Gravitational Waves from Second-Order Phase Transition and Domain Walls during Inflation

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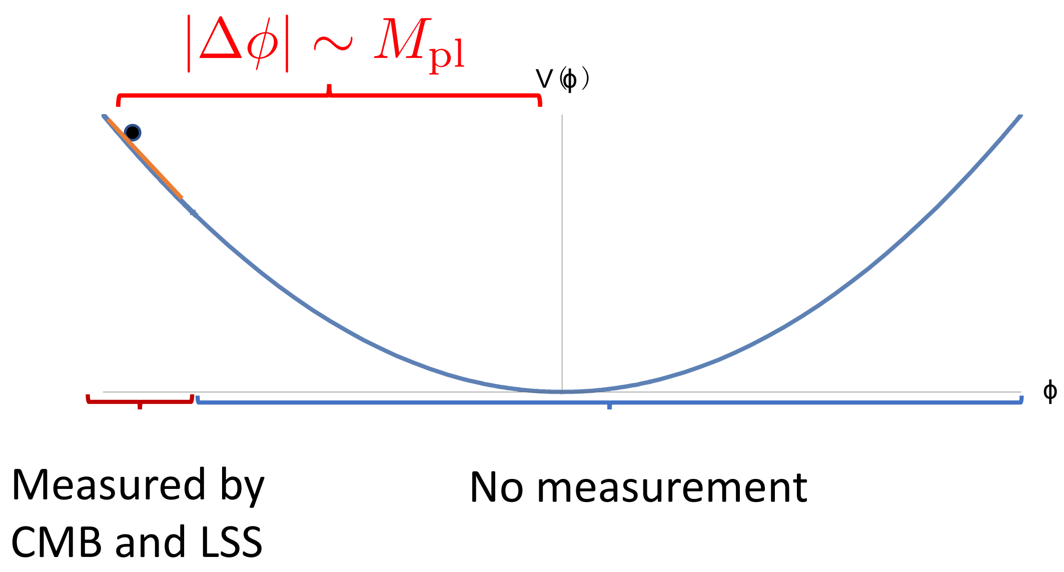
Outline

- Inflation and phase transition
- Second-order phase transition during inflation
- Summary and outlook

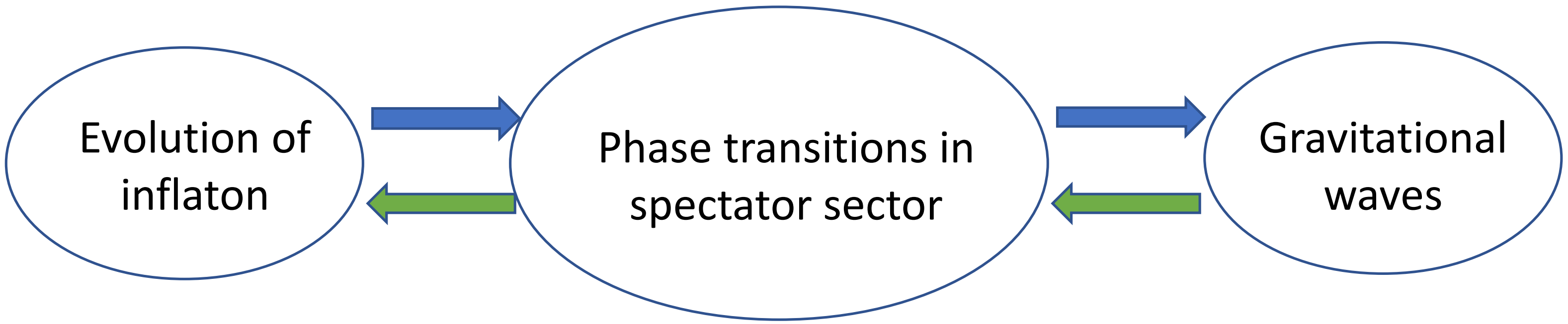
Inflation

- Slow roll inflation
- We usually assume a potential.
- Use it to calculate n_s, r …



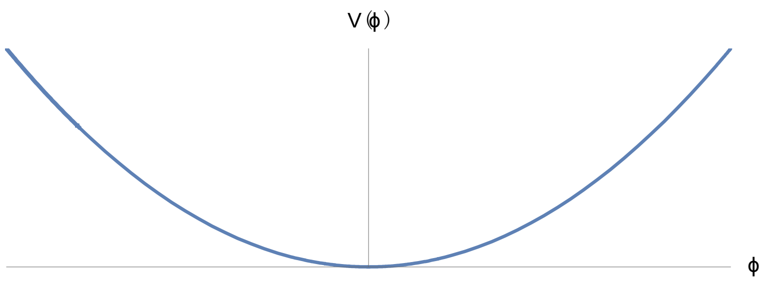
- The inflaton must couple to some spectator field.
- The masses or couplings in the spectator sector can be changed drastically due to the evolution of the inflaton field.

