



河南省科学院
HENAN ACADEMY OF SCIENCES

Explaining The New CDF II W-Boson Mass Data in the “GM” Framework

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11/09/2022

Based on Xk Du, Z Li, Fei Wang, Yk Zhang. arXiv:2204.05760, 2204.04286

第十六届TeV物理工作组学术研讨会暨邝宇平院士学术思想研讨会

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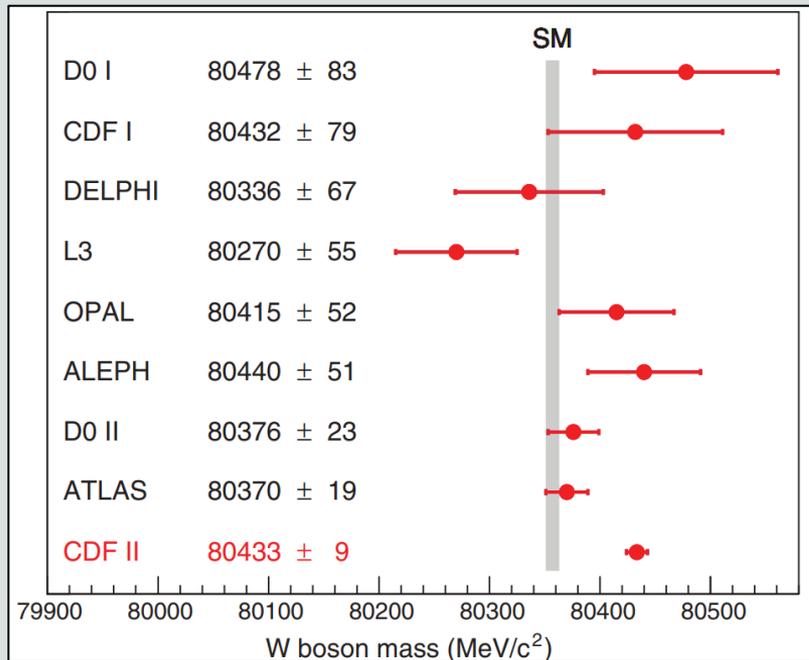
Summary
and Q&A

1、CDF-II Results

CDF-II Results on W Boson Mass



Science 376, 170-176 (2022)



$$\text{SM: } M_W = 80357 \pm 6 \text{ MeV}$$

$$\text{CDF-II: } M_W = 80433.5 \pm 9.4 \text{ MeV}$$

1、CDF-II Results

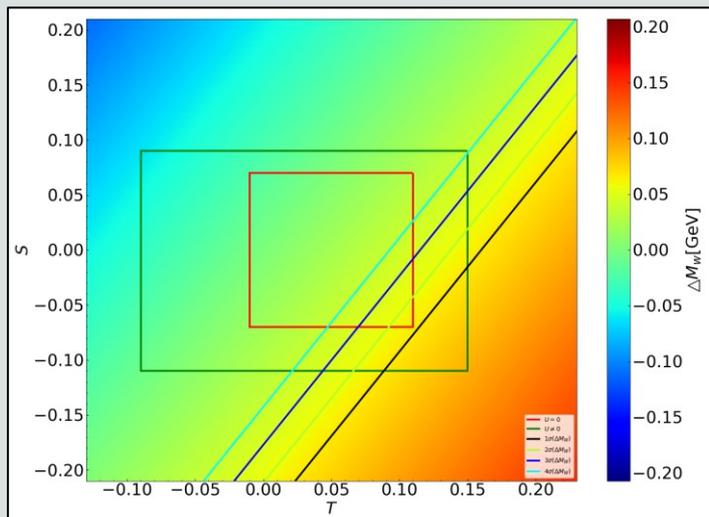
Δm_W ?

$$\Delta m_W = \frac{\alpha M_W}{2(c_W^2 - s_W^2)} \left(-\frac{1}{2}S + c_W^2 T + \frac{c_W^2 - s_W^2}{4s_W^2} U \right),$$

$$\alpha S = 4s_W^2 c_W^2 \left[\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W} \Pi'_{ZY}(0) - \Pi'_{YY}(0) \right],$$

$$\alpha T = \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2},$$

$$\alpha U = 4s_W^2 \left[\Pi'_{WW}(0) - c_W^2 \Pi'_{ZZ}(0) - 2s_W c_W \Pi'_{ZY}(0) - s_W^2 \Pi'_{YY}(0) \right]$$



$$\begin{aligned} S &= 0.00 \pm 0.07, \\ T &= 0.05 \pm 0.06, \\ U &= 0 \end{aligned}$$

$$\begin{aligned} S &= -0.01 \pm 0.10, \\ T &= 0.03 \pm 0.012, \\ U &= 0.02 \pm 0.11 \end{aligned}$$

Phys. Rev. D 46 (1992) 381, PTEP 2020 (2020) 8, 083C01(PDG2020)

2、 Explaining W-Boson Mass In the Georgi-Machacek Model

The Georgi-Machacek Model

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$$

Fields	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$
ϕ	1/2	2	1
χ	1	3	1
ξ	0	3	1

$$\mathcal{L}_{GM} = \mathcal{L}_{kin} + \mathcal{L}_Y + \mathcal{L}_\nu - V_H$$

$$\mathcal{L}_\nu \supset h_{ij} \overline{L_L^{ic}} i\tau_2 \chi L_L^j + h.c.$$

$SU(2)_L \times SU(2)_R$

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \Delta = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 \end{pmatrix}$$

$$\begin{aligned} V(\Phi, \Delta) = & \frac{1}{2} m_\Phi^2 \text{tr}[\Phi^\dagger \Phi] + \frac{1}{2} m_\Delta^2 \text{tr}[\Delta^\dagger \Delta] + \lambda_1 (\text{tr}[\Phi^\dagger \Phi])^2 + \lambda_2 (\text{tr}[\Delta^\dagger \Delta])^2 \\ & + \lambda_3 \text{tr}[(\Delta^\dagger \Delta)^2] + \lambda_4 \text{tr}[\Phi^\dagger \Phi] \text{tr}[\Delta^\dagger \Delta] + \lambda_5 \text{tr} \left[\Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2} \right] \text{tr}[\Delta^\dagger T^a \Delta T^b] \\ & + \mu_1 \text{tr} \left[\Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2} \right] (P^\dagger \Delta P)_{ab} + \mu_2 \text{tr}[\Delta^\dagger T^a \Delta T^b] (P^\dagger \Delta P)_{ab} \end{aligned}$$

2、 Explaining W-Boson Mass In the Georgi-Machacek Model

$$\boxed{SU(2)_L \times SU(2)_R} \xrightarrow[\langle \chi^0 \rangle = \langle \xi^0 \rangle = v_\Delta]{\langle \phi^0 \rangle = v_\phi / \sqrt{2}} \boxed{SU(2)_c}$$

$$v_{EW}^2 = \sum_i [4T_i(T_i + 1) - Y_i^2] |v_i|^2 c_i = v_\phi^2 + 4v_\chi^2 + 4v_\xi^2 = v_\phi^2 + 8v_\Delta^2 = \frac{1}{\sqrt{2}G_F} \approx (246\text{GeV})^2$$

$$c_{T,Y} = \begin{cases} 1, & (T, Y) \in \text{complex representation} \\ 1/2, & (T, Y = 0) \in \text{real representation} \end{cases}$$

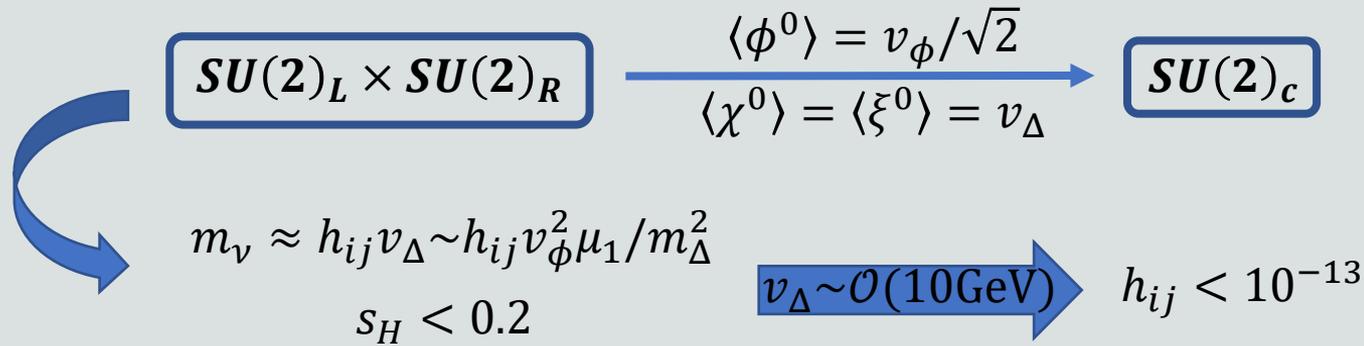
$$\rho_{tree} \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = \sum_i [4T_i(T_i + 1) - Y_i^2] |v_i|^2 c_i / \sum_i 2Y_i |v_i|^2$$

$$\Delta\rho_{tree} \equiv \rho - 1 = \frac{v_\phi^2 + 4v_\chi^2 + 4v_\xi^2}{v_\phi^2 + 8v_\Delta^2} - 1 \approx \frac{4v_\chi^2 - 4v_\xi^2}{v_{EW}^2}$$

$$\tan \theta = 2\sqrt{2}v_\Delta / v_\phi, s_H = \sin \theta < 0.2$$

2、 Explaining W-Boson Mass In the Georgi-Machacek Model

$$\mathcal{L}_{type-II} \supset h_{ij} \overline{L}_L^{ic} i\tau_2 \chi L_L^j + \mu_1 \text{tr} \left[\Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2} \right] (P^\dagger \Delta P)_{ab}$$



Vacuum stability

$$\begin{aligned} \lambda_1 > 0, \lambda_2 + \lambda_3 > 0, \lambda_2 + \frac{1}{2} \lambda_3 > 0, \\ -|\lambda_4| + 2\sqrt{\lambda_1(\lambda_2 + \lambda_3)} > 0, \\ \lambda_4 - \frac{1}{4} |\lambda_5| + \sqrt{2\lambda_1(2\lambda_2 + \lambda_3)} > 0. \end{aligned}$$

Perturbative unitarity

$$\begin{aligned} |\lambda_4 - \lambda_5| < 2\pi, \quad |2\lambda_3 + \lambda_2| < \pi, \\ |6\lambda_1 + 7\lambda_3 + 11\lambda_2| + \sqrt{(6\lambda_1 - 7\lambda_3 - 11\lambda_2)^2 + 36\lambda_4^2} < 4\pi, \\ |2\lambda_1 - \lambda_3 + 2\lambda_2| + \sqrt{(2\lambda_1 + \lambda_3 - 2\lambda_2)^2 + \lambda_5^2} < 4\pi. \end{aligned}$$

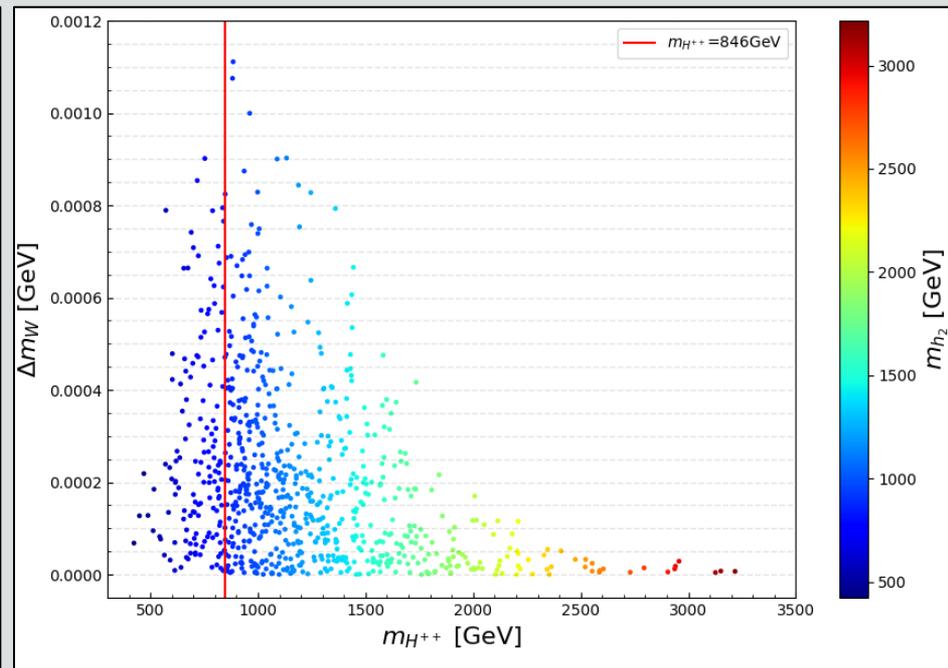
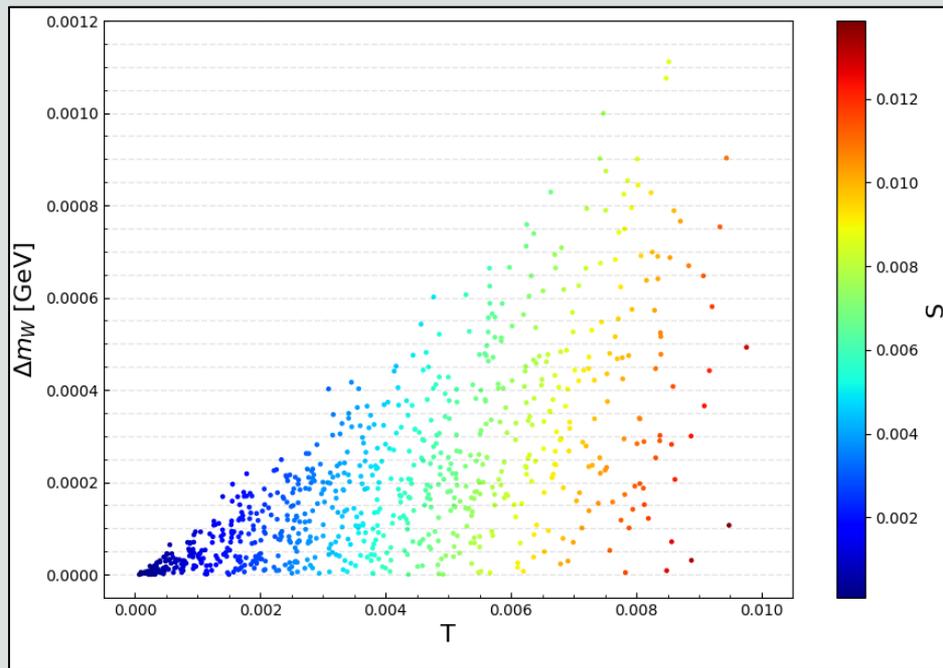
Nucl.Phys.B 262 (1985) 463-477, *JHEP* 01 (2013) 026, *JHEP* 01 (2016) 120

2、 Explaining W-Boson Mass In the Georgi-Machacek Model

$SU(2)_L \times SU(2)_R$

$$\begin{aligned} \langle \phi^0 \rangle &= v_\phi / \sqrt{2} \\ \langle \chi^0 \rangle &= \langle \xi^0 \rangle = v_\Delta \end{aligned}$$

$SU(2)_c$



2、 Explaining W-Boson Mass In the Georgi-Machacek Model

1

$$\Delta v = v_\chi - v_\xi > 0$$



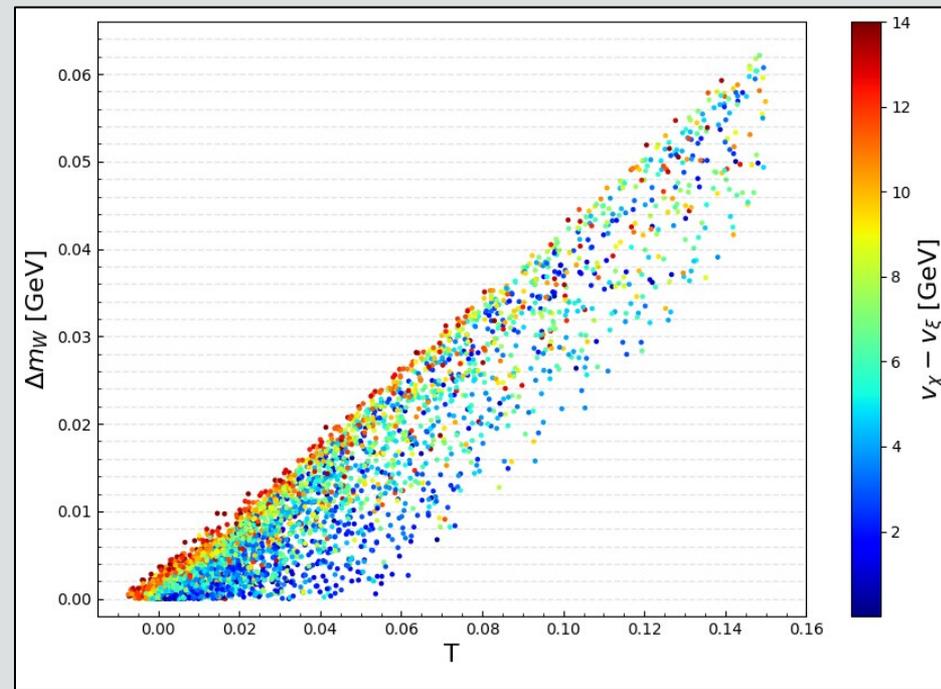
~~$SU(2)_c$~~



Δm_W



1. Choices of inputs to generate non-custodial symmetry preserving vacuum from $SU(2)_L \times SU(2)_R$ symmetry preserving scalar potential is possible.
2. hypercharge gauge boson loops as a consequence of $SU(2)_L \times SU(2)_R$ breaking effects in the kinetic term.
3. Loop contributions contain ultra-violet (UV) divergences which cannot be cancelled by counterterms associated with the V_{cust} part alone.
4. ...

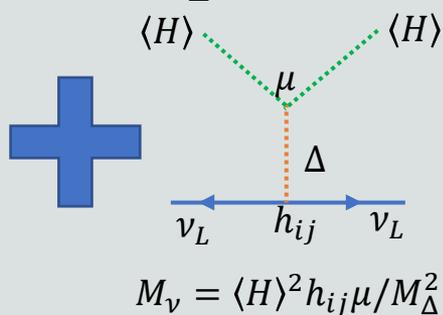
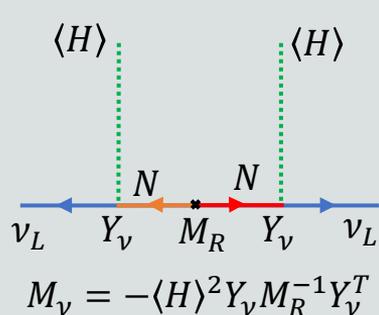


JHEP 03 (2021) 221; *Phys.Rev.D* 98 (2018) 1, 013008; *Phys.Lett.B* 774 (2017) 119-122; *Phys.Rev.D* 106 (2022) 5, 055035

2、Explaining W-Boson Mass In the Georgi-Machacek Model



$$-\mathcal{L}_\nu \supset y_{ij}^N \bar{L}_{L,i} \phi N_{R,j} + \frac{1}{2} (M_R)_{ij} N_{R,i}^T C N_{R,j} + h_{ij} \bar{L}_L^{ic} i\tau_2 \chi L_L^j + h.c.$$



$$M_\nu = \begin{pmatrix} h_{ij} v_\Delta & (y_{ij}^N)^T v_\phi \\ y_{ij}^N v_\phi & (M_R)_{ij} \end{pmatrix}$$

$$M_R \gg v_\phi \gg v_\Delta$$

$$m_\nu \approx h_{ij} v_\Delta - v_\phi^2 (y_{ij}^N)^T M_{R,j}^{-1} (y_{ij}^N), \quad h_{ij} v_\Delta \approx (y^N v_\phi)^2 / M_{R,j}^{-1}$$

$$h_{ij} = 2\sqrt{2} (V_{PMNS}^T)^{-1} \left(\frac{v(1-s_H^2)}{s_H M_{R,i}} \right) \delta_{ij} (V_{PMNS})^{-1}, \quad y_{ij}^N = (V_{PMNS})^{-1}$$

2、 Explaining W-Boson Mass In the Georgi-Machacek Model

2

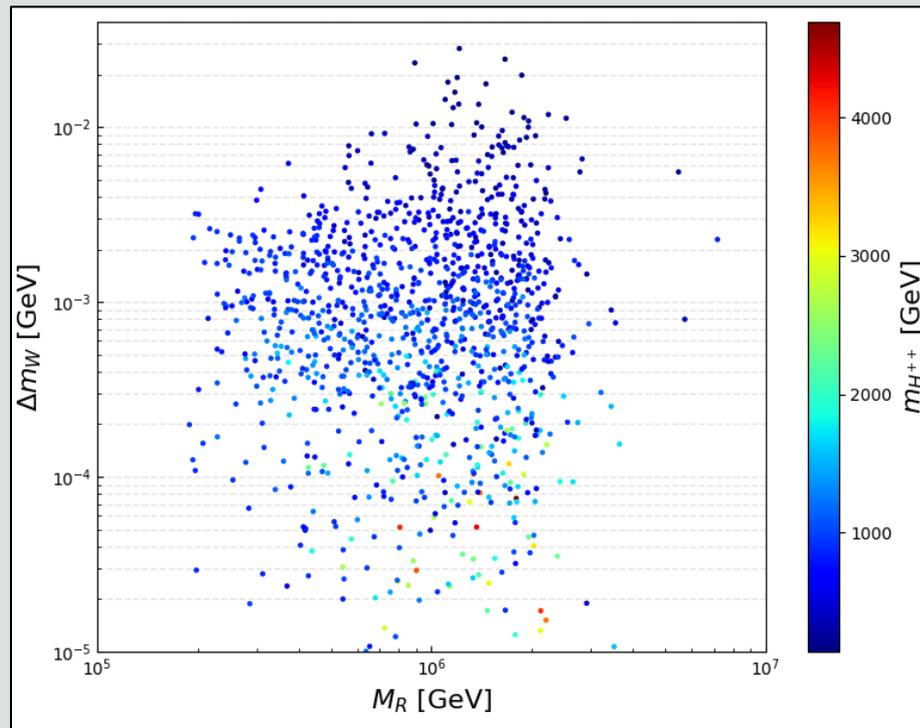
Type-I + Type-II

$$BR(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13},$$
$$BR(\mu \rightarrow 3e) < 10^{-12}$$

$$BR(\mu \rightarrow e\gamma) \sim \frac{\alpha_{EM}}{192\pi} |h_{ij}|^4 \left(\frac{m_W}{M_{H^{++}}} \right)^4$$



$$h_{ij} < 10^{-2}, \quad M_R > 50\text{TeV}$$



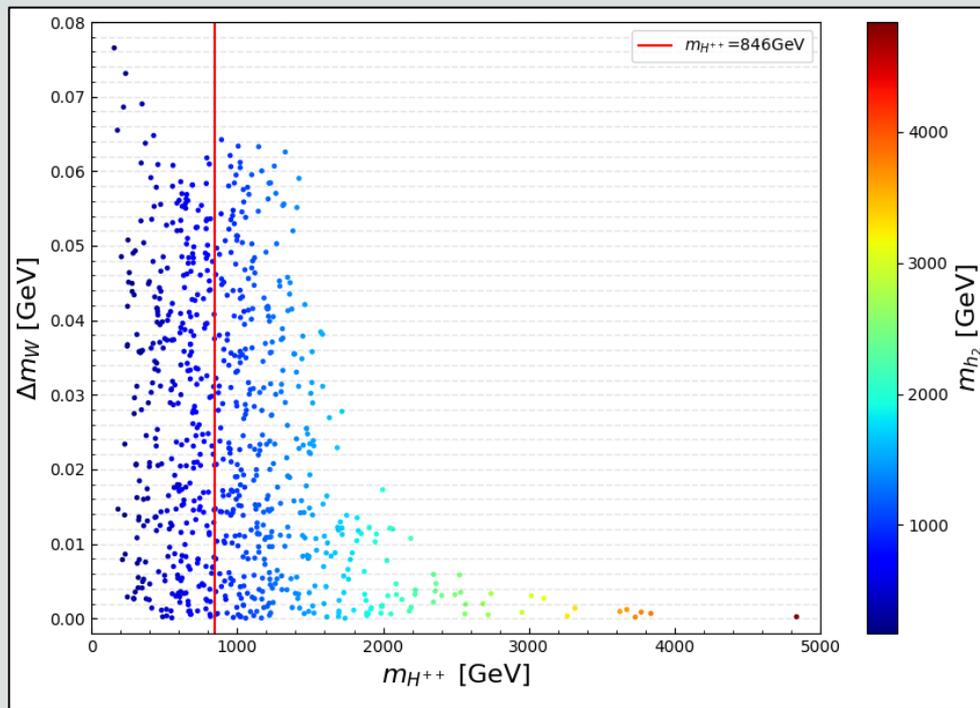
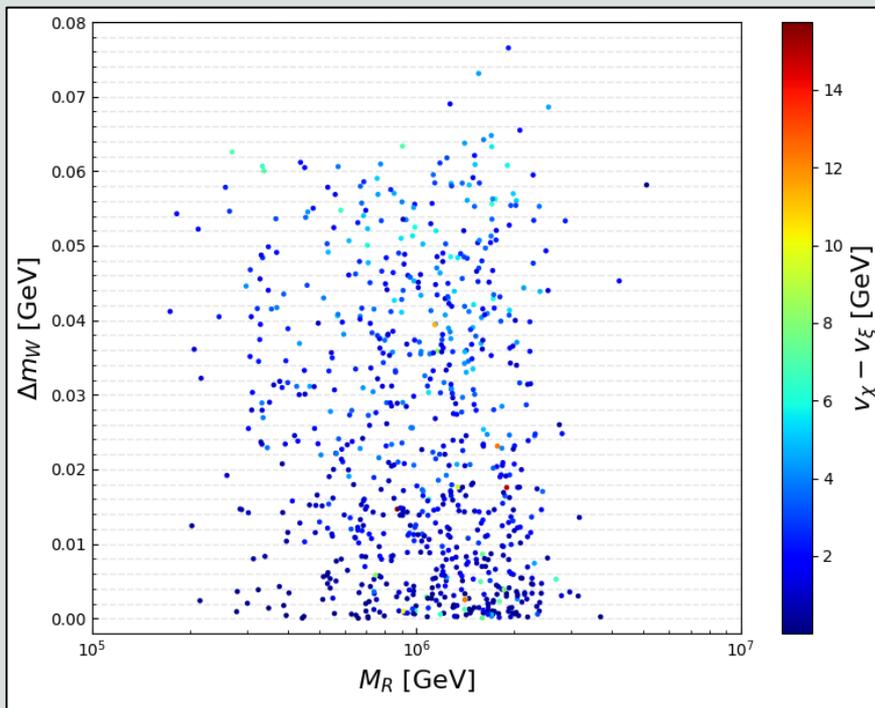
2、 Explaining W-Boson Mass In the Georgi-Machacek Model

3

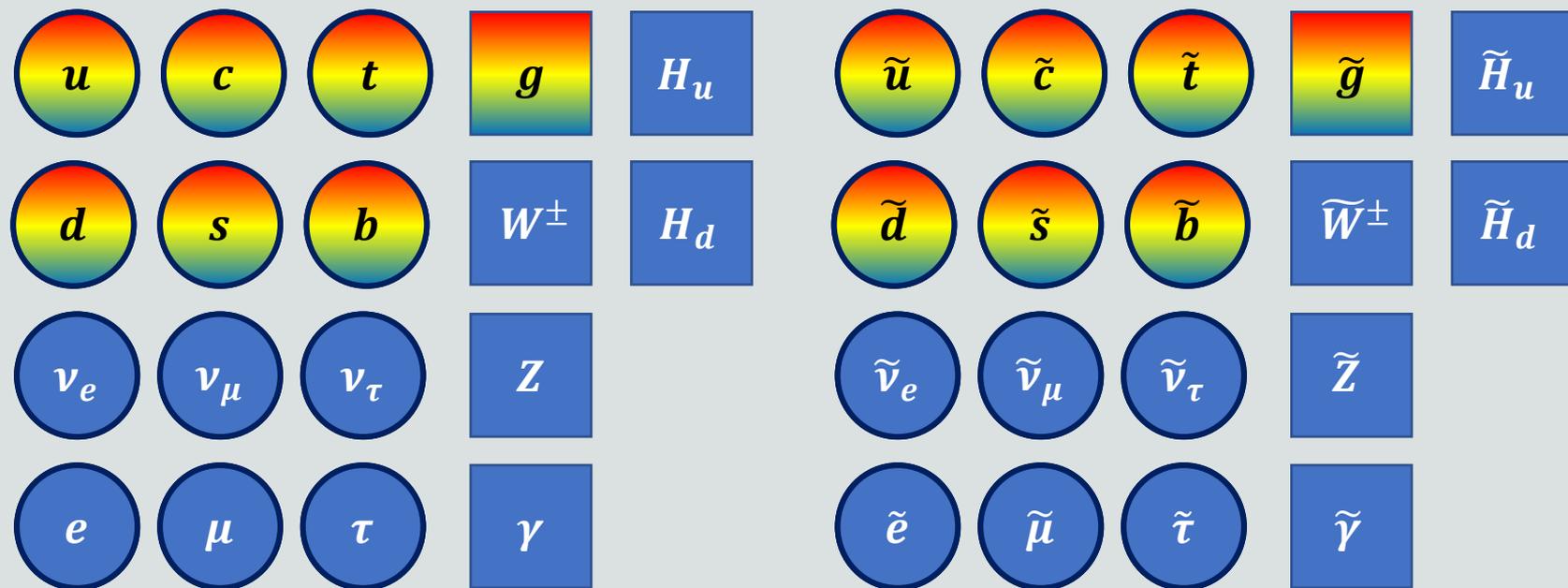
$$\Delta v = v_\chi - v_\xi > 0$$



Type-I + Type-II



3、 Explaining W-Boson Mass In the Gauge Mediation Scenarios



SUSY Broken
origin
(Hidden Sector)

AMSB, GMSB, etc

MSSM/NMSSM
(Visible Sector)

Nucl.Phys.B 34 (1971) 632-639, *Phys.Lett.B* 46 (1973) 109-110, *Nucl.Phys.B* 70 (1974) 39-50, *Adv.Ser.Direct.High Energy Phys.* 21 (2010) 1-153, *Phys.Rept.* 496 (2010) 1-77

3、 Explaining W-Boson Mass In the Gauge Mediation Scenarios

01 the minimal Gauge Mediation (MGM)

singlet field X (spurion): $\langle X \rangle = X + \theta F$

Messenger fields (in $5 \oplus \bar{5}$ representation of $SU(5)$): $\phi_i, \tilde{\phi}_i$

$$W_m = \sum_i \lambda_i X \phi_i \tilde{\phi}_i$$

$$M_i = \frac{\alpha_i}{4\pi} \Lambda N_5,$$

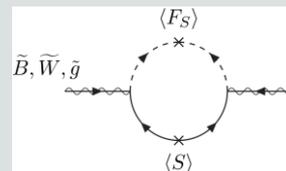
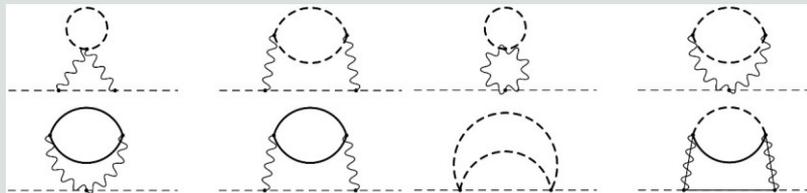
$$m_{\tilde{f}}^2 = 2 \sum_{i=1}^3 C_{\tilde{f}}^i \left(\frac{\alpha_i}{4\pi} \right)^2 \Lambda^2 N_5,$$

$$A_{u,d,l} = 0,$$

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} = C$$

$$\longrightarrow M_1 : M_2 : M_3 = 1 : 2 : 6 \text{ (EW)}$$

$$\left. \begin{aligned} \Lambda &\sim F/M. \\ C_3 &= \begin{cases} 4/3 & \text{when } \Phi = Q, \bar{u}, \bar{d} \\ 0 & \text{when } \Phi = L, \bar{e}, H_u, H_d \end{cases} \\ C_2 &= \begin{cases} 3/4 & \text{when } \Phi = Q, L, H_u, H_d \\ 0 & \text{when } \Phi = \bar{u}, \bar{d}, \bar{e} \end{cases} \\ C_1 &= Y^2 * 3/5 \end{aligned} \right\}$$



Nucl.Phys.B 34 (1971) 632-639, *Phys.Lett.B* 46 (1973) 109-110, *Nucl.Phys.B* 70 (1974) 39-50, *Adv.Ser.Direct.High Energy Phys.* 21 (2010) 1-153, *Phys.Rept.* 496 (2010) 1-77

3、 Explaining W-Boson Mass In the Gauge Mediation Scenarios

02 the (Extra)Ordinary Gauge Mediation (EOGM)

$$W_m = M_{ij}^k \lambda_i X_k \phi_i \tilde{\phi}_j = (\lambda_{ij}^k X_k + m_{ij}) \phi_i \tilde{\phi}_j$$

$$M_i = \frac{\alpha_i}{4\pi} \Lambda_G,$$

$$m_{\tilde{f}}^2 = 2 \sum_{i=1}^3 C_{\tilde{f}}^i \left(\frac{\alpha_i}{4\pi}\right)^2 \Lambda_S^2,$$

$$A_{u,d,l} = 0,$$

$$\left\{ \begin{array}{l} \Lambda_G = F \partial_X \log \det M_{ij}^k = \frac{nF}{X} \\ \Lambda_S^2 = \frac{1}{2} |F|^2 \frac{\partial^2}{\partial X \partial X^*} \sum_{k=1}^N \left(\log |M^k|^2 \right)^2 \end{array} \right. \Rightarrow N_{eff} \equiv \frac{\Lambda_G^2}{\Lambda_S^2}$$

$$5 = (1,2)(L) \oplus (3,1)(D), \quad \bar{5} = (1, \bar{2})(\bar{L}) \oplus (\bar{3}, 1)(\bar{D}),$$

$$W_m = \left(\lambda_{2ij} X + m_{2ij} \right) L_i \tilde{L}_j + \left(\lambda_{3ij} X + m_{3ij} \right) D_i \tilde{D}_j$$

$$m_{\tilde{f}}^2 = 2 \sum_{r=1}^3 C_{\tilde{f}}^r \left(\frac{\alpha_i}{4\pi}\right)^2 \Lambda_{S;r}^2$$

$$\left\{ \begin{array}{l} \Lambda_{S;r=2,3}^2 = \Lambda_G^2 N_{eff}^{-1} \\ \Lambda_{S1}^2 = \frac{2}{5} \Lambda_{S3}^2 + \frac{3}{5} \Lambda_{S2}^2 \end{array} \right.$$

$$N_{eff,1}^{-1} = \frac{2}{5} N_{eff,3}^{-1} + \frac{3}{5} N_{eff,2}^{-1}$$

Cheung, Clifford etc, JHEP 07 (2008) 054 • arXiv: 0710.3585

3、 Explaining W-Boson Mass In the Gauge Mediation Scenarios

02 the (Extra)Ordinary Gauge Mediation (EOGM)

with large $\tan \beta$:

$$\mu^2 \approx -\frac{1}{2} m_Z^2 - m_{H_u}^2 (SUSY)$$

$$m_{H_u}^2 \propto \frac{3}{4} \alpha_2 (M_{mess})^2 N_{eff,2}^{-1}, \quad m_{\tilde{t}}^2 \propto \frac{4}{3} \alpha_3 (M_{mess})^2 N_{eff,3}^{-1}$$

with stop loops:

$$m_{H_u}^2 (SUSY) \approx m_{H_u}^2 - \frac{3}{4\pi^2} y_t^2 m_{\tilde{t}}^2 \log \frac{M_{mess}}{m_{\tilde{t}}}$$

◆ for $N_{eff,3}^{-1} \ll N_{eff,2}^{-1}$:



$$\begin{aligned} \mu^2 &\sim m_Z^2, \\ m_{\tilde{t}} &\sim m_{\tilde{q}}, \\ M_1 : M_2 : M_3 &\sim 1 : 2 : 6 \end{aligned}$$

◆ for $N_{eff,3}^{-1} \gg N_{eff,2}^{-1}$:

$$\begin{aligned} m_{\tilde{t}} &\ll m_{\tilde{q}}, \\ \mu^2 &\sim \frac{\alpha_3}{4\pi^2} \frac{\Lambda_G}{\sqrt{N_{eff,3}}} \gg M_Z^2, \\ M_1 : M_2 : M_3 &\sim 1 : 2 : 6 \end{aligned}$$



$$\begin{aligned} |\mu \tan \beta_{eff}| &< 56.9 \sqrt{m_{\tilde{L}} m_{\tilde{\tau},R}} \\ &+ 57.1 (m_{\tilde{L}} + 1.03 m_{\tilde{\tau},R}) \\ &- 1.28 \times 10^4 \text{ GeV} + \frac{10^6 \text{ GeV}^2}{m_{\tilde{L}} + m_{\tilde{\tau},R}} \\ &- 6.41 \times 10^7 \text{ GeV}^3 \left(\frac{1}{m_{\tilde{L}}^2} + \frac{0.983}{m_{\tilde{\tau},R}^2} \right) \end{aligned}$$

Cheung, Clifford etc, JHEP 07 (2008) 054, arXiv: 0710.3585; JHEP 05 (2013) 035, arXiv: 1303.0461

3、 Explaining W-Boson Mass In the Gauge Mediation Scenarios

03 the General Gauge Mediation (GGM)

several singlets and a single set of $\mathbf{5} \oplus \bar{\mathbf{5}}$ messengers:

$$\langle X_i \rangle = X_i + \theta^2 F_i$$

$$W_m = X_i (\lambda_L^i L \tilde{L} + \lambda_D^i D \tilde{D}) + F_i X^i$$

$$M_1 = \frac{\alpha_1}{4\pi} \left[\frac{2}{3} \Lambda_D^2 + \Lambda_L^2 \right], \quad \Lambda_D = \frac{\lambda_D^i F_i}{\lambda_D^j X_j},$$

$$M_2 = \frac{\alpha_2}{4\pi} \Lambda_L,$$

$$M_3 = \frac{\alpha_3}{4\pi} \Lambda_D$$

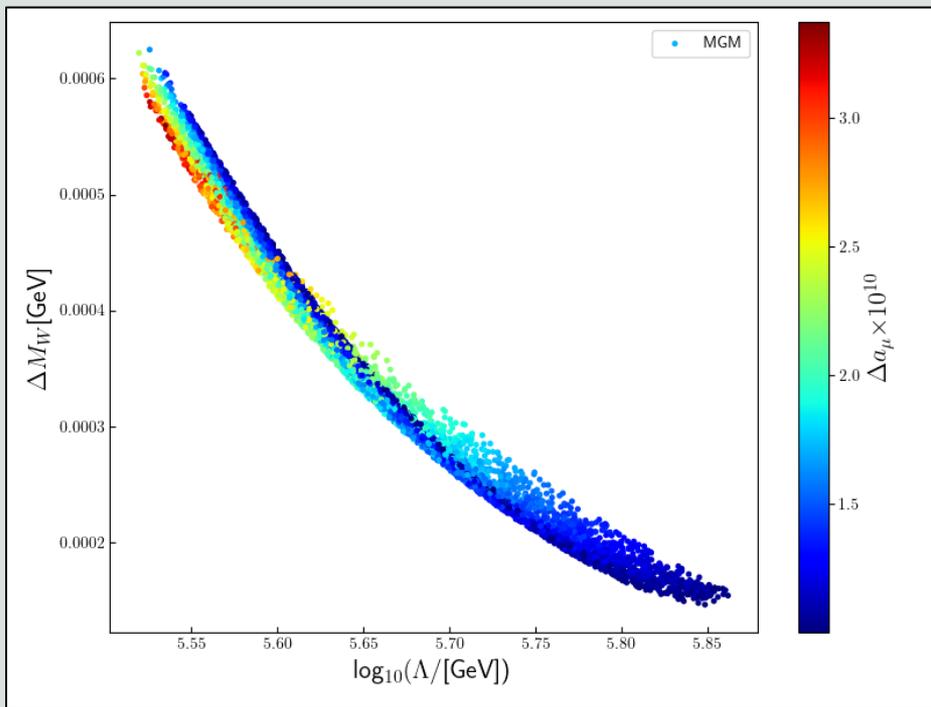
$$\Lambda_L = \frac{\lambda_L^i F_i}{\lambda_L^j X_j}$$

~~$$M_1 : M_2 : M_3 \sim 1 : 2 : 6$$~~

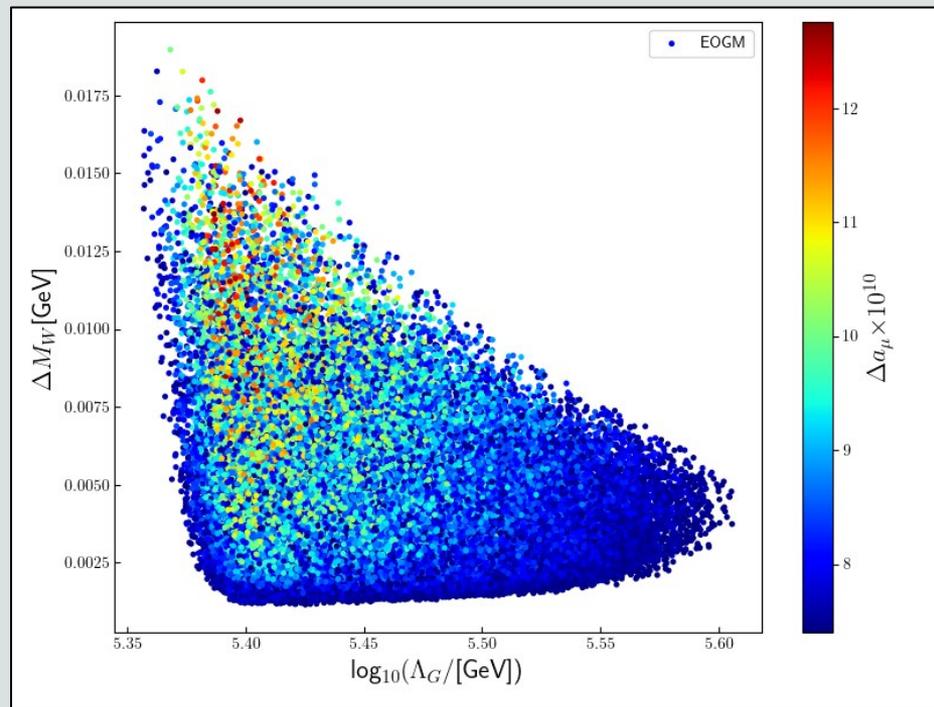
Phys.Rev.D 77 (2008) 016005, arXiv: hep-ph/0611312; Phys.Rev.D 79 (2009) 035002, arXiv: 0805.2944

3、 Explaining W-Boson Mass In the Gauge Mediation Scenarios

MGM

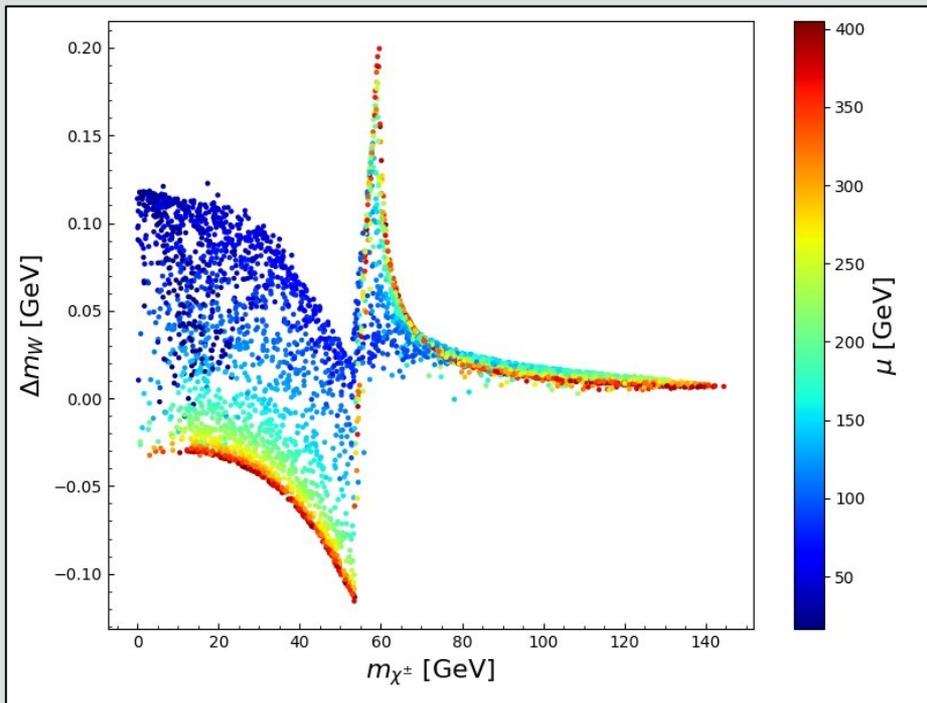


EOGM

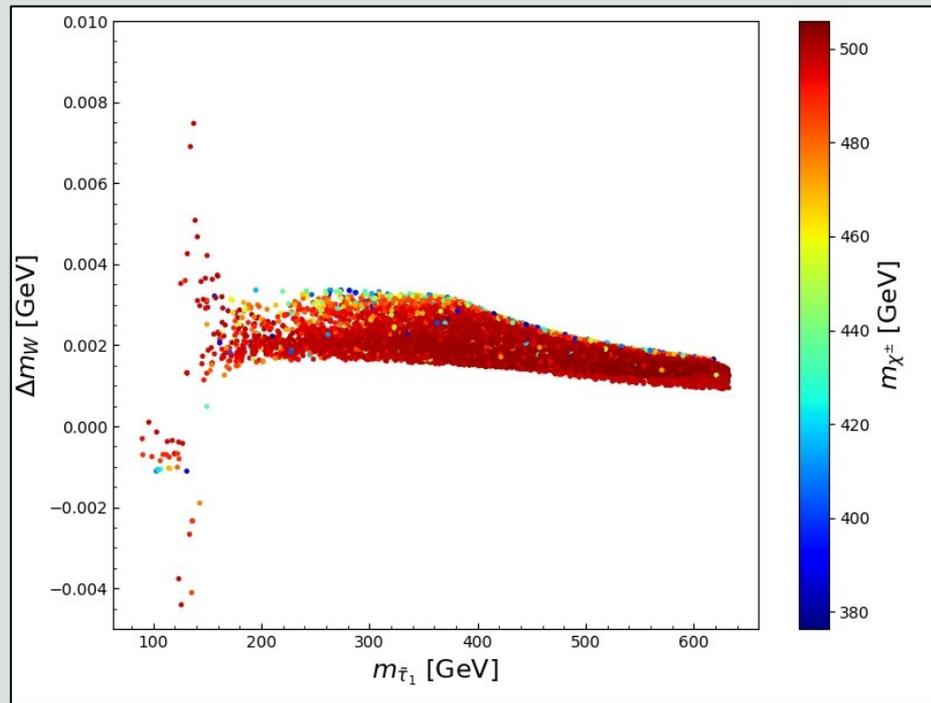


3、 Explaining W-Boson Mass In the Gauge Mediation Scenarios

GGM (light wino)

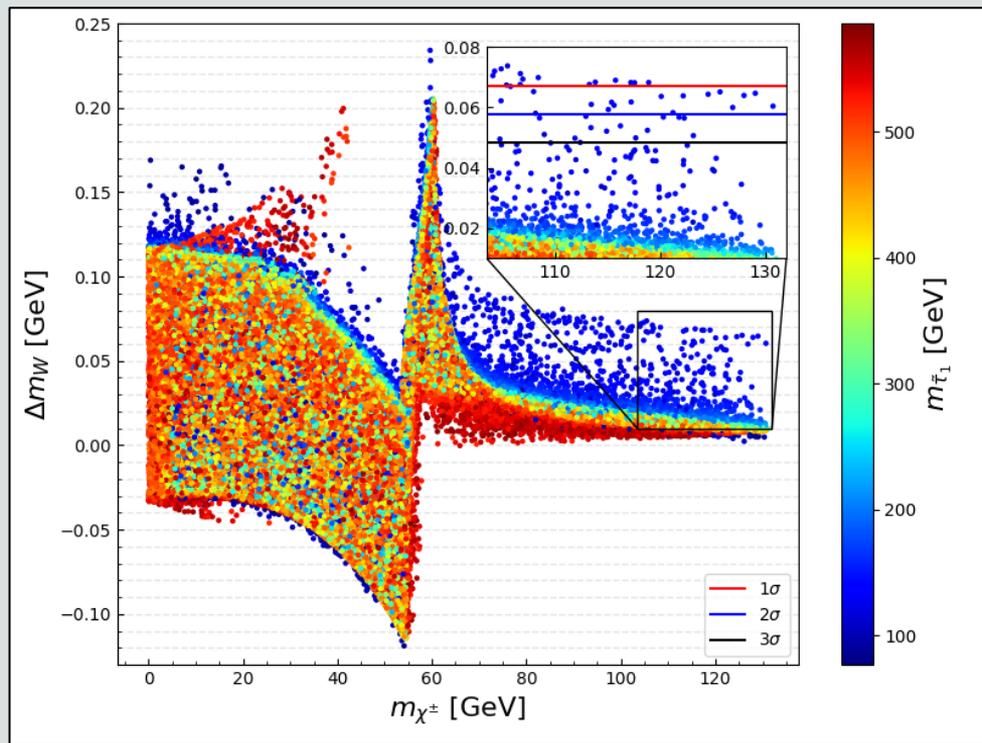


GGM (light slepton)



3、 Explaining W-Boson Mass In the Gauge Mediation Scenarios

MGM (light wino and slepton)



4、 Summary and Q&A

1. Taken a general discussion about the contribution to W boson mass in the original GM Model;
2. Explaining CDF-II results in the GM Extension Models.
Misalignment among the triplet VEVs and large h_{ij} couplings allowed with RH neutrino sector;
3. Taken a general discussion about the contribution to W boson mass and $g_\mu - 2$ in the MGM and EOGM;
4. Explaining CDF-II results in the GGM framework.

THANKS

Q&A

03 the General Gauge Mediation (GGM)

several singlets and a single set of $\mathbf{10} \oplus \overline{\mathbf{10}}$ messengers ($Q, \bar{Q}, U, \bar{U}, E, \bar{E}$)

$$\langle X_i \rangle = X_i + \theta^2 F_i$$

$$W_m = X_i (\lambda_Q^i Q \tilde{Q} + \lambda_U^i U \tilde{U} + \lambda_E^i E \tilde{E})$$

$$M_1 = \frac{\alpha_1}{4\pi} \left(\frac{4}{3} \Lambda_Q^2 + 2\Lambda_E^2 + \frac{8}{3} \Lambda_U \right), \quad \Lambda_Q = \frac{\lambda_Q^i F_i}{\lambda_Q^j X_j},$$

$$M_2 = \frac{\alpha_2}{4\pi} \Lambda_Q, \quad \Lambda_U = \frac{\lambda_U^i F_i}{\lambda_U^j X_j},$$

$$M_3 = \frac{\alpha_3}{4\pi} (2\Lambda_Q + \Lambda_U) \quad \Lambda_E = \frac{\lambda_E^i F_i}{\lambda_E^j X_j}$$