精确理论预测对W玻色子质量测量的影响

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7 September, 2022

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"THREE CLOUDS" IN PARTICLE PHYSICS SINCE 2021

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Statistics from iNSPIRE-HEP by 22-07-2022

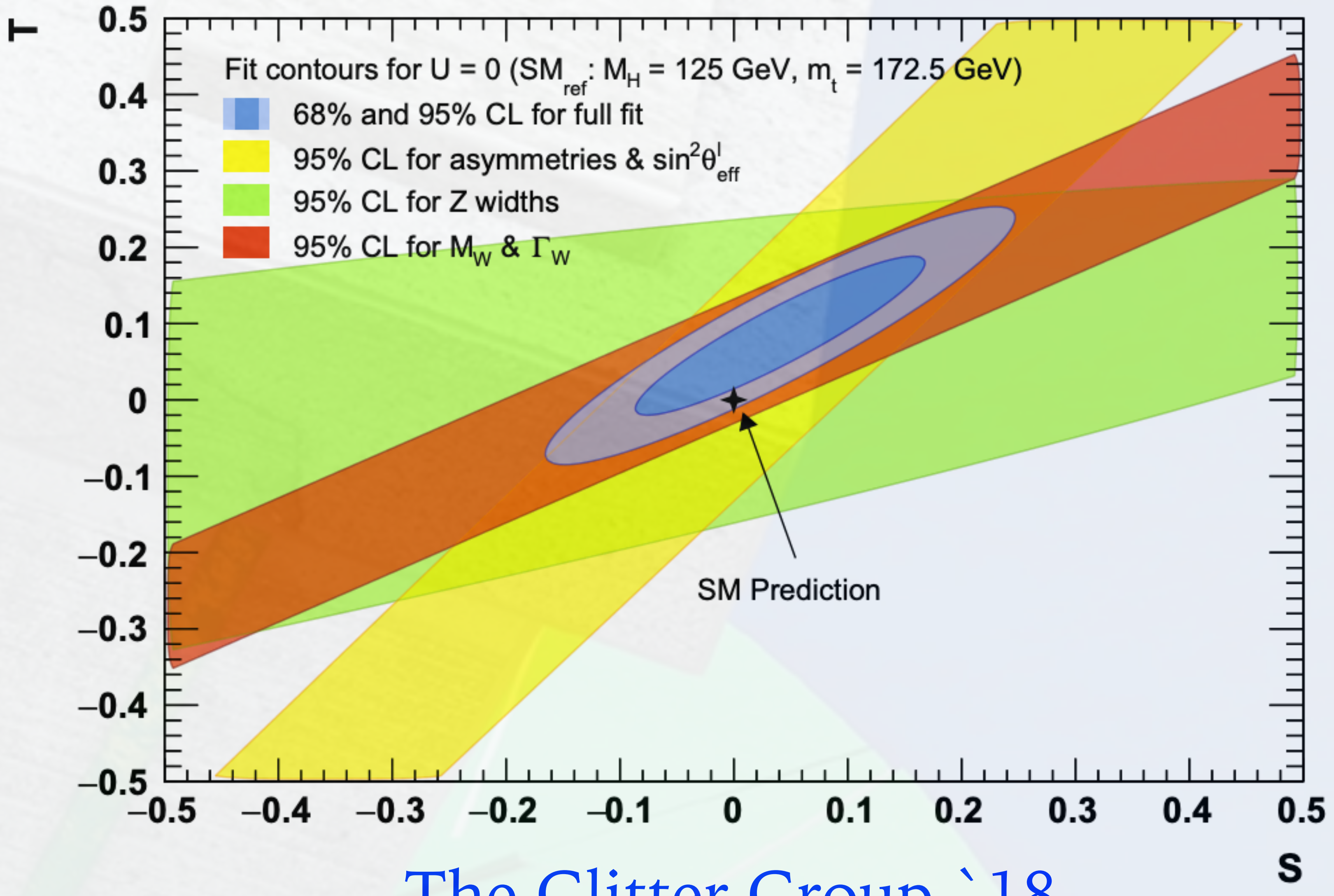
- Fitting the elephant with BSM free parameters
  - The "oblique corrections" S-T-U in vacuum polarisation:

$$\alpha S = 4e^2[\Pi'_{33}(0) - \Pi'_{3Q}(0)]$$

$$\alpha T = \frac{e^2[\Pi_{11}(0) - \Pi_{33}(0)]}{\sin^2(\theta_W)\cos^2(\theta_W)m_Z^2}$$

$$\alpha U = 4e^2[\Pi'_{11}(0) - \Pi'_{33}(0)]$$

Peskin and Takeuchi `92



The Glitter Group `18

- Further experimental confirmation
  - Fermilab Run 2 ~ Run 5 analysis
  - LHCb Upgrade I (2025) and II (2030)
  - ATLAS, LHCb, CMS all have on-going analysis of W mass.

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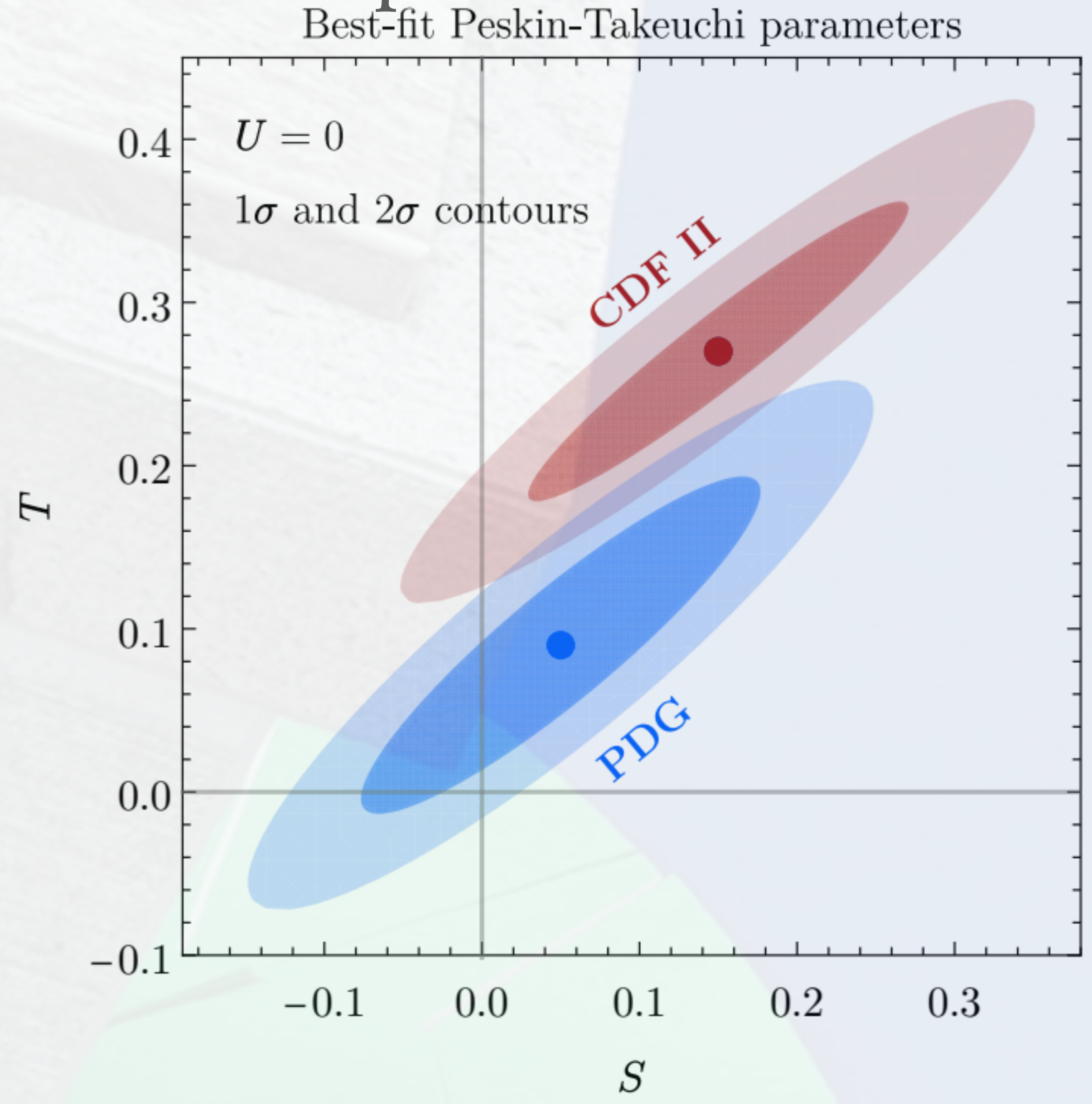

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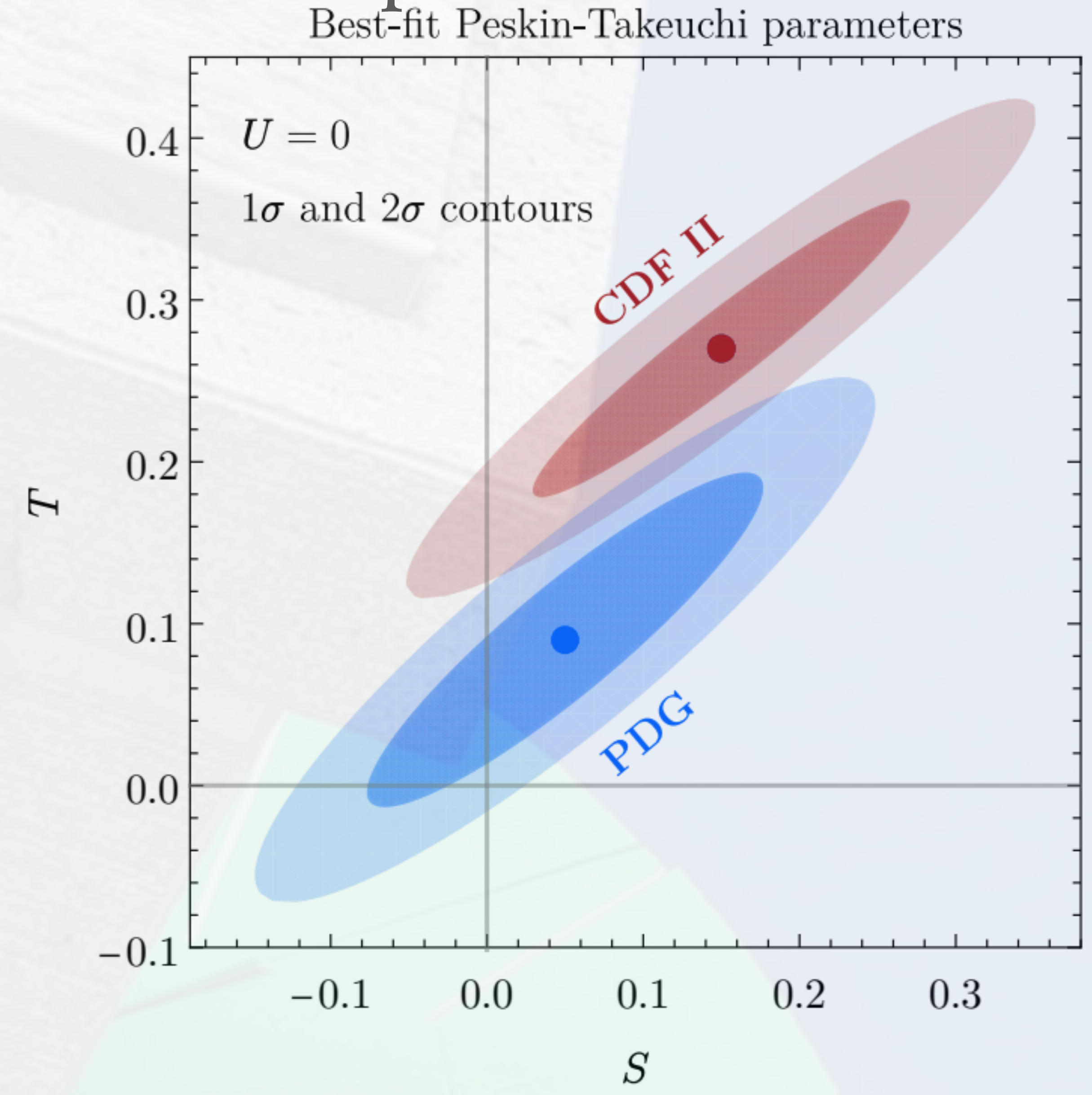
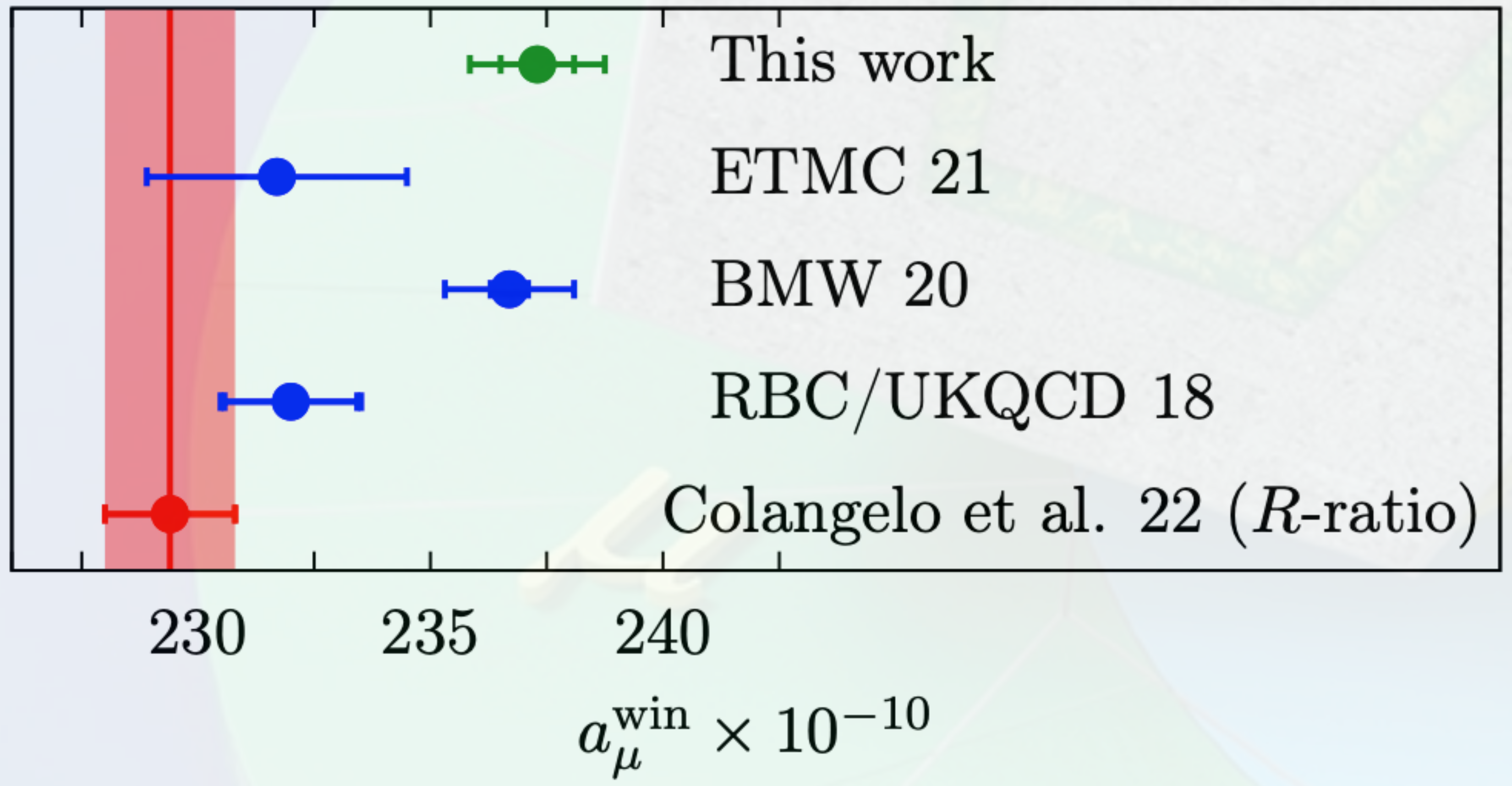
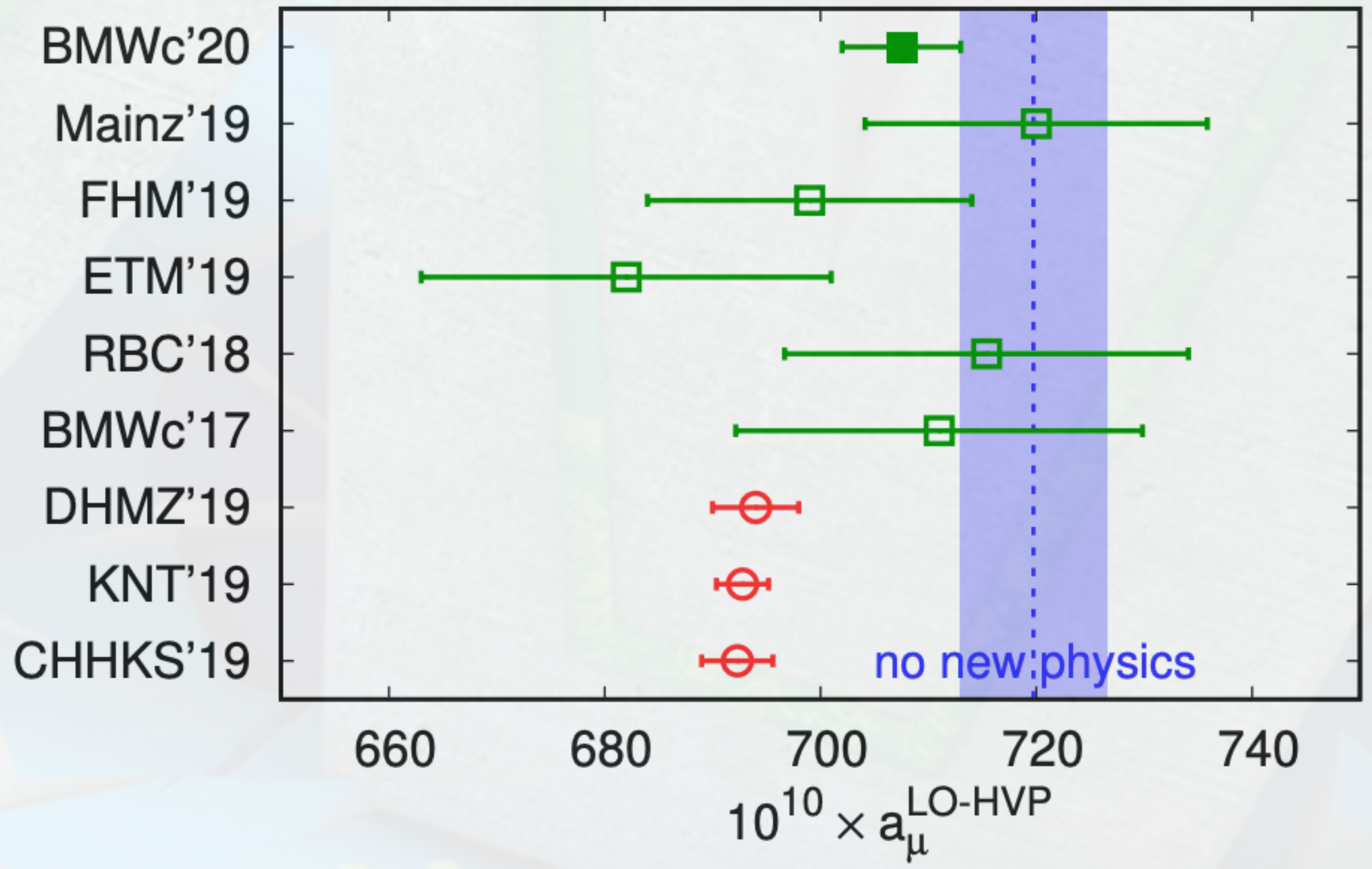


Ce, Gerardin, et. al. `22



Borsanyi, Fodor et. al. `20



- Challenge experiment with better/alternative predictions
  - Lattice prediction of HVP in g-2
  - Improve template fit in CDFII (ResBos@NLO+NNLL)

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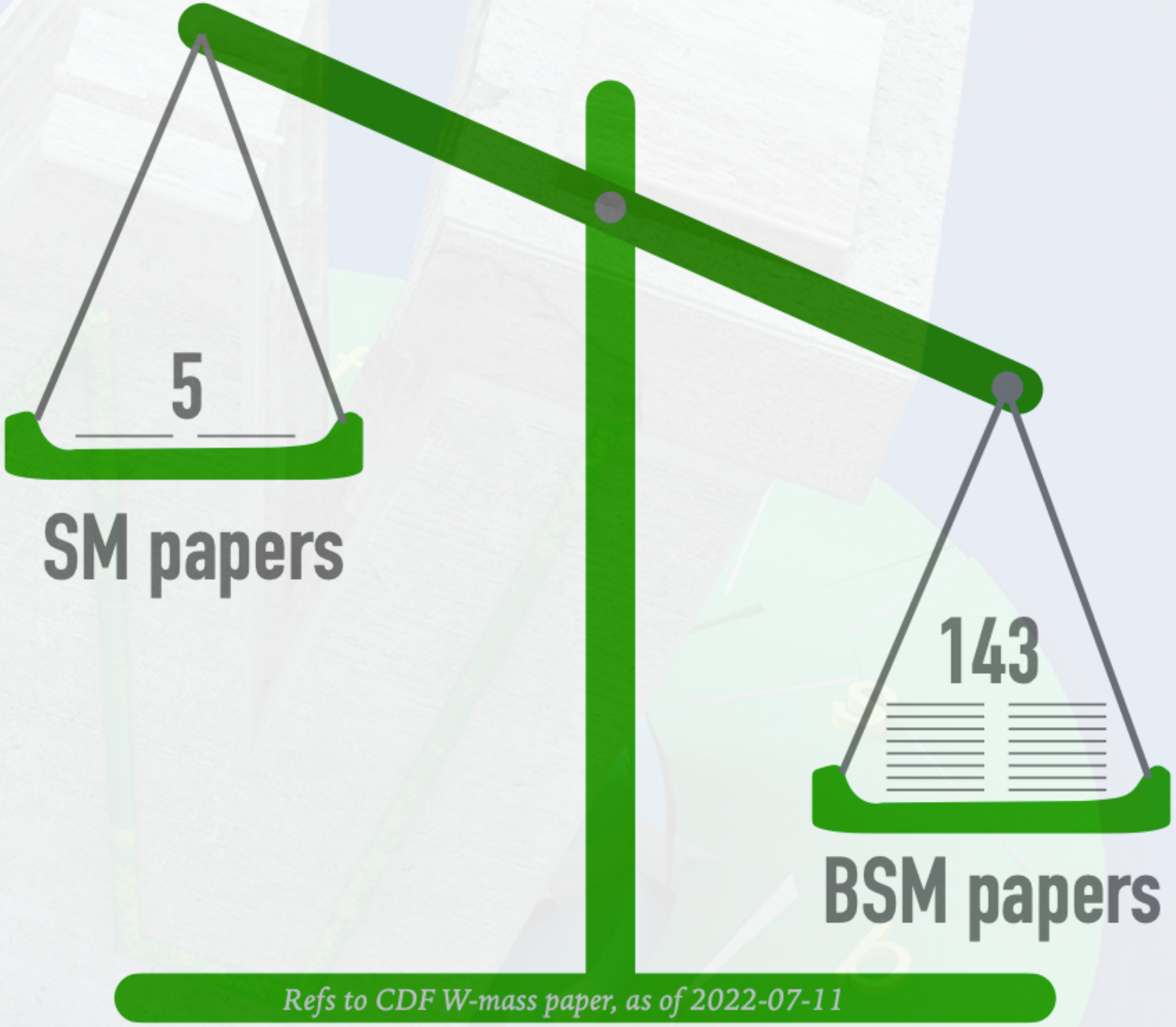
Slide by Gavin Salam ICHEP 2022 →



W MASS IN CDFII MEASUREMENT

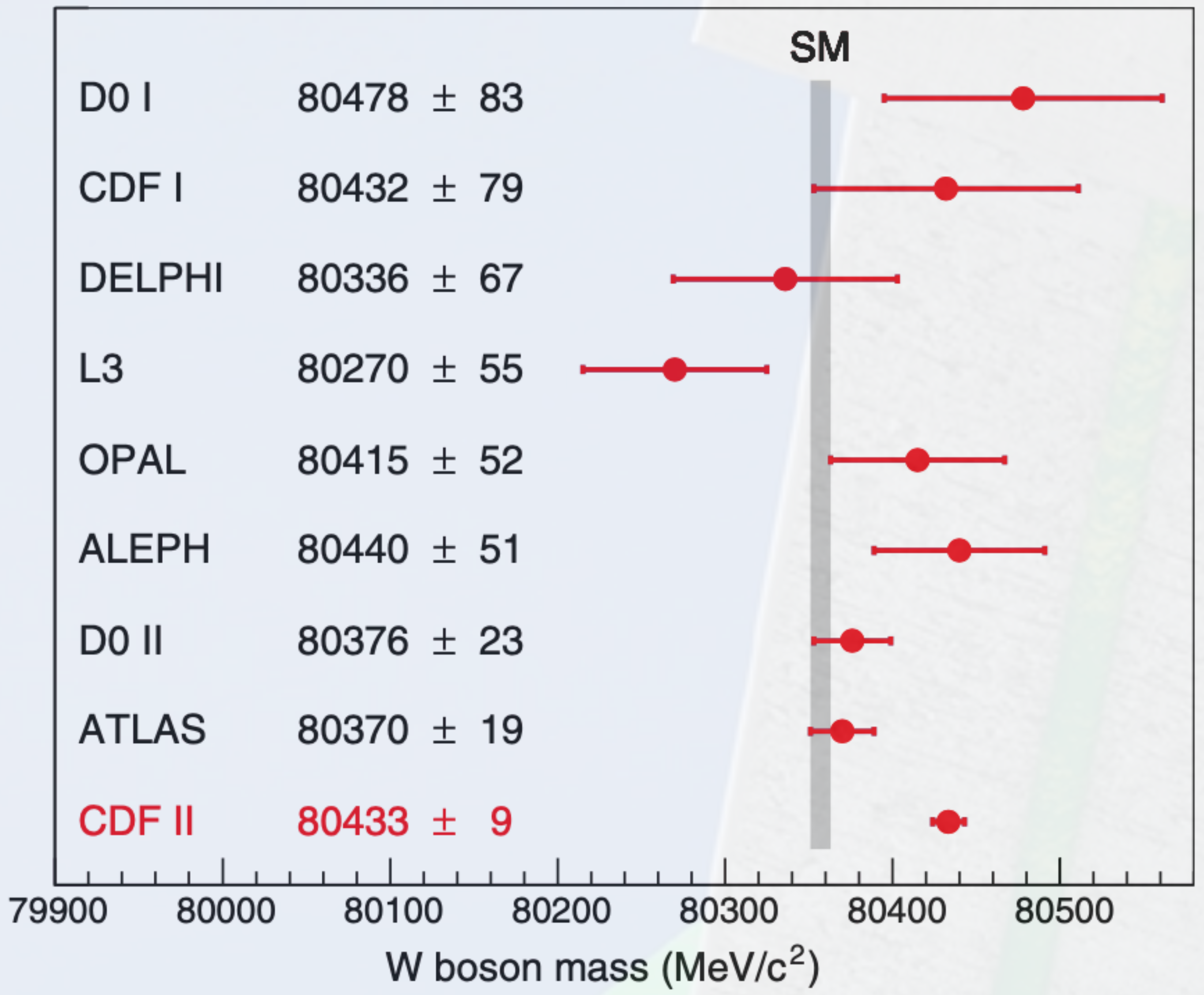


Table 1. Individual fit results and uncertainties for the M_W measurements. The fit ranges are 65 to 90 GeV for the m_T fit and 32 to 48 GeV for the p_T^ℓ and p_T^ν fits. The χ^2 of the fit is computed from the expected statistical uncertainties on the data points. The bottom row shows the combination of the six fit results by means of the best linear unbiased estimator (66).

Distribution	W boson mass (MeV)	χ^2/dof
$m_T(e,\nu)$	$80{,}429.1 \pm 10.3_{stat} \pm 8.5_{syst}$	39/48
$p_T^\ell(e)$	$80{,}411.4 \pm 10.7_{stat} \pm 11.8_{syst}$	83/62
$p_T^\nu(e)$	$80{,}426.3 \pm 14.5_{stat} \pm 11.7_{syst}$	69/62
$m_T(\mu,\nu)$	$80{,}446.1 \pm 9.2_{stat} \pm 7.3_{syst}$	50/48
$p_T^\ell(\mu)$	$80{,}428.2 \pm 9.6_{stat} \pm 10.3_{syst}$	82/62
$p_T^\nu(\mu)$	$80{,}428.9 \pm 13.1_{stat} \pm 10.9_{syst}$	63/62
Combination	$80{,}433.5 \pm 6.4_{stat} \pm 6.9_{syst}$	7.4/5

- PDG world average: $m_W = 80379 \pm 12\ MeV$ (Particle Data Group `20)
- CDFII latest result: $m_W = 80433 \pm 9\ MeV$ (CDF `22)
  - Indirect measurement of m_T^W, p_T^l, p_T^ν distributions

$$p_T^{l(\nu)} = \sqrt{(p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2}$$

$$E_T^{l(\nu)} = \sqrt{m^2 + (p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2} \approx p_T^{l(\nu)}$$

$$m_T^W = \sqrt{2E_T^l E_T^\nu(1-\cos\Delta\phi)}$$

  - Template fit to the best parameter values
  - Full error = Experiment + Theory model
  - Experiment statistics: $\pm 6.4\ MeV$
  - Experiment systematic: $\pm 5.3\ MeV$
  - Theory model: $\pm 5.2\ MeV$ $\pm ??\ MeV$

ResBos, DYqT, PHOTOS, HORACE

Table 2. Uncertainties on the combined M_W result.

Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p_T^Z model	1.8
p_T^W/p_T^Z model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4

W MASS IN CDFII MEASUREMENT

➤ $d\sigma/dp_T^{l(\nu)}$, $d\sigma/dm_T^W$ at hadron colliders (p_a, $p_b \to p_1$, p_2 at centre of mass frame):

$$p_a = (\sqrt{p^2 + m_a^2}, 0, 0, p)$$

$$p_b = (\sqrt{p^2 + m_b^2}, 0, 0, -p)$$

$$p_1 = (\sqrt{q^2 + m_1^2}, q\sin\theta\cos\phi, q\sin\theta\sin\phi, q\cos\theta)$$

$$p_2 = (\sqrt{q^2 + m_2^2}, -q\sin\theta\cos\phi, -q\sin\theta\sin\phi, -q\cos\theta)$$

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$$

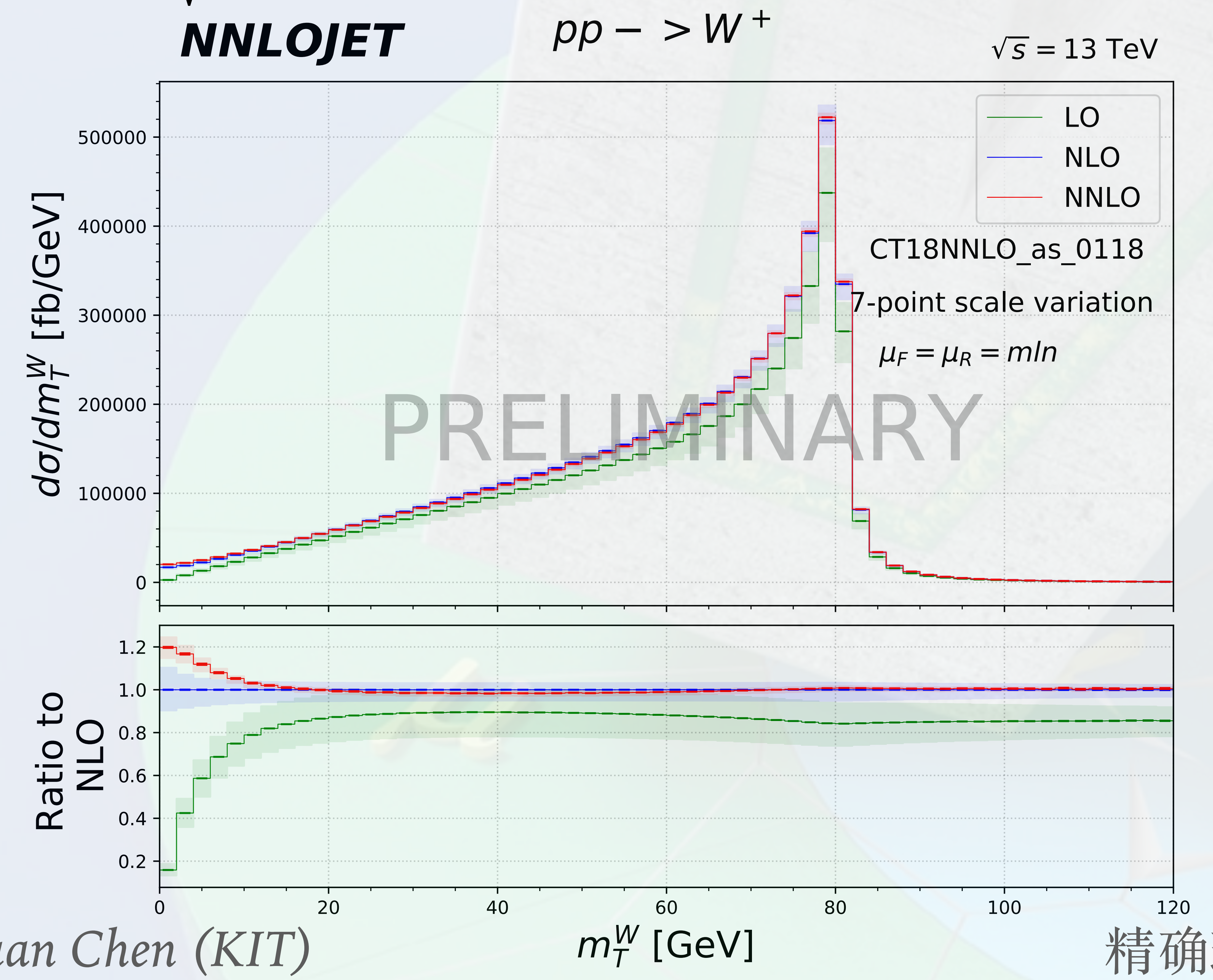
$$p = \frac{\lambda^{\frac{1}{2}}(s, m_a^2, m_b^2)}{2\sqrt{s}},$$

$$q = \frac{\lambda^{\frac{1}{2}}(s, m_1^2, m_2^2)}{2\sqrt{s}},$$

$$p_T^{l(\nu)} = \sqrt{(p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2} = \frac{\sqrt{s}}{2}\sin\theta \to m_W/2$$

$$E_T^{l(\nu)} = \sqrt{m^2 + (p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2} \approx p_T^{l(\nu)} \to m_W/2$$

$$m_T^W = \sqrt{2E_T^l E_T^\nu(1 - \cos\Delta\phi)} \to m_W$$



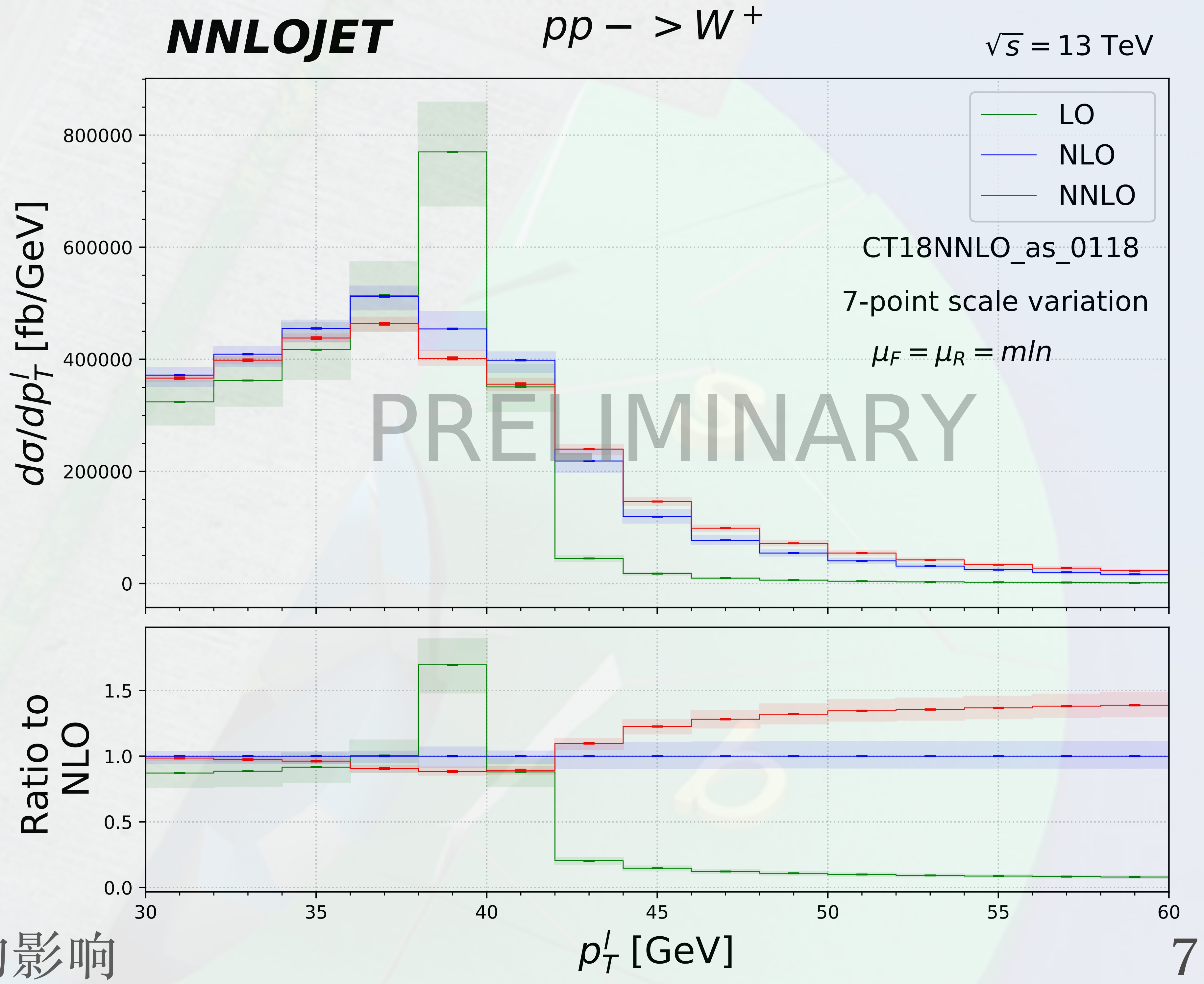
Observables linearly sensitive to m_W

➤ Γ_W sensitive region

➤ m_W sensitive region

➤ PDF sensitive

➤ Resumation region



W MASS IN CDFII MEASUREMENT

- $d\sigma/dm_T^W$ Template fit to best best parameter values:

CDFII: 0.2 MeV granularity of m_W with fixed Γ_W

W MASS IN CDFII MEASUREMENT

- $d\sigma/dm_T^W$ Template fit to best best parameter values:
  - Relativistic Breit-Wigner form: $(s^2 - m_W^2 + is^2\Gamma_W/m_W)^{-1}$ with fixed Γ_W
  - Binned maximum-likelihood fit: (Poisson distribution cross bins)

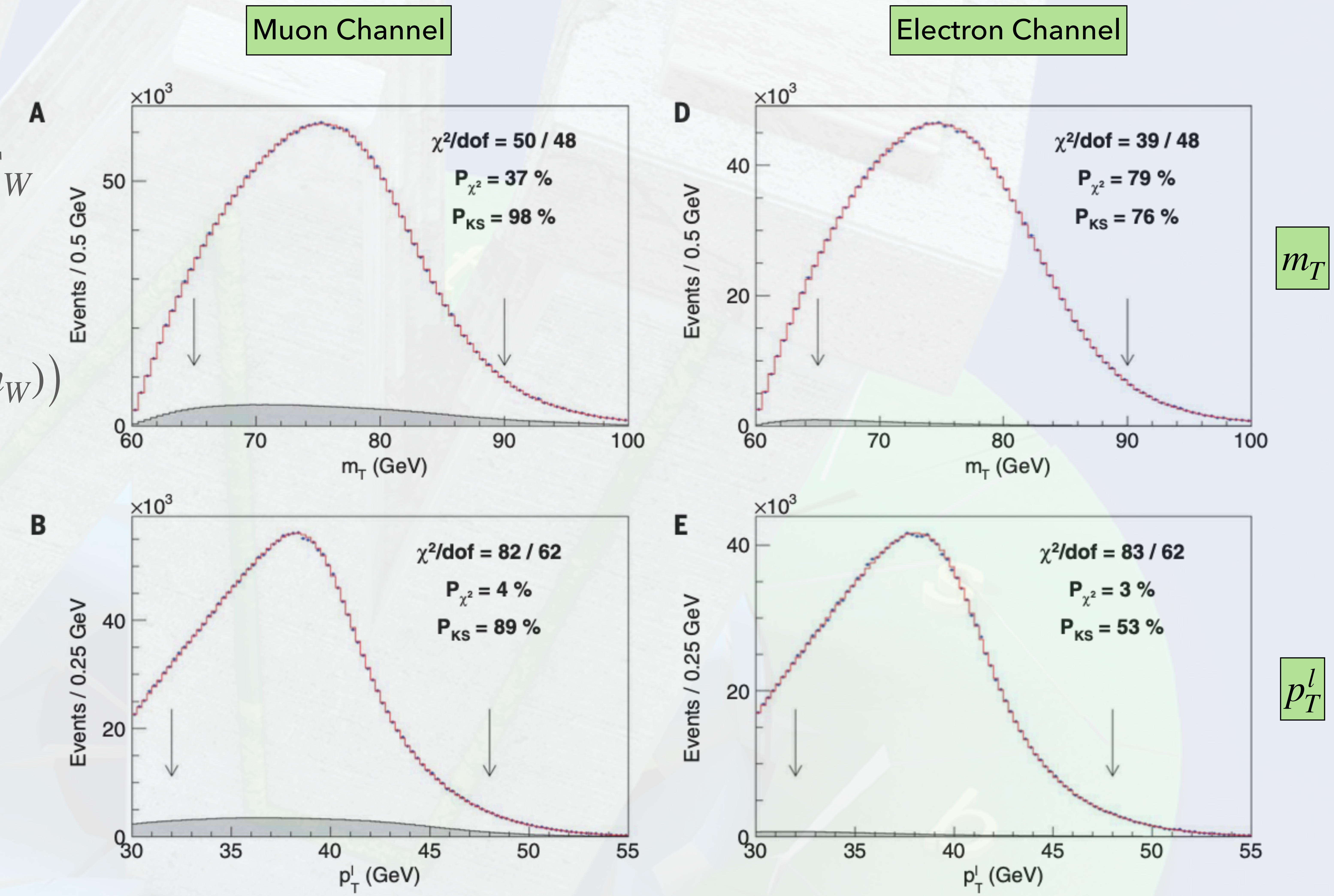
$$-\ln\mathscr{L}_b(m_W) = -\sum_b \left(n_b \ln\left(\Delta\sigma_b(m_W)\right) - \Delta\sigma_b(m_W)\right)$$

  n_b: observed event, $\Delta\sigma_b(m_W)$: predicted

  - The best linear unbiased estimator to combine each observable:
    - $\chi^2/\text{dof} = 7.4/5 \rightarrow$ p-value = 20%
    - Weight distribution:

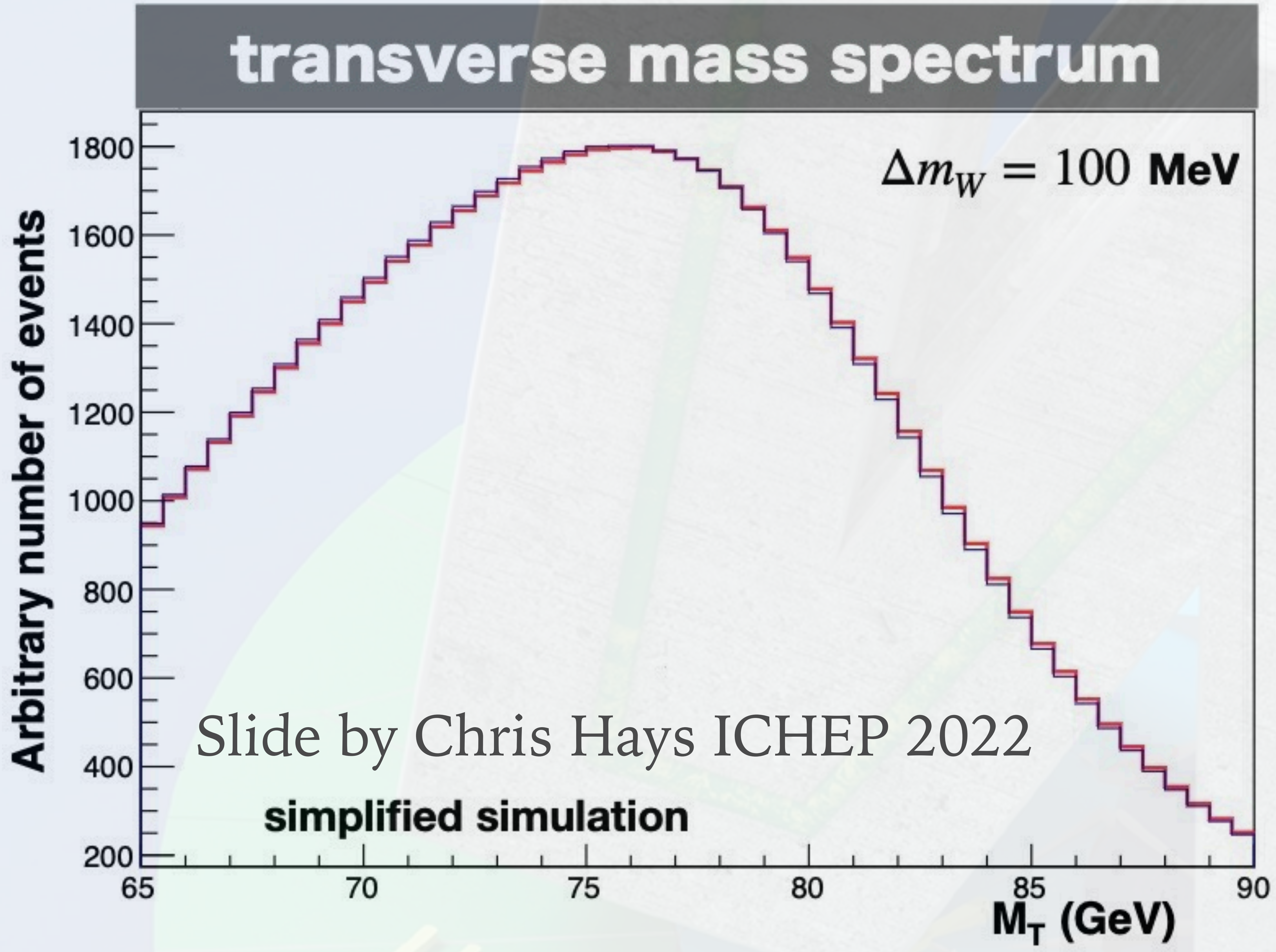
$m_T^W \sim 64.2\,\%$, $p_T^l \sim 25.4\,\%$, $p_T^\nu \sim 10.4\,\%$

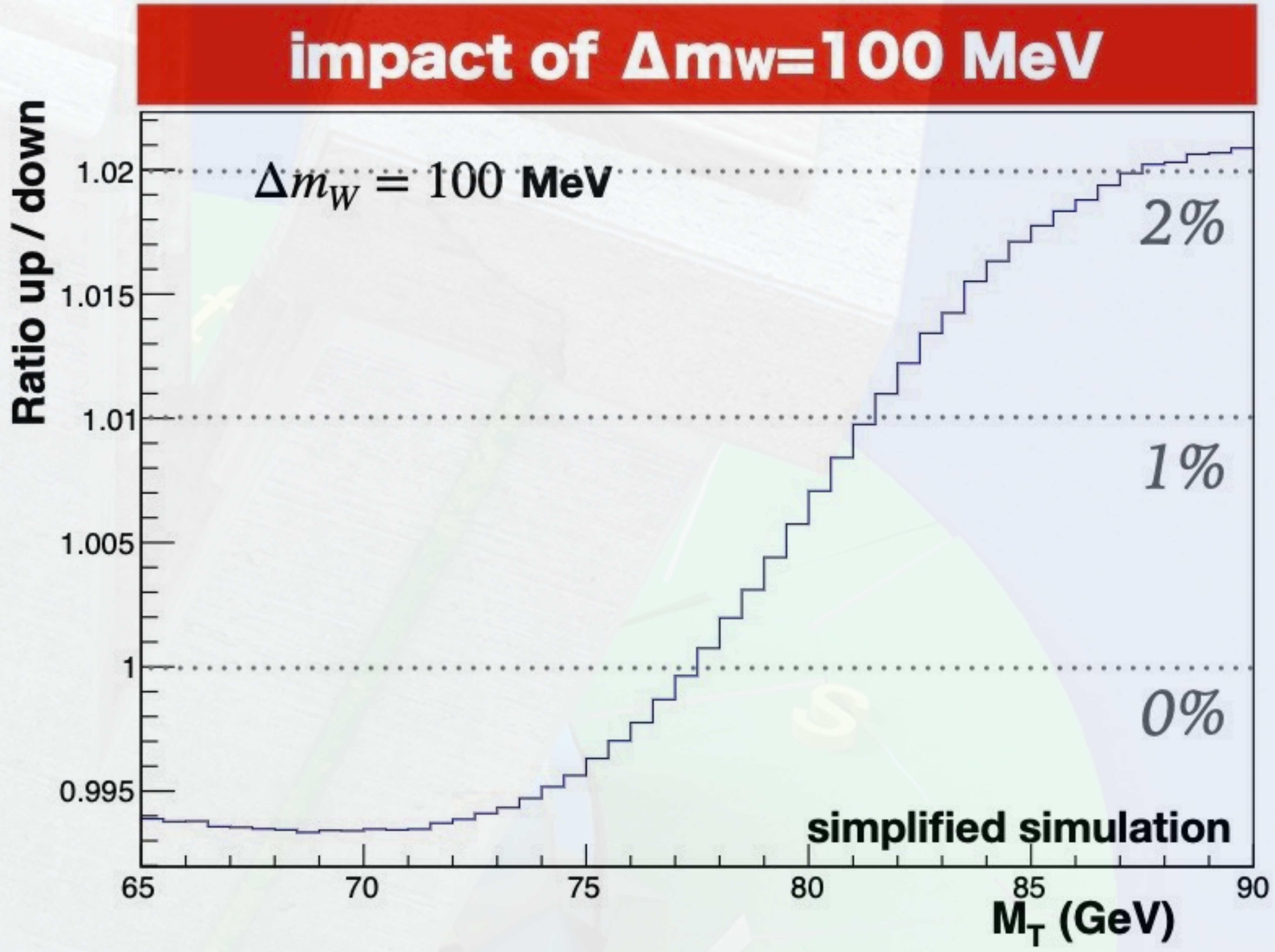
CDFII: Best fitted results for m_T^W, p_T^l



W MASS IN CDFII MEASUREMENT

➤$d\sigma/dm_T^W$ two templates with $\Delta m_W = 100$ MeV



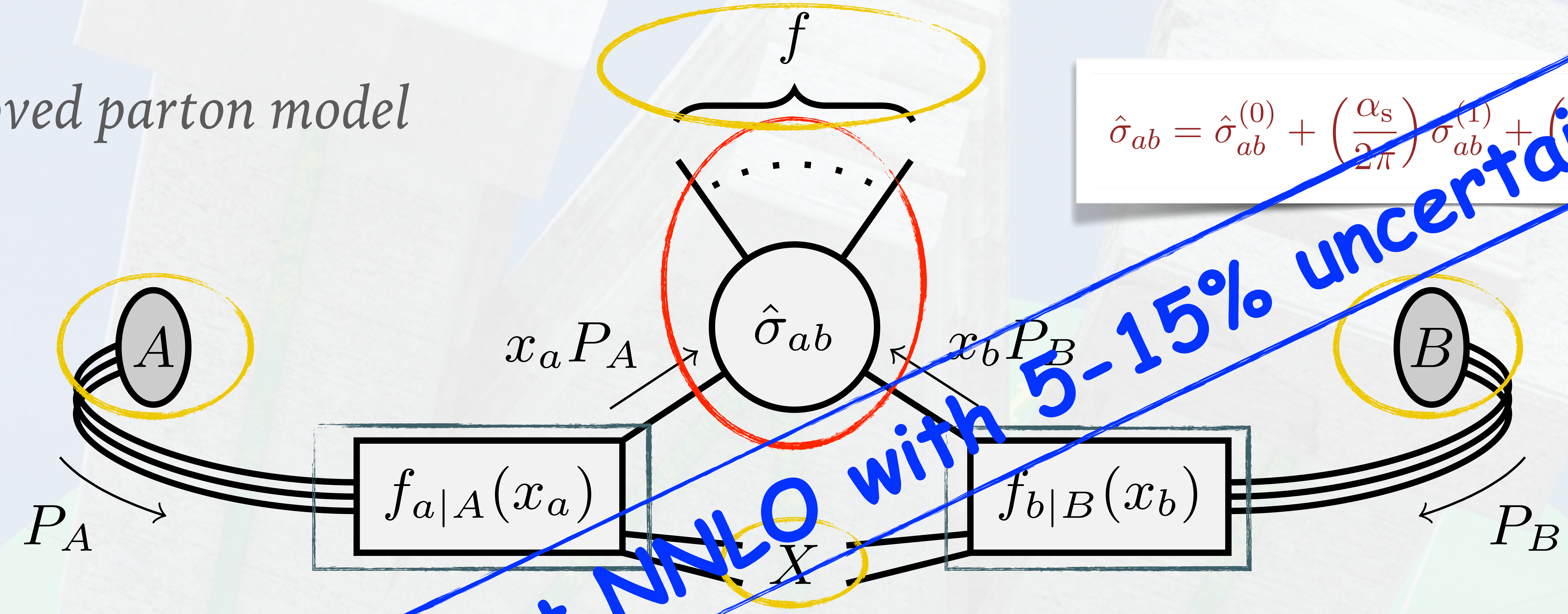


$\Delta m_W = 100$ MeV ~ 0.5-2% change in $d\sigma/dm_T^W$ → $\Delta m_W = 10$ MeV ~ 0.1% precision in $d\sigma/dm_T^W$

PRECISION PREDICTIONS AT HADRON COLLIDER

QCD improved parton model

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)\hat{\sigma}^{(1)}_{ab} + \left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}^{(2)}_{ab} + \ldots$$



Typical precision at NNLO with 5-15% uncertainties

$$\sigma_{AB} = \sum_{ab}\int_0^1 dx_a \int_0^1 dx_b\, f_{a|A}(x_a)\, f_{b|B}(x_b)\, \hat{\sigma}_{ab}(x_a, x_b)\, (1+\mathcal{O}(\Lambda_{QCD}/Q))$$

Parton distribution functions
(Energy evolution from all exp.)
± 3-5 % at LHC energy

Hard scattering
(Perturbative quantum field theory)
± 10 % level!

non-perturbative effects
(Fragmentation, hadronisation)
± 1.2 GeV/13 TeV

PRECISION PREDICTIONS AT HADRON COLLIDER

➤Beyond QCD improved parton model

➤pQCD describes the tail of spectrum

➤Large logarithmic divergence

$$\ln\frac{p_T}{Q} \text{ as } \quad p_T \to 1 \text{ GeV}$$

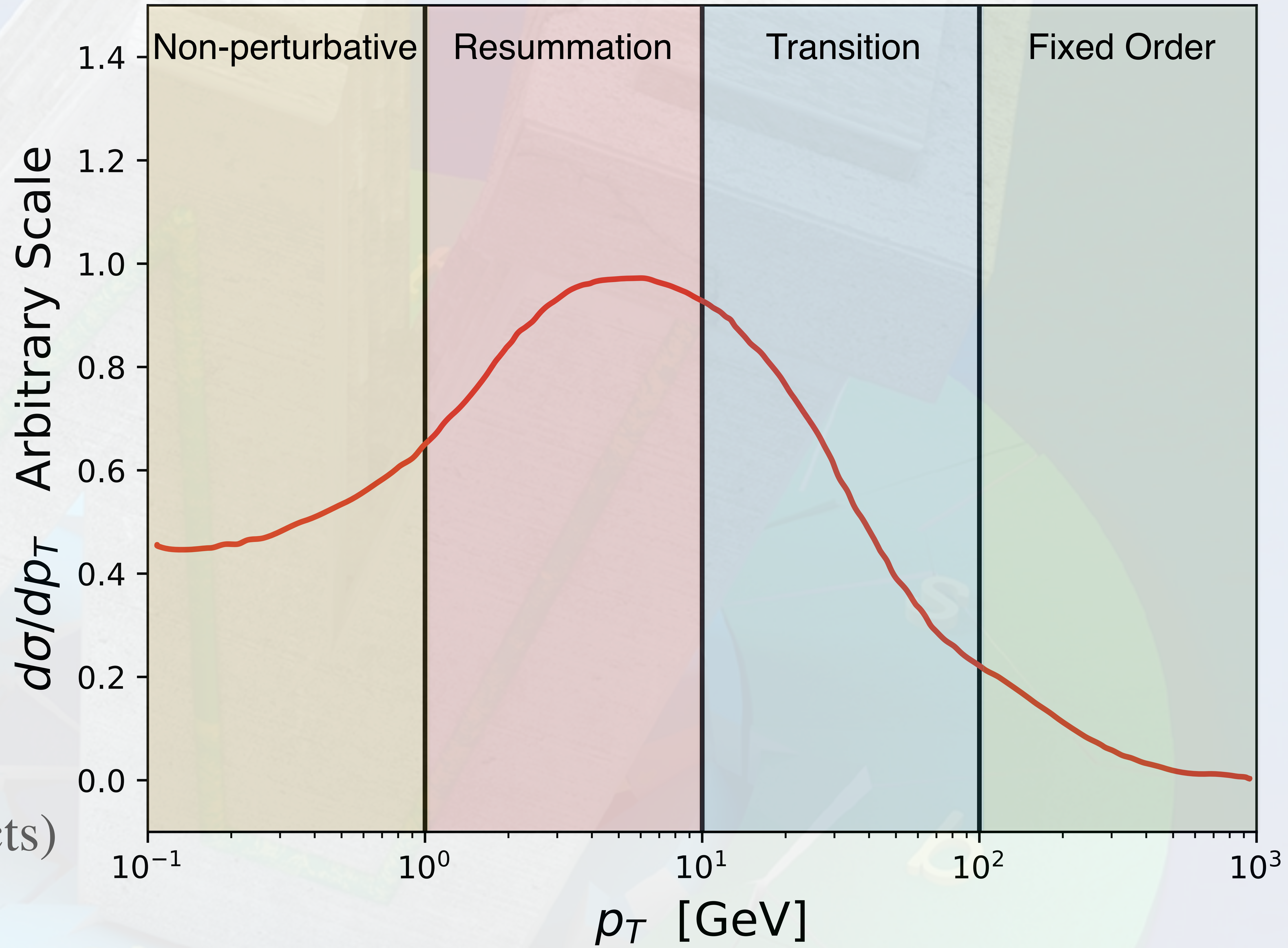
➤Various LP resummation schemes

➤Multiple solutions in transition region

➤Non-perturbative effects ~ 1 GeV

(Short distance and long distance effects)

p_T Spectrum = multi-scale problem



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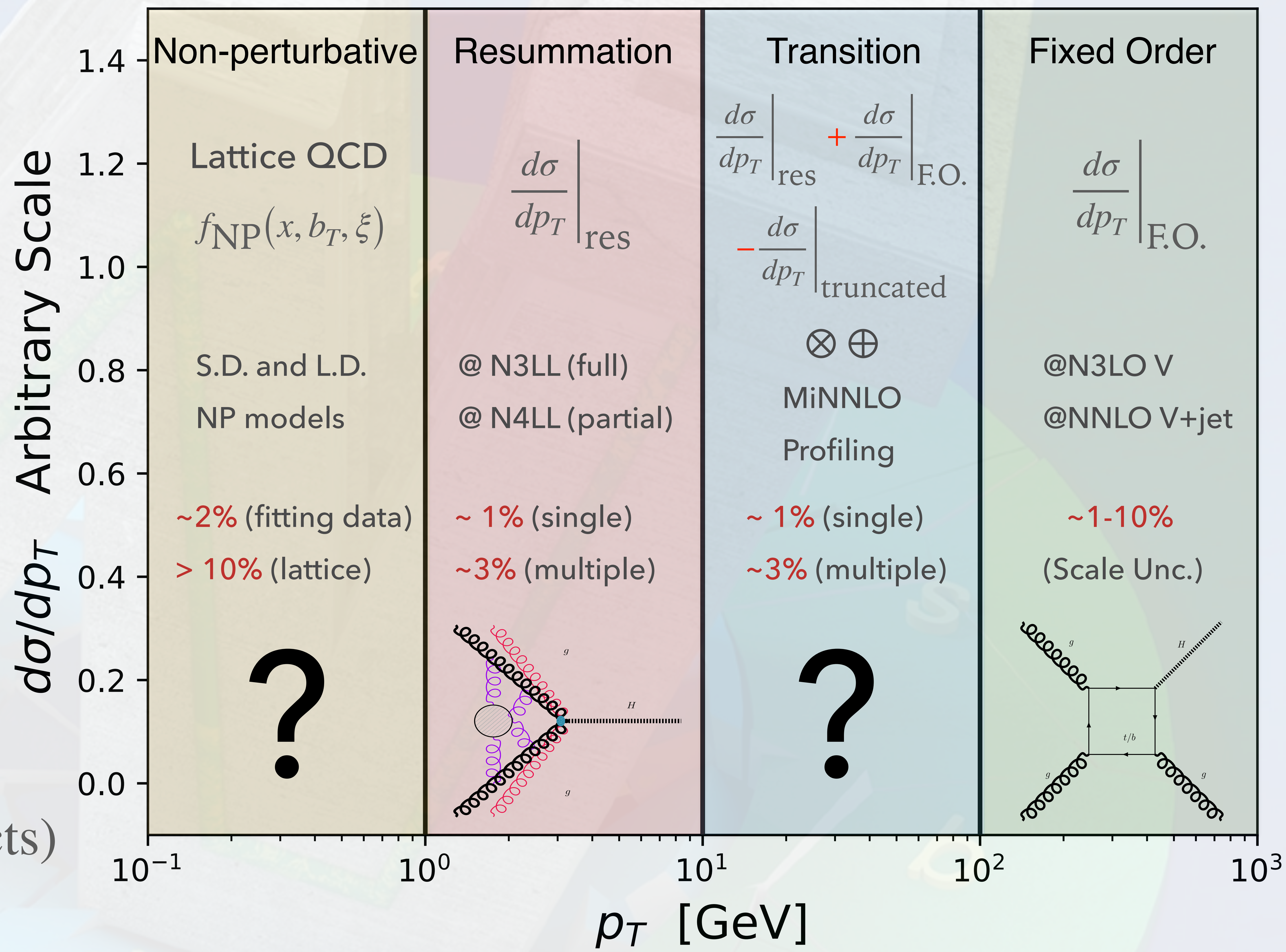

PRECISION PREDICTIONS IN CDFII

➤CDF II use ResBos to generate theory templates

➤NLO+NNLL accuracy for W/Z production

Balazs, Brock, Landry, Nadolsky and Yuan`97 to`03

➤CSS factorisation and resummation of p_T in b space:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}^2\vec{p}_T\,\mathrm{d}y\,\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \sigma_0 \int \frac{\mathrm{d}^2 b}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} e^{-S(b)}$$

$$\times\, C \otimes f(x_1,\mu)\, C \otimes f(x_2,\mu) + Y(Q,\vec{p}_T,x_1,x_2,\mu_R,\mu_F)$$

Collins, Soper and Sterman`85

➤Non-perturbative effects at $\alpha_s(\Lambda)$ and large b:

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$$S(b) = S_{\mathrm{NP}} S_{\mathrm{Pert}}\,,$$

Collins and Soper `77

$$S_{\mathrm{Pert}}(b) = \int_{C_1^2/(b^*)^2}^{C_2^2 Q^2} \frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2 Q^2}{\bar{\mu}^2}\right) A(\bar{\mu},C_1) + B(\bar{\mu},C_1,C_2)\right]$$

$$S_{\mathrm{NP}} = \left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln(100 x_1 x_2)\right] b^2$$

S_{NP} assumes the BLNY functional form

Brock, Landry, Nadolsky and Yuan `02

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Brock, Landry, Nadolsky and Yuan `02

➤Use data driven method:

Fix	g1	g2	g3	α_s
p_T^Z	Global fit `03	CDFII fit	Global fit `03	CDFII fit
p_T^Z/p_T^W			Global fit `03	

Global fit by Brock, Landry, Nadolsky and Yuan `03

$m_T^W \sim 0.7$ MeV, $p_T^l \sim 2.3$ MeV, $p_T^\nu \sim 0.9$ MeV

CDF supplementary materials `22

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CDF supplementary materials `22

➤Scale uncertainty of p_T^Z/p_T^W by DYQT

Bozzi, Catani, Ferrera, de Florian, Grazzini `09 `11

$m_T^W \sim 3.5$ MeV, $p_T^l \sim 10.1$ MeV, $p_T^\nu \sim 3.9$ MeV

Not included in final result CDF sm`22

PRECISION PREDICTIONS IN CDFII

- ResBos → ResBos2
  - NNLO+N3LL accuracy for W/Z production

    Isaacson Ph.D. thesis`17
  - Upgrade CSS formalism to N3LL
  - Rescale NLO to NNLO from MCFM:

    Campbell, Ellis and Giele `15

$$\frac{d\sigma_{NLO}}{dp_T dy dQ} \to K_{\frac{NNLO}{NLO}}(p_T, y, Q)\frac{d\sigma_{NLO}}{dp_T dy dQ}$$

