

# Analytical Calculation of Inflation Correlators

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Based on 2205.01692 and 2208.13790 with Zhong-Zhi Xianyu



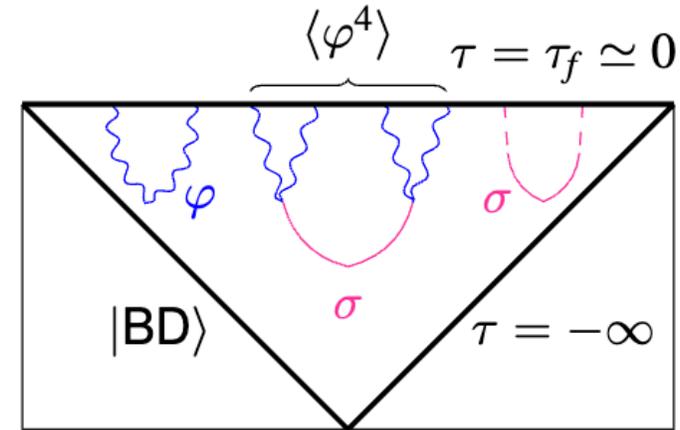
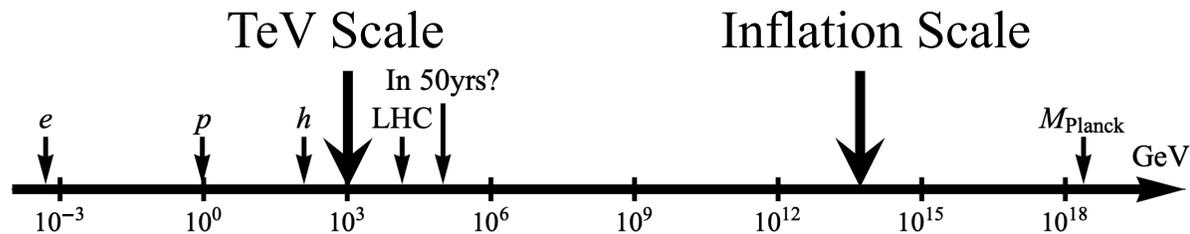
- Cosmological Collider Physics
  - Helical chemical potential
- Analytical Approaches to Inflation Correlators
  - Seed integrals
  - Partial Mellin-Barnes representation
  - Bootstrap equations
- Main results
- Summary & Outlooks

# Cosmological Collider Physics

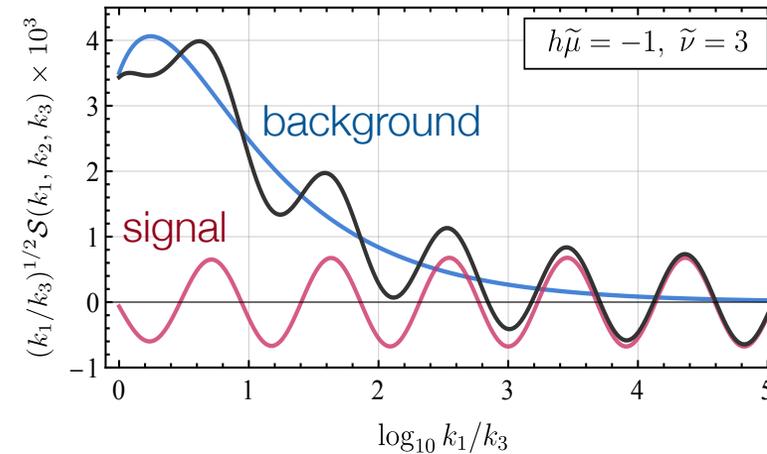
Inflation seems to be the highest energy observable process, during which particles with mass  $m \gtrsim H \sim 10^{14} \text{ GeV}$  can be produced from vacuum.

When coupled to the inflaton, the oscillating mode function will give rise to characteristic  $n$ -point functions of inflaton at late-time boundary, known as the CC signal.

By measuring the frequency of this oscillating signal, we can reveal the mass of the massive particle.



Particle production in dS.



An example of bispectrum.  $Q_{\text{in}}, \text{ZZX}, 2208.13790$ .

Spacetime metric:

$$ds^2 = a^2(\tau)[-d\tau^2 + dx^2], \quad a(\tau) = -\frac{1}{H\tau}.$$

# Cosmological Collider Physics

“Cosmological Collider”

Arkani-Hamed & Maldacena: 1503.08043.



