Institute of Theoretical Physics
Chinese Academy of Sciences

高能物理研究中心
CENTER FOR HIGH ENERGY PHYSICS

有效场论基础和前沿介绍
Effective Field Theories

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高能物理前沿系列讲座

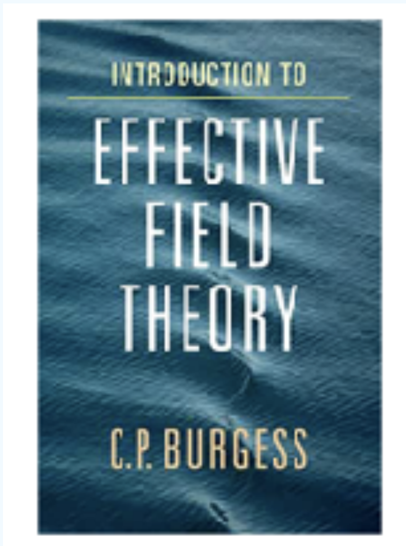
October 08, 2022 @ CHEP-PKU

Outline

- Conceptual Overview on Effective Field Theory
- Practical Calculation on Matching and Running
- Quest for New Physics in the Standard Model EFT
- Chiral Lagrangian for QCD and Electroweak Theory

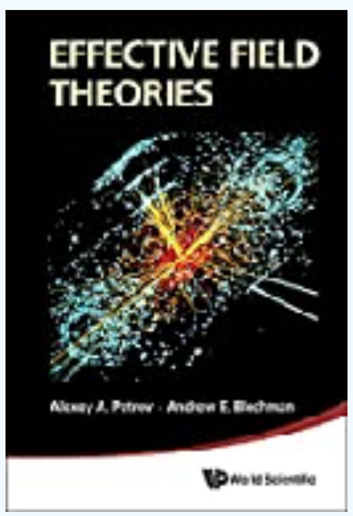
Apologize for not discussing HQET, SCET, Gravity EFT, etc

Books on EFT



Introduction to Effective Field Theory: Thinking Effectively about Hierarchies of Scale
by C. P. Burgess | Jan 21, 2021

SMEFT, Chiral Lag, NRQED, Gravity



Effective Field Theories
by Alexey A Petrov and Andrew E Blechman | Nov 18, 2015

SMEFT, HQET, SCET, Gravity

Effective Field Theories
by Ulf-G Meißner and Akaki Rusetsky | Aug 25, 2022

Chiral Lag, Nuclear EFT

References

Some slides are taken from these refs.

Introduction to Effective Field Theories #3

Aneesh V. Manohar (UC, San Diego) (Apr 16, 2018)

Published in: *Les Houches Lect.Notes* 108 (2020) • Contribution to: Les Houches summer school • e-Print: 1804.05863 [hep-ph]

As Scales Become Separated: Lectures on Effective Field Theory

Timothy Cohen (Oregon U.) (Mar 8, 2019)

Published in: *PoS* TASI2018 (2019) 011 • Contribution to: TASI 2018, 011 • e-Print: 1903.03622

Effective Field Theory and Precision Electroweak Measurements

Witold Skiba (Yale U.) (Jun, 2010)

Published in: • Contribution to: TASI 2009, 5-70 • e-Print: 1006.2142 [hep-ph]

Effective Field Theory with Nambu-Goldstone Modes

Antonio Pich (Valencia U., IFIC) (Apr 16, 2018)

Published in: *Les Houches Lect.Notes* 108 (2020) • Contribution to: Les Houches summer school • e-Print: 1804.05664

Saclay Lectures on Effective Field Theories

Adam Falkowski (IJCLab, Orsay) (June 14, 2017)

Unpublished lecture notes

Basic Concepts on EFT

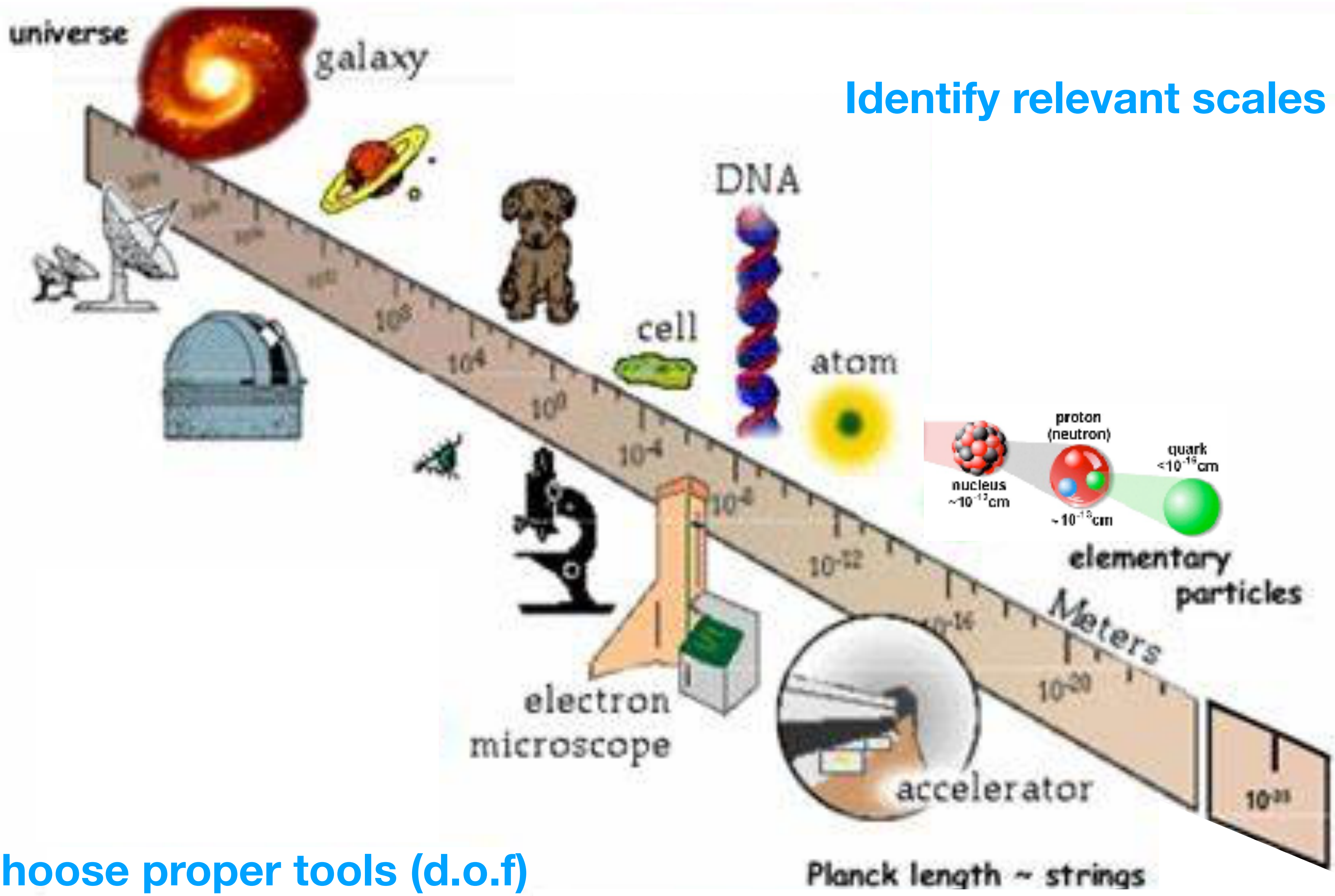
Bottom-up

Bottom-up EFT

Top-down

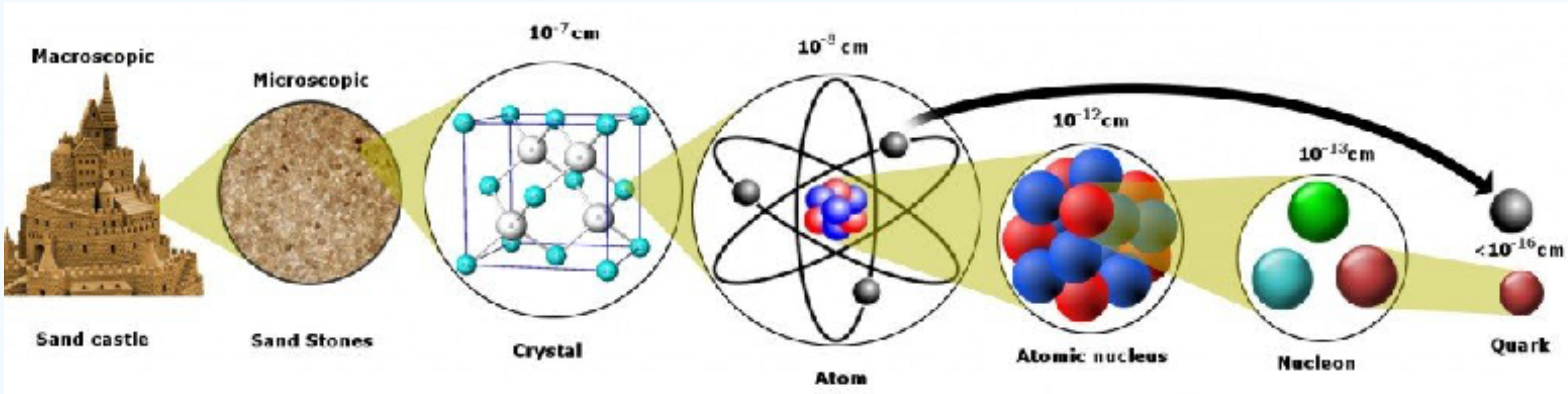
Scales of Nature

Identify relevant scales (scale)

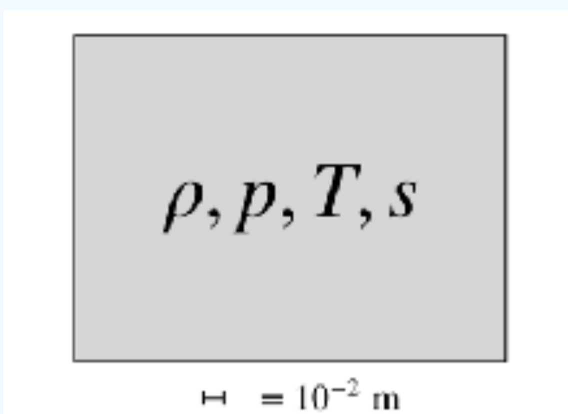


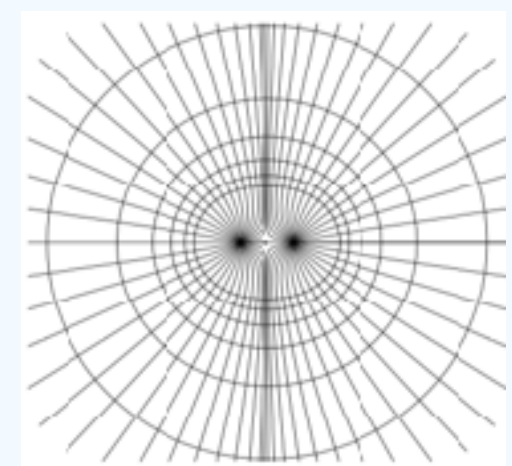
Choose proper tools (d.o.f)

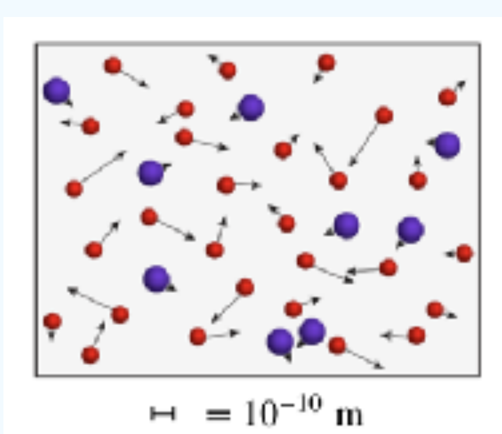
Decoupling Among Scales

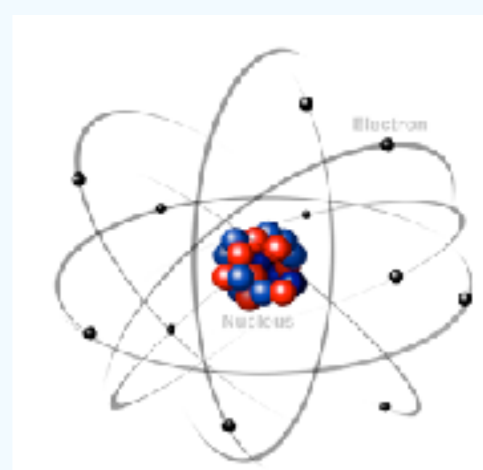


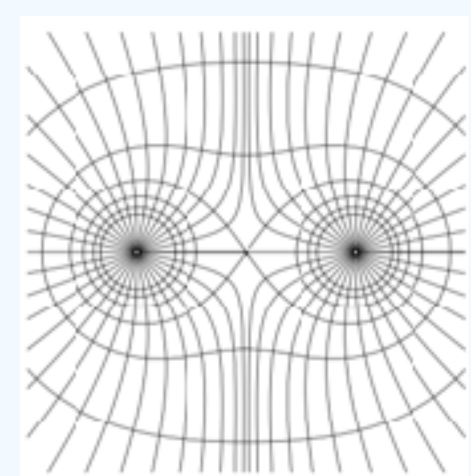
Not all scales relevant at the energy of interest











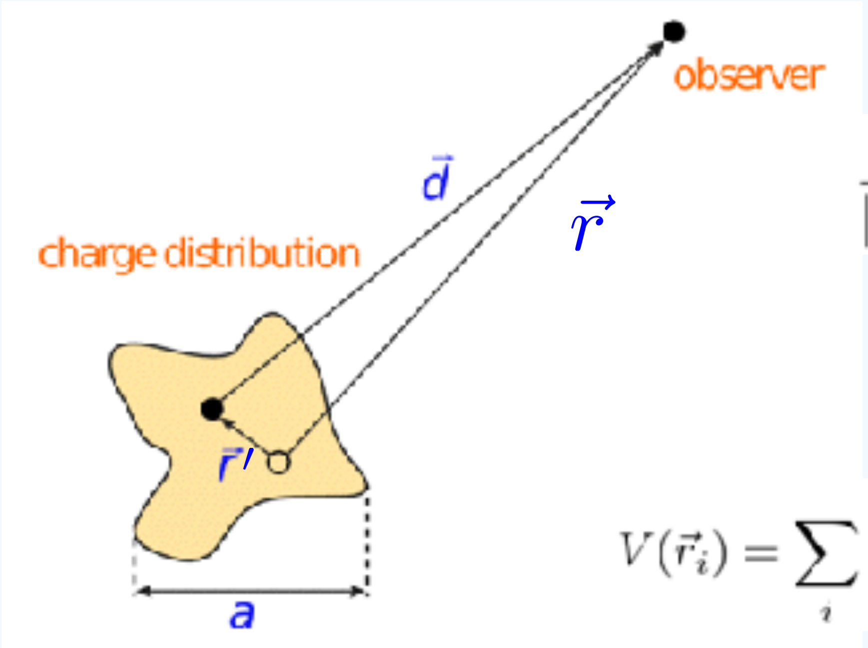
Statistical

Hyperfine splitting

Multipole expansion

Scale Separation

When involving multiple, disparate scales two scales r and a, with $r \gg a$



Tayler expansion on $\delta = r'/r$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r}\frac{1}{[1+(\frac{r'}{r})^2 - \frac{2\vec{r}\cdot\vec{r}'}{r^2}]^{1/2}} \qquad \frac{2\vec{r}\cdot\vec{r}'}{r^2} = 2(r'/r)cos\theta$$

$$= \frac{1}{r}[1 + \frac{r'}{r}cos\theta + (\frac{r'}{r})^2(\frac{3cos^2\theta}{2} - \frac{1}{2}) + (\frac{r'}{r})^3(\frac{5cos^3\theta}{2} - \frac{3cos\theta}{2}) + O((\frac{r}{r'})^4$$

$$V(\vec{r}_i) = \sum_i \frac{kq_i}{|\vec{r}-\vec{r}_i|} = \sum_i \frac{kq_i}{r} + \sum_i \frac{kq_i r_i cos\theta_i}{r^2} + \sum_i \frac{kq_i r_i^2}{2r^3}[3cos^2\theta_i - 1] + O(\frac{1}{r^4})$$

For unknown charge distribution, parametrize

charge dipole quadrupole

$$= \frac{kQ}{r} + \frac{k\vec{p}\cdot\hat{r}}{r^2} + \frac{k\hat{r}\cdot\tilde{Q}_2\cdot\hat{r}}{r^3} + O(1/r^4)$$

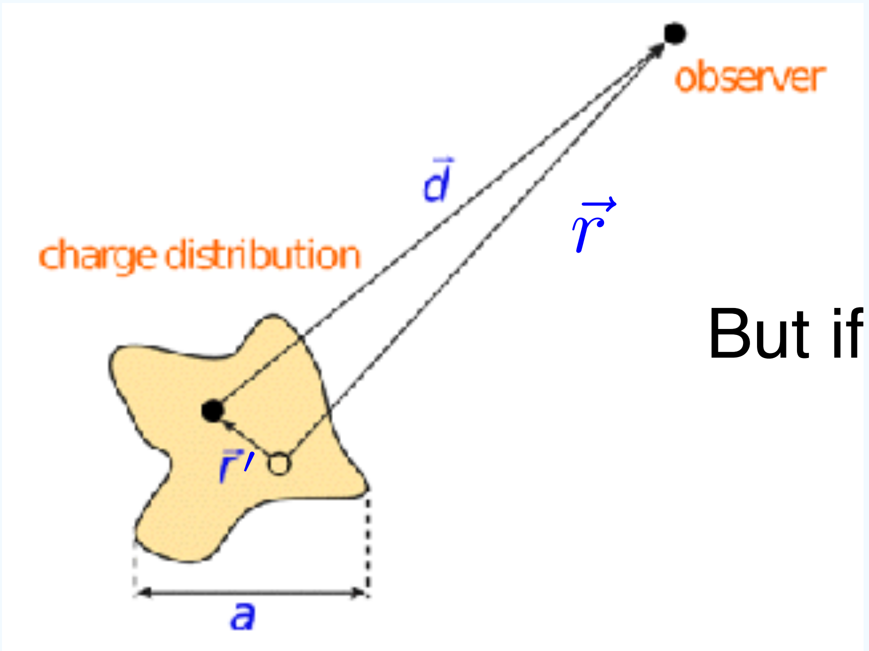
Determine these unknown coefficients up to certain moments

for finite experimental resolution

Then predict electric potential up to certain accuracy

Decoupling

Short distance scale a is not important since moments determined by exp



$$V(\mathbf{r}) = \frac{1}{r}\sum_{l,m} b_{lm}\frac{1}{r^l}Y_{lm}(\Omega)$$

But if we know a, then we can predict b (matching)

$$V(\mathbf{r}) = \frac{1}{r}\sum_{l,m} c_{lm}\left(\frac{a}{r}\right)^l Y_{lm}(\Omega)$$

$$b_{lm} \equiv c_{lm}a^l$$

Matching

$$V(\mathbf{r}) = \frac{1}{r}\sum_{l,m} b_{lm}\frac{1}{r^l}Y_{lm}(\Omega)$$

Momentum space via Fourier transformation

$$\text{Short distance } a \longleftrightarrow \text{UV scale } \Lambda \sim 1/a$$

$$\text{Long distance } r \longleftrightarrow \text{IR scale } p \sim 1/r$$

$$\text{Tayler expansion } \frac{a}{r} \longleftrightarrow \text{power counting } \frac{p}{\Lambda}$$

d.o.f

Effective Field Theory (EFT)

In scattering, the relevant scales are particle masses, scattering momenta

When heavy mass scale M involved into the low energy process

Take scattering amplitude (two mass scales m, M) at CM energy E

$E^2 \sim m^2 \ll M^2$

- Describe by an **expansion** in $(m/M)^n$, $(E/M)^n$ (power counting)
- Effects of heavy physics with mass M, 'decouple' at low momenta, p
- When applied at the right scale, EFTs can predict with arbitrary precision

Range of validity

Why using EFT?

A. If 'full theory' is **known**: greatly simplify calculations — Top-down

B. If 'full theory' is **unknown**: universally parametrise UV effects — Bottom-up

How to Build an EFT?

Start with QED theory in QFT course

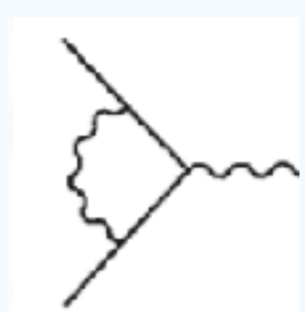
$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}\gamma^\mu\psi A_\mu$$

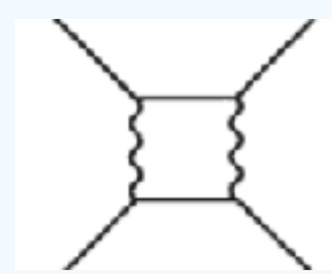
Fock, London, Dirac, Weyl 1927-1929

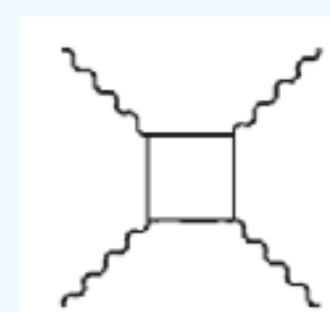
Add higher dimensional terms

$$+\frac{c_5}{\Lambda}\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu} + \frac{c_6}{\Lambda^2}(\bar{\psi}\psi)^2 + \frac{c_8}{\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2 + \cdots$$

Generate such terms from renormalizible QED?

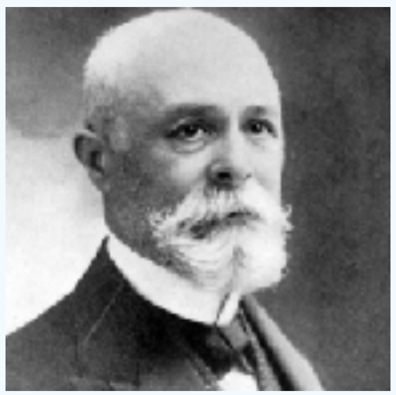


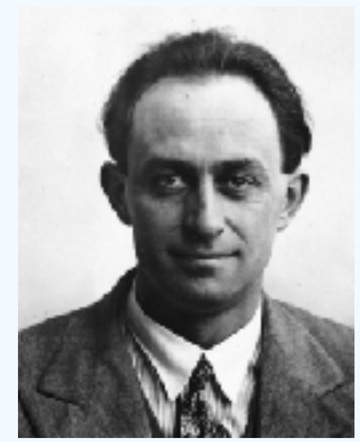




Why not generate new effective theory?

The First EFTs!



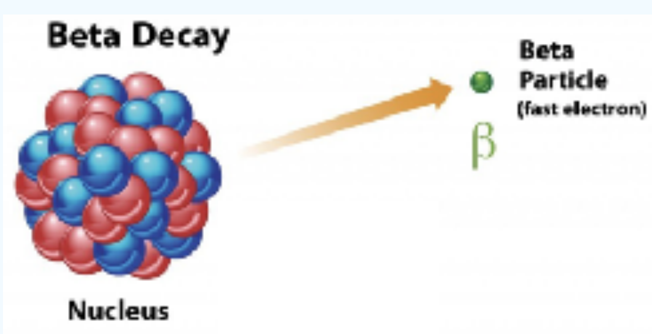


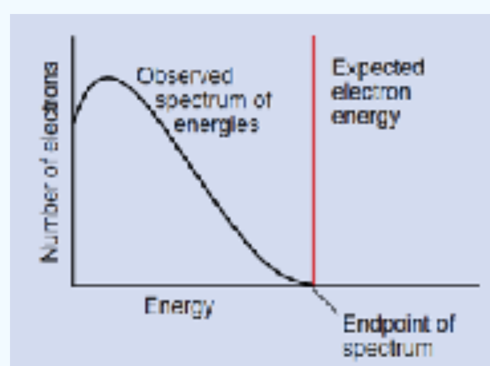

Becquerel
1896

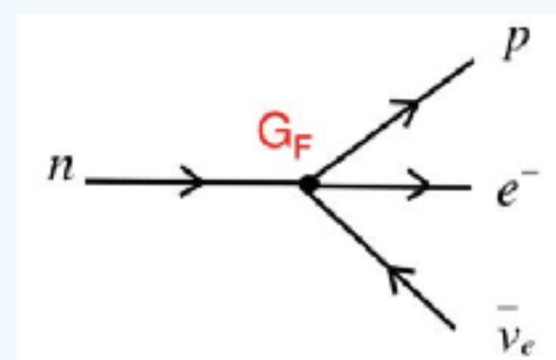
Pauli
1933

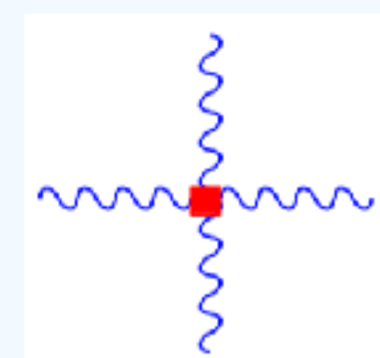
Fermi
1934

Euler-Heisenberg
1936







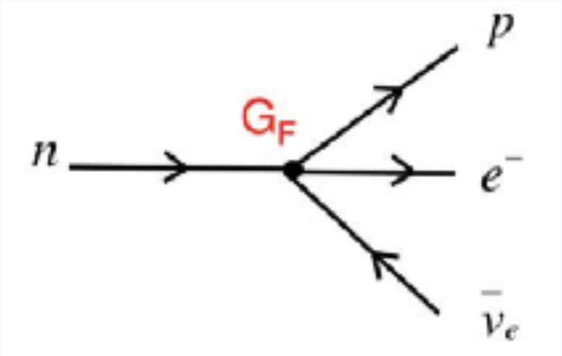


$$G_F\,[\overline{\psi_n}\gamma^\mu\psi_p]\,[\overline{\psi_e}\gamma^\mu\psi_\nu]$$

$$(F_{\mu\nu}F^{\mu\nu})^2$$

Four-Fermion Theory

According to the Fermi's Golden rule, predict the electron energy spectra



$$W_{fi} = 2\pi G_F^2 |M_{if}|^2 \; p_e^2 (E_f - E_e)^2 dp_e$$

$$M_{if} = \langle p|\, J_\mu^{wk} \,|n\rangle \, \langle e\nu|\, J_\mu^{wk} \,|0\rangle$$

$$E_e \sqrt{E_e^2 - m_e^2 c^4}\, E_\nu^2 \delta(E_e + E_\nu - E_0) dE_e dE_\nu$$

$$G_F \left[\overline{\psi_n} \gamma^\mu \psi_p\right] \left[\overline{\psi_e} \gamma^\mu \psi_\nu\right]$$

$$G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$$

First higher dimensional operator (1934 rejected by nature)!

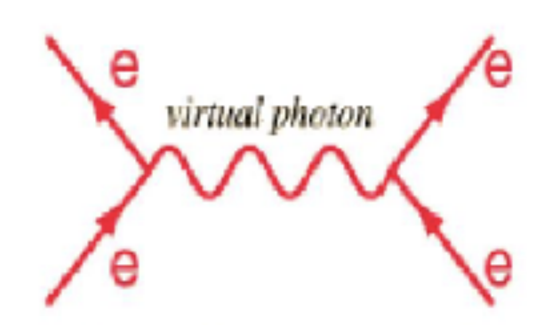
Why does G_F has dimensions of GeV^{-2}?

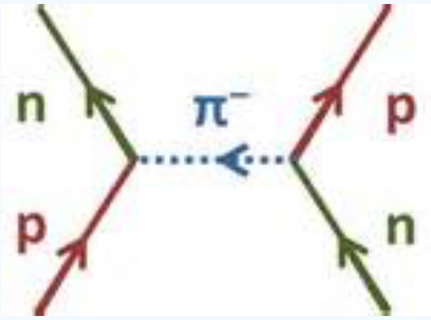
Suppose the four-fermion theory were right

$$\sigma(e\nu \to e\nu) \propto G_F^2 s$$ $\sqrt{s} \sim 500$ GeV, unitary it violated

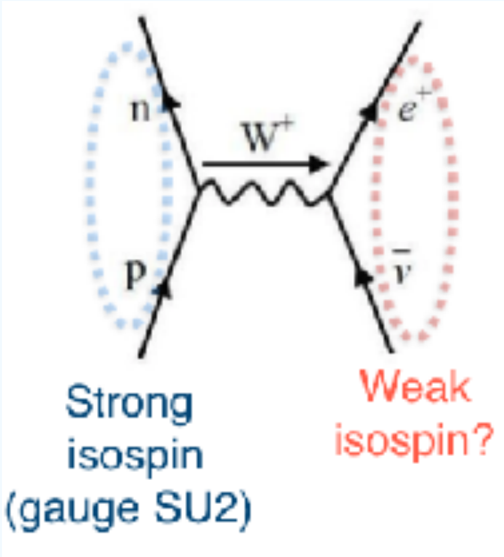
Intermediate Vector Boson

Current-current interaction through exchange of mediator boson





Strong
isospin



$$-e[\bar{\psi}\gamma_\mu\psi]\frac{g^{\mu\nu}}{q^2}[\bar{\psi}\gamma_\mu\psi] \longrightarrow -g[\bar{\psi}\gamma_\mu\psi]\frac{g^{\mu\nu}}{q^2 - m_W^2}[\bar{\psi}\gamma_\mu\psi]$$

$$\downarrow$$

$$\frac{g}{m_W^2}[\bar{\psi}\gamma_\mu\psi][\bar{\psi}\gamma_\mu\psi]$$

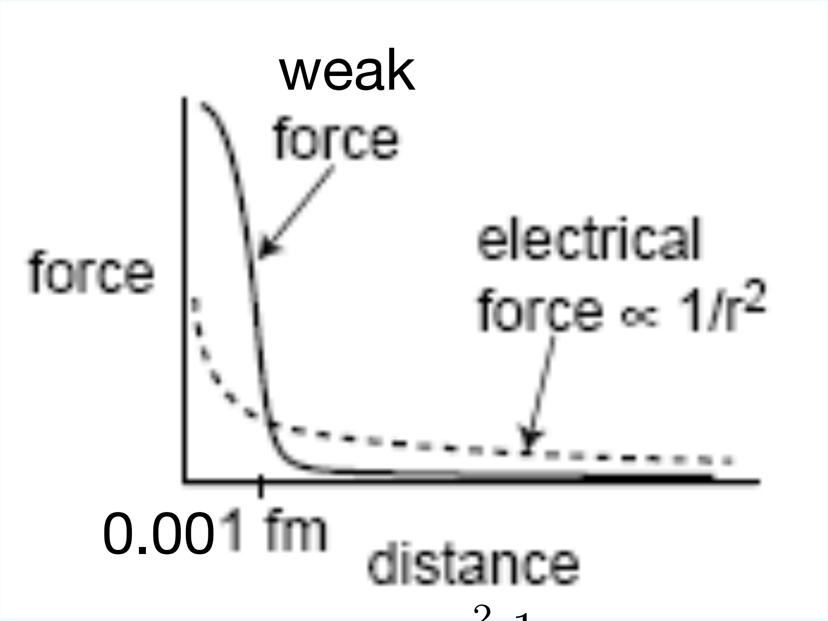
Explain GF $G_F = \frac{g}{m_W^2}$

Preserve unitarity

EFT vs UV Theory

$$\lambda \sim \frac{1}{p} < \Delta r_W \sim \frac{1}{m_W}$$

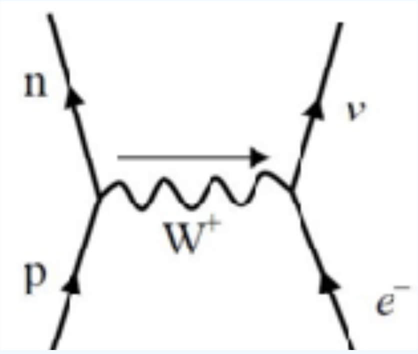
Short wave



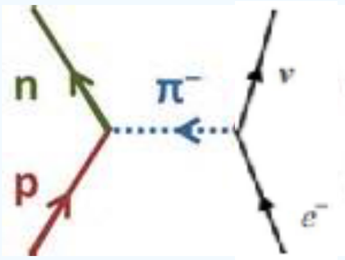
$$V(r) = -\frac{g^2}{4\pi}\frac{1}{r}e^{-m_\phi r}$$

$$\lambda \sim \frac{1}{p} > \Delta r_W \sim \frac{1}{m_W}$$

Long wave

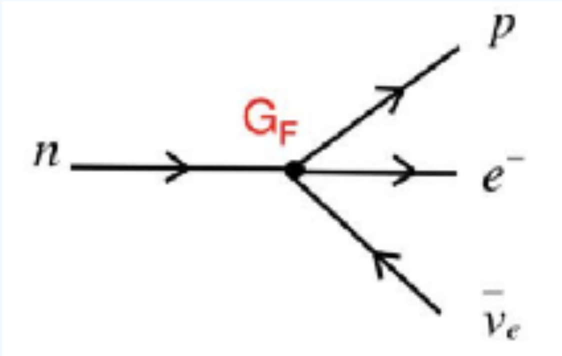


Top-down



Long-range interaction at UV

Other UV?



Bottom-up

EFT contact interaction at low energy

Euler-Heisenberg EFT

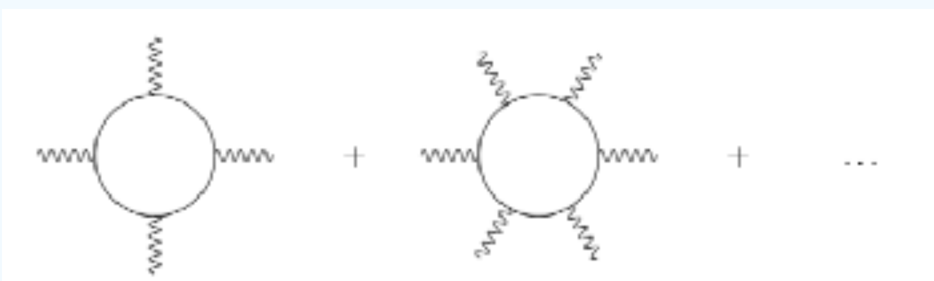
Light-by-light scattering at very low energy scale

$(\mathbf{E}_\gamma \ll \mathbf{m}_\mathbf{e})$

- Gauge, Lorentz, Charge Conjugation & Parity
- Energy expansion (E_γ / m_e)

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{a}{m_e^4} (F^{\mu\nu} F_{\mu\nu})^2 + \frac{b}{m_e^4} F^{\mu\nu} F_{\nu\sigma} F^{\sigma\rho} F_{\rho\mu} + \mathcal{O}(F^6 / m_e^8)$$

Give the UV theory (QED)



$$a = -\frac{1}{36}\alpha^2 \quad , \quad b = \frac{7}{90}\alpha^2$$

$$\sigma(\gamma\gamma \to \gamma\gamma) \propto \frac{\alpha^4 E^6}{m_e^8}$$

photon does interact with itself

Rayleigh scattering

Low-energy scattering of photons with neutral atoms

$$E_\gamma \ll \Delta E \sim \alpha^2 m_e \ll a_0^{-1} \sim \alpha m_e \ll M_A$$

- Neutral atom + gauge invariance → $F^{\mu\nu} = (\vec{E}, \vec{B})$
- Non-relativistic description: $\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{1}{2M} \vec{\nabla}^2 \right) \psi + \mathcal{L}_{\rm int}$

$$\mathcal{L}_{\rm int} = a_0^3\, \psi^\dagger \psi \left(c_1 \vec{E}^2 + c_2 \vec{B}^2 \right) + \dots \quad , \quad c_i \sim \mathcal{O}(1)$$

$$\mathcal{M} \sim c_i\, a_0^3\, E_\gamma^2 \quad \rightarrow \quad \sigma \propto a_0^6\, E_\gamma^4$$

Blue light is scattered more strongly than red one

Dimensional Analysis

Fermi interaction is a higher dimensional operator

4D QFT functional integral: $Z = \int \mathcal{D}\phi \, e^{iS[\phi]}, \quad S = \int d^4x \, \mathcal{L}[\phi(x)]$

Natural units, h=c=1: [Length] = Mass^{-1}

From kinetic terms

$$[\mathcal{L}] = 4: \quad [\phi] = 1, \ [\psi] = \frac{3}{2}, \ [D_\mu] = 1, \ [A_\mu] = 1, \ [g] = 0$$

Renormalisable interactions have couplings [c] ≥ 0

$$\mathcal{L}_{\text{int.}} = c\,\mathcal{O}, \quad [\mathcal{O}] \le 4$$

- Renormalisable: need a **finite number** of counter-terms (CT) to absorb divergences in loop computations to **all orders** in perturbation theory

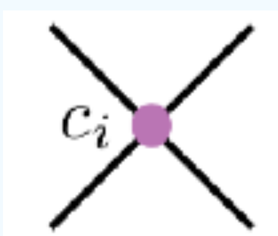
$[\mathcal{O}] < 4, \ [c] > 0$	$[\mathcal{O}] = 4, \ [c] = 0$	$[\mathcal{O}] > 4, \ [c] < 0$
'Relevant'	'Marginal'	'Irrelevant'
$I, \ \phi^2, \ \phi^3, \ \bar\psi\psi$	$\phi^4, \ \phi\bar\psi\psi, \ V_\mu \bar\psi\gamma^\mu\psi$	$\bar\psi\psi\bar\psi\psi, \ \partial_\mu\phi\bar\psi\gamma^\mu\psi, \ \phi^2\bar\psi\psi, \ \cdots$

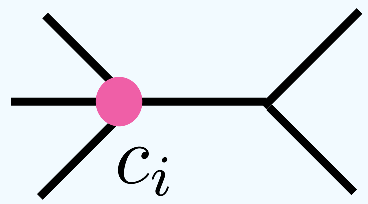
Power Counting

EFT Lagrangian expansion based on the canonical dimension

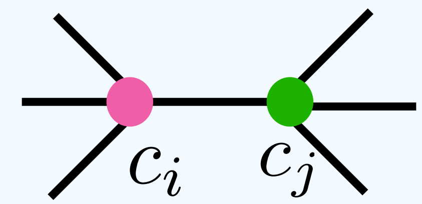
$$\mathscr{L}_{\rm EFT} = \mathscr{L}_{\mathscr{D}\leq 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \ldots = \sum_i \frac{c_i}{\Lambda^{d_i-4}} \mathcal{O}^{d_i}$$

Normalized scattering amplitude follows the EFT power counting formula





$$\mathcal{A} \sim c_i \left(\frac{p}{\Lambda}\right)^{d_i-4}$$



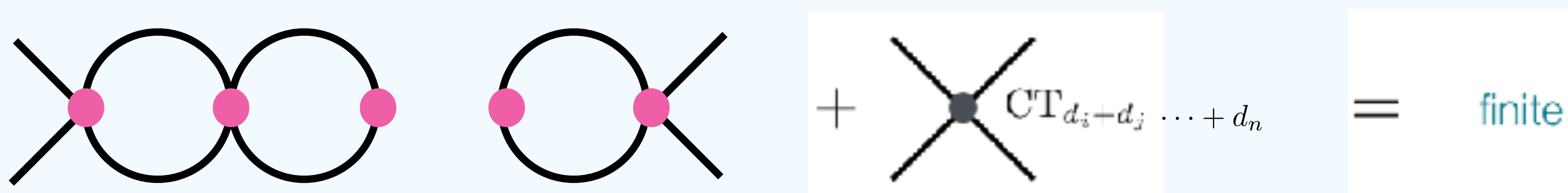
$$\mathcal{A} \sim c_i c_j \left(\frac{p}{\Lambda}\right)^{n_i+n_j} \qquad n_i = d_i - 4$$

How about beyond tree-level?

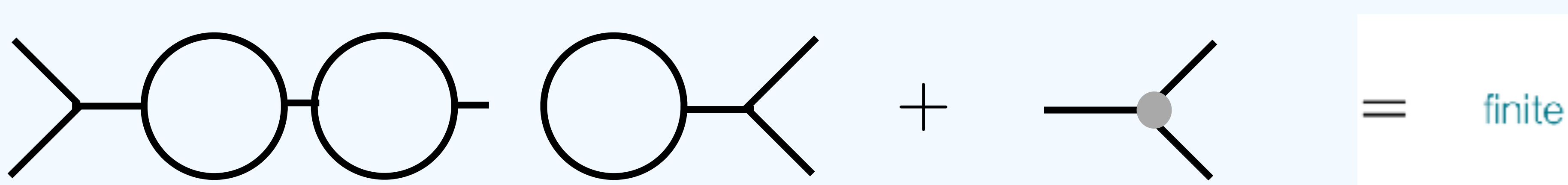
$$\longrightarrow \quad \mathcal{A} \sim \left(\frac{p}{\Lambda}\right)^{n_i+n_j}$$

Non-Renormalizable!

Counterterms from higher dim operators



Infinite counter terms, formally **non-renormalizable**!

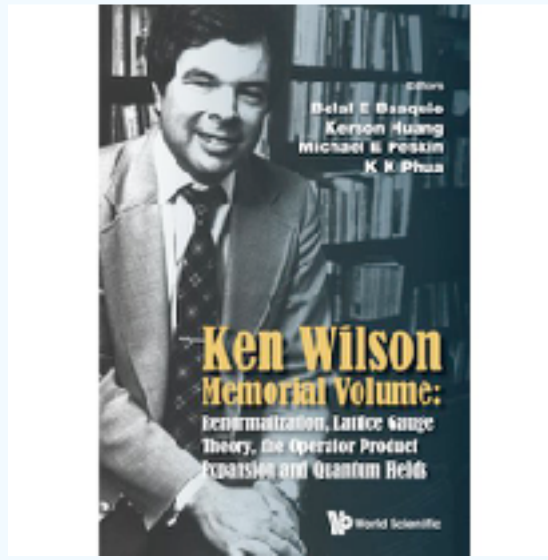


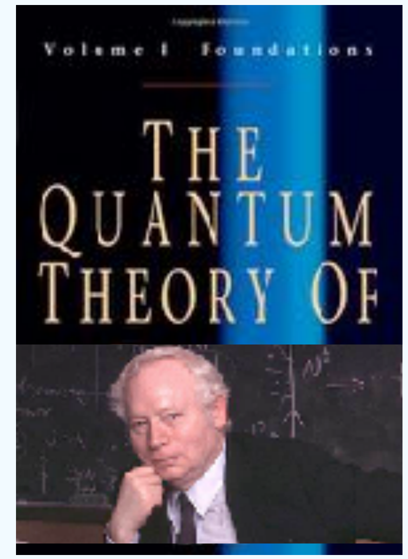
$\left(\frac{p}{\Lambda}\right)^{d_i-4\ \leq 0}$ $\qquad$ $d_i \ \leq 4$

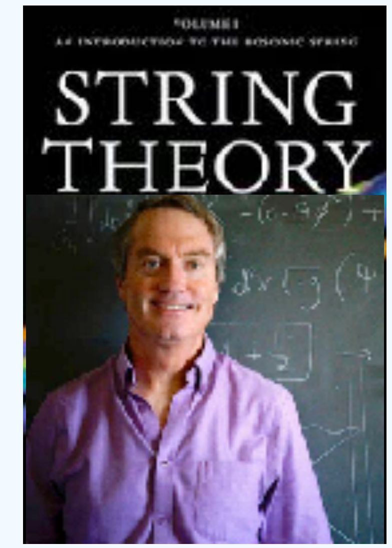
$$\mathrm{div} = 4 - B - \frac{3}{2}F - V + \sum_i n_i(\Delta_i - 4)$$

If only dim-3,4 operators, then renormalizable!

Progress on QFT and EFT







1949~1970
QFT should be
Renormalizable

1970
Wilsonian EFT

1979
Folk theorem

1984
EFT should be
Renormalizable

"What bothered me was that the proofs that renormalization works seemed extremely combinatoric and technical, but the results in the end came down to dimensional analysis. What I realized was that things would become nearly trivial if, instead of describing the path integral order by order in perturbation theory, as nearly always done, we described it scale-by-scale in energy. As soon as I thought those words, I knew I could prove them...It took just three weeks for me to work out the proof and write it up."

New understanding of QFT needs the understanding of the bottom-up EFT

Renormalizable EFT

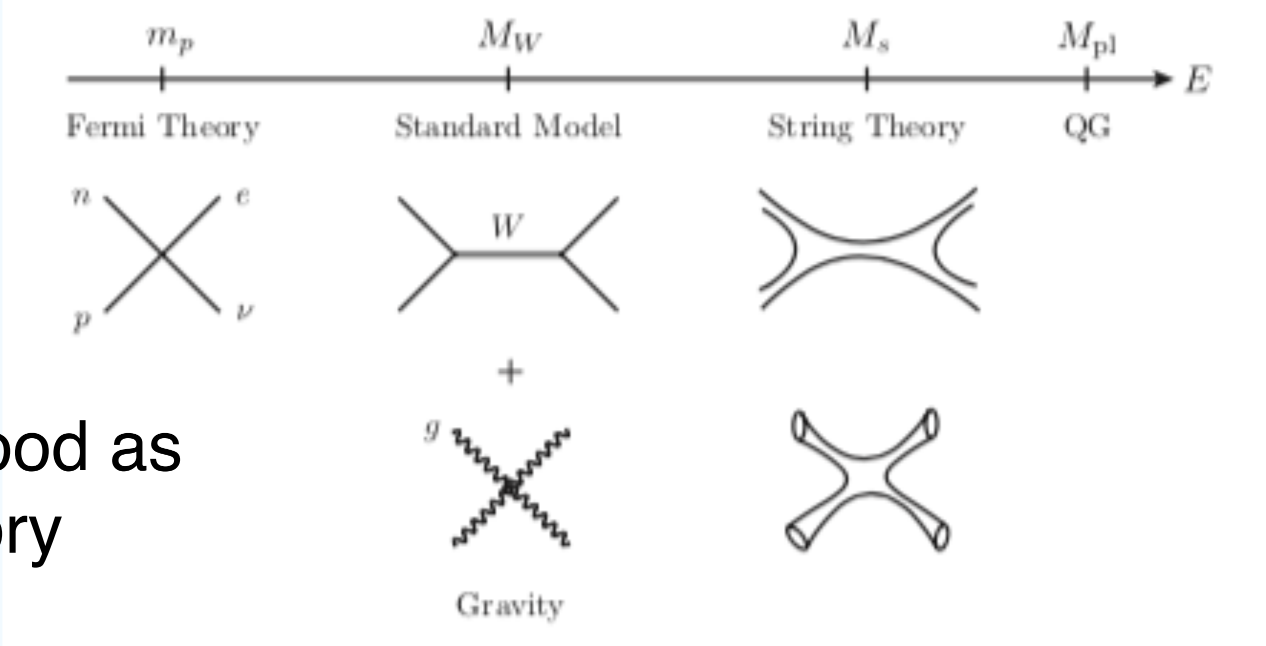
The EFT describes the low energy physics, to a given precision,
in terms of a finite set of parameters

$$(E/M)^{d_i-4} \gtrsim \epsilon \quad \Longleftrightarrow \quad d_i \lesssim 4 + \frac{\log(1/\epsilon)}{\log(M/E)}$$

Renormalization of non-renormalizable theory:

1. Write the most general operator up to certain truncated order
2. All the div. in such order can be absorbed into redefinition of Wilson coeff.
3. Make predictions on observables using the truncated theory

To a given precision, EFT is renormalizable and predictive!



Fermi/Gravity as good as
underlying theory

Wilsonian Renormalization

Choose a cutoff $\Lambda < M$ and divide all quantum fields

$$Z[J] = \int d[\phi]\, e^{-S[\phi] - \int J\phi}$$

$$\phi = \phi_H + \phi_L$$
$$\phi_H : \ \omega > \Lambda$$
$$\phi_L : \ \omega < \Lambda$$

integrate out high momentum modes
generate Wilson effective action

Integrate-out

$$\int \mathcal{D}\phi_L\, \mathcal{D}\phi_H\, e^{iS(\phi_L,\phi_H)} = \int \mathcal{D}\phi_L\, e^{iS_\Lambda(\phi_L)}$$

$$Z[J_L] \equiv \int \mathcal{D}\phi_L\, e^{iS_\Lambda(\phi_L) + i\int d^Dx\, J_L(x)\,\phi_L(x)}$$

$$e^{iS_\Lambda(\phi_L)} = \int \mathcal{D}\phi_H\, e^{iS(\phi_L,\phi_H)}$$

$$S[\varphi_\Lambda] = \int \mathrm{d}^D x \left(\frac{1}{2}\partial_\mu(\phi_0 + \hat{\phi}_0)\partial^\mu(\phi_0 + \hat{\phi}_0) + \frac{1}{2} m^2 (\phi_0 + \hat{\phi}_0)^2 + \frac{\lambda}{4!}(\phi_0 + \hat{\phi}_0)^4 \right)$$

$$\int D\hat{\phi}_0\, e^{-\int d^D x [\frac{1}{2}(\partial\hat{\phi}_0)^2 + \frac{1}{2} m_0^2 \hat{\phi}_0^2 + \frac{\lambda_0}{4!}(\hat{\phi}_0^4 + 4\phi_0^3\hat{\phi}_0 + 4\phi_0\hat{\phi}_0^3 + 6\phi_0^2\hat{\phi}_0^2)]}$$

Effective action

$$S_\Lambda(\phi_L) = \int d^Dx\, \mathcal{L}_\Lambda^{\text{eff}}(x)$$

$$S_{eff}[\phi_0] = \int d^D x\, [\frac{1}{2}(1 + \Delta Z)(\partial\phi_0)^2 + \frac{1}{2}(m_0^2 + \Delta m^2)\phi_0^2 + \frac{1}{4!}(\lambda_0 + \Delta\lambda)\phi_0^4 + \ldots]$$

Local EFT

$$\mathcal{L}_\Lambda^{\text{eff}}(x) = \sum_i g_i\, Q_i(\phi_L(x))$$

coupling constants (Wilson coefficients)

local operators built out of fields ϕ_L and their derivatives

Wilson effective action (counter term) =
Modern understanding of renormalization

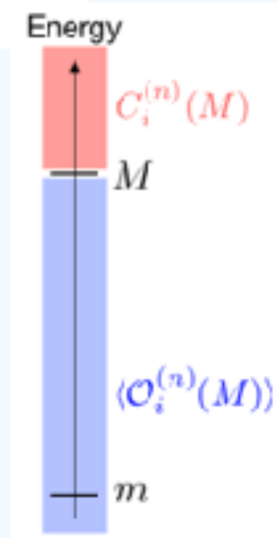
Wilsonian RGE

Lowering the cutoff, getting the same effective action = Wilson RGE

$$Z[J_L] \equiv \int \mathcal{D}\phi_L \, e^{iS_\Lambda(\phi_L) + i\int d^Dx\, J_L(x)\,\phi_L(x)}$$

$$S_\Lambda(\phi_L) = \int d^Dx\, \mathcal{L}_\Lambda^{\rm eff}(x)$$

$$\mathcal{L}_\Lambda^{\rm eff}(x) = \sum C_i(\Lambda) Q_i(\phi_L(x))$$



Integrate-out

$$\Lambda'^2 < k^2 < \Lambda^2$$

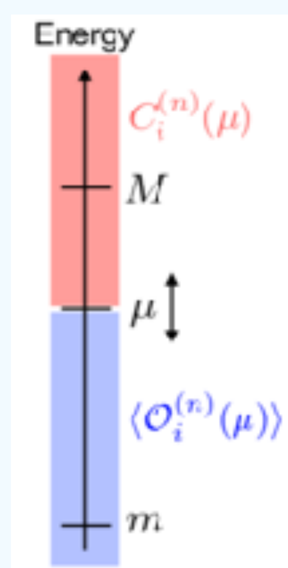
$$Z[J] = \int \left(\prod_{k^2<\Lambda'^2} d\tilde{\phi}_k \right) e^{-S'[\phi] - \int J\phi}$$

Effective action

$$e^{-S'[\phi]} \overset{\rm def}{=} \int \left(\prod_{\Lambda'^2<k^2<\Lambda^2} d\tilde{\phi}_k \right) e^{-S[\phi]}$$

$$S_{\Lambda'}(\phi_L) = \int d^Dx\, \mathcal{L}_{\Lambda'}^{\rm eff}(x)$$

$$\mathcal{L}_{\Lambda'}^{\rm eff}(x) = \sum C_i(\Lambda') Q_i(\phi_L(x))$$



$$\Delta S[\phi] = \int d^4x \left[\frac{\lambda_4}{4!}\phi^4 + \frac{\lambda_6}{6!}\phi^6 + \cdots \right].$$

$$\Lambda \frac{dm^2}{d\Lambda} = \Lambda^2 \beta_m(m^2/\Lambda^2, \lambda_4, \Lambda^2\lambda_6, \ldots),$$

$$\Lambda \frac{d\lambda_4}{d\Lambda} = \beta_4(\lambda_4, \Lambda^2\lambda_6, \ldots),$$

$$\Lambda \frac{d\lambda_6}{d\Lambda} = \frac{1}{\Lambda^2}\beta_6(\lambda_4, \Lambda^2\lambda_6, \ldots),$$

$$\vdots$$

$$\beta_6 \sim \frac{\hbar}{16\pi^2}\left[\hat{\lambda}_4^3 + \hat{\lambda}_4\hat{\lambda}_6 + \hat{\lambda}_8\right]$$

$$\hat{\lambda}_6(\Lambda') \simeq \left(\frac{\Lambda'}{\Lambda}\right)^2 \hat{\lambda}_6(\Lambda) \qquad \Lambda \frac{d\hat{\lambda}_6}{d\Lambda} = 2\hat{\lambda}_6 + \beta_6(\hat{\lambda}_4, \hat{\lambda}_6, \ldots)$$

Lowering the cutoff, lambda6 decrease

$$\lambda_4(\Lambda') = \frac{\lambda_4(\Lambda)}{1 - \beta_4 \log\left(\frac{\Lambda'}{\Lambda}\right)} \qquad \beta_4 \sim \frac{\hbar}{16\pi^2}\left[\hat{\lambda}_4^2 + \hat{\lambda}_6^2\right]$$

lambda6 has large effects on lambda4
But absorbed into redefinition of couplings

Wilson RG Renormalization

Relevant or Irrelevant?

Consider the leading scaling behavior

$$S_{\rm EFT}(\phi) = \int d^4x \left[(\partial_\mu \phi)^2 - m^2\phi^2 - \kappa\mu\phi^3 - \lambda\phi^4 - \sum_{n+d>4} \frac{c_{n,d}}{\Lambda^{n+d-4}} \phi^{n-1}\partial^d\phi \right]$$

Scaling $x_\mu \to \xi x'_\mu$ $\phi \to \phi' \xi^{-1}$

$$S_{\rm EFT}(\phi) = \int d^4x' \left[(\partial_\mu \phi')^2 - m^2\xi^2(\phi')^2 - \kappa(\xi\mu)(\phi')^3 - \lambda(\phi')^4 - \sum_{n+d>4} \frac{c_{n,d}}{(\xi\Lambda)^{n+d-4}} (\phi')^{n-1}\partial^d\phi' \right]$$

$$\xi^{-\gamma} = \left(\frac{E}{\Lambda}\right)^\gamma$$

$$\delta_i = [Q_i] = D + \gamma_i$$

Dimension	Importance for $E \to 0$	Terminology
$\delta_i < D,\ \gamma_i < 0$	grows	relevant operators (super-renormalizable)
$\delta_i = D,\ \gamma_i = 0$	constant	marginal operators (renormalizable)
$\delta_i > D,\ \gamma_i > 0$	falls	irrelevant operators (non-renormalizable)

$$c_i \left(\frac{E}{M}\right)^{\gamma_i} = \begin{cases} O(1); & \text{if } \gamma_i = 0 \\ \ll 1; & \text{if } \gamma_i > 0 \\ \gg 1; & \text{if } \gamma_i < 0 \end{cases}$$

only operators with $\gamma_i \leq 0$ are important for $E \ll M$

Only a **finite number** of relevant and marginal operators exist!

“marginal” operators are all there is in “renormalizable” QFTs

“irrelevant” operators are the most interesting ones, since they tell us something about the fundamental scale *M*

EFT Ladder

Every QFT is an EFT!

High-energy theory	Fundamental scale	Low-energy theory
11-d M theory (?)	?	String theory (?)
String theory (?)	$M_S \sim 10^{18}$ GeV	QFT /Gravity
GUT (?)	$M_{GUT} \sim 10^{16}$ GeV	SUSY (?)
SUSY (?)	$M_{SUSY} \sim 10^{4+x}$ GeV	SMEFT

Short Summary: EFT as Modern QFT

"Theorem of modesty":

- no QFT ever is complete on all length and energy scales
- all QFTs are low-energy effective theories valid in some energy range, up to some cutoff Λ

Give up renormalizability as a construction criterion for "decent" QFTs:

Forget the folklore about "cancellations of infinities"

- at low energy, any effective theory will automatically reduce to a "renormalizable" QFT, meaning that "non-renormalizable" interactions give rise to small contributions ~$(E/M)^n$

- low-energy physics depends on the **short-distance dynamics** of the fundamental theory only through a small number of **relevant and marginal couplings,** and possibly through some irrelevant couplings if our measurements are sufficiently precise

- this finite number of couplings can be renormalized (i.e., infinities can be removed consistently) using a finite number of experimental data

Matching and Running

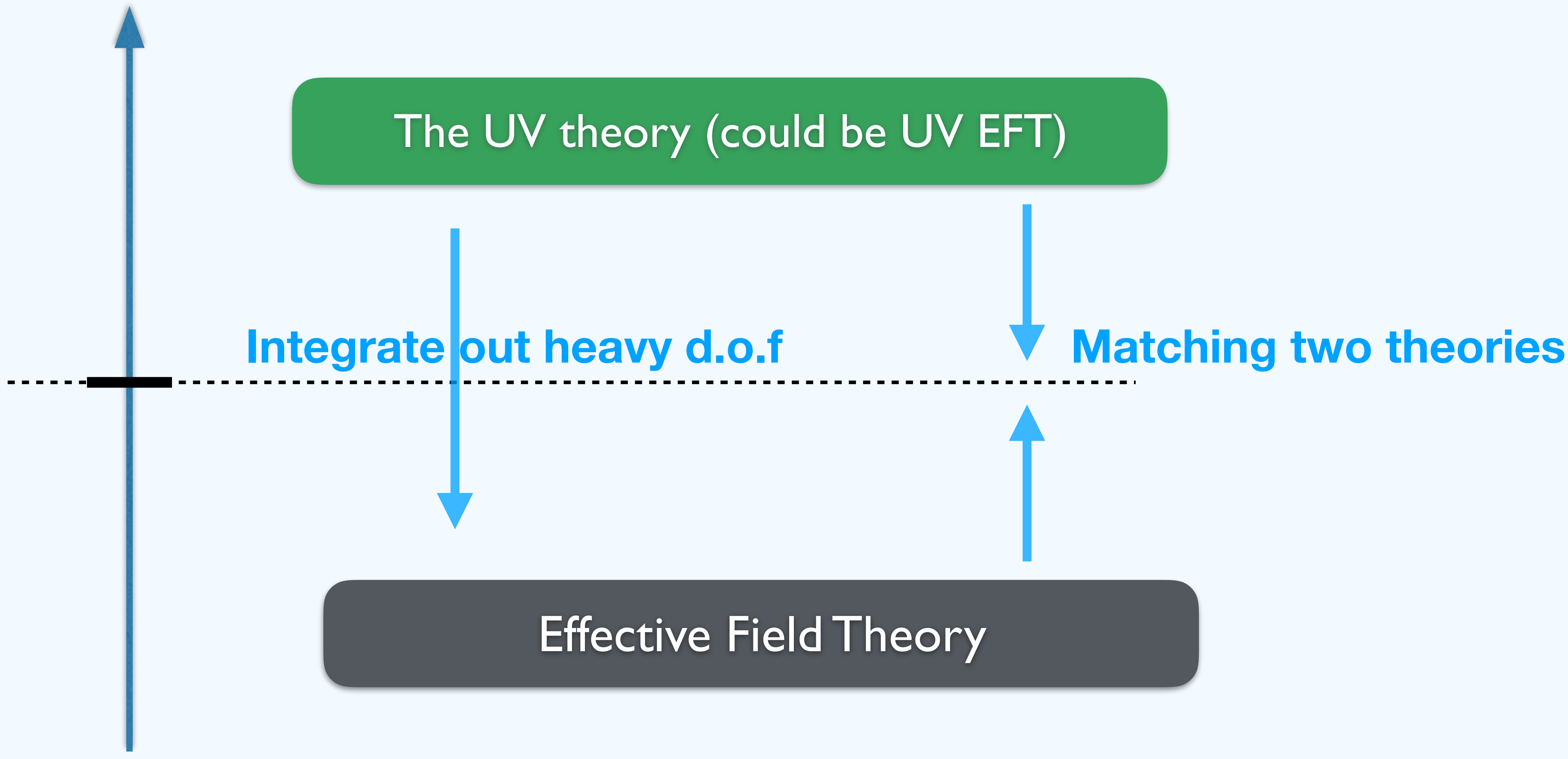
Bottom-up

Top-Down EFT

EFT and UV Theories

The previous section discusses bottom-up EFT, now focus on Top-down EFT

Suppose the UV theory is known, build connection between UV and EFT



Matching Procedure

The UV theory (could be UV EFT)

$$Z_{\rm UV}[J_\phi, J_H] = \int [D\phi][DH] \exp\left[i \int d^4x \left(\mathcal{L}_{\rm UV}(\phi, H) + J_\phi \phi + J_H H\right)\right]$$

$$\Gamma_{\rm UV}[\phi_b, H_b] = -i \log Z_{\rm UV}[J_\phi, J_H] - \int d^4x J_\phi(x)\phi_b(x) - \int d^4x J_H(x) H_b(x)$$

$$\phi_b(x) = \frac{\delta \log Z_{\rm UV}[J_\phi, J_H]}{\delta J_\phi(x)}, \quad H_b(x) = \frac{\delta \log Z_{\rm UV}[J_\phi, J_H]}{\delta J_H(x)}$$

Matching two theories

$$Z_{\rm EFT}[J_\phi] = Z_{\rm UV}[J_\phi, 0] \qquad \Gamma_{\rm UV}[\phi, 0] = \Gamma_{\rm EFT}[\phi]$$

Effective Field Theory

$$Z_{\rm EFT}[J_\phi] = \int [D\phi] \exp\left[i \int d^4x \left(\mathcal{L}_{\rm EFT}(\phi) + J_\phi \phi\right)\right]$$

$$\Gamma_{\rm EFT}[\phi_b] = -i \log Z_{\rm EFT}[J_\phi] - \int d^4x J_\phi(x)\phi_b(x) \qquad \phi_b(x) = \frac{\delta \log Z_{\rm EFT}[J_\phi]}{\delta J_\phi(x)}$$

Matching @ Tree-Level

The UV theory (could be UV EFT)

$$Z_{\rm UV}[J_\phi, J_H] = \int [D\phi][DH] \exp\left[i \int d^4x \left(\mathcal{L}_{\rm UV}(\phi, H) + J_\phi \phi + J_H H\right)\right]$$

$$Z_{\rm UV}[J_\phi, 0] = \int [D\phi] \exp\left[i \int d^4x (\mathscr{L}_{\rm UV}(\phi, H_{\rm cl}(\phi)) + J_\phi \phi)\right]$$

$$Z_{\rm EFT}[J_\phi] = Z_{\rm UV}[J_\phi, 0]$$

$$0 = \left.\frac{\delta S}{\delta H}\right|_{H=H_{\rm cl}(\phi)}$$

the classical equations of motion in the UV Lagrangian

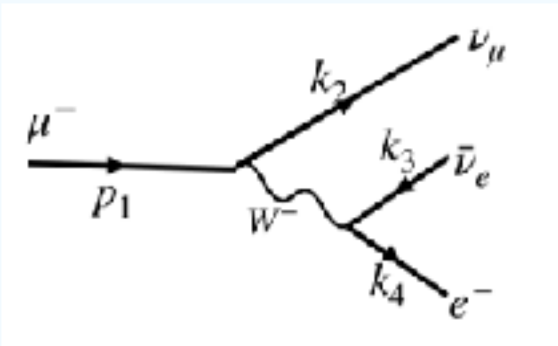
$$\mathscr{L}_{\rm EFT}(\phi) = \mathscr{L}_{\rm UV}(\phi, H_{\rm cl}(\phi))$$

Effective Field Theory

$$Z_{\rm EFT}[J_\phi] = \int [D\phi] \exp\left[i \int d^4x \left(\mathcal{L}_{\rm EFT}(\phi) + J_\phi \phi\right)\right]$$

Matching @ Tree-Level

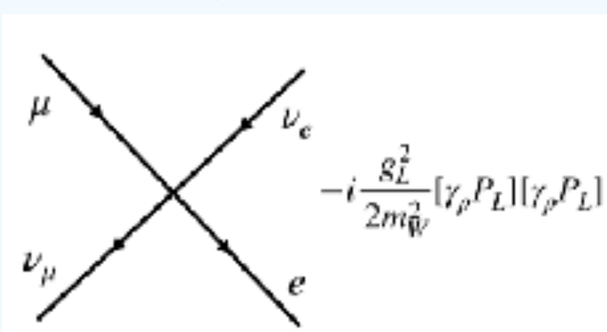
Diagrammatic approach



$$\mathcal{M} = \frac{g_L^2}{2}\bar{u}(k_2)\gamma_\rho P_L u(p_1)\frac{1}{q^2 - m_W^2}\bar{u}(k_4)\gamma_\rho P_L v(k_3)$$

$$q^2 \lesssim m_\mu^2 \ll m_W^2$$

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \ldots\right)$$



$$\mathcal{M} = -\frac{g_L^2}{2m_W^2}\bar{u}(k_2)\gamma_\rho P_L u(p_1)\bar{u}(k_4)\gamma_\rho P_L v(k_3) + \mathcal{O}(q^2/m_W^4)$$

Path Integral approach

$$\mathcal{L}_{\text{UV}} \supset -W_\rho^+(\Box - m_W^2)W_\rho^- + \frac{g_L}{\sqrt{2}}\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right]W_\rho^+ + \text{h.c.}$$

$$-(\Box - m_W^2)W_\rho^- + \frac{g_L}{\sqrt{2}}\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right] = 0$$

$$W_\rho^- = \frac{g_L}{\sqrt{2}}(\Box - m_W^2)^{-1}\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right]$$

(Non-local) Effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{g_L^2}{2}\left[\bar{e}_L\gamma_\rho\nu_e + \bar{\mu}_L\gamma_\rho\nu_\mu\right](\Box - m_W^2)^{-1}\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right]$$

$$\frac{1}{\Box - m_W^2} = -\frac{1}{m_W^2} - \frac{\Box}{m_W^4} - \frac{\Box^2}{m_W^6} - \ldots$$

Leading (local) Effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = -\frac{g_L^2}{2m_W^2}\left[\bar{e}_L\gamma_\rho\nu_e + \bar{\mu}_L\gamma_\rho\nu_\mu\right]\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right] + \mathcal{O}\left(\frac{1}{m_W^4}\right)$$

$$-\frac{g_L^2}{2m_W^4}\left[\bar{e}_L\gamma_\rho\nu_e + \bar{\mu}_L\gamma_\rho\nu_\mu\right]\Box\left[\bar{\nu}_e\gamma_\rho e_L + \bar{\nu}_\mu\gamma_\rho\mu_L\right] + \ldots$$

Matching @ Loop-Level

The UV theory (could be UV EFT)

$$Z_{\rm UV}[J_\phi, J_H] = \int [D\phi][DH] \exp\left[i \int d^4x \left(\mathcal{L}_{\rm UV}(\phi, H) + J_\phi\phi + J_H H\right)\right]$$

$$\Gamma_{\rm UV}[\phi] = \int d^4x \mathcal{L}_{\rm UV}|_{H=H_c(\phi)} + \frac{i}{2}\log\det Q_{\rm UV} + \ldots$$

$$Q_{\rm UV} \equiv \begin{pmatrix} \Delta_H & X_{LH} \\ X_{LH} & \Delta_L \end{pmatrix} = \begin{pmatrix} -\frac{\delta^2\mathcal{L}_{\rm UV}}{\delta H^2} & -\frac{\delta^2\mathcal{L}_{\rm UV}}{\delta\phi\delta H} \\ -\frac{\delta^2\mathcal{L}_{\rm UV}}{\delta\phi\delta H} & -\frac{\delta^2\mathcal{L}_{\rm UV}}{\delta\phi^2} \end{pmatrix}\Bigg|_{H=H_c(\phi)}$$

$$\Gamma_{\rm UV}[\phi, 0] = \Gamma_{\rm EFT}[\phi]$$

$$\int d^4x \mathcal{L}^{(1)}_{\rm EFT} = \log\det Q_{\rm UV} - \log\det Q_{\rm EFT}$$

$$= \frac{i}{2}\log\det\left(\Delta_H - X_{LH}\Delta_L^{-1}X_{LH}\right)_{\rm hard}$$

Effective Field Theory

$$Z_{\rm EFT}[J_\phi] = \int [D\phi] \exp\left[i \int d^4x \left(\mathcal{L}_{\rm EFT}(\phi) + J_\phi\phi\right)\right]$$

$$Q_{\rm EFT} = -\frac{\delta^2\mathcal{L}_{\rm EFT}}{\delta\phi^2}\Big|_{\phi=\phi_b}$$

$$= \frac{\delta^2}{\delta\phi^2}\left(\hat{\mathcal{L}}_{\rm UV}(\phi, H_c(\phi))\right)$$

$$\phi = \phi_b + \phi'$$

$$= e^{i\int d^4x\left(\mathcal{L}_{\rm EFT}|_{\phi=\phi_b} + J_\phi\phi_b\right)} \int [D\phi'] \exp\left[-\frac{i}{2}\int d^4x \phi'^T Q_{\rm EFT}\phi'\right] + \ldots$$

$$\Gamma_{\rm EFT}[\phi] = \int d^4x \mathcal{L}_{\rm EFT} + \frac{i}{2}\log\det Q_{\rm EFT} + \ldots$$

Example: Real Scalar Theory

The UV theory (could be UV EFT)

$$\mathcal{L}_{\rm UV} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m_L^2\phi^2 + (\partial_\mu H)^2 - M^2 H^2\right] - \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2}M\phi^2 H - \frac{\lambda_2}{4}\phi^2 H^2$$

Integrate out heavy d.o.f

Matching two theories

Effective Field Theory

$$\mathscr{L}_{\rm EFT} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

Real Scalar EFT

EFT for a single real scalar with Z2 symmetry

$$\mathscr{L}_{\rm EFT} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

$$\hat{O}_6 \equiv (\Box\phi)^2, \quad \tilde{O}_6 \equiv \phi\Box\phi^3, \quad \tilde{O}_6' \equiv \phi^2\Box\phi^2, \quad \tilde{O}_6'' \equiv \phi^2\partial_\mu\phi\partial_\mu\phi, \quad \ldots$$

Use Leibniz rule + integration by parts:

$$\phi^2\partial_\mu\phi\partial_\mu\phi = -2\phi\partial_\mu\phi\partial_\mu\phi\phi - \phi^3\Box\phi \quad \Rightarrow \quad \tilde{O}_6'' = -\frac{1}{3}\phi^3\Box\phi = -\frac{1}{3}\tilde{O}_6$$

$$\phi^2\Box\phi^2 = 2\phi^2\partial_\mu(\phi\partial_\mu\phi) = 2\phi^3\Box\phi + 2\phi^2(\partial_\mu\phi)^2 \quad \Rightarrow \quad \tilde{O}_6' = 2\tilde{O}_6 + 2\tilde{O}_6'' = \frac{4}{3}\tilde{O}_6$$

Use equations of motion: $\Box\phi = -m^2\phi - \frac{C_4}{6}\phi^3 + \mathcal{O}(\Lambda^{-2})$

$$\tilde{O}_6 \equiv \phi^3\Box\phi = -m^2\phi^4 - \frac{C_4}{6}\phi^6 = -m^2 O_4 - \frac{C_4}{6}O_6$$

$$\hat{O}_6 \equiv (\Box\phi)^2 = m^4\phi^2 + \frac{m^2C_4}{3}\phi^4 + \frac{C_4^2}{36}\phi^6 = m^4 O_2 + \frac{m^2C_4}{3}O_4 + \frac{C_4^2}{36}O_6$$

Equivalent Lagrangian

$$\mathscr{L}_{\rm EFT} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - \tilde{C}_4\frac{\phi^4}{4!} - \frac{\tilde{C}_6}{\Lambda^2}\frac{\phi^3\Box\phi}{4!} + \mathcal{O}(\Lambda^{-4})$$

$$\tilde{C}_6 = -\frac{C_6}{5C_4}$$

$$\tilde{C}_4 = C_4 - \frac{m^2}{\Lambda^2}\frac{C_6}{5C_4}$$

Operator Bases

Consider 2 to 2 scattering in the two kinds of Lagrangian

$$\mathscr{L}_{\text{EFT}} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - C_4\frac{\phi^4}{4!} - \frac{C_6}{\Lambda^2}\frac{\phi^6}{6!} + \mathcal{O}(\Lambda^{-4})$$

$$\mathscr{L}_{\text{EFT}} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right] - \tilde{C}_4\frac{\phi^4}{4!} - \frac{\tilde{C}_6}{\Lambda^2}\frac{\phi^3\Box\phi}{4!} + \mathcal{O}(\Lambda^{-4})$$

$$\mathcal{M}^{\text{unbox}}_{EFT} = -C_4 + \mathcal{O}(\Lambda^{-4})$$

$$\begin{aligned}\mathcal{M}^{\text{box}}_{EFT} &= -\tilde{C}_4 + \frac{\tilde{C}_6}{4\Lambda^2}(p_1^2 + p_2^2 + p_3^2 + p_4^2) + \mathcal{O}(\Lambda^{-4}) \\ &= -\tilde{C}_4 + \tilde{C}_6\frac{m^2}{\Lambda^2} + \mathcal{O}(\Lambda^{-4})\end{aligned}$$

$$\tilde{C}_6 = -\frac{C_6}{5C_4}$$

$$\tilde{C}_4 = C_4 - \frac{m^2}{\Lambda^2}\frac{C_6}{5C_4}$$

$$\mathcal{M}^{\text{unbox}}_{EFT} = \mathcal{M}^{\text{box}}_{EFT} + \mathcal{O}(\Lambda^{-4})$$

Origin: field redefinition!

Field Redefinition

With the field redefinition, the two Lagrangian gives the same S-matrix

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda^2}\phi^3\partial^2\phi + \frac{c_6}{\Lambda^2}\phi^6 + \ldots .$$

$$\phi \to \phi + \frac{c_1}{\Lambda^2}\phi^3$$

$$\begin{aligned}\mathscr{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + \frac{c_1}{\Lambda^2}\phi^3\partial^2\phi + \frac{c_6}{\Lambda^2}\phi^6 \\ &\quad + \frac{c_1}{\Lambda^2}\phi^3\left[-\partial^2\phi - m^2\phi - \frac{\lambda}{3!}\phi^3\right] + \ldots \\ &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \left[\frac{1}{4!}\lambda + \frac{c_1}{\Lambda^2}m^2\right]\phi^4 + \left[\frac{c_6}{\Lambda^2} - \frac{c_1}{\Lambda^2}\frac{\lambda}{3!}\right]\phi^6 + \ldots .\end{aligned}$$

EOM (Field redefinition) can be applied beyond the tree-level

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}O_i = \gamma_{ij}O_j + \Gamma_{ik}E_k$$

It is often stated that the use of the classical equations of motion to eliminate operators from the basis is not justified beyond tree level. This statement is false!

Field Redefinition

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i \exp\left(i \int d^4x \left[\mathcal{L}_0 + \eta \mathcal{L}_1 + \sum_i j_i \varphi_i + \mathcal{O}(\eta^2)\right]\right)$$

$$\phi^\dagger = (\phi')^\dagger + \eta T[\varphi]$$

$T[\varphi]$ is any local function of any of the fields φ

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i' \left|\frac{\delta \phi^\dagger}{\delta(\phi')^\dagger}\right| \exp\left(i \int d^4x \left[\mathcal{L}_0' + \delta\mathcal{L}_0' + \eta \mathcal{L}_1' + \eta \delta \mathcal{L}_1' + \sum_i j_i \varphi_i + j_{\phi^\dagger} \eta T + \mathcal{O}(\eta^2)\right]\right)$$

$$\mathcal{L}_i' \equiv \mathcal{L}_i\left((\phi')^\dagger, \partial_\mu (\phi')^\dagger\right) \qquad \delta\phi^\dagger \equiv \phi^\dagger - (\phi')^\dagger = \eta T[\varphi]$$

$$\delta \mathcal{L}_i' = \frac{\delta \mathcal{L}_i'}{\delta(\phi')^\dagger}\delta\phi^\dagger - \frac{\delta \mathcal{L}_i'}{\delta \partial_\mu (\phi')^\dagger}\delta \partial_\mu \phi^\dagger = \left(\frac{\delta \mathcal{L}_i'}{\delta(\phi')^\dagger} - \partial_\mu \frac{\delta \mathcal{L}_i'}{\delta \partial_\mu (\phi')^\dagger}\right)\delta\phi^\dagger = \left(\frac{\delta \mathcal{L}_i'}{\delta(\phi')^\dagger} - \partial_\mu \frac{\delta \mathcal{L}_i'}{\delta \partial_\mu (\phi')^\dagger}\right)\eta T[\varphi]$$

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i' \left|\frac{\delta \phi^\dagger}{\delta(\phi')^\dagger}\right| \exp\left(i \int d^4x \left[\mathcal{L}_0' + \left(\frac{\delta \mathcal{L}_0'}{\delta(\phi')^\dagger} - \partial_\mu \frac{\delta \mathcal{L}_0'}{\delta \partial_\mu (\phi')^\dagger}\right)\eta T[\varphi] + \eta \mathcal{L}_1' + \sum_i j_i \varphi_i + j_{\phi^\dagger}\eta T + \mathcal{O}(\eta^2)\right]\right)$$

the source term and the Jacobian can be neglected

Gaussian theorem on action

Equation of Motion (EOM)

Two equivalent operators related by EOM

Integration by part (IBP) $\partial_\mu \mathcal{O}^\mu$

Total derivatives are removed

Exercise: Independent Operator

How many independent operators of the form $\partial^{2n}\phi^4$?

$$\partial^{2n}\phi^2,\ \partial^{2n}\phi^3,\ \partial^{2n}\phi^4,\ \cdots$$

$2n$	0	2	4	6	8	10	12	14	16
# independent $\partial^{2n}\phi^4$ operators	1	0	1	1	1	1	2	1	2

1

$s+t+u=0$

$s^2+t^2+u^2$

$s^3+t^3+u^3 \sim stu$

$(s^2+t^2+u^2)^2$

$stu\,(s^2+t^2+u^2)$

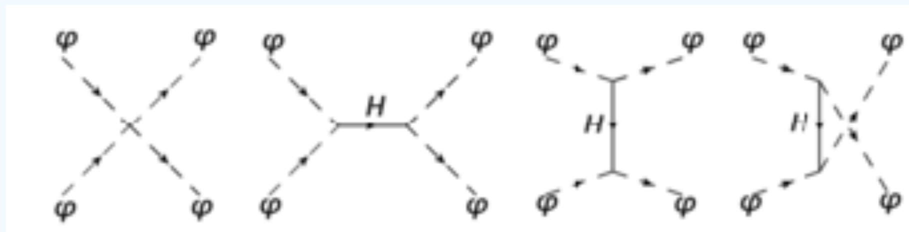
$(s^2+t^2+u^2)^3$ & $(stu)^2$

$stu\,(s^2+t^2+u^2)^2$

$(s^2+t^2+u^2)^4$ & $(stu)^2(s^2+t^2+u^2)$

Magic things happen?!

Matching @ Tree-Level



$$= -\lambda_0 - \lambda_1^2 M^2 \left[\frac{1}{s-M^2} + \frac{1}{t-M^2} + \frac{1}{u-M^2}\right]$$
$$\approx -\lambda_0 + 3\lambda_1^2 + \frac{\lambda_1^2}{M^2}(s+t+u) + \mathcal{O}(M^{-4})$$
$$\approx -\lambda_0 + 3\lambda_1^2 + \frac{4m_L^2\lambda_1^2}{M^2} + \mathcal{O}(M^{-4})$$

$$m^2 = m_L^2.$$

$$= -C_4$$

$$= -\tilde{C}_4 + \frac{m^2}{M^2}\tilde{C}_6$$

$$C_4 = \lambda_0 - 3\lambda_1^2 - 4\lambda_1^2\frac{m_L^2}{M^2}$$

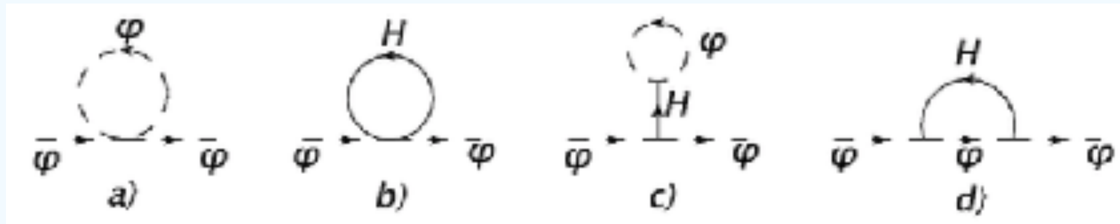
$$\tilde{C}_4 - \frac{m^2}{M^2}\tilde{C}_6 = \lambda_0 - 3\lambda_1^2 - 4\lambda_1^2\frac{m_L^2}{M^2}$$

Obtain C6 via calculating 6-point function at tree-level

$$\tilde{C}_4 = \lambda_0 - 3\lambda_1^2 - \frac{9m_L^2}{M^2}\frac{\lambda_1^2\lambda_2}{\lambda_0 - 3\lambda_1^2},$$
$$\tilde{C}_6 = 4\lambda_1^2 - 9\frac{\lambda_1^2\lambda_2}{\lambda_0 - 3\lambda_1^2}.$$

$$C_6 = 45\lambda_1^2\lambda_2 - 20\lambda_0\lambda_1^2 + 60\lambda_1^4$$

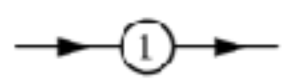
2-Point Matching @ One-Loop



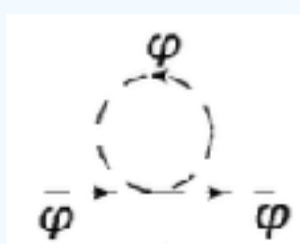
$$= \lambda_0 \frac{m_L^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m_L^2}\right) + 1\right] \qquad \lambda_2 \frac{M^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{M^2}\right) + 1\right] \qquad + \lambda_1^2 \frac{m_L^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m_L^2}\right) + 1\right]$$

$$\lambda_1^2 \frac{M^2}{16\pi^2}\left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{M^2}\right) + 1\right] + \lambda_1^2 \frac{m_L^2}{32\pi^2}\left[-2\log\left(\frac{M^2}{m_L^2}\right) + 1\right] + \lambda_1^2 \frac{m_L^4}{48\pi^2 M^2}\left[-6\log\left(\frac{M^2}{m_L^2}\right) + 5\right].$$

UV and EFT UV div. different!



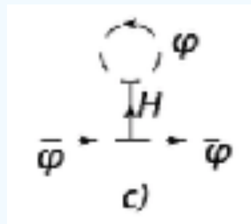
$$= \left[\begin{array}{c}\text{1-loop} \\ \text{full theory}\end{array}\right]_{\overline{MS}} - \left[\begin{array}{c}\text{1-loop} \\ \text{EFT}\end{array}\right]_{\overline{MS}}$$



$$= C_4 \frac{m^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right]$$

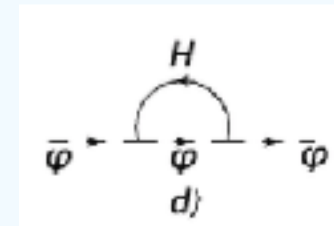
UV and EFT IR log same!

2-Point Matching @ One-Loop



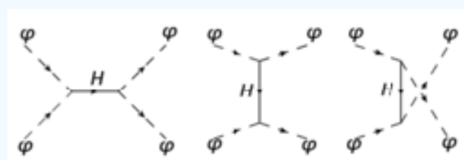
$$= (-i)(-i\lambda_1 M)^2 \frac{1}{0^2 - M^2} \int \frac{d^dk}{(2\pi)^d} \frac{i}{k^2 - m^2}$$

$$= -\lambda_1^2 \frac{m_L^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m_L^2}\right) + 1\right]$$



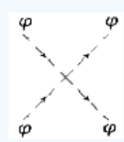
$$= (-i)(-i\lambda_1 M)^2 \int \frac{d^dk}{(2\pi)^d} \frac{i^2}{(k^2 - M^2)((k+p)^2 - m_L^2)}$$

$$\lambda_1^2 \frac{M^2}{16\pi^2}\left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{M^2}\right) + 1\right] - \lambda_1^2 \frac{m_L^2}{32\pi^2}\left[-2\log\left(\frac{M^2}{m_L^2}\right) + 1\right] + \lambda_1^2 \frac{m_L^4}{48\pi^2 M^2}\left[-6\log\left(\frac{M^2}{m_L^2}\right) + 5\right]$$

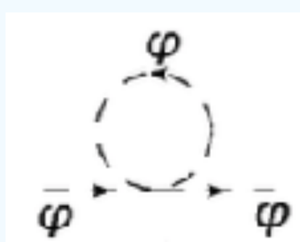


First integration and then expansion

$$\frac{1}{k^2 - M^2} = -\frac{1}{M^2}\left(1 + \frac{k^2}{M^2} + \frac{k^4}{M^4} + \ldots\right)$$



First expansion and then integration



$$= (-i)\frac{-iC_4}{2} \int \frac{d^dk}{(2\pi)^d} \frac{i}{k^2 - m^2}$$

$$= C_4 \frac{m^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right]$$

$$C_4 = \lambda_0 - 3\lambda_1^2 - 4\lambda_1^2 \frac{m_L^2}{M^2}$$

Two results are different: Integration and expansion can not be exchanged!

Non-analytic Log M term does not present in EFT calculation (first expansion)!

Method of Region

$$I_0 = \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2 - m^2} \frac{1}{k^2 - M^2}$$

First integration and then expansion

$$I_0 = \frac{i}{16\pi^2}\left[\frac{1}{\bar{\epsilon}} + \frac{m^2}{M^2 - m^2}\log\left(\frac{m^2}{M^2}\right) + \log\left(\frac{\mu^2}{M^2}\right) + 1\right]$$

$$\frac{1}{k^2 - M^2} = -\frac{1}{M^2}\left(1 + \frac{k^2}{M^2} + \frac{k^4}{M^4} + \ldots\right)$$

$$= \frac{i}{16\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{M^2}\right) + 1 + \frac{m^2}{M^2}\log\left(\frac{m^2}{M^2}\right)\right] + \mathcal{O}(M^{-4})$$

Log m/M

First expansion and then integration

$$k^2 \sim m^2 \ll M^2 \qquad \frac{1}{k^2 - M^2} = -\frac{1}{M^2}\left(1 + \frac{k^2}{M^2} + \frac{k^4}{M^4} + \ldots\right)$$

Expand high energy integrand around low energy limit

$$I_{\rm soft} = -\frac{1}{M^2}\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2 - m^2} + \mathcal{O}(M^{-4})$$
$$= -\frac{i}{16\pi^2}\frac{m^2}{M^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right] + \mathcal{O}(M^{-4})$$

Soft region **Log m**

Exact seperation:

$$I_0 = I_{\rm soft} + I_{\rm hard}$$

$k \sim m$ $k \sim M$

Full theory calculation can be separated into two parts

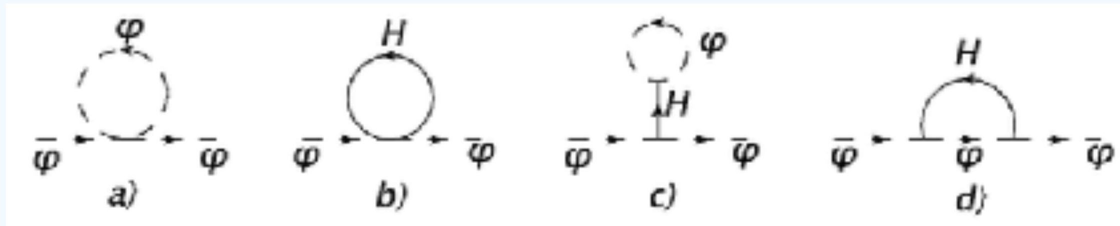
$$k^2 \sim M^2 \gg m^2 \qquad \frac{1}{k^2 - m^2} \sim \frac{1}{k^2} + \frac{m^2}{k^4} + \ldots$$

Expand low energy integrand around high energy limit

$$I_{\rm hard} = \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2}\frac{1}{k^2 - M^2} + m^2\int \frac{d^dk}{(2\pi)^d}\frac{1}{k^4}\frac{1}{k^2 - M^2} + \mathcal{O}(M^{-4})$$
$$= \frac{i}{16\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{M^2}\right) + 1\right] + \frac{i}{16\pi^2}\frac{m^2}{M^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{M^2}\right) + 1\right] + \mathcal{O}(M^{-4}).$$

Hard region **Log M**

2-Point Matching @ One-Loop

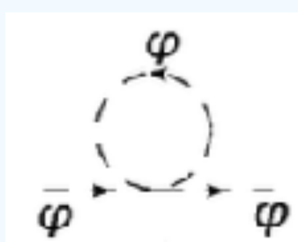


$$= \lambda_0 \frac{m_L^2}{32\pi^2}\left[\frac{1}{\bar\epsilon} + \log\left(\frac{\mu^2}{m_L^2}\right) + 1\right] \qquad \lambda_2 \frac{M^2}{32\pi^2}\left[\frac{1}{\bar\epsilon} + \log\left(\frac{\mu^2}{M^2}\right) + 1\right] \qquad -\lambda_1^2 \frac{m_L^2}{32\pi^2}\left[\frac{1}{\bar\epsilon} + \log\left(\frac{\mu^2}{m_L^2}\right) + 1\right]$$

$$\lambda_1^2 \frac{M^2}{16\pi^2}\left[\frac{1}{\epsilon} + \log\left(\frac{\mu^2}{M^2}\right) + 1\right] + \lambda_1^2 \frac{m_L^2}{32\pi^2}\left[-2\log\left(\frac{M^2}{m_L^2}\right) + 1\right] + \lambda_1^2 \frac{m_L^4}{48\pi^2 M^2}\left[-6\log\left(\frac{M^2}{m_L^2}\right) + 5\right].$$

$$m^2(\mu) = m_L^2(\mu) - \frac{1}{32\pi^2}\log\left(\frac{\mu^2}{M^2}\right)\left[M^2\left(\lambda_2 + 2\lambda_1^2\right) + 2\lambda_1^2 m_L^2 + 4\lambda_1^2 \frac{m_L^4}{M^2}\right] - \frac{1}{32\pi^2}\left[M^2\left(\lambda_2 + 2\lambda_1^2\right) + 3\lambda_1^2 m_L^2 + \frac{22}{3}\lambda_1^2 \frac{m_L^4}{M^2}\right]$$

$$m^2(M) = m_L^2(M) - \frac{1}{32\pi^2}\left[M^2\left(\lambda_2 + 2\lambda_1^2\right) + 3\lambda_1^2 m_L^2 + \frac{22}{3}\lambda_1^2 \frac{m_L^4}{M^2}\right]$$



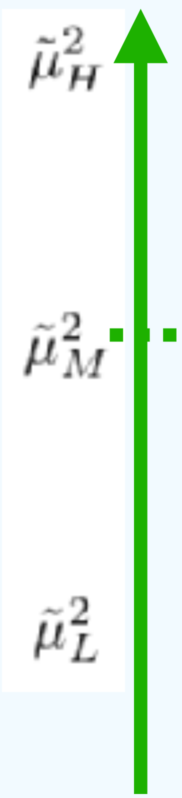
$$= C_4 \frac{m^2}{32\pi^2}\left[\frac{1}{\bar\epsilon} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right]$$

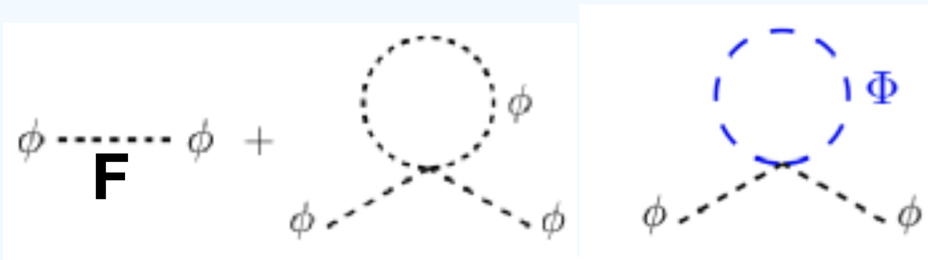
$$\frac{dm^2}{d\log\mu} = \frac{m^2 C_4}{16\pi^2} \qquad m^2(\mu) = m^2(M)\left(\frac{\mu}{M}\right)^{\frac{C_4}{16\pi^2}} \approx m^2(M)\left[1 + \frac{C_4}{16\pi^2}\log\left(\frac{\mu}{M}\right)\right]$$

Only contains suppressed m/M (log m cancelled out during matching)

UV log M are absorbed in redefinition of Wilson coefficients

Hierarchy Problem





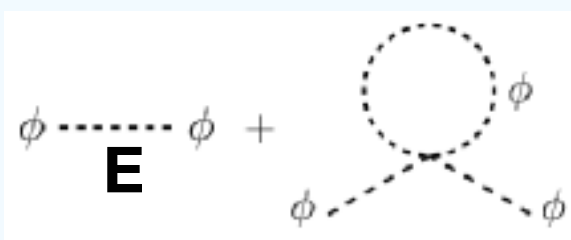
$$\frac{i\kappa}{32\pi^2} M^2 \left[\frac{1}{\epsilon} + \log\frac{\tilde{\mu}_M^2}{M^2} + 1 + \mathcal{O}(\epsilon)\right]$$

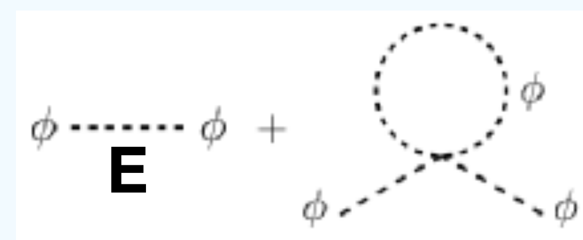
$$-\frac{iy^2}{4\pi^2}\left\{\frac{1}{\varepsilon}(p^2+2M^2) + \frac{p^2}{2}\log\left(\frac{\mu^2}{M^2}\right) + M^2\left[1+\log\left(\frac{\mu^2}{M^2}\right)\right]\right\}$$

(1) matching

$$m_{\rm E}^2(\mu) = m_{\rm F}^2(\mu) - \frac{\kappa}{32\pi^2}M^2\left[\log\frac{\mu^2}{M^2}+1\right]$$

$$m_E^2(\mu) = m_F^2(\mu) + \frac{y^2}{4\pi^2}M^2\left[1+\log\left(\frac{\mu^2}{M^2}\right)\right]$$





(2) running down to low scale

$$m_{\rm E}^2(\mu) = m_{\rm E}^2(M) + \frac{C_4}{32\pi^2} m_{\rm E}^2 \log\frac{\mu^2}{M^2}$$

(3) Hierarchy problem: matching at M

SUSY!

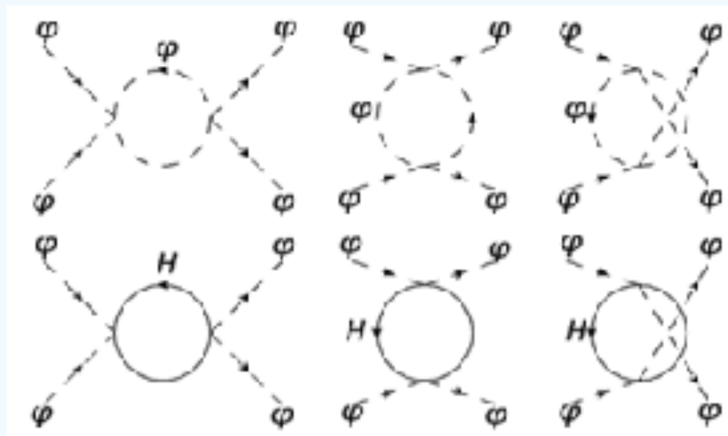
$$m_{\rm F}^2(M) - \frac{\kappa}{32\pi^2}M^2 = m_{\rm E}^2(m) - \frac{C_4}{32\pi^2}m_{\rm E}^2\left(1+\log\frac{M^2}{m_{\rm E}^2}\right) = m_F^2(M) + \frac{y^2}{4\pi^2}M^2$$

$$y^2 = \frac{\kappa}{8}$$

$$m_{\rm E}^2(M) = m_{\rm E}^2(\mu) - \frac{C_4}{32\pi^2} m_{\rm E}^2\log\frac{\mu^2}{M^2}$$

4-Point Matching @ One-Loop

in the limit $\lambda_1 = 0$.

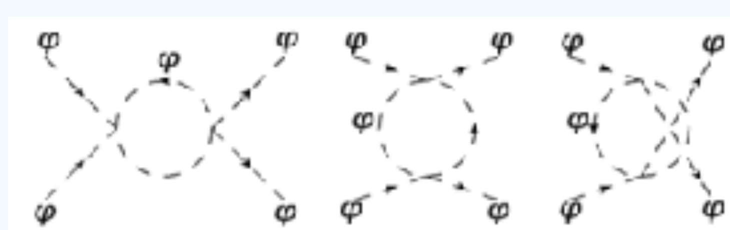


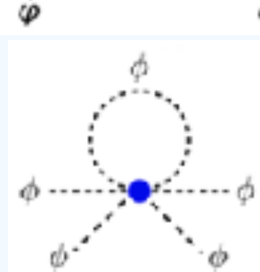
$$= -\lambda_0 + \frac{3\lambda_0^2}{32\pi^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 2\right) + \frac{3\lambda_2^2}{32\pi^2}\left(\frac{1}{\bar{\epsilon}} - \log\left(\frac{\mu^2}{M^2}\right) + 2\right)$$
$$+ \frac{\lambda_0^2}{32\pi^2}\left[f(s,m) + f(t,m) + f(u,m)\right] - \frac{\lambda_2^2}{32\pi^2}\left[f(s,M) + f(t,M) + f(u,M)\right]$$

$$f(s,m) = \sqrt{1 - \frac{4m^2}{s}}\log\left(\frac{2m^2 - s + \sqrt{s(s-m^2)}}{2m^2}\right)$$

Scattering amplitude from UV Lag has large log term

UV and EFT UV div. different!



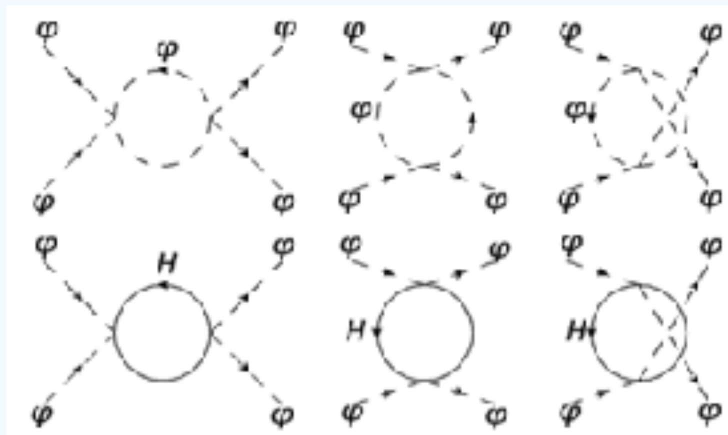


$$= -C_4 + \frac{C_4^2}{32\pi^2}\left[f(s,m) + f(t,m) + f(u,m)\right]$$
$$+ \frac{3C_4^2}{32\pi^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 2\right) + \frac{C_6 m^2}{32\pi^2 M^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right)$$

UV and EFT IR log same!

4-Point Matching @ One-Loop

in the limit $\lambda_1 = 0$.

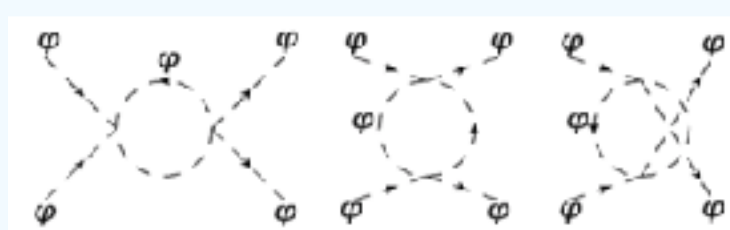


$$= -\lambda_0 + \frac{3\lambda_0^2}{32\pi^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 2\right) + \frac{3\lambda_2^2}{32\pi^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{M^2}\right) + 2\right)$$
$$+ \frac{\lambda_0^2}{32\pi^2}\left[f(s,m) + f(t,m) + f(u,m)\right] + \frac{\lambda_2^2}{32\pi^2}\left[f(s,M) + f(t,M) + f(u,M)\right]$$

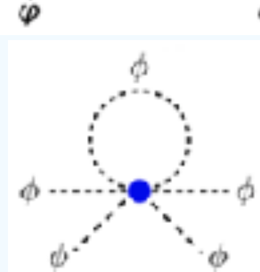
$$f(s,m) \equiv \sqrt{1 - \frac{4m^2}{s}} \log\left(\frac{2m^2 - s + \sqrt{s(s-m^2)}}{2m^2}\right)$$

$$C_4 = \lambda_0 - \frac{3\lambda_2^2}{32\pi^2}\log\left(\frac{\mu^2}{M^2}\right) - \frac{\lambda_2^2 m^2}{48\pi^2 M^2}$$

$$C_4(M) = \lambda_0(M) - \frac{\lambda_2^2 m^2}{48\pi^2 M^2}$$



$$= -C_4 + \frac{C_4^2}{32\pi^2}\left[f(s,m) + f(t,m) + f(u,m)\right]$$
$$+ \frac{3C_4^2}{32\pi^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 2\right) + \frac{C_6 m^2}{32\pi^2 M^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right)$$

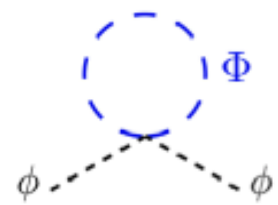


$$\frac{dC_4}{d\log\mu} = \frac{3}{16\pi^2}C_4^2 + \frac{m^2}{16\pi^2 M^2}C_6$$

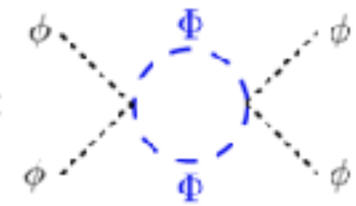
$$C_4(\mu) \approx C_4(M) + \frac{3C_4^2}{16\pi^2}\log\left(\frac{\mu}{M}\right) + \frac{C_6 m^2}{16\pi^2 M^2}\log\left(\frac{\mu}{M}\right)$$

Large Log needs running

Running for UV and EFT

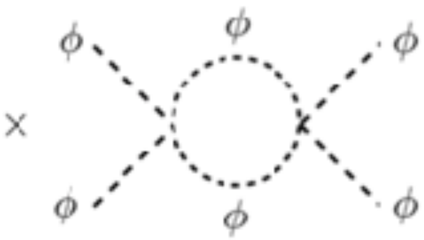


$$= \frac{i\kappa}{32\pi^2} M^2 \left[\frac{1}{\epsilon} + \log\frac{\tilde{\mu}_M^2}{M^2} + 1 + \mathcal{O}(\epsilon)\right]$$

3 ×


$$= \frac{3i}{32\pi^2}\mu^{2\epsilon}\kappa^2\left(\frac{1}{\epsilon} + \log\frac{\tilde{\mu}^2}{M^2}\right) + \mathcal{O}(\lambda^2)$$

$$= \frac{i\eta}{32\pi^2} m_{\rm F}^2 \left[\frac{1}{\epsilon} + \log\frac{\tilde{\mu}_M^2}{m_{\rm F}^2} + 1 + \mathcal{O}(\epsilon)\right]$$

3 ×


$$= \frac{3i}{32\pi^2}\mu^{2\epsilon}\eta^2\left(\frac{1}{\epsilon} + \log\frac{\tilde{\mu}^2}{m^2} + \frac{2}{3}\right)$$

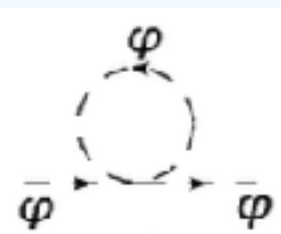
Running up from matching scale

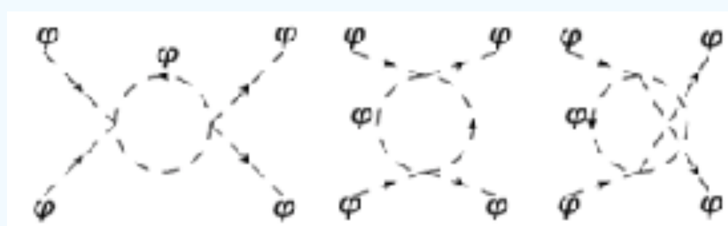
$$\frac{\mathrm{d}}{\mathrm{d}\log\tilde{\mu}^2} m_{\rm F}^2 = \gamma_{m_{\rm F}^2} m_{\rm F}^2$$

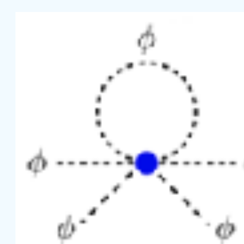
$$\gamma_{m_{\rm F}^2} = \frac{\eta}{32\pi^2} + \frac{\kappa}{32\pi^2}\frac{M^2}{m_{\rm F}^2}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}\log\tilde{\mu}^2} = \frac{3}{32\pi^2}\eta^2 + \frac{3}{32\pi^2}\kappa^2$$

$$\frac{\mathrm{d}\kappa}{\mathrm{d}\log\tilde{\mu}^2} = \frac{1}{8\pi^2}\kappa^2 + \frac{1}{32\pi^2}\kappa\,\eta$$







Running down from matching scale

$$\frac{dm^2}{d\log\mu} = \frac{m^2 C_4}{16\pi^2}$$

$$\frac{dC_4}{d\log\mu} = \frac{3}{16\pi^2}C_4^2 + \frac{m^2}{16\pi^2 M^2}C_6$$

UV Non-analytic Log

Calculate the 2 to 2 scattering amplitude using UV theory at low energy scale E

$$i\mathcal{A}^{\text{Full}} = -i\lambda_0(\mu) + i\frac{3}{32\pi^2}(\lambda_0(\mu))^2\left(\log\frac{\mu^2}{m^2}+\frac{2}{3}\right) + i\frac{3}{32\pi^2}(\lambda_2(\mu))^2\log\frac{\mu^2}{M^2}$$

Large log problem!

1. Perform matching by expressing the EFT parameters using those of UV theory

$$m^2(M) = m_L^2(M) - \frac{1}{32\pi^2}\left[M^2\left(\lambda_2+2\lambda_1^2\right)+3\lambda_1^2 m_L^2+\frac{22}{3}\lambda_1^2\frac{m_L^4}{M^2}\right]$$

$$C_4(M) = \lambda_0(M) - \frac{\lambda_2^2 m^2}{48\pi^2 M^2}$$

2. Evolve the Wilson coefficients down to process energy E using EFT RGE

$$\frac{dm^2}{d\log\mu} = \frac{m^2 C_4}{16\pi^2}$$

$$\frac{dC_4}{d\log\mu} = \frac{3}{16\pi^2}C_4^2 + \frac{m^2}{16\pi^2 M^2}C_6$$

$$m^2(\mu) = m^2(M)\left(\frac{\mu}{M}\right)^{\frac{C_4}{16\pi^2}} \approx m^2(M)\left[1+\frac{C_4}{16\pi^2}\log\left(\frac{\mu}{M}\right)\right]$$

$$C_4(\mu) \approx C_4(M) + \frac{3C_4^2}{16\pi^2}\log\left(\frac{\mu}{M}\right) + \frac{C_6 m^2}{16\pi^2 M^2}\log\left(\frac{\mu}{M}\right)$$

3. Calculate the amplitude using the EFT Lagrangian with parameters at scale E

$$i\mathcal{A}^{\text{EFT}} = -iC_4(\tilde{\mu}_L) + i\frac{3}{32\pi^2}(C_4(\tilde{\mu}_L))^2\left(\log\frac{\tilde{\mu}_L^2}{m^2}+\frac{2}{3}\right) + \frac{iC_6}{32\pi^2}\frac{m^2}{M^2}\left[\log\frac{\tilde{\mu}_L^2}{m^2}+1\right]$$

$$= -i\lambda_0(\tilde{\mu}_H) + \frac{3i}{32\pi^2}\left[\lambda_0^2\log\frac{\tilde{\mu}_H^2}{\tilde{\mu}_M^2} + C_4^2\left(\log\frac{\tilde{\mu}_M^2}{m^2}+\frac{2}{3}\right)\right] + \frac{3i}{32\pi^2}\lambda_2^2\log\frac{\tilde{\mu}_H^2}{M^2}$$

Recover the UV (with UV RGE)

Resummation Procedure

Separate two scale problem to two one-scale problems

$$\underbrace{\log\frac{m^2}{M^2}}_{\text{UV}} = \underbrace{-\log\frac{M^2}{\bar{\mu}^2}}_{\text{matching}} + \underbrace{\log\frac{m^2}{\bar{\mu}^2}}_{\text{EFT}}$$

Avoid large logs

Step 1: Determine the Wilson coefficients at a high scale $\mu \approx M$, where they are free of large logarithms

$I_M(\bar{\mu} \sim M)$ $\quad C_t(\mu_t)$

$\bar{\mu}$ changes with RGE $\quad \mu \frac{d}{d\mu} C_t = \gamma_t(\alpha_s)\, C_t$

Step 2: Evolve the Wilson coefficients to a low scale $\mu \approx E$, which is characteristic for the observable at hand

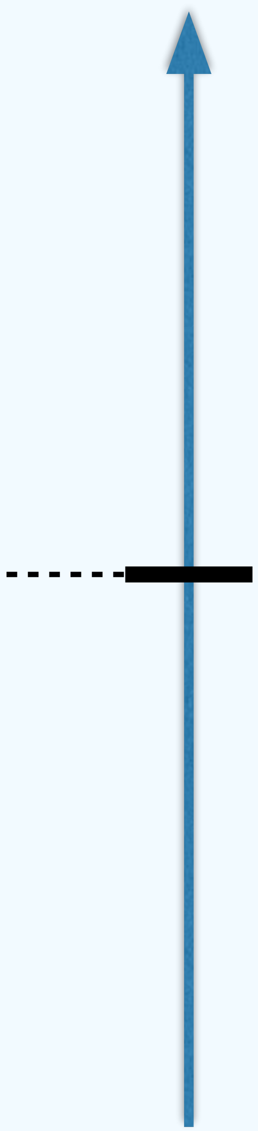
$I_{\text{EFT}}(\bar{\mu} \sim m)$ $\quad C_t(\mu_l) = C_t(\mu_t)\, U(\mu_t, \mu_l)$

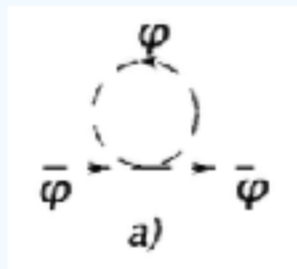
Step 3: Evaluate the matrix elements of the EFT operators at the scale $\mu \approx E$, where they are free of large logarithms

UV Non-analytic log M absorbed into Wilson coeff. at matching scale

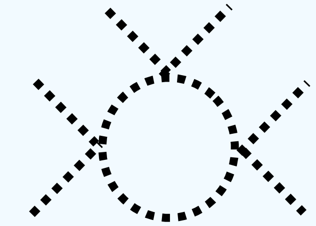
IR Non-analytic Log

Effects of light d.o.f (soft region) at UV and EFT

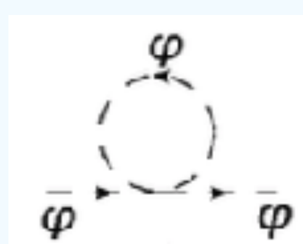




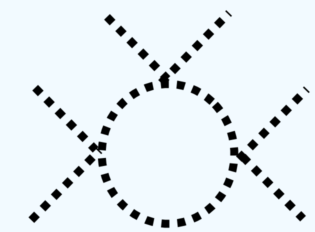
$$= \lambda_0 \frac{m_L^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m_L^2}\right) + 1\right]$$



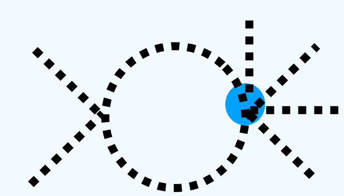
$$= -\lambda_0 + \frac{3\lambda_0^2}{32\pi^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 2\right) + \frac{\lambda_0^2}{32\pi^2}\left[f(s,m) + f(t,m) + f(u,m)\right]$$



$$= C_4 \frac{m^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 1\right]$$



$$= -C_4 + \frac{C_4^2}{32\pi^2}\left[f(s,m) + f(t,m) + f(u,m)\right] + \frac{3C_4^2}{32\pi^2}\left(\frac{1}{\bar{\epsilon}} + \log\left(\frac{\mu^2}{m^2}\right) + 2\right)$$



Light d.o.f effects (IR non-analytic log m) cancel out during matching!

(Only running effects kept)

$C_4 = \lambda_0$

Simplified Matching Procedure

Non-analytic IR log m dropped out during matching

$$I_M = [I_F + I_{F,\text{c.t.}}] - [I_{\text{EFT}} + I_{\text{EFT,c.t.}}]$$
$$= \frac{ig^2}{16\pi^2}\left[\left(\log\frac{\bar{\mu}^2}{M^2}+1\right) + \frac{m^2}{M^2}\left(\log\frac{\bar{\mu}^2}{M^2}+1\right)+\ldots\right]$$

$$\underbrace{I_M(m)}_{\text{analytic}} = \underbrace{I_F(m)}_{\text{non-analytic}} - \underbrace{I_{\text{EFT}}(m)}_{\text{non-analytic}}$$

Only polynomial m/M kept

We can perform a simplified calculation if only matching is needed

$k \gg m,$ $\quad k^2 \sim M^2 \gg m^2$

$$I_F^{(\exp)} = g^2\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2-M^2}\left[\frac{1}{k^2}+\frac{m^2}{k^4}+\ldots\right]$$

$$\frac{1}{k^2-m^2} \sim \frac{1}{k^2}+\frac{m^2}{k^4}+\ldots$$

Hard region for UV

$k \gg m, k \ll M$ Hard region for EFT

$$I_{\text{EFT}}^{(\exp)} = g^2\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\left[\frac{1}{k^2}+\frac{m^2}{k^4}+\ldots\right]\left[-\frac{1}{M^2}-\frac{k^2}{M^4}-\ldots\right]$$

$$I_M = I_F^{(\exp)} - I_{\text{EFT}}^{(\exp)}$$

No need to calculate soft region

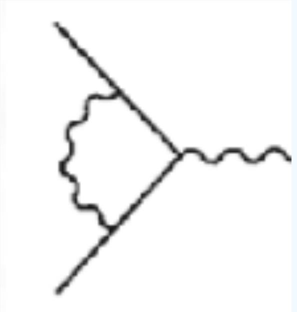
Back to QED Theory

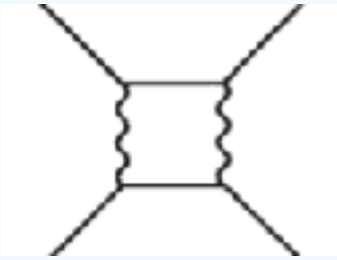
Start with QED theory in QFT course

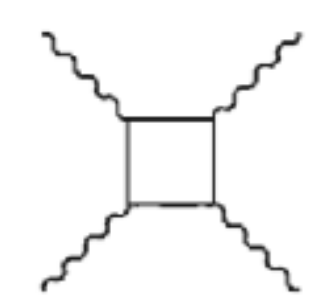
$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}\gamma^\mu\psi A_\mu$$

$$+ \frac{c_5}{\Lambda}\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu} + \frac{c_6}{\Lambda^2}(\bar{\psi}\psi)^2 + \frac{c_8}{\Lambda^4}(F_{\mu\nu}F^{\mu\nu})^2 + \cdots$$

Generate such terms from renormalizible QED?







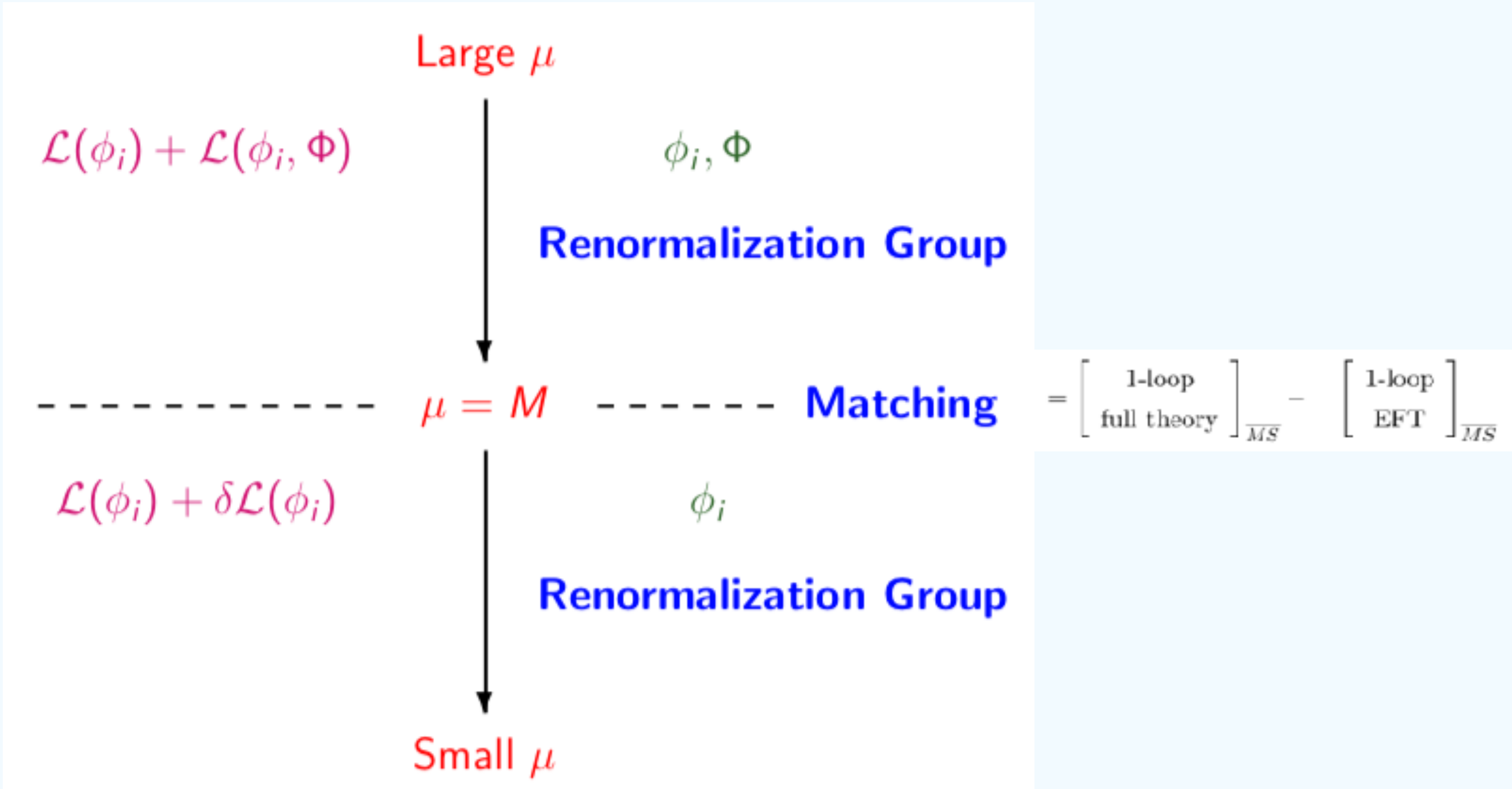
The same IR behavior for both UV and IR theory

Only if beyond QED physics (EW, SUSY, etc) considered, generate such operators

If no heavy particles (heavy particle masses infinity heavy)

These diagrams would cancel at matching, so no EFT operators generated

Decoupling Theorem



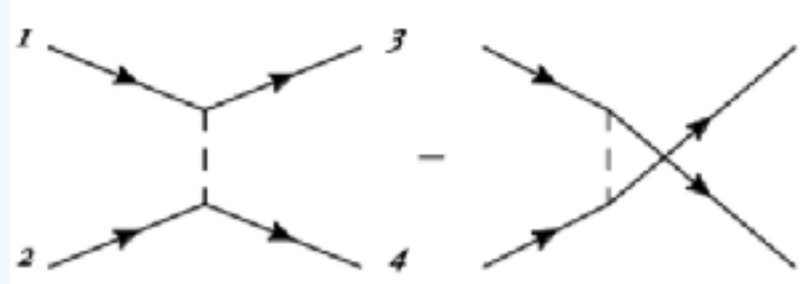
Decoupling: Appelquist–Carazzone

The low-energy effects of heavy particles are either suppressed by inverse powers of the heavy masses, or they get absorbed into renormalizations of the couplings and fields of the EFT obtained by removing the heavy particles

4-Fermi EFT

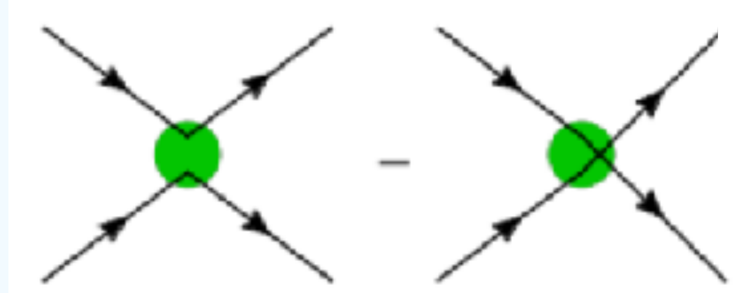
$$\mathcal{L}_{\rm Full} = i\bar\psi\partial\!\!\!/\psi - \sigma\bar\psi\psi + \frac{1}{2}(D_\mu\Phi)^2 - \frac{1}{2}M^2\Phi^2 - \lambda\Phi\bar\psi\psi$$

Heavy mass scale (M^2) Yukawa interaction ($\lambda\Phi\bar\psi\psi$)



Full:

$$\mathcal{M} = \bar u(p_3)(-i\lambda)u(p_1)\bar u(p_4)(-i\lambda)u(p_2)\left[\frac{i}{(p_1-p_3)^2-M^2}\right] - (3\leftrightarrow 4)$$



$$(-i\lambda)^2\frac{i}{(p_3-p_1)^2-M^2} = i\frac{\lambda^2}{M^2}\frac{1}{1-\frac{(p_3-p_1)^2}{M^2}} \approx i\frac{\lambda^2}{M^2}\left(1+\frac{(p_3-p_1)^2}{M^2}+\mathcal{O}(\frac{p^4}{M^4})\right)$$

EFT

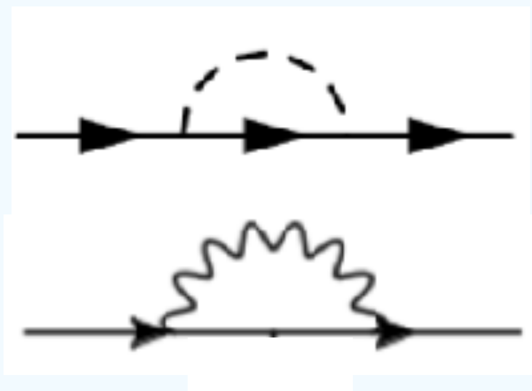
$$\mathcal{M} = \bar u(p_3)u(p_1)\bar u(p_4)u(p_2)\left[\frac{ic_S}{M^2} + \left(\frac{-ic^{(8)}}{M^4}\right)(p_1\cdot p_3 + p_2\cdot p_4)\right] - (3\leftrightarrow 4)$$

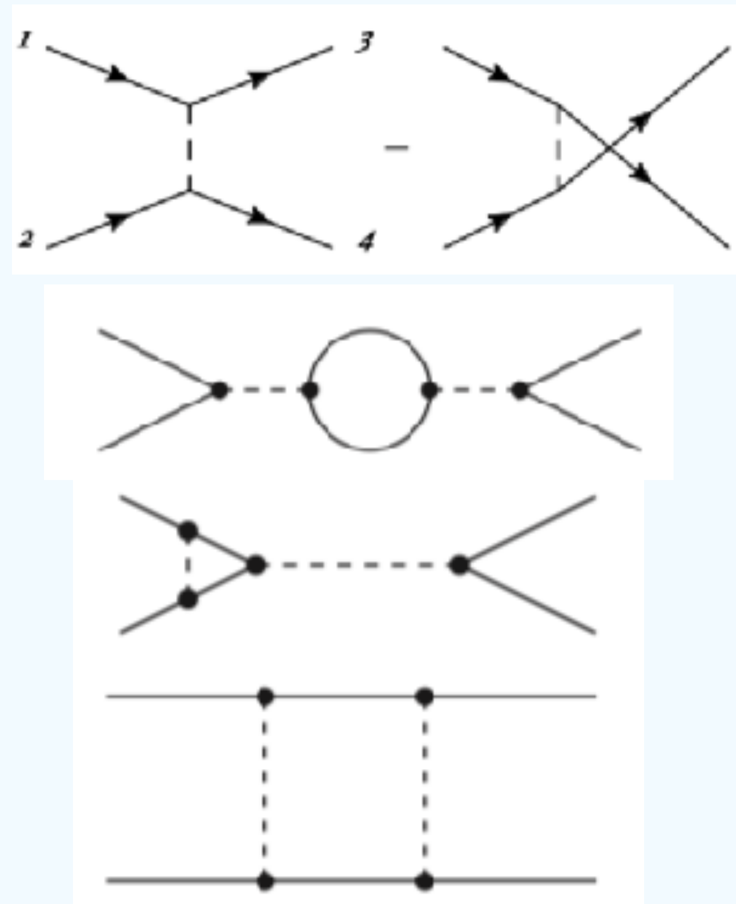
$$\mathcal{L}_{\rm EFT} = i\bar\psi\partial\!\!\!/\psi - \sigma\bar\psi\psi + \frac{c_S}{M^2}\frac{1}{2}(\bar\psi\psi)(\bar\psi\psi) + \frac{c^{(8)}}{M^4}\left(\partial_\mu\bar\psi\partial^\mu\psi\right)\left(\bar\psi\psi\right)$$

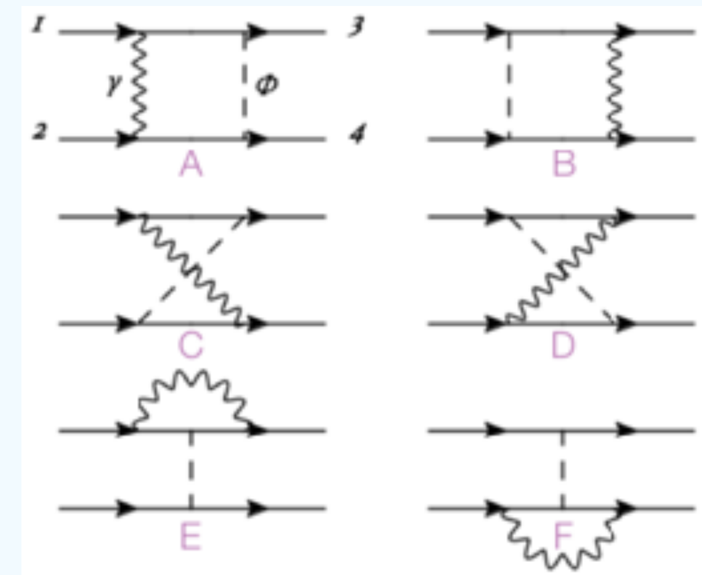
$$c_S = \lambda^2$$

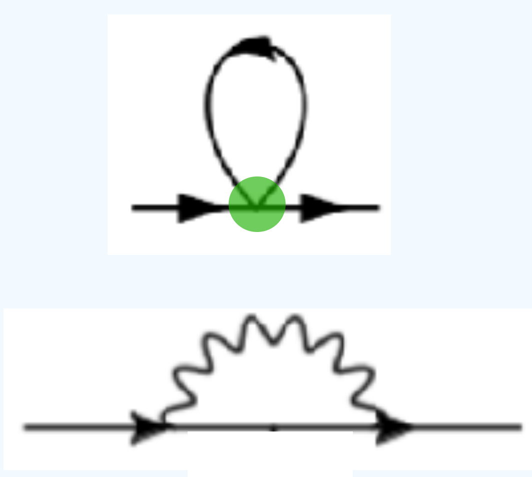
$$c^{(8)} = \lambda^2$$

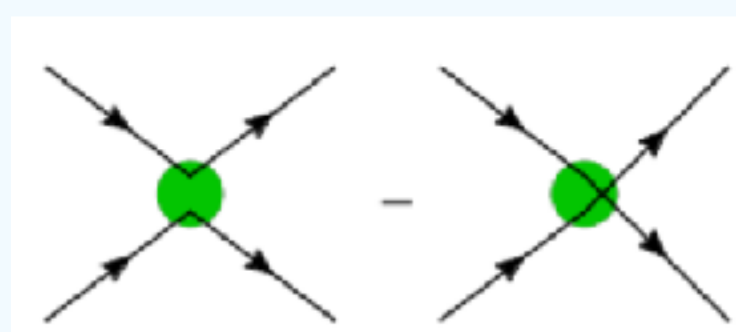
Matching at One-loop

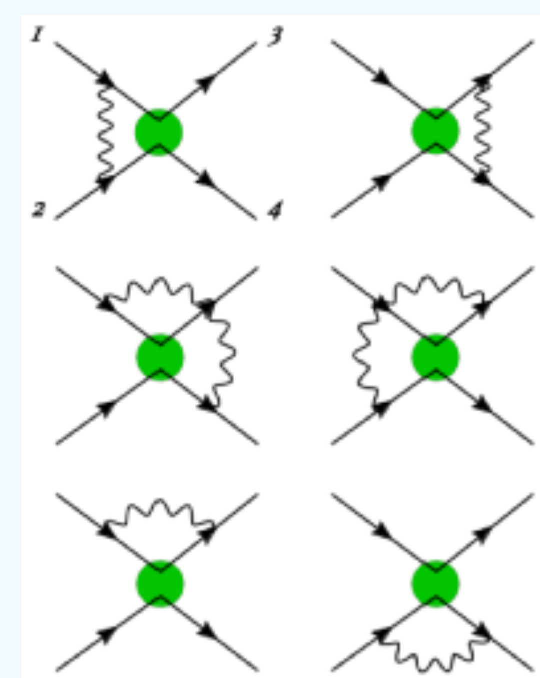










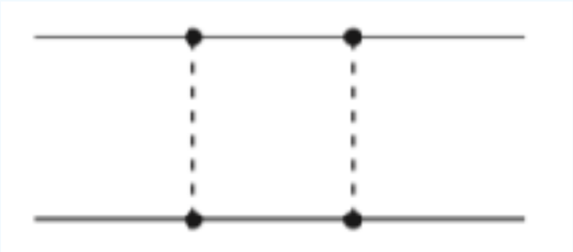


Exercise: Matching and Running

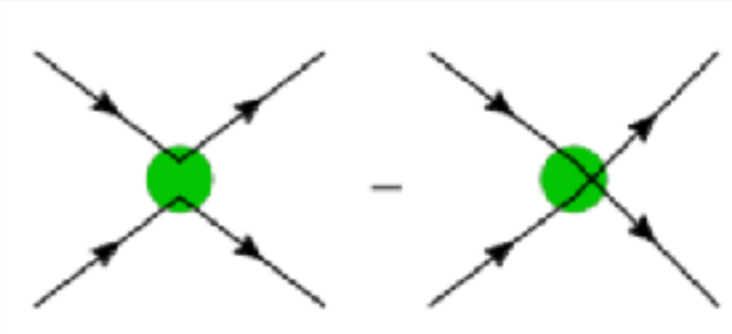
$$\Gamma^{(4)}(0) = \frac{3g^2(\mu)}{M^2} + \frac{3g^4(\mu)}{8\pi^2 M^4}\left(3 + 2\log\frac{m^2}{M^2} + \frac{1}{4}\log\frac{m^2}{\mu^2}\right)$$

$$+\text{ perms.} = \left(\frac{1}{2}\times 3\right)(-ig)^4\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\left(\frac{i}{k^2-m^2}\right)^2\left(\frac{-i}{M^2}\right)^2 = \frac{3ig^4}{32\pi^2 M^4}\log\frac{m^2}{\mu^2},$$

$$+\text{ perms.} = (6)(-ig)^4\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\left(\frac{i}{k^2-m^2}\right)^2\frac{-i}{M^2}\frac{i}{k^2-M^2} = \frac{3ig^4}{8\pi^2 M^4}\left(1+\log\frac{m^2}{M^2}\right),$$



$$+\text{ perms.} = (6)(-ig)^4\mu^{2\epsilon}\int\frac{d^dk}{(2\pi)^d}\left(\frac{i}{k^2-m^2}\right)^2\left(\frac{i}{k^2-M^2}\right)^2 = \frac{6ig^4}{8\pi^2 M^4}\left(1+\frac{1}{2}\log\frac{m^2}{M^2}\right),$$

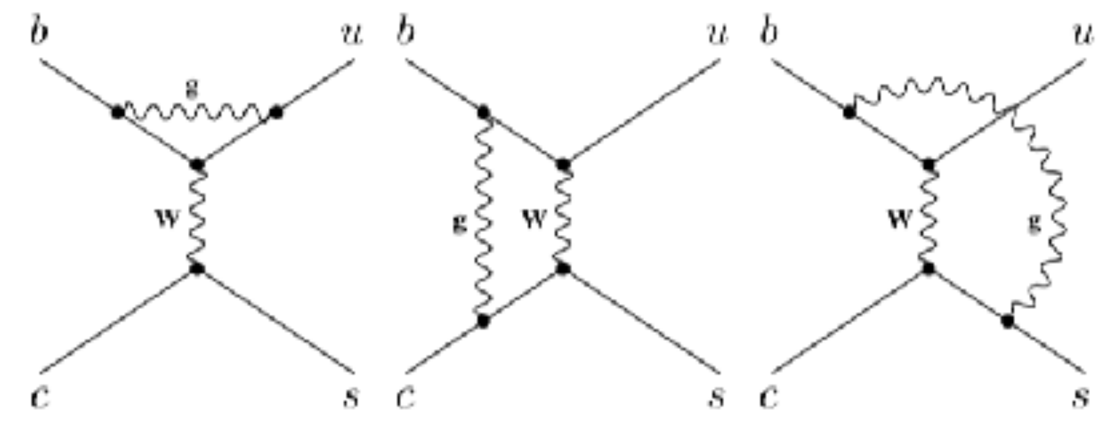


$$\lambda(\mu) = -\frac{3g^2(\mu)}{M^2} - \frac{3g^4}{8\pi^2 M^4}\left(3 + 2\log\frac{\mu^2}{M^2}\right)$$

$$\lambda(M) = -\frac{3g^2(M)}{M^2} - \frac{9g^4}{8\pi^2 M^4}$$

Exercise: b decay

SM:



Calculate one-loop matching and RGE in EFT?

$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ub} \left[C_1(\mu)\, \bar{s}^j_L \gamma_\mu c^j_L\, \bar{u}^i_L \gamma^\mu b^i_L + C_2(\mu)\, \bar{s}^i_L \gamma_\mu c^j_L\, \bar{u}^j_L \gamma^\mu b^i_L \right]$$

$$\bar{s}_L \gamma_\mu t_a c_L\, \bar{u}_L \gamma^\mu t_a b_L = \frac{1}{2} \bar{s}^i_L \gamma_\mu c^j_L\, \bar{u}^j_L \gamma^\mu b^i_L - \frac{1}{2N_c} \bar{s}^j_L \gamma_\mu c^j_L\, \bar{u}^i_L \gamma^\mu b^i_L$$

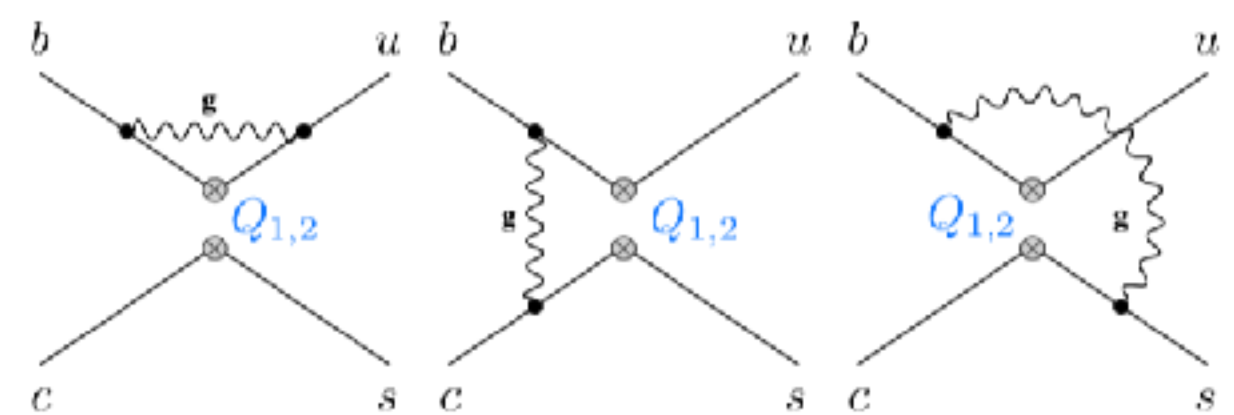
$$C_1(\mu) = 1 + \frac{3}{N_c} \frac{\alpha_s(\mu)}{4\pi} \left(\ln \frac{M_W^2}{\mu^2} - \frac{11}{6} \right) + O(\alpha_s^2)$$

$$C_2(\mu) = -3 \frac{\alpha_s(\mu)}{4\pi} \left(\ln \frac{M_W^2}{\mu^2} - \frac{11}{6} \right) + O(\alpha_s^2)$$

$$\boldsymbol{Z}(\mu) = \boldsymbol{1} + \frac{\alpha_s(\mu)}{4\pi\epsilon} \begin{pmatrix} \frac{3}{N_c} & -3 \\ -3 & \frac{3}{N_c} \end{pmatrix}$$

$$\gamma_{ij} = \frac{\alpha_c}{2\pi} \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix}$$

EFT:



Short Summary

Diagrammatic approach

Feynman diagrams

Easy to miss diagrams

Once matching is done

Running could be easy

[Carmona, et.al, 2112.10787]

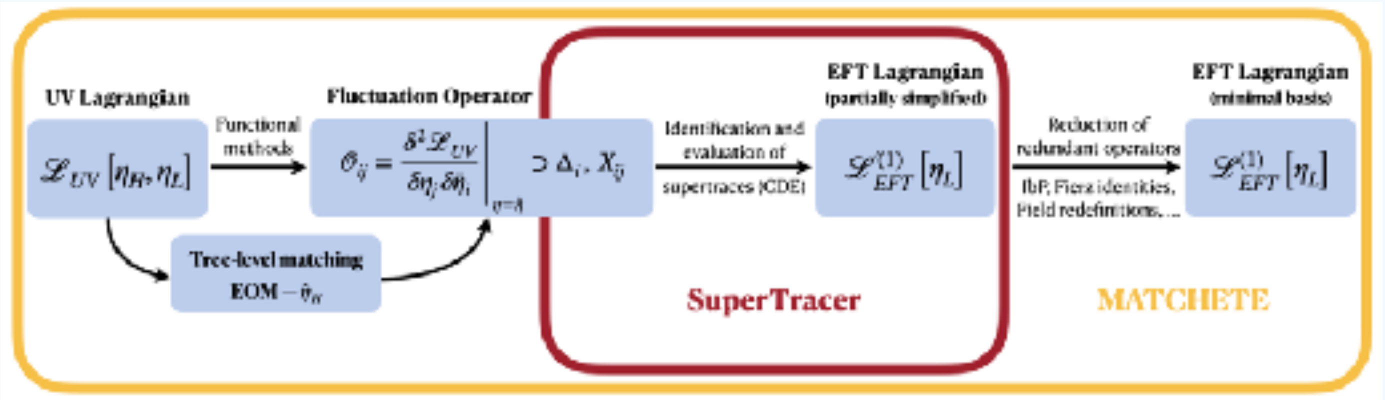
Path Integral approach

Covariant Derivative Expansion

Systematic but difficult

Matching with loop expansion

Running additional treatment



[Fuentes-Martin, et.al., 2012.08506]

[Cohen, Lu, Zhang, 2012.07851]

Exercise: Scalar CDE Matching

$$\mathcal{L}_{\rm UV} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m_L^2\phi^2 + (\partial_\mu H)^2 - M^2 H^2\right] - \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2}M\phi^2 H - \frac{\lambda_2}{4}\phi^2 H^2$$

$$H_c(\phi) = -\frac{\lambda_1 M}{2}\left[M^2 + \Box + \frac{\lambda_2}{2}\phi^2\right]^{-1}\phi^2$$

$$\begin{aligned}\mathcal{L}^{(0)}_{\rm EFT}(\phi) &= \mathcal{L}_{\rm UV}(\phi, H_c(\phi)) \\ &= \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m_L^2}{2}\phi^2 - \frac{\lambda_0}{4!}\phi^4 - \frac{\lambda_1}{2}M\phi^2 H_c(\phi) - \frac{1}{2}H_c(\phi)\left[\Box + M^2 + \frac{\lambda_2}{2}\phi^2\right]H_c(\phi) \\ &= \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m_L^2}{2}\phi^2 - \frac{\lambda_0}{4!}\phi^4 + \frac{\lambda_1^2 M^2}{8}\phi^2\left[M^2 + \Box + \frac{\lambda_2}{2}\phi^2\right]^{-1}\phi^2.\end{aligned}$$

$$\mathcal{L}^{(0)}_{\rm EFT} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m_L^2}{2}\phi^2 - \left(\lambda_0 - 3\lambda_1^2\right)\frac{\phi^4}{4!} - 45\lambda_1^2\lambda_2\frac{\phi^6}{6!M^2} - 4\lambda_1^2\frac{\phi^3\Box\phi}{4!M^2} + \mathcal{O}(M^{-4}).$$

$$\begin{aligned}m^2 &= m_L^2, \\ C_4 &= \lambda_0 - 3\lambda_1^2 - 4\lambda_1^2\frac{m_L^2}{M^2}, \\ C_6 &= 45\lambda_1^2\lambda_2 - 20\lambda_0\lambda_1^2 + 60\lambda_1^4.\end{aligned}$$

$$\int d^4x\,\mathcal{L}^{(1)}_{\rm EFT} = \frac{i}{2}\log\det\left(\Delta_H - X_{LH}\Delta_L^{-1}X_{LH}\right)_{\rm hard}$$

$$X_H = \frac{\lambda_2}{2}\phi^2, \qquad X_{LH} = 0.$$

Exercise: Scalar CDE Matching

$$\mathcal{L}^{(1)}_{\rm EFT} = -\frac{i}{2}\sum_{n=1}^{\infty} n^{-1}\int\frac{d^dq}{(2\pi)^d}\left(\frac{2q\hat{P}-\hat{P}^2+\frac{\lambda_2}{2}\phi^2}{q^2-M^2}\right)^n$$

Only keep the hard part

$$\begin{aligned}\mathcal{L}^{(1)}_{\rm EFT} &\supset -\frac{i}{2}\frac{\lambda_2\phi^2}{2}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2-M^2}\\ &= \frac{\lambda_2 M^2}{32\pi^2}\left[\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{M^2}\right)+1\right]\frac{\phi^2}{2}\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{(1)}_{\rm EFT} &\supset -\frac{i}{2}\frac{\lambda_2^2\phi^4}{4}\frac{1}{2}\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2-M^2)^2}\\ &= \frac{3\lambda_2^2}{32\pi^2}\left[\frac{1}{\bar{\epsilon}}+\log\left(\frac{\mu^2}{M^2}\right)\right]\frac{\phi^4}{4!}\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{(1)}_{\rm EFT} &\supset -\frac{i}{2}\frac{1}{3}\left(\frac{\lambda_2^3\phi^6}{8}+\frac{7}{2}\phi^2(\partial_\mu\phi)^2+\frac{3}{2}\phi^3\Box\phi\right)\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2-M^2)^3}\\ &= -\frac{i}{2}\frac{1}{3}\left(\frac{\lambda_2^3\phi^6}{8}+\frac{1}{3}\phi^3\Box\phi\right)\int\frac{d^dq}{(2\pi)^d}\frac{1}{(q^2-M^2)^3}\\ &= -\frac{15\lambda_2^3}{32\pi^2}\frac{\phi^6}{6!M^2}-\frac{\lambda_2^2}{24\pi^2}\frac{\phi^3\Box\phi}{24M^2}\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{(1)}_{\rm EFT} &\supset -\frac{i}{2}\frac{1}{4}\left(-\frac{4\lambda^2}{3}\phi^3\partial_\mu\partial_\nu\phi\right)\int\frac{d^dq}{(2\pi)^d}\frac{q_\mu q_\nu}{(q^2-M^2)^4}\\ &= \frac{\lambda_2^2}{48\pi^2M^2}\frac{\phi^3\Box\phi}{24}\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{(0)}_{\rm EFT}+\mathcal{L}^{(1)}_{\rm EFT} &\supset \frac{1}{2}(\partial_\mu\phi)^2-\frac{\phi^2}{2}\left(m_L^2-\frac{\lambda_2M^2}{32\pi^2}\left[\log\left(\frac{\mu^2}{M^2}\right)+1\right]\right)\\ &- \frac{\phi^4}{4!}\left[\lambda_0-\frac{3\lambda_2^2}{32\pi^2}\log\left(\frac{\mu^2}{M^2}\right)\right]\\ &- \frac{\phi^6}{6!M^2}\frac{15\lambda_2^3}{32\pi^2}-\frac{\phi^3\Box\phi}{24M^2}\frac{\lambda_2^2}{48\pi^2}.\end{aligned}$$

$$\frac{1}{4!M^2}\phi^3\Box\phi = -\frac{m^2}{4!M^2}\phi^4-\frac{5C_4}{6!M^2}\phi^6+\mathcal{O}(M^{-4})$$

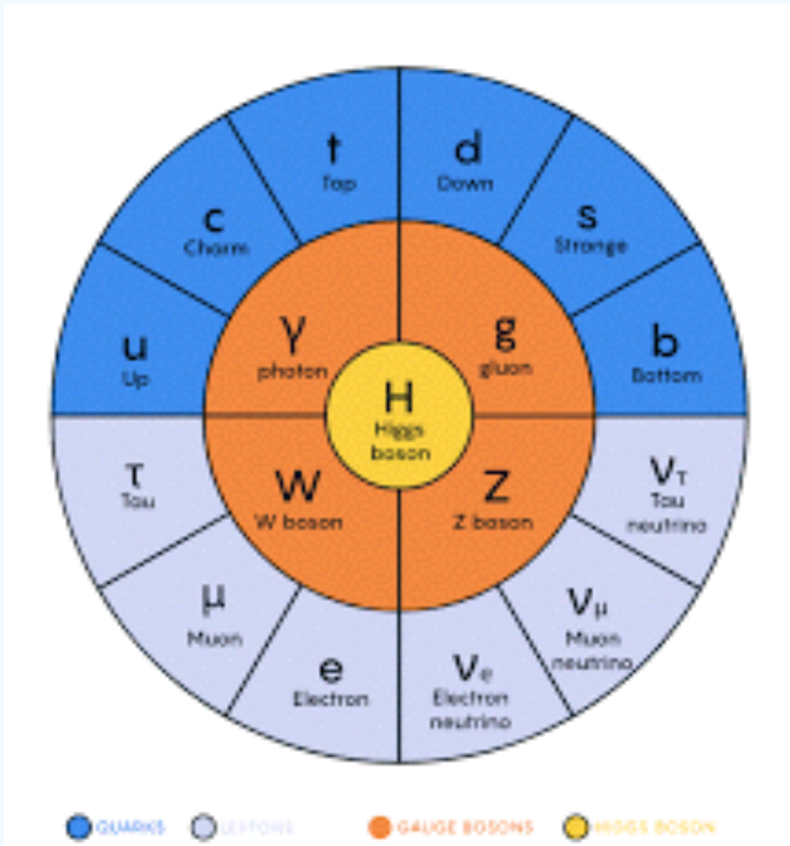
$$\begin{aligned}m^2 &= m_L^2-\frac{\lambda_2M^2}{32\pi^2}\left[\log\left(\frac{\mu^2}{M^2}\right)+1\right]\\ C_4 &= \lambda_0-\frac{3\lambda_2^2}{32\pi^2}\log\left(\frac{\mu^2}{M^2}\right)-\frac{\lambda_2^2m_L^2}{48\pi^2M^2}\\ C_6 &= \frac{15\lambda_2^3}{32\pi^2}-\frac{5\lambda_0\lambda_2^2}{48\pi^2}\end{aligned}$$

Standard Model EFT

Bottom-up

New Physics

The Standard Model



$$
\begin{aligned}
\mathcal{L}_{SM} = & \underbrace{\frac{1}{4} W_{\mu\nu}\cdot W^{\mu\nu} - \frac{1}{4} B_{\mu\nu}B^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu}G^{\mu\nu}_a}_{\text{kinetic energies and self-interactions of the gauge bosons}} \\
& + \underbrace{\bar{L}\gamma^\mu \left(i\partial_\mu - \frac{1}{2} g\tau\cdot W_\mu - \frac{1}{2} g' Y B_\mu\right) L + \bar{R}\gamma^\mu \left(i\partial_\mu - \frac{1}{2} g' Y B_\mu\right) R}_{\text{kinetic energies and electroweak interactions of fermions}} \\
& + \underbrace{\frac{1}{2}\left|\left(i\partial_\mu - \frac{1}{2} g\tau\cdot W_\mu - \frac{1}{2} g' Y B_\mu\right)\phi\right|^2 - V(\phi)}_{W^\pm, Z, \gamma \text{ and Higgs masses and couplings}} \\
& + \underