



# Title: Unitarity bound on extensions of Higgs sector

**Speaker:** 卢伯强 (Bo-Qiang Lu)  
@Huzhou University (湖州师范学院)

**Cooperate with:** Da Huang (黄达)

Based on: arXiv:xxx (in progress)

**Email:** [bqlu@zjhu.edu.cn](mailto:bqlu@zjhu.edu.cn)

16th TeV物理及邝宇平院士学术思想研讨会

# Outline

## 1. Background

- Dark matter
- Electroweak phase transition (EWPT)

## 2. Perturbative Unitarity

## 3. Bases of scattering matrices

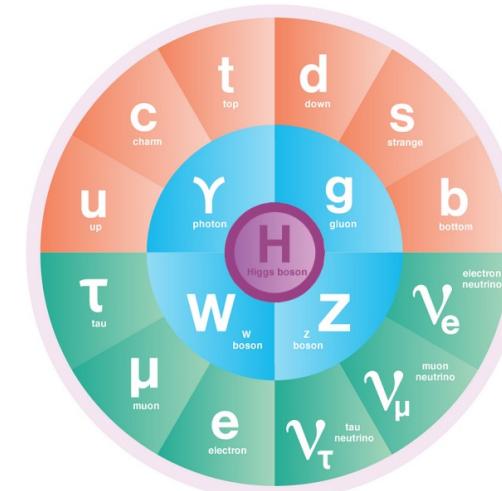
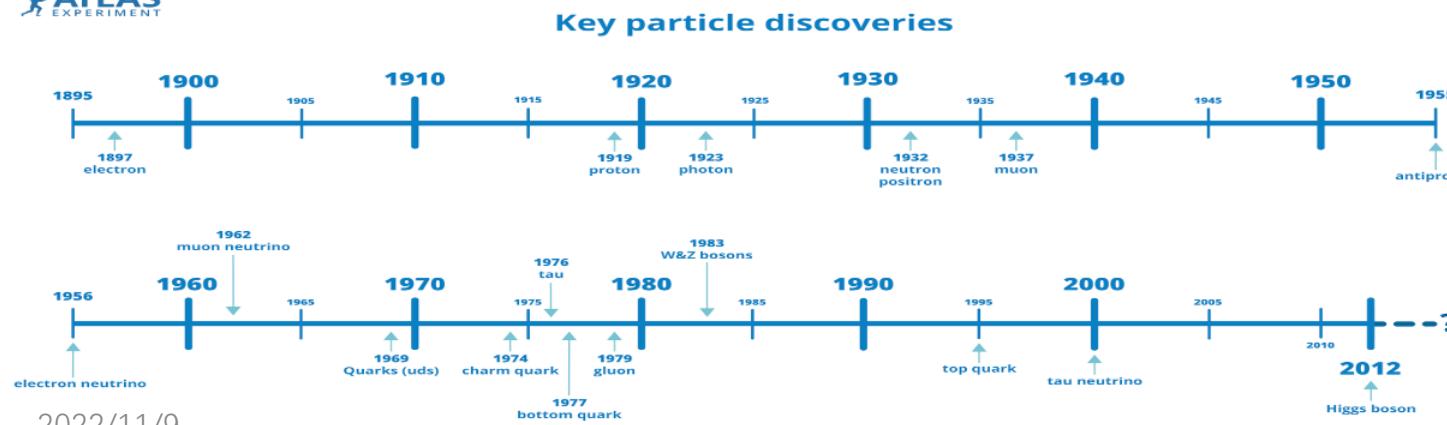
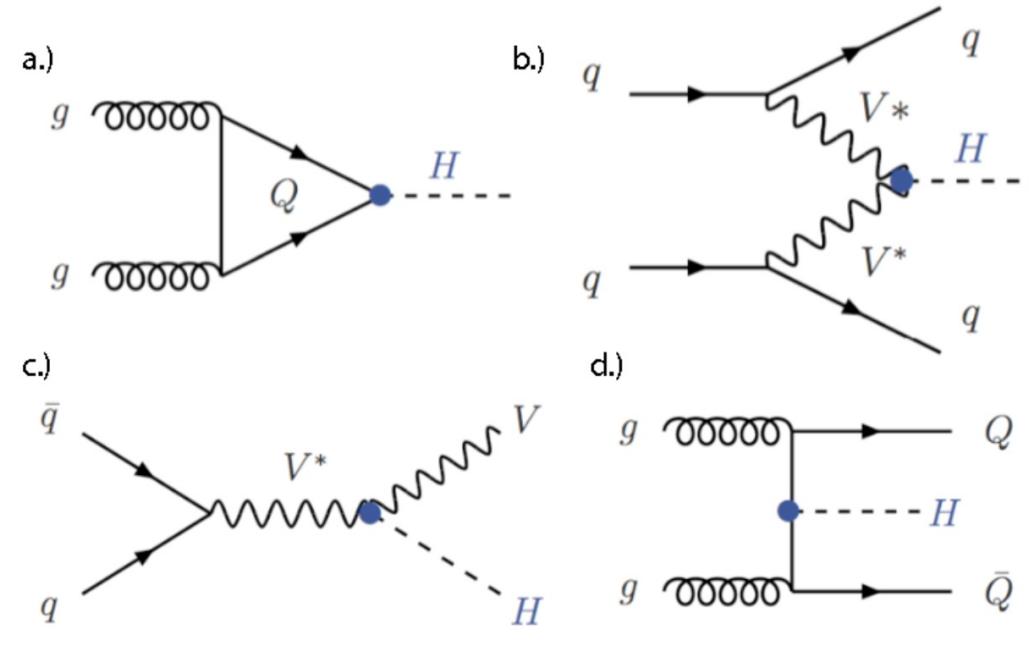
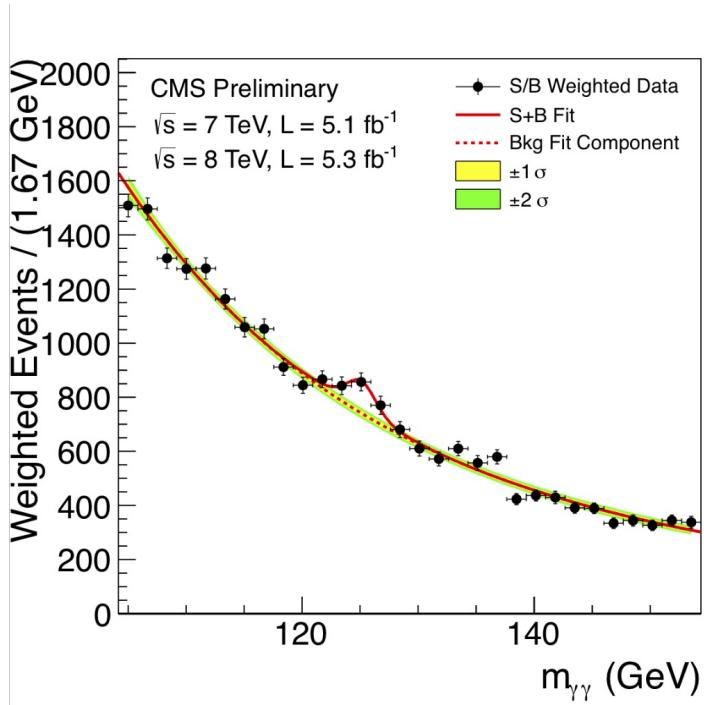
## 4. Scattering matrix

- The extension of 2HDM with a real triplet
- The extension of 2HDM with a complex triplet