# Cosmological Collider Physics

In general the 4pt function has the form:

$$\langle \varphi_{k_1} \varphi_{k_2} \varphi_{k_3} \varphi_{k_4} \rangle' = J(k_1, k_2, k_3, k_4, k_s, k_t) \times K(\theta_i).$$

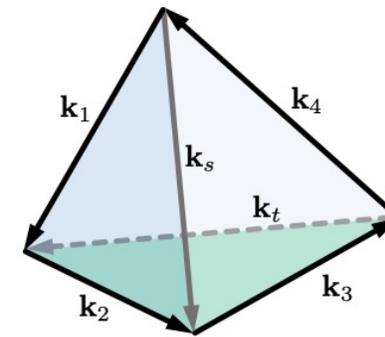
In the squeezed limit  $k_s \rightarrow 0$ , the dynamic piece can be divided into three part:

$$\lim_{k_s \rightarrow 0} J = J_{\text{EFT}} + J_{\text{L}} + J_{\text{NL}}.$$

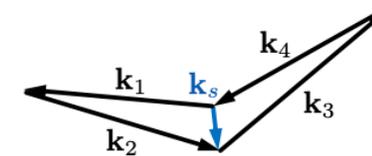
- $J_{\text{EFT}}$ : EFT term, or the background piece. Fully analytic in both  $r_1$  and  $r_2$ .
- $J_{\text{L}}$ : Local signal, proportional to  $(r_1/r_2)^{\pm i\omega}$ .
- $J_{\text{NL}}$ : Nonlocal signal, proportional to  $(r_1 r_2)^{\pm i\omega}$ .

Oscillation:

$$A (r_1 r_2)^{i\omega} + \text{c. c.} = 2|A| \cos[\omega \log(r_1 r_2) + \vartheta].$$



Momentum configuration.



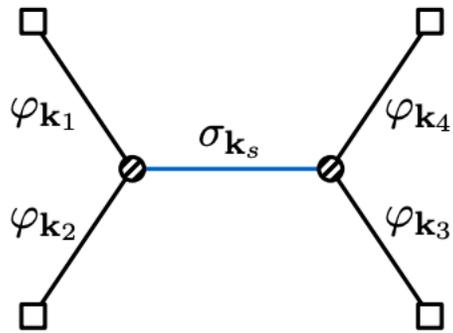
Squeezed limit configuration.

Momentum ratios:

$$r_1 = \frac{k_s}{k_1 + k_2}, \quad r_2 = \frac{k_s}{k_3 + k_4}.$$

# Cosmological Collider Physics

Inflation correlators are hard to calculate!



$\sim \frac{i}{E_S^2 - k_S^2 - m^2}$  in Minkowski.

$$= -\frac{\lambda_T^2}{16k_1 \cdots k_4} \sum_{a,b} ab \int_{-\infty}^0 d\tau_1 d\tau_2 e^{iak_{12}\tau_1 + ibk_{34}\tau_2} D_{ab}(k_s; \tau_1, \tau_2). \text{ in de Sitter.}$$

$$D_{-+}(k_s; \tau_1, \tau_2) = \frac{\pi}{4} e^{-\pi\tilde{\nu}} (\tau_1\tau_2)^{3/2} H_{i\tilde{\nu}}^{(1)}(-k_s\tau_1) H_{-i\tilde{\nu}}^{(2)}(-k_s\tau_2)$$

$$D_{+-}(k_s; \tau_1, \tau_2) = D_{-+}^*(k_s; \tau_1, \tau_2),$$

$$D_{++}(k_s; \tau_1, \tau_2) = D_{-+}(k_s; \tau_1, \tau_2)\theta(\tau_1 - \tau_2) + D_{+-}(k_s; \tau_1, \tau_2)\theta(\tau_2 - \tau_1),$$

$$D_{--}(k_s; \tau_1, \tau_2) = D_{+-}(k_s; \tau_1, \tau_2)\theta(\tau_1 - \tau_2) + D_{-+}(k_s; \tau_1, \tau_2)\theta(\tau_2 - \tau_1).$$

(See Xingang Chen, Yi Wang, ZZX: 1703.10166 for SK formalism.)

Recent progress:

- Cosmological bootstrap: Symmetry perspective.

Baumann, et al: 1811.00024, 1910.14051, 2005.04234.

- Mellin space approach: From AdS to dS.

Sleight, et al: 1906.12302, 1907.01143, 2007.09993, 2019.02725.

- Spectral decomposition: From tree to loop.

Beyond dS-covariance:

(dS-boost breaking)

- Non-unit sound speed.
- Non-covariant couplings.
- Chemical potentials. (Also P-violating)

# Helical Chemical Potential

$$S = \int d^4x \left[ \sqrt{-g} \left( -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu \right) + \frac{\phi}{4\Lambda} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right]$$

Lian-Tao Wang, ZZX, 2004.02887.