  - Dependence of angular coefficients recently included with more rescaling: $\frac{d\sigma}{d\cos\theta d\phi} \sim$

$$(1+\cos^2\theta) + \frac{1}{2}A_0\left(1-3\cos^2\theta\right) + A_1 \sin 2\theta\cos\phi + A_2 \sin^2\theta\cos 2\phi$$
$$+A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi$$

Isaacson, Fu and Yuan`22

PRECISION PREDICTIONS IN CDFII

- ResBos → ResBos2
  - NNLO+N3LL accuracy for W/Z production

    Isaacson Ph.D. thesis`17
  - Upgrade CSS formalism to N3LL
  - Rescale NLO to NNLO from MCFM:

    Campbell, Ellis and Giele `15

$$\frac{d\sigma_{NLO}}{dp_T dy dQ} \rightarrow K_{\frac{NNLO}{NLO}}(p_T, y, Q)\frac{d\sigma_{NLO}}{dp_T dy dQ}$$

  - Dependence of angular coefficients recently included with more rescaling: $\frac{d\sigma}{d\cos\theta d\phi} \sim$

$$(1+\cos^2\theta) + \frac{1}{2}A_0\left(1-3\cos^2\theta\right) + A_1 \sin 2\theta\cos\phi + A_2\sin^2\theta\cos 2\phi$$
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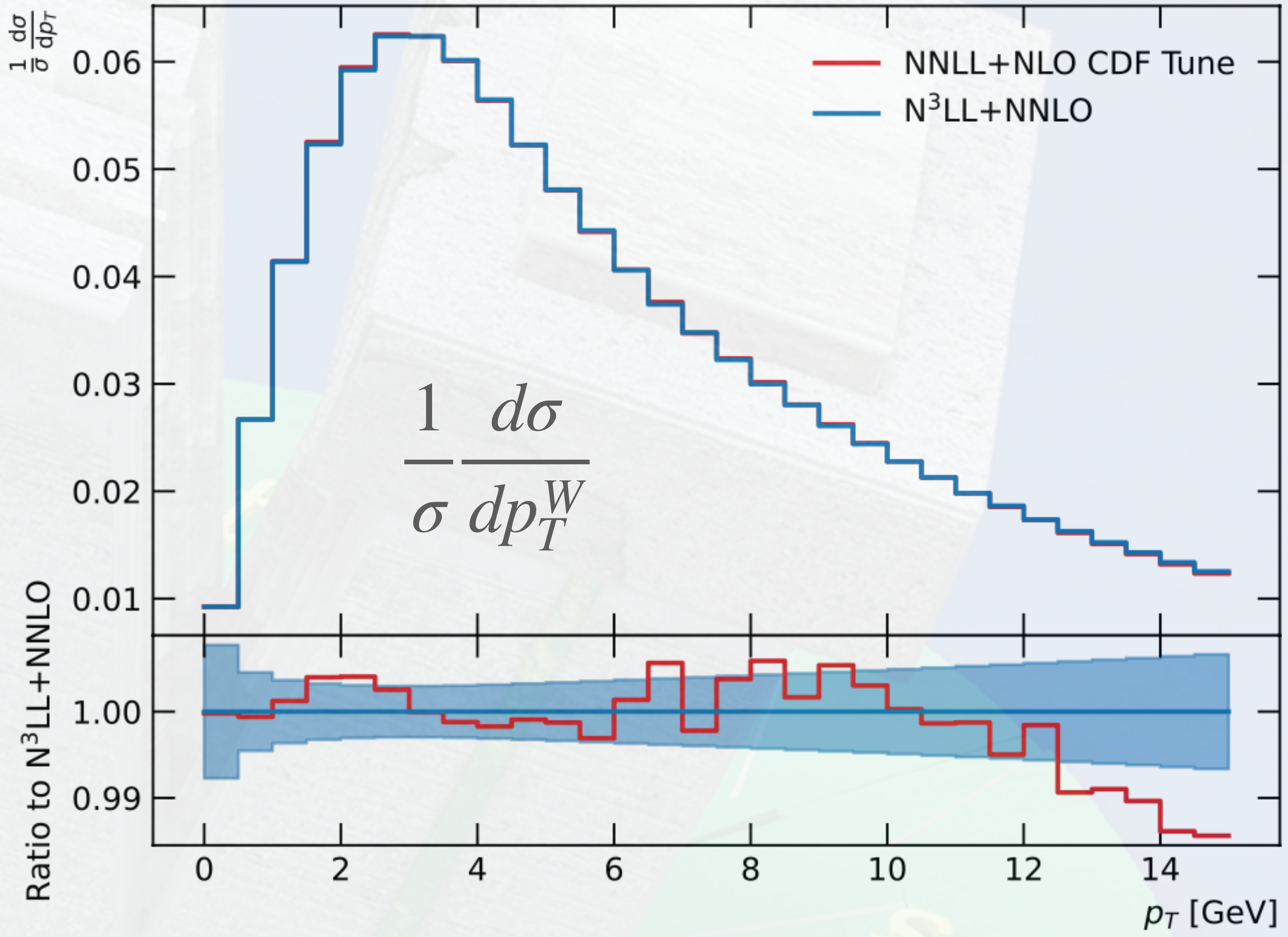
Isaacson, Fu and Yuan`22



- Pseudo data: NNLO+N3LL p_T^Z with global fit
- Fit g2, α_s in NLO+NNLL p_T^Z to pseudo data
- Use fitted g2, α_s in NLO+NNLL W templates

PRECISION PREDICTIONS IN CDFII

- ResBos → ResBos2
  - NNLO+N3LL accuracy for W/Z production
    Isaacson Ph.D. thesis`17
  - Upgrade CSS formalism to N3LL
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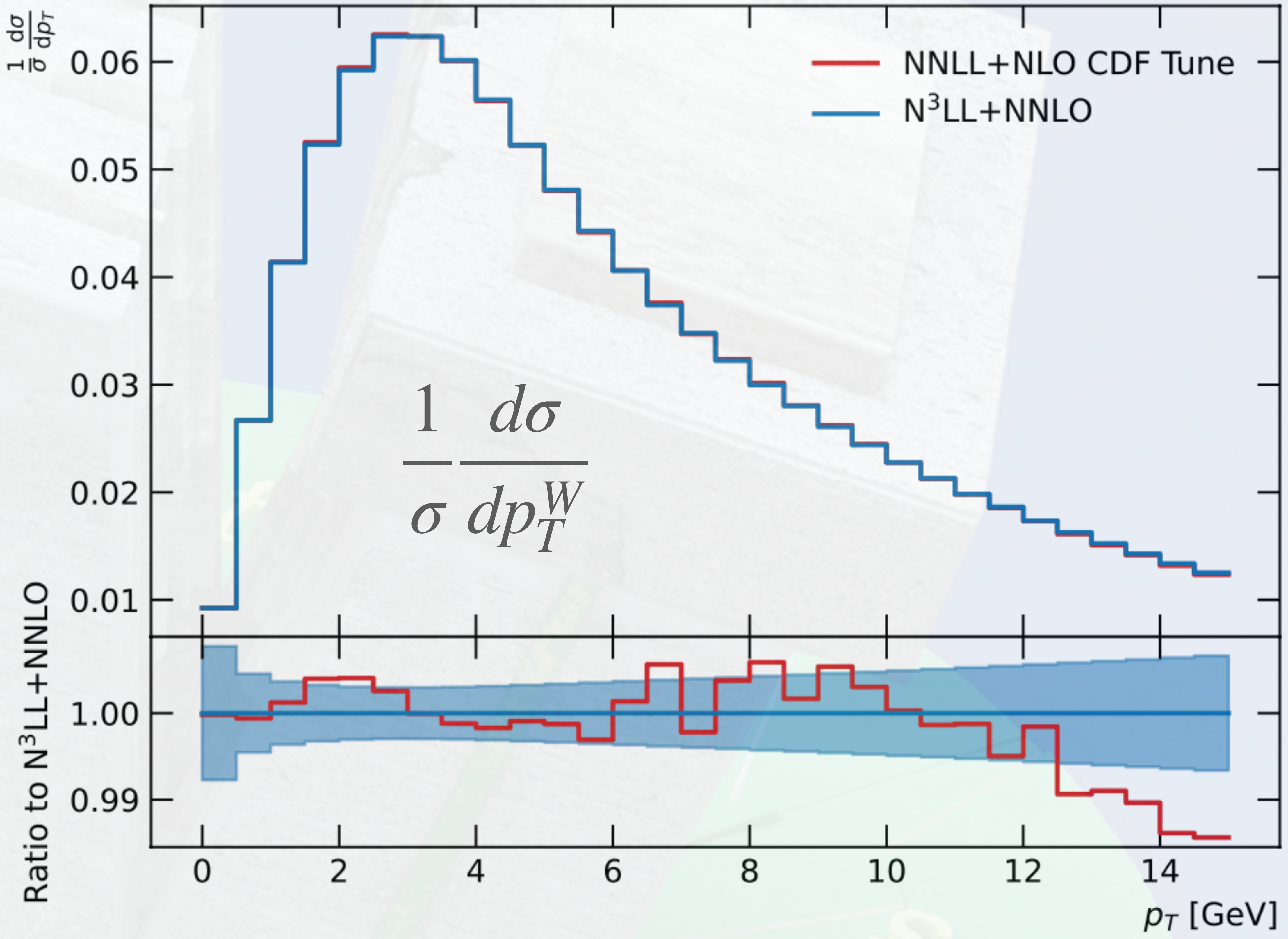
$$+A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi$$

Isaacson, Fu and Yuan`22



We determine that the data-driven techniques used by CDF capture most of the higher order corrections, and using higher order corrections would result in a decrease in the value reported by CDF by at most 10 MeV

PRECISION PREDICTIONS IN CDFII

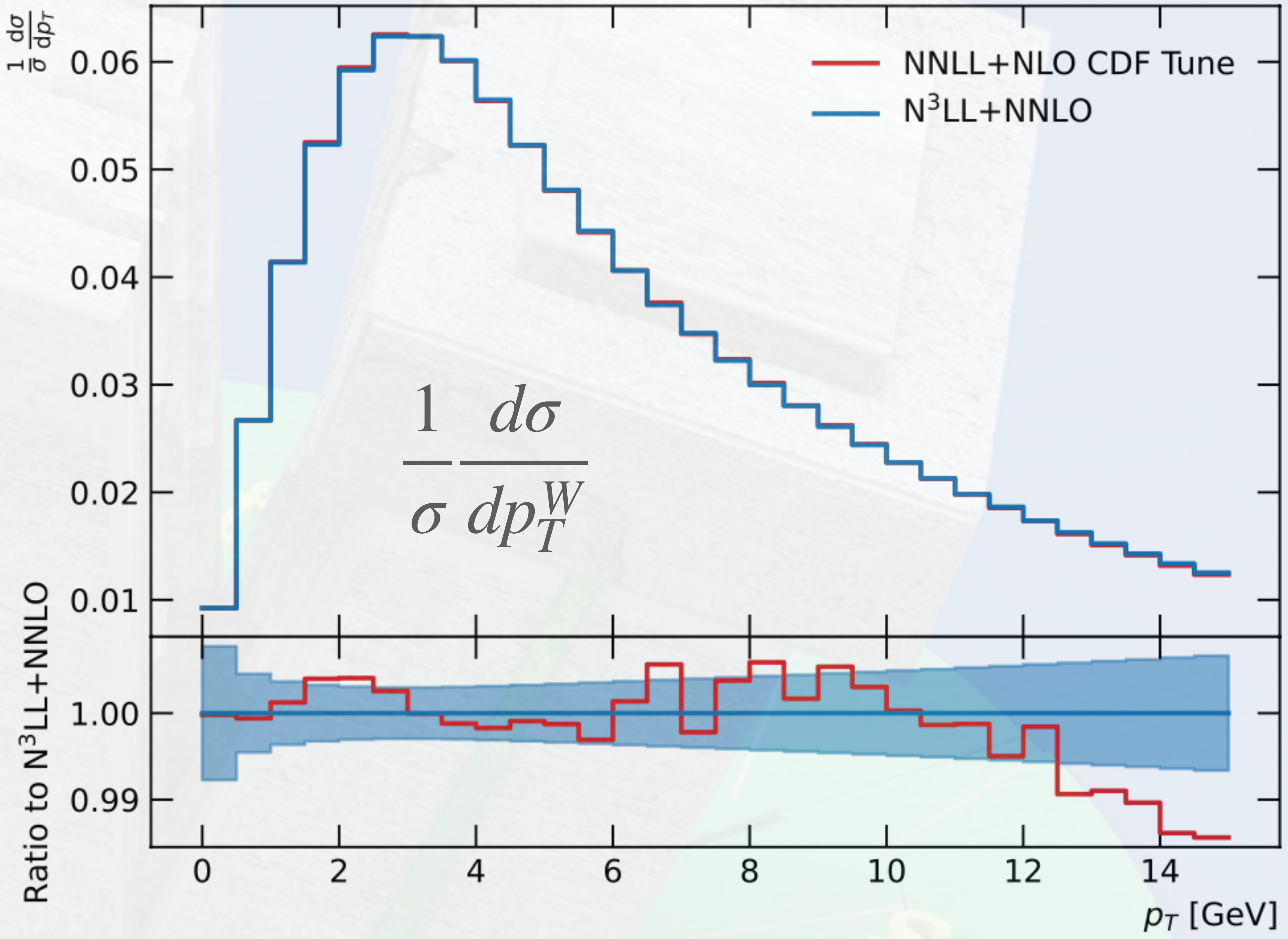
- ➤ResBos → ResBos2
  - ➤NNLO+N3LL accuracy for W/Z production
    Isaacson Ph.D. thesis`17
  - ➤Upgrade CSS formalism to N3LL
  - ➤Rescale NLO to NNLO from MCFM:
    Campbell, Ellis and Giele `15

$$\frac{d\sigma_{NLO}}{dp_T dy dQ} \to K_{\frac{NNLO}{NLO}}(p_T, y, Q)\frac{d\sigma_{NLO}}{dp_T dy dQ}$$

  - ➤Dependence of angular coefficients recently included with more rescaling: $\frac{d\sigma}{d\cos\theta d\phi} \sim$

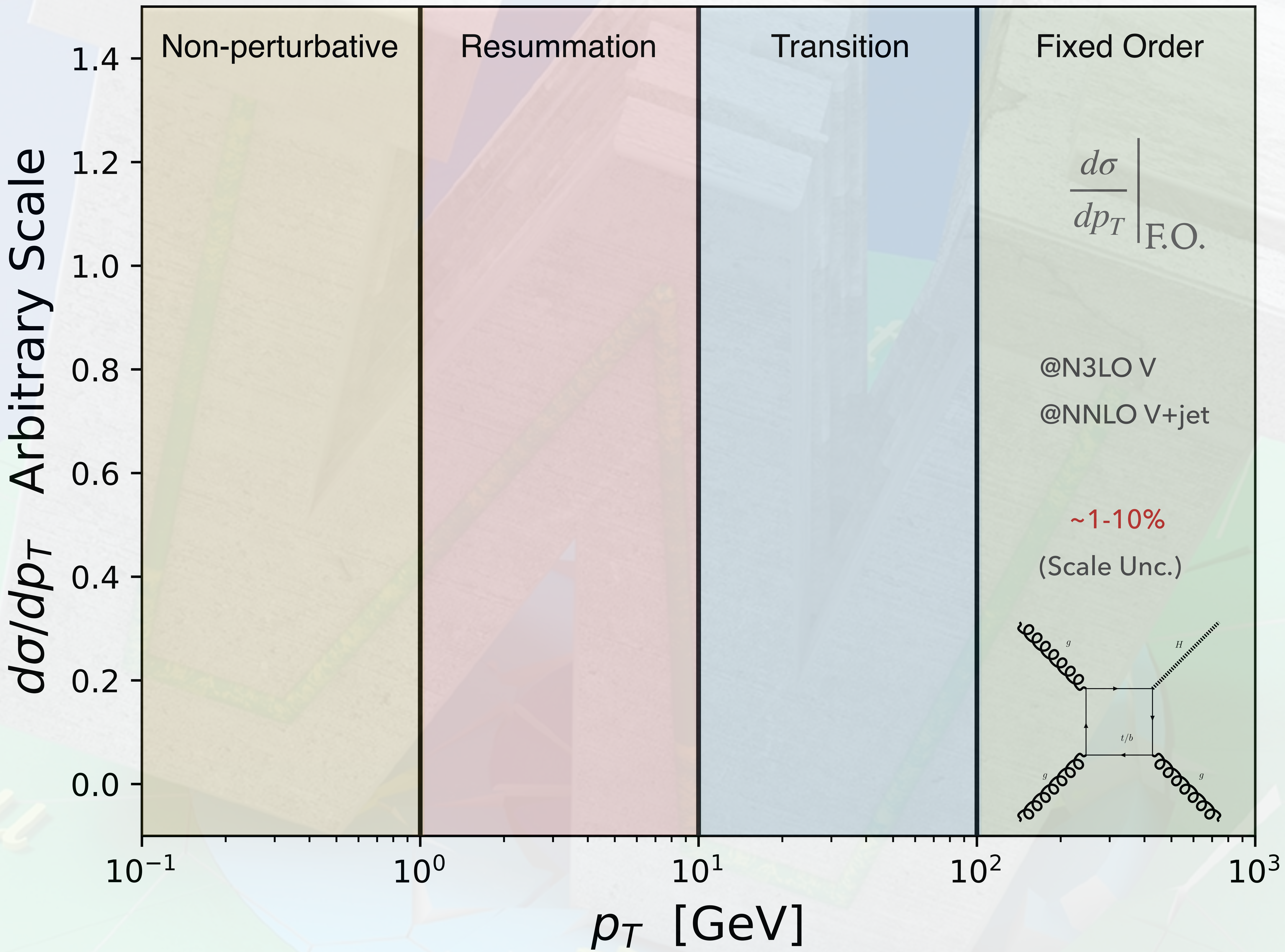
$$(1+\cos^2\theta) + \frac{1}{2}A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi + A_2\sin^2\theta\cos 2\phi$$
$$+A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi$$

Isaacson, Fu and Yuan`22

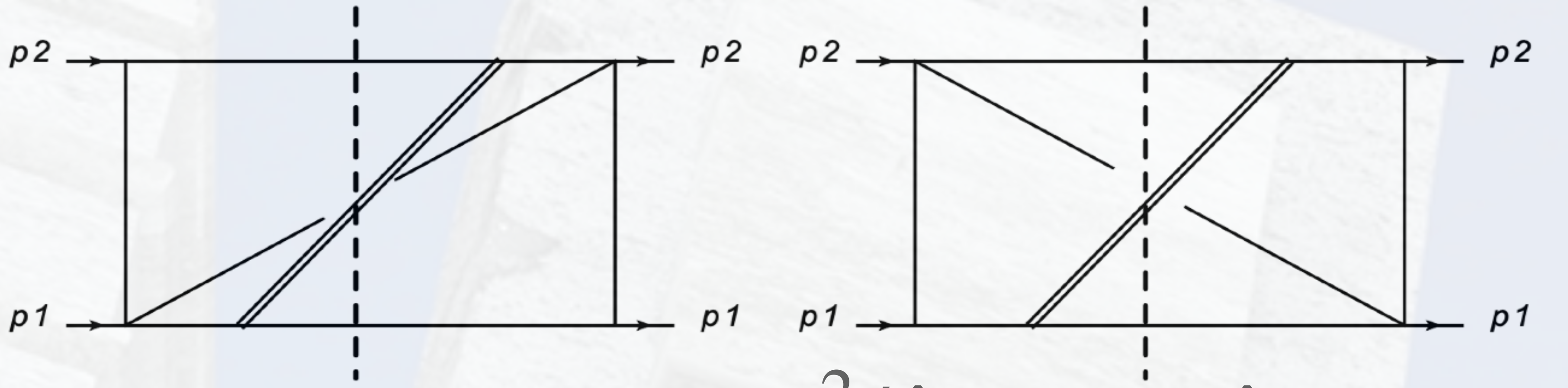


Is it safe to fit perturbative corrections into non-perturbative effect ?

STATE-OF-THE-ART PREDICTIONS FOR σ_{N^3LO}



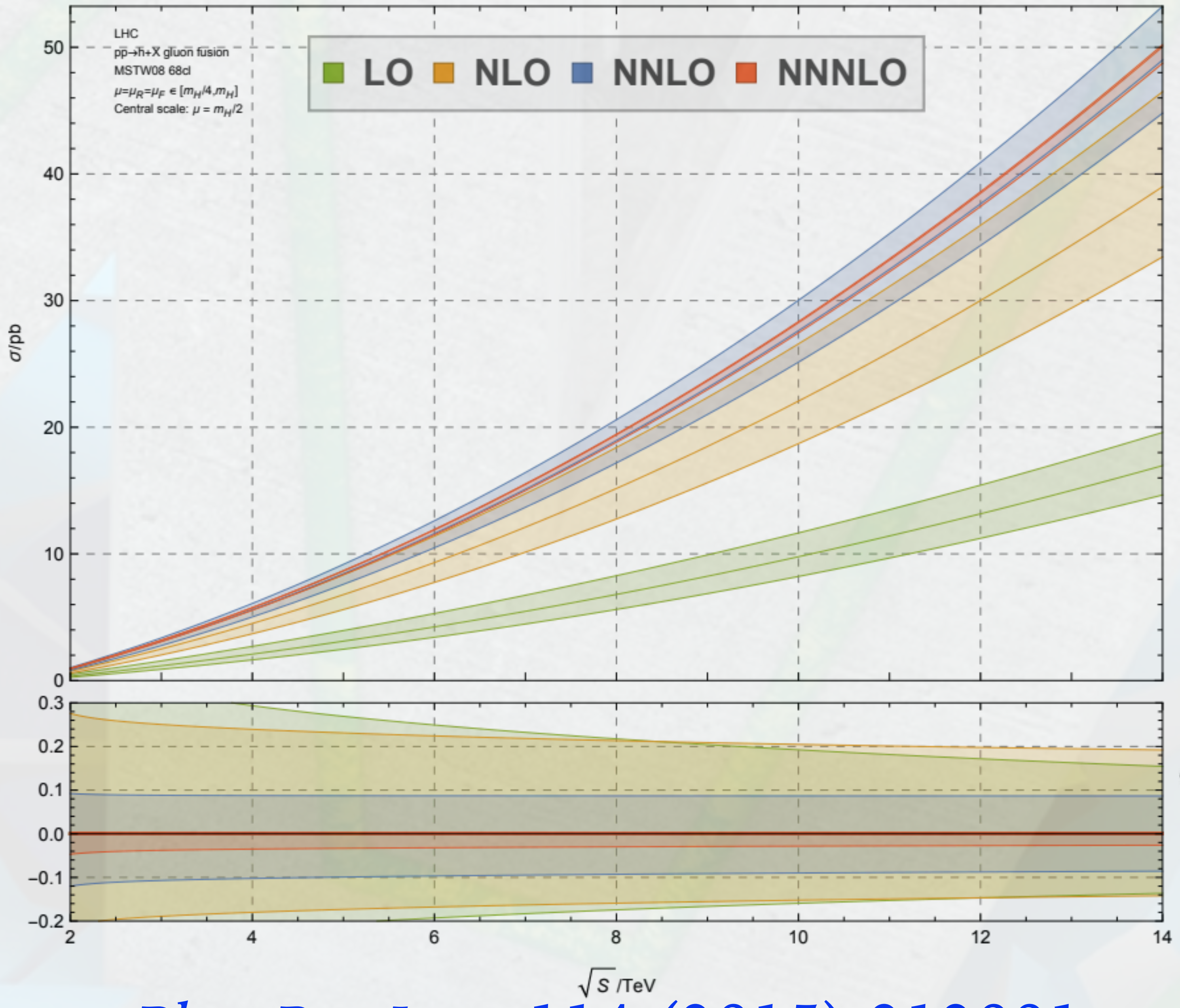
STATE-OF-THE-ART PREDICTIONS FOR σ_{N^3LO}



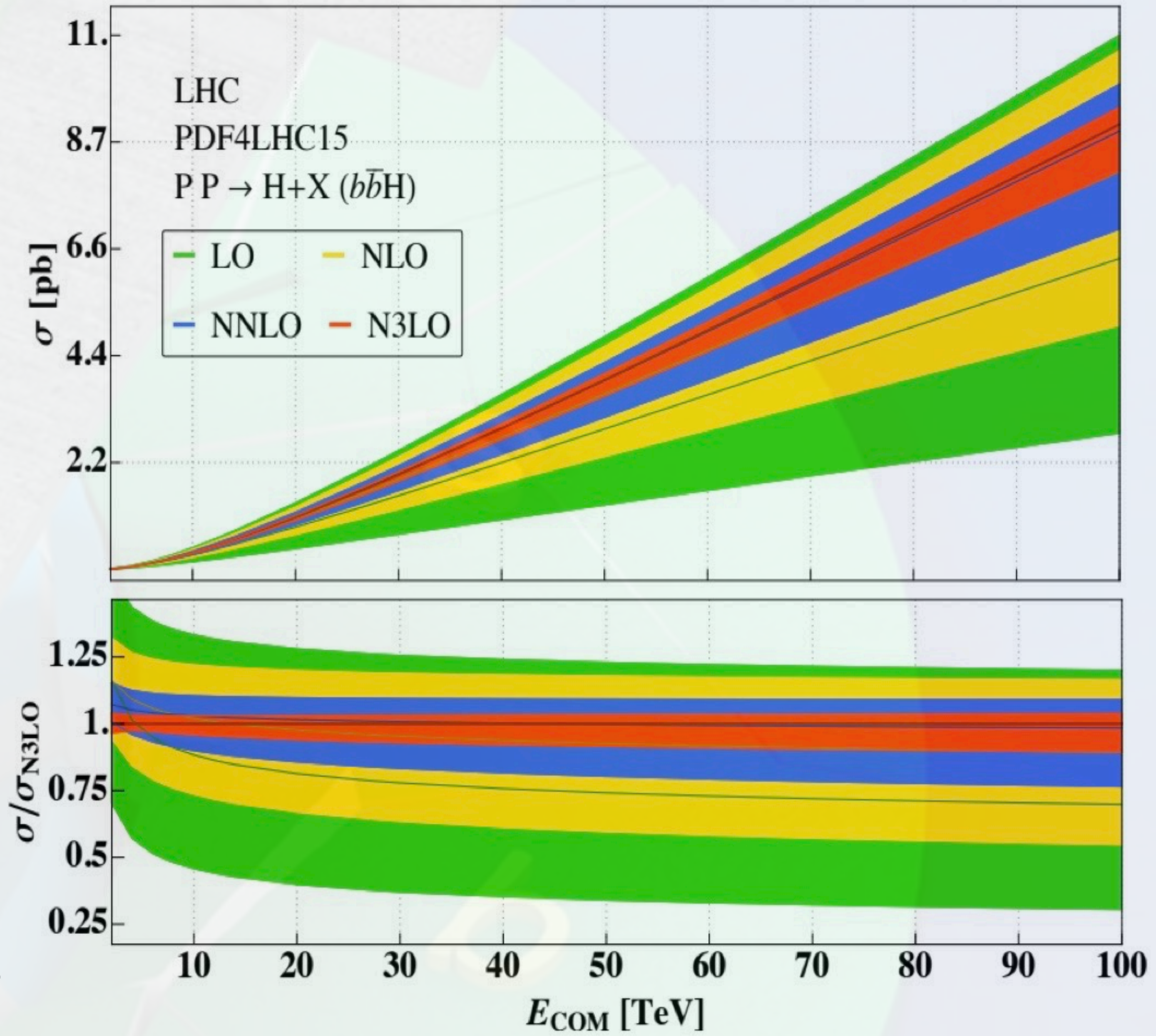
- Assemble each $\hat{\sigma}_{ab}(x_a, x_b)$ at N3LO
  - Integration of QCD radiation with unitarity cuts
  - Standard treatment of multi-loop calculations except elliptic integrals with $\tau = m^2/\hat{s}$ where $\hat{s} = x_a x_b s$
  - Use threshold expansion at different region of τ and truncate at sufficiently high orders ($\mathcal{O}(100)$). (Mistlberger `18)
  - Use generalised power series ansatz to test the approximation and match coeff. of overlapping regions.

Not exact analytical solution of elliptic integrals but numerically precise enough for phenomenology

- Application of ggF Higgs production
  - Remarkable precision of the first N3LO XS (Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger `15 to `18)
  - Available in public code iHixs 2 (Dulat, Lazopoulos, Mistlberger `18)
  - Further application to bbF Higgs (Dulat, Lazopoulos, Mistlberger `19)
- VBF to Higgs and HH using DIS structure function (Dreyer, Karlberg `17 `19)



Phys.Rev.Lett. 114 (2015) 212001

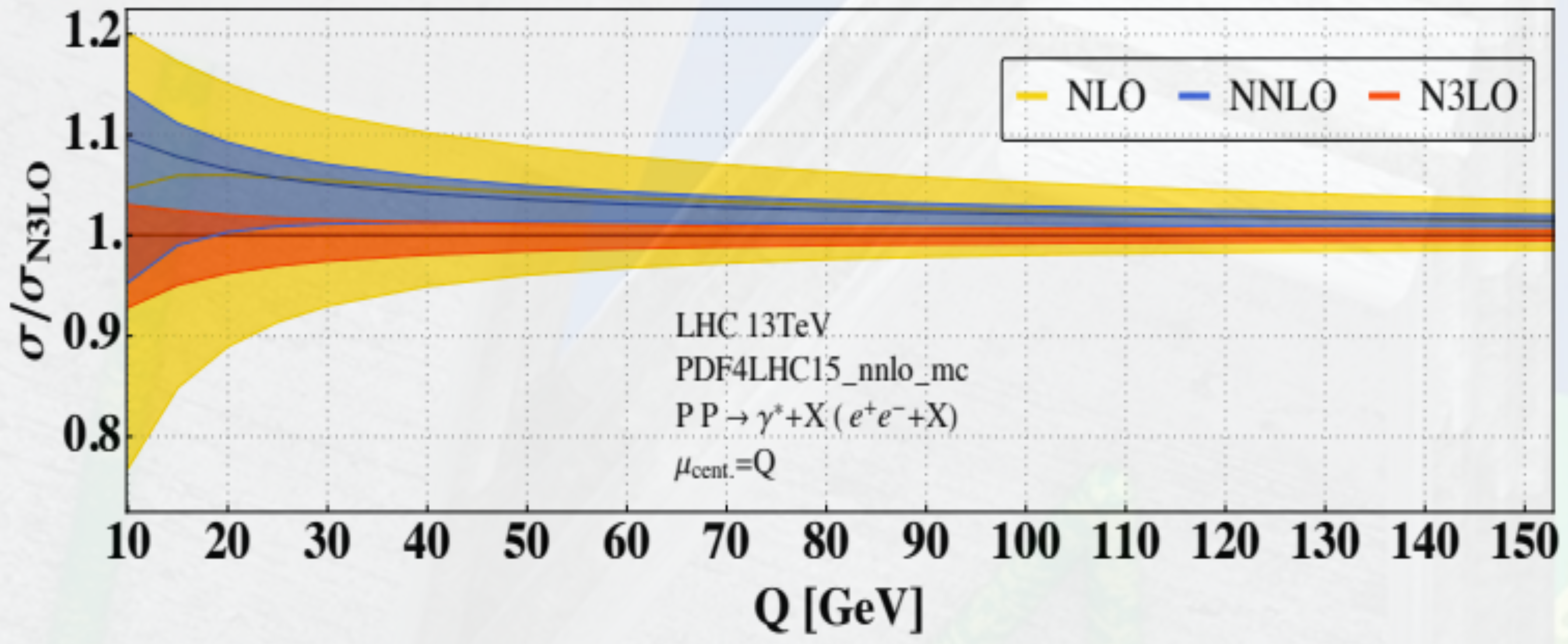


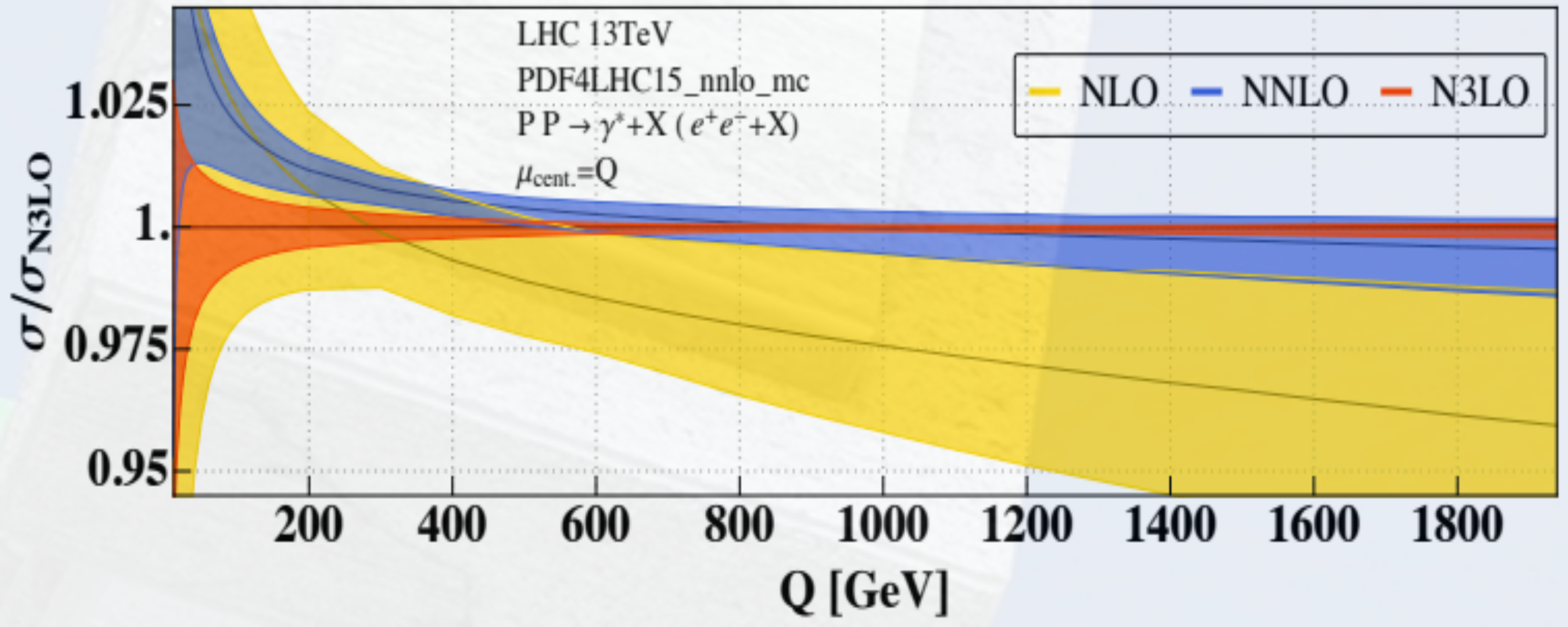
Phys.Rev.Lett. 125 (2020) 5, 051804

STATE-OF-THE-ART PREDICTIONS FOR σ_{N^3LO}

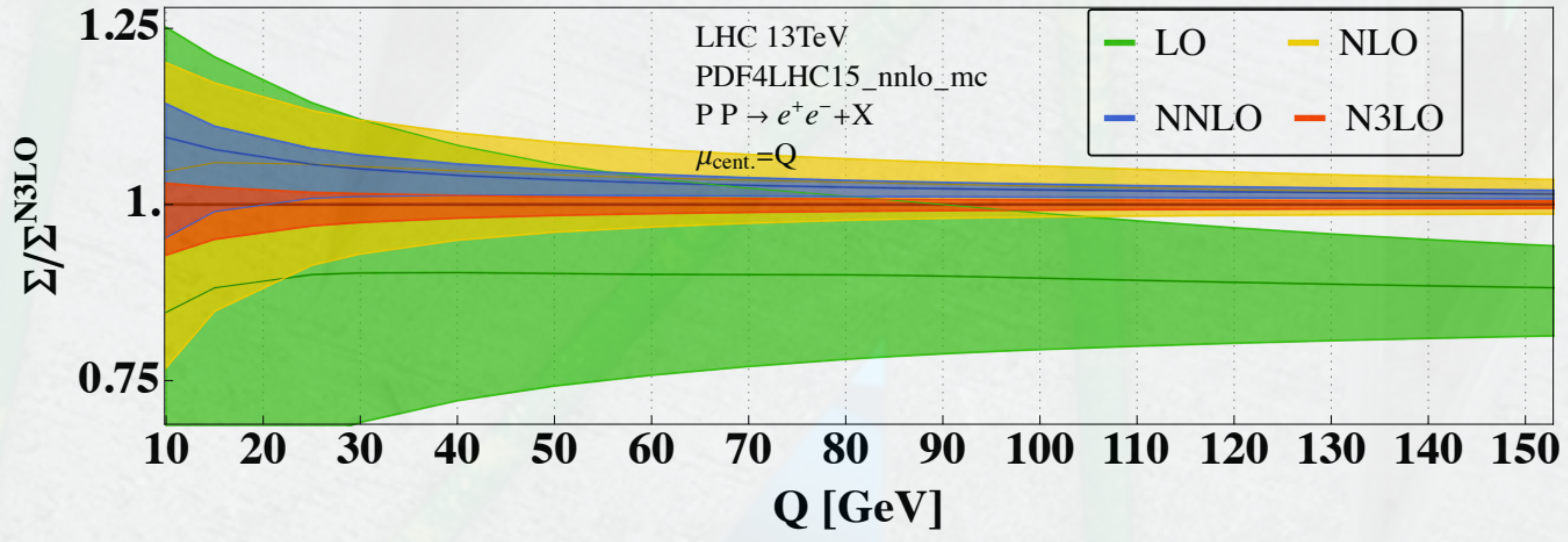
➤Application to 2 → 1 colour singlet production at the LHC (Duhr, Dulat, Mistlberger `20 `21)

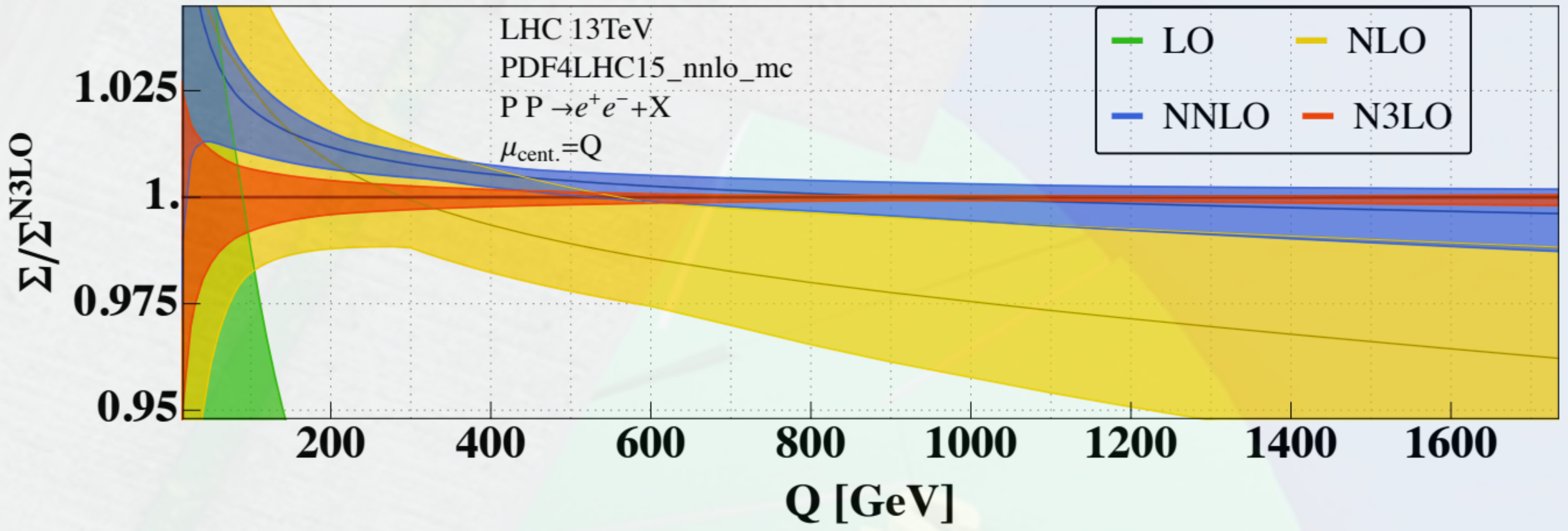
$pp \to \gamma^*$



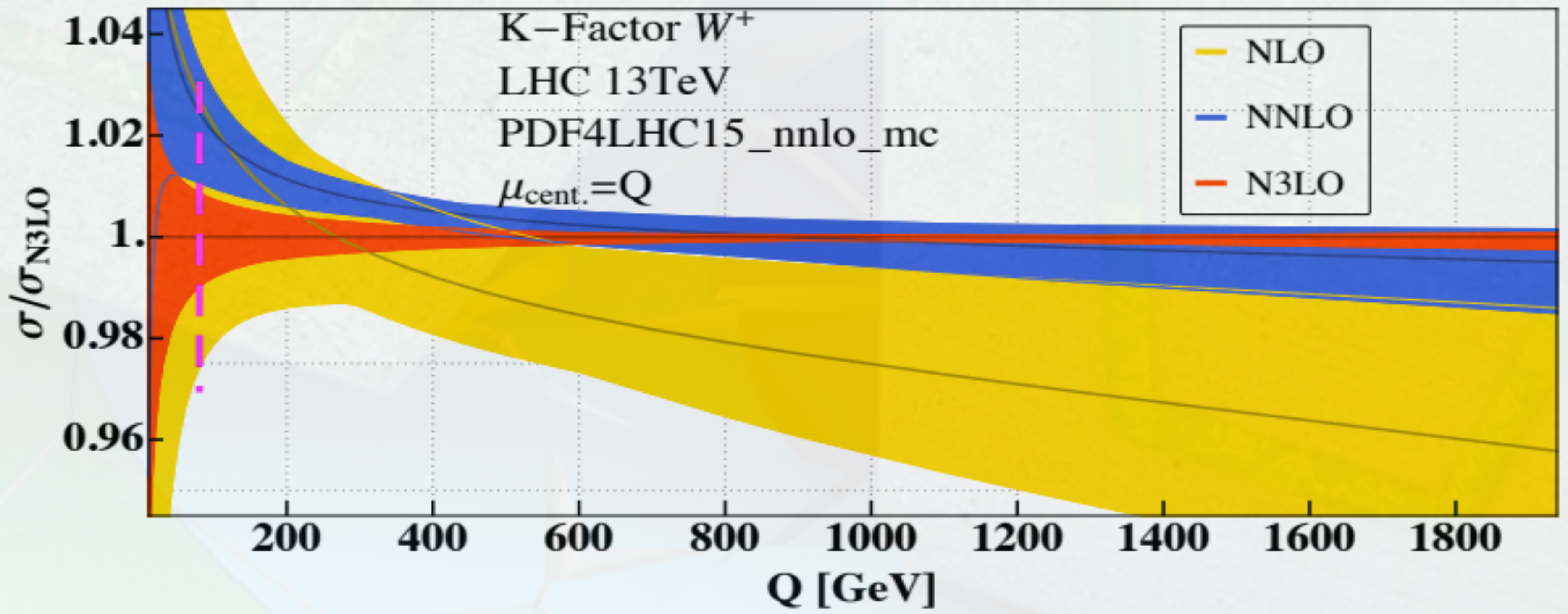


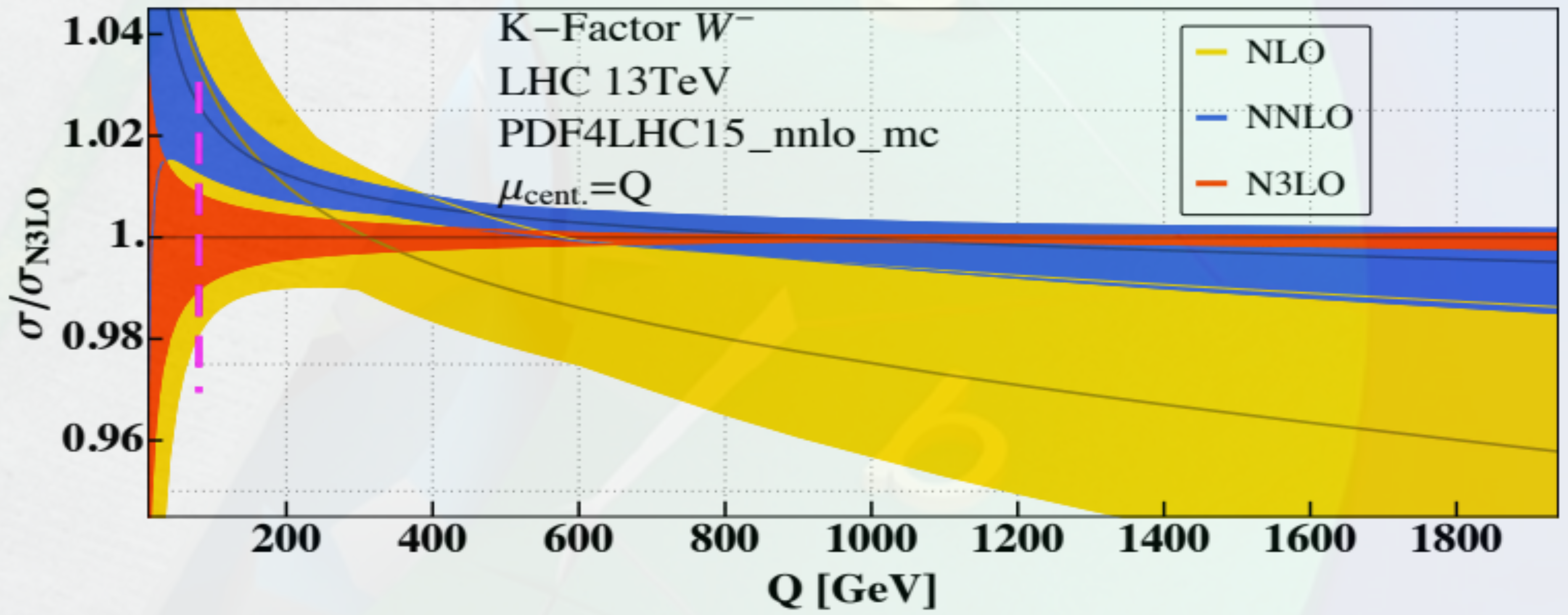
$pp \to Z/\gamma^* \to e^+e^-$





$pp \to W^\pm$





GOING DIFFERENTIAL

$\sigma_{tot}^{pp \to H}$

$d\sigma^{pp \to H}$

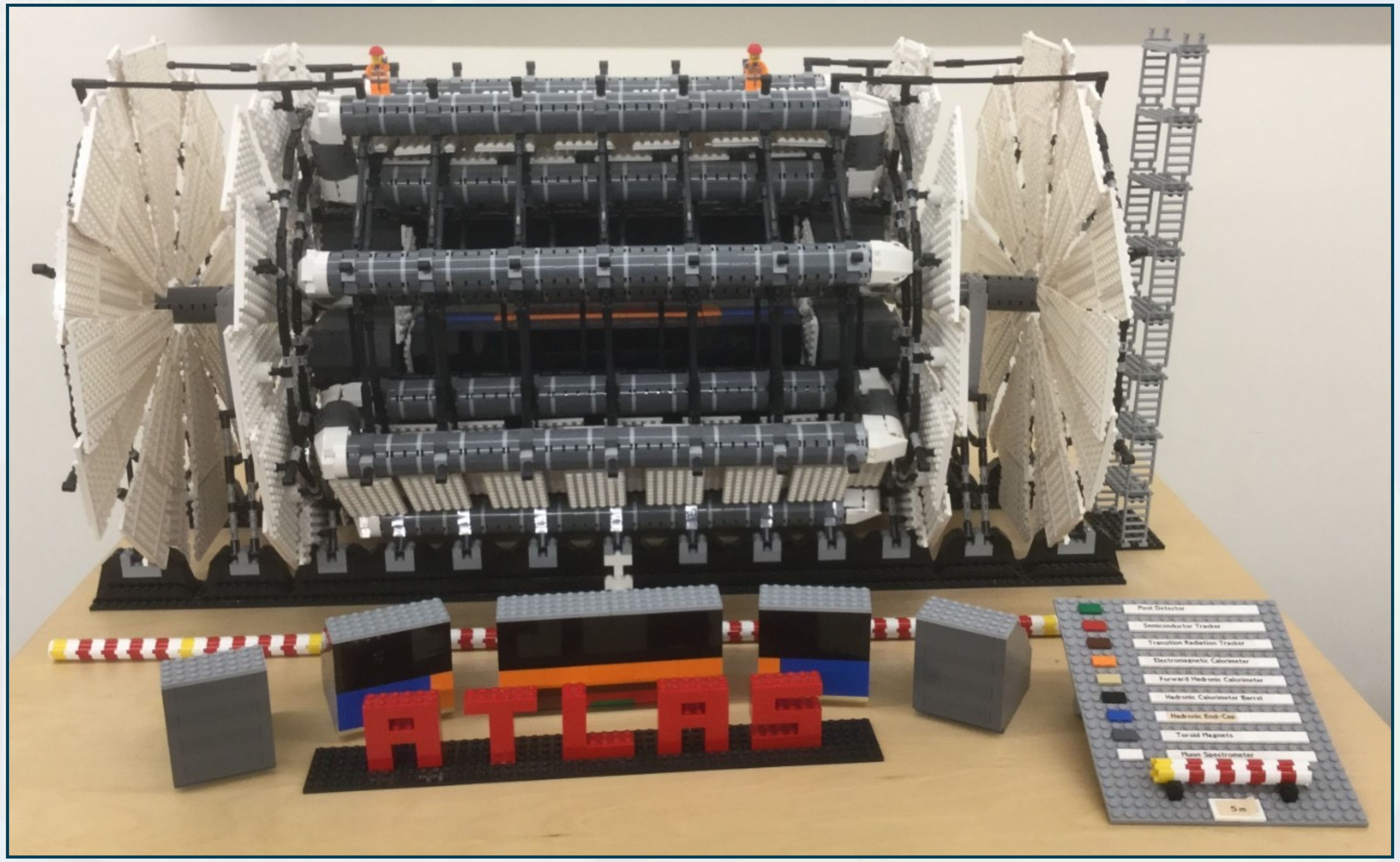
- What is the probability of producing a Higgs boson?

… where the Higgs decays into a pair of photons, $H \to \gamma\gamma$, and the leading and sub-leading photon have a transverse momentum that is larger than 35% and 25% of the Higgs boson mass, respectively, and are produced within the rapidity interval $|y_\gamma| < 2.37$, where the barrel-endcap region $1.37 < |y_\gamma| < 1.52$ is excluded. Photons are further required to be isolated from additional QCD activity by requiring that the scalar sum of the transverse momenta of hadrons in a cone of $\Delta R = 0.2$ around the photons is less than $5\,\%$ of the photon transverse energy E_T.

Measurements are done within a fiducial volume:

want direct comparison

(extrapolation ⇝ source of uncertainties)



STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

- Differential N3LO accuracy
  - Projection to Born

$$\frac{d\sigma^F_{N^kLO}}{d\mathcal{O}} = \left(\frac{d\sigma^{F+jet}_{N^{k-1}LO}}{d\mathcal{O}} - \frac{d\sigma^{F+jet}_{N^{k-1}LO}}{d\tilde{\mathcal{O}}}\right) + \frac{d\sigma^F_{N^kLO}}{d\tilde{\mathcal{O}}}$$

    - Jet production in DIS (NNLOJET) Currie, Gehrmann, Glover, Huss, Niehues `18
    - Higgs decay to $b\bar{b}$ (MCFM) Mondini, Schiavi, Williams `19
    - Higgs production via ggF (RapidiX+NNLOJET) XC, Gehrmann, Glover, Huss, Mistlberger, Pelloni `21
  - qT slicing

$$d\sigma^F_{N^kLO} = \mathcal{H}^F_{N^kLO} \otimes d\sigma^F_{LO}\Big|_{\delta(\tau)} + \left[d\sigma^{F+jet}_{N^{k-1}LO} - d\sigma^{F\ CT}_{N^kLO}\right]_{\tau>\tau_{cut}} + \mathcal{O}(\tau^2_{cut}/Q^2)$$

    - Higgs production via ggF (HN3LO+NNLOJET) Cieri, XC, Gehrmann, Glover, Huss `18
    - Higgs pair production via ggF (with modified iHixs2) Chen, Li, Shuo, Wang `19
    - Drell-Yan production (NNLOJET) XC, Gehrmann, Glover, Huss, Yang, Zhu `21 `22
- Combined with resummation (N3LL at small qT)
  - Drell-Yan production (DYTurbo) Camarda, Cieri, Ferrera `21 (RadISH+NNLOJET) XC, Gehrmann, Glover, Huss, Monni, Re, et. al. `18 `19 `22 (CuTe-MCFM with partial N4LL) Neumann and Campbell `22
  - Higgs production via ggF (SCET+NNLOJET) XC, Gehrmann et. al.`18 (SCETlib) Billis, Dehnadi, et. al.`21

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

- Differential N3LO predictions for neutral current production
  - Fully differential N3LO Drell-Yan production (XC, Gehrmann, Glover, Huss, Yang, Zhu `21)
  - Apply qt-slicing at N3LO with SCET factorisation and expand to N3LO:

$$\frac{d^3\sigma}{dQ^2 d^2\vec{q}_T dy} = \int \frac{d^2 b_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} \sum_q \sigma_{\rm LO}^{\gamma^*} H_{q\bar{q}} \Big[\sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}\left(z_1, b_T^2, \mu\right) f_{k/h_1}(x_1/z_1, \mu)$$
$$\times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}j}\left(z_2, b_T^2, \mu\right) f_{j/h_2}(x_2/z_2, \mu) \mathcal{S}\left(b_\perp, \mu\right) + (q \leftrightarrow \bar{q}) \Big] + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

  - All factorised functions are recently known up to N3LO:
    1) 3-loop hard function $H_{q\bar{q}}^{(3)}$ (Gehrmann, Glover, Huber, Ikizlerli, Studerus `10)
    2) Transverse-momentum-dependent (TMD) soft function $S(b_\perp, \mu)$ at α_s^3 (Li, Zhu `16)
    3) Matching kernel of TMD beam function I_{qk} at α_s^3 (Luo, Yang, Zhu, Zhu `19, Ebert, Mistlberger, Vita `20)
  - Apply qt cut to factorise N3LO contribution into two parts:

$$\mathrm{d}\sigma^{\gamma^*}_{N^3LO} = [\mathscr{H}^{\gamma^*} \otimes \mathrm{d}\sigma^{\gamma^*}]_{N^3LO}\Big|_{\delta(p_{T,\gamma^*})} + \left[\mathrm{d}\sigma^{\gamma^*+jet}_{NNLO} - \mathrm{d}\sigma^{\gamma^*\ CT}_{N^3LO}\right]_{p_{T,\gamma^*} > q_T^{cut}} + \mathcal{O}((q_T^{cut}/Q)^2)$$

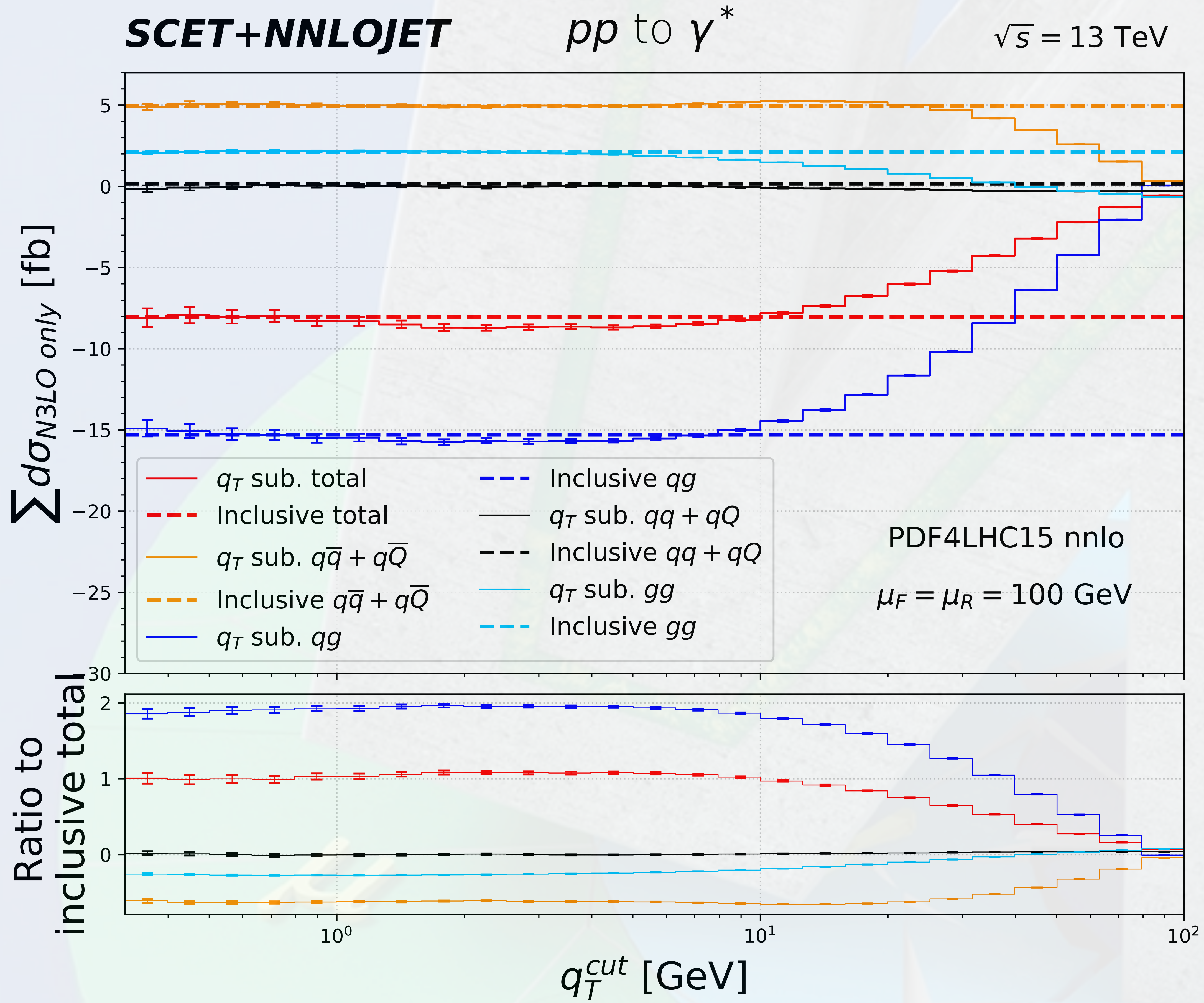
STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

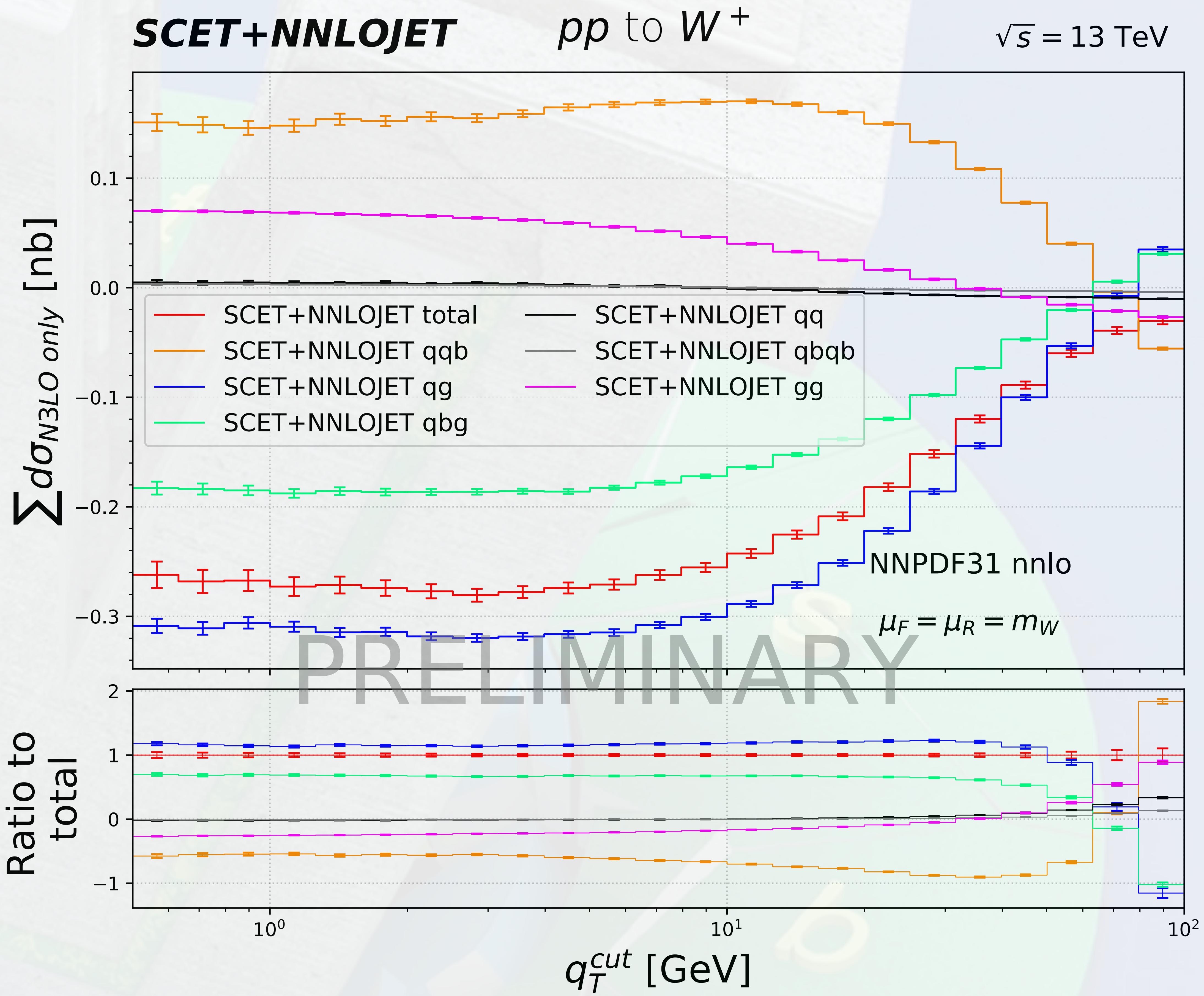
- Differential N3LO predictions for neutral and charged current production $\mathcal{O}(\alpha\alpha_s^3)$
  - Computational setup for $pp \to \gamma^* \to l^+l^-$
    (identical setup in the inclusive calculation by Durh, Dulat and Mistlberger in *Phys.Rev.Lett.* 125 (2020) 17, 172001)
    - Fix Q value for γ^* at 100 GeV (NNLO and N3LO scale variations deviate)
    - Use central value of PDF4LHC15_nnlo_mc as benchmark input
    - $\mu_R = \mu_F = 100$ GeV for central QCD scale and use 7-point variations for uncertainty estimation
    - Apply $p_{T,\gamma^*} > 0.25$ GeV constrain for NNLO $\gamma^* + Jet$ without jet definition
  - Computational setup for $pp \to W^\pm \to l^\pm\nu$
    - Dynamic QCD scale $\mu_R = \mu_F = m_{l\nu}$ with 7 variations and $m_{l\nu} \in [0, +\infty]$
    - Use NNPDF31_nnlo PDFs and $p_{T,l\nu} > 0.5$ GeV
  - Common setup
    - Consider LO decay with $m_e = m_\mu = 0$
    - $\alpha_s(m_Z) = 0.118$, G_μ EW-scheme with fixed α value

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

➤Differential N3LO predictions for neutral and charged current production

$$\sum \frac{\mathrm{d}\sigma^{V}_{N3LO}}{\mathrm{d}p_{T,V}} \equiv \sum \mathrm{d}\sigma^{V+jet}_{NNLO}/\mathrm{d}p_{T,V}\big|_{p_{T,V}>q_T^{cut}} + \sum \mathrm{d}\sigma^{V\,SCET}_{N^3LO}/\mathrm{d}p_{T,V}\big|_{p_{T,V}\in[0,q_T^{cut}]}$$



