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第十六届TeV物理工作组学术研讨会
暨邝宇平院士学术思想研讨会

微扰场论研究进展

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北京大学

2022/11/07-11, 北京·友谊宾馆

Outline

I. Higgs — as an introduction

II. Phenomenology — $H, W, t\ldots$

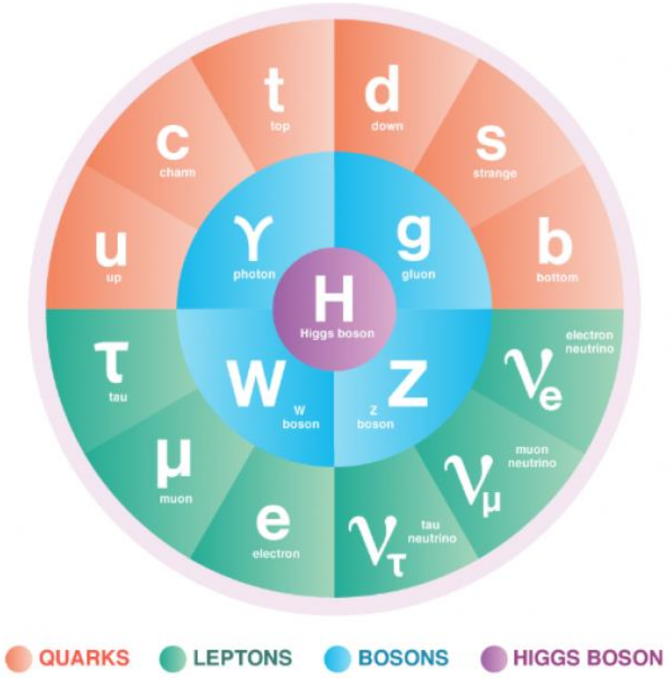
III. Methodology — Feynman integrals

IV. Summary

Higgs

➢ At the central of the Standard Model

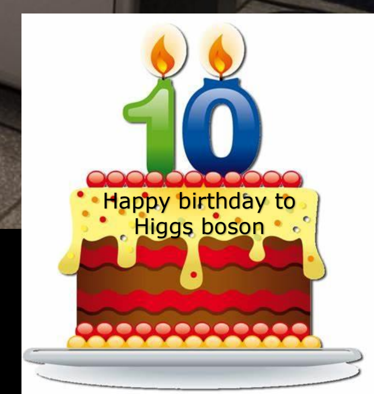
- It is the only spin-0 particle
- The source all masses of elementary particle
- That is the reason we exist ($v \sim \alpha$)
- Sensitive to new physics



➢ But

- We do not know dynamics behand the Higgs condensation
- We do not know its potential
- …

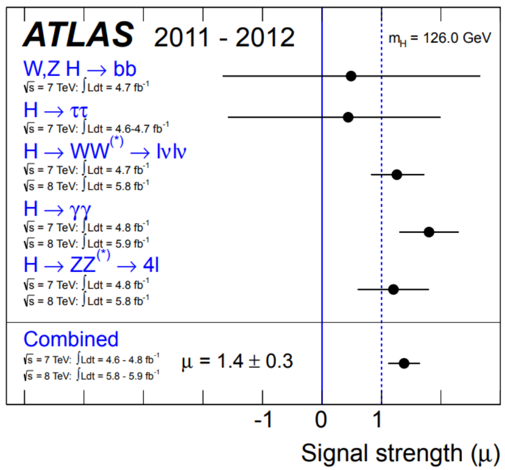
Announcement of Higgs discovery: July 4, 2012

