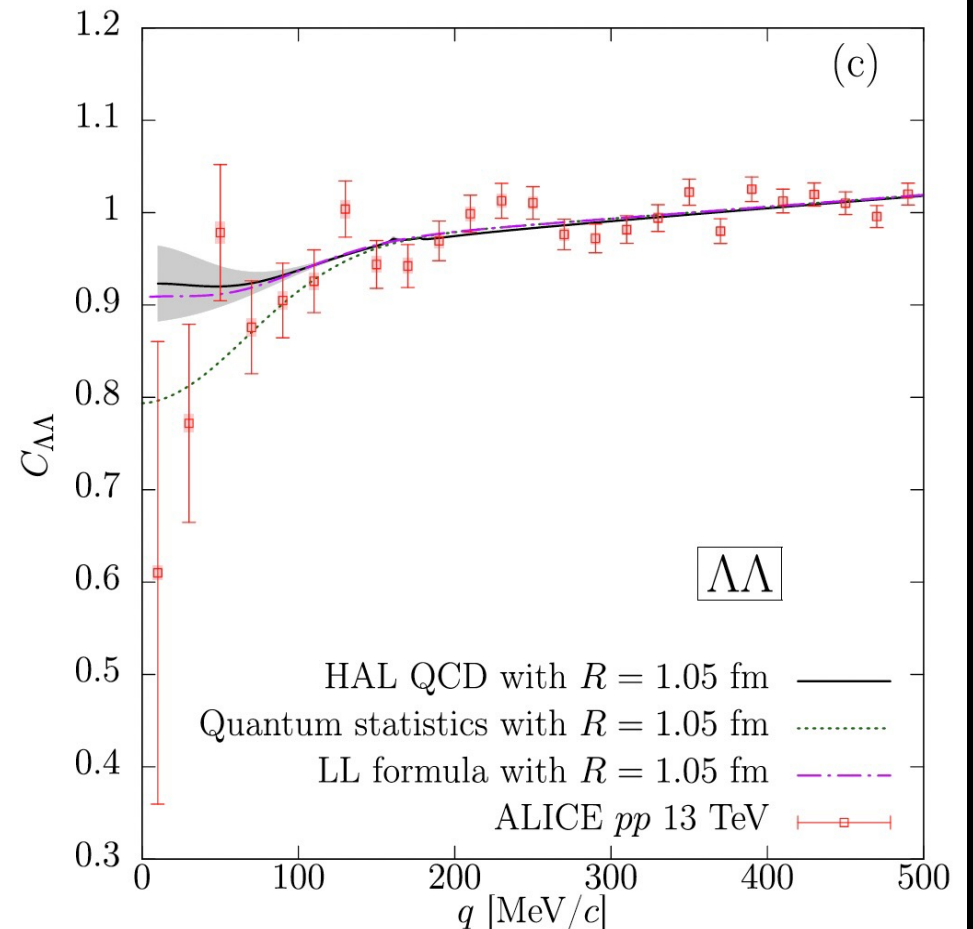
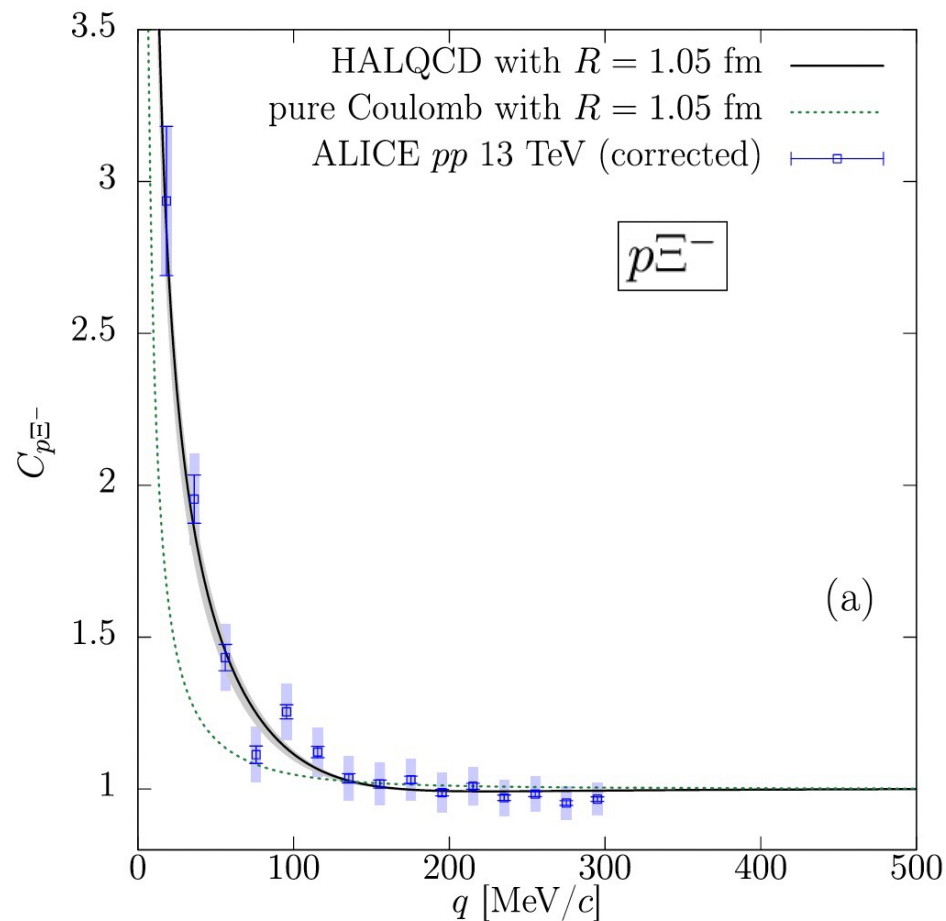
$N\Xi$ correlation in pp

ALICE Coll., Nature 588 (2020) 232



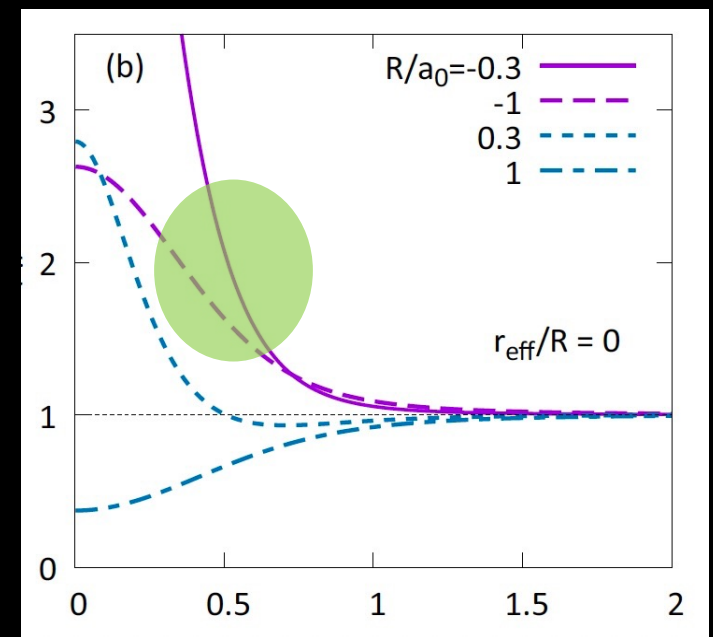
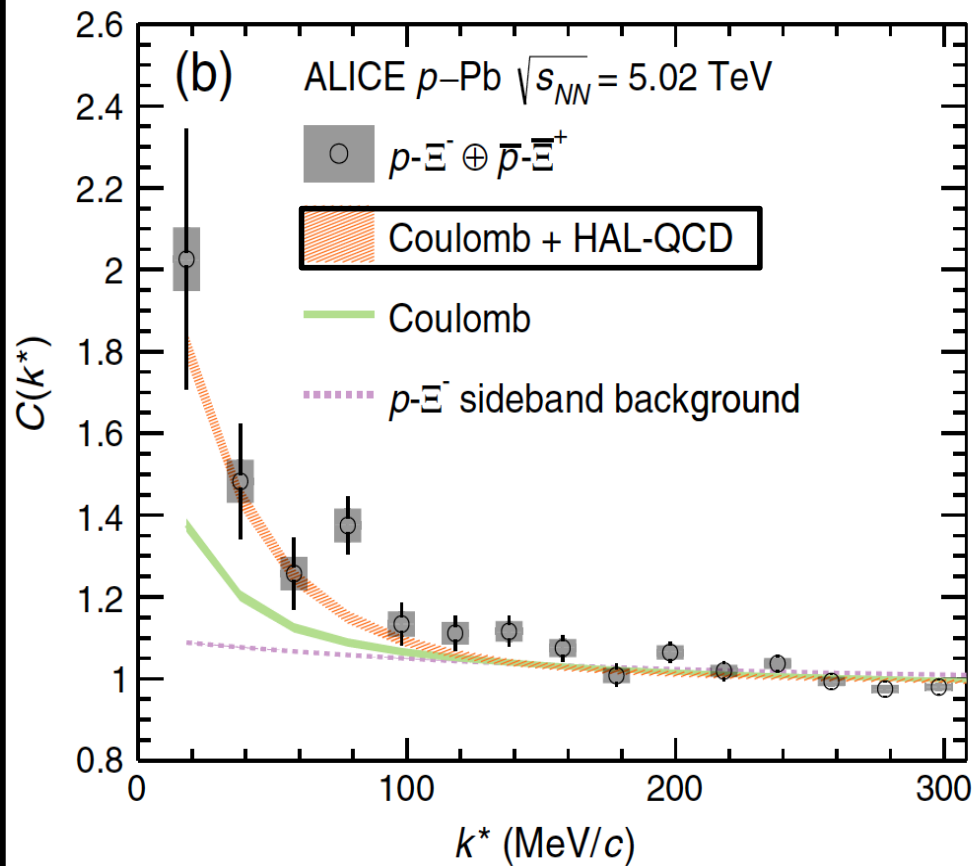
Fully coupled channel analysis