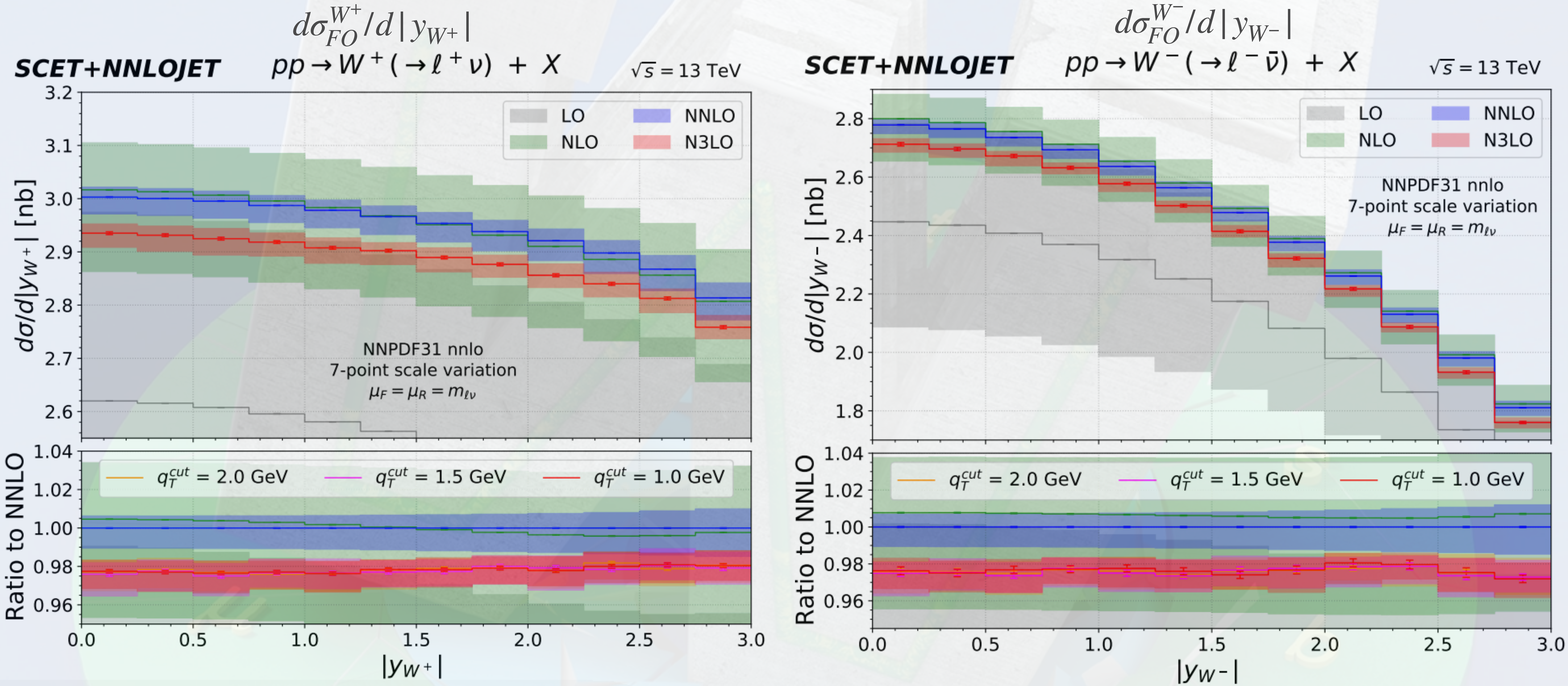


XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Rev.Lett.* 128 (2022) 5

XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

➤Differential N3LO predictions for charged current production

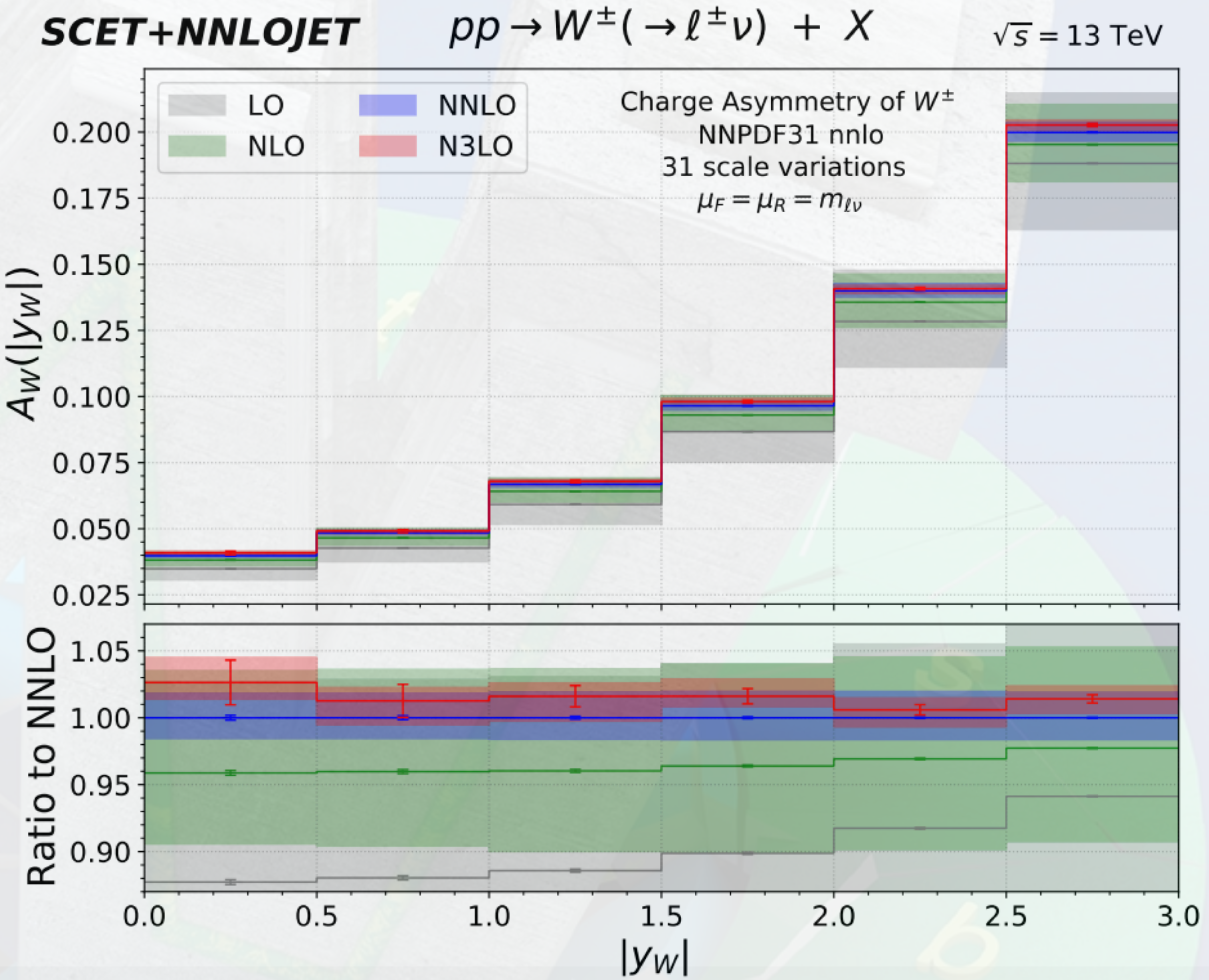


XC, Gehrmann, Glover, Huss, Yang, Zhu `22

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

➤Differential N3LO predictions for charged current production

$$A_W(|y_W|) = \frac{d\sigma/d|y_{W^+}| - d\sigma/d|y_{W^-}|}{d\sigma/d|y_{W^+}| + d\sigma/d|y_{W^-}|}$$



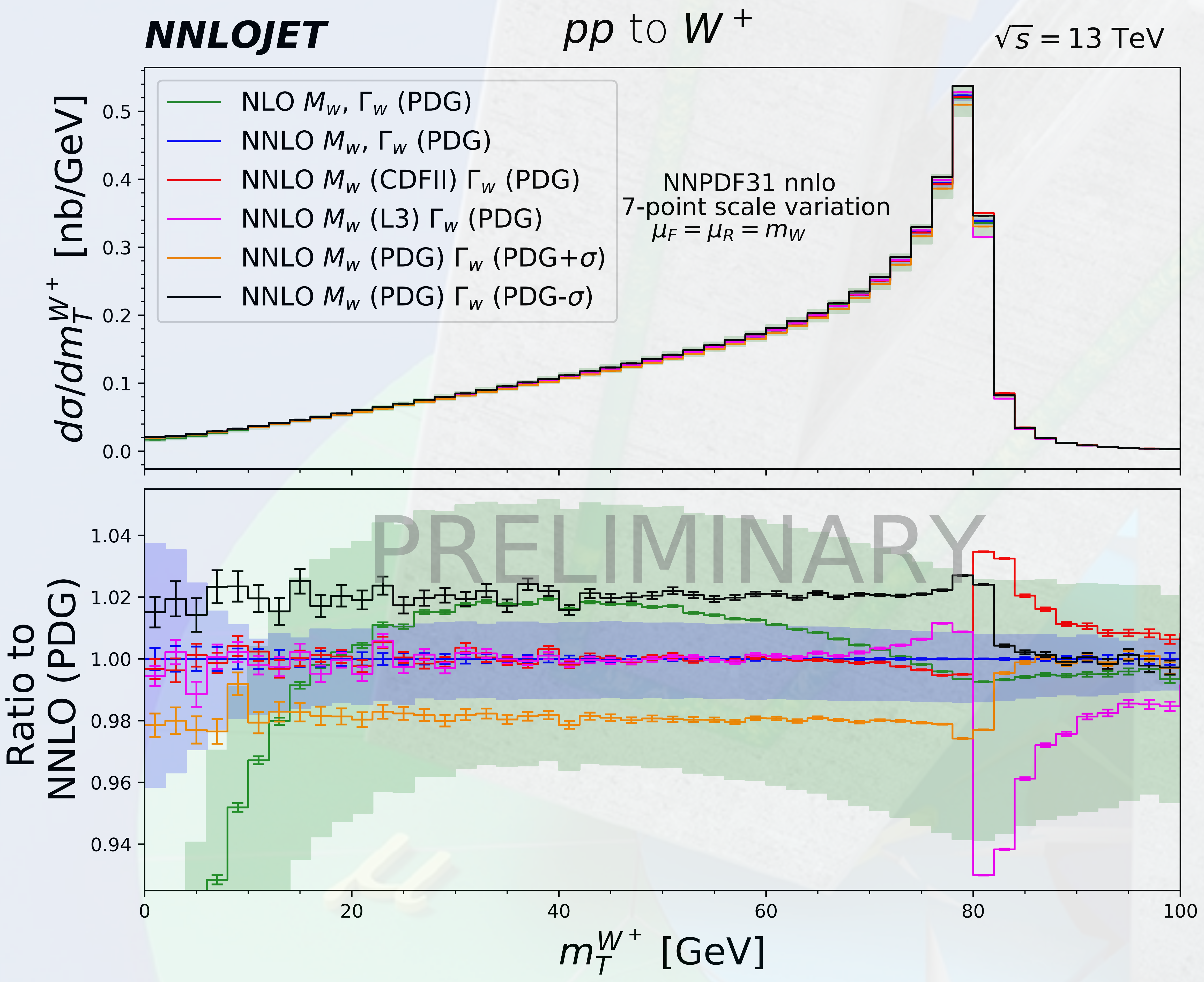
- ➤Scale dependence very similar in the ratio → Artificial small uncertainty if correlated
- ➤Here we use 31 combinations of uncorrelated scale variations (maybe over estimating theory error)

XC, Gehrmann, Glover, Huss, Yang, Zhu `22

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

➤Differential N3LO predictions for charged current production

$$m_T = (E_T^l + E_T^\nu)^2 - (\vec{p}_T^{\,l} + \vec{p}_T^{\,\nu})^2 = \sqrt{2E_T^l E_T^\nu(1 - cos\phi)}$$



Breit-Wigner form (running decay width):

$$\frac{1}{s^2 - m_W^2 + is^2\Gamma_W/m_W}$$

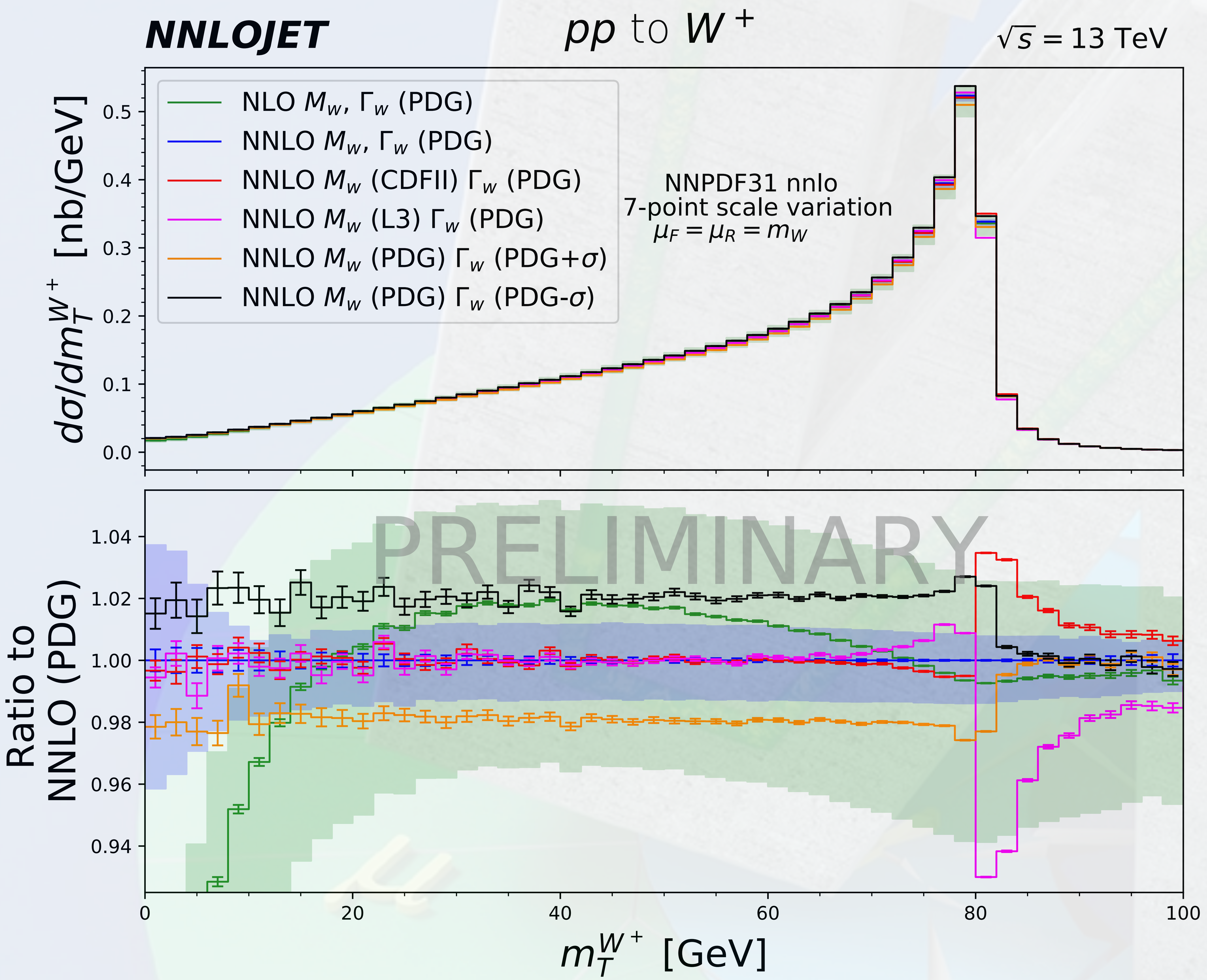
(MeV)	W mass	W width
PDG (2020)	80379±12	2085±42
CDFII	80433±9	2089.5 ± 0.6
L3	80270±55	2180 ± 14

XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation

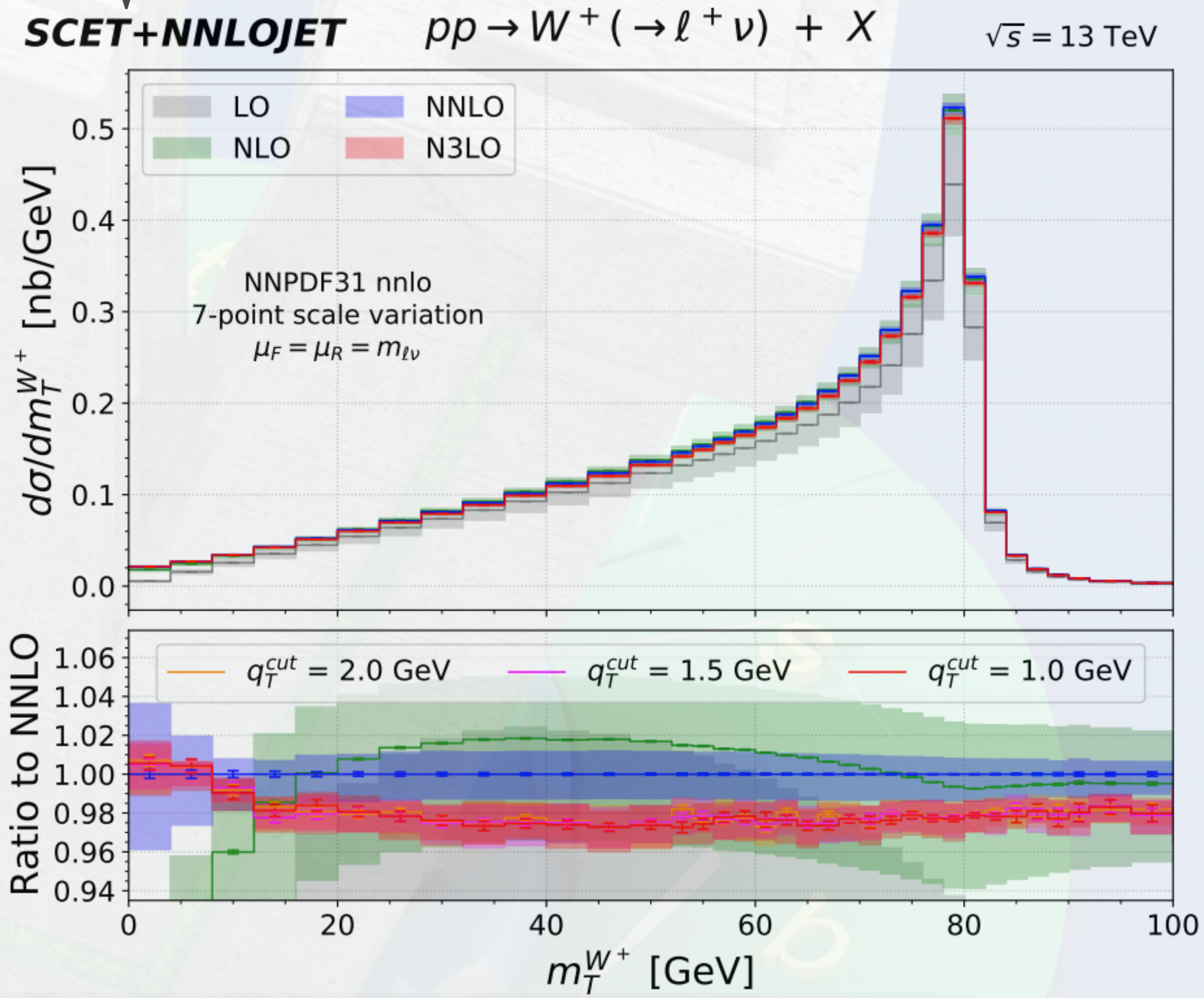
STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

➤Differential N3LO predictions for charged current production

$$m_T = (E_T^l + E_T^\nu)^2 - (\vec{p}_T^{\,l} + \vec{p}_T^{\,\nu})^2 = \sqrt{2E_T^l E_T^\nu(1 - cos\phi)}$$



XC, Gehrmann, Glover, Huss, Yang, Zhu in preparation

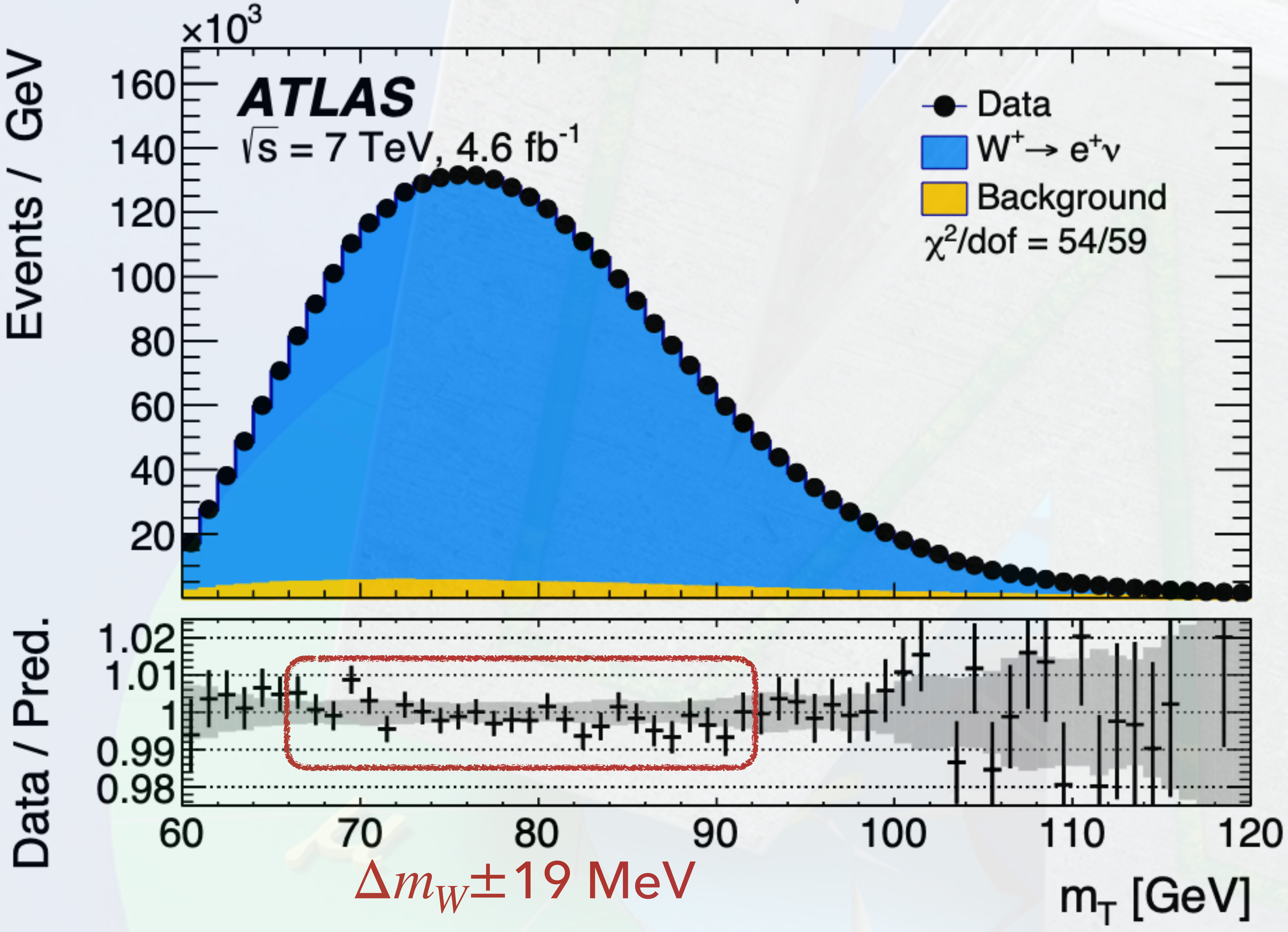


XC, Gehrmann, Glover, Huss, Yang, Zhu `22

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO}$

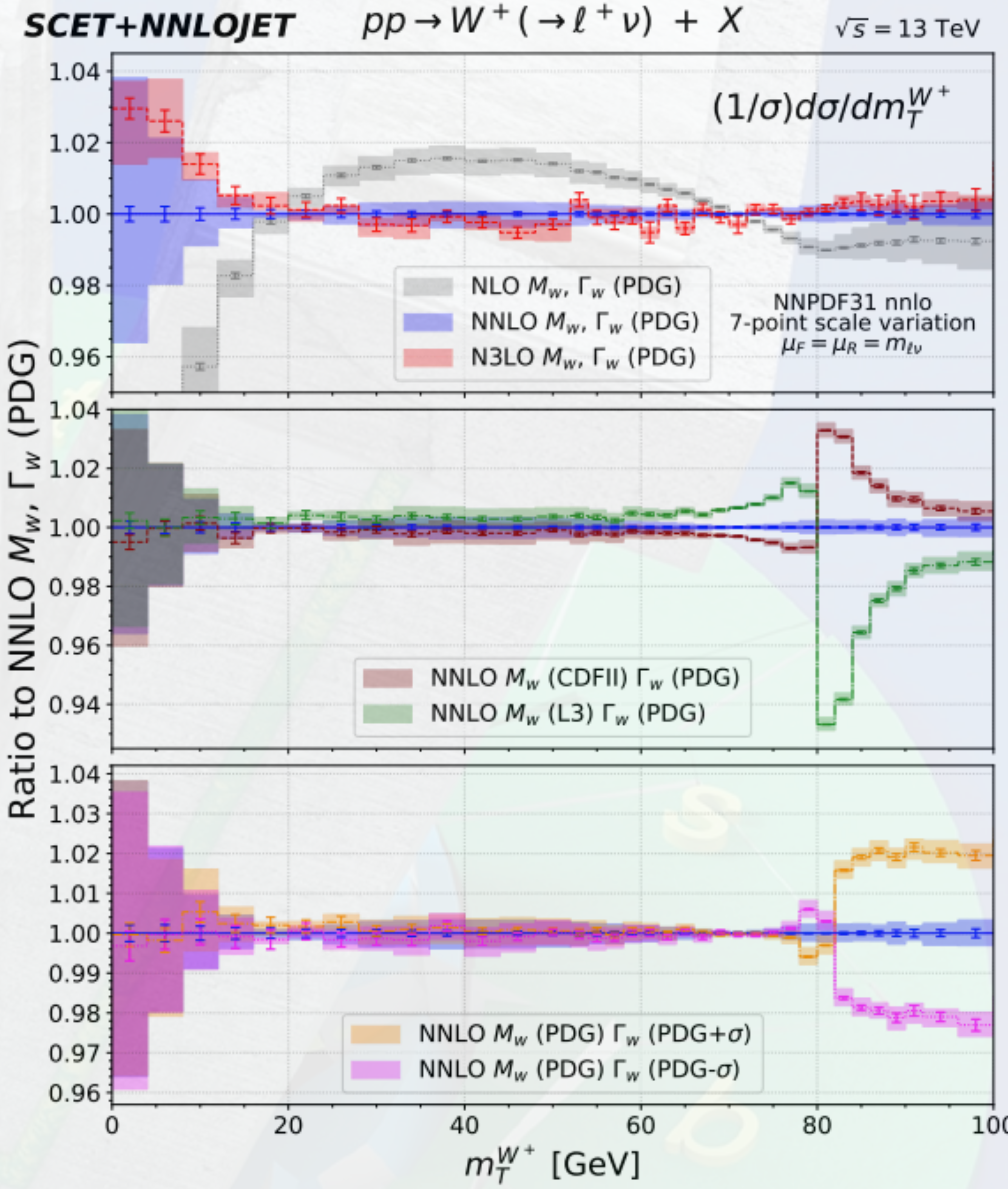
➤Differential N3LO predictions for charged current production

$$m_T = (E_T^l + E_T^\nu)^2 - (\vec{p}_T^{\,l} + \vec{p}_T^{\,\nu})^2 = \sqrt{2E_T^l E_T^\nu (1 - cos\phi)}$$



$\Delta m_W \pm 19$ MeV

ATLAS `17



±0.1%

XC, Gehrmann, Glover, Huss, Yang, Zhu `22

STATE-OF-THE-ART PREDICTIONS: $\alpha_{EW} \times \alpha_s$

➤NNLO QCD-EW mixed corrections

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0,0)} + \hat{\sigma}_{ab}^{(1,0)} + \hat{\sigma}_{ab}^{(2,0)} + \hat{\sigma}_{ab}^{(3,0)} + \ldots \quad \text{QCD}$$

$$+ \hat{\sigma}_{ab}^{(0,1)} + \ldots \quad \text{EW}$$

$$+ \hat{\sigma}_{ab}^{(1,1)} + \ldots \quad \text{QCD-EW}$$

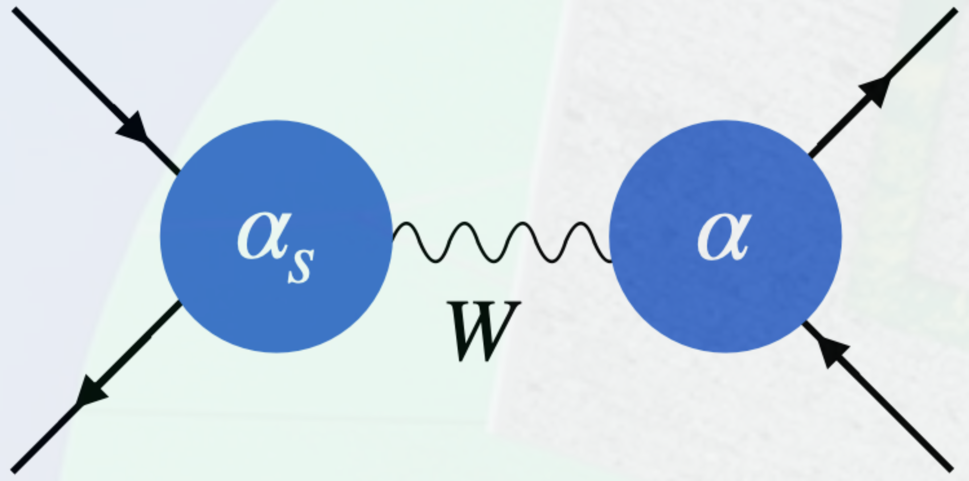
Talk by L. Buonocore @ L&L2022

σ [pb]	$\sigma_{\rm LO}$	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	5029.2	970.5(3)	−143.61(15)	251(4)	− 7.0(1.2)
qg	—	−1079.86(12)	—	−377(3)	39.0(4)
$q(g)\gamma$	—	—	2.823(1)	—	0.055(5)
$q(\bar{q})q'$	—	—	—	44.2(7)	1.2382(3)
gg	—	—	—	100.8(8)	—
tot	5029.2	−109.4(4)	−140.8(2)	19(5)	33.3(1.3)
$\sigma^{(m,n)}/\sigma_{\rm LO}$		$-2.2\,\%$	$-2.8\,\%$	$+0.4\,\%$	$+0.6\,\%$ $\mu = m_W$
$\sigma^{(m,n)}/\sigma_{\rm LO}$		$+10\,\%$	$-2.9\,\%$	$+4.2\,\%$	$+0.8\,\%$ $\mu = \frac{m_W}{2}$

Buonocore, Grazzini, Kallweit, Savoini, Tramontano \`21

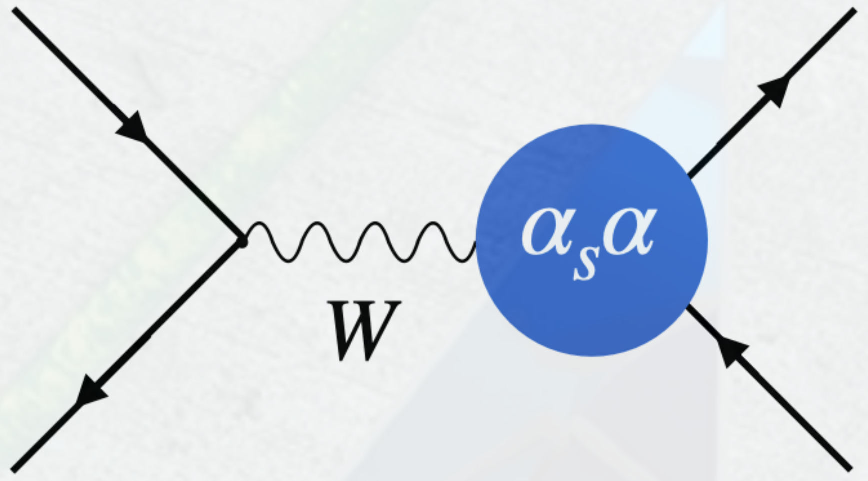
➤NLO EW correction from W decay is large

➤Scale variation from EW power counting is small



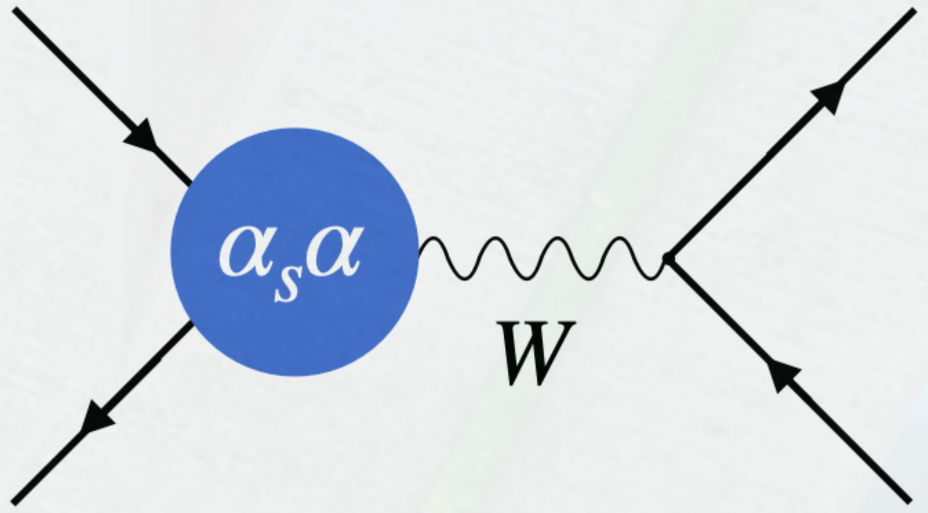
Initial-Final contribution from automation tools

Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini \`16



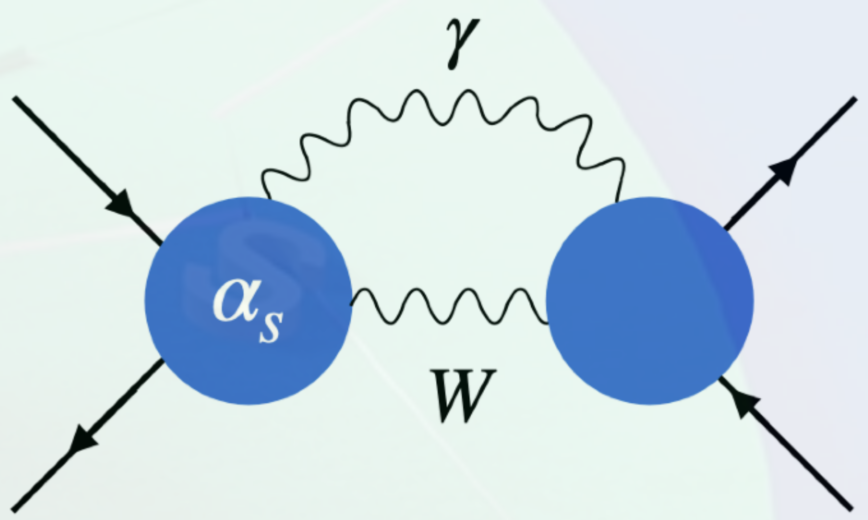
Final-Final finite renormalisation constant

