

Improving the constraints of high-redshift Cosmography by using PAge approximation

Yong Zhou(周勇)

Institute of Theoretical Physics, Chinese Academy of Sciences
Collaborated with Rong-Gen Cai, Zong-Kuan Guo, Shao-Jiang Wang, Wang-Wei Yu

第十六届 TeV 物理工作组学术研讨会暨邝宇平院士学术思想研讨会
November 9, 2022



Outline

1 Background

2 Cosmography

3 Hubble diagram

4 MCMC analysis

Hubble's law

"Hubble's law, is the observation in physical cosmology that galaxies are moving away from Earth at speeds proportional to their distance."

$$v = H_0 D$$

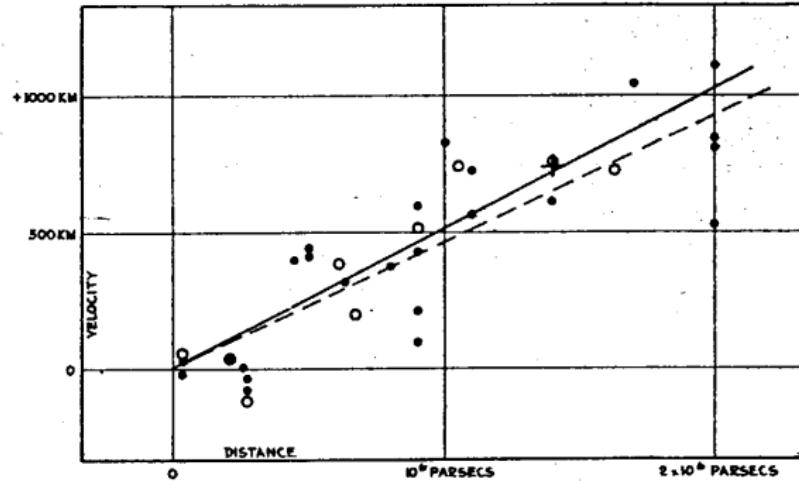
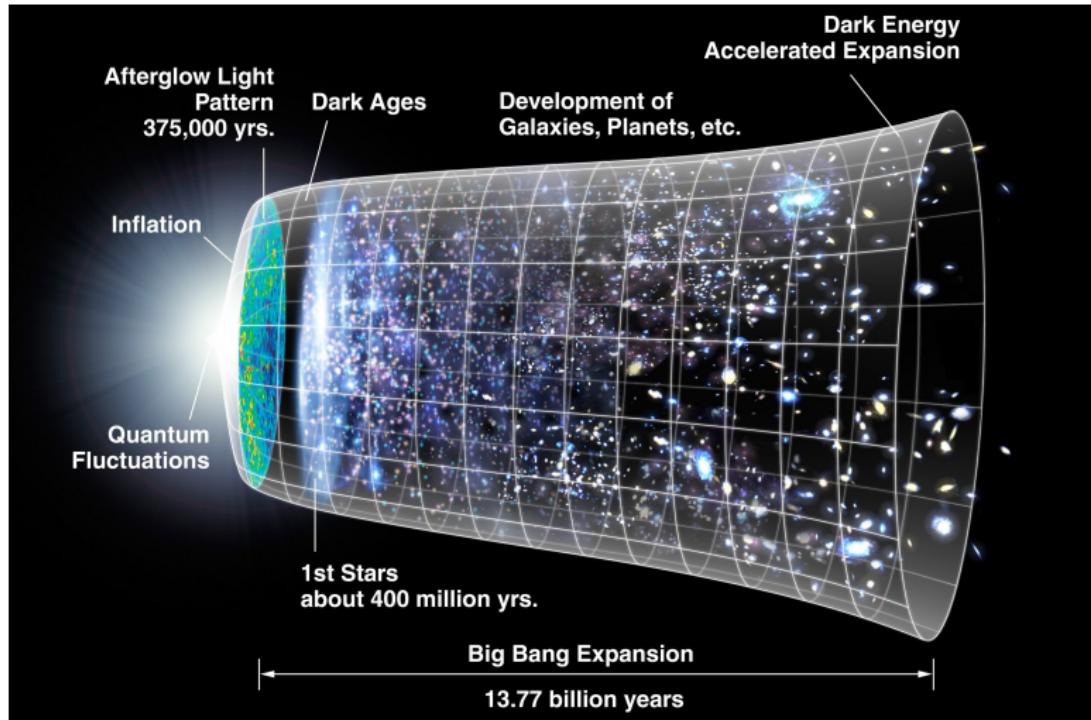