Kamiya+, Phys. Rev. C 105 (2022) 014915



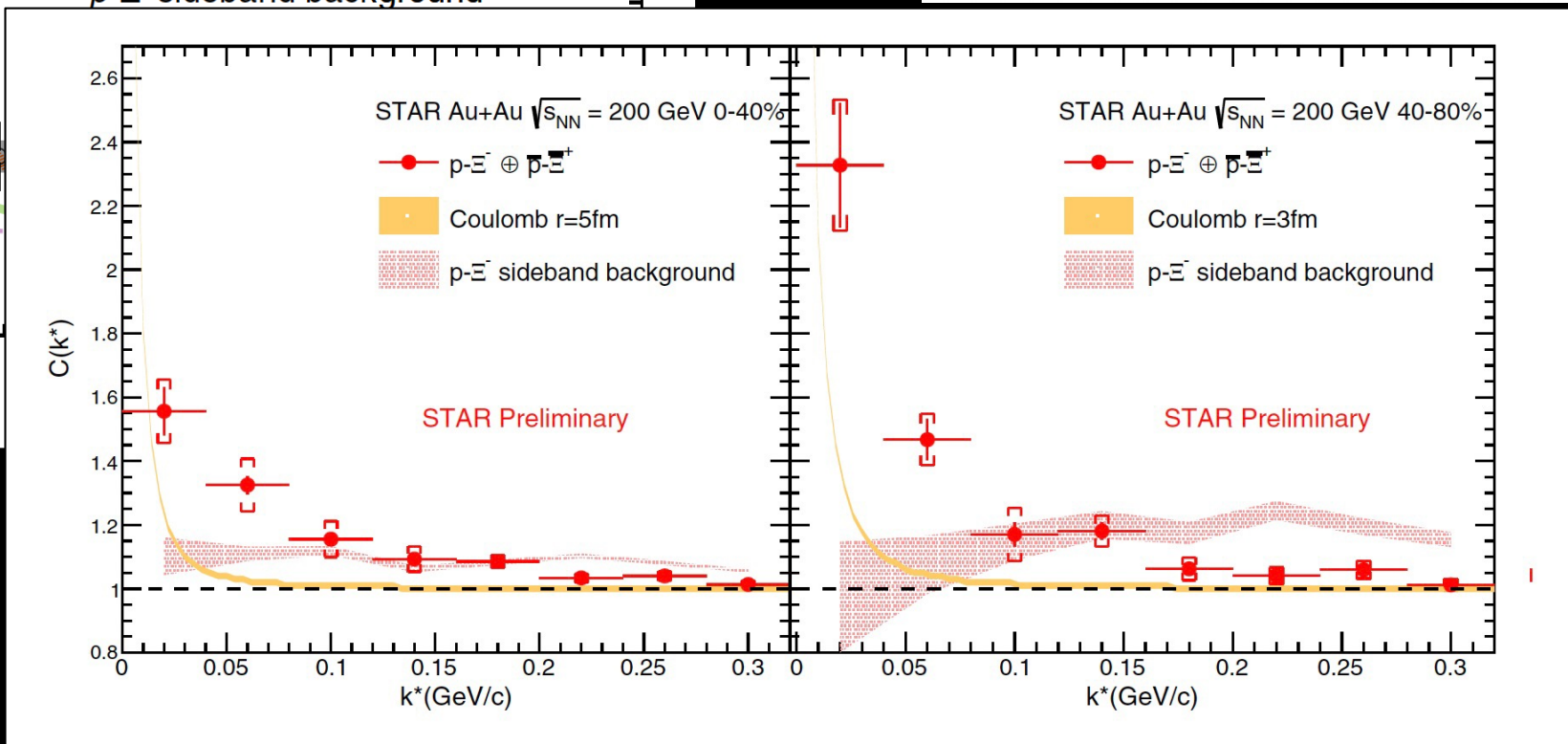
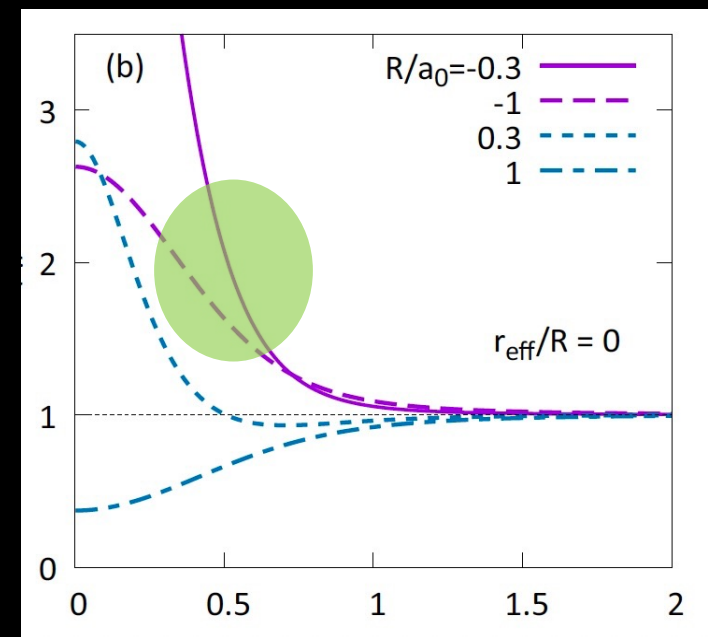
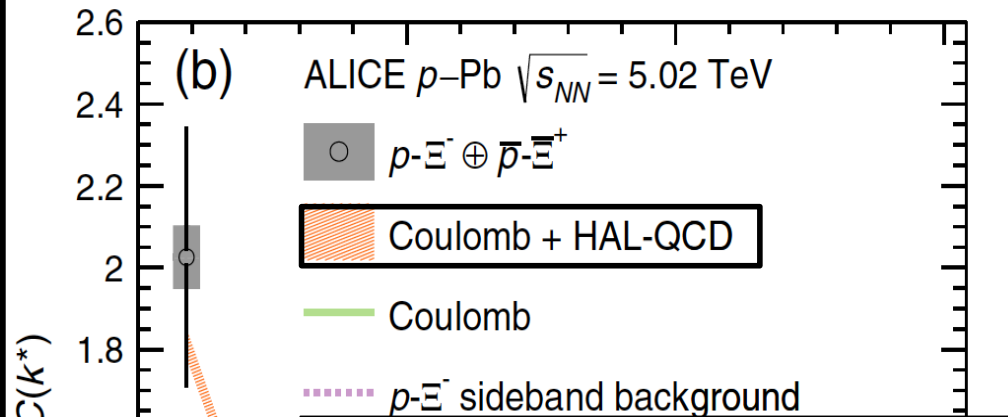
$N\Xi$ correlation in pA and AA

ALICE Coll., Phys.Rev.Lett. 123 (2019)