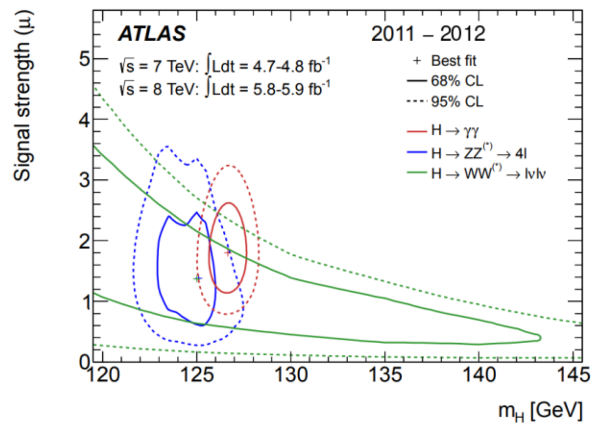


At the time of discovery

$$\mu \equiv \frac{\sigma_{\text{Exp}}}{\sigma_{\text{SM}}}$$

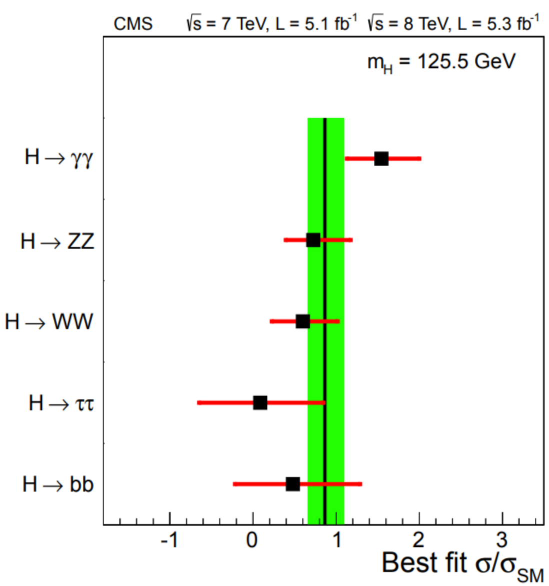
Theoretical error: negligible

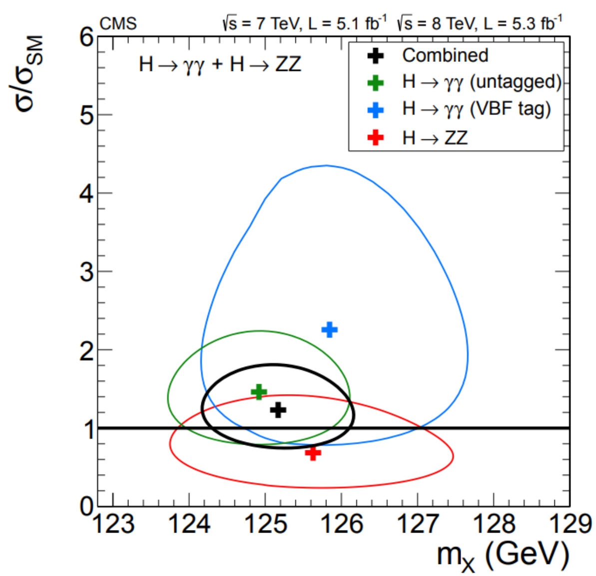




ATLAS ,1207.7214

$$\mu = 1.4 \pm 0.3$$





CMS, 1207.7235

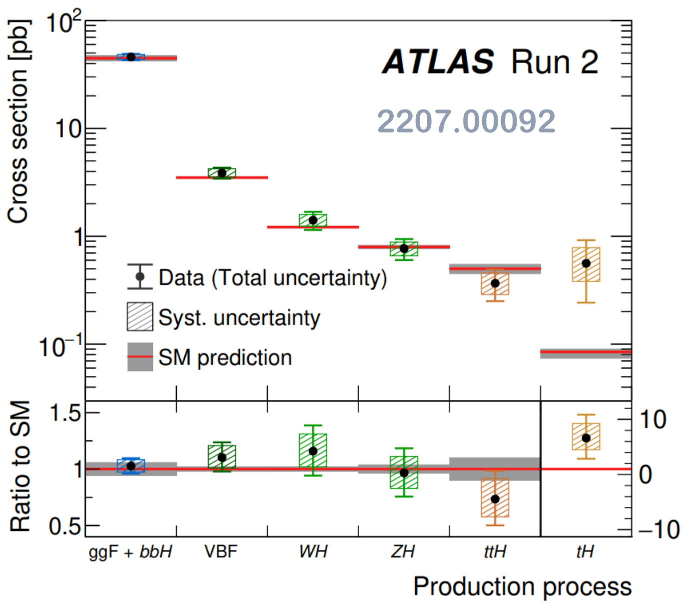
$$\mu = 0.87 \pm 0.23$$

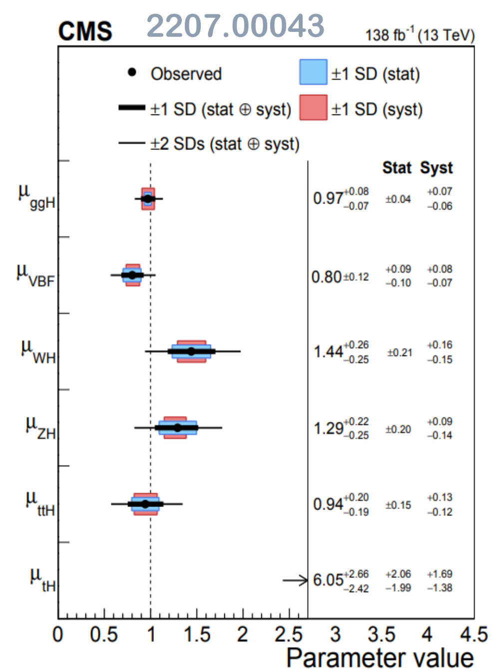
Current status

ATLAS: $\mu = 1.05 \pm 0.06 = 1.05 \pm \boxed{0.04(\text{th})} \pm 0.03(\text{exp}) \pm 0.03(\text{stat})$

CMS: $\mu = 1.002 \pm 0.057 = 1.002 \pm \boxed{0.036(\text{th})} \pm 0.033(\text{exp}) \pm 0.029(\text{stat})$

- **Theoretical error dominant!**
- **Theorists also working hard, but experimentalists are excellent…**

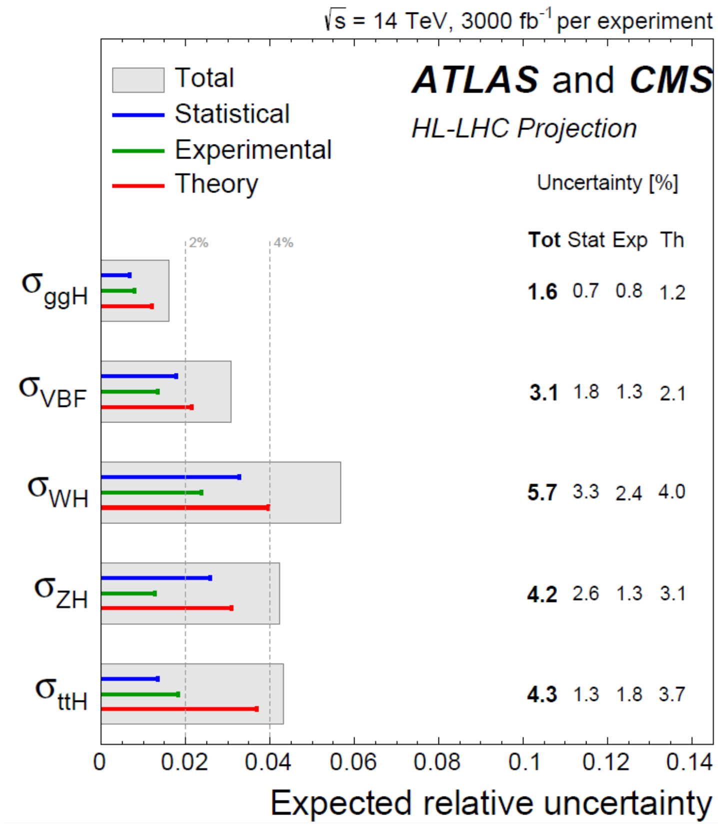




Looking ahead

➢ High-precision data expected

- Run III of LHC
- High luminosity LHC: expect $O(2\%)$ uncertainty
- Requirement: reducing theoretical uncertainties by at least a factor of 4 (one higher order in α_s !)



1902.00134

Assuming theoretical errors halved

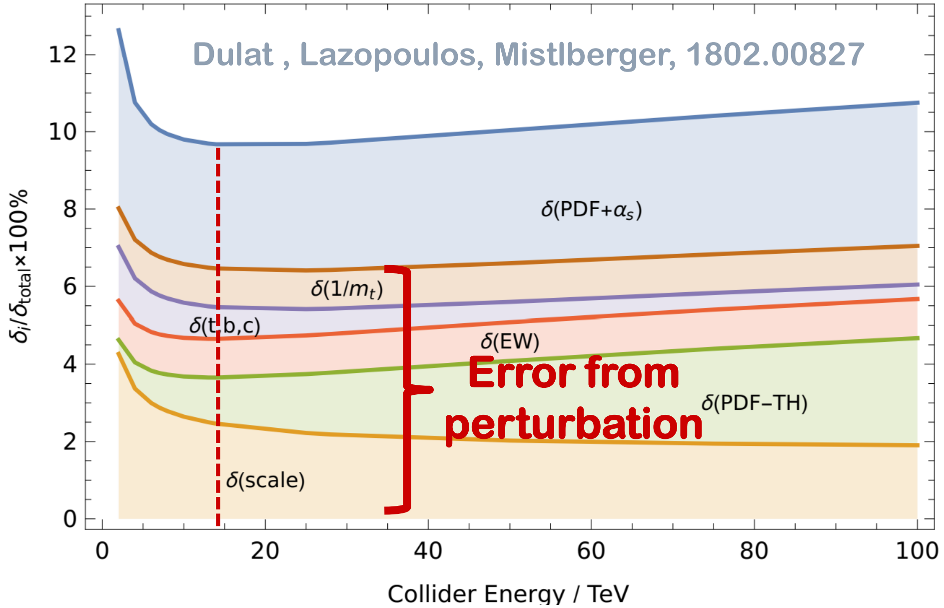
Can theory keep up?

➢ A “billion-dollar project”

- Halving total uncertainty ≈ building another LHC
- Note: LHC cost about 10 billion

Uncertainty budget

$$
\begin{aligned}
\delta(\text{theory}) &= {}^{+0.13pb}_{-1.20pb} & \left({}^{+0.28\%}_{-2.50\%}\right) && \delta(\text{scale}) \\
&+ \pm 0.56pb & (\pm 1.16\%) && \delta(\text{PDF-TH}) \\
&+ \pm 0.49pb & (\pm 1.00\%) && \delta(\text{EWK}) \\
&+ \pm 0.41pb & (\pm 0.85\%) && \delta(\text{t,b,c}) \\
&+ \pm 0.49pb & (\pm 1.00\%) && \delta(1/m_t) \\
&= {}^{+2.08pb}_{-3.16pb} & \left({}^{+4.28\%}_{-6.5\%}\right), \\
\delta(\text{PDF}) &= \pm 0.89\text{pb} & (\pm 1.85\%), \\
\delta(\alpha_S) &= {}^{+1.25pb}_{-1.26pb} & \left({}^{+2.59\%}_{-2.62\%}\right).
\end{aligned}
$$



$$\sigma = \sum_{i,j} dx_1 dx_2 \boxed{f_{i/p}(x_1) f_{j/p}(x_2)} \boxed{\hat{\sigma}_{ij}} + O(\Lambda^2/M^2)$$

PDFs
More input data, Better fitting methods

$\alpha_s = 0.118 \pm 0.001$
Lattice is the main method, other better way?