FIGURE 1

Expansion of the Universe



Λ CDM model

Cosmological principle

In modern physical cosmology, the cosmological principle is the notion that the spatial distribution of matter in the universe is homogeneous and isotropic when viewed on a large enough scale.

FLRW metric

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]$$

Einstein field equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Λ CDM model

Friedmann equation

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3}$$

Friedmann acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

Λ CDM model

$$H(t) = H_0 \sqrt{\Omega_r a^{-4}(t) + \Omega_m a^{-3}(t) + \Omega_k a^{-2}(t) + (1 - \Omega_r - \Omega_m - \Omega_k)}$$

Luminosity distance

$$d_L = \frac{a(t_0)}{a(t)} r = \frac{a(t_0)}{a(t)} \int_t^{t_0} \frac{cdt'}{a(t')} = (1+z) \int_0^z \frac{cdz'}{H(z')}$$

Cosmography²

- “Indeed, a surprising amount of modern cosmology is pure kinematics, what Weinberg¹ refers to as cosmography, and is completely independent of the underlying dynamics governing the evolution of the universe.”
- “Whereas pure cosmography by itself will not predict anything about the scale factor $a(t)$, in the cosmographic scenario we can to some extent infer the history of the scale factor $a(t)$ from the observational data while steadfastly avoiding use of the Einstein equations.”

¹Weinberg, S.: Gravitation and cosmology: Principles and Applications of the General Theory of relativity. Wiley, New York (1972)

²Visser, M. Cosmography: Cosmology without the Einstein equations. Gen Relativ Gravit 37, 1541–1548

Cosmography

scale factor

$$a(t) = a(t_0)[1 + H_0(t - t_0) - \frac{1}{2!}q_0 H_0^2(t - t_0)^2 + \frac{1}{3!}j_0 H_0^3(t - t_0)^3 + \frac{1}{4!}s_0 H_0^4(t - t_0)^4 + \frac{1}{5!}l_0 H_0^5(t - t_0)^5 + \mathcal{O}((t - t_0)^6)]$$

Hubble and deceleration parameter

$$H(t) = +\frac{1}{a} \frac{\mathrm{d}a(t)}{\mathrm{d}t}, \quad q(t) = -\frac{1}{aH^2} \frac{\mathrm{d}^2a(t)}{\mathrm{d}t^2}$$

jerk, snap and lerk parameter

$$j(t) = +\frac{1}{aH^3} \frac{\mathrm{d}^3a(t)}{\mathrm{d}t^3}, \quad s(t) = +\frac{1}{aH^4} \frac{\mathrm{d}^4a(t)}{\mathrm{d}t^4}, \quad l(t) = +\frac{1}{aH^5} \frac{\mathrm{d}^5a(t)}{\mathrm{d}t^5}$$

Cosmography

scale factor

$$a(x) = 1 - x - \frac{1}{2!}q_0x^2 - \frac{1}{3!}j_0x^3 + \frac{1}{4!}s_0x^4 - \frac{1}{5!}l_0x^5 + \mathcal{O}(x^6), x = H_0(t_0 - t)$$

$$y = 1 - a$$

$$y(x) = x + \frac{1}{2!}q_0x^2 + \frac{1}{3!}j_0x^3 - \frac{1}{4!}s_0x^4 + \frac{1}{5!}l_0x^5 + \mathcal{O}(x^6),$$

$$\begin{aligned} x(y) &= y - \frac{1}{2!}q_0y^2 + \frac{1}{3!}(3q_0^2 - j_0)y^3 + \frac{1}{4!}(-15q_0^3 + 10q_0j_0 + s_0)y^4 \\ &\quad + \frac{1}{5!}(105q_0^4 - 105q_0^2j_0 + 10j_0^2 - 15q_0s_0 - l_0)y^5 + \mathcal{O}(y^6) \end{aligned}$$