$N\Xi$ correlation in pA and AA

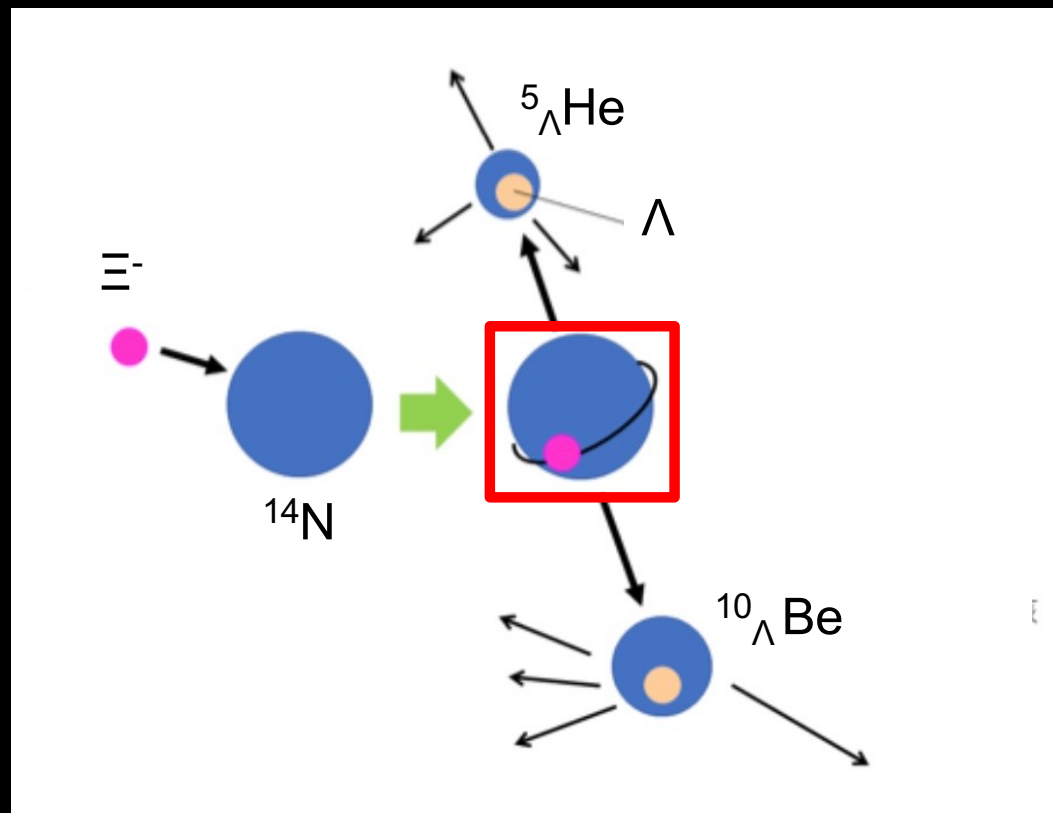
ALICE Coll., Phys.Rev.Lett. 123 (2019)



No bound $N\Xi$?

Ξ hypernuclei at J-PARC

E07 Coll. at J-PARC,
Phys.Rev.Lett. 126 (2021) 062501

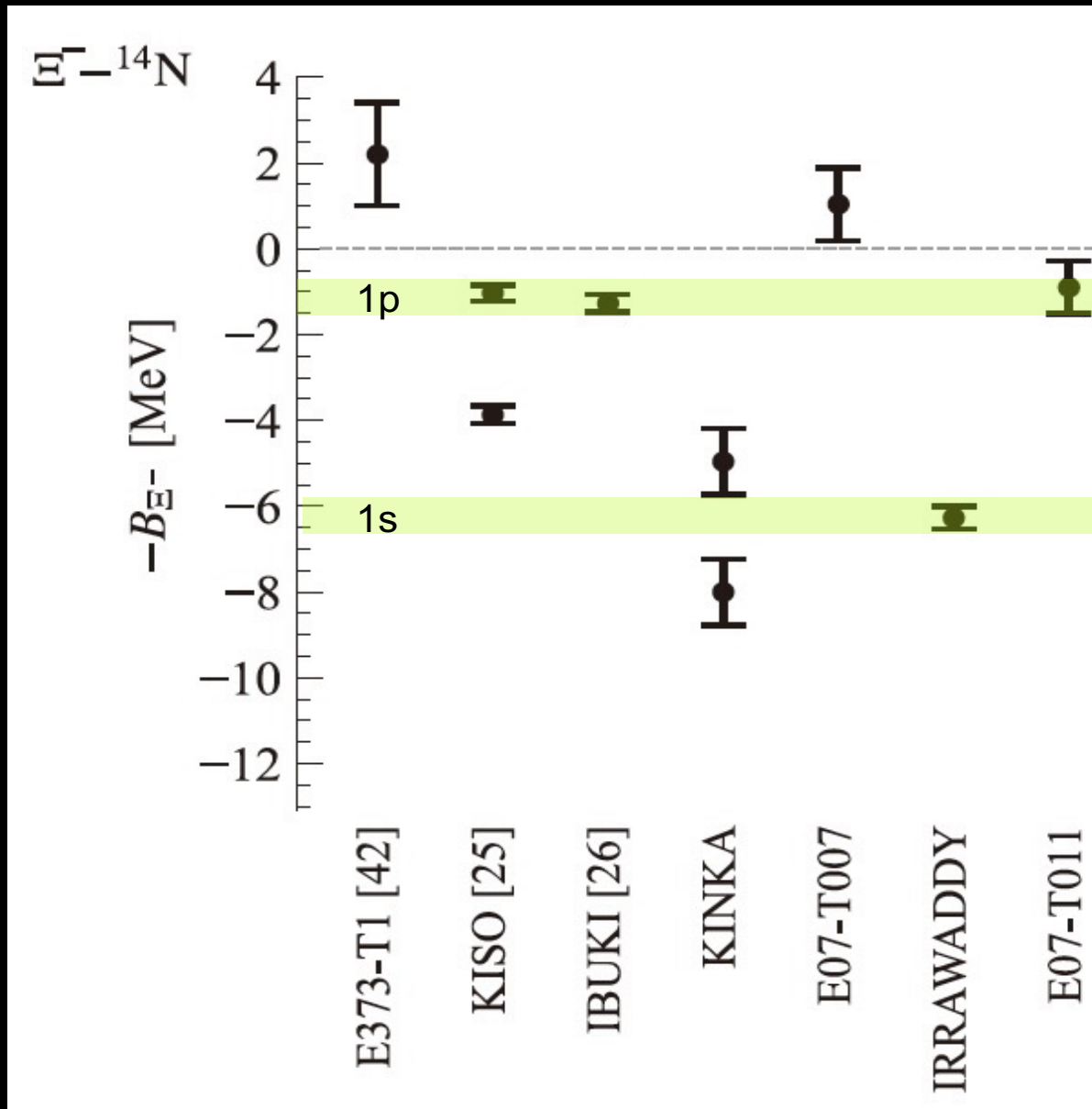


Attraction in $N\Xi$
Weak $N\Xi - \Lambda$ coupling

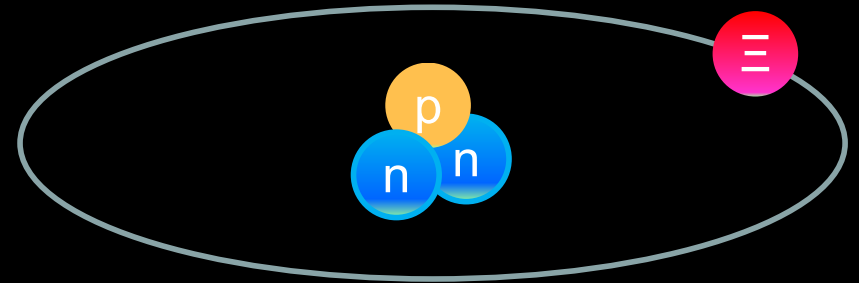
Consistent with HAL QCD prediction

Ξ hypernuclei found so far

Yoshimoto+, PTEP 2021 (2021) 073D02

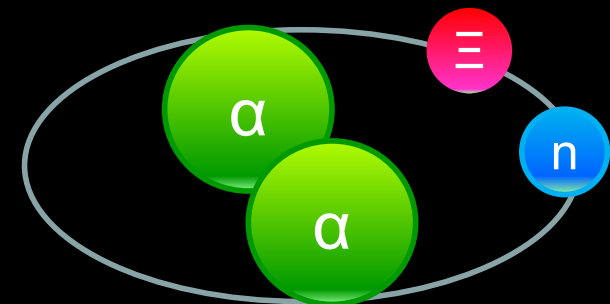


Q1. What is the lightest Ξ hypernuclei?



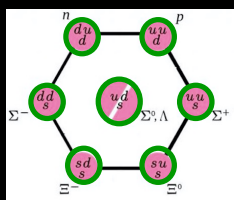
[Hiyama, Sasaki, Miyamoto, Doi, Hatsuda, Yamamoto, Rijken,](#)
[Phys.Rev.Lett. 124 \(2020\) 092501](#)

Q2. Can we test the spin-isospin dependence of ΞN int.?