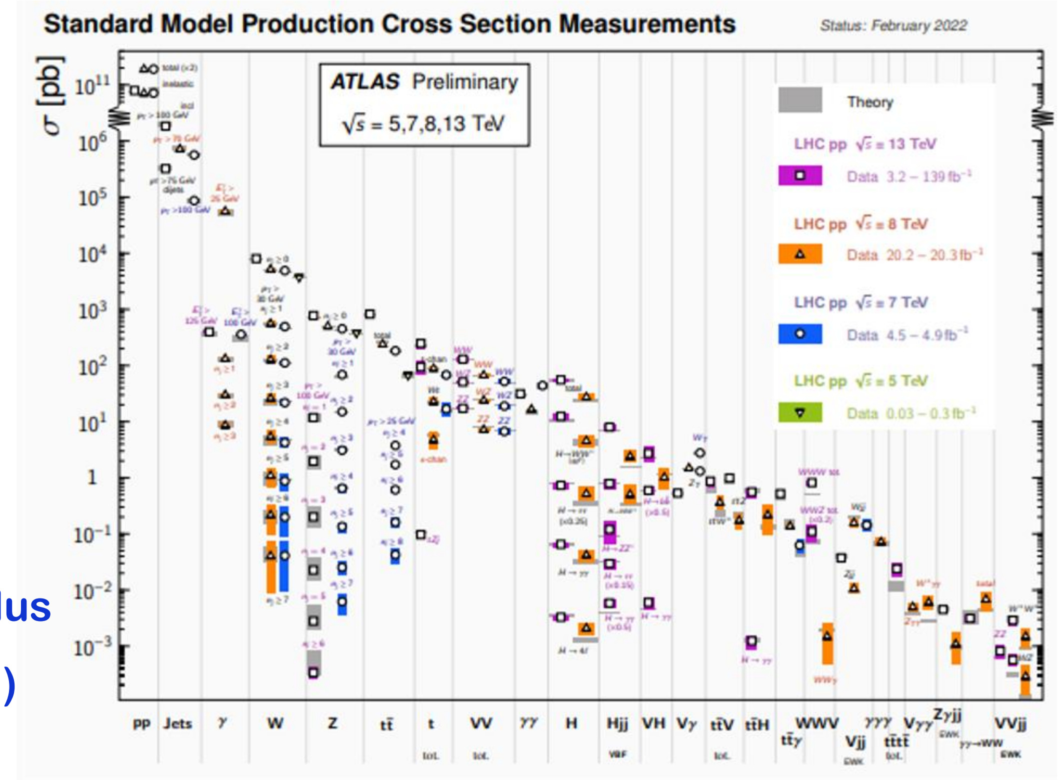
Theory
Calculate higher orders in perturbation theory

Era of precision physics at the LHC

➢ High-precision data

ATL-PHYS-PUB-2022-009

- Many observables probed at precent level precision
- Discovery via precision: search anomalous deviations from SM



- At least NNLO QCD and NLO EW corrections generally required (plus parton shower, resummation, etc.)

Automatic NNLO correction is highly demanded

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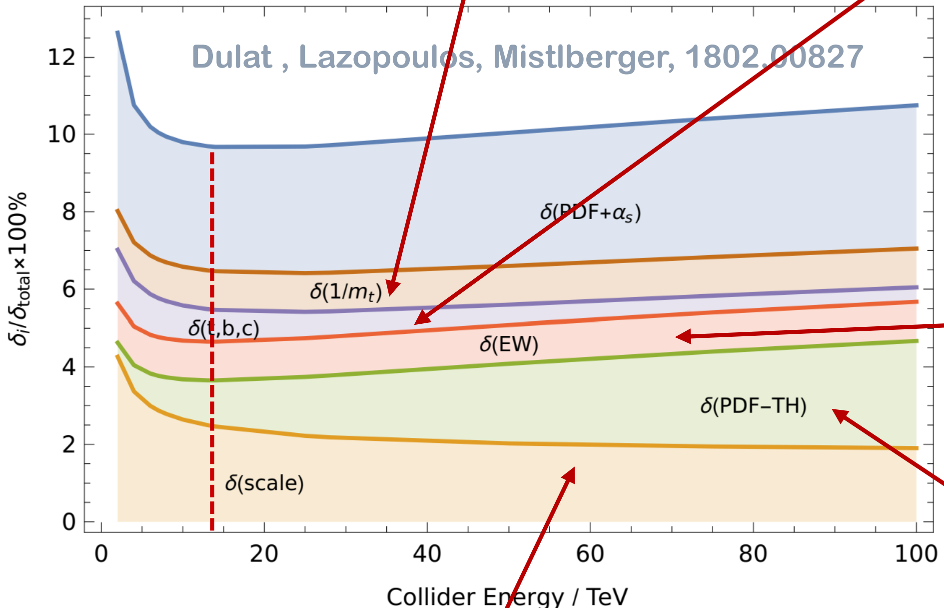
$pp \to H$

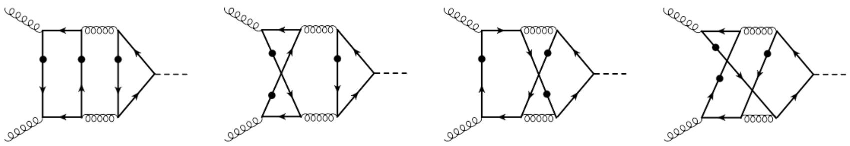


- **Eliminated: exact NNLO calculation**

Czakon, Harlander, Klappert, Niggetiedt, 2105.04436

- **Reduced by calculating leading contribution in power log expansion**

T. Liu, Modi, Penin, 2111.01820



- **Reduced by calculating light-quark mixed QCD-EW contributions**

Becchetti, et. al., 2010.09451

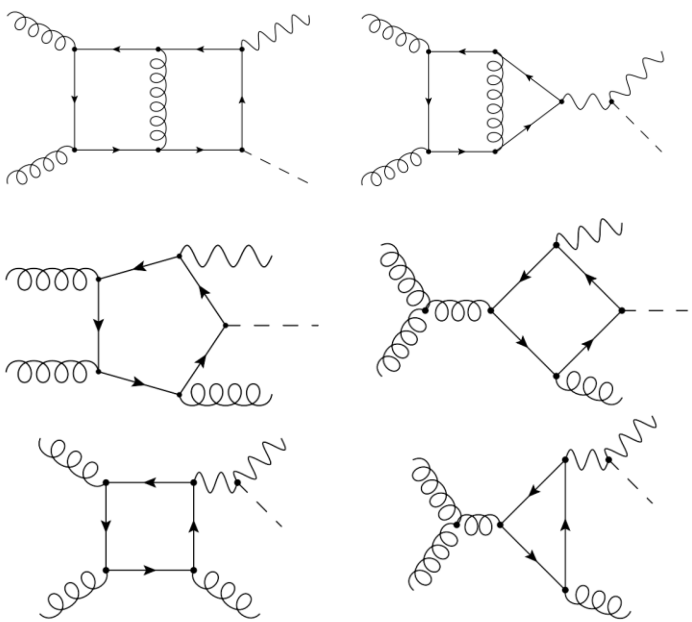
Require $\mathrm{N^3LO}$ PDFs, and thus
4-loop splitting functions
(Low moments) Moch, et. al., 2111.15561