This action is both P-violating & dS-boost breaking.

With a rolling background:  $\phi = \dot{\phi}_0 t + \text{const.}$ , and defining the chemical potential:  $\mu = \dot{\phi}_0/\Lambda$ , the EoM reads:

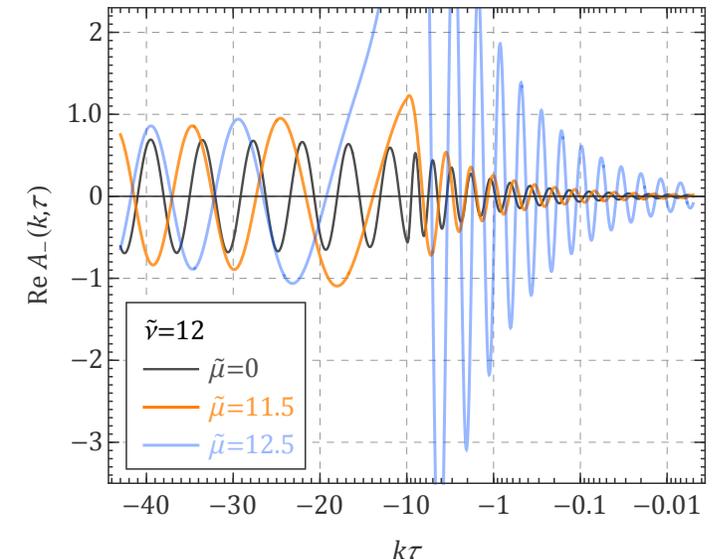
$$A_0'' - \partial_j^2 A_0 + (2aH A_0)' + a^2 m^2 A_0 = 0,$$

$$A_i'' - \partial_j^2 A_i + 2aH \partial_i A_0 + a^2 m^2 A_i + 2a\mu \epsilon_{ijk} \partial^j A^k = 0.$$

Mode functions:

$$B^{(\pm)}(k, \tau) = \frac{e^{\mp\pi\tilde{\mu}/2}}{\sqrt{2k}} W_{\pm i\tilde{\mu}, i\tilde{\nu}}(2ik\tau),$$

$$B^{(T)}(k, \tau) = \frac{\sqrt{\pi}k}{2m} e^{-\pi\tilde{\nu}/2} H(-\tau)^{3/2} H_{i\tilde{\nu}}^{(1)}(-k\tau)$$



CC signal for one helicity state is enhanced exponentially!

# Seed Integrals

- Scalar seed:

$$\mathcal{I}_{ab}^{p_1 p_2}(r_1, r_2) \equiv -ab k_s^{5+p_1+p_2} \int_{-\infty}^0 d\tau_1 d\tau_2 (-\tau_1)^{p_1} (-\tau_2)^{p_2} e^{iak_{12}\tau_1 + ibk_{34}\tau_2} D_{ab}(k_s; \tau_1, \tau_2)$$

$$\Delta\mathcal{L} = \frac{1}{2} \lambda a^2 \varphi'^2 \sigma \quad \mathcal{T}_\varphi = \frac{\lambda^2}{16k_1 k_2 k_3 k_4 k_s^5} \sum_{a,b=\pm} \mathcal{I}_{ab}^{00}(r_1, r_2)$$

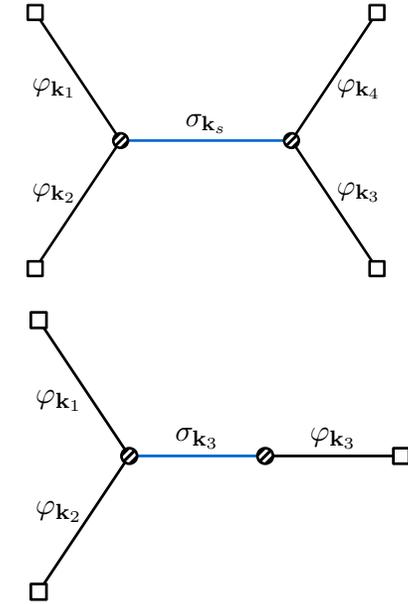
$$\Delta\mathcal{L} = \frac{1}{2} \lambda_c a^4 \phi_c^2 \sigma \quad \mathcal{T}_{\phi_c} = \frac{\lambda_c^2 \tau_f^4}{16k_1 k_2 k_3 k_4 k_s} \sum_{a,b=\pm} \mathcal{I}_{ab}^{-2,-2}(r_1, r_2)$$

$$\mathcal{B}_\phi = \langle \varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2} \varphi_{\mathbf{k}_3} \rangle'_3 = \frac{\lambda \lambda_2}{8k_1 k_2 k_3^4} \lim_{r_2 \rightarrow 1^-} \sum_{a,b=\pm} \mathcal{I}_{ab}^{0,-2}(r_1, r_2)$$

- (Helical) Vector seed:

$$\mathcal{I}_{ab}^{(\lambda)p_1 p_2} \equiv -ab k_s^{3+p_1+p_2} \int_{-\infty}^0 d\tau_1 d\tau_2 (-\tau_1)^{p_1} (-\tau_2)^{p_2} e^{iak_{12}\tau_1 + ibk_{34}\tau_2} D_{ab}^{(\lambda)}(k_s; \tau_1, \tau_2)$$

$$D_{>}^{(\pm)}(k; \tau_1, \tau_2) = \frac{e^{\mp\pi\tilde{\mu}}}{2k} W_{\pm i\tilde{\mu}, i\tilde{\nu}}(2ik\tau_1) W_{\mp i\tilde{\mu}, i\tilde{\nu}}(-2ik\tau_2)$$



# Partial Mellin-Barnes representation

- Mellin Transform:

$$F(s) = \int_0^\infty dz z^{s-1} f(z), \quad f(z) = \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} z^{-s} F(s).$$

- Partial MB Rep: Only perform the inverse Mellin transform to the internal modes. The time integrals and the loop momentum integrals are trivialized. Then we integrate out the Mellin variables  $s_i$  using the residue theorem.
- Scalar propagator:

$$D_{+-}(k_s; \tau_1, \tau_2) = \frac{1}{4\pi} \int_{-i\infty}^{+i\infty} \frac{ds_1}{2\pi i} \frac{ds_2}{2\pi i} e^{i\pi(s_1-s_2)} \left(\frac{k_s}{2}\right)^{-2s_1-2s_2} (-\tau_1)^{\frac{3}{2}-2s_1} (-\tau_2)^{\frac{3}{2}-s_2} \\ \times \Gamma\left(s_1 - \frac{i\nu}{2}\right) \Gamma\left(s_1 + \frac{i\nu}{2}\right) \Gamma\left(s_2 - \frac{i\nu}{2}\right) \Gamma\left(s_2 + \frac{i\nu}{2}\right).$$

# Bootstrap equations

- Original bootstrap: Symmetry implies differential equations.

Baumann, et al: 1811.00024,  
1910.14051, 2005.04234.

- Our bootstrap: EoM of propagators (mode functions) implies ODE for correlators. Applicable to dS-boost breaking cases.

$$(\tau_1^2 \partial_{\tau_1}^2 - 2\tau_1 \partial_{\tau_1} + k_s^2 \tau_1^2 + m^2) D_{\pm\mp}(k_s; \tau_1, \tau_2) = 0$$

$$(\tau_1^2 \partial_{\tau_1}^2 - 2\tau_1 \partial_{\tau_1} + k_s^2 \tau_1^2 + m^2) D_{\pm\pm}(k_s; \tau_1, \tau_2) = \mp i \tau_1^2 \tau_2^2 \delta(\tau_1 - \tau_2)$$

Using  $\mathcal{I}_{ab}^{-2,-2}(r_1, r_2) \equiv -ab k_s \int_{-\infty}^0 \frac{d\tau_1}{\tau_1^2} \frac{d\tau_2}{\tau_2^2} e^{iak_{12}\tau_1 + ibk_{34}\tau_2} D_{ab}(k_s; \tau_1, \tau_2)$ .

$$\left[ (r_1^2 - r_1^4) \partial_{r_1}^2 - 2r_1^3 \partial_{r_1} + \left( \tilde{\nu}^2 + \frac{1}{4} \right) \right] \mathcal{I}_{\pm\mp}^{-2,-2}(r_1, r_2) = 0$$

$$\left[ (r_1^2 - r_1^4) \partial_{r_1}^2 - 2r_1^3 \partial_{r_1} + \left( \tilde{\nu}^2 + \frac{1}{4} \right) \right] \mathcal{I}_{\pm\pm}^{-2,-2}(r_1, r_2) = \frac{r_1 r_2}{r_1 + r_2}$$



Commute the differential operator with the integral.

# Bootstrap equations

$$\left[ (r_1^2 - r_1^4) \partial_{r_1}^2 - 2r_1^3 \partial_{r_1} + \left( \tilde{\nu}^2 + \frac{1}{4} \right) \right] \mathcal{I}_{\pm\mp}^{-2,-2}(r_1, r_2) = 0$$

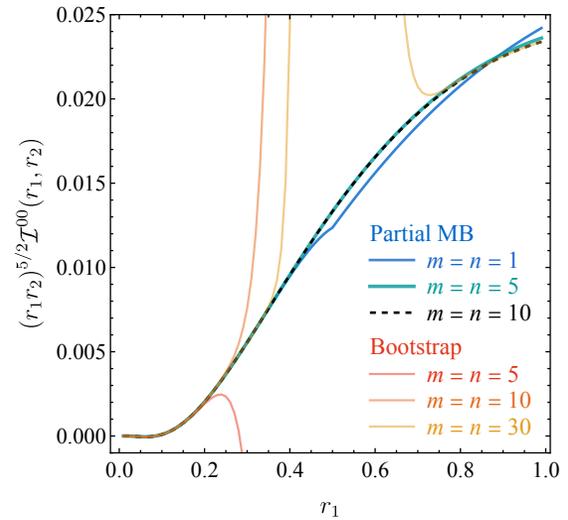
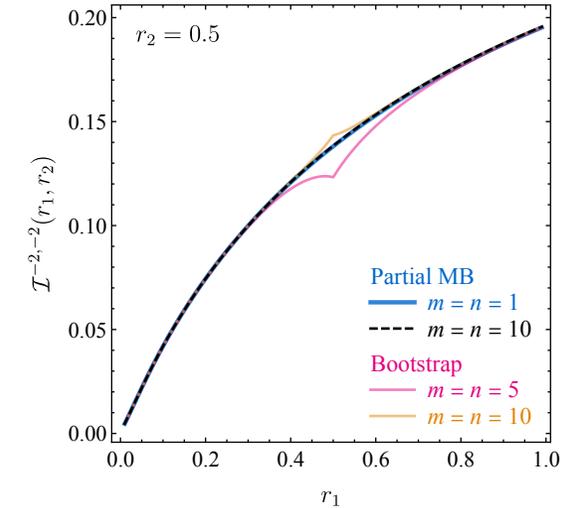
$$\left[ (r_1^2 - r_1^4) \partial_{r_1}^2 - 2r_1^3 \partial_{r_1} + \left( \tilde{\nu}^2 + \frac{1}{4} \right) \right] \mathcal{I}_{\pm\pm}^{-2,-2}(r_1, r_2) = \frac{r_1 r_2}{r_1 + r_2}$$

- Homogeneous solutions: Local and nonlocal signals.
- Inhomogeneous solution: Background.

A change of variables:  $u_{1,2} = 2r_{1,2}/(1 + r_{1,2})$  is crucial for solving the bootstrap equation for vector seed integral. It is also useful for deriving closed-form formula for 3pt and 2pt correlators.

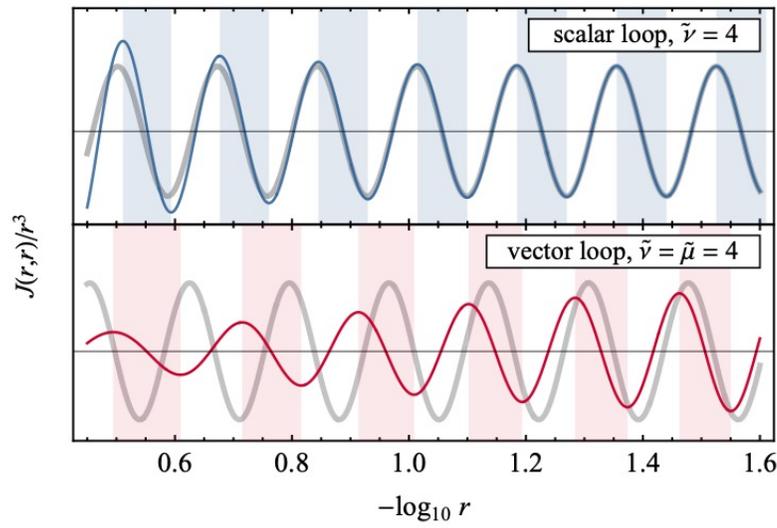
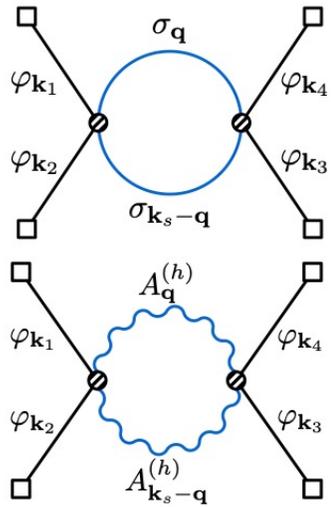
$$\left[ (u_1^2 - u_1^3) \partial_{u_1}^2 - (1 \pm i h \tilde{\mu}) u_1^2 \partial_{u_1} + \left( \tilde{\nu}^2 + \frac{1}{4} \right) \right] \mathcal{I}_{\pm\mp}^{(h)-1,-1}(u_1, u_2) = 0$$

$$\left[ (u_1^2 - u_1^3) \partial_{u_1}^2 - (1 \pm i h \tilde{\mu}) u_1^2 \partial_{u_1} + \left( \tilde{\nu}^2 + \frac{1}{4} \right) \right] \mathcal{I}_{\pm\pm}^{(h)-1,-1}(u_1, u_2) = \frac{1}{2} \frac{u_1 u_2}{u_1 + u_2 - u_1 u_2}$$

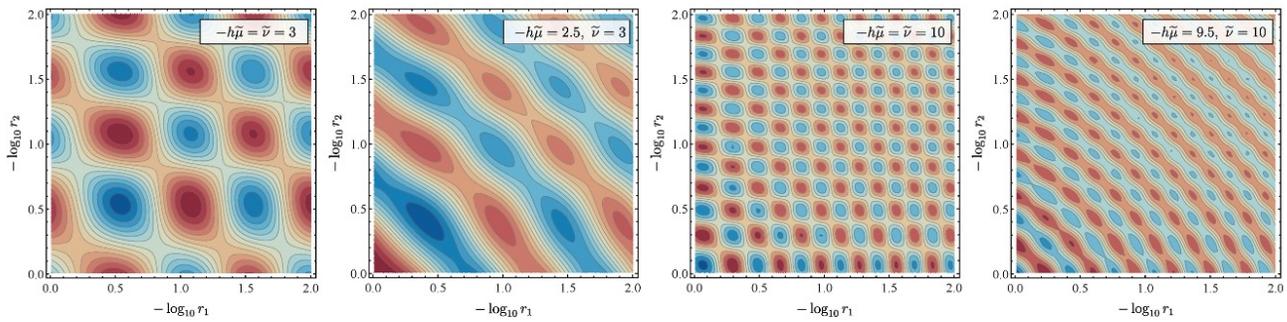


PMB vs Bootstrap

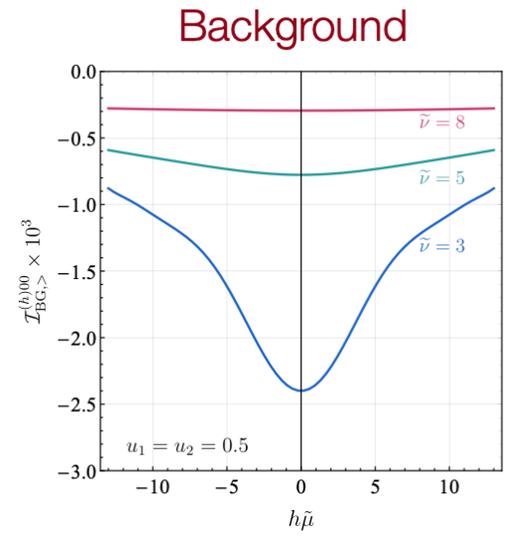
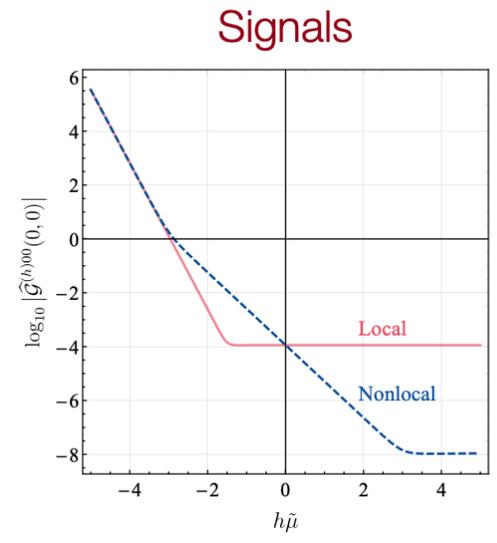
# Main results



Different particle species and different types of interaction give rise to different phases.  
Qin, ZZX, 2205.01692.



Local and nonlocal signals with chemical potential.  
Qin, ZZX, 2208.13790.



Signals exponentially sensitive to chemical potential. Background insensitive.  
Qin, ZZX, 2208.13790.

# Summary & Outlooks

- Calculation of inflation correlators are important (but difficult).
  - Useful for particle model buildings, parameter scanning, template design, ...
  - Amplitudes in dS are least understood among the three maximally symmetric spacetimes.
  - Inflation provides a very high energy scale at  $H \sim 10^{14}$  GeV.
- Two very useful methods are developed, both applicable to helical correlators, and tree level calculations are basically solved.
  - Partial MB rep & Bootstrap.
- Cutting rule as a byproduct. (Xi Tong, Yi Wang, Yuhang Zhu: 2112.03448.)
- Outlooks:
  - PMB works well for nonlocal signals of bubble and triangle and box diagrams.
  - For covariant cases, spectral decomposition is useful for bubble diagram. (Hongyu Zhang, ZZX: 2211.03810.)
  - Correlators with higher-spin or half-integer-spin.

*Thanks*

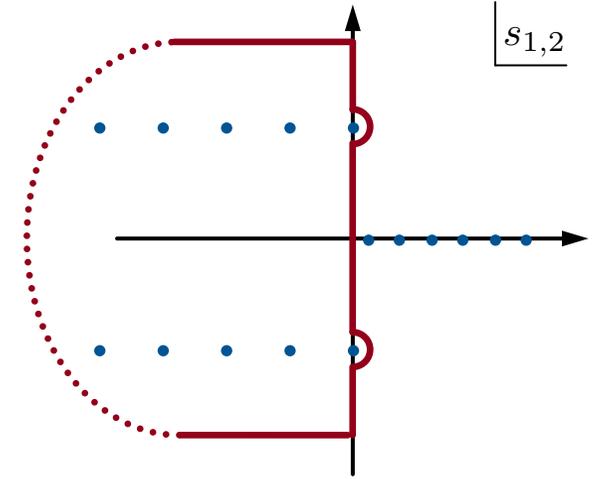
# Partial Mellin-Barnes representation

- $I_{++}^{-2,-2}$  as an example:

$$\mathcal{I}_{\pm\pm}^{p_1 p_2}(r_1, r_2) = \mathcal{I}_{\pm\pm, F}^{p_1 p_2}(r_1, r_2) + \mathcal{I}_{\pm\pm, TO}^{p_1 p_2}(r_1, r_2)$$

$$\begin{aligned} \mathcal{I}_{++, F, >}^{-2,-2}(r_1, r_2) &\equiv -k_s \int_{-\infty}^0 \frac{d\tau_1}{\tau_1^2} \frac{d\tau_2}{\tau_2^2} e^{\pm i(k_{12}\tau_1 + k_{34}\tau_2)} D_{>}(k_s; \tau_1, \tau_2) \\ &= \frac{(r_1 r_2)^{1/2}}{4\pi} \int_{-i\infty}^{i\infty} \frac{ds_1}{2\pi i} \frac{ds_2}{2\pi i} (ie^{2i\pi s_1}) \left(\frac{r_1}{2}\right)^{-2s_1} \left(\frac{r_2}{2}\right)^{-2s_2} \\ &\quad \times \Gamma\left[\frac{1}{2} - 2s_1, \frac{1}{2} - 2s_2, s_1 - \frac{i\tilde{\nu}}{2}, s_1 + \frac{i\tilde{\nu}}{2}, s_2 - \frac{i\tilde{\nu}}{2}, s_2 + \frac{i\tilde{\nu}}{2}\right] \end{aligned}$$

$$\begin{aligned} \mathcal{I}_{++, TO, >}^{-2,-2} &= \frac{(r_1 r_2)^{1/2}}{4\pi} \int_{-i\infty}^{i\infty} \frac{ds_1}{2\pi i} \frac{ds_2}{2\pi i} (-ie^{2i\pi s_1} + ie^{2i\pi s_2}) \left(\frac{r_1}{2}\right)^{-2s_{12}} \\ &\quad \times \Gamma\left[\frac{1}{2} - 2s_2, 1 - 2s_{12}, s_1 - \frac{i\tilde{\nu}}{2}, s_1 + \frac{i\tilde{\nu}}{2}, s_2 - \frac{i\tilde{\nu}}{2}, s_2 + \frac{i\tilde{\nu}}{2}\right] \\ &\quad \times {}_2\tilde{F}_1\left[\frac{1}{2} - 2s_2, 1 - 2s_{12} \middle| -\frac{r_1}{r_2}\right] \end{aligned}$$



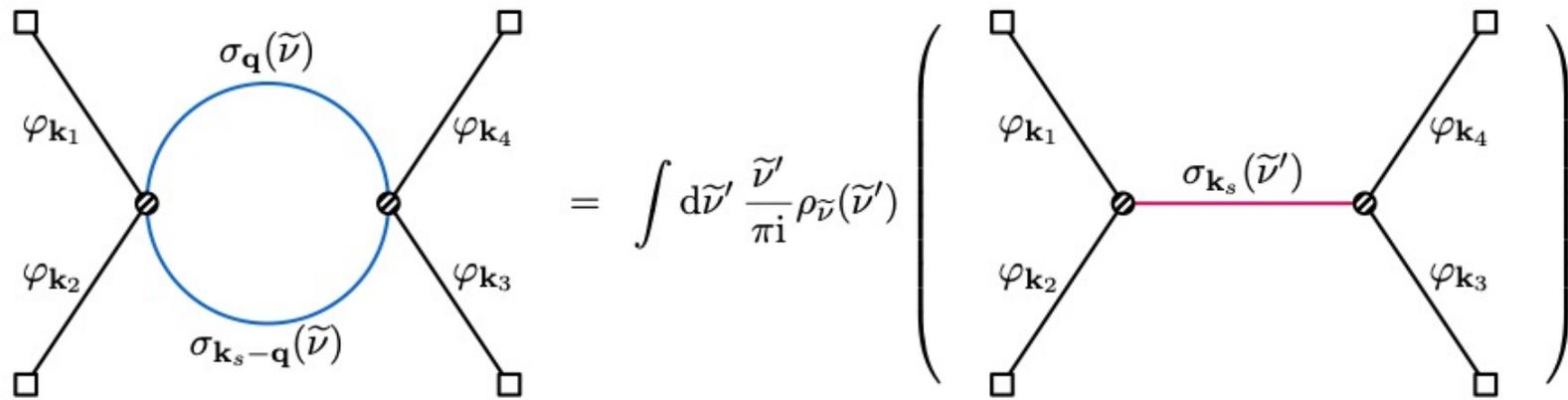
$$\begin{aligned} s_1 &= -n_1 \mp \frac{i\tilde{\nu}}{2}, & s_2 &= -n_2 \pm \frac{i\tilde{\nu}}{2} \\ s_1 &= -n_1 \mp \frac{i\tilde{\nu}}{2}, & s_2 &= -n_2 \mp \frac{i\tilde{\nu}}{2} \end{aligned}$$

Contributing to  
local/nonlocal signals in the  
Factorized part;  
and to background/0 in the  
Time-Ordered part.

# Backup

- Spectral decomposition

$$\int \frac{d^d \mathbf{q}}{(2\pi)^d} D_{\tilde{\nu},ab}(q; \tau_1, \tau_2) D_{\tilde{\nu},ab}(|\mathbf{k}_s - \mathbf{q}|; \tau_1, \tau_2) = \int_{-\infty}^{+\infty} d\tilde{\nu}' \frac{\tilde{\nu}'}{\pi i} \rho_{\tilde{\nu}}^{\text{dS}}(\tilde{\nu}') D_{\tilde{\nu}',ab}(k_s; \tau_1, \tau_2)$$



# Backup

- PMB for one loop bubble:

$$\begin{aligned}
 \mathcal{T}_{\text{NL},\varphi'^2\sigma^2} &= \frac{\lambda^2}{256\pi^{7/2}k_1 \cdots k_4 (k_{12}k_{34})^{5/2}} (1 - \cosh 2\pi\tilde{\nu}) \left(\frac{r_1 r_2}{4}\right)^{3+2i\tilde{\nu}} \\
 &\times \sum_{n_1, n_2, n_3, n_4=0}^{\infty} \frac{(-1)^{n_{1234}}}{n_1! n_2! n_3! n_4!} \left(\frac{r_1}{2}\right)^{2n_{13}} \left(\frac{r_2}{2}\right)^{2n_{24}} \\
 &\times \Gamma\left[4 + 2n_{13} + 2i\tilde{\nu}, 4 + 2n_{24} + 2i\tilde{\nu}, -n_1 - i\tilde{\nu}, -n_2 - i\tilde{\nu}, -n_3 - i\tilde{\nu}, -n_4 - i\tilde{\nu}\right] \\
 &\times \Gamma\left[\frac{3}{2} + n_{12} + i\tilde{\nu}, \frac{3}{2} + n_{34} + i\tilde{\nu}, -n_{1234} - \frac{3}{2} - 2i\tilde{\nu}\right. \\
 &\quad \left.-n_{12} - i\tilde{\nu}, -n_{34} - i\tilde{\nu}, 3 + n_{1234} + 2i\tilde{\nu}\right] \\
 &+ \text{c.c.}
 \end{aligned}$$