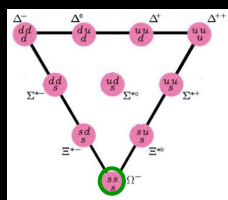


[Hiyama, Isaka, Doi, Hatsuda,](#)
[arXiv:2209.06711 \[nucl-th\]](#)

SU(3)_F classification of BB system

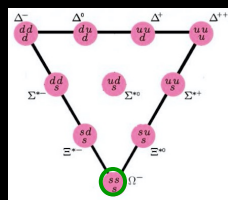


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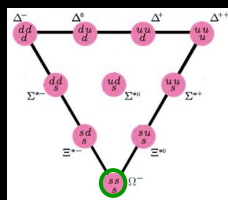


$$8 \times 10 = 35 + 8 + 10 + 27$$

$N\Omega({}^5S_2)$ Goldman+ (1987), Oka (1988)



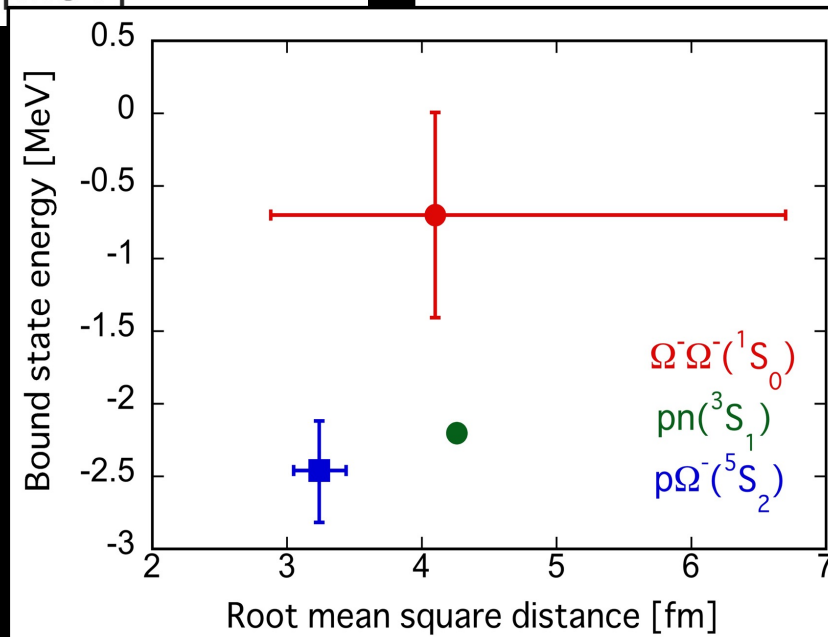
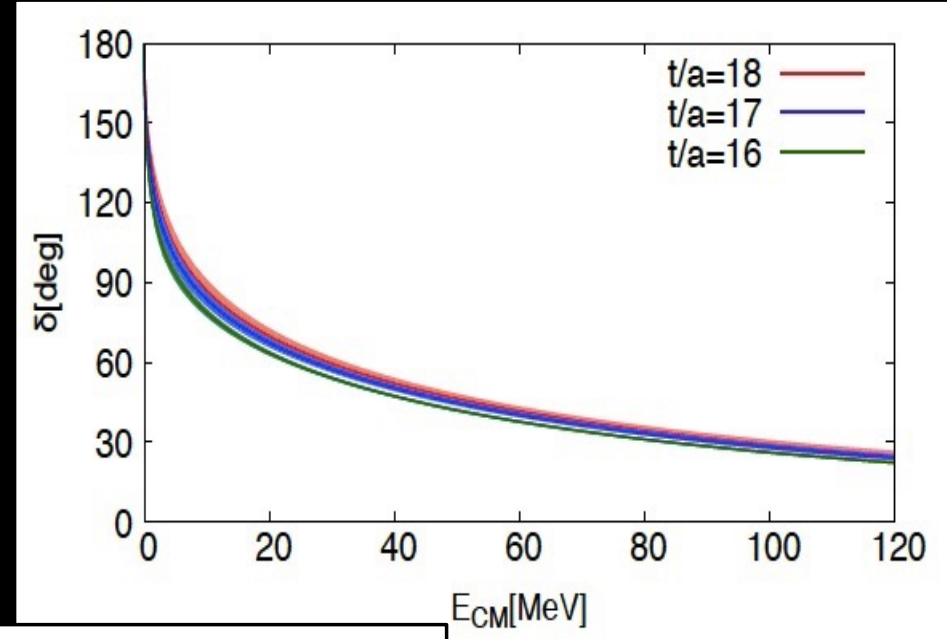
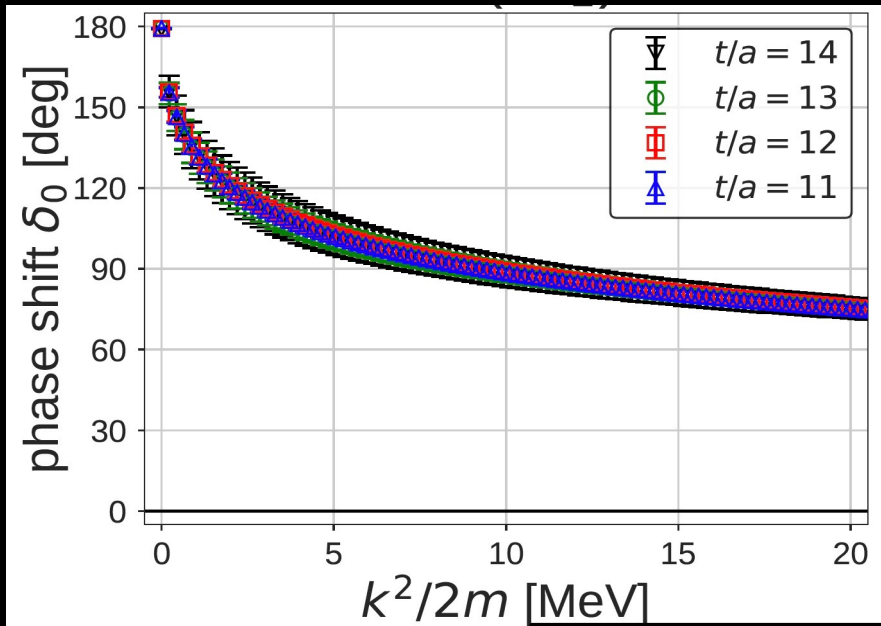
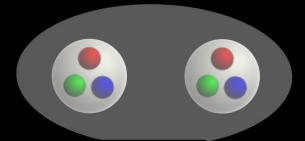
×



$$10 \times 10 = 28 + 27 + 35 + 10^*$$

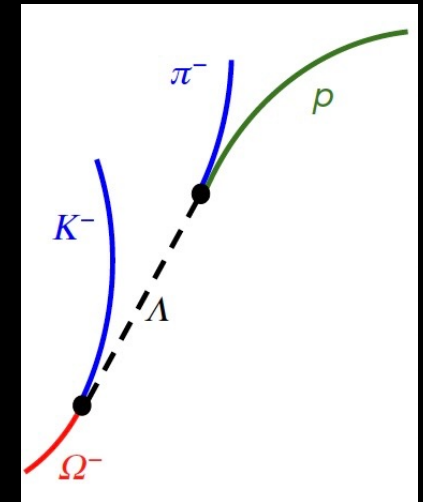
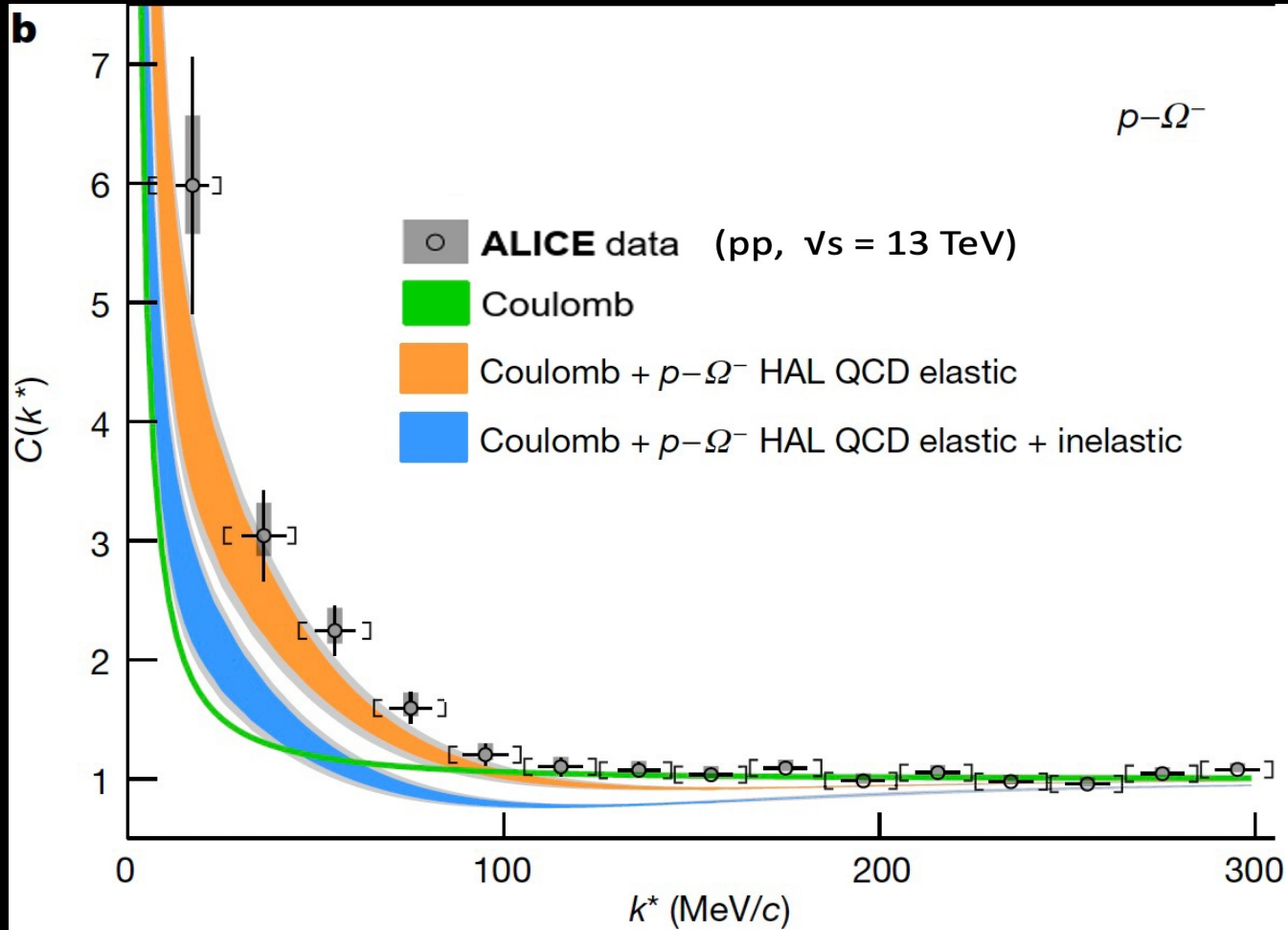
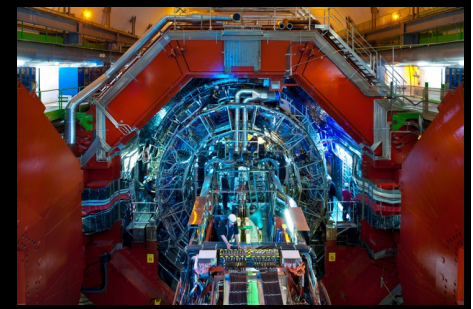
Kopeliovich+ (1990) $\Omega\Omega({}^1S_0)$ $\Delta\Delta({}^7S_3)$ Dyson+ (1964)