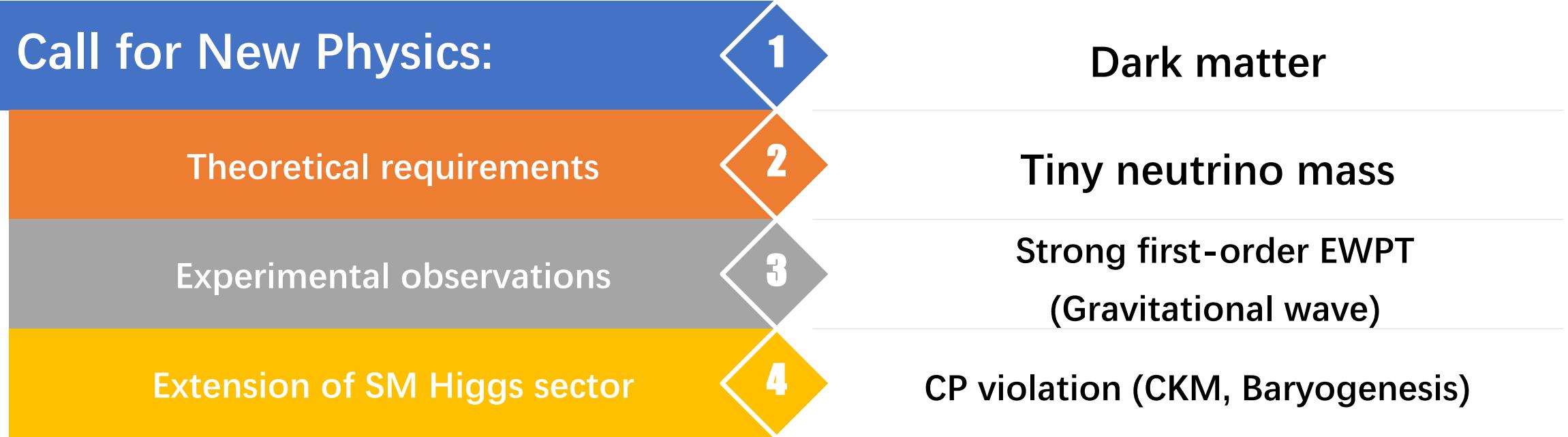
## 5. Application

## 6. Summary

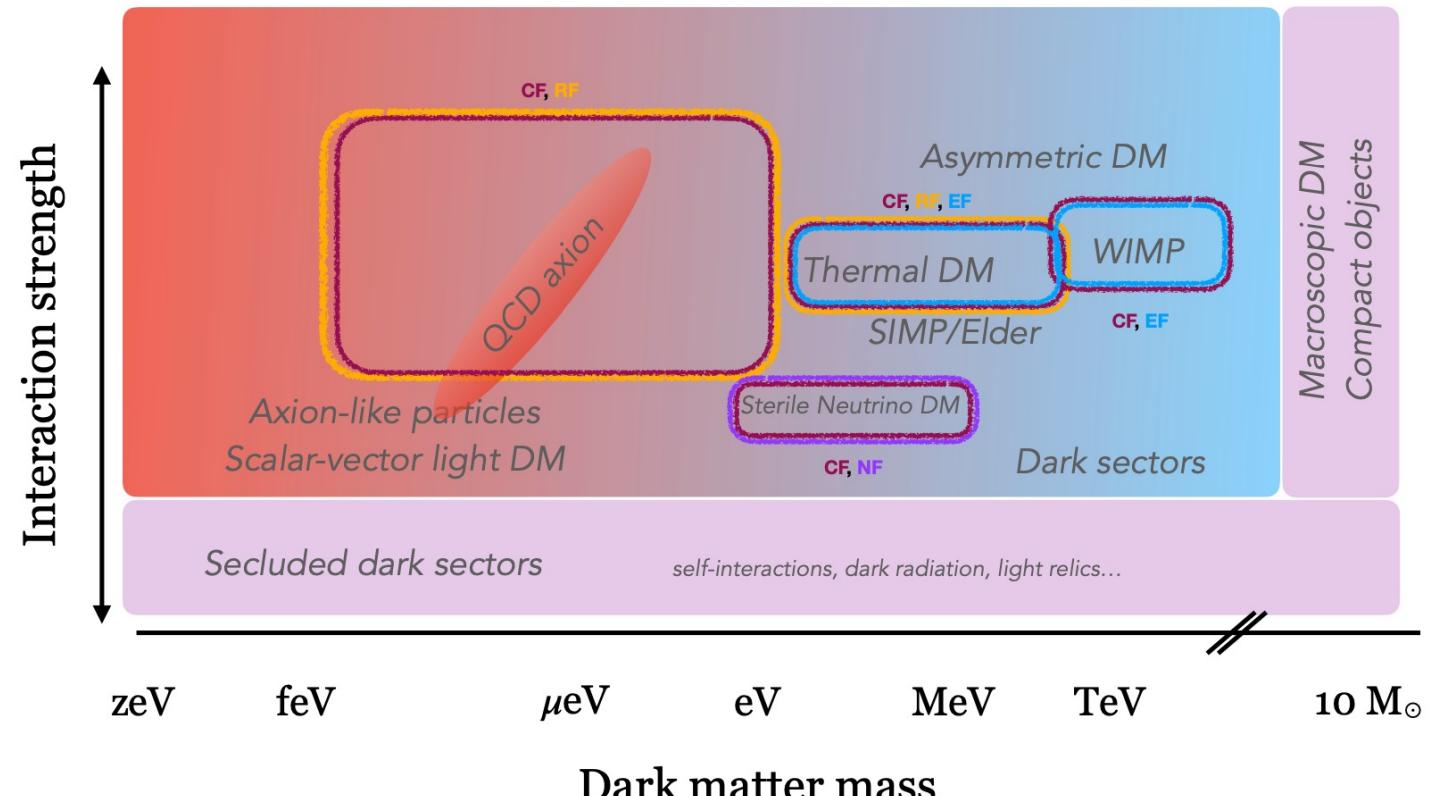
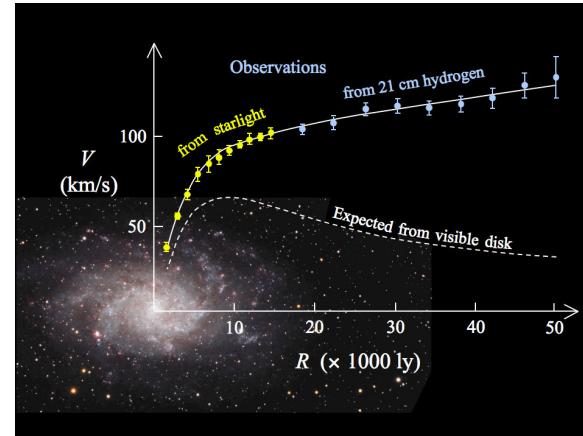
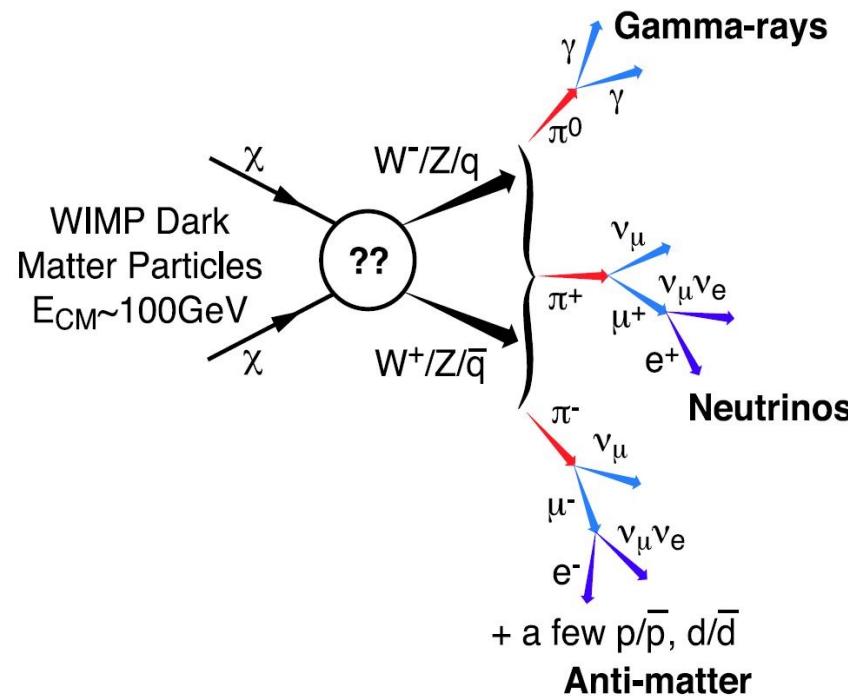
# Background



## Call for New Physics:



# WIMP dark matter

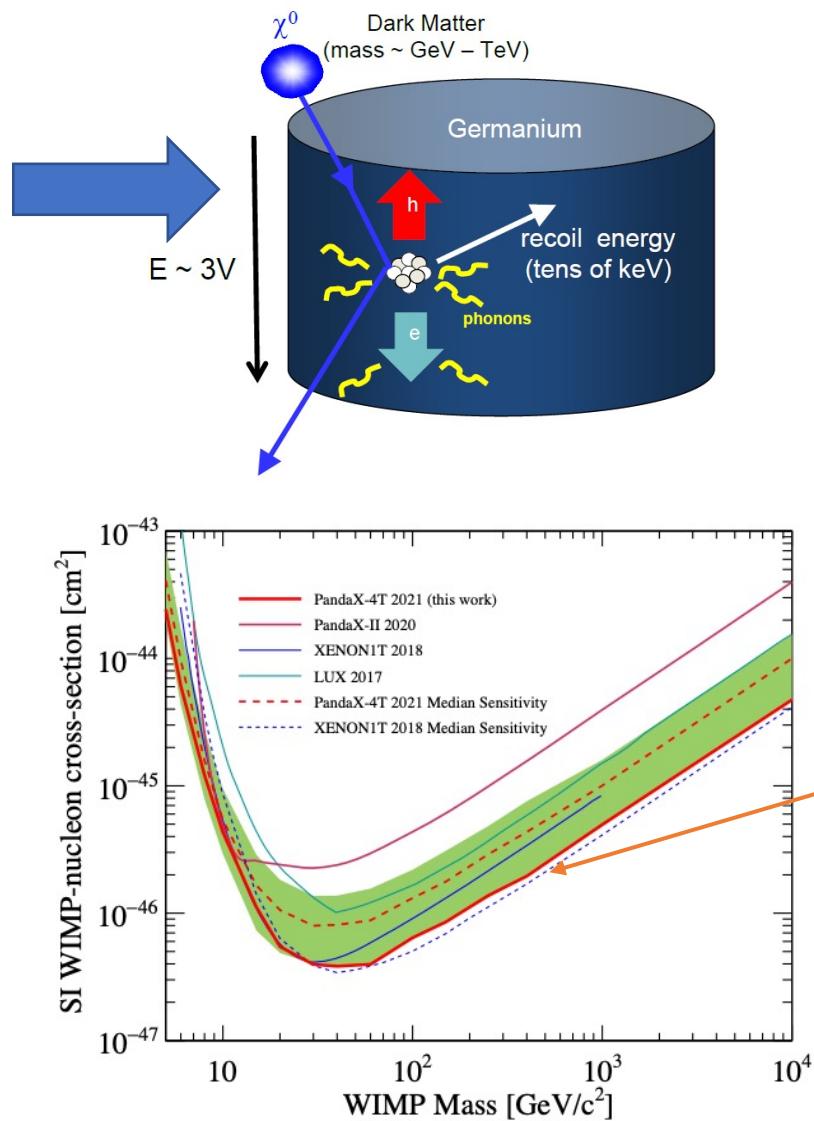


# WIMP direct detections



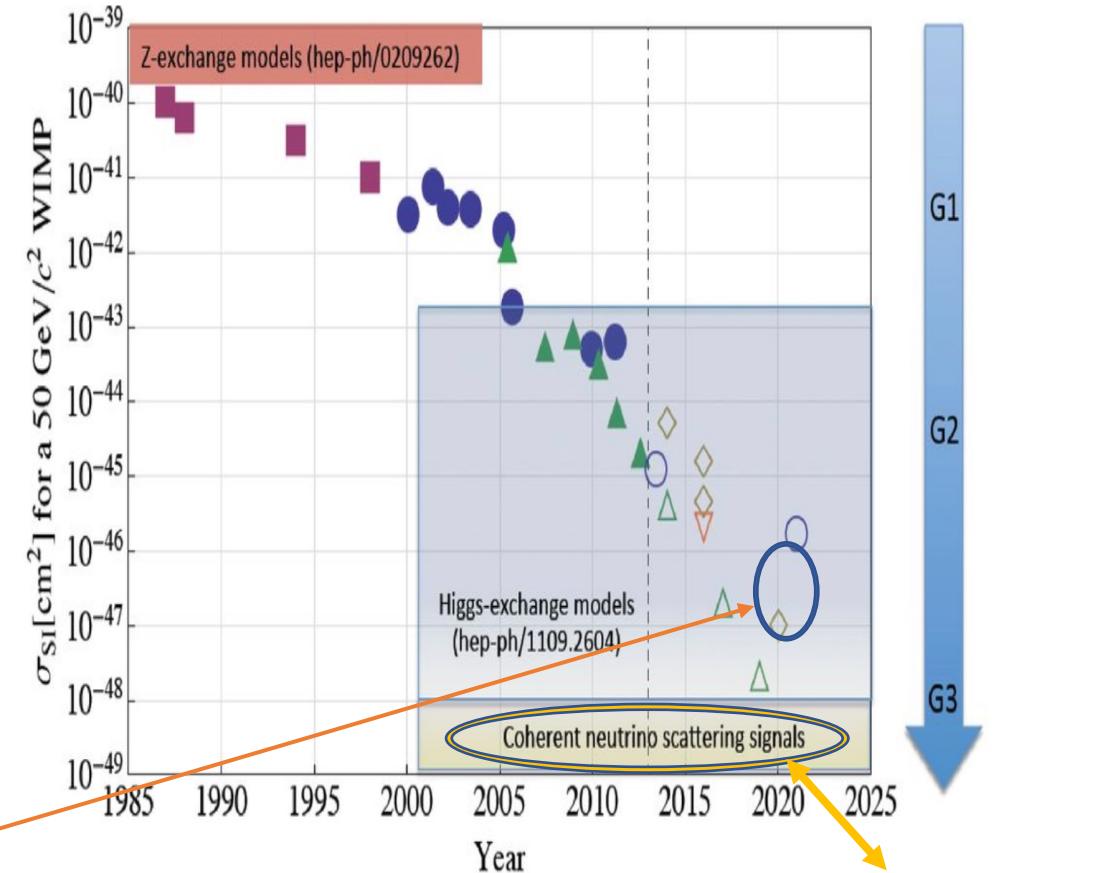
Experiments:

1. PandaX
2. CDEX
3. XENON
4. LUX
5. ICECUBE
6. ...



PandaX-4T, PRL 127, 261802 (2021).

Evolution of the WIMP-Nucleon  $\sigma_{\text{SI}}$



直接探测的摩尔定律

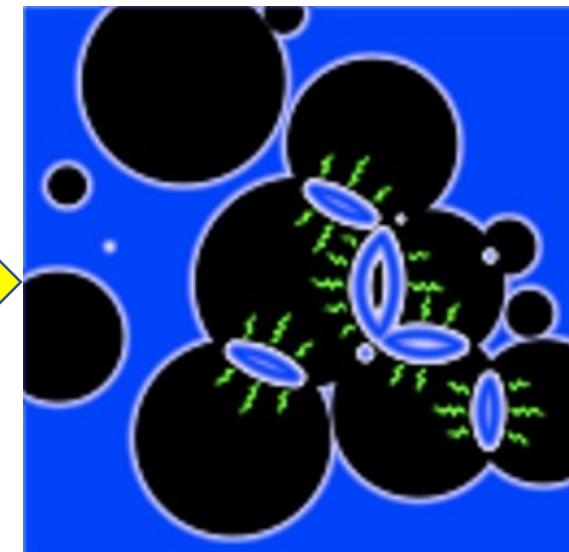
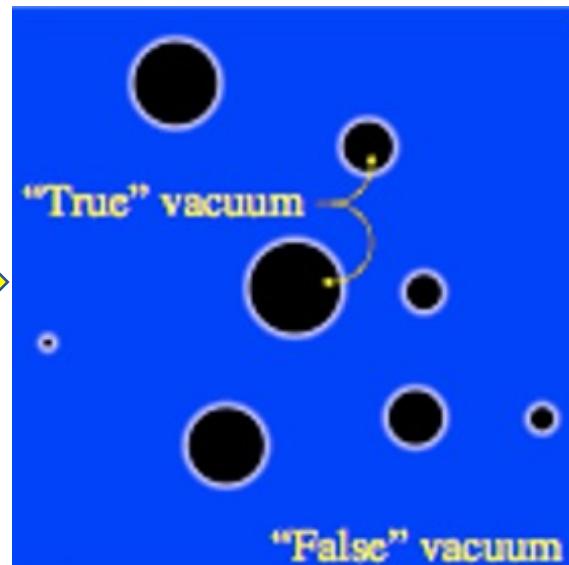
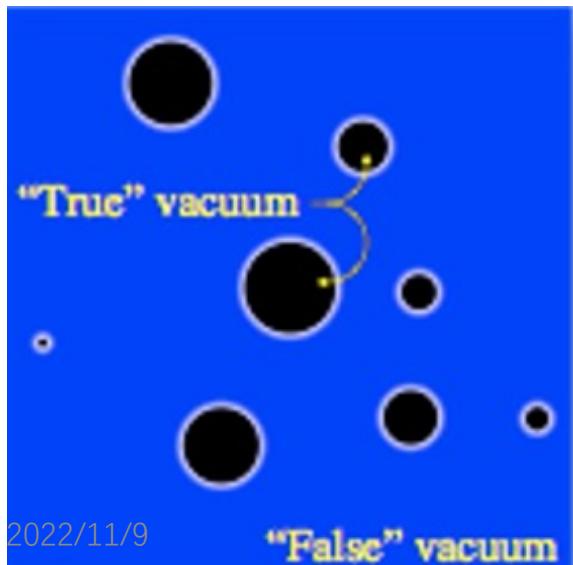
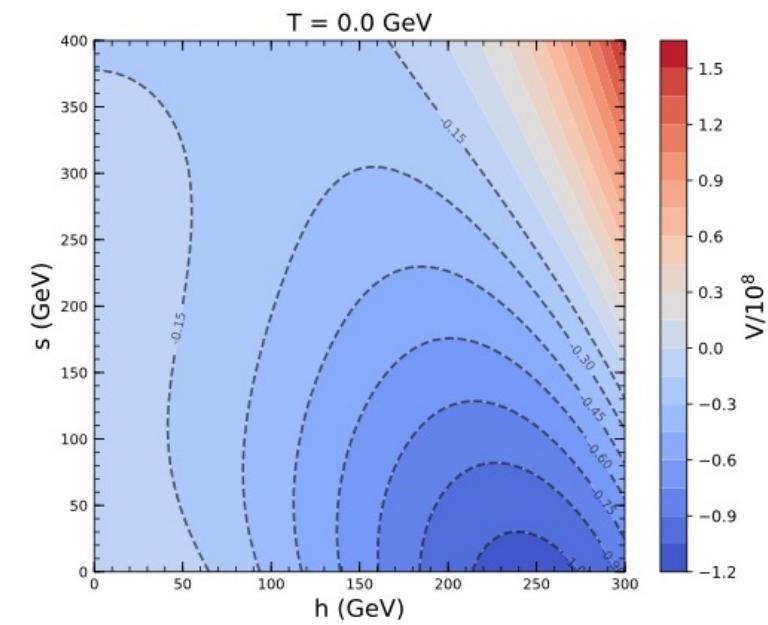
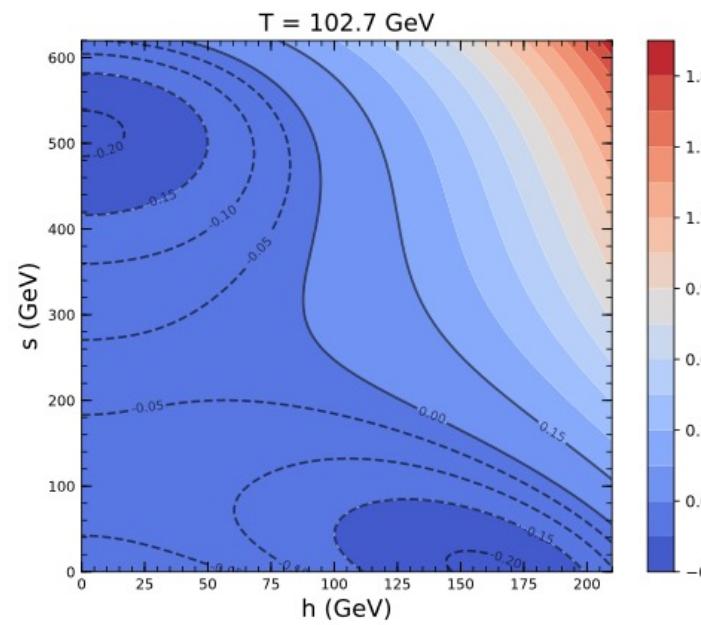
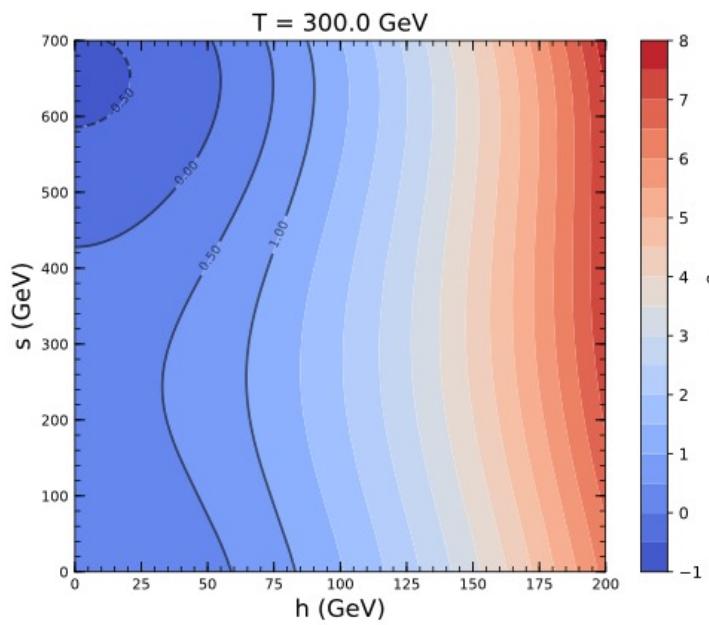
中微子本底

Jia Liu, Xiao-Ping Wang, and Ke-Pan Xie,  
arXiv:2203.10046.

# First-order electroweak phase transition (EWPT)



Evolution of the Universe

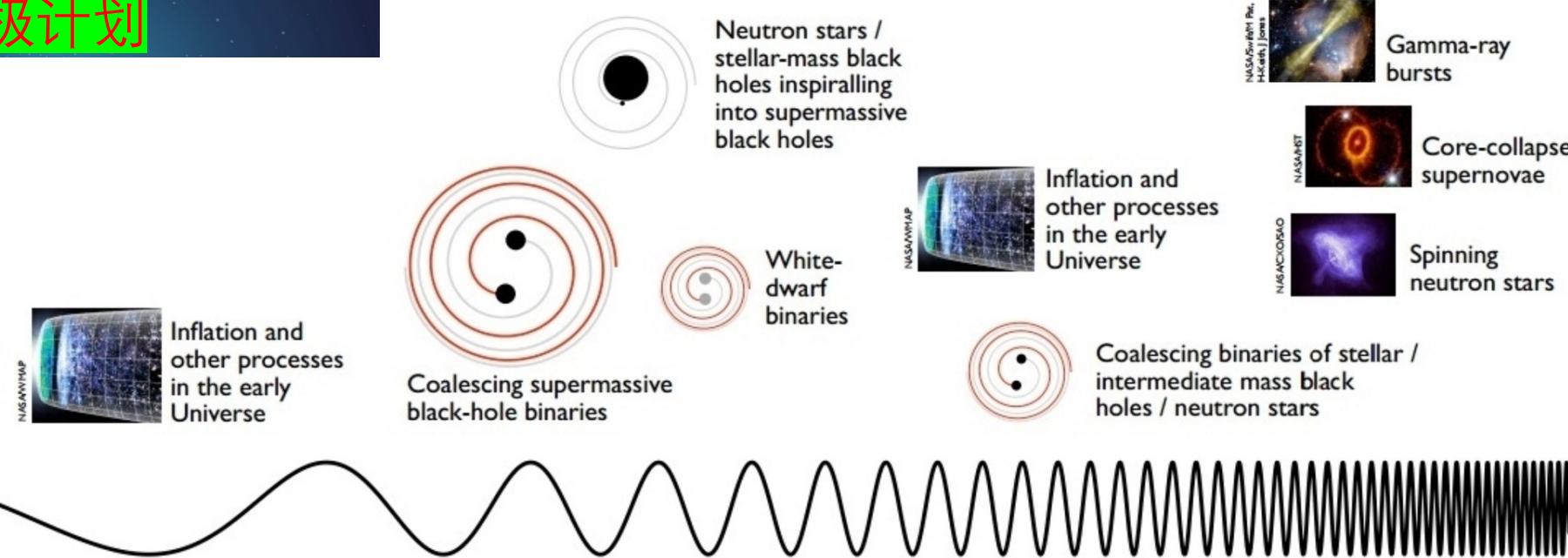
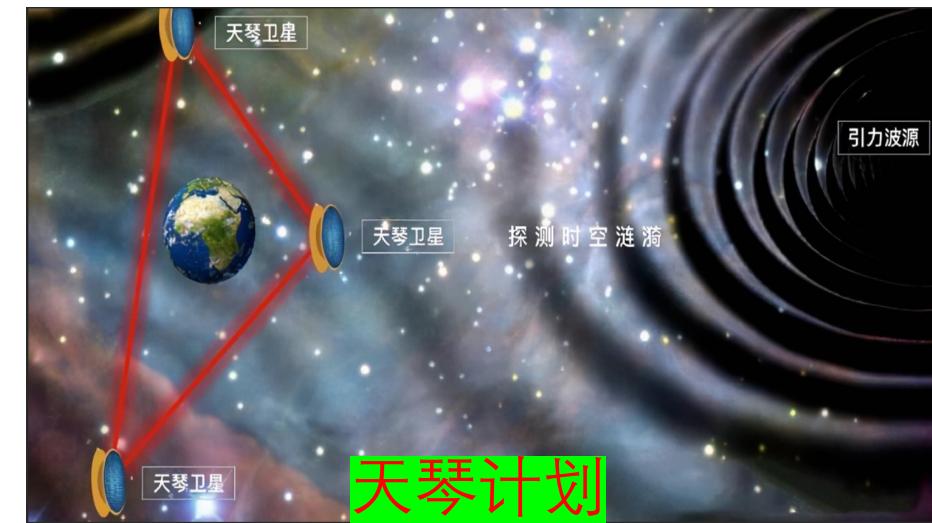