Phase transitions in the spectator sector

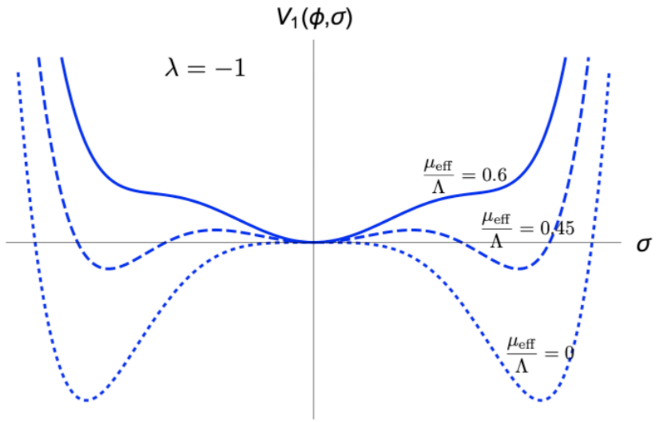


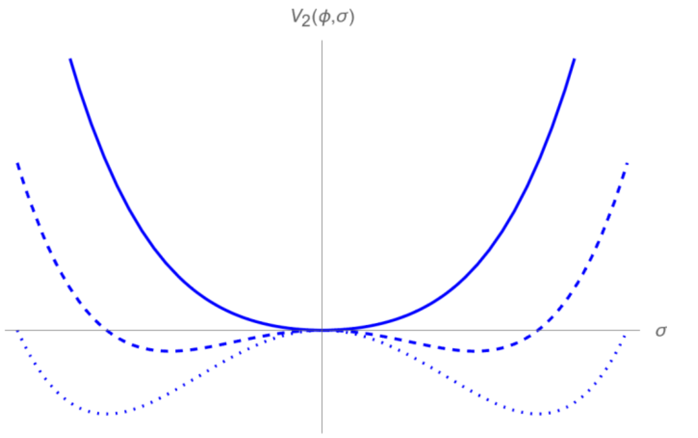
For first-order phase transitions, see 2009.12381, 2201.05171.
In this talk we focus on second-order phase transition.

ϕ: inflaton field



σ: order parameter in the spectator sector



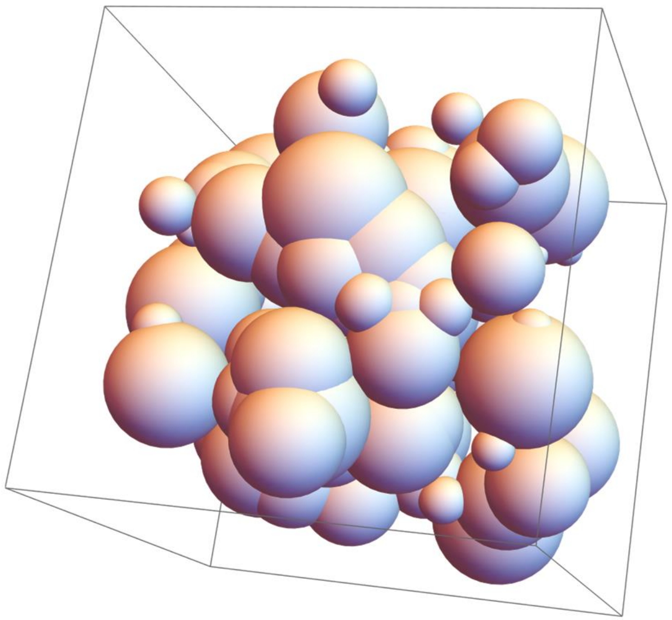


$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

$$V_2(\phi,\sigma) = -\frac{1}{2}\left(\mu^2 - c^2\phi^2\right)\sigma^2 + \frac{\lambda}{4}\sigma^4$$

First-order vs second-order

First-order phase transition

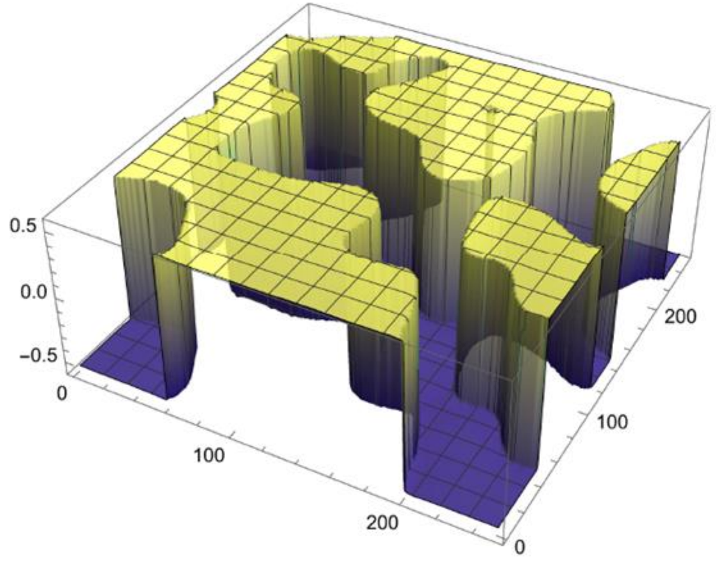


$$\frac{\Gamma}{V} = I_0 m_\sigma^4 e^{-S_4}$$

$\beta = -\frac{dS_4}{dt}$,
determines the rate of the phase transition.

Phase transition completes if $\beta \gg H$.

Second-order phase transition



$$\xi(t) = \tau(t) = |m_\sigma(t)|^{-1}$$

ξ_q determines the density of topological defects.

Phase transition always completes.