quasi-bound $N\Omega$ and bound $\Omega\Omega$?

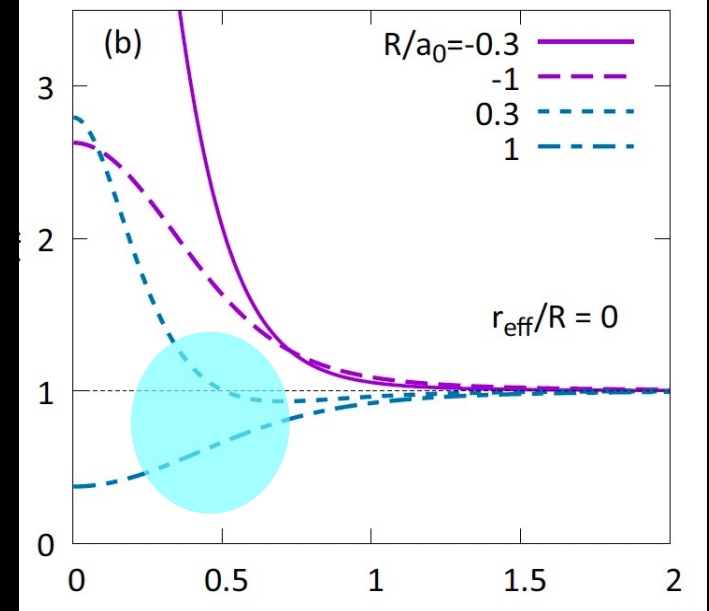
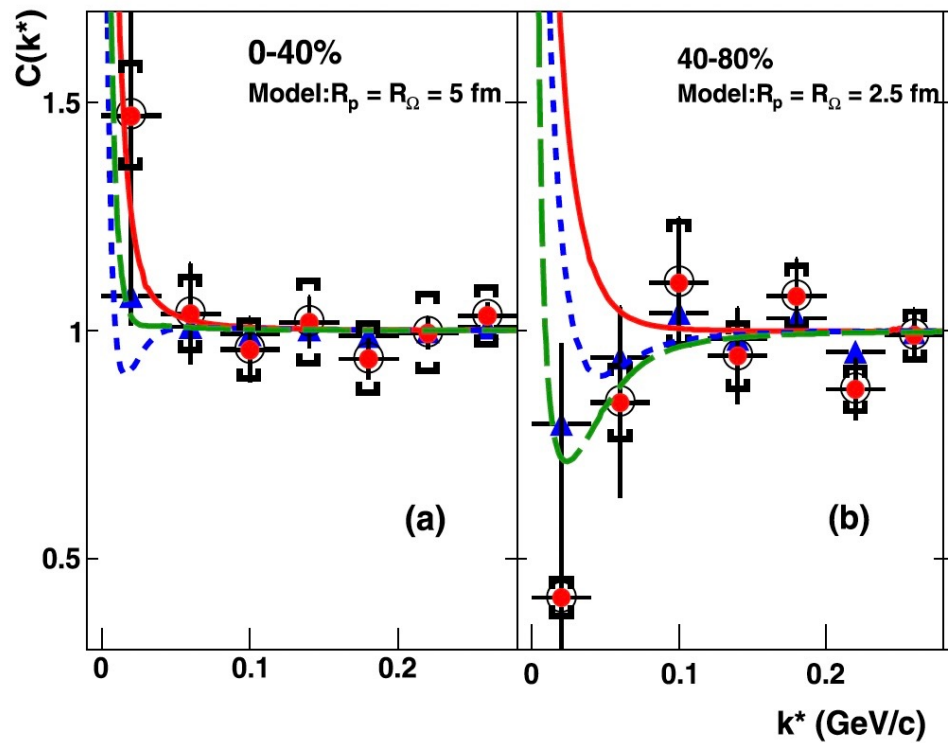


N Ω correlation in pp

LHC ALICE Coll., Nature 588 (2020) 232



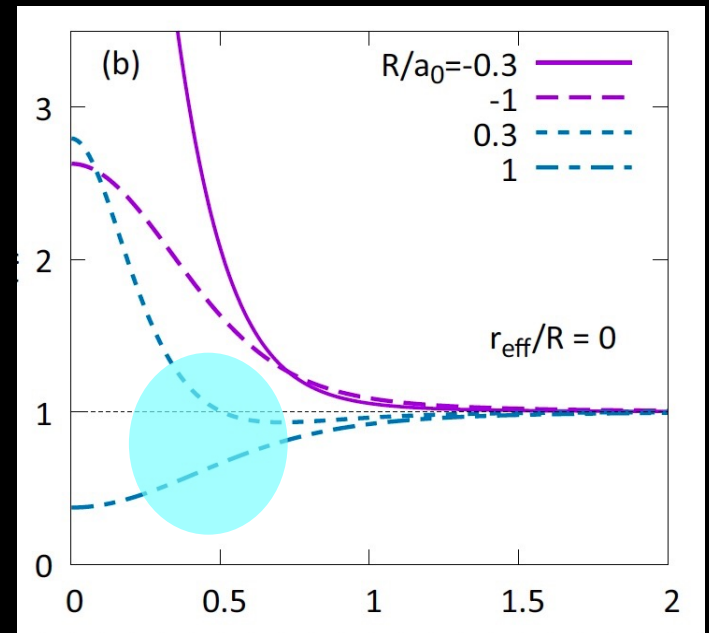
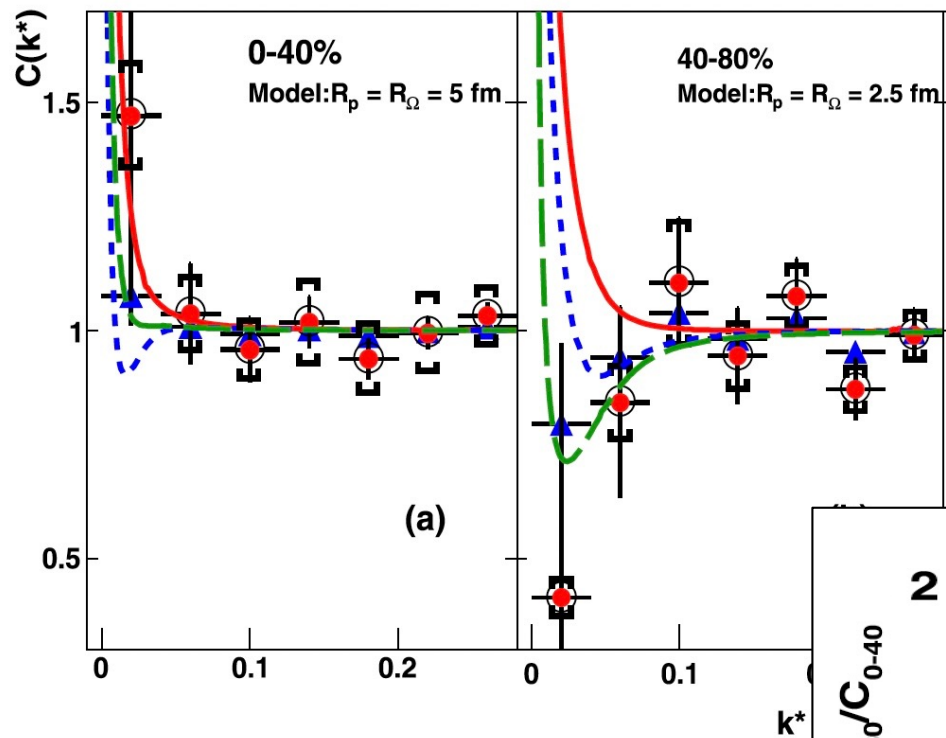
$N\Omega$ correlation in AA