Cosmography

scale factor

$$a(x) = 1 - x - \frac{1}{2!}q_0x^2 - \frac{1}{3!}j_0x^3 + \frac{1}{4!}s_0x^4 - \frac{1}{5!}l_0x^5 + \mathcal{O}(x^6), \quad x = H_0(t_0 - t)$$

$$z = 1/a - 1$$

$$z(x) =$$

$$\begin{aligned} & x + \frac{1}{2!}(2 + q_0)x^2 + \frac{1}{3!}(6 + 6q_0 + j_0)x^3 + \frac{1}{4!}(24 + 36q_0 + 6q_0^2 + 8j_0 - s_0)x^4 \\ & + \frac{1}{5!}(120 + 240q_0 + 90q_0^2 + 20q_0j_0 + 60j_0 - 10s_0 + l_0)x^5 + \mathcal{O}(x^6), \end{aligned}$$

$$x(z) =$$

$$\begin{aligned} & z - \frac{1}{2!}(2 + q_0)z^2 + \frac{1}{3!}(6 + 6q_0 + 3q_0^2 - j_0)z^3 + \frac{1}{4!}(-24 - 36q_0 - 36q_0^2 - 15q_0^3 + 10q_0j_0 + 12j_0 + s_0)z^4 \\ & + \frac{1}{5!}(120 + 240q_0 + 360q_0^2 + 300q_0^3 + 105q_0^4 - 200q_0j_0 - 105q_0^2j_0 - 120j_0 + 10j_0^2 - 15q_0s_0 - 20s_0 - l_0)z^5 \\ & + \mathcal{O}(z^6) \end{aligned}$$

xyz parameter space

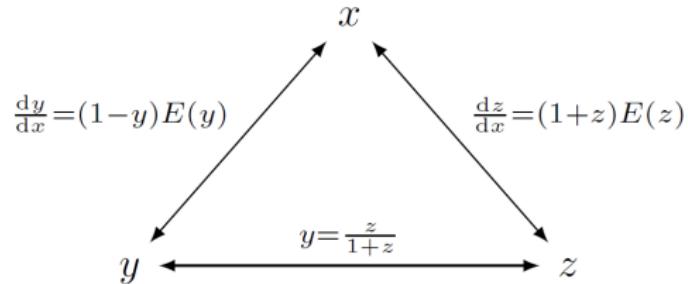


FIG. 1. The schematic diagram of xyz space and their conversion relations.

Cosmography

Dimensionless Hubble parameter $E = H/H_0$ on xyz space

$$E(x) =$$

$$\begin{aligned} & 1 + (1 + q_0)x + \frac{1}{2!}(2 + 3q_0 + j_0)x^2 + \frac{1}{3!}(6 + 12q_0 + 3q_0^2 + 4j_0 - s_0)x^3 \\ & + \frac{1}{4!}(24 + 60q_0 + 30q_0^2 + 10q_0j_0 + 20j_0 - 5s_0 + l_0)x^4 + \mathcal{O}(x^5), \end{aligned}$$

$$E(y) =$$

$$\begin{aligned} & 1 + (1 + q_0)y + \frac{1}{2!}(2 + 2q_0 - q_0^2 + j_0)y^2 + \frac{1}{3!}(6 + 6q_0 - 3q_0^2 + 3q_0^3 - 4q_0j_0 + 3j_0 - s_0)y^3 \\ & + \frac{1}{4!}(24 + 24q_0 - 12q_0^2 + 12q_0^3 - 15q_0^4 - 16q_0j_0 + 25q_0^2j_0 + 12j_0 - 4j_0^2 + 7q_0s_0 + 8s_0 + l_0)y^4 + \mathcal{O}(y^5), \end{aligned}$$

$$E(z) =$$

$$\begin{aligned} & 1 + (1 + q_0)z + \frac{1}{2!}(-q_0^2 + j_0)z^2 + \frac{1}{3!}(3q_0^2 + 3q_0^3 - 4q_0j_0 - 3j_0 - s_0)z^3 \\ & + \frac{1}{4!}(-12q_0^2 - 24q_0^3 - 15q_0^4 + 32q_0j_0 + 25q_0^2j_0 + 12j_0 - 4j_0^2 + 7q_0s_0 + 8s_0 + l_0)z^4 + \mathcal{O}(z^5) \end{aligned}$$