Require $\mathrm{N^4LO}$ perturbative calculation

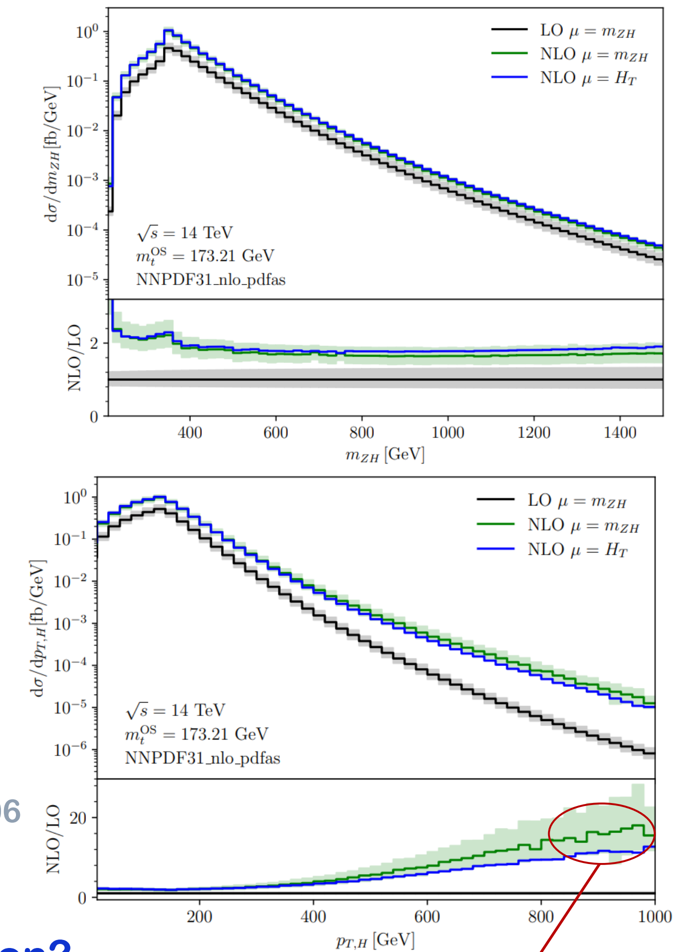
Require: $2 \to 1$ processes at 4 loops!!!

$pp \to H + Z$

- **Probe Higgs-Z coupling**



L. Chen, et. al., 2204.05225

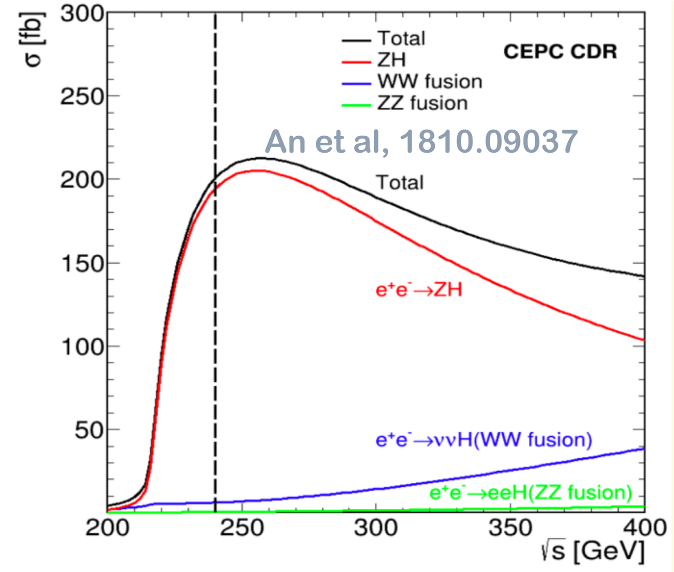


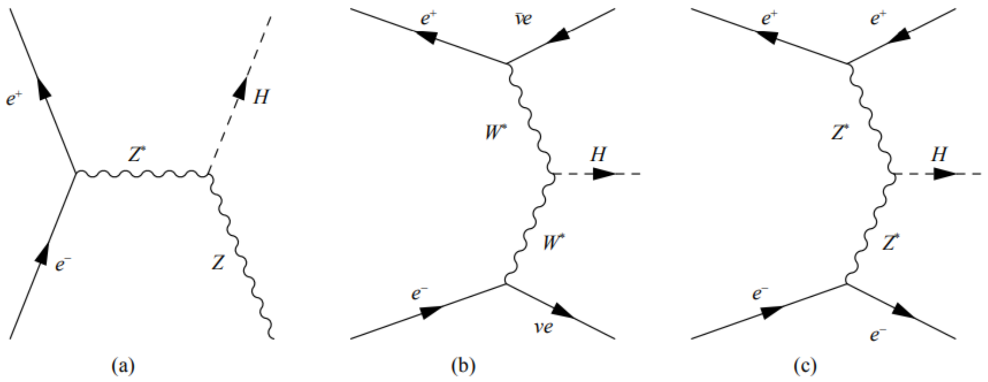
K factor~10-20

- **Total XS agree with** Wang, Xu, Xu, L.L.Yang, 2107.08206
- **Huge corrections at high p_T!**
- **Contribution from low p_T Z, due to fragmentation?**

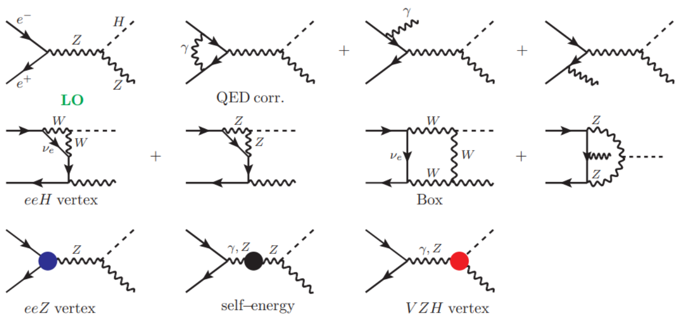
$e^+e^- \to H + Z$

See also Prof. Xinchou Lou's talk





- $\delta\sigma_{HZ} \sim 0.51\%$ CEPC Study Group, 1811.10545



➢ Up to EW+QCD correction

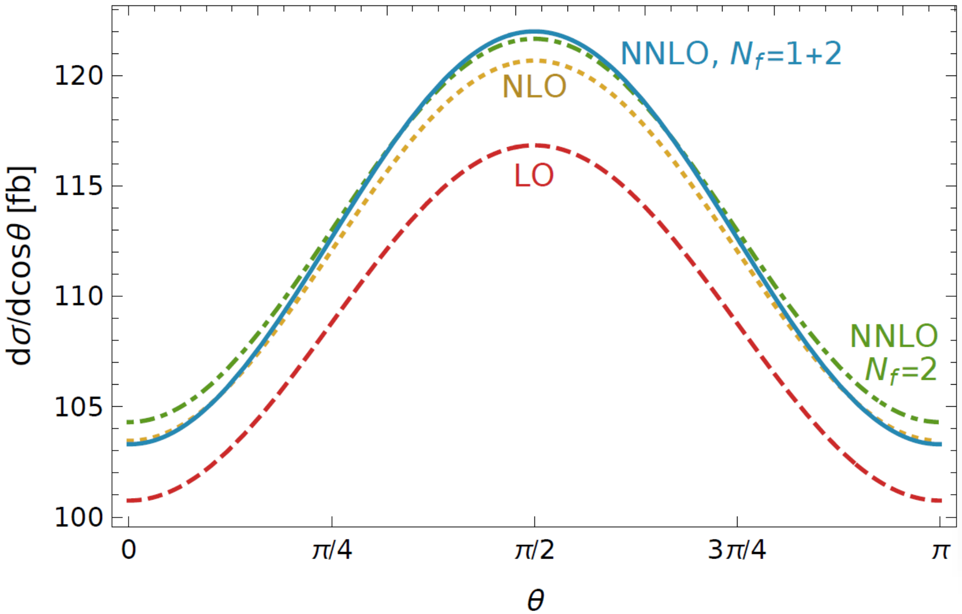
- 1.3% **($\overline{MS}, \alpha(M_Z)$)** Gong, Li, Xu, Yang, Zhao,1609.03955
- $0.4 - 1.3\%$ **(Scheme dependence: $\alpha(0), \alpha(M_Z), G_\mu$)** Sun, Feng, Jia, Sang, 1609.03995
- $O(\alpha^2)$ **EW correction is indispensable!**

$e^+e^- \to H + Z$

➢ The first two-loop EW correction with 4 legs

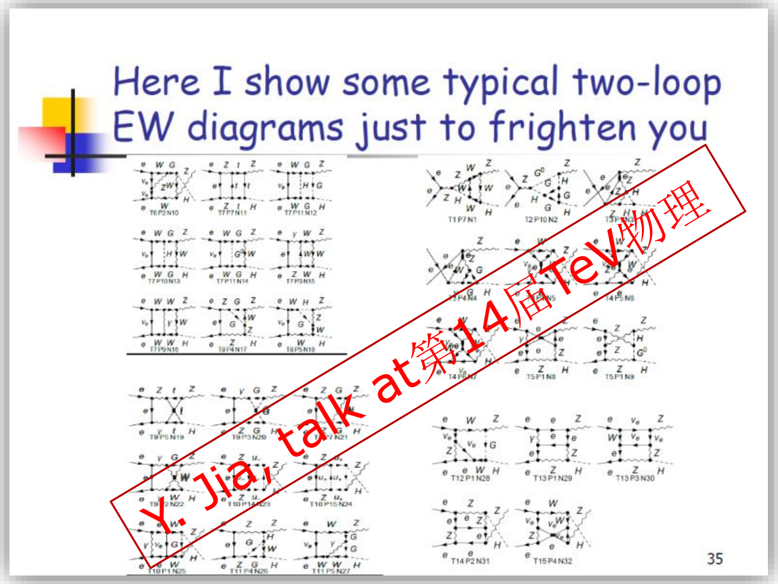
Freitas, Song, 2209.07612

- **With Fermion loops**



Chen, Guan, He, Li, Liu, YQM, 2209.14953

- **Complete EW two-Loop (25377 Feynman diagrams)**



Calculation of real emissions are under going, full NNLO EW corrections will be obtained soon.

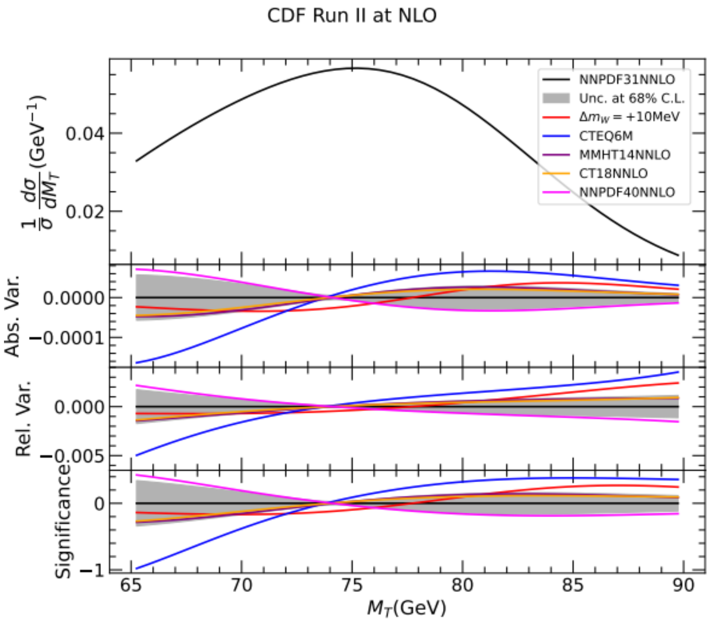
$pp \to W$

➢ 7 standard deviations for W mass!

Science 376 (2022) 6589, 170-176

- **CDF:** $M_W = 80,433.5 \pm 6.4_{\text{stat}} \pm 6.9_{\text{syst}}$ MeV
- **SM precision fit:** $M_W = 80,357 \pm 6$ MeV

Gao, Liu, Xie, 2205.03942



➢ PDF uncertainties

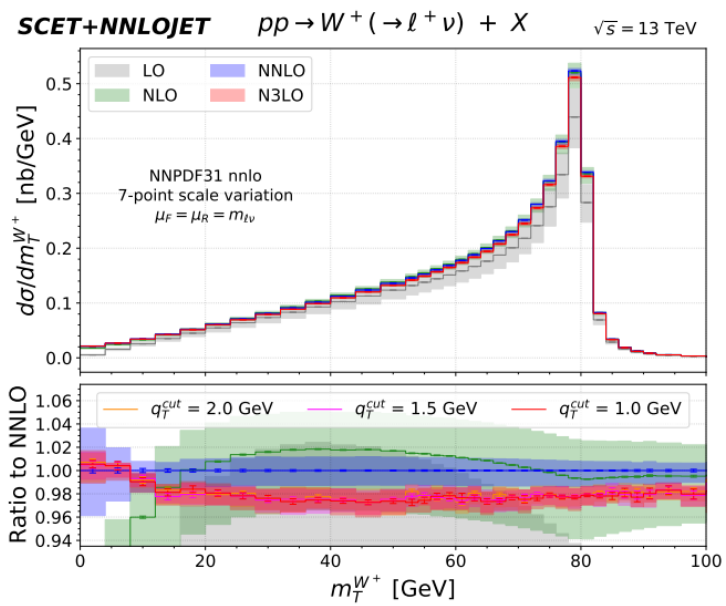
- Spread of predictions from different PDF sets can be much larger than the PDF uncertainty predicted by a specific PDF set
- PDF variation cannot explain CDF deviations

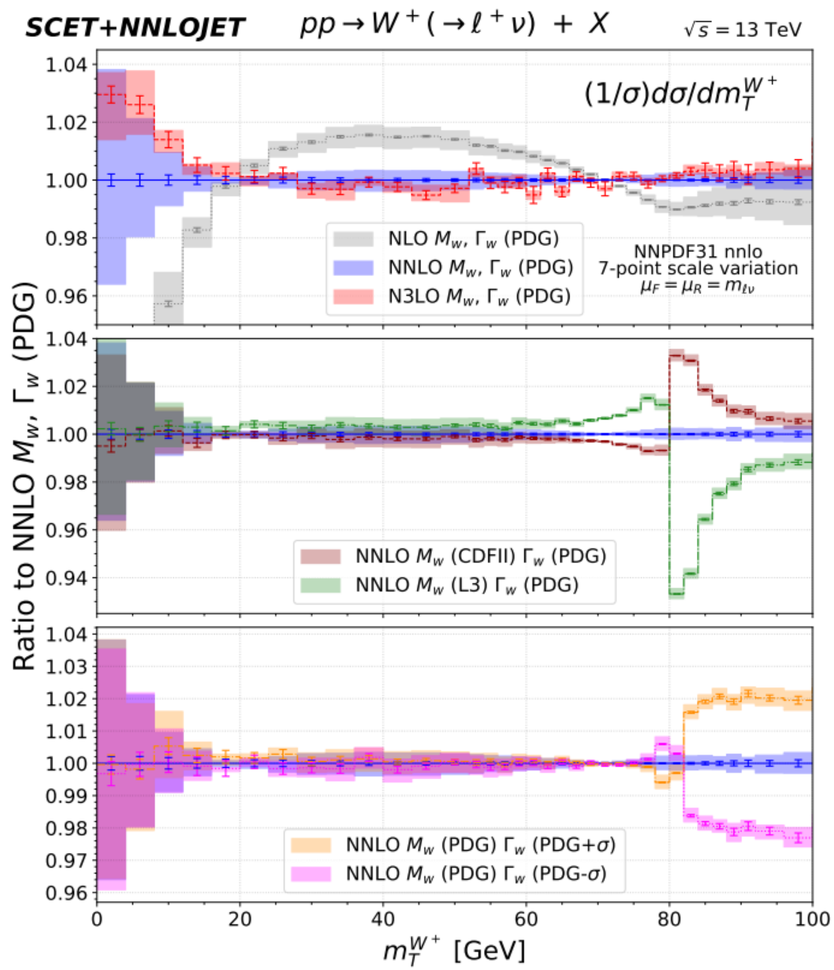
$pp \to W$

➢ N3LO prediction

X.Chen, Gehrmann, Glover, Huss, Yang, Zhu, 2205.11426

- Reduce W mass uncertainty to 100MeV
- Not enough to explain CDF anomaly
- LHC aims for 10MeV uncertainty, further (non) perturbative improvements needed



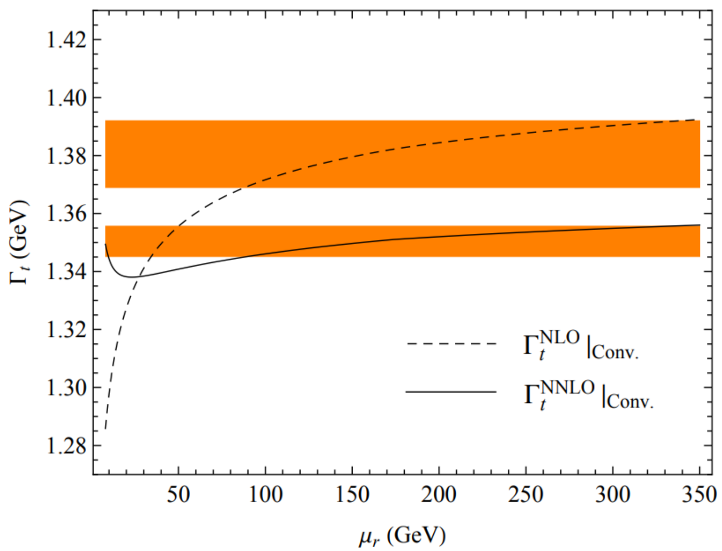


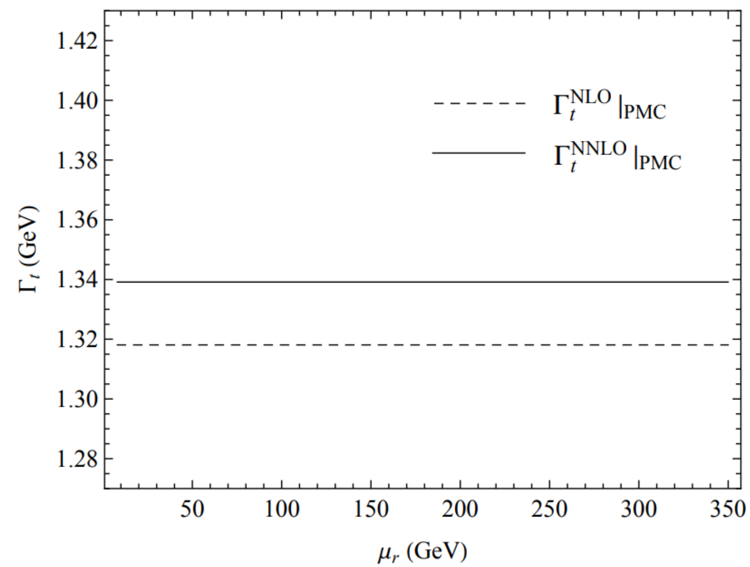
Better choice of scale

Meng, Wang, Sun, Luo, Shen, Wu, 2202.09978

➢ Top quark decay

- Principle of Maximal Conformality: absorbing all-order beta functions to α_s
- Error dominated by $\Delta m_t = \pm 1$GeV
- The PMC improved predictions for the top-quark decay are helpful for detailed studies of properties of the top-quark.





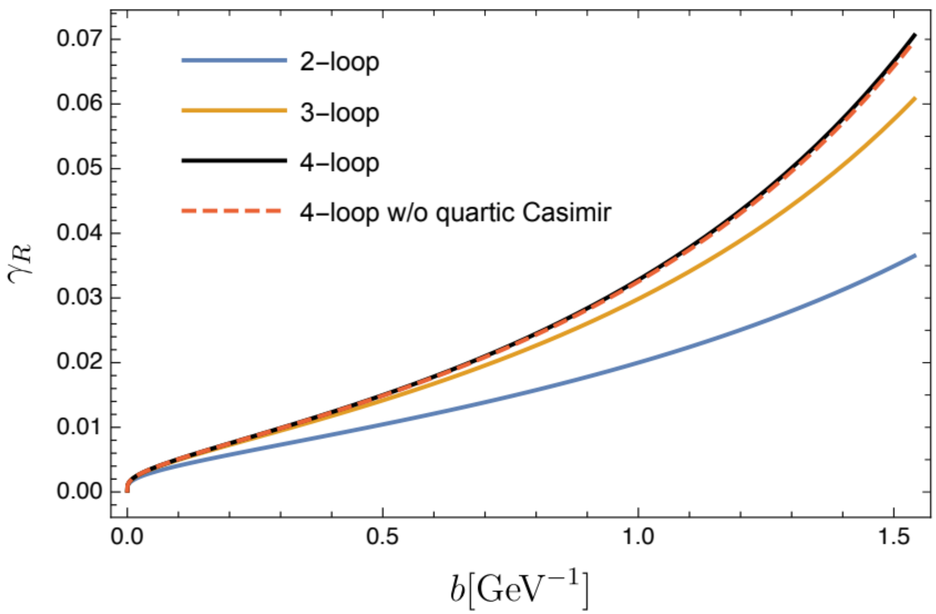
$$\Gamma_t^{\mathrm{tot}}|_{\mathrm{PMC}} = 1.3383^{+0.0275+0.0016}_{-0.0271-0.0017} \pm 0.0023 \text{ GeV}$$

Anomalous dim. and resummation

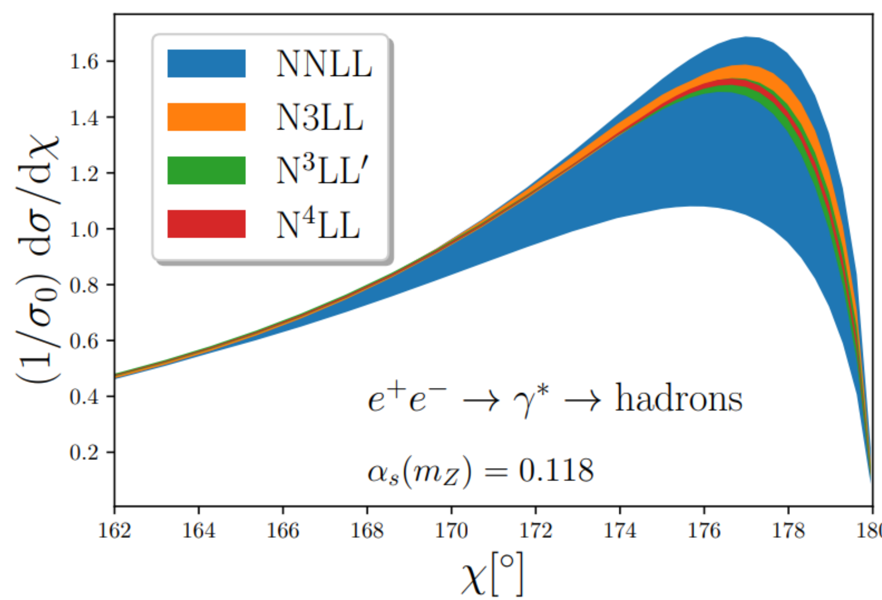
➢ 4-loop rapidity anomalous dimension

- Result enables the resummation of the Energy-Energy Correlators (EEC) in the back-to-back region at N4LL
- A reduction of perturbative uncertainties on the resummed cross section to below 1%.

Moult, Zhu, Zhu, 2205.02249



Duhr, Mistlberger, Vita, 2205.02242



Quarkonia

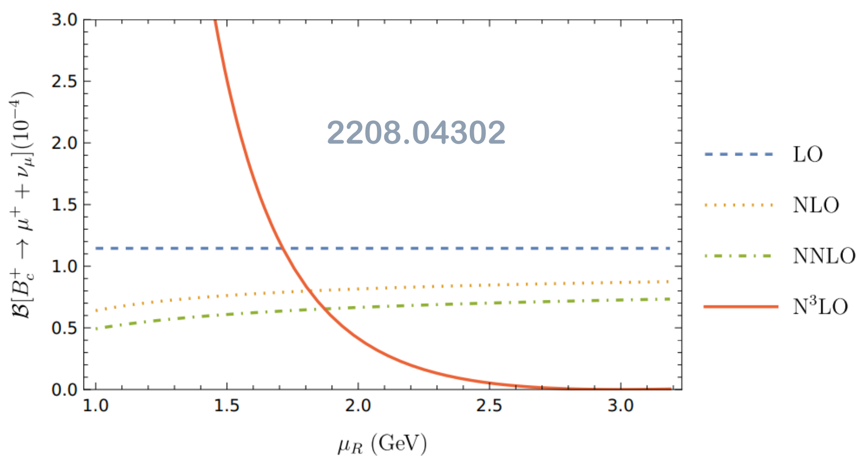
➢ Incredible progress!!!

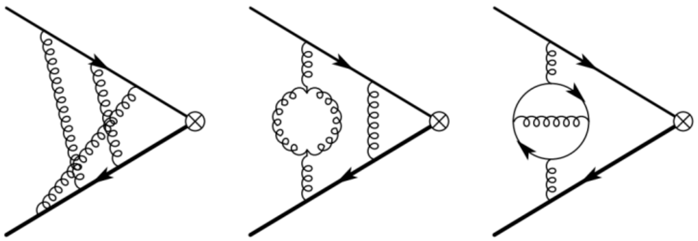
- Many important processes have been pushed to N2LO or N3LO

1. 2202.11615: Sang, Feng, Jia, Mo, Zhang, N2LO for $e^+e^- \to J/\psi + \chi_{cJ}$
2. 2205.06124: Zhang, Sang, Zhang, Two-loop for $\Upsilon \to$double charmonia
3. 2207.14259: Feng, Jia, Mo, Pan, Sang, Zhang, N3LO for leptonic width of vector quarkonium
4. 2208.04302: Feng, Jia, Mo, Pan, Sang, Zhang, N3LO for the decay constant of B_c
5. 2208.10118: Sang, Yang, Zhang, N2LO $Z \to P$ wave quarkonium
6. 2209.15521: Tao, Zhu, Xiao, N2LO B_c and B_c^* decay constant
7. 2210.02979: Sang, Zhang, Zhou, N3LO B_c^* decay constant

- Taking advantage of AMFlow package Liu, YQM, 2201.11669

➢ Strange high order behavior





- Breaking down of perturbation theory?
- Unknown mechanism to be explored?

State-of-the-art computation

- 2→2 process with massive particles at two-loop order: almost done

$$g+g\to t+\bar{t}, \qquad g+g\to H+H(g), \qquad e^+e^- \to HZ$$

- Frontier in the following decade:
  - 2→3 processes at two loops (3j/γ, V/H+2j $t\bar{t}$+j, $t\bar{t}H$,…)
  - 2→2 processes at three loops (2j/γ, V/H+j, $t\bar{t}$, HH, …)
  - 2→1 processes at four loops (j, V/H, AP kernel)
  - Very a few obtained, usually no exact pure virtual contribution

- Very challenging

Davies, Herren, Steinhauser, 1911.10214

  - Four-loop $g+g\to H$ (NNLP in HTL): 860 days (wall time!)
  - Bottleneck: computing multi-loop Feynman integrals

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Main strategy

- **Reduce all FIs to Master Integrals (bases), mainly integration-by-parts. Improvements:**

  1) Finite field: solving intermediate express swell
  Manteuffel, Schabinger, 1406.4513

  2) Syzygy equations: trimming IBP system
  Gluza, Kajda, Kosower, 1009.0472
  Larsen, Zhang, et. al., 1511.01071, 1805.01873, 2104.06866

  3) Block-triangular form: minimize IBP system
  Liu, YQM, 1801.10523, Guan, Liu, YQM, 1912.09294

- **Calculate MIs**

  - Differential equations
    - Analytical : if canonical integrals exist
      Henn, 1304.1806
    - Numerical: auxiliary mass flow
      Liu, YQM, Wang, 1711.09572
  - Difference equations
  - Sector decomposition
  - Mellin-Barnes representation
  - ...

Improvement for tensor reduction

➢ Reduction using auxiliary momenta

Feng, Li, 2203.16881
Feng, Gong, Li, 2204.03190
Feng, 2209.09517

- Changing tensor integrals to scalar integrals
- Applied to one loop and two-loop sunrise diagram

$$I^{(r_1,r_2)}_{a_1,a_2,a_3} \equiv \int \frac{d^D\ell_1}{i\pi^{D/2}} \frac{d^D\ell_2}{i\pi^{D/2}} \frac{(2\ell_1\cdot R_1)^{r_1}(2\ell_2\cdot R_2)^{r_2}}{D_1^{a_1}D_2^{a_2}D_3^{a_3}}$$

where the propagators are

$$D_1 \equiv \ell_1^2 - M_1^2\,,\quad D_2 \equiv \ell_2^2 - M_2^2\,,\quad D_3 \equiv (\ell_1+\ell_2-K)^2 - M_3^2$$

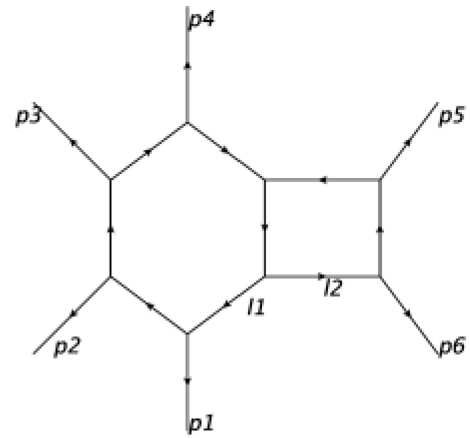
➢ Reduction using generation function

- In principle can reduce any degree of tensor integrals

$$I_{bub}(t,R) \equiv \int d\ell \frac{e^{t(2\ell\cdot R)}}{(\ell^2-M_0^2)((\ell-K)^2-M_1^2)}$$

How to find canonical integrals?

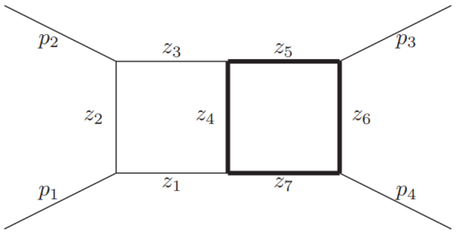
➢ Using leading singularity

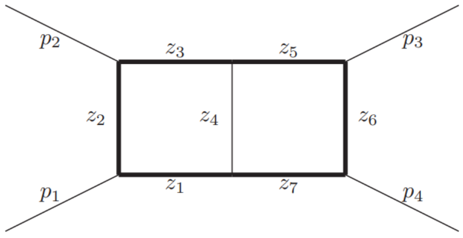


Henn, Peraro, Xu, Zhang, 2112.10605

- Find canonical integrals with 6 legs for the first time
- An important step in the analytic calculation of planar two-loop six-particle Feynman integrals

➢ Using intersection theory





Chen, Jiang, Ma, Xu, Yang,2202.08127

- A novel method, very promising
- Success in examples with massive propagators

See also Yifan Wang’s talk for tW production at two loops

Determining FIs by linear algebra

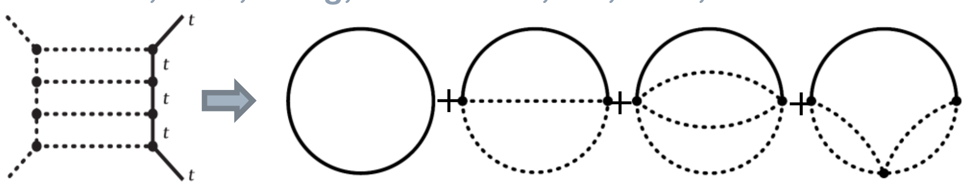
- **Since 90s': IBP reduction**

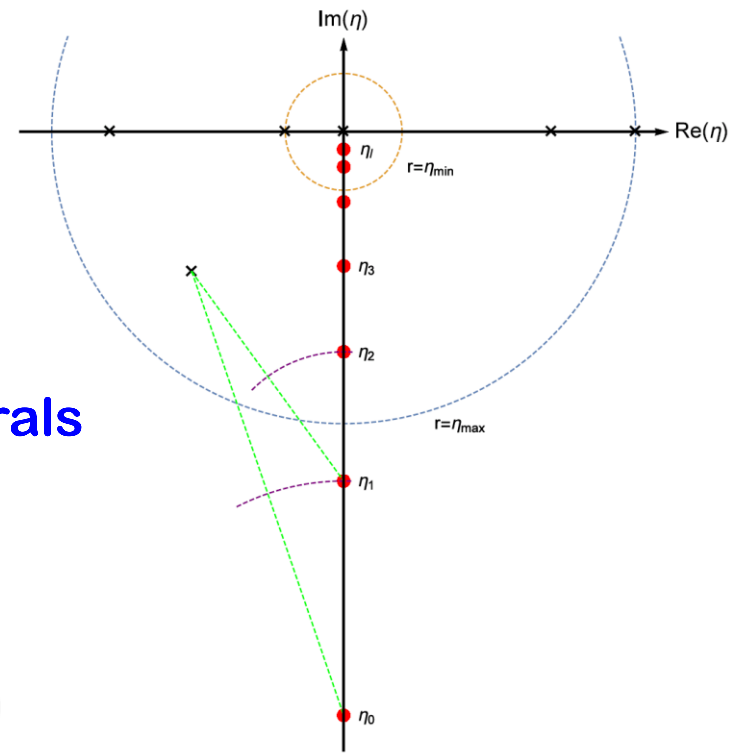
Laporta, Remiddi, 9602417
Gehrmann, Remiddi, 9912329

FIs ≜ Linear algebra ⊕ Master integrals

- **2017-2021: Auxiliary mass flow**

Liu, YQM, Wang, 1711.09572, Liu, YQM, 2107.01864

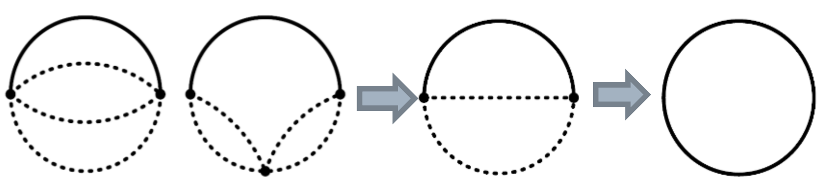




FIs ≜ Linear algebra ⊕ Vacuum integrals

- **Since 2022: loop reduction**

Liu, YQM, 2201.11637 :



FIs ≜ Linear algebra

AMFlow package

- Automatic calculation of general multi-loop integrals for the first time!

Liu, YQM, 2201.11669

https://gitlab.com/multiloop-pku/amflow

AMFlow: A Mathematica package for Feynman integrals computation via auxiliary mass flow

Xiao Liu (Peking U., SKLNPT and Oxford U., Theor. Phys.), Yan-Qing Ma (Peking U., SKLNPT and Peking U., CHEP and CICQM, Beijing)

Jan 27, 2022

30 pages

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Published: Feb, 2023

e-Print: 2201.11669 [hep-ph]

DOI: 10.1016/j.cpc.2022.108565 (publication)

View in: ADS Abstract Service

pdf cite claim reference search 40 citations

- Already been used in many physical processes

Packages on the way

	Generate amplitudes	Manipulate amplitudes	Integral reduction	Master integrals calculation
Package used	FeynArts or Qgraf	LoopCalc	Blade	AMFlow
Notes	Open source	(Since 2009) To be released	(Since 2020) To be released	Open source

Implementing block-triangular form, usually improves efficiency by $O(10^2)$

- Fully automatic, valid to any-loop order
- Main challenge: integral reduction is time/resource consuming

The dawn of automatic multi-loop calculation!

Automatic NLO correction obtained more than 10 years ago: MadGraph, Helac, etc.

Summary

- Understanding QCD (& EW) corrections is crucial for drawing conclusions from precision measurements and direct searches for new physics
- Remarkable progress from experimental side, big pressure for theorists
- Significant progress in perturbative calculation, still huge gap from requirements
- Feynman integrals calculation remains the main bottle neck, stay tune

Thank you!