Morita et al., [PRC101 \(2020\) 015201](#)

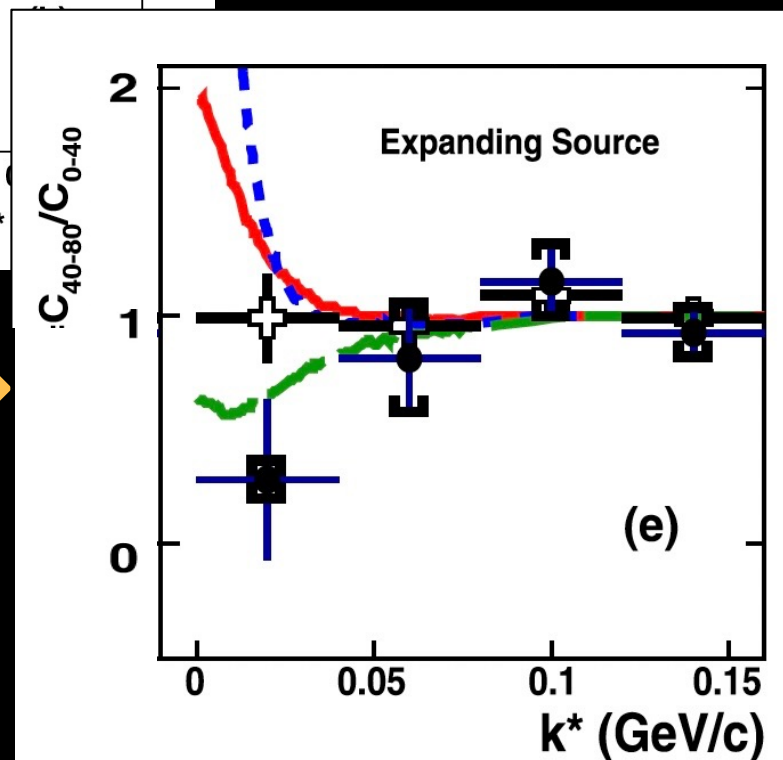
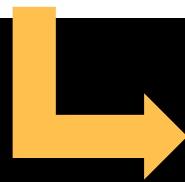
STAR Coll.,
[Phys. Lett. 790 \(2019\) 490](#)

N Ω correlation in AA



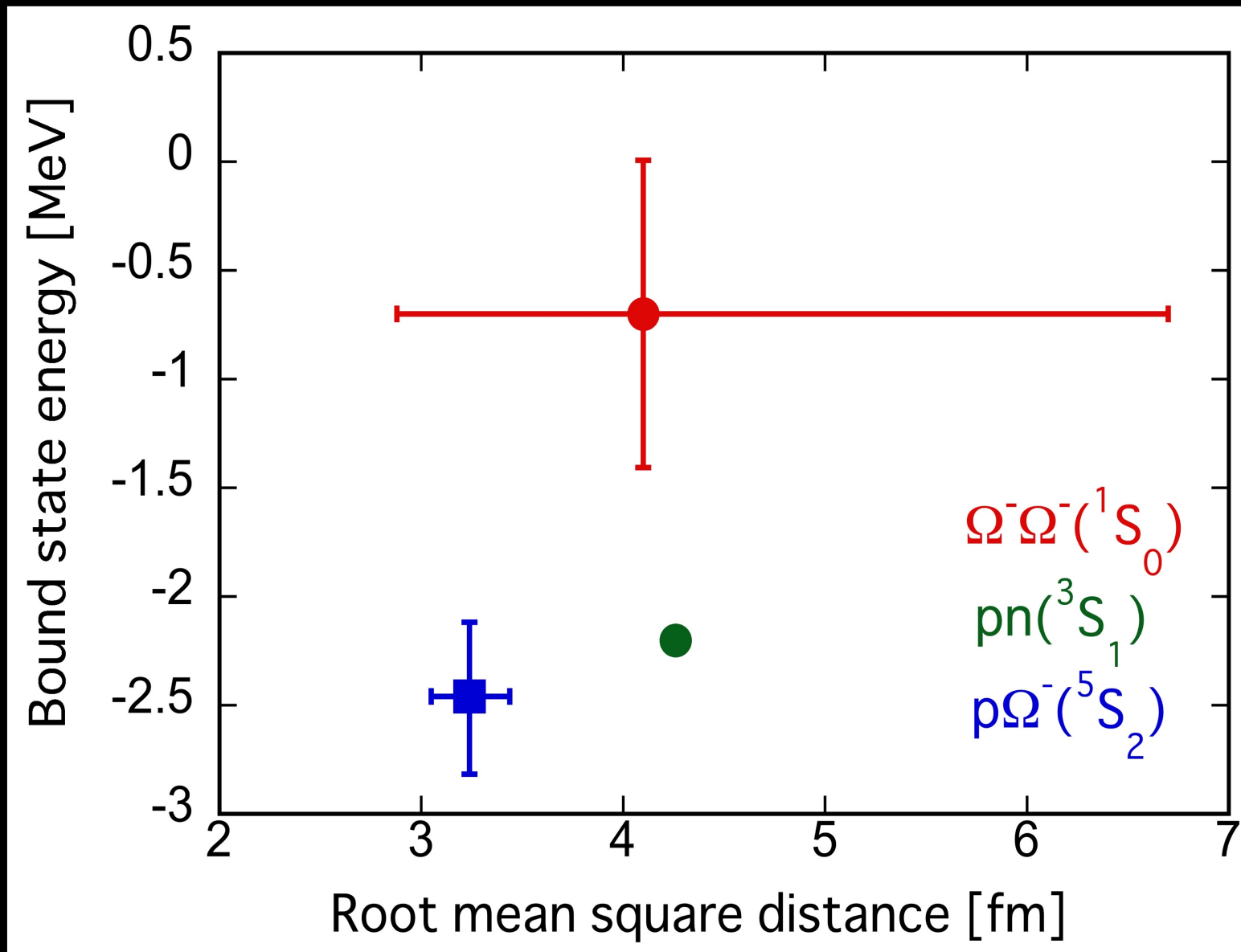
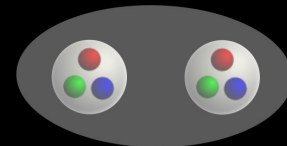
Morita et al., PRC101 (2020) 015201

STAR Coll.,
Phys. Lett. 790 (2019) 490



**Bound
N Ω ???**

quasi-bound $N\Omega$ and bound $\Omega\Omega$?



e-mail from Freeman Dyson,
(Sep. 13, 2017)



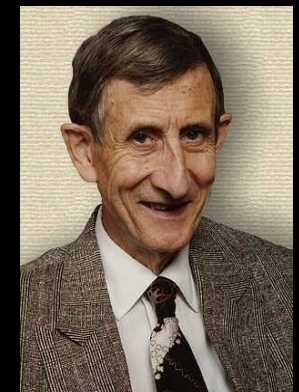
Thank you very much for sending me your paper on the Omega-Omega calculation. This is a beautiful piece of work, and it will be a big step forward if the experimenters are able to confirm it.

Thank you also for referring to our 1964 paper. I am amazed that you remember that paper after 53 years. The predictions that we made in that paper turned out to be wrong, and the SU6 theory was soon abandoned. Luckily you did not assume SU6 symmetry when you made your prediction.

Now I wish you the joy of seeing it confirmed.

Yours sincerely,

Freeman Dyson.