Dittmaier, Huss, Schwinn \`15



Initial-Initial QCD-EW 2-loop form factors

Behring, Buccioni, Caola, Delto, Jaquier, Melkikov, Röntsch \`20



Non-factorizable contribution

Dittmaier, Huss, Schwinn \`14 (PA)

Buccioni, Caola, Chawdhry et. al. \`22 (NC-DY)

STATE-OF-THE-ART PREDICTIONS: $\alpha_{EW} \times \alpha_s$

[adapted from Gavin Salam @ ICHEP 2022]

➤Full study of fit to distribution is not easy at fixed order

➤Instead study **mass determination from mean lepton pT**, inclusive or fiducial

(Here just the **production corrections**; decay corrections should factorise)

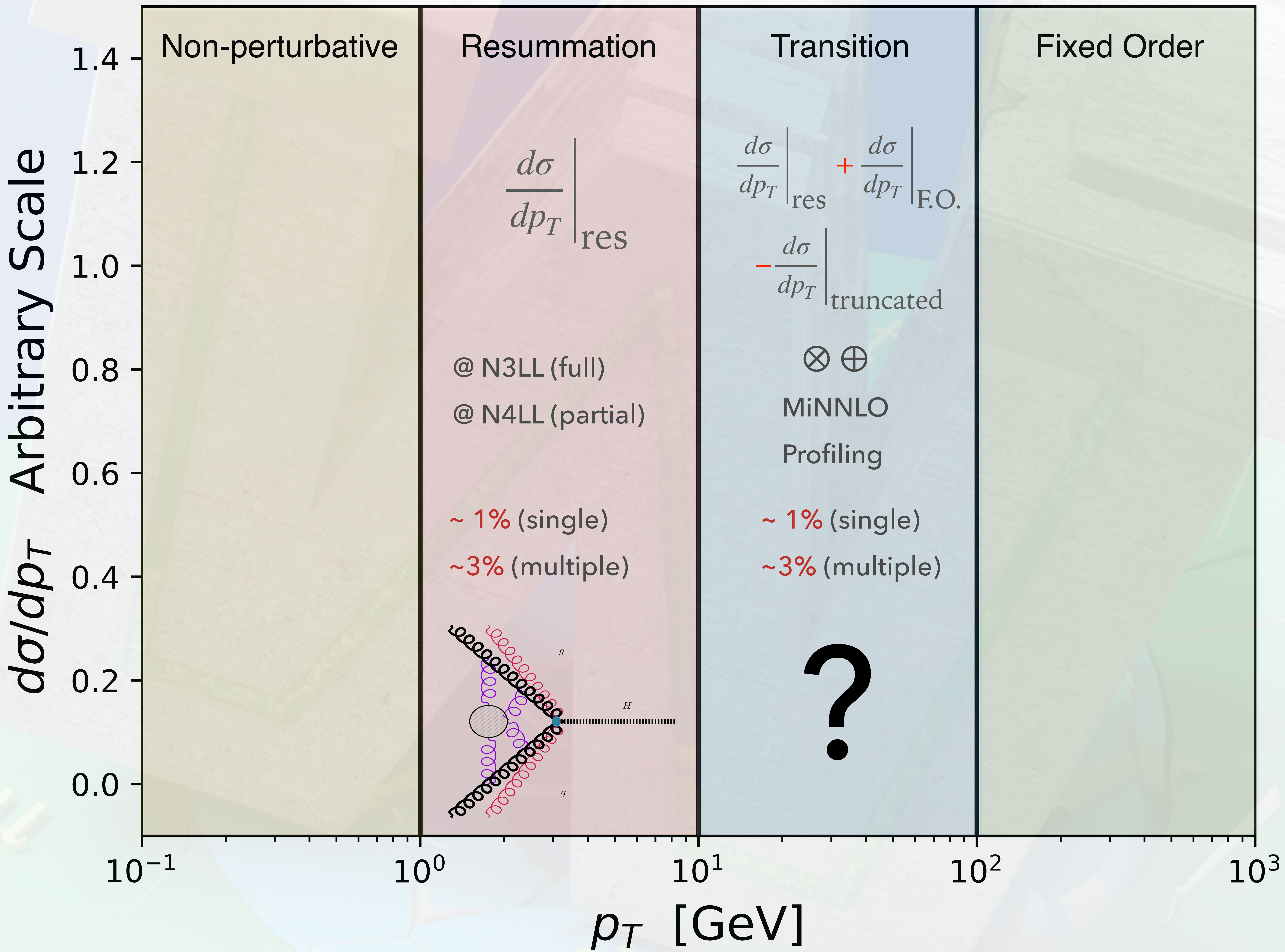
$$\frac{\delta m_W^{\text{meas}}}{m_W^{\text{meas}}} = \frac{\delta C_{\text{th}}}{C_{\text{th}}} = \frac{\delta \langle p_\perp^{l,Z} \rangle^{\text{th}}}{\langle p_\perp^{l,Z} \rangle^{\text{th}}} - \frac{\delta \langle p_\perp^{l,W} \rangle^{\text{th}}}{\langle p_\perp^{l,W} \rangle^{\text{th}}}$$

Behring, Buccioni, Caola, Delto et. al. `21

	δm_Z (scaled by m_W/m_Z)	δm_W	difference
inclusive $\langle p_{t\ell} \rangle$ @ α_{EW}	–32 MeV	–32 MeV	0.3 MeV
inclusive $\langle p_{t\ell} \rangle$ @ $\alpha_{EW}\,\alpha_s$	+62 MeV	+55 MeV	–7 MeV
fiducial $\langle p_{t\ell} \rangle$ @ $\alpha_{EW}\,\alpha_s$	[ATLAS cuts]		–17±2 MeV

➤Relevant for both Z-calibrated methods & standalone W methods. (Impact by fiducial cuts)

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^3LL}$



STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^3LL}$

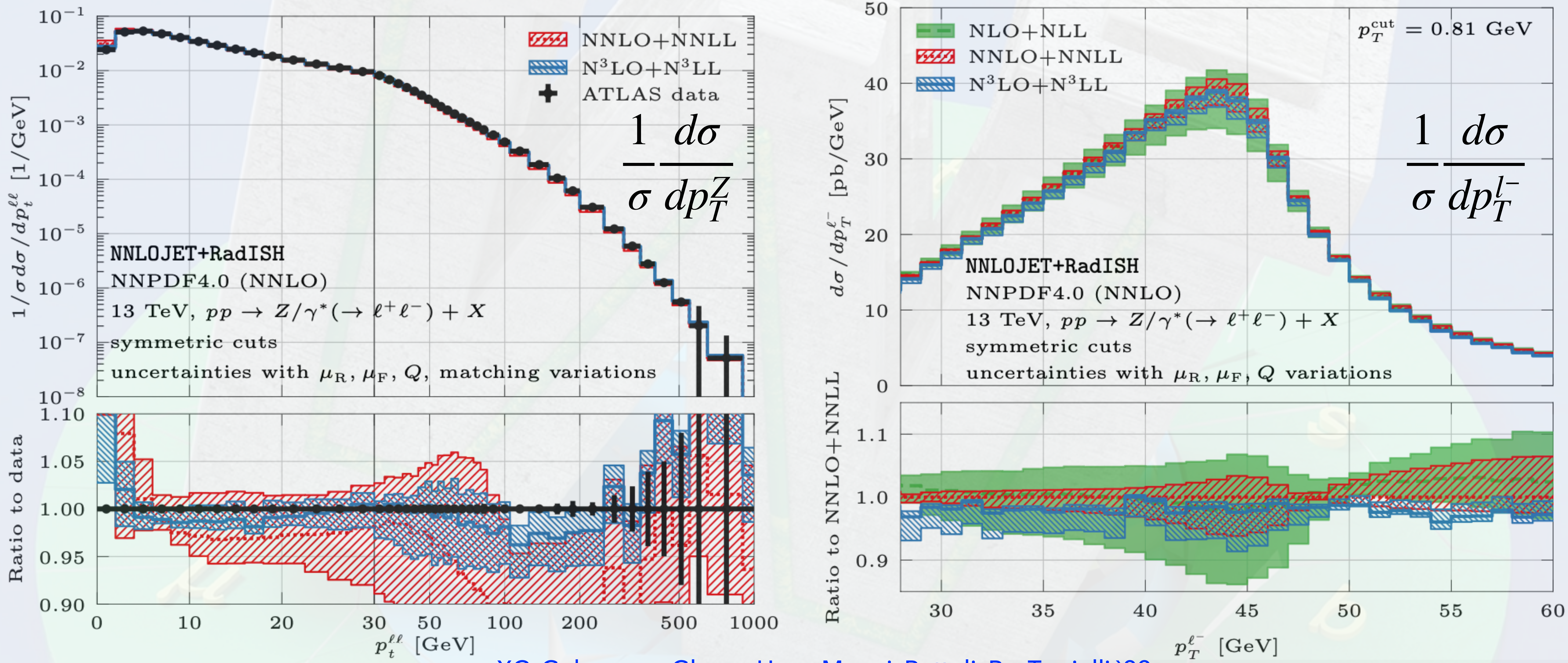
- Resummation kernels (Higgs pT example): $d\sigma^H = \sigma^H_{LO} \otimes H \otimes B \otimes B \otimes S$
  - In SCET: $$\frac{d\sigma}{dp_T^2} = \pi\sigma^H_{LO}\int dx_a dx_b \delta\left(x_a x_b - \frac{m_H^2}{E_{CM}^2}\right)\int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} W(x_a, x_b, m_H, \vec{b}),$$
$$W(x_a, x_b, m_H, \vec{b}) = H(m_H, \mu_h)U_h(m_H, \mu_B, \mu_h)S_\perp(\vec{b}, \mu_s, \nu_s)U_s(b, \mu_B, \mu_s; \nu_B, \nu_s)\prod_{\gamma=a,b} B^{\alpha\beta}_{g/N_\gamma}(x_\gamma, \vec{b}, m_H, \mu_B, \nu_B),$$
$$U_s(b, \mu, \mu_s; \nu, \nu_s) = \exp\left[2\int_{\mu_s}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\left(\Gamma_{\text{cusp}}\left(\alpha_s(\bar{\mu})\right)\ln\frac{b^2\bar{\mu}^2}{b_0^2} - \gamma_s\left(\alpha_s(\bar{\mu})\right)\right)\right]\left(\frac{\nu^2}{\nu_s^2}\right)^{\int_\mu^{b_0/b}\frac{d\bar{\mu}}{\bar{\mu}}2\Gamma_{\text{cusp}}\left[\alpha_s(\bar{\mu})\right]+\gamma_r\left[\alpha_s(b_0/b)\right]}.$$
  - In qT (CSS): $$S_c(M, b) = \exp\left[-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left(A_c\left(\alpha_s(q^2)\right)\ln\frac{M^2}{q^2} + B_c\left(\alpha_s(q^2)\right)\right)\right]$$
$$\frac{d\sigma}{dp_T^2 dy} = \frac{m_H^2}{s}\sigma^H_{LO}\int_0^{+\infty} db\frac{b}{2}J_0(bp_T)S_g(m_H, b)\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\left[HC_1C_2\right]_{gg;a_1a_2}\prod_{i=1,2} f_{a_i/h_i}(x_i/z_i, b_0^2/b^2)$$
  - In momentum space (RadISH):
$$\Sigma(p_T) = \int_0^{p_T} dk_T\frac{d\sigma(k_T)}{dk_t} = \sigma^H_{LO}\int_0^\infty [dk_1]R'(m_H, k_{t,1})\exp\left(-R(m_H, \epsilon k_{t,1})\right)\sum_{n=0}^\infty\frac{1}{n!}\prod_{i=2}^{n+1}\int_{\epsilon k_{t,1}}^{k_{t,1}}[dk_i]R'(m_H, k_{t,i})\Theta\left(p_T - \left|\sum_{j=1}^{n+1}\vec{k}_{t,j}\right|\right)$$

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^3LL}$

FO	α_s^n	$P_{ab}^{(n)}(x)$	$\ln W(x_a, x_b, m_V, \vec{b}, \mu = b_0/b) \sim \int_{\mu_h}^{\mu} d\bar{\mu}/\bar{\mu}\left(A(\alpha_s(\bar{\mu}))\ln\frac{m_V^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu}))\right)$						
$\frac{d\hat{\sigma}^V_{NLO}}{dq_T}$	1	✓	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1				
$\frac{d\hat{\sigma}^V_{NNLO}}{dq_T}$	2	✓	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1			
$\frac{d\hat{\sigma}^V_{N^3LO}}{dq_T}$	3	✓	$\ln^4(b^2m_V^2)$	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1		
$\frac{d\hat{\sigma}^V_{N^4LO}}{dq_T}$	4	✗	$\ln^5(b^2m_V^2)$	$\ln^4(b^2m_V^2)$	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1	
…	…		…	…	…	…		…	…
$\frac{d\hat{\sigma}^V_{N^kLO}}{dq_T}$	K		$\ln^{k+1}(b^2m_V^2)$	$\ln^k(b^2m_V^2)$	$\ln^{k-1}(b^2m_V^2)$	$\ln^{k-2}(b^2m_V^2)$	$\ln^{k-3}(b^2m_V^2)$	…	…
…	…		…	…	…	…		…	…
	Resum		LL	NLL	NNLL	N3LL	N4LL	…	$N^{k+1}LL$
	A		A1 ✓	A2 ✓	A3 ✓	A4 ✓	A5 ✗	…	A_{k+2}
	B			B1 ✓	B2 ✓	B3 ✓	B4 ✓	…	B_{k+1}

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^3LL}$

- ➤Differential predictions for neutral current production with fiducial cuts
  - ➤ Resum all order contributions at N3LL using RadISH and matched to N3LO

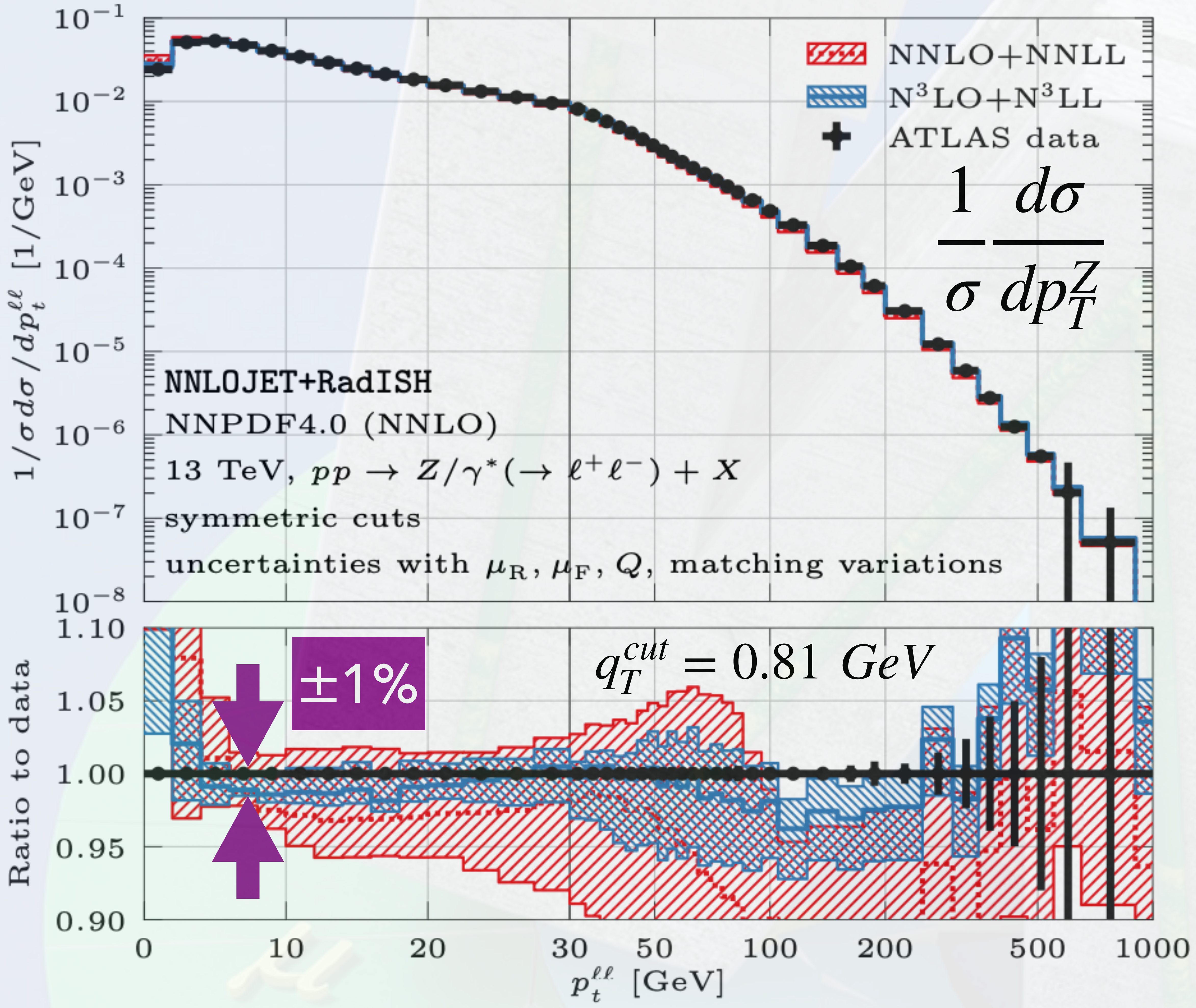


XC, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli `22

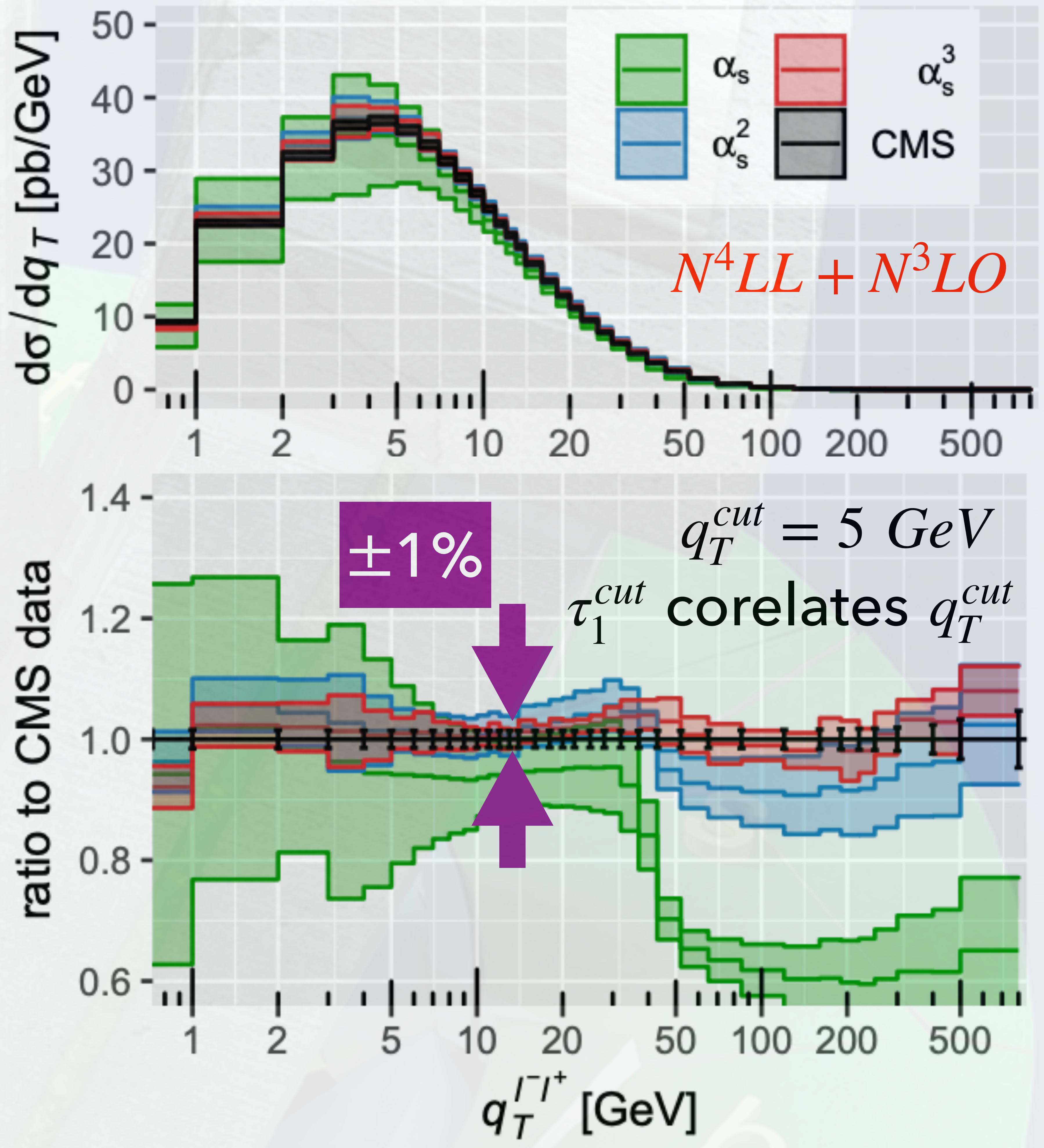
STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^3LL}$

➤Differential predictions for neutral current production with fiducial cuts

XC, Gehrmann, Glover, Huss, Monni, Rottoli, et. al.`22



Neumann and Campbell `22



➤Different in pQCD convergence, theory uncertainties, agreement with data → systematic error cross schemes

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^3LL}$

➤Differential predictions for neutral current production with fiducial cuts

- Evaluate $\alpha_S(\mu)$ independently (green):
  - Production at hard scale
  - Logarithmic terms at soft scale (resummation formalism dependent)
- PDF evaluation at small μ_F
  - Switch on/off charm quark PDF at 0.5 GeV (dark red)
  - Freeze PDF at $\mu_F \sim 1.5$ GeV (default)
- Central value of resummation scale 2xQ
- 3 matching schemes in transition region (orange band)
- Correlated Impact with independent $\alpha_S(\mu)$ evaluation (bright green)

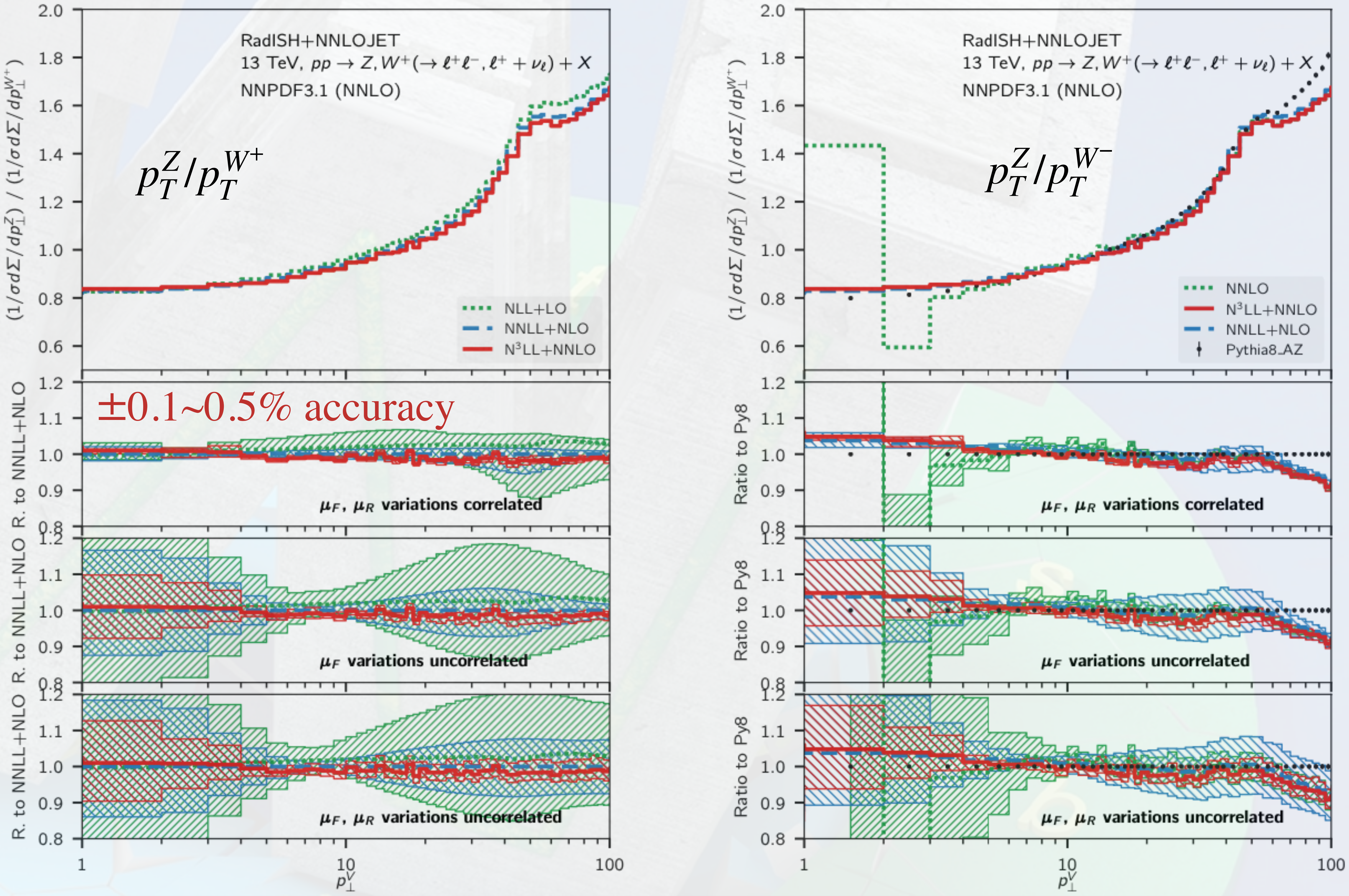
XC, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli `22

STATE-OF-THE-ART PREDICTIONS FOR $d\sigma_{N^3LO+N^3LL}$

- Differential N3LL +N3LO predictions for charged current production with fiducial cuts
  - NNLO W/Z+jet matched with N3LL pT resummation
  - Precise W measurement with calibration against Z.
  - Improved QCD uncertainties through out pT.
  - What is a reasonable estimation of uncertainties?

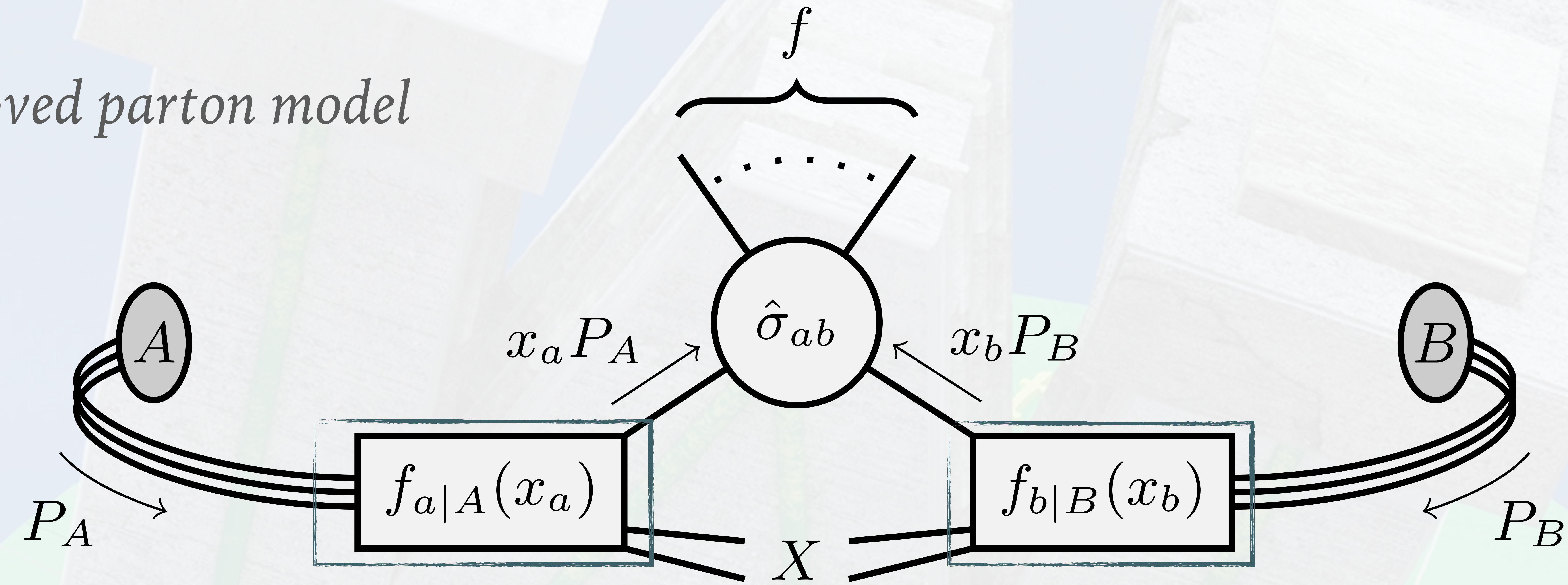
    $\pm 0.1\% \rightarrow 10$ MeV (CDF)
  - Different EW and QCD-EW correction between Z and W are not yet considered.

Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Walker `19



PRECISION PREDICTIONS AT HADRON COLLIDER

QCD improved parton model



$$\sigma_{AB} = \sum_{ab} \int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \, f_{a|A}(x_a)\, f_{b|B}(x_b)\, \hat{\sigma}_{ab}(x_a, x_b)\, \left(1+\mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)\right)$$

Parton distribution functions
(Energy evolution from all exp.)
± 3-5 % at LHC energy

ANATOMY OF PDFS

- Theory input
  - Option A: solve proton wave function using non-perturbative QCD Lagrangian (use Lattice QCD in finite space-time volume)
  - Option B: collinear factorisation $f_a \to f_a(z, \mu)$ with pQCD evolution of factorisation scale

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} f_q \\ f_g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} f_q \\ f_g \end{pmatrix}$$

DGLAP evolution 1970's

$$p_{a\leftarrow b} = \frac{\alpha_s}{\pi} P^{(0)}_{a\leftarrow b} + \frac{\alpha_s^2}{\pi^2} P^{(1)}_{a\leftarrow b} + \frac{\alpha_s^3}{\pi^3} P^{(2)}_{a\leftarrow b} + \cdots$$

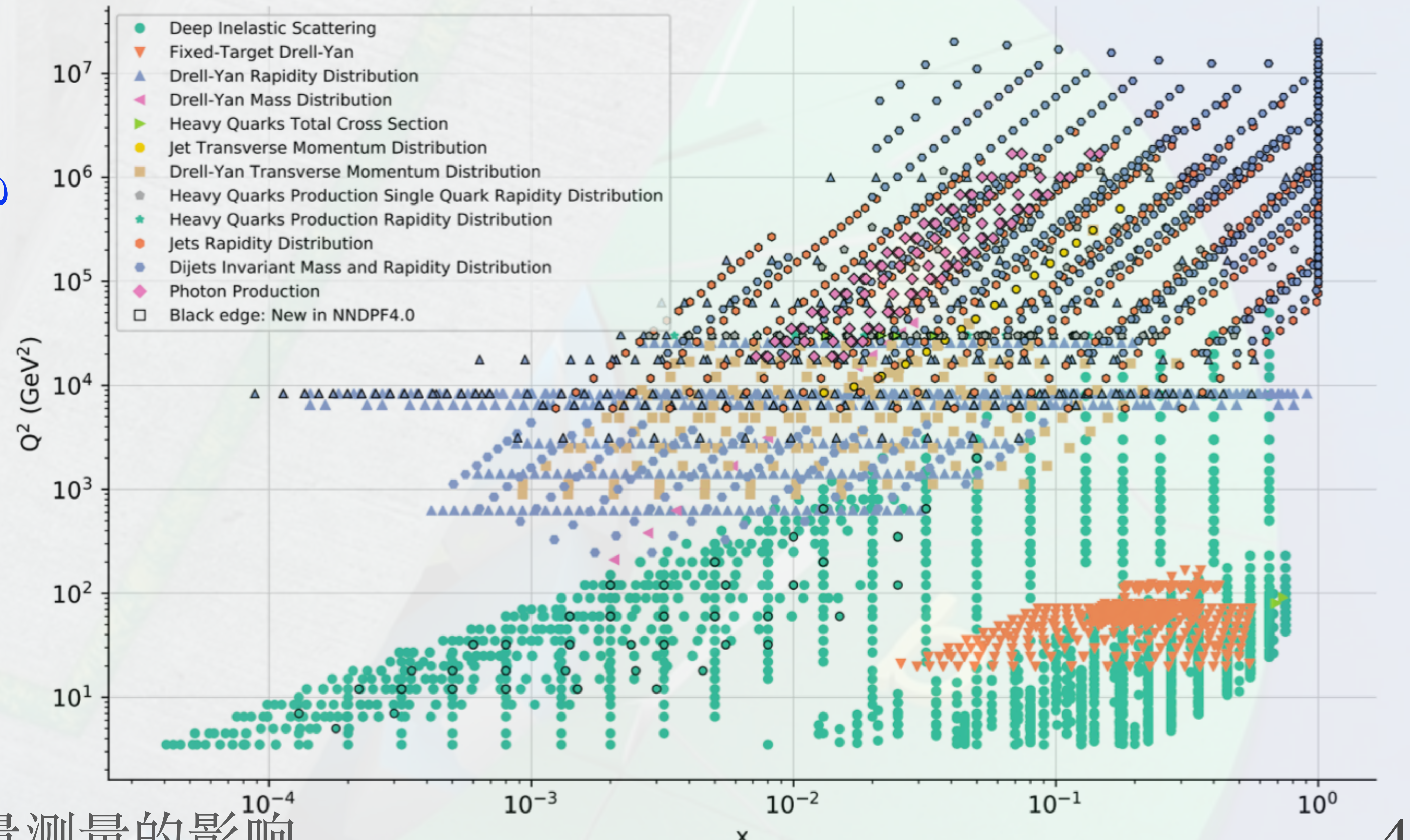
Full $P^{(1)}_{a\leftarrow b}$ G. Curci, W. Furmanski, R. Petronzio 1980

Full $P^{(2)}_{a\leftarrow b}$ S. Moch , J.A.M. Vermaseren, A. Vogt 2004

Partial $P^{(3)}_{a\leftarrow b}$ F. Herzog, S. Moch, B. Ruijl, T. Ueda et al. 2018-21

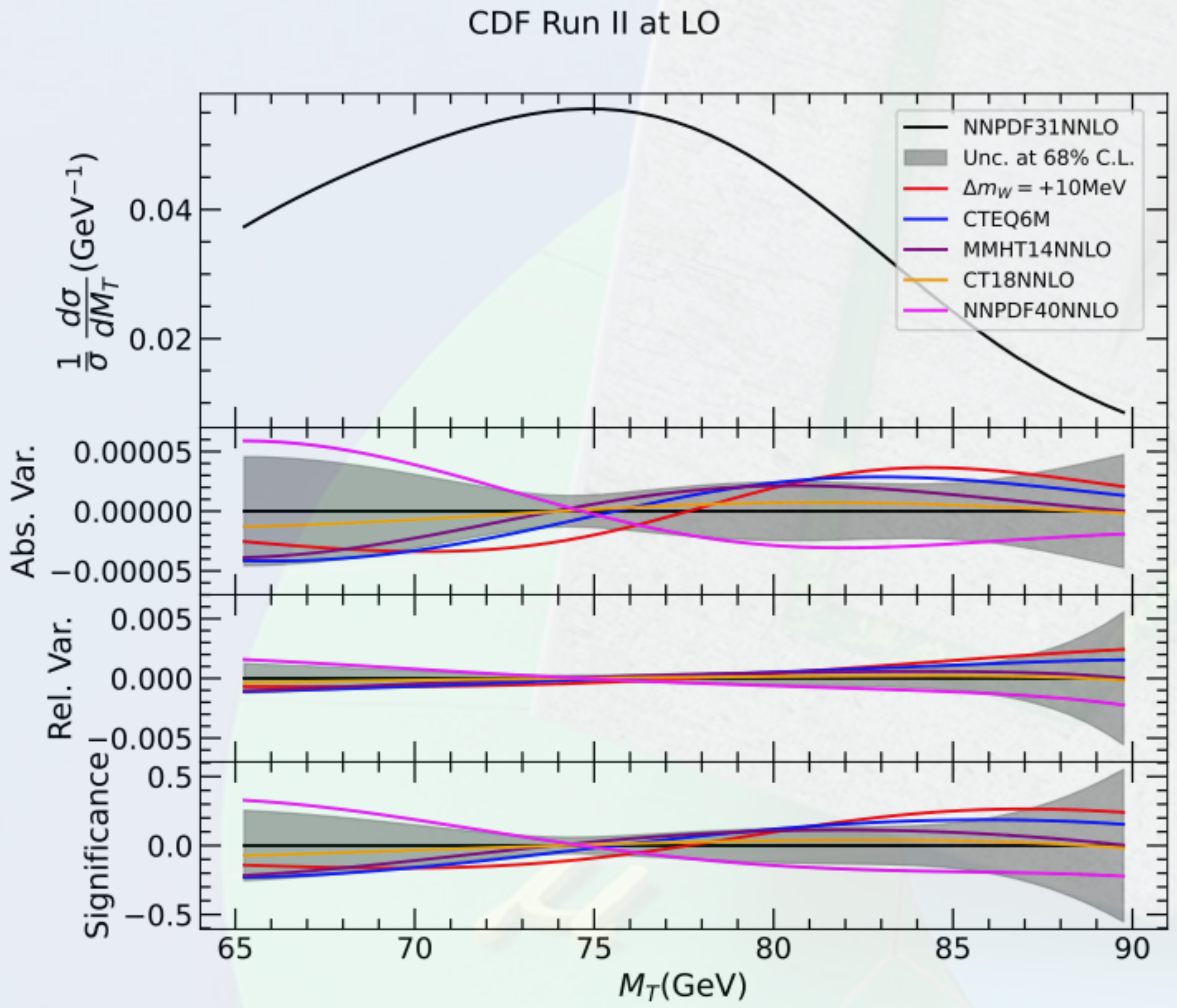
- Experiment input
  - All past and current measurements of DIS, DY, jets etc. provide fitting targets of $f_a(z, Q)$
  - Differential and total cross sections provide sensitivity in different regions of $z \in [0,1]$
  - Various technology for fitting: functional form, neural network, fast evaluation grids etc.

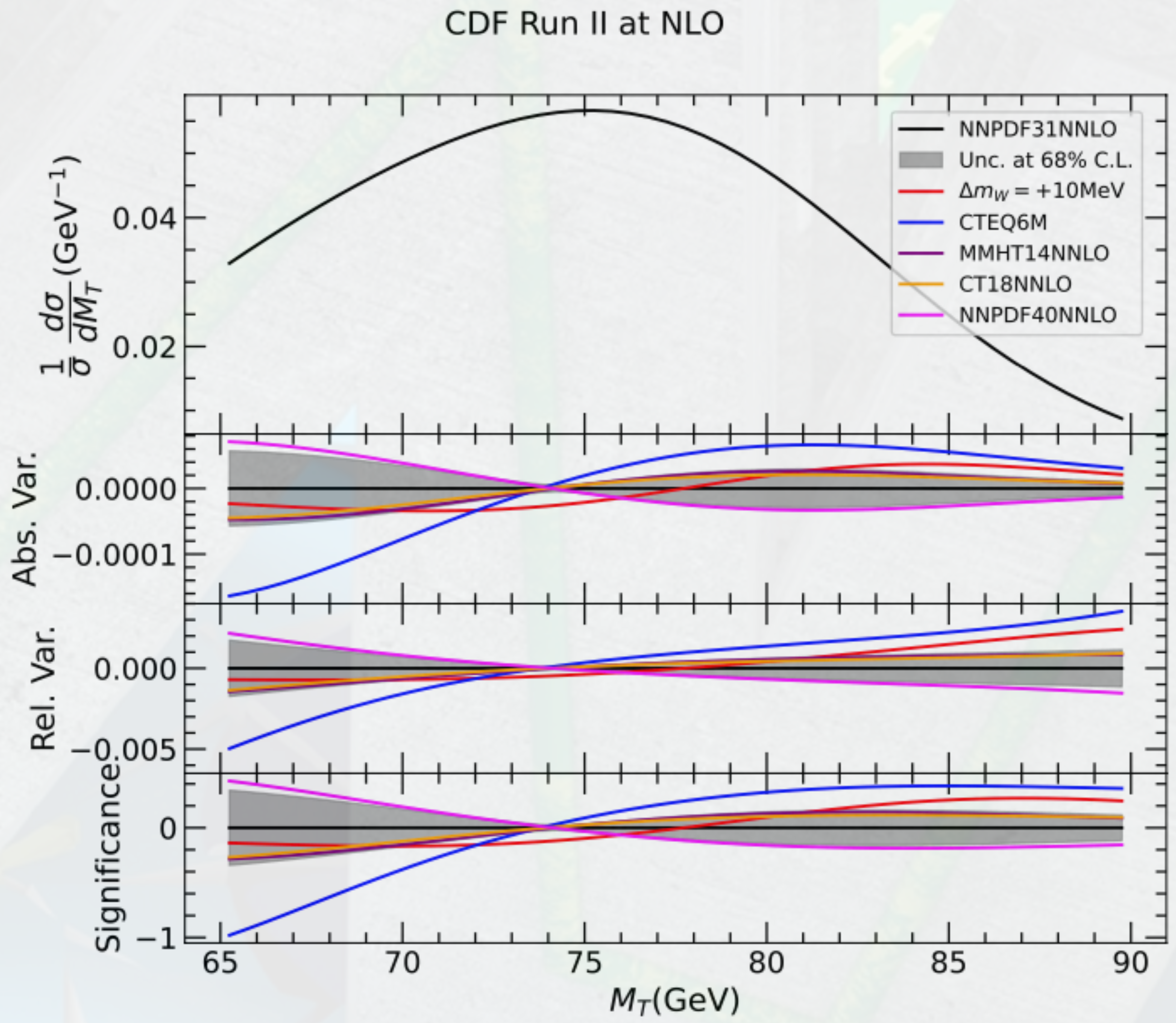
NNPDF4.0 Coverage



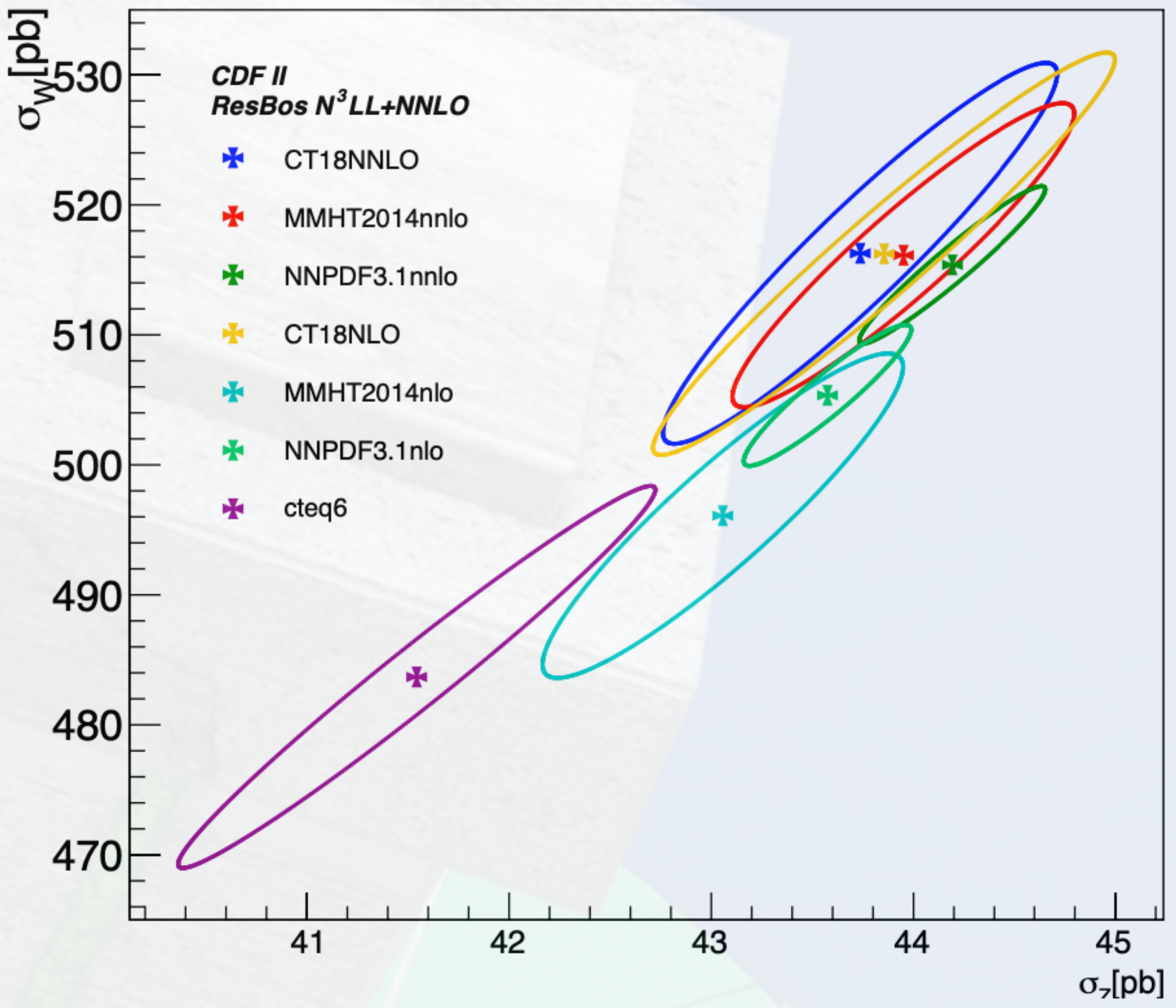
PDF UNCERTAINTY

- PDF from Collinear factorisation
  - NNLO evolution accuracy is the current standard
  - 1~1.5% impact on fiducial σ_W and σ_Z predictions
  - ~0.25% impact on m_T distribution at CDFII ($\Delta m_W \pm 3.9$ MeV)





Gao, Liu and Xie`22



Isaacson, Fu and Yuan`22

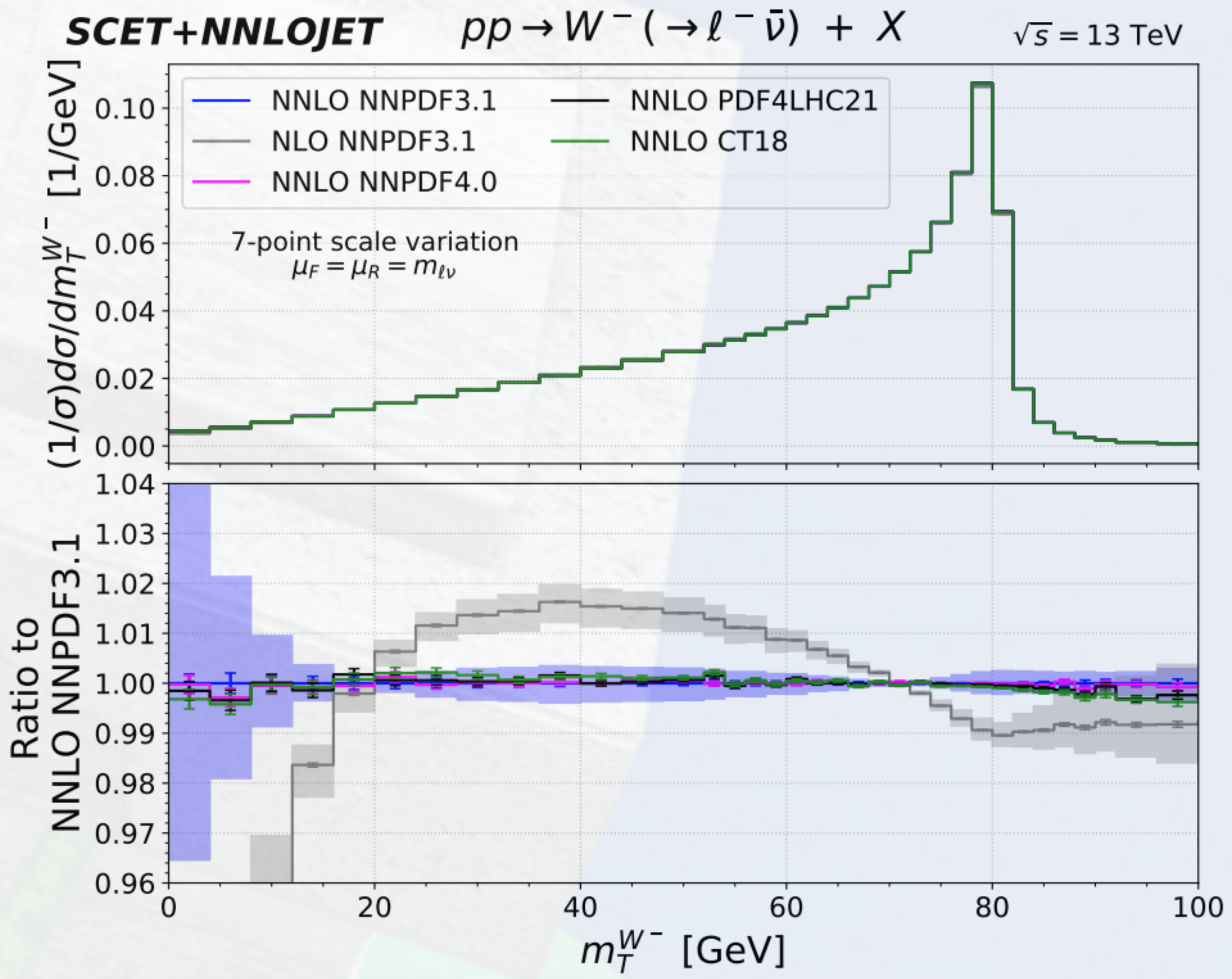
- PDF uncertainties:
  - Among different sets
  - Fitting error within a set

PDF UNCERTAINTY

- Impact of PDF on Δm_W via exam of $< M_T >$ and χ^2 of $d\sigma/dM_T$
  - LO to NNLO predictions consistent with CDF reported error

δM_W in MeV	sta.	NNPDF3.1	CT18	MMHT14	NNPDF4.0	MSHT20
$\langle M_T \rangle$(LO)	–	$0^{+8.3}_{-8.3}$	$-1.0^{+8.3}_{-11.4}$	$-3.3^{+7.4}_{-4.2}$	$+7.8^{+5.1}_{-5.1}$	$-3.1^{+6.7}_{-5.7}$
χ^2 fit (LO)	8.0	$0^{+7.6}_{-7.6}$	$-1.0^{+5.4}_{-8.6}$	$-3.3^{+6.1}_{-3.0}$	$+8.0^{+3.7}_{-3.7}$	$-3.0^{+5.0}_{-4.0}$
$\langle M_T \rangle$(NLO)	–	$0^{+5.9}_{-5.9}$	$-4.2^{+8.8}_{-13.3}$	$-5.0^{+6.7}_{-5.3}$	$+6.9^{+6.2}_{-6.2}$	$-7.6^{+7.9}_{-6.7}$
χ^2 fit (NLO)	8.0	$0^{+4.2}_{-4.2}$	$-4.3^{+5.4}_{-10.1}$	$-5.1^{+4.8}_{-3.4}$	$+7.1^{+4.5}_{-4.5}$	$-7.8^{+5.7}_{-4.5}$
CDF	9.2	$0^{+3.9}_{-3.9}$	–	–	–	–

Gao, Liu and Xie`22



XC, Gehrmann, Glover, Huss, Yang, Zhu `22

PDF UNCERTAINTY

- Impact of PDF on Δm_W via exam of $< M_T >$ and χ^2 of $d\sigma/dM_T$
  - LO to NNLO predictions consistent with CDF reported error

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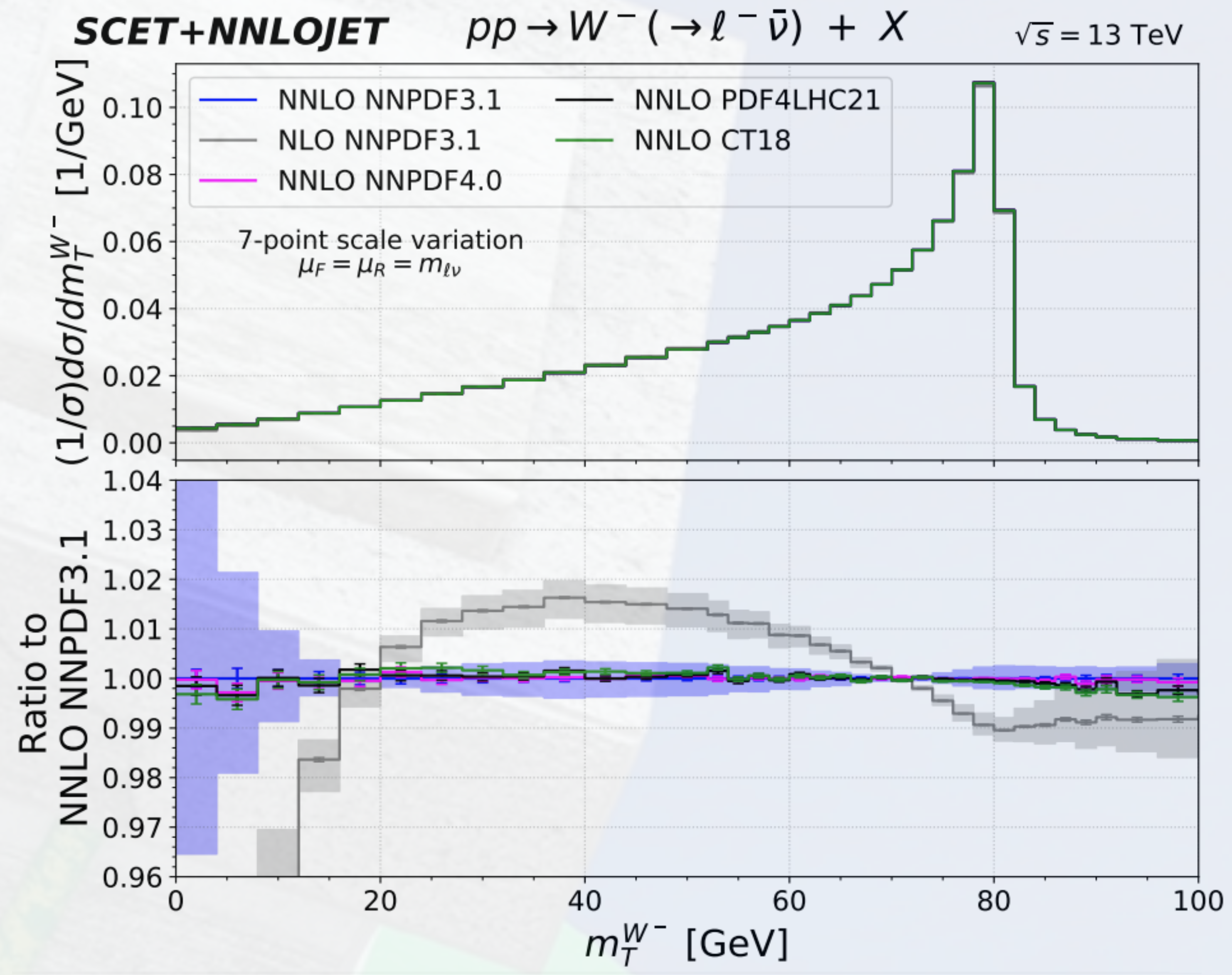
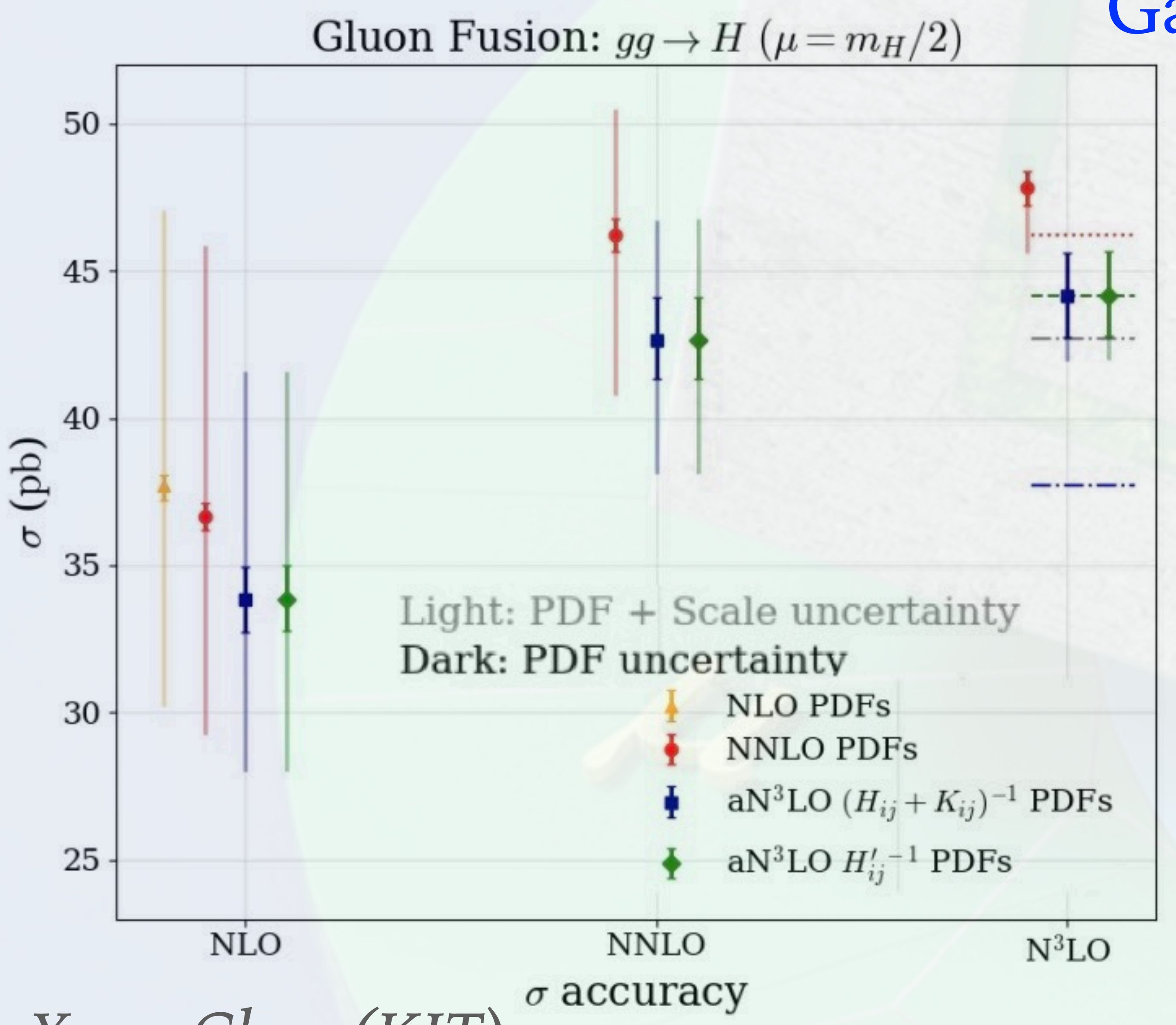


Gao, Liu and Xie`22

XC, Gehrmann, Glover, Huss, Yang, Zhu `22



- Approximated N3LO PDF based on 4-loop DGLAP anomalous dimensions
  - Approximation and data driven fits to unknown N3LO contributions
  - 7.6% decrease in gluon fusion to Higgs cross section at N3LO
  - PDF uncertainty goes up by 3 times
  - Require many future investigations but needed in N3LO predictions