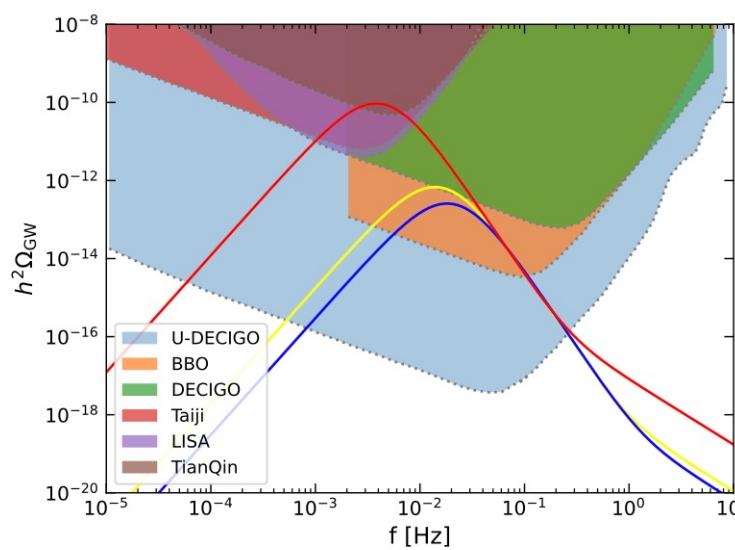
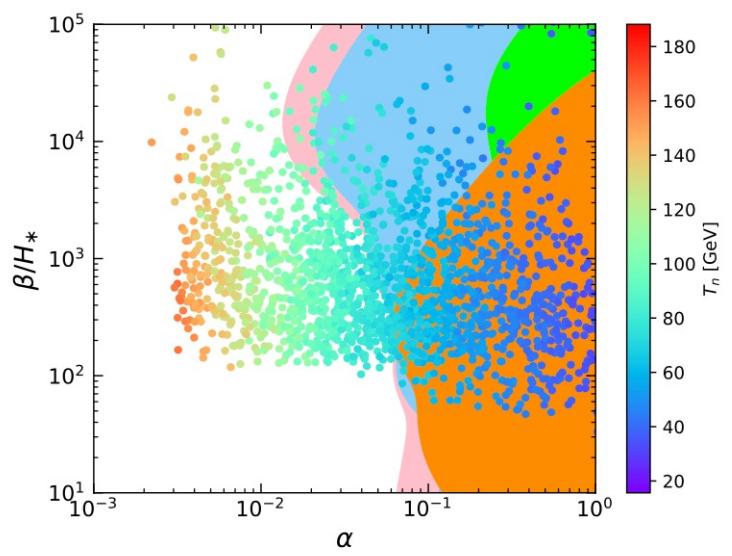
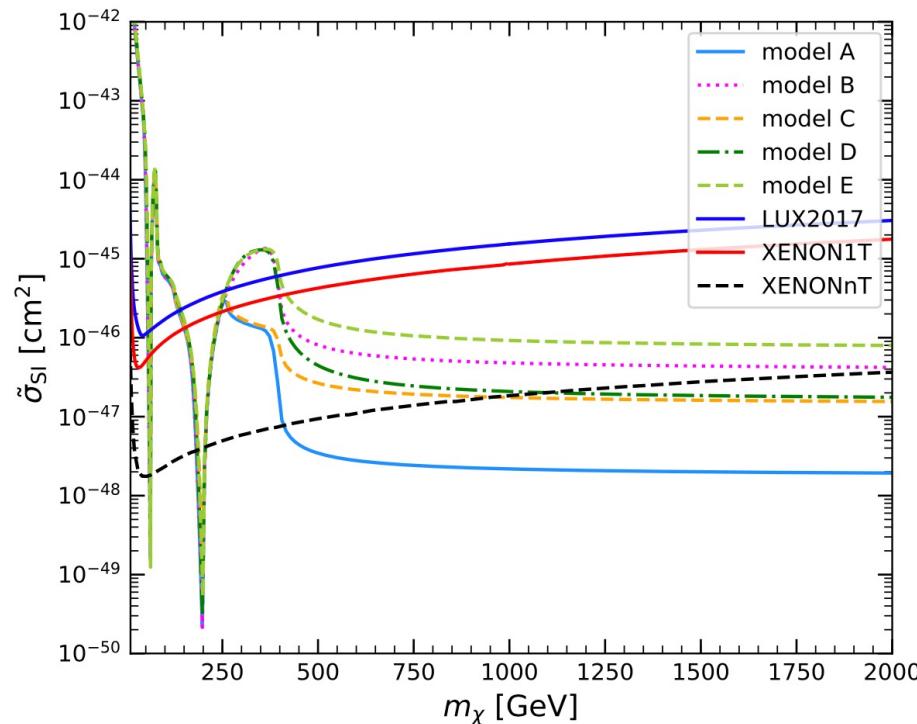
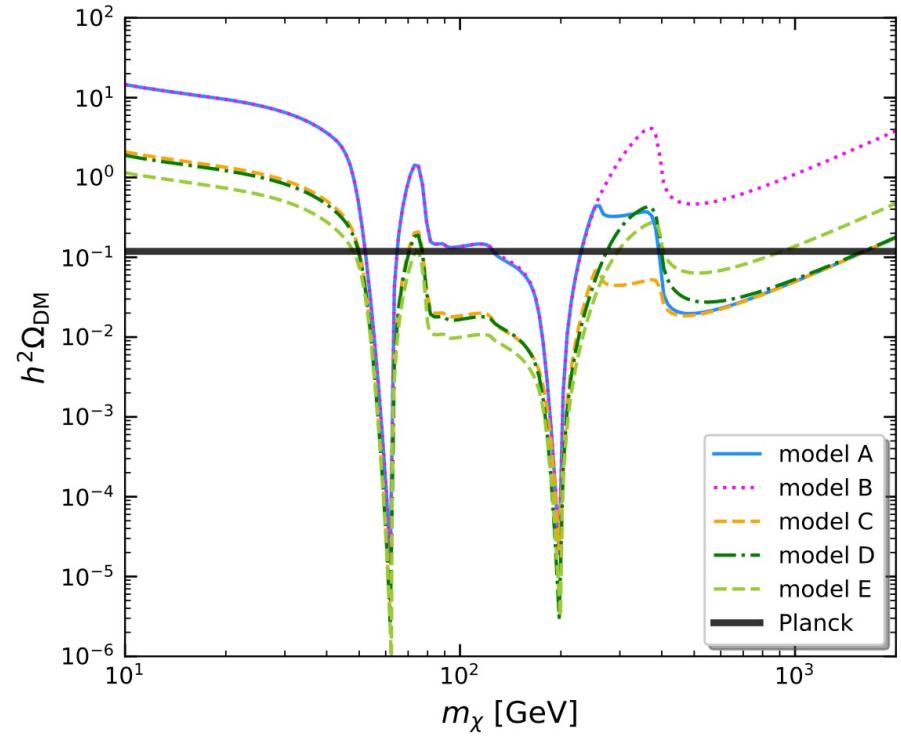
太极计划

## 空间引力波探测：

1. 太极计划
2. 天琴计划
3. LISA
4. BBO...



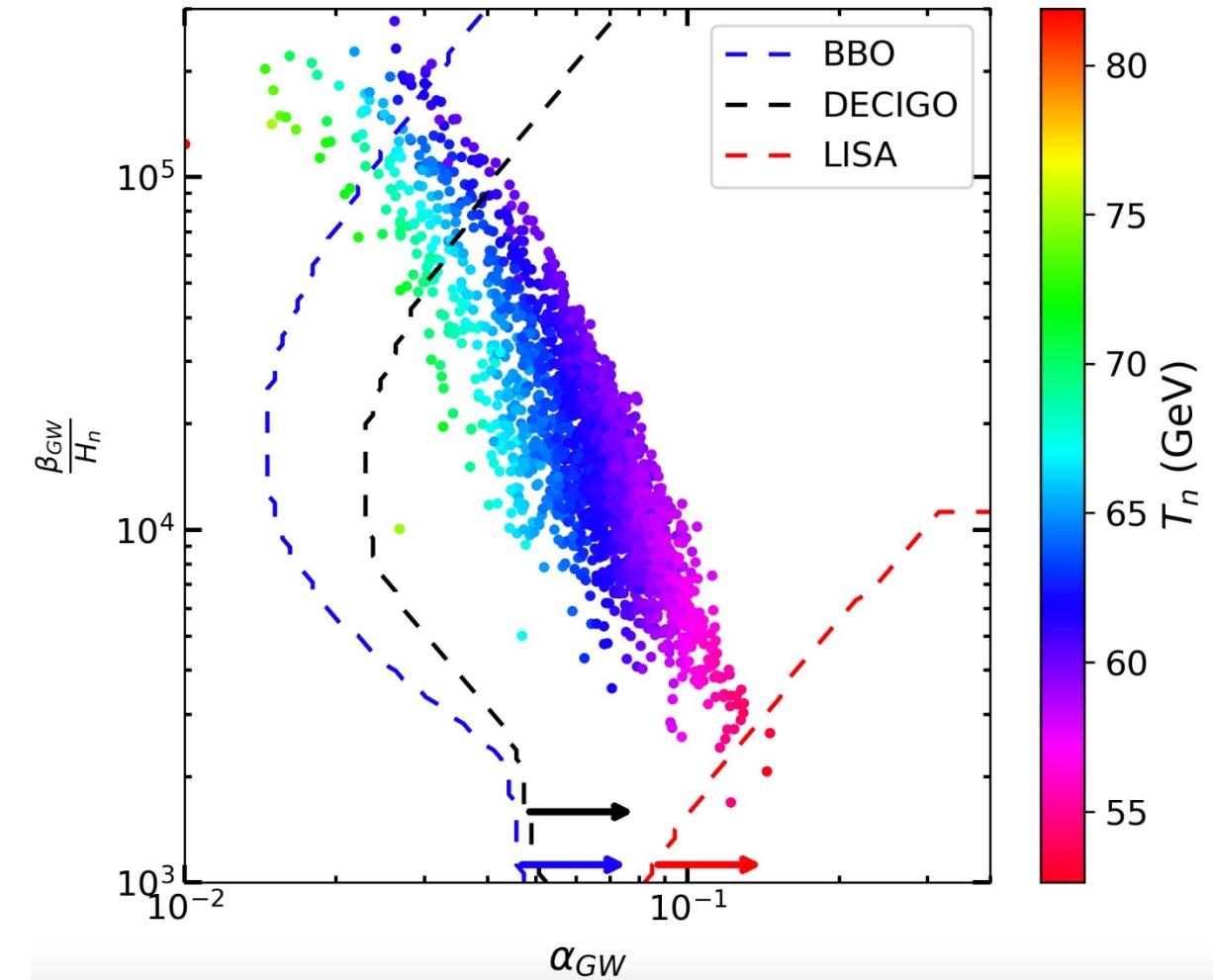
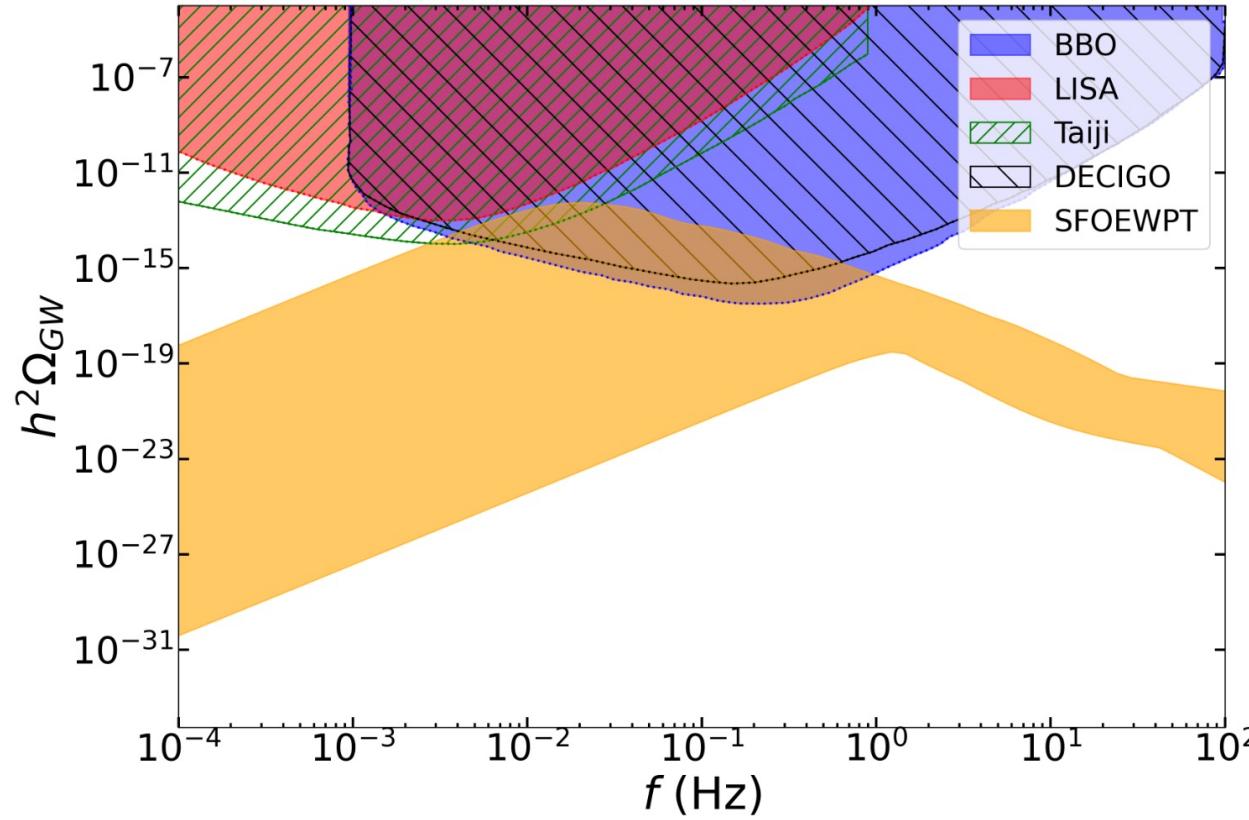
Frequency	$10^{-16}$ Hz	$10^{-9} - 10^{-6}$ Hz	$10^{-5} - 10^{-1}$ Hz	$10^{-1} - 1$ Hz	$1 - 10^4$ Hz
Wavelength	$10^{21}$ km	$10^{14} - 10^{11}$ km	$10^{10} - 10^6$ km	$10^6 - 10^5$ km	$10^5 - 10$ km
Detection	CMB Polarization	Pulsar timing	LISA	BBO/DECIGO	LIGO/Virgo/LCGT/ET