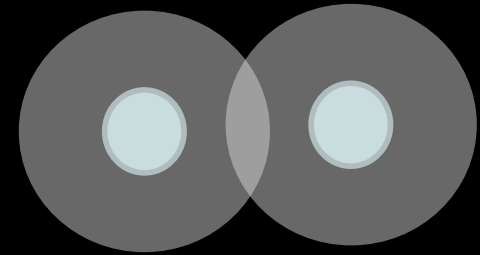
Y=2 States in Su(6) Theory

Freeman J. Dyson and Nguyen-Huu Xuong

Phys. Rev. Lett. **13**, 815 – Published 28 December 1964; Erratum Phys. Rev. Lett. **14**, 339 (1965)

Question:

What is the force between
“neutral particles” at long range ?



Answer:

Atoms

2-photon exchange force

= van der Waals (Casimir-Polder) force $\rightarrow -1/r^7$

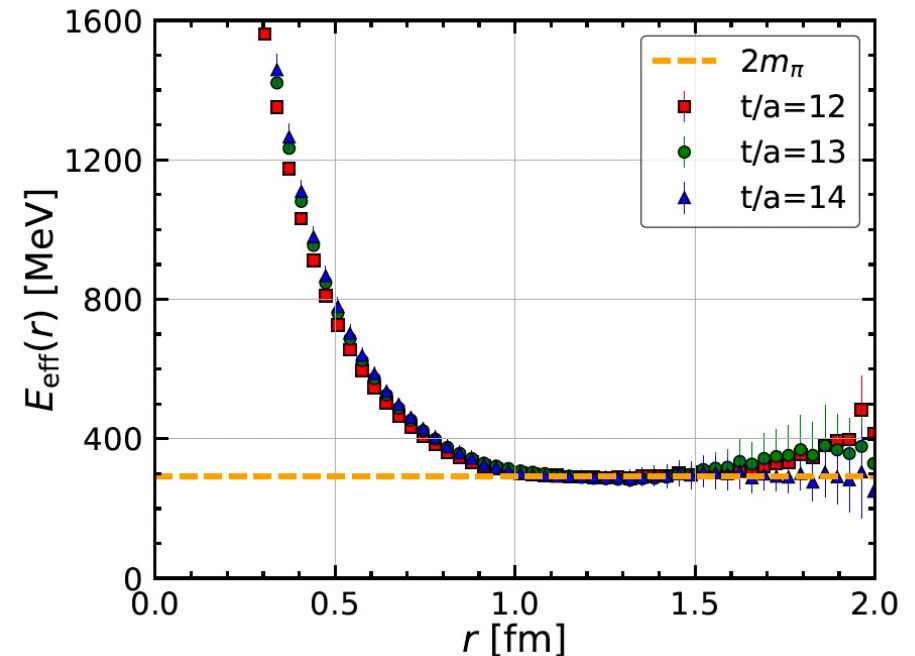
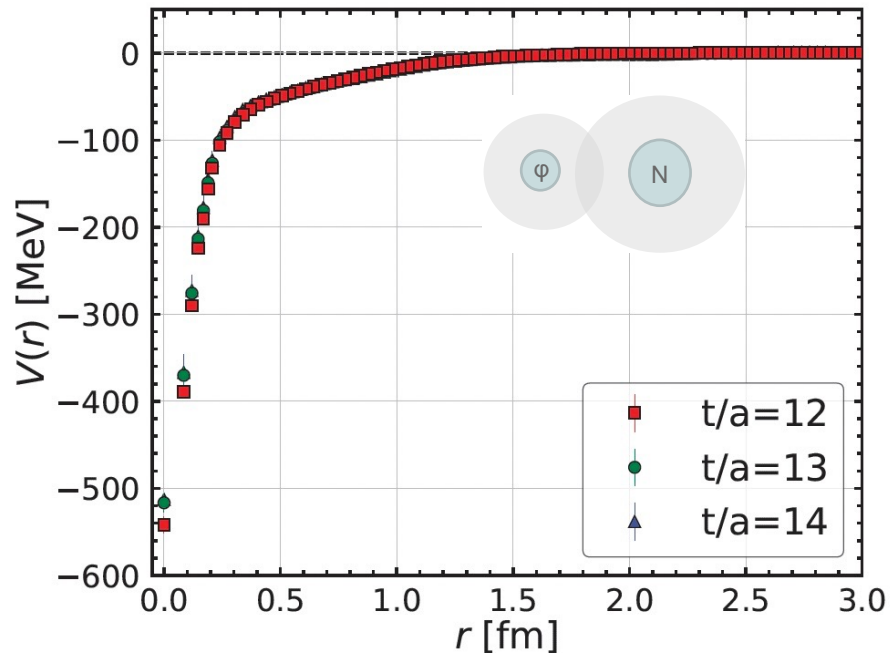
QCD

2-pion exchange force

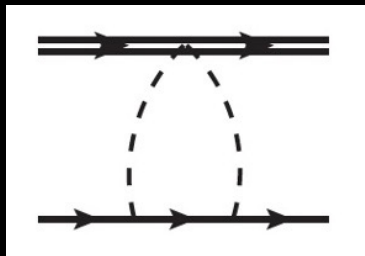
Bhanot and Peskin, Nucl. Phys. B156 (1979) 391
Fujii and Kharzeev, Phys. Rev. D60 (1999) 114039
Brambilla et al., Phys. Rev. D93 (2006) 054002

Attractive N - ϕ interaction and two-pion tail from lattice QCD near physical point

Yan Lyu ^{1,2,*} Takumi Doi ^{2,†} Tetsuo Hatsuda ^{2,‡} Yoichi Ikeda ^{3,§} Jie Meng ^{1,4,||}
 Kenji Sasaki ^{3,¶} and Takuya Sugiura ^{2,**}



Two-pion Tail at $r > (2m_\pi)^{-1}$

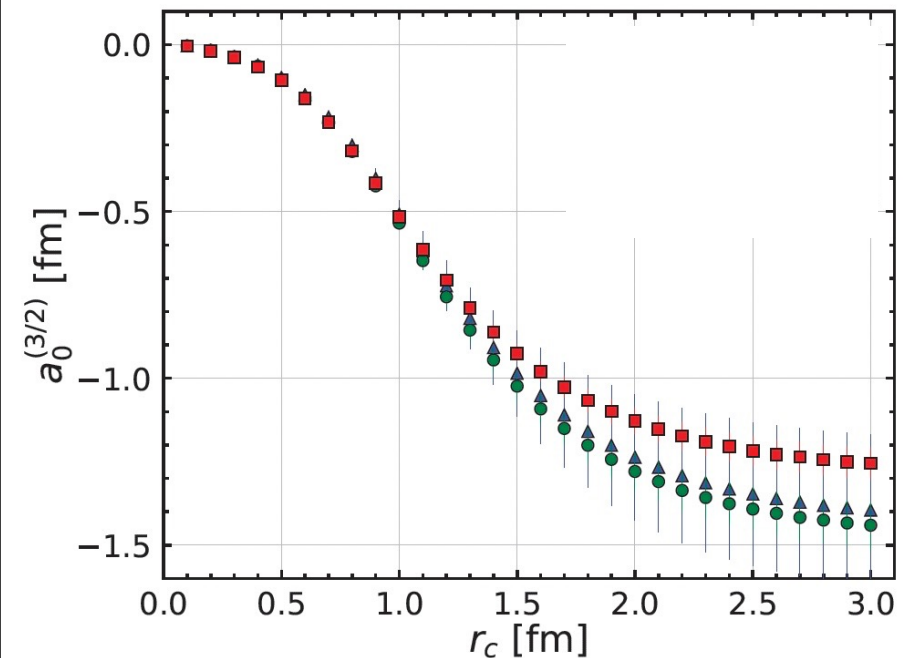
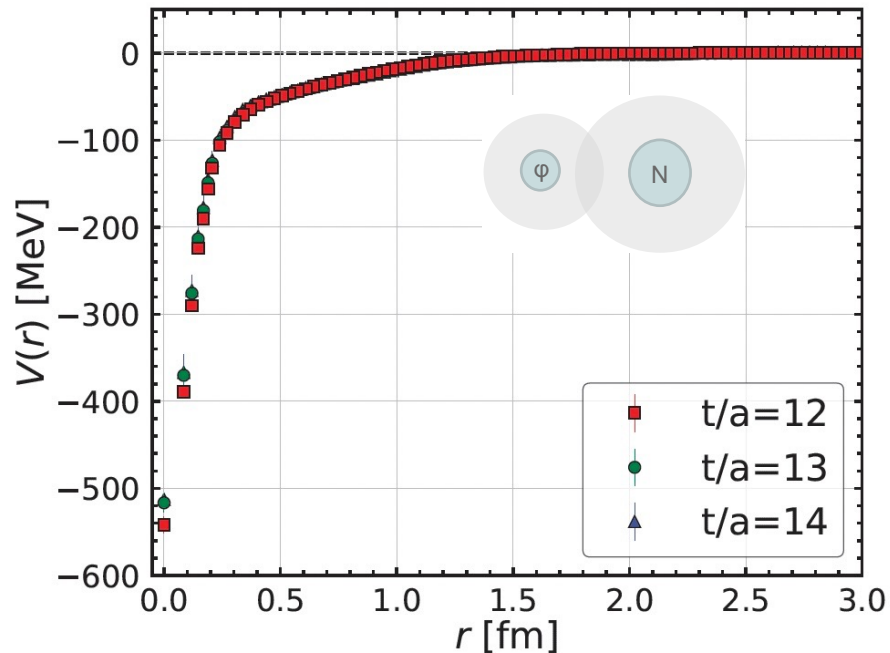


$$V(r) = \frac{3g_A^2 m_\pi^4 (c_{di} + c_m)}{128\pi^2 F^2} \frac{e^{-2m_\pi r}}{r^2}.$$

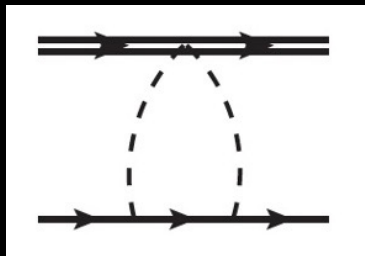
Krein and Castella, Phys. Rev. D98 (2018) 0140289.

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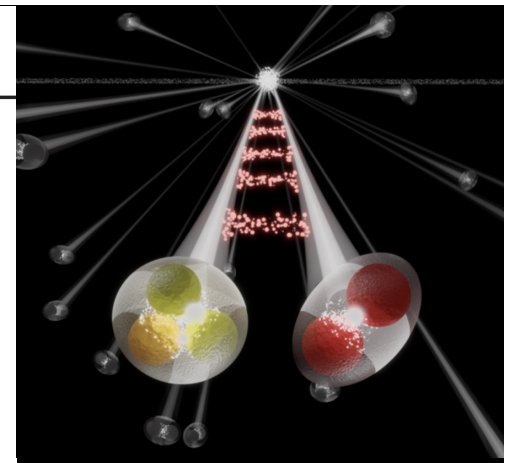
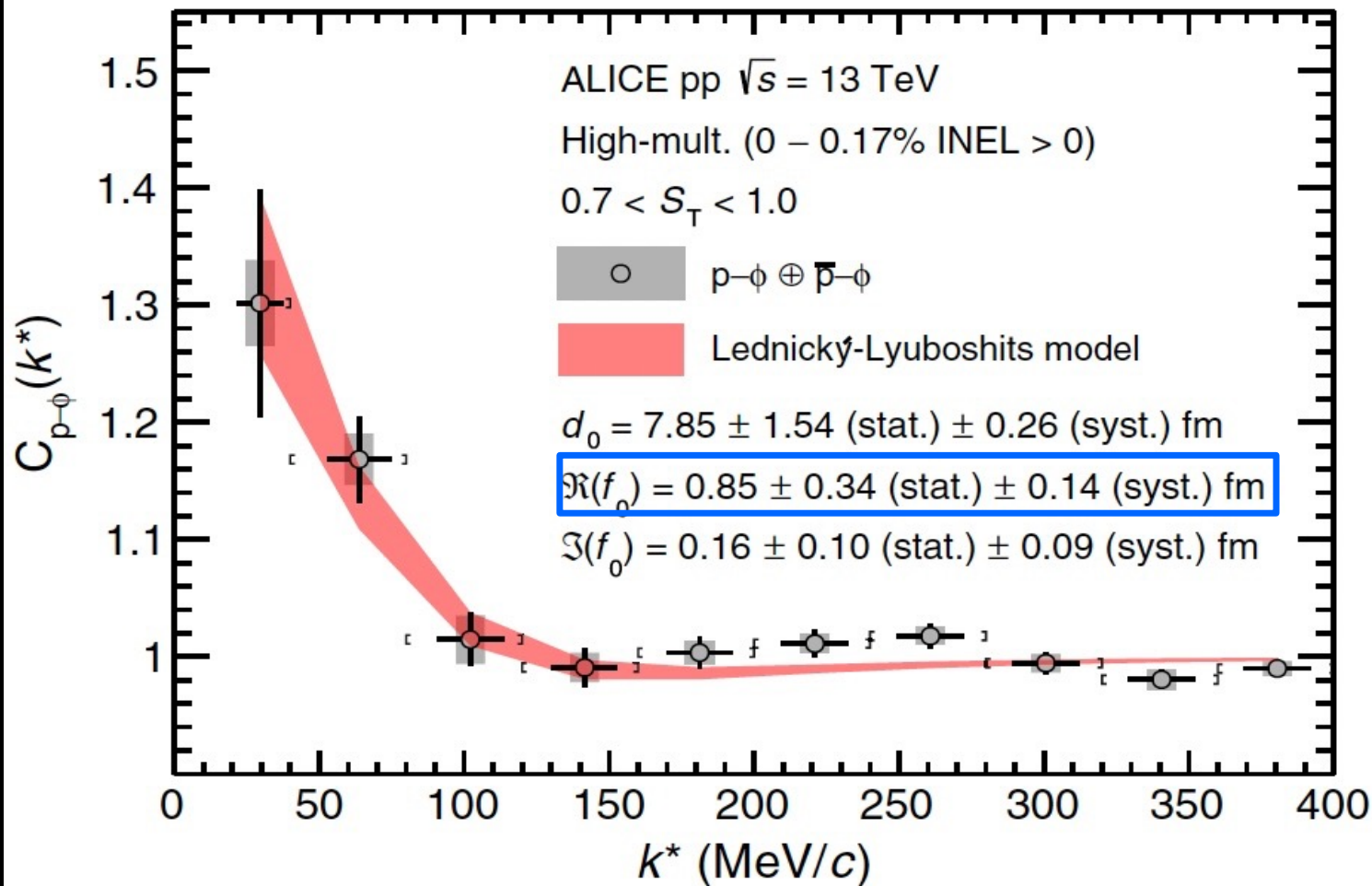


$$V(r) = \frac{3g_A^2 m_\pi^4 (c_{di} + c_m)}{128\pi^2 F^2} \frac{e^{-2m_\pi r}}{r^2}.$$

Krein and Castella, Phys. Rev. D98 (2018) 0140289.

Experimental Evidence for an Attractive p - ϕ Interaction

S. Acharya *et al.**
(ALICE Collaboration)



<https://home.cern/news/news/physics/alice-takes-next-step-understanding-interaction-between-hadrons>

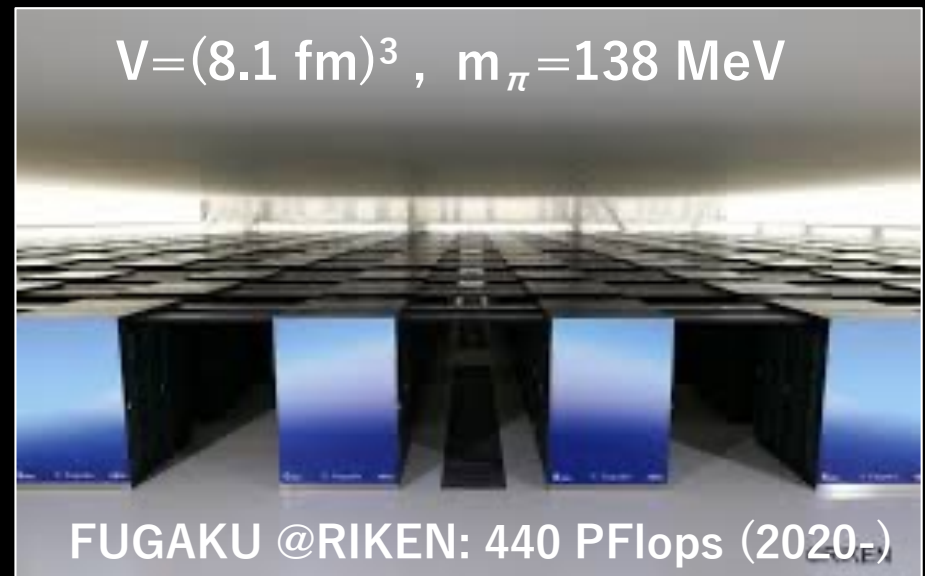
Summary and future

1. “LQCD \rightarrow realistic hadron-hadron interactions” became possible.
2. Comparison of LQCD data with femtoscopy and hypernuclei has been started.
3. $H(uuddss)$ dibaryon is not likely to exist below $\Lambda\Lambda$.
4. $\Omega\Omega(ssssss)$ dibaryon may exist and should be searched experimentally.
5. Hyperon–nucleon interaction & 3-baryon force. \leftrightarrow neutron star structure.
6. Heavy tetraquarks (e.g. T_{cc}, T_{bb}) are highly interesting in both Exp. and LQCD.

More LQCD results at
the physical pion mass will come soon.

Thank you!

$$V=(8.1 \text{ fm})^3, m_\pi=138 \text{ MeV}$$



FUGAKU @RIKEN: 440 PFlops (2020-)