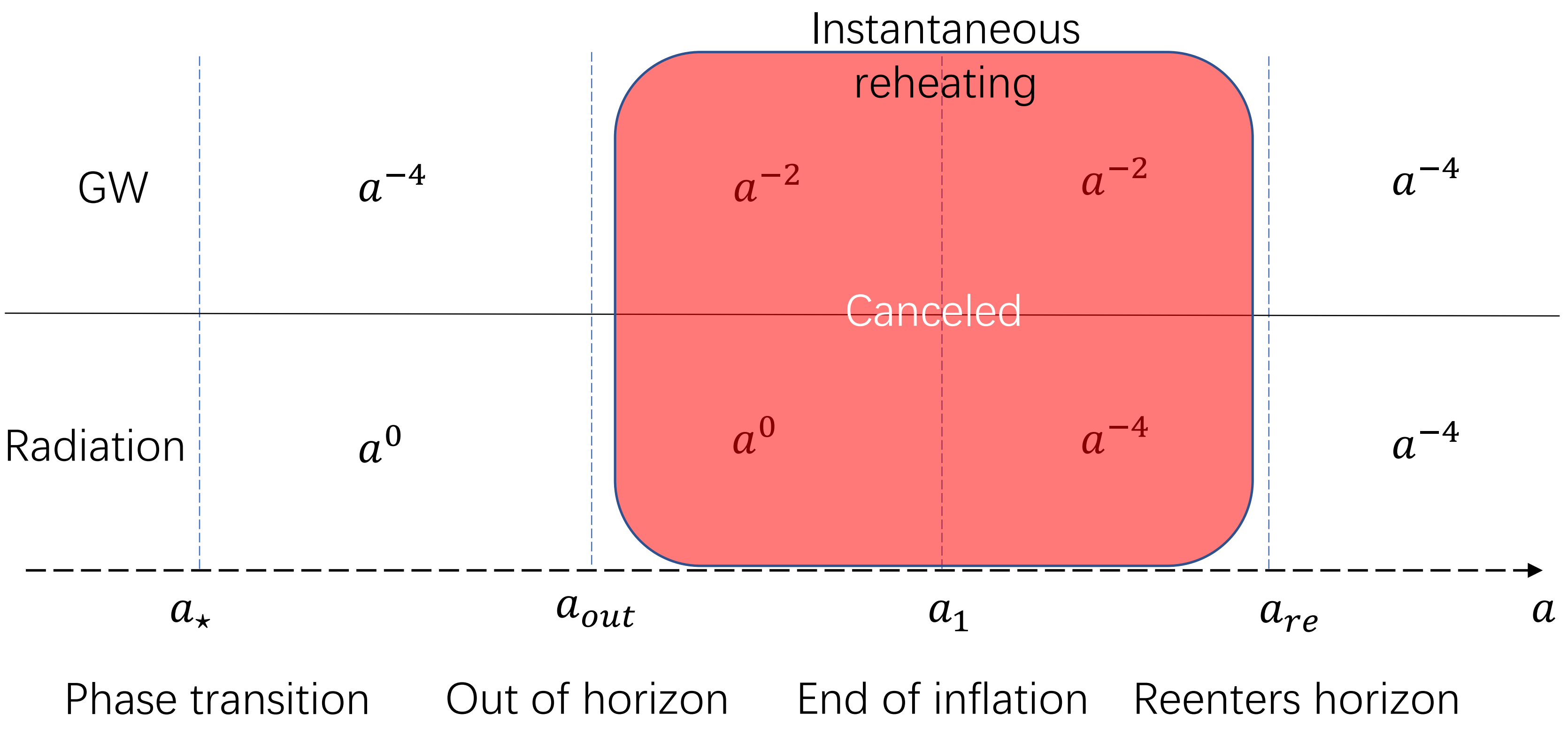
$\rho_{\rm GW}$ can be estimated by the gravity potential between the bubble-like structure:

$$\rho_{\rm GW} \sim G M_{\rm B}^2 / R_{\rm B}^4 \sim G \rho_{\rm B}^2 R_{\rm B}^2.$$

For thermal phase transition during radiation domination:
FOPT: $R_{\rm B} \approx \beta^{-1}$, with $H/\beta \approx O(10^{-2})$.
SOPT: $R_{\rm B} \approx \xi_q$, with $H\xi_q \approx O(10^{-10})$.

During inflation, however, H/β and $H\xi_q$ can be the same order.

Redshifts of the GW signal



$$\left(\frac{\Omega_{\rm GW}}{\Omega_{\rm R}}\right)_{\rm Inf} \sim \left(\frac{H}{k_p}\right)^4 \left(\frac{\Omega_{\rm GW}}{\Omega_{\rm R}}\right)_{\rm RD}$$

Gravitational waves during inflation

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

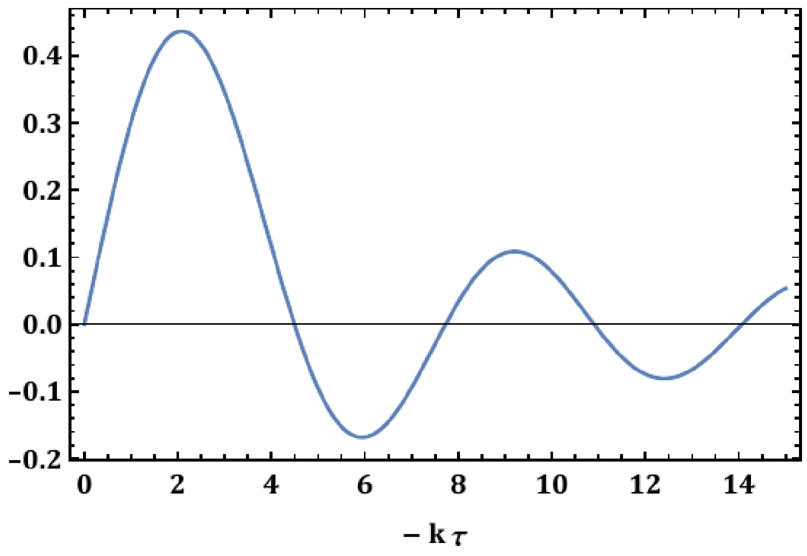
$$h''_{ij}(\tau, \mathbf{x}) + 2\frac{a'}{a}h'_{ij}(\tau, \mathbf{x}) - \nabla^2 h_{ij} = 16\pi G T_{ij}$$

For de Sitter inflation:

$$\tilde{h}^f_{ij}(\mathbf{k}) = 16\pi G\frac{H}{k^2}\int_{\tau_i}^{0} \mathrm{d}\tau' a(\tau')\left(\frac{\sin[-k\tau']}{-k\tau'} - \cos[-k\tau']\right)\tilde{T}_{ij}(\mathbf{k}, \tau').$$

For constant anisotropy stress tensor:

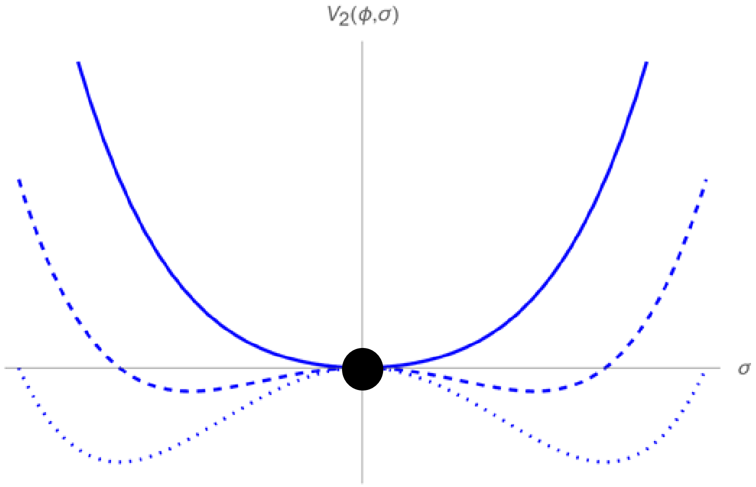
$$\tilde{h}^f_{ij}(\mathbf{k}) = \frac{16\pi G}{k}\left(1 - \frac{\sin[-k\tau_i]}{-k\tau_i}\right)\tilde{T}_{ij}(\mathbf{k}) \neq 0$$



During inflation constant anisotropy stress tensor can also induce tensor perturbation!

The model

$$V(\phi,\sigma) = \frac{1}{2}\left(g^2\phi^2 - m^2\right)\sigma^2 + \frac{\lambda}{4}\sigma^4$$



Low energy modes suffer tachyonic instability.

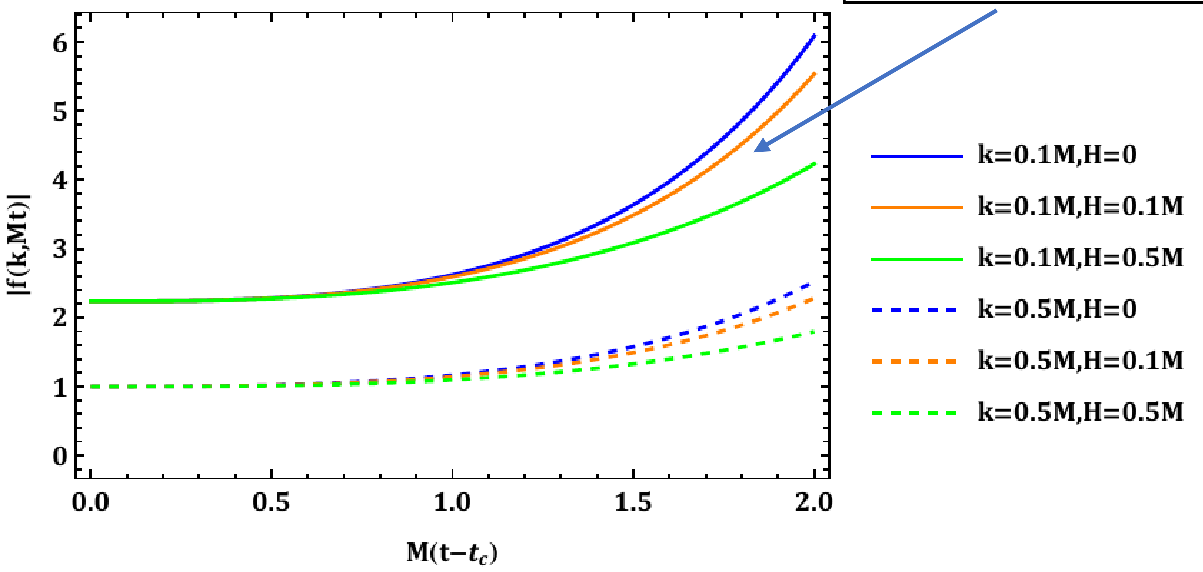
Around critical point, $\phi_c = m/g$:

$$V_c = -\frac{1}{m}\frac{\dot{\phi}_c}{\phi_c}, \qquad \xi(t_q) = (2V_c)^{-1/3} m^{-1} \equiv M^{-1}$$

$$\ddot{\sigma}_{\mathbf{k}} + 3H\dot{\sigma}_{\mathbf{k}} + \left(\mathrm{e}^{-2H(t-t_c)}k^2 - 2M^3(t-t_c)\right)\sigma_{\mathbf{k}} = 0$$

$$\sigma_{\mathbf{k}}(t) = a_{\mathbf{k}} f(\mathbf{k},t) + a^{\dagger}_{-\mathbf{k}} f^*(-\mathbf{k},t)$$

Exponential growth

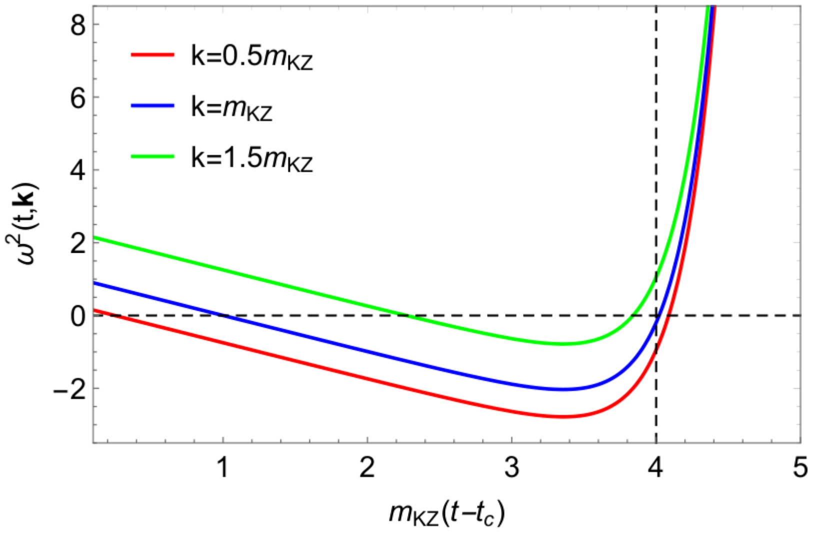


The model

$$\ddot{\sigma}_{\mathbf{k}} + 3H\dot{\sigma}_{\mathbf{k}} + \left(e^{-2H(t-t_c)}k^2 - 2M^3(t-t_c) + \frac{\lambda}{2}\langle\sigma^2\rangle\right)\sigma_{\mathbf{k}} = 0$$

$\omega^2(t, \boldsymbol{k})$

Interaction induced mass

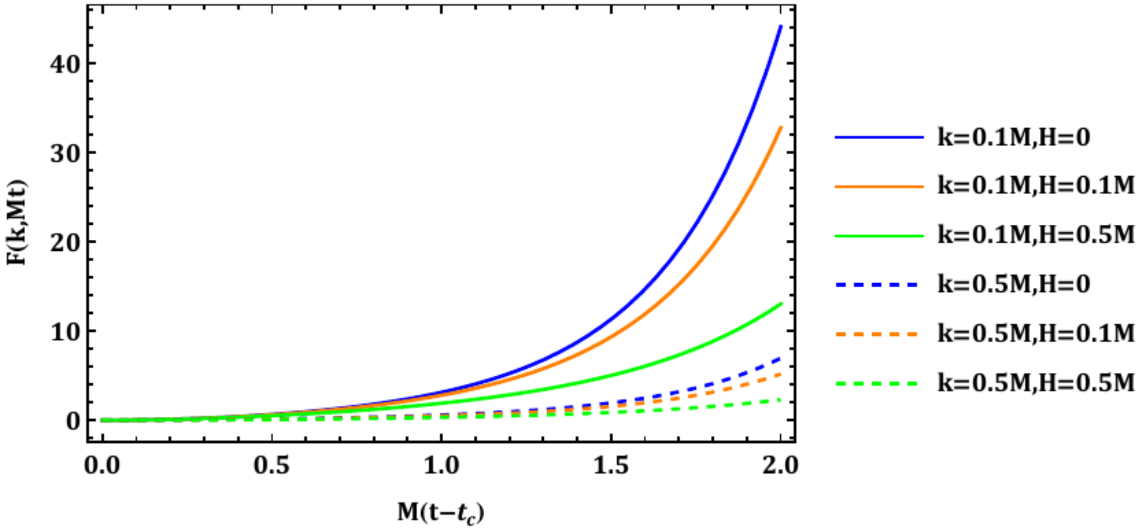


Interaction can induce a effective mass which stop the growth.

Quantum-classical transition

$$|\langle\Omega, t|[\hat{\sigma}_{\mathbf{k}}, \hat{\pi}_{\mathbf{k}}]_+|\Omega, t\rangle| \gg |\langle\Omega, t|[\hat{\sigma}_{\mathbf{k}}, \hat{\pi}_{\mathbf{k}}]|\Omega, t\rangle|$$

$F(k,t)$ $\quad$ 1



Wigner function:

$$W_{\mathbf{k}}(\sigma_{\mathbf{k}}, \pi_{\mathbf{k}}) = \frac{1}{\pi^2}\exp\left[-\frac{|\sigma_{\mathbf{k}}|^2}{|f(\mathbf{k},\tau)|^2} - 4|f(\mathbf{k},\tau)|^2\left|\pi_{\mathbf{k}} - \frac{F(\mathbf{k},\tau)}{|f(\mathbf{k},\tau)|^2}\sigma_{\mathbf{k}}\right|^2\right]$$

Numerical simulation

$$\sigma'' + 2\frac{a'}{a}\sigma' - \nabla^2\sigma + a^2\left(g^2\phi^2 + \sigma^2 - m^2\right)\sigma = 0$$

$$\Phi = g\frac{\phi}{m}, \quad \Sigma = \sqrt{\lambda}\frac{\sigma}{m}, \quad \eta = m\tau, \quad \mathbf{X} = m\mathbf{x}, \quad \tilde{H} = H/m.$$

$$\Sigma'' - \frac{2}{\eta}\Sigma' - \nabla^2\Sigma + \frac{1}{(\tilde{H}\eta)^2}\left(\Phi^2 + \Sigma^2 - 1\right)\Sigma = 0$$

$\lambda, V_c, \tilde{H}$

$\Sigma(\eta_0), \Sigma'(\eta_0)$

$\Phi(\eta)$

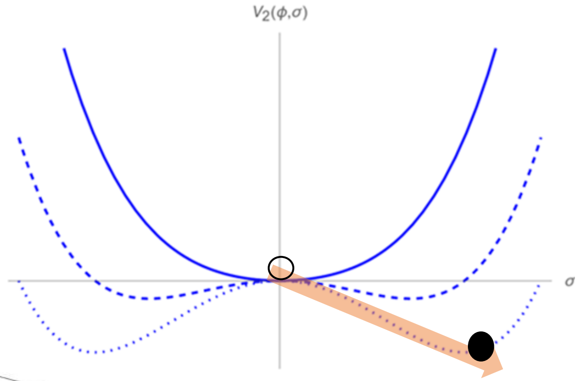
$$W_{\mathbf{k}}(\sigma_{\mathbf{k}}, \pi_{\mathbf{k}}) = \frac{1}{\pi^2}\exp\left[-\frac{|\sigma_{\mathbf{k}}|^2}{|f(\mathbf{k},\tau)|^2} - 4|f(\mathbf{k},\tau)|^2\left|\pi_{\mathbf{k}} - \frac{F(\mathbf{k},\tau)}{|f(\mathbf{k},\tau)|^2}\sigma_{\mathbf{k}}\right|^2\right]$$

Lattice grid 251^3
6th Runge-Kutta

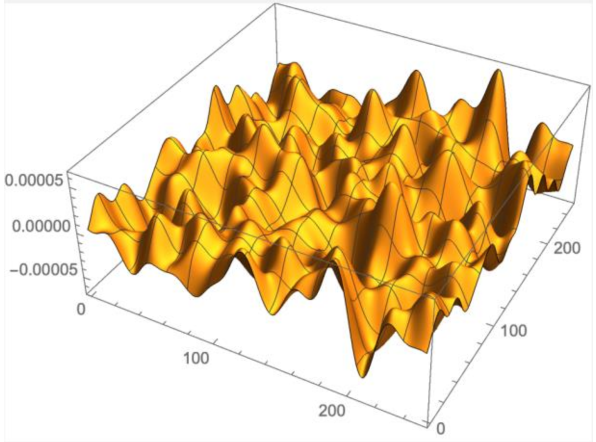
We match the quantum evolution and classical evolution at $t_0 - t_c = 2M^{-1}$.

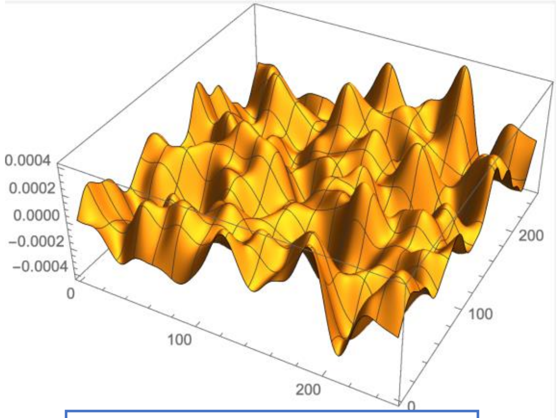
For small enough g, the backreaction from σ is negligible. During phase transition, inflaton can be regarded as a time-dependent parameter $\phi(t) = \phi_c + \dot{\phi}_c\,(t - t_c)$

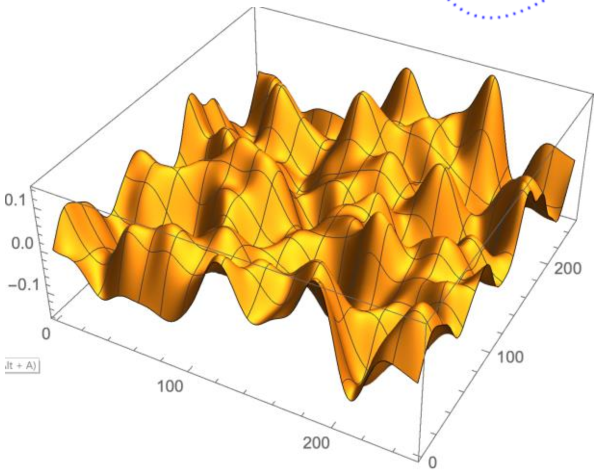
Nonlinear evolution



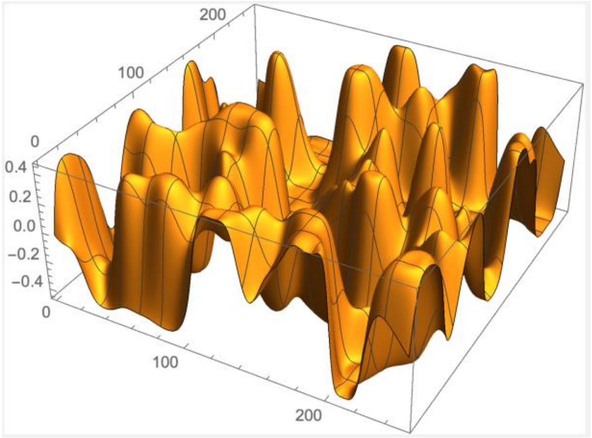
Tachyonic growth

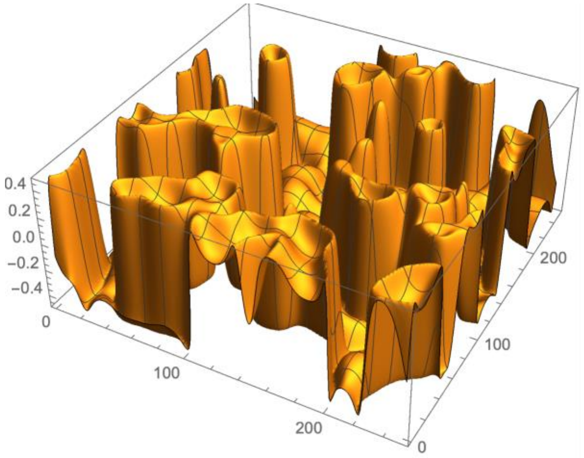


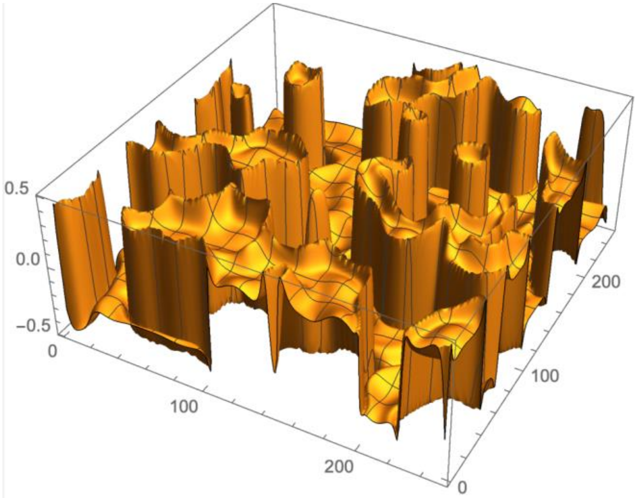




“bubble collision”

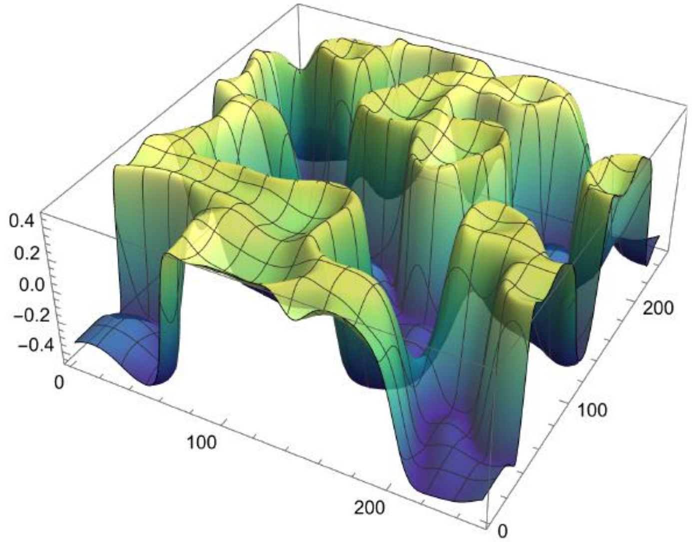


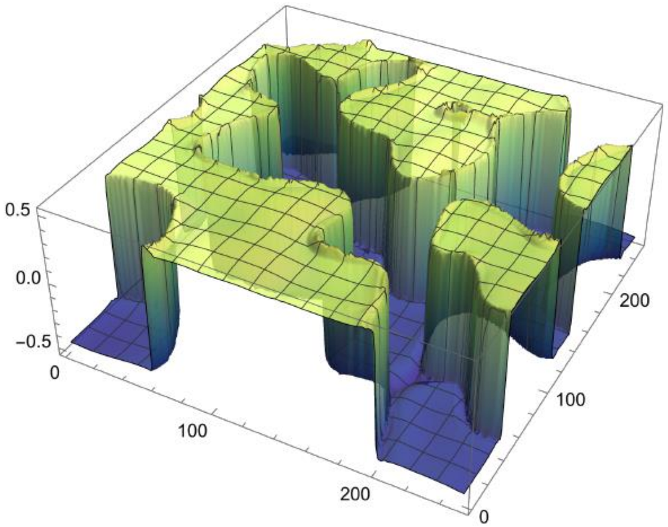


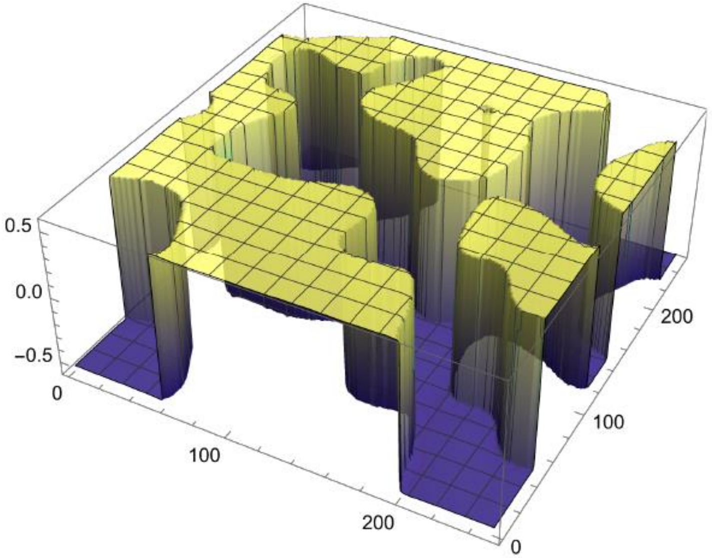


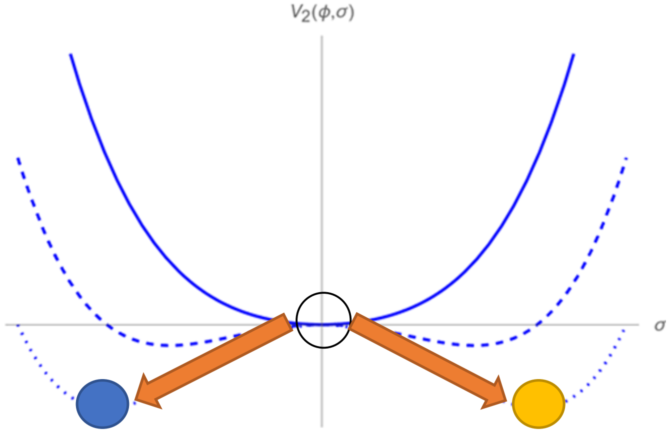
Nonlinear evolution II

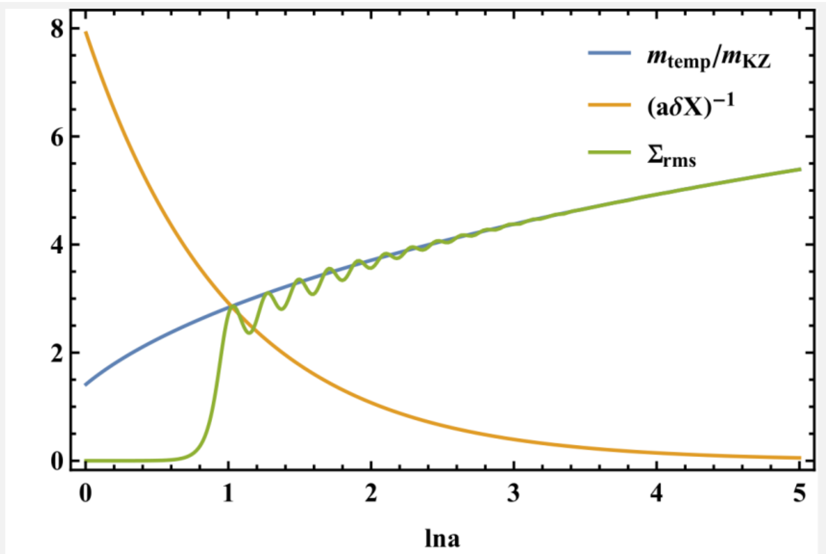
Formation of domain walls



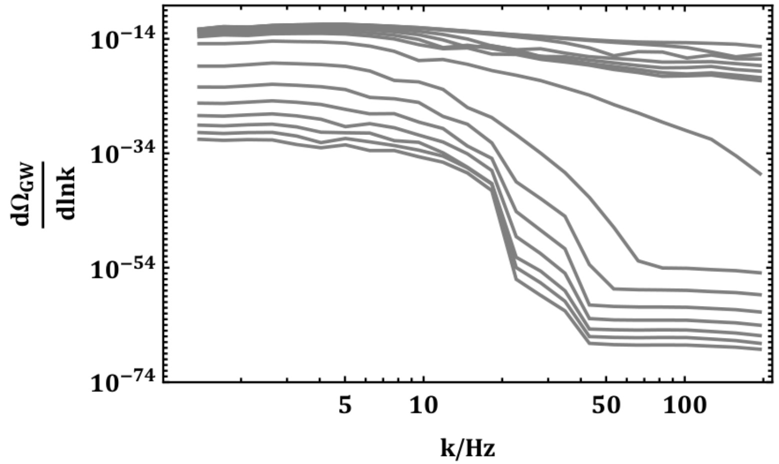








Power spectra of gravitational waves

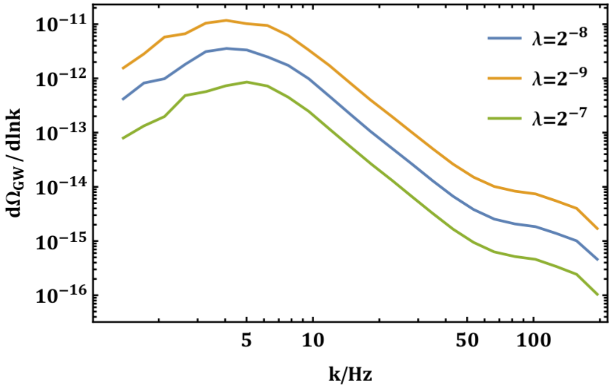


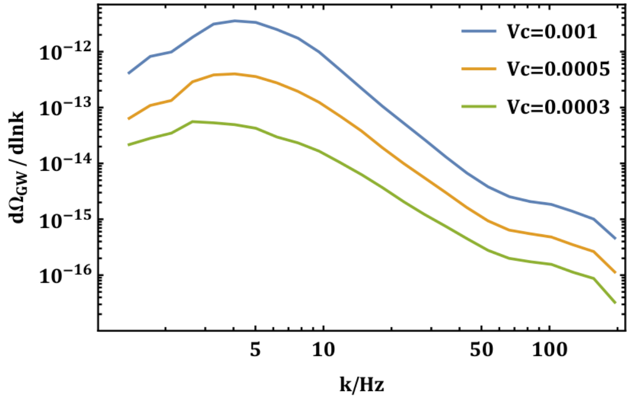
$$\frac{\mathrm{d}\Omega_{\mathrm{GW}}}{\mathrm{d}\ln k} = \frac{\Omega_{\mathrm{R}}}{\rho_0} \frac{H^4}{k^4} \frac{8\pi G}{(2\pi)^2} \frac{k^3}{V} \int \frac{\mathrm{d}\Omega}{4\pi} \left| C_{ij}^{\mathrm{TT}}(\mathbf{k}) \right|^2$$

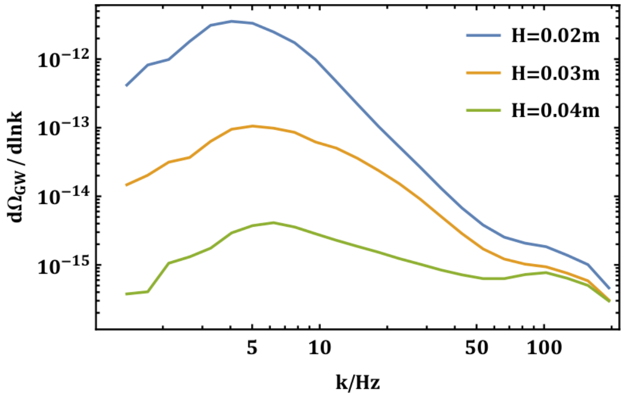
Distortion by inflation

$$C_{ij}(\mathbf{k}) = \int_{\tau_i}^{\tau_f} \mathrm{d}\tau' a(\tau') \left(\frac{\sin[-k\tau']}{-k\tau'} - \cos[-k\tau'] \right) \tilde{T}_{ij}(\mathbf{k}, \tau')$$

For $\lambda = 2^{-8}, V_c = 10^{-3}, \frac{H}{m} = 0.02, \frac{M_{Pl}}{m} = 200, N_e = 22.$







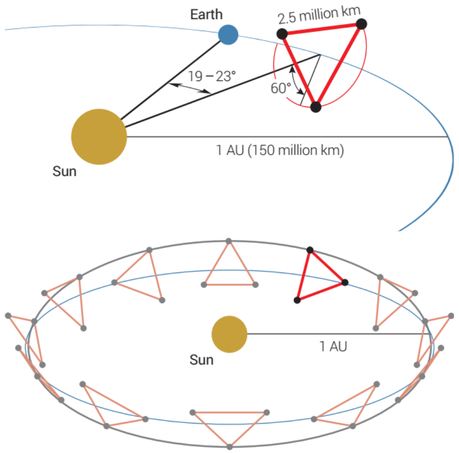
Detectability of gravitational waves

For sources happen at different e-folds, today's GW has different frequencies

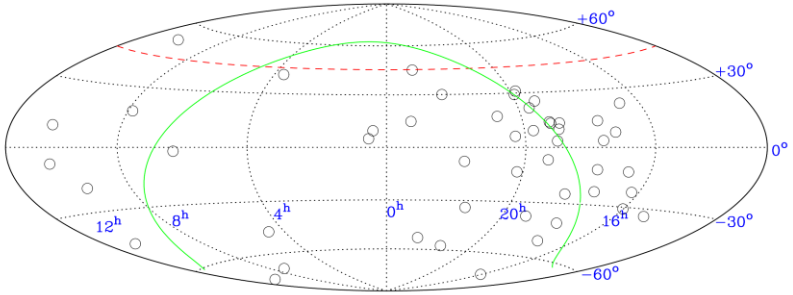
$$k_{\mathrm{today}} \sim \frac{a_*}{a_{\mathrm{today}}} M$$

$N_e \approx 20$

$N_e \approx 29$



$N_e \approx 43$



Summary and outlook

- We study the features of classical GWs produced from second-order phase transition and domain walls during inflation.
- If we are lucky enough, such a signal can be detected by future GW detectors.

Thanks for listening