$$\begin{aligned}
 V(H, S) = & -\mu_h^2 |H|^2 + \lambda_h |H|^4 - \mu_1^2 (S^* S) - \frac{1}{2} \mu_2^2 (S^2 + S^{*2}) + \lambda_1 (S^* S)^2 + \frac{1}{4} \lambda_2 (S^2 + S^{*2})^2 \\
 & + \frac{1}{2} \lambda_3 (S^* S) (S^2 + S^{*2}) + \kappa_1 |H|^2 (S^* S) + \frac{1}{2} \kappa_2 |H|^2 (S^2 + S^{*2}) + \frac{1}{\sqrt{2}} c_1^3 (S + S^*) \\
 & + \frac{1}{2\sqrt{2}} b_m |H|^2 (S + S^*) + \frac{\sqrt{2}}{3} c_1 (S^* S) (S + S^*) + \frac{\sqrt{2}}{3} c_2 (S^3 + S^{*3}),
 \end{aligned} \quad (3.1)$$

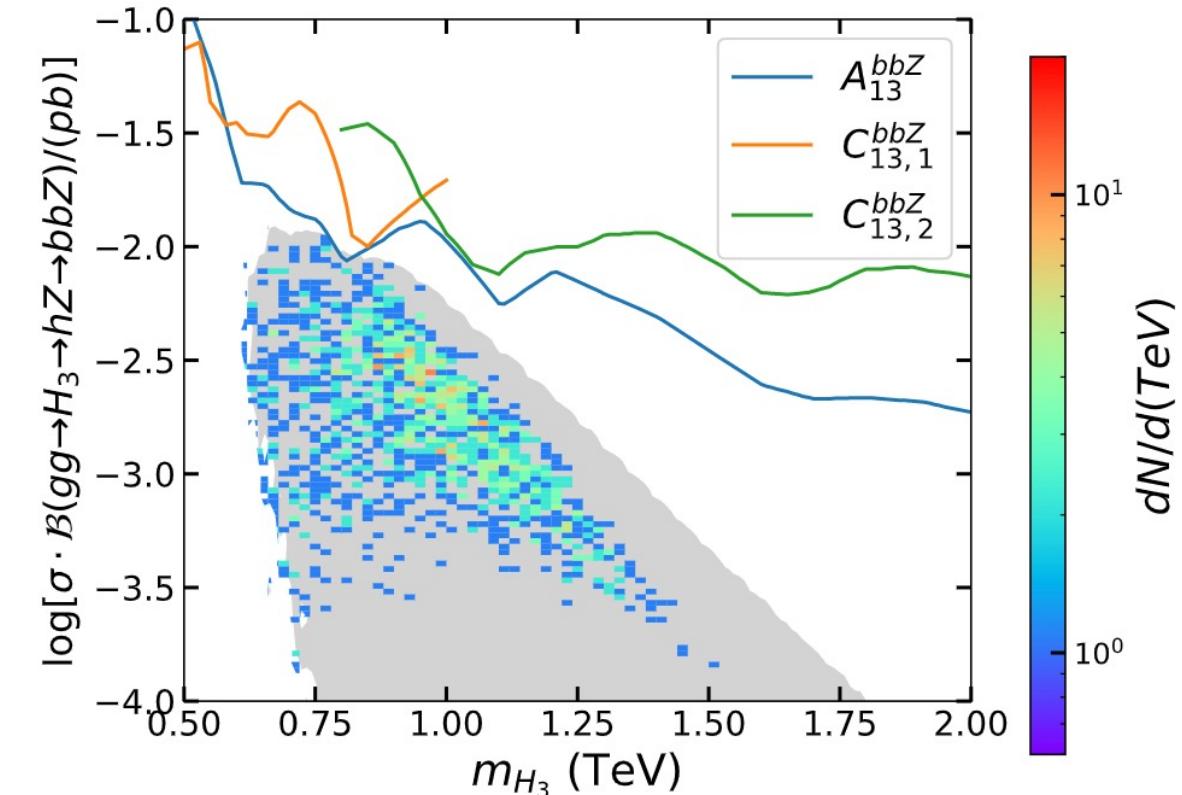
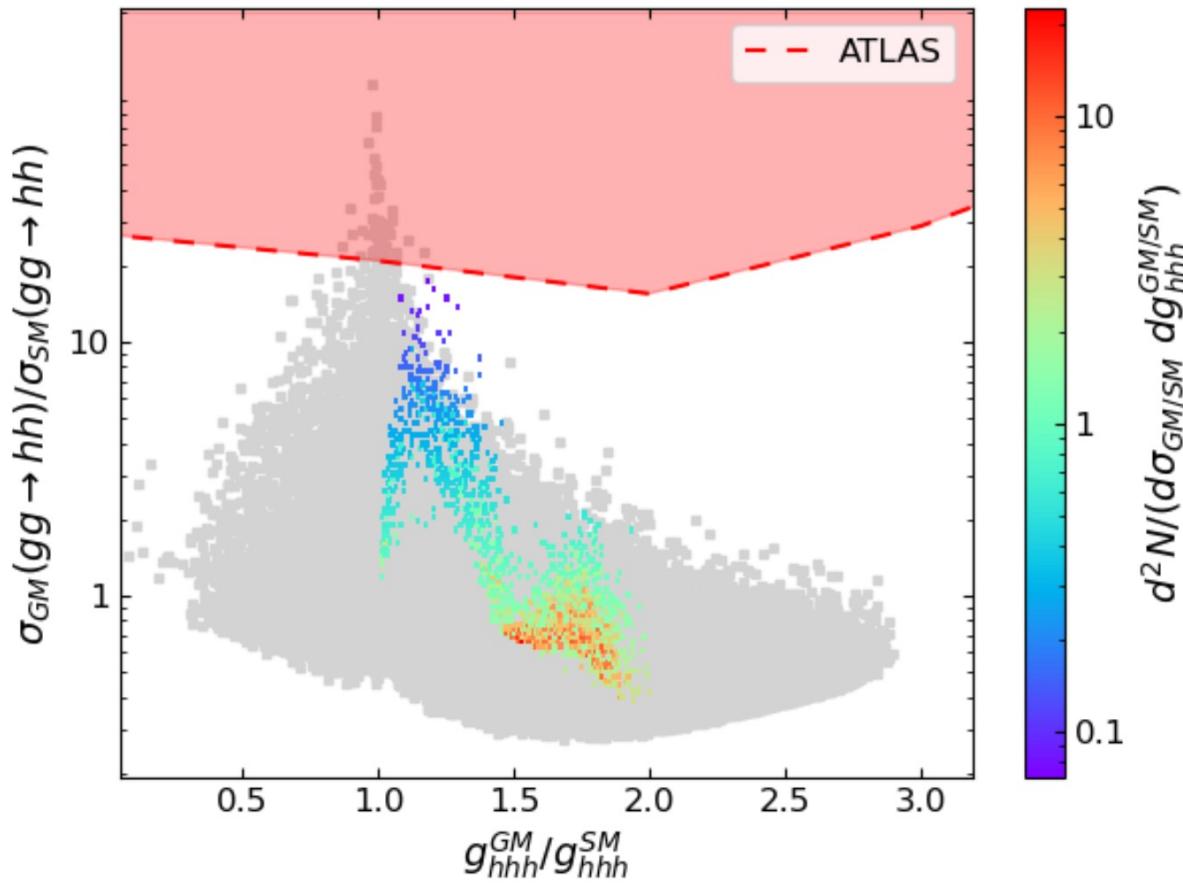
C. W. Chiang, D. Huang, and B. Q. Lu,  
JCAP 01 (2021) 035

# ➤ PT in Georgi-Machacek model: Gravitational wave



T. K. Chen, C. W. Chiang, C. T. Huang, and B. Q. Lu PRD 106 055019 (2022).

## ➤ PT in GM model: LHC constraints and detections



T. K. Chen, C. W. Chiang, C. T. Huang, and B. Q. Lu PRD 106 055019 (2022).

# Perturbative Unitarity

1. Consider the scalar scattering processes  $S_1S_2 \rightarrow S_3S_4$ .
2. In terms of the partial wave decomposition, the scattering amplitude is given by:

$$\mathcal{M}(s, t, u) = 16\pi \sum_{l=0}^{\infty} (2l + 1) P_l(\cos \theta) a_l(s)$$

3. The scattering cross section:

$$\sigma = \int \frac{1}{64\pi^2 s} |\mathcal{M}|^2 d\Omega = \frac{16\pi}{s} \sum_{l=1}^{\infty} (2l + 1) |a_l(s)|^2$$

# Perturbative Unitarity

- The unitarity constraint

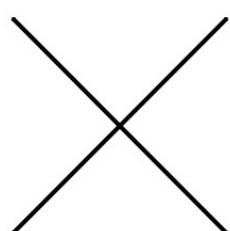
$$\Re(a_l)^2 + \Im(a_l)^2 = |a_l|^2 = \Im(a_l) \text{ or } |\Re(a_l)| < \frac{1}{2} \quad \text{for all } l$$

with

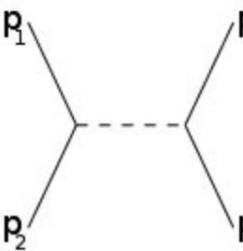
$$a_l(s) = \frac{1}{32\pi} \int_{-1}^1 d(\cos \theta) P_l(\cos \theta) \mathcal{M}(s, t, u)$$

- For  $J=0$  s-wave

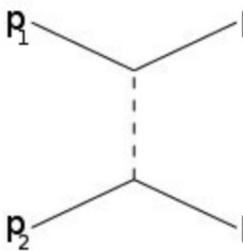
$$a_0(s) = \frac{1}{16\pi} [Q + \{T_h^{12}T_h^{34} \frac{1}{s - M_h^2} - \frac{1}{s} (c_t T_h^{13}T_h^{24} + c_u T_h^{14}T_h^{23}) \ln(1 + \frac{s}{m_h^2})\}]$$



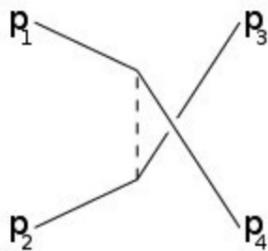
+



s-channel



t-channel



u-channel

# Perturbative Unitarity

- In the high energy limit the contributions from s-, t-, u-channel can be neglected

$$a_0(s) = \frac{1}{16\pi} Q$$

- **Equivalence Theorem:** at very high energy, the amplitude of scattering processes involving longitudinal gauge bosons in the initial and final states are equivalent to those in which these gauge bosons are replaced by the corresponding Nambu-Goldstone bosons.

J. M. Cornwall, et al., PRL 30, 1268 (1973); PRD 10, 1145 (1974).

Y. P. Yao, C. P. Yuan, PRD 38, 2237 (1988).

Z. H. G. J. Veltman, PRD 41, 2294 (1990).

H. J. He, Y. P. Kuang, and X. Li, PRL 69, 2619 (1992).

- Unitarity bound:  $|\Lambda_{(Y,I)}| \leq 8\pi$

# Bases of scattering matrices

- We decompose the direct product of two Higgs multiplets into its direct sum of the irreducible representation

$$2 \otimes 2 = 1 \oplus 3, \quad 2 \otimes 3 = 2 \oplus 4, \quad \text{and} \quad 3 \otimes 3 = 1 \oplus 3 \oplus 5.$$

**1**  $\mathbb{D} \times \mathbb{D}$

For the singlet state,  $I = 0$ ,

Not normalization!

$$\varepsilon^{ab} \mathbb{D}_a^i \mathbb{D}_b^j = -\mathbb{D}_1^i \mathbb{D}_2^j + \mathbb{D}_2^i \mathbb{D}_1^j$$

For the triplet states,  $I = 1$ ,

$$\mathbb{D}_{(a}^i \mathbb{D}_{b)}^j = \frac{1}{2} \left( \mathbb{D}_a^i \mathbb{D}_b^j + \mathbb{D}_b^i \mathbb{D}_a^j \right).$$

# Bases of scattering matrices

2  $\mathbb{D} \times \mathbb{T}$

Not normalization!

For the doublet state,  $I = \frac{1}{2}$ ,

$$\varepsilon^{ab} \mathbb{D}_a \mathbb{T}_{bc} = -\mathbb{D}_1 \mathbb{T}_{2c} + \mathbb{D}_2 \mathbb{T}_{1c}.$$

For the quadruplet states,  $I = \frac{3}{2}$ ,

$$\mathbb{D}_{(a} \mathbb{T}_{bc)} = \frac{1}{3} (\mathbb{D}_a \mathbb{T}_{bc} + \mathbb{D}_b \mathbb{T}_{ac} + \mathbb{D}_c \mathbb{T}_{ab}).$$

# Bases of scattering matrices

3  $\mathbb{T} \times \mathbb{T}$

Not normalization!

For the singlet state,

$$\varepsilon^{ac}\varepsilon^{bd}\mathbb{T}_{ab}^i\mathbb{T}_{cd}^j = \mathbb{T}_{11}^i\mathbb{T}_{22}^j - 2\mathbb{T}_{12}^i\mathbb{T}_{12}^j + \mathbb{T}_{22}^i\mathbb{T}_{11}^j.$$

For the triplet state,

$$\varepsilon^{ac}\mathbb{T}_{a(b}^i\mathbb{T}_{cd)}^j = -\frac{1}{3} \left( \mathbb{T}_{1b}^i\mathbb{T}_{2d}^j + \mathbb{T}_{1d}^i\mathbb{T}_{2b}^j - \mathbb{T}_{2b}^i\mathbb{T}_{1d}^j - \mathbb{T}_{2d}^i\mathbb{T}_{1b}^j \right).$$

For the quintuplet (5-plet) state,  $I = 2$ ,

$$\mathbb{T}_{(ab}^i\mathbb{T}_{cd)}^j = \frac{1}{6} \left( \mathbb{T}_{ab}^i\mathbb{T}_{cd}^j + \mathbb{T}_{bc}^i\mathbb{T}_{ad}^j + \mathbb{T}_{ac}^i\mathbb{T}_{bd}^j + \mathbb{T}_{cd}^i\mathbb{T}_{ab}^j + \mathbb{T}_{ad}^i\mathbb{T}_{bc}^j + \mathbb{T}_{bd}^i\mathbb{T}_{ac}^j \right).$$

# Bases of scattering matrices

- Classify the bases according to their total hypercharge  $Y$  and total isospin  $I$  ( $Y$  and  $I$  are conserved quantum numbers in the scattering at high energy).

$$S_{(Y,I)} = \left\langle (\phi\phi)_{Y,I}^f | \hat{S} | (\phi\phi)_{Y,I}^i \right\rangle.$$

Field	$\Phi$	$\tilde{\Phi}$	$\Sigma$	$\Delta$	$\tilde{\Delta}$
$SU(2)_L$ isospin	2	2	3	3	3
Hypercharge	1	-1	0	2	-2

# Bases of scattering matrices

	$I = 0$	$I = 1$
$Y = 0$	$\frac{1}{\sqrt{2}} \left( w_i^+ w_j^- + H_i^0 H_j^{0*} \right)$	$w_i^+ H_j^{0*}$ $\frac{1}{\sqrt{2}} \left( -w_i^+ w_j^- + H_i^0 H_j^{0*} \right)$ $-H_i^0 w_j^-$
$Y = 2$	$\frac{1}{\sqrt{2}} \left( -w_i^+ H_j^0 + H_i^0 w_j^+ \right)$	$w_i^+ w_j^+ \left( \times \frac{1}{\sqrt{2}} \text{ for } i = j \right)$ $\frac{1}{\sqrt{2}} \left( w_i^+ H_j^0 + H_i^0 w_j^+ \right)$ $H_i^0 H_j^0 \left( \times \frac{1}{\sqrt{2}} \text{ for } i = j \right)$

**Table 2.** The bases of the irreducible representation for the two Higgs doublets direct product. The bases in the first and second row are corresponding to the direct product  $\Phi_i \times \tilde{\Phi}_j$  ( $Y = 0$ ) and  $\Phi_i \times \Phi_j$  ( $Y = 2$ ), respectively. Note that  $i$  and  $j$  indicate the Higgs doublet. We observe that the bases with ( $Y = 2$ ,  $I = 2$ ) vanish when the two Higgs doublets are identical, i.e.,  $i = j$ .

# Bases of scattering matrices

	$I = \frac{1}{2}$	$I = \frac{3}{2}$
$Y = 1$	$\sqrt{\frac{2}{3}} \left( -\frac{1}{\sqrt{2}} H^{0*} \delta^+ + w^- \delta^{++} \right)$ $\sqrt{\frac{2}{3}} \left( -H^{0*} \delta^0 - \frac{1}{\sqrt{2}} w^- \delta^+ \right)$	$-H^{0*} \delta^{++}$ $\frac{1}{\sqrt{3}} (\sqrt{2} H^{0*} \delta^+ + w^- \delta^{++})$ $\frac{1}{\sqrt{3}} (H^{0*} \delta^0 - \sqrt{2} w^- \delta^+)$ $-w^- \delta^0$
$Y = 3$	$\sqrt{\frac{2}{3}} \left( -\frac{1}{\sqrt{2}} w^+ \delta^+ - H^0 \delta^{++} \right)$ $\sqrt{\frac{2}{3}} \left( -w^+ \delta^0 + \frac{1}{\sqrt{2}} H^0 \delta^+ \right)$	$-w^+ \delta^{++}$ $\frac{1}{\sqrt{3}} (\sqrt{2} w^+ \delta^+ - H^0 \delta^{++})$ $\frac{1}{\sqrt{3}} (w^+ \delta^0 + \sqrt{2} H^0 \delta^+)$ $H^0 \delta^0$

**Table 4.** The bases of the irreducible representation for the direct product of a Higgs doublet and a complex Higgs triplet scalar. The bases in the first and second row are corresponding to the direct product  $\tilde{\Phi} \times \Delta$  ( $Y = 1$ ) and  $\Phi \times \Delta$  ( $Y = 3$ ), respectively.

# Bases of scattering matrices

	$I = 0$	$I = 1$	$I = 2$
$Y = 0$	$\frac{1}{\sqrt{3}} (\delta^{++}\delta^{--} + \delta^+\delta^- + \delta^0\delta^{0*})$	$\frac{1}{\sqrt{2}} (-\delta^{++}\delta^- + \delta^+\delta^{0*})$ $-\frac{1}{\sqrt{2}} (\delta^{++}\delta^{--} - \delta^0\delta^{0*})$ $-\frac{1}{\sqrt{2}} (-\delta^+\delta^{--} + \delta^-\delta^0)$	$-\delta^{++}\delta^{0*}$ $\frac{1}{\sqrt{2}} (\delta^{++}\delta^- + \delta^+\delta^{0*})$ $\frac{1}{\sqrt{6}} (-2\delta^+\delta^- + \delta^{++}\delta^{--} + \delta^0\delta^{0*})$ $\frac{1}{\sqrt{2}} (-\delta^+\delta^{--} - \delta^0\delta^-)$ $-\delta^{--}\delta^0$
$Y = 4$	$\sqrt{\frac{2}{3}} (-\delta^{++}\delta^0 - \frac{1}{2}\delta^+\delta^+)$	0	$\frac{1}{\sqrt{2}} \delta^{++}\delta^{++}$ $-\delta^{++}\delta^+$ $\frac{1}{\sqrt{3}} (\delta^+\delta^+ - \delta^{++}\delta^0)$ $\delta^0\delta^+$ $\frac{1}{\sqrt{2}} \delta^0\delta^0$

**Table 6.** The bases of the irreducible representation for the direct product of two complex Higgs triplet scalars. The bases in the first and second row are corresponding to the direct product  $\Delta \times \tilde{\Delta}$  ( $Y = 0$ ) and  $\Delta \times \Delta$  ( $Y = 4$ ), respectively. We observe again that the bases with ( $Y = 4$ ,  $I = 1$ ) vanish because the two triplets are identical.

# Scattering matrix

$$\Phi_i = \begin{pmatrix} w_i^+ \\ H_i^0 \end{pmatrix}, \text{ and } \Sigma = \begin{pmatrix} \sigma^0/\sqrt{2} & \sigma^+ \\ \sigma^- & -\sigma^0/\sqrt{2} \end{pmatrix},$$

- The extension of 2HDM with a real triplet

$$V_r = V(\Phi_1, \Phi_2) + V(\Sigma) + V(\Phi_1, \Phi_2, \Sigma), \quad V(\Sigma) = \frac{1}{2}m_\Sigma^2 \operatorname{Tr} \Sigma^2 + \frac{1}{4}\lambda_\Sigma \operatorname{Tr} \Sigma^4.$$

$$\begin{aligned} V(\Phi_1, \Phi_2) = & m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.}) + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 \\ & + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \left[ \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 \right. \\ & \left. + \lambda_6 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \text{H.c.} \right]. \end{aligned}$$

$$\begin{aligned} V(\Phi_1, \Phi_2, \Sigma) = & \frac{1}{\sqrt{2}} \left[ a_1 \Phi_1^\dagger \Sigma \Phi_1 + a_2 \Phi_2^\dagger \Sigma \Phi_2 + (a_{12} \Phi_1^\dagger \Sigma \Phi_2 + \text{H.c.}) \right] \\ & + \frac{1}{2} \operatorname{Tr} \Sigma^2 \left[ \lambda_8 \Phi_1^\dagger \Phi_1 + \lambda_9 \Phi_2^\dagger \Phi_2 + (\lambda_{10} \Phi_1^\dagger \Phi_2 + \text{H.c.}) \right]. \end{aligned}$$

# Scattering matrix

$$16\pi S_{(0,0)} = \begin{pmatrix} 3\lambda_1 & 2\lambda_3 + \lambda_4 & 3\lambda_6 & 3\lambda_6^* & \sqrt{3}\lambda_8 \\ 2\lambda_3 + \lambda_4 & 3\lambda_2 & 3\lambda_7 & 3\lambda_7^* & \sqrt{3}\lambda_9 \\ 3\lambda_6^* & 3\lambda_7^* & \lambda_3 + 2\lambda_4 & 3\lambda_5^* & \sqrt{3}\lambda_{10}^* \\ 3\lambda_6 & 3\lambda_7 & 3\lambda_5 & \lambda_3 + 2\lambda_4 & \sqrt{3}\lambda_{10} \\ \sqrt{3}\lambda_8 & \sqrt{3}\lambda_9 & \sqrt{3}\lambda_{10} & \sqrt{3}\lambda_{10}^* & 5\lambda_\Sigma \end{pmatrix}$$

$$16\pi S_{(0,1)} = \begin{pmatrix} \lambda_1 & \lambda_4 & \lambda_6 & \lambda_6^* \\ \lambda_4 & \lambda_2 & \lambda_7 & \lambda_7^* \\ \lambda_6^* & \lambda_7^* & \lambda_3 & \lambda_5^* \\ \lambda_6 & \lambda_7 & \lambda_5 & \lambda_3 \end{pmatrix}$$

$$16\pi S_{(0,2)} = 2\lambda_\Sigma$$

$$16\pi S_{(1,\frac{1}{2})} = 16\pi S_{(1,\frac{3}{2})} = \begin{pmatrix} \lambda_8 & \lambda_{10}^* \\ \lambda_{10} & \lambda_9 \end{pmatrix}$$

$$16\pi S_{(2,0)} = \lambda_3 - \lambda_4$$

$$16\pi S_{(2,1)} = \begin{pmatrix} \lambda_1 & \lambda_5^* & \sqrt{2}\lambda_6^* \\ \lambda_5 & \lambda_2 & \sqrt{2}\lambda_7 \\ \sqrt{2}\lambda_6 & \sqrt{2}\lambda_7^* & \lambda_3 + \lambda_4 \end{pmatrix}$$

# Scattering matrix

- The extension of 2HDM with a complex triplet

$$V_c = V(\Phi_1, \Phi_2) + V(\Delta) + V(\Phi_1, \Phi_2, \Delta), \quad \Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

$$V(\Delta) = m_\Delta^2 \operatorname{Tr} \Delta^\dagger \Delta + \lambda_{\Delta 1} \left( \operatorname{Tr} \Delta^\dagger \Delta \right)^2 + \lambda_{\Delta 2} \operatorname{Tr} \left( \Delta^\dagger \Delta \right)^2.$$

$$\begin{aligned} V(\Phi_1, \Phi_2, \Delta) = & \left( \mu_1 \Phi_1^T i\tau_2 \Delta^\dagger \Phi_1 + \mu_2 \Phi_2^T i\tau_2 \Delta^\dagger \Phi_2 + \mu_3 \Phi_1^T i\tau_2 \Delta^\dagger \Phi_2 + \text{H.c.} \right) \\ & + \left[ \lambda_8 \Phi_1^\dagger \Phi_1 + \lambda_9 \Phi_2^\dagger \Phi_2 + \left( \lambda_{10} \Phi_1^\dagger \Phi_2 + \text{H.c.} \right) \right] \operatorname{Tr} \Delta^\dagger \Delta \\ & + \lambda_{11} \Phi_1^\dagger \Delta \Delta^\dagger \Phi_1 + \lambda_{12} \Phi_2^\dagger \Delta \Delta^\dagger \Phi_2 + \left( \lambda_{13} \Phi_1^\dagger \Delta \Delta^\dagger \Phi_2 + \text{H.c.} \right). \end{aligned}$$

# Scattering matrix

$$16\pi S_{(0,0)} = \begin{pmatrix} 3\lambda_1 & 2\lambda_3 + \lambda_4 & 3\lambda_6 & 3\lambda_6^* & \lambda_a \\ 2\lambda_3 + \lambda_4 & 3\lambda_2 & 3\lambda_7 & 3\lambda_7^* & \lambda_b \\ 3\lambda_6^* & 3\lambda_7^* & \lambda_3 + 2\lambda_4 & 3\lambda_5^* & \lambda_c^* \\ 3\lambda_6 & 3\lambda_7 & 3\lambda_5 & \lambda_3 + 2\lambda_4 & \lambda_c \\ \lambda_a & \lambda_b & \lambda_c & \lambda_c^* & \lambda_\Delta \end{pmatrix} \text{ with } \begin{cases} \lambda_a = \sqrt{\frac{3}{2}}(2\lambda_8 + \lambda_{11}) \\ \lambda_b = \sqrt{\frac{3}{2}}(2\lambda_9 + \lambda_{12}) \\ \lambda_c = \sqrt{\frac{3}{2}}(2\lambda_{10} + \lambda_{13}) \\ \lambda_\Delta = 2(4\lambda_{\Delta 1} + 3\lambda_{\Delta 2}) \end{cases}$$

$$16\pi S_{(0,1)} = \begin{pmatrix} \lambda_1 & \lambda_4 & \lambda_6 & \lambda_6^* & \lambda_{11} \\ \lambda_4 & \lambda_2 & \lambda_7 & \lambda_7^* & \lambda_{12} \\ \lambda_6^* & \lambda_7^* & \lambda_3 & \lambda_5^* & \lambda_{13}^* \\ \lambda_6 & \lambda_7 & \lambda_5 & \lambda_3 & \lambda_{13} \\ \lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{13}^* & 2\lambda_{\Delta 1} + 4\lambda_{\Delta 2} \end{pmatrix}$$

$$16\pi S_{(0,2)} = 2\lambda_{\Delta 1}$$

$$16\pi S_{(1,\frac{1}{2})} = \begin{pmatrix} \lambda_8 + 3\lambda_{11}/2 & \lambda_{10} + 3\lambda_{13}/2 \\ \lambda_{10}^* + 3\lambda_{13}^*/2 & \lambda_9 + 3\lambda_{12}/2 \end{pmatrix}$$

$$16\pi S_{(1,\frac{3}{2})} = \begin{pmatrix} \lambda_8 & \lambda_{10} \\ \lambda_{10}^* & \lambda_9 \end{pmatrix}$$

$$16\pi S_{(2,0)} = \lambda_3 - \lambda_4$$

$$16\pi S_{2,1} = \begin{pmatrix} \lambda_1 & \lambda_5^* & \sqrt{2}\lambda_6 \\ \lambda_5 & \lambda_2 & \sqrt{2}\lambda_7^* \\ \sqrt{2}\lambda_6^* & \sqrt{2}\lambda_7 & \lambda_3 + \lambda_4 \end{pmatrix}$$

$$16\pi S_{(3,\frac{1}{2})} = \begin{pmatrix} \lambda_8 - \lambda_{11}/2 & \lambda_{10}^* - \lambda_{13}^*/2 \\ \lambda_{10} - \lambda_{13}/2 & \lambda_9 - \lambda_{12}/2 \end{pmatrix}$$

$$16\pi S_{(3,\frac{3}{2})} = \begin{pmatrix} \lambda_8 + \lambda_{11} & \lambda_{10}^* + \lambda_{13}^* \\ \lambda_{10} + \lambda_{13} & \lambda_9 + \lambda_{12} \end{pmatrix}$$

$$16\pi S_{(4,0)} = 2\lambda_{\Delta 1} - \lambda_{\Delta 2}$$

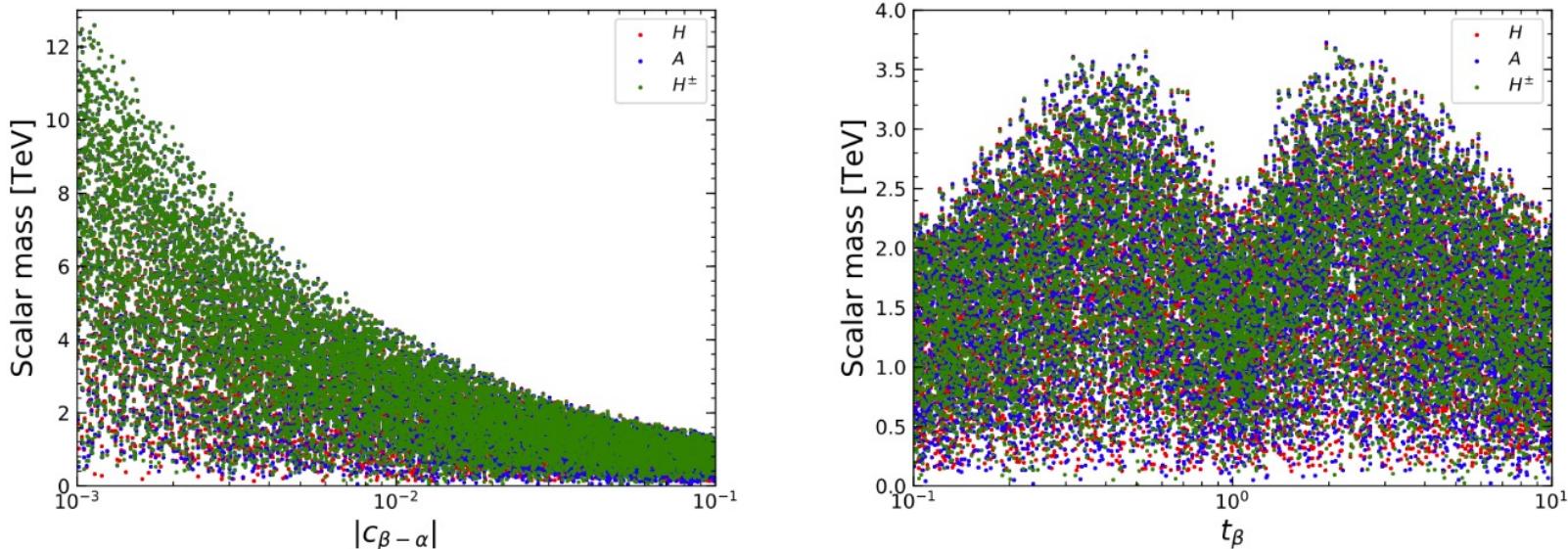
$$16\pi S_{(4,2)} = 2(\lambda_{\Delta 1} + \lambda_{\Delta 2})$$

# Application

- CP-conserving 2HDM with softly broken Z2 symmetry (R2HDM)

The recent global fit to the Higgs signal strengths and flavor observations have restricted:

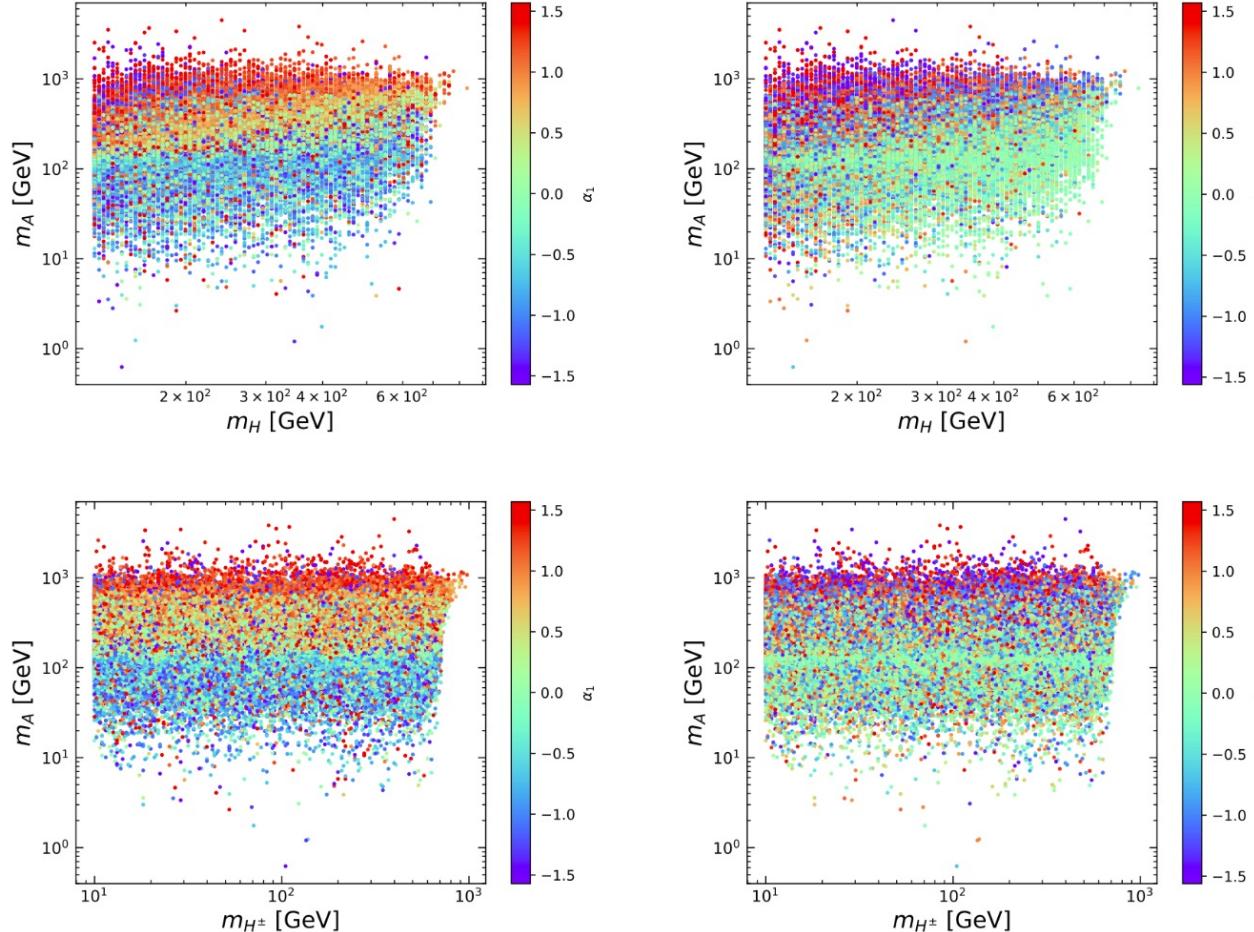
$$c_{\beta-\alpha} \lesssim 10^{-1}$$



**Figure 1.** Unitarity bound on the scalar masses. Left: scalar mass distributions as a function of  $c_{\beta-\alpha}$ . Right: scalar mass distributions as a function of  $t_{\beta}$ . The red, blue, and green scatter points denote the mass distributions for  $H$ ,  $A$ , and  $H^\pm$ , respectively.

# Application

- CP-violation 2HDM with softly broken Z2 symmetry (C2HDM)



**Figure 3.** The perturbative unitarity bound on the scalar masses. The colorbars in the left and right two plots represent the values of mixing angle  $\alpha_1$  and  $\alpha_3$ , respectively.

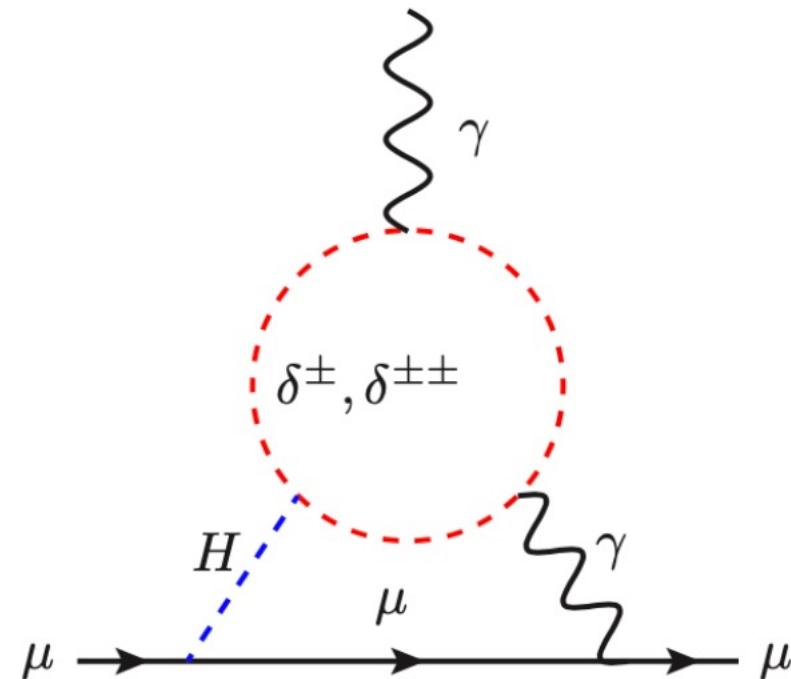
# Application

- Muon g-2 in extension of 2HDM with a complex triplet

$$\Delta a_\mu = \sum_{\phi_i} \frac{\alpha m_\mu^2}{8\pi^3 m_H^2} \operatorname{Re} (y_f^H) \lambda_{H\phi_i\phi_i^*} \mathcal{F} \left( \frac{m_{\phi_i}^2}{m_H^2} \right),$$

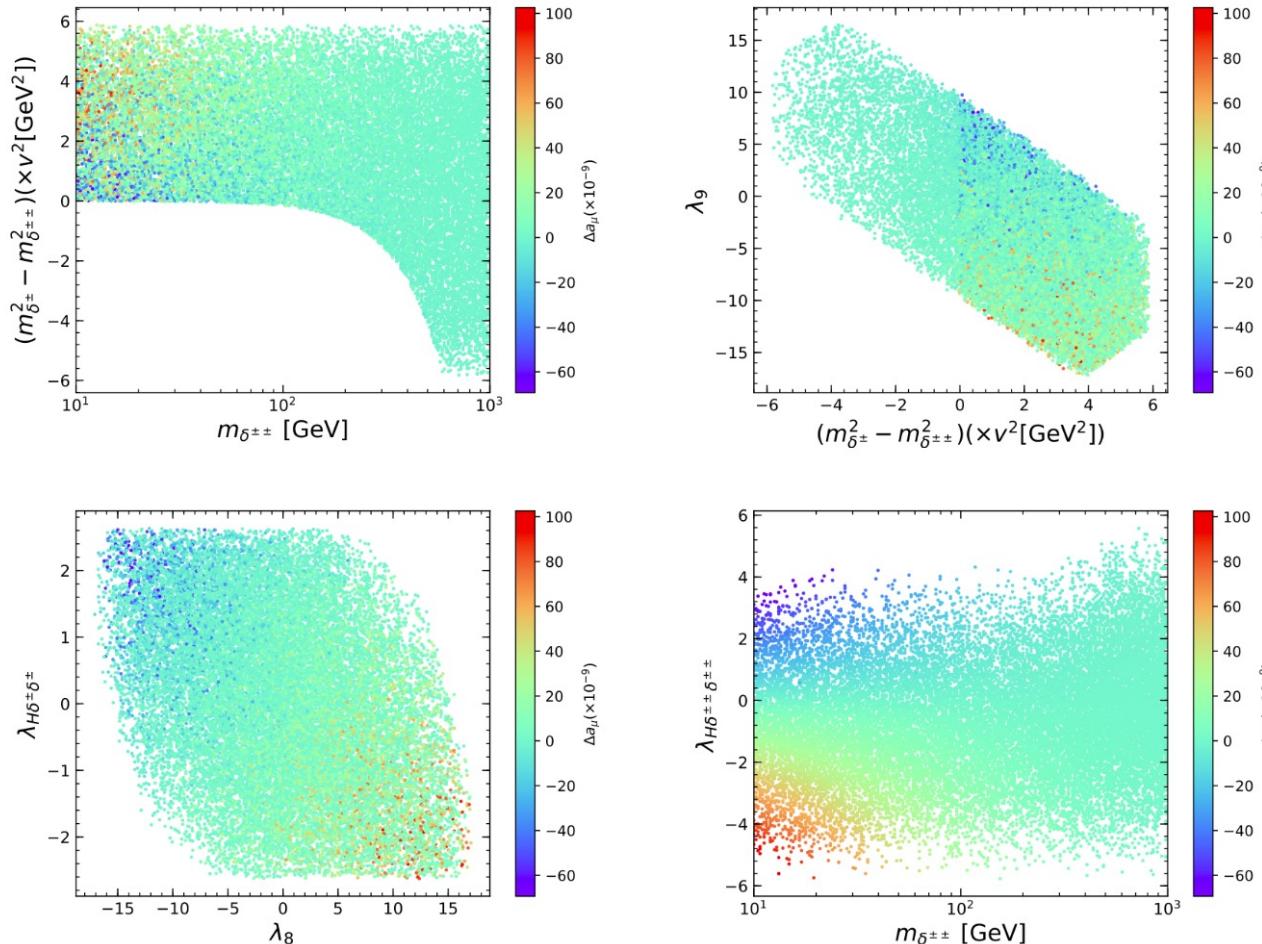
$$\mathcal{F}(\omega) = \frac{1}{2} \int_0^1 dx \frac{x(x-1)}{\omega - x(1-x)} \ln \left( \frac{\omega}{x(1-x)} \right).$$

C. H. Chen, C. W. Chiang and T. Nomura,  
Phys. Rev. D 104, no.5, 055011 (2021)



# Application

- Muon g-2 in extension of 2HDM with a complex triplet



**Figure 5.** Upper: unitarity bound on the mass difference of  $m_{\delta^\pm} - m_{\delta^{\pm\pm}}$ . Lower: unitarity bound on the trilinear couplings  $\lambda_{H\delta^\pm\delta^\pm}$  and  $\lambda_{H\delta^{\pm\pm}\delta^{\pm\pm}}$ . The colorbar represents the values of  $\Delta a_\mu (\times 10^{-9})$ .

# Summary

- Extensions of Higgs sector are the promising direction for resolving the problems of dark matter, first-order EWPT, neutrino mass...
- Constraints on the extensions of Higgs sector from the perturbative Unitarity.
- We construct the bases and scattering matrices for the model with two Higgs doublets and a (real/complex) triplet.
- Applications.

Thanks a lot for your attention!

谢 谢!