McGowan, Cridge, Harland-Lang and Thorne `22

PDF UNCERTAINTY

➤ Impact of PDF on Δm_W via exam of $< M_T >$ and χ^2 of $d\sigma/dM_T$

➤ LO to NNLO predictions consistent with CDF reported error

δM_W in MeV	sta.	NNPDF3.1	CT18	MMHT14	NNPDF4.0	MSHT20
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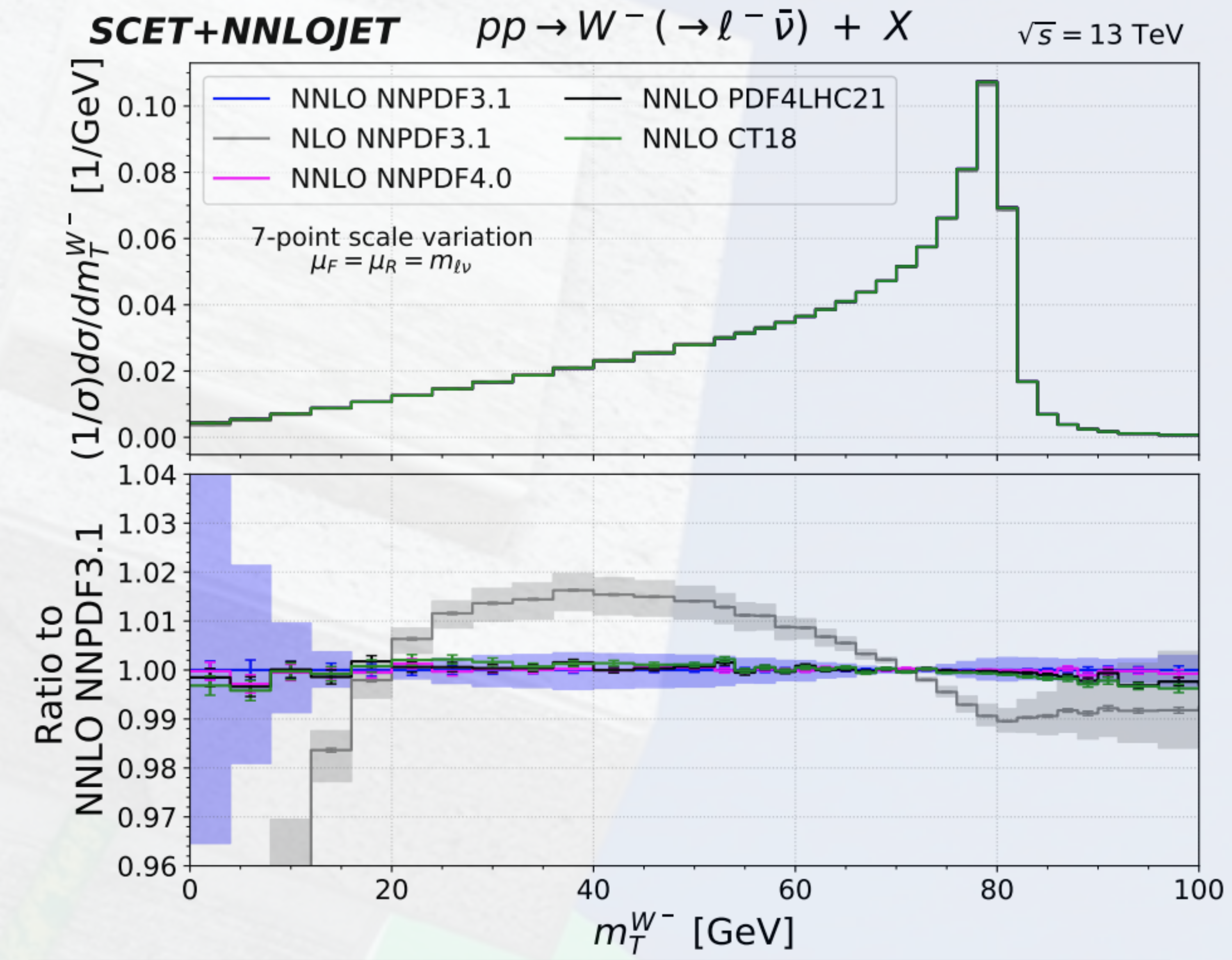
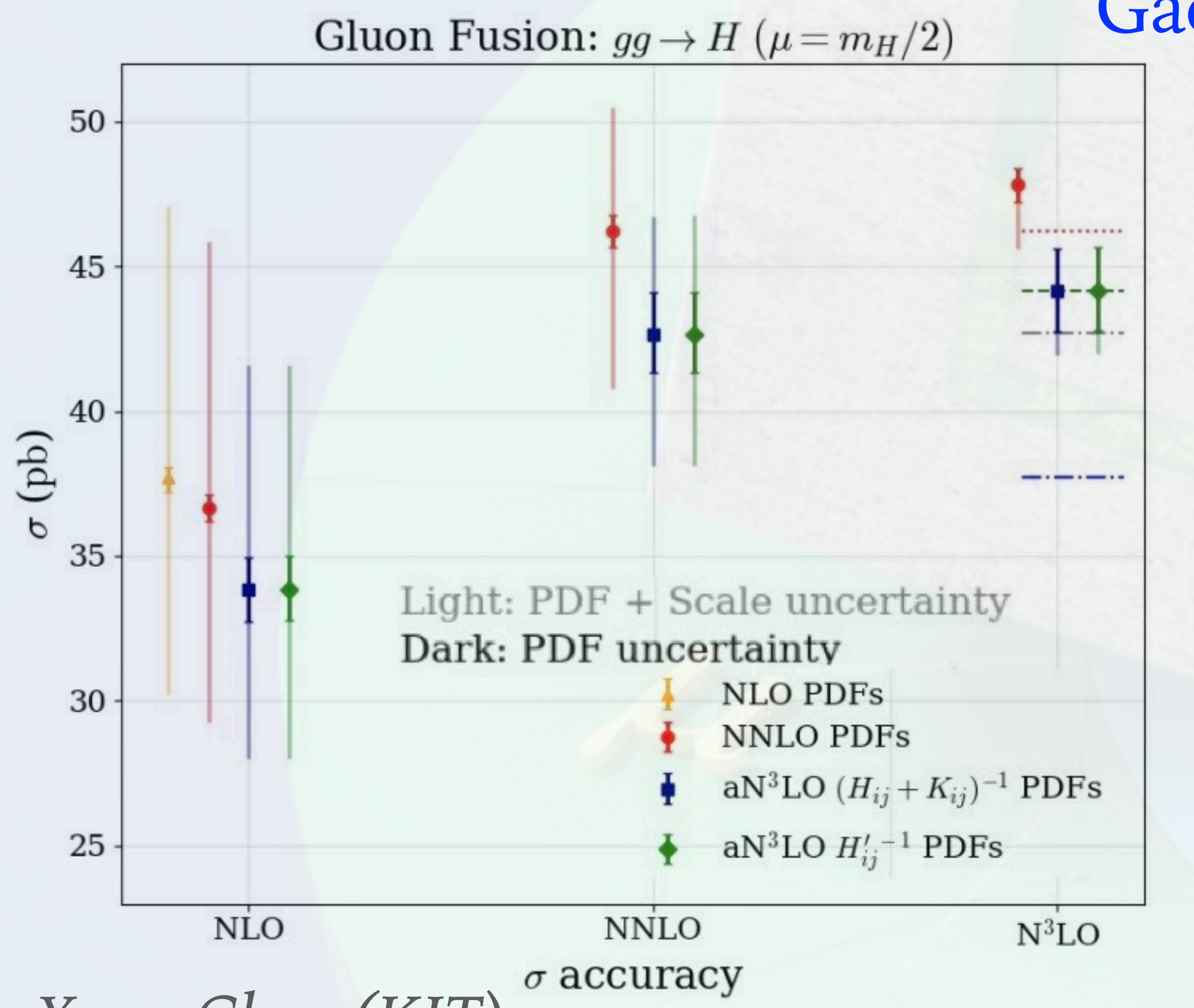
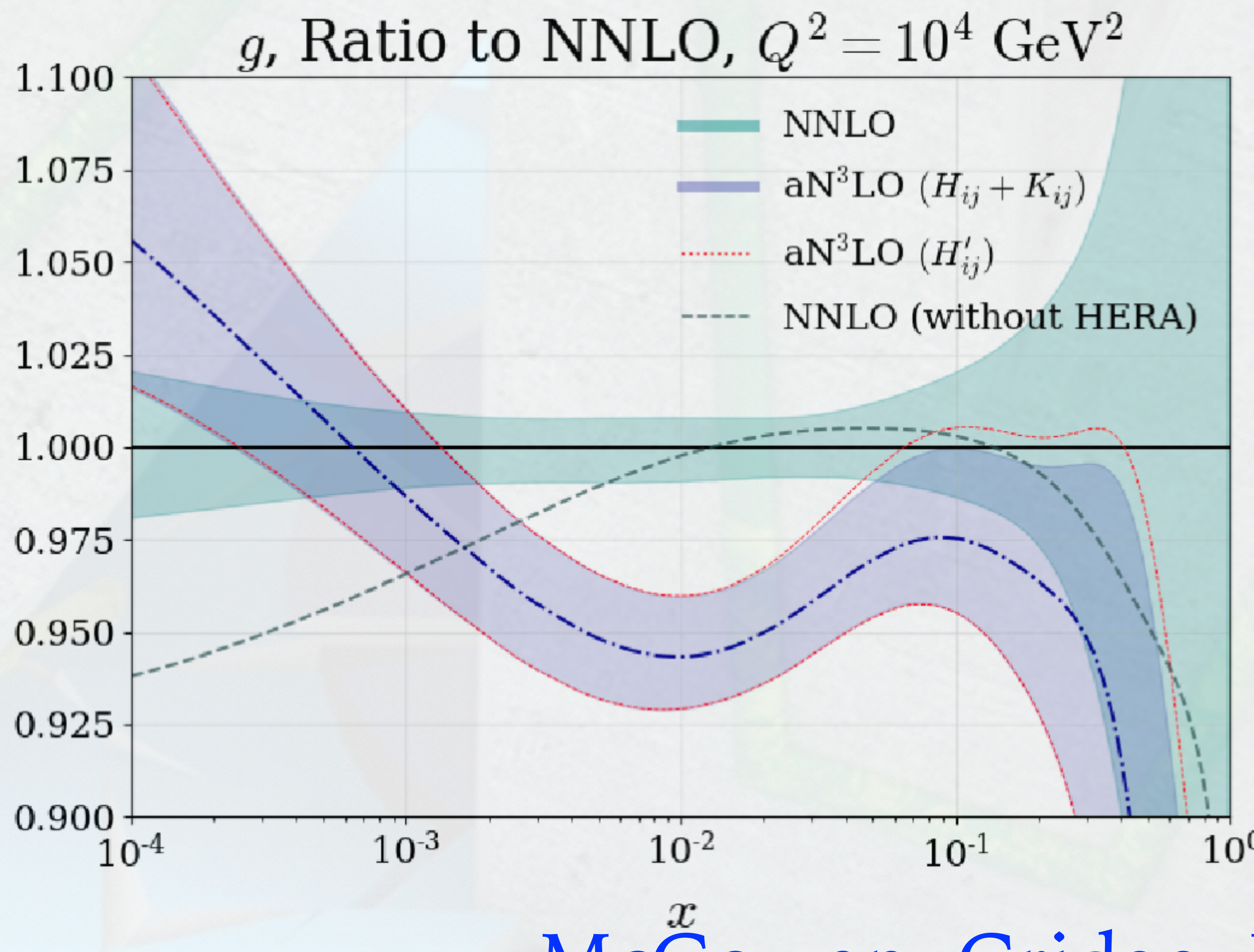
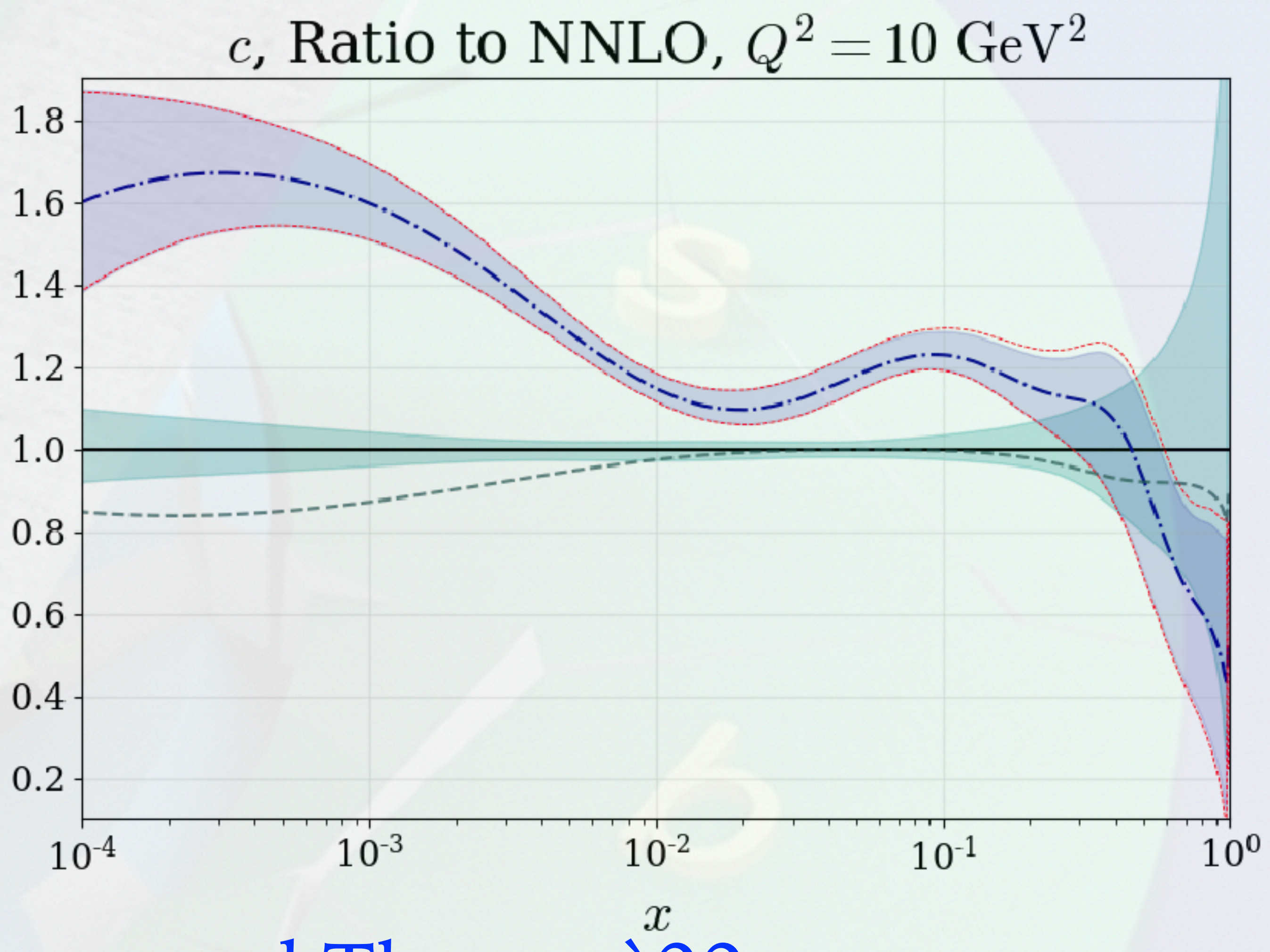
Gao, Liu and Xie`22



XC, Gehrmann, Glover, Huss, Yang, Zhu `22







McGowan, Cridge, Harland-Lang and Thorne `22

CONCLUSION AND OUTLOOK

- The determination of W boson mass requires delicate treatment and thorough understanding of experiment and theory uncertainties.
- Theoretical uncertainties at 0.1% level is required to achieve 10 MeV accuracy in m_W
- Best predictions for CC DY production at N3LL+N3LO QCD achieves 1% accuracy
- Thorough study of resummation schemes, transition region profiling, non-perturbative effects indicates 2-3% extra error
- EW-QCD mixed corrections and NNLO PDF variations each have $\mathcal{O}(10)$ MeV error
- Choices of correlated, uncorrelated, MHO uncertainty analysis can make theoretical uncertainties artificial small at 0.1% level.
- As the LHC entering precision era, fascinating progress is ahead to understand the m_W puzzle. Many uncertainties and opportunities!

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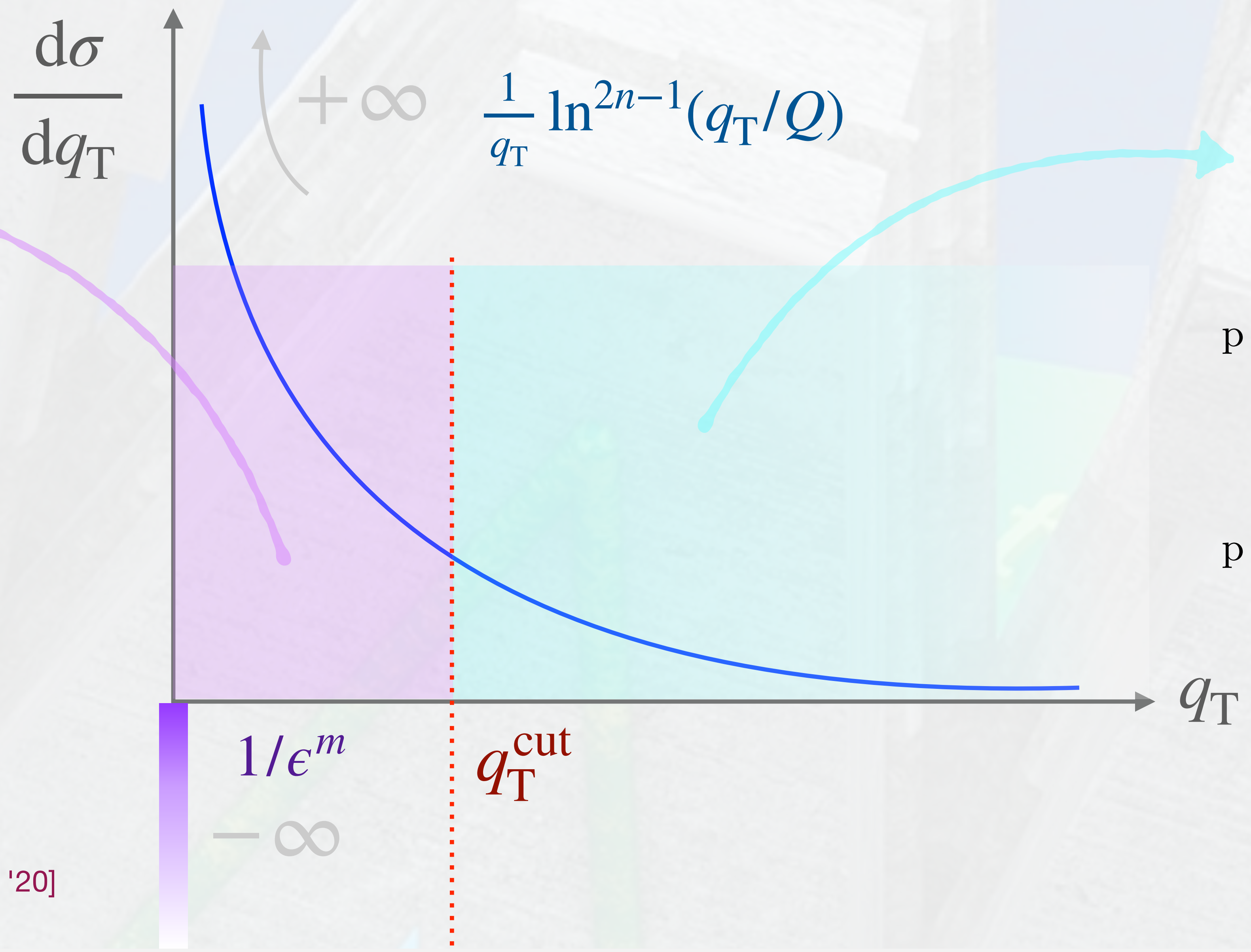
Thank You for Your Attention

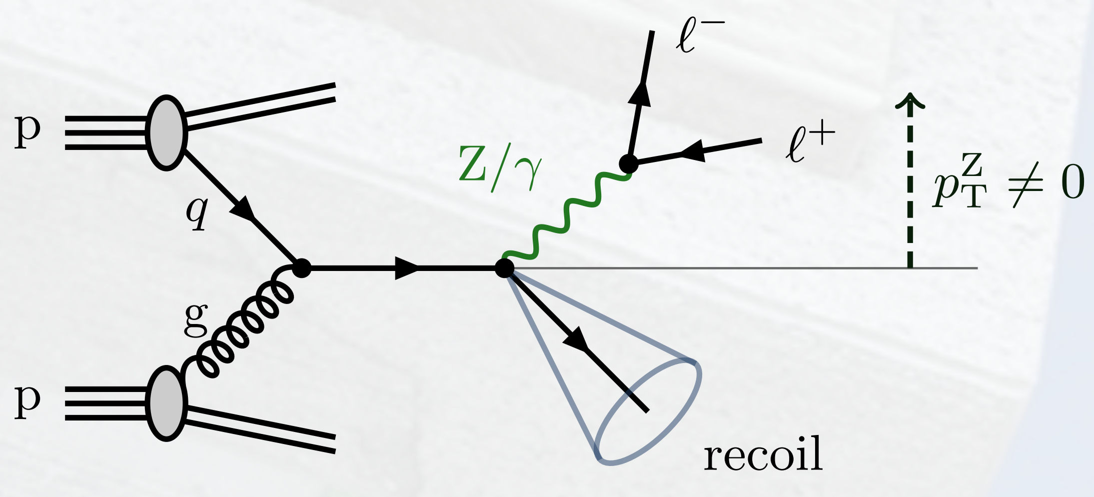
q_T Subtraction @ N^{3}LO

q_T resummation

- expand to fixed order
- $\mathcal{O}(\alpha_s^3)$ ingredients:
  - hard function $H_{q\bar{q}}$
    [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]
  - soft function $S(\mathbf{b}_\perp)$
    [Li, Zhu '16]
  - beam function $B_q(\mathbf{b}_\perp)$
    [Luo, Yang, Zhu, Zhu '19] [Ebert, Mistlberger, Vita '20]



V+jet @ NNLO



[Catani, Grazzini '07]

$$d\sigma^V_{N^3LO} = d\sigma^V_{N^3LO}\Big|_{q_T<q_T^{cut}} + d\sigma^V_{N^3LO}\Big|_{q_T>q_T^{cut}}$$

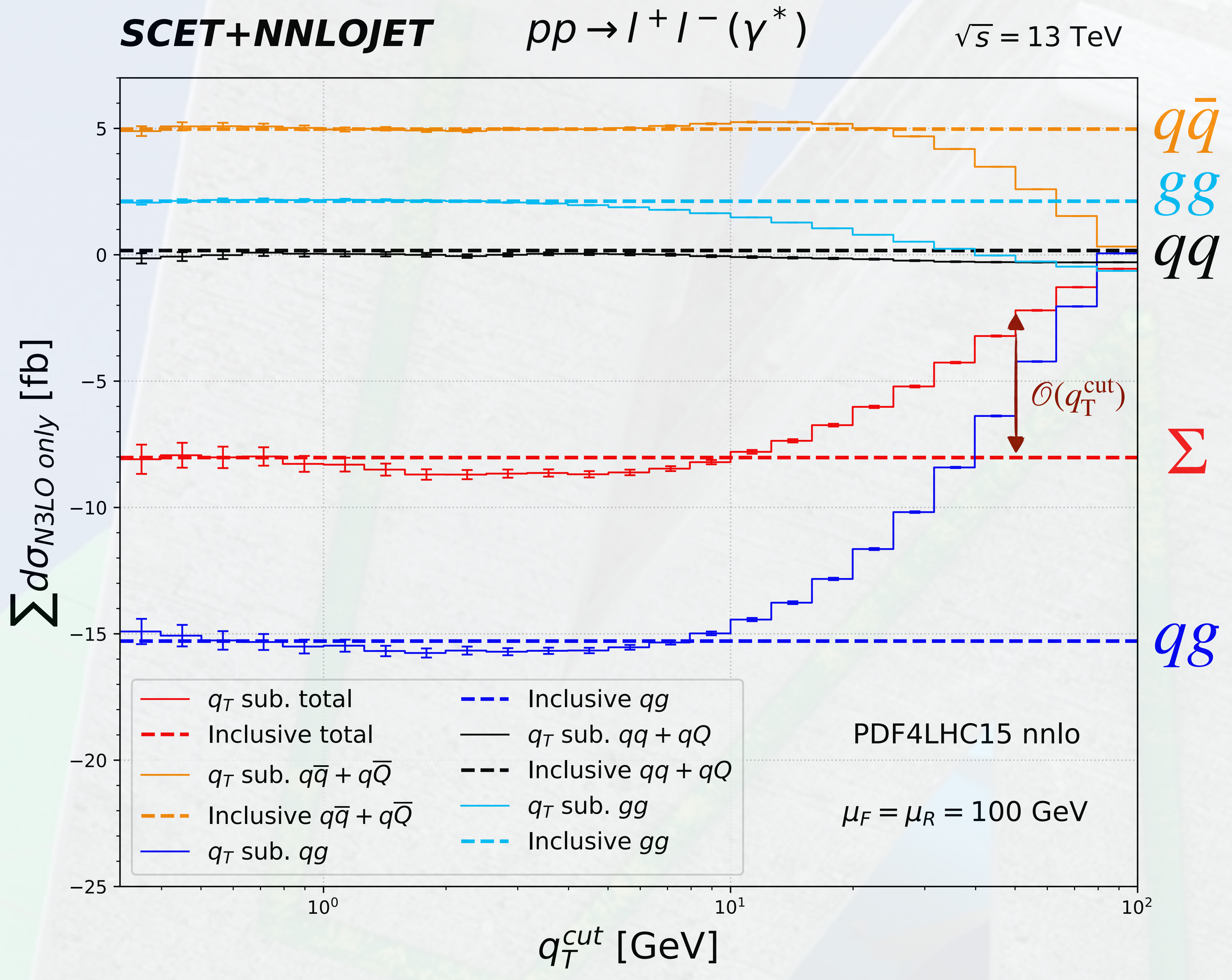
$$= \mathcal{H}^V_{N^3LO} \otimes d\sigma^V_{LO} + \left[d\sigma^{V+jet}_{NNLO} - d\sigma^{V,CT}_{N^3LO}\right]_{q_T>q_T^{cut}} + \mathcal{O}\left((q_T^{cut}/Q)^n\right)$$

Competing interests: q_T^{cut} as small as possible ⇝ q_T^{cut} as large as possible

↪ suppress power corrections ↪ numerical stability & efficiency

INCLUSIVE $pp \to \gamma^*$ @ N³LO

[Chen, Gehrmann, Glover, AH, Yang Zhu '21]



Fixed order	$\sigma_{pp\to\gamma^*}$ (fb)		
LO	$339.62^{+34.06}_{-37.48}$		
NLO	$391.25^{+10.84}_{-16.62}$		
NNLO	$390.09^{+3.06}_{-4.11}$		
N^{3}LO	$382.08^{+2.64}_{-3.09}$ [14]		
N^{3}LO only	$q_T^{cut} = 0.63$ GeV	$q_T^{cut} \to 0$ fit	[14]
qg	$-15.32(32)$	$-15.34(54)$	-15.29
$q\bar{q}+q\bar{Q}$	$+5.06(12)$	$+5.05(12)$	$+4.97$
gg	$+2.17(6)$	$+2.19(6)$	$+2.12$
$qq+qQ$	$+0.09(13)$	$+0.09(17)$	$+0.17$
Total	$-7.98(36)$	$-8.01(58)$	-8.03

- validation against analytic result ( ) [Duhr, Dulat, Mistlberger '20]
- fully independent calculation
  - ⇝ confirmation of large negative N³LO corrections (–2% & outside of NNLO band)

The Projection-to-Born Method — Master Formula

$$\frac{\mathrm{d}\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}} = \frac{\mathrm{d}\sigma_{F,\,\mathrm{inc.}}^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}_B} + \left\{ \frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}} - \left.\frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{k-1}\mathrm{LO}}}{\mathrm{d}\mathcal{O}}\right|_{\mathcal{O}\to\mathcal{O}_B} \right\}$$

observables projected to Born
fully local counter term

- real-emission phase space: $\mathrm{d}\Phi_{\mathrm{H}+n}$

$$p_a + p_b \to p_{\mathrm{H}} + k_1 + k_2 + \ldots + k_n$$

- projection to Born: $\mathrm{d}\widetilde{\Phi}_{\mathrm{H}}$

$$\tilde{p}_a + \tilde{p}_b \to \tilde{p}_{\mathrm{H}} \qquad (\tilde{p}_a = \xi_a p_a,\quad \tilde{p}_b = \xi_b p_b)$$

on-shell: $\tilde{p}_{\mathrm{H}}^2 \equiv p_{\mathrm{H}}^2 = M_{\mathrm{H}}^2 \quad \Rightarrow \quad \xi_a\,\xi_b = \dfrac{2p_a p_b - 2(p_a+p_b)k_{1\ldots n} + k_{1\ldots n}^2}{2p_a p_b}$

rapidity: $\tilde{y}_{\mathrm{H}} \equiv y_{\mathrm{H}} \quad \Rightarrow \quad \xi_a/\xi_b = \dfrac{2p_b p_{\mathrm{H}}}{2p_a p_{\mathrm{H}}}$

↪ decay products: $p_{\mathrm{H}} \to p_1 + \ldots + p_m \qquad (p_i^\mu \to \tilde{p}_i^\mu = \Lambda^\mu{}_\nu\, p_i^\nu)$

$$\Lambda^\mu{}_\nu(p_{\mathrm{H}},\tilde{p}_{\mathrm{H}}) = g^\mu{}_\nu - \frac{2(p_{\mathrm{H}}+\tilde{p}_{\mathrm{H}})^\mu(p_{\mathrm{H}}+\tilde{p}_{\mathrm{H}})_\nu}{(p_{\mathrm{H}}+\tilde{p}_{\mathrm{H}})^2} + \frac{2\tilde{p}_{\mathrm{H}}^\mu p_{\mathrm{H},\nu}}{p_{\mathrm{H}}^2}$$

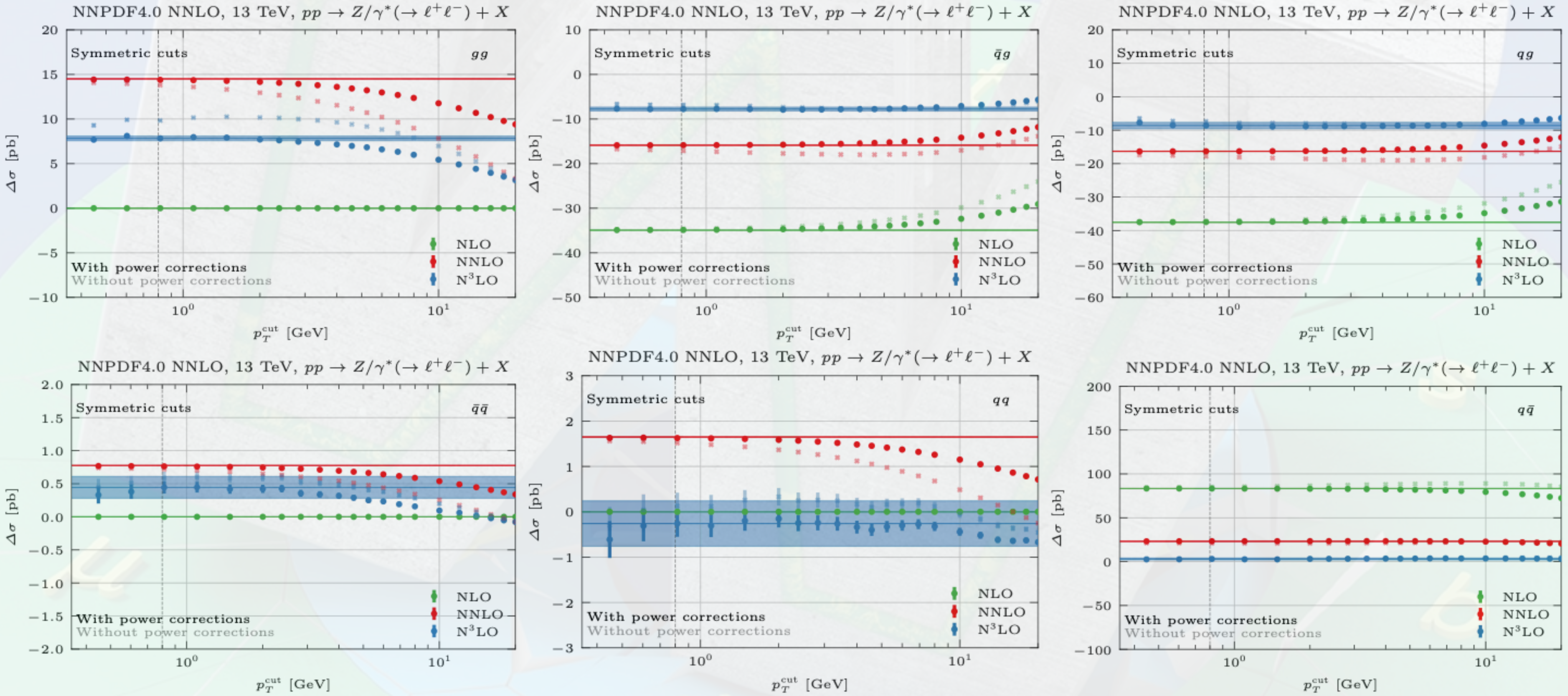
- sub-divergences
  - dealt with F+jet @ $\mathrm{N}^{k\text{-}1}\mathrm{LO}$
- $\mathrm{N}^k\mathrm{LO}$ divergences
  - fully local prescription
  - P2B (“ideal” subtraction)

BACKUP SLIDES

- Differential N3LO predictions for neutral current production with fiducial cuts
  - Resum all order contributions at N3LL using RadISH and matched to N3LO

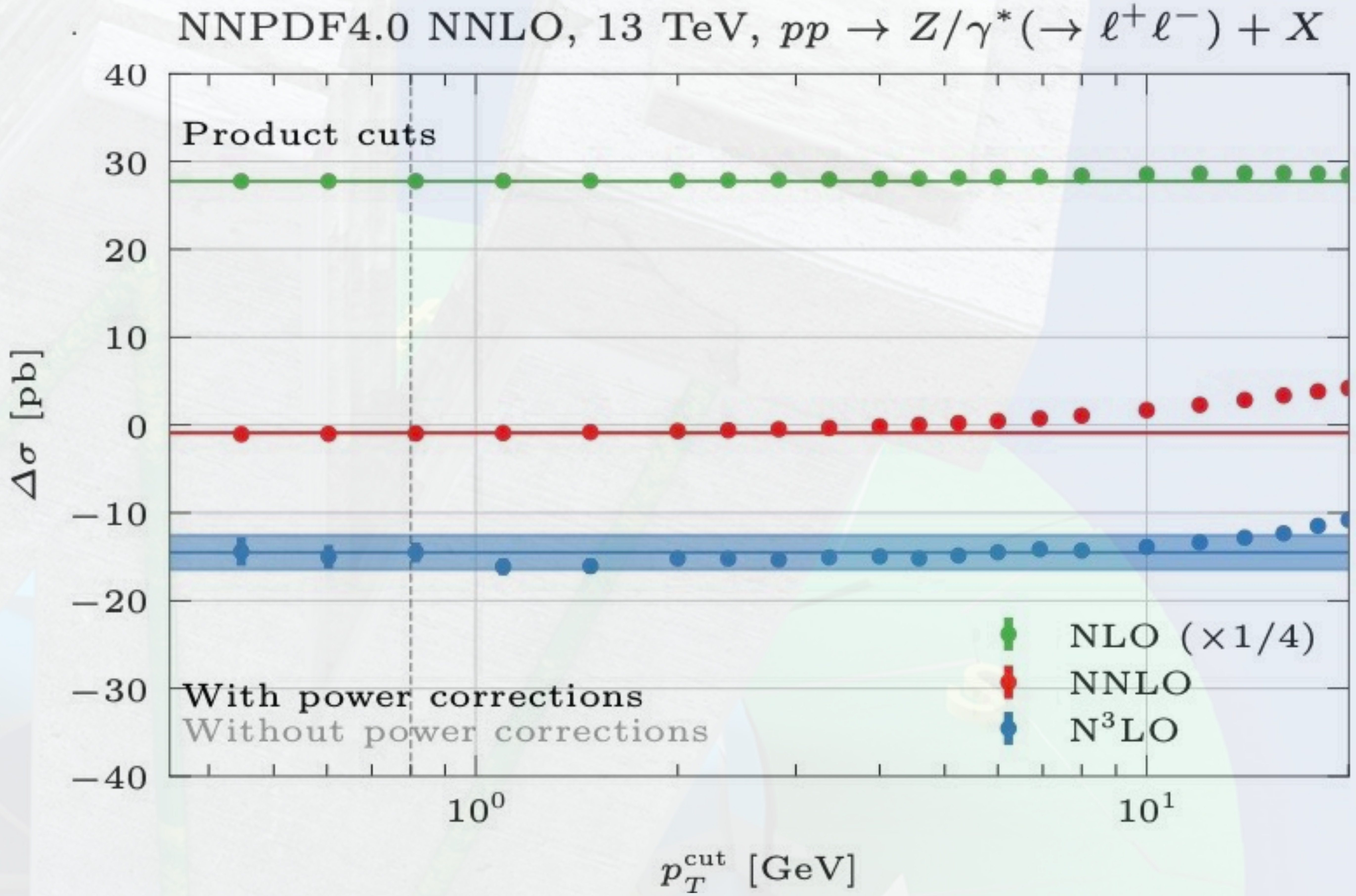
Order	σ [pb] Symmetric cuts		σ [pb] Product cuts	
k	N^{k}LO	N^{k}LO+N^{k}LL	N^{k}LO	N^{k}LO+N^{k}LL
0	$721.16^{+12.2\%}_{-13.2\%}$	—	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

XC, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli `22



BACKUP SLIDES

- Differential N3LO predictions for neutral current production with fiducial cuts
  - Apply ATLAS fiducial cuts at 13 TeV
    - Dynamical scale $\mu_F = \mu_R = \sqrt{m_{ll}^2 + p_T^{ll2}}$
    - $m_{ll} \in [66{,}116]$ GeV, $|\eta^{l^\pm}| < 2.5$
    - Symmetric cuts: $|p_T^{l^\pm}| > 27$ GeV
      Introduce power correction at $\mathcal{O}(q_T^{cut}/m_{ll})$
    - Solution:
      - Apply Lorentz Boost below q_T^{cut}
        Buonocore, Rottoli, Kallweit, Wiesemann \`21
        Camarda, Cieri, Ferrera \`21
      - Product cuts: $\sqrt{p_T^{l^+} p_T^{l^-}} > 27$ GeV
        Salam, Slade \`21
        $\min\{p_T^{l^+}, p_T^{l^-}\} > 20$ GeV
  - Typical fiducial cuts for m_T^V, p_T^V in DY production
    - Large log terms appear in $p_T^l \sim m_V/2$, $m_T^V \sim 2 \times \min[p_T^l]$, $p_T^V \sim 0$



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