Cosmography

Dimensionless luminosity distance on xyz space

$$D_L(x) =$$

$$x + \frac{3}{2!}x^2 + \frac{1}{3!}(11 + 4q_0)x^3 + \frac{5}{4!}(10 + 8q_0 + j_0)x^4 + \frac{1}{5!}(274 + 346q_0 + 46q_0^2 + 63j_0 - 6s_0)x^5 + \mathcal{O}(x^6),$$

$$D_L(y) =$$

$$y + \frac{1}{2!}(3 - q_0)y^2 + \frac{1}{3!}(11 - 5q_0 + 3q_0^2 - j_0)y^3 + \frac{1}{4!}(50 - 26q_0 + 21q_0^2 - 15q_0^3 + 10q_0j_0 - 7j_0 + s_0)y^4 \\ + \frac{1}{5!}(274 - 154q_0 + 141q_0^2 - 135q_0^3 + 105q_0^4 + 90q_0j_0 - 105q_0^2j_0 - 47j_0 + 10j_0^2 - 15q_0s_0 + 9s_0 - l_0)y^5 + \mathcal{O}(y^6),$$

$$D_L(z) =$$

$$z + \frac{1}{2!}(1 - q_0)z^2 + \frac{1}{3!}(-1 + q_0 + 3q_0^2 - j_0)z^3 + \frac{1}{4!}(2 - 2q_0 - 15q_0^2 - 15q_0^3 + 10q_0j_0 + 5j_0 + s_0)z^4 \\ + \frac{1}{5!}(-6 + 6q_0 + 81q_0^2 + 165q_0^3 + 105q_0^4 - 110q_0j_0 - 105q_0^2j_0 - 27j_0 + 10j_0^2 - 15q_0s_0 - 11s_0 - l_0)z^5 + \mathcal{O}(z^6)$$

Cosmography

Dimensionless luminosity distance on logarithmic xyz space

$$D_L(\ln(1+x)) =$$

$$\begin{aligned} & \ln(1+x) + 2\ln^2(1+x) + \frac{1}{3!}(21+4q_0)\ln^3(1+x) + \frac{1}{4!}(138+64q_0+5j_0)\ln^4(1+x) \\ & + \frac{1}{5!}(1095+846q_0+46q_0^2+113j_0-6s_0)\ln^5(1+x) + \mathcal{O}(\ln^6(1+x)), \end{aligned}$$

$$D_L(\ln(1+y)) =$$

$$\begin{aligned} & \ln(1+y) + \frac{1}{2!}(4-q_0)\ln^2(1+y) + \frac{1}{3!}(21-8q_0+3q_0^2-j_0)\ln^3(1+y) \\ & + \frac{1}{4!}(138-63q_0+39q_0^2-15q_0^3+10q_0j_0-13j_0+s_0)\ln^4(1+y) + \frac{1}{5!}(1095-554q_0+426q_0^2 \\ & -285q_0^3+105q_0^4+190q_0j_0-105q_0^2j_0-142j_0+10j_0^2-15q_0s_0+19s_0-l_0)\ln^5(1+y) + \mathcal{O}(\ln^6(1+y)), \end{aligned}$$

$$D_L(\ln(1+z)) =$$

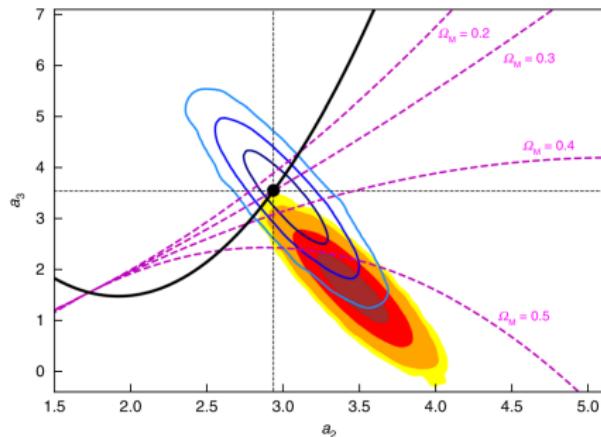
$$\begin{aligned} & \ln(1+z) + \frac{1}{2!}(2-q_0)\ln^2(1+z) + \frac{1}{3!}(3-2q_0+3q_0^2-j_0)\ln^3(1+z) \\ & + \frac{1}{4!}(4-3q_0+3q_0^2-15q_0^3+10q_0j_0-j_0+s_0)\ln^4(1+z) + \frac{1}{5!}(5-4q_0+6q_0^2+15q_0^3+105q_0^4 \\ & -10q_0j_0-105q_0^2j_0-2j_0+10j_0^2-15q_0s_0-s_0-l_0)\ln^5(1+z) + \mathcal{O}(\ln^6(1+z)) \end{aligned}$$

Cosmography³

Expansion on $\ln(1 + z)$

$$d_L(z) = \frac{c}{H_0} [\ln(1 + z) + a_2 \ln^2(1 + z) + a_3 \ln^3(1 + z) + \mathcal{O}(\ln^4(1 + z))]$$

$$a_2 = \frac{1}{2!}(2 - q_0), a_3 = \frac{1}{3!}(3 - 2q_0 - 3q_0^2 - j_0)$$



³Risaliti, G., Lusso, E. 2019, Nat. Astron., 195

Cosmography

- $a(t) \propto t^{1/2}$, radiation-dominated era
- $a(t) \propto t^{2/3}$, matter-dominated era
- $H(t) \propto 1/t$

Expansion on t

$$\begin{aligned} H(t)t = E(x)(p_{age} - x) &= p_{age} - (1 - p_{age} - p_{age}q_0)x \\ &\quad + \frac{1}{2!}(-2 + 2p_{age} + (3p_{age} - 2)q_0 + p_{age}j_0)x^2 \\ &\quad - \frac{1}{3!}(6 - 6p_{age} - (-3 + 4p_{age} + p_{age}q_0)3q_0 \\ &\quad + (3 - 4p_{age})j_0 + p_{age}s_0)x^3 + \mathcal{O}(x^4) \end{aligned}$$

$$x = H_0(t_0 - t), \quad p_{age} = H_0 t_0$$

PAge approximation⁴

PPage

Parameterization based on the cosmic Age

Hubble parameter

$$\frac{H(t)}{H_0} = 1 + \frac{2}{3} \left(1 - \eta \frac{H_0 t}{p_{age}} \right) \left(\frac{1}{H_0 t} - \frac{1}{p_{age}} \right)$$

$$H(t)t = p_{age} + \frac{(2 - 3p_{age} - 2\eta)}{3p_{age}}x + \frac{2\eta}{3p_{age}^2}x^2$$

deceleration parameter

$$\eta = 1 - \frac{3}{2}p_{age}^2(1 + q_0)$$

⁴Huang, Z. 2020, ApJL, 892, L28

PAge approximation

PAge approximation

$$H(t)t = p_{age} - (1 - p_{age} - p_{age}q_0)x - \left(1 + q_0 - \frac{2}{3p_{age}^2}\right)x^2$$

Expansion on t

$$H(t)t = p_{age} - (1 - p_{age} - p_{age}q_0)x - \left(1 + q_0 - \frac{p_{age}}{2}(2 + 3q_0 + j_0)\right)x^2 + \mathcal{O}(x^3)$$

Cosmography

Λ CDM model

$$H(t) = H_0 \sqrt{\Omega_m a^{-3}(t) + (1 - \Omega_m)} = H_0 \sqrt{\Omega_m(1+z)^3 + (1 - \Omega_m)}$$

$$q_0 = -1 + \frac{3}{2}\Omega_m, \quad j_0 = 1, \quad s_0 = 1 - \frac{9}{2}\Omega_m, \quad l_0 = 1 + \frac{3}{2}\Omega_m(2 + 9\Omega_m)$$

$$p_{age} = H_0 t_0 = \frac{1}{3\sqrt{1-\Omega_m}} \ln \left(\frac{1+\sqrt{1-\Omega_m}}{1-\sqrt{1-\Omega_m}} \right)$$

Hubble diagram

Supernovae

$$\mu = m - M = 5 \log \frac{d_L}{\text{Mpc}} + 25$$

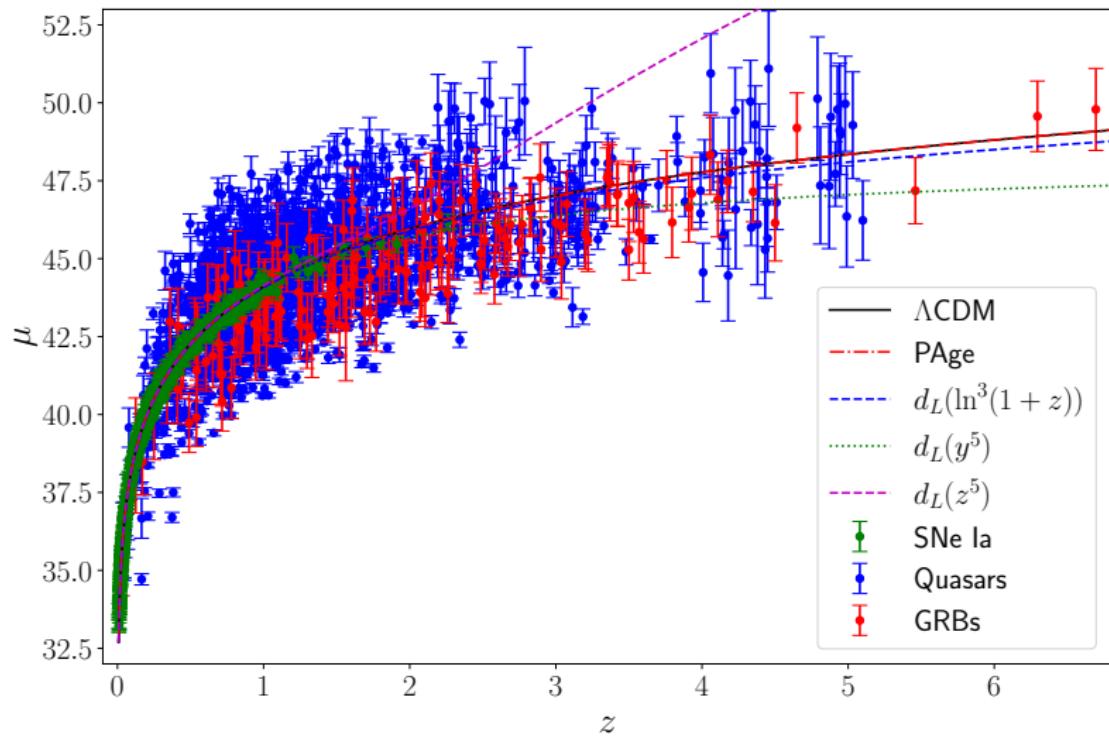
Quasars

$$\log d_L = \frac{1}{2 - 2\gamma} (\gamma F_{UV} - \log F_X) + \beta, \quad \log L_{UV} = \gamma \log L_X + \beta$$

GRBs

$$\log \left(\frac{E_{iso}}{1 \text{ erg}} \right) = a \log \left(\frac{E_{p,i}}{300 \text{ keV}} \right) + b, \quad E_{iso} = 4\pi d_L^2 S_{bolo} / (1+z)$$

Hubble diagram



MCMC($\ln(1 + z)$)

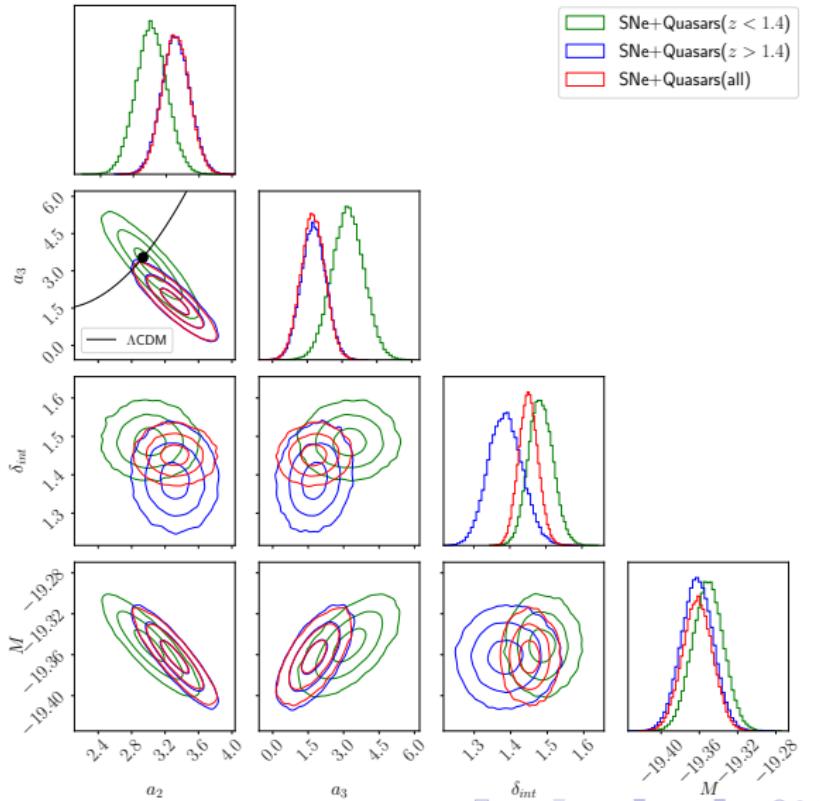
Matching condition

$$a_2 = \frac{3}{2} - \frac{3}{4}\Omega_m$$

$$a_3 = \frac{7}{6} - 2\Omega_m + \frac{9}{8}\Omega_m^2$$

Tension with Λ CDM

Sample	Tension
SNe+Quasars($z < 1.4$)	0.4σ
SNe+Quasars($z > 1.4$)	2.8σ
SNe+Quasars(all)	2.9σ



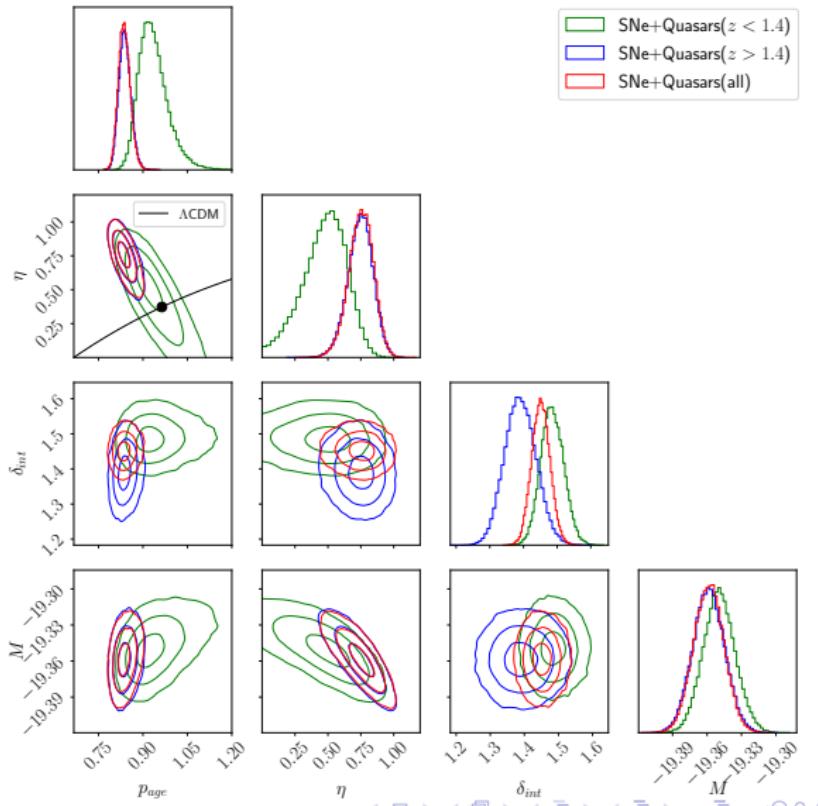
MCMC(PAge)

Matching condition

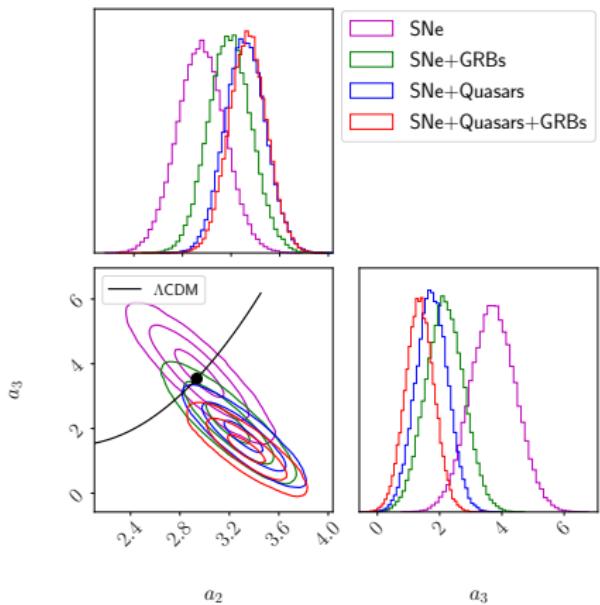
$$p_{age} = \frac{1}{3\sqrt{1-\Omega_m}} \ln \left(\frac{1+\sqrt{1-\Omega_m}}{1-\sqrt{1-\Omega_m}} \right)$$
$$\eta = 1 - \frac{9}{4} p_{age}^2 \Omega_m$$

Tension with Λ CDM

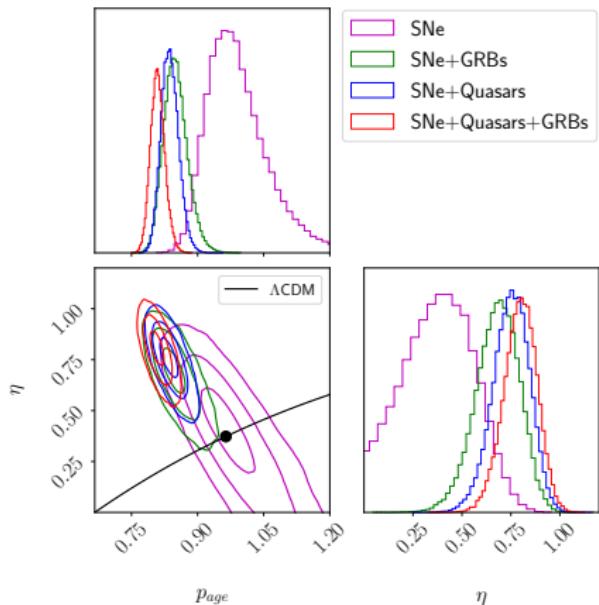
Sample	Tension
SNe+Quasars($z < 1.4$)	0.9σ
SNe+Quasars($z > 1.4$)	4.6σ
SNe+Quasars(all)	4.7σ



MCMC(2D posterior)



(a). $\ln(1 + z)$



(b). PAge approximation

MCMC(2D posterior)

Tension with Λ CDM

Sample	$\log(1 + z)$	PAge approximation
SNe	0.1σ	0.3σ
SNe+GRBs	1.9σ	3.2σ
SNe+Quasars	2.9σ	4.7σ
SNe+Quasars+GRBs	3.6σ	6.1σ

Question

- Unknown systematic error?
- New physics beyond Λ CDM?

Thanks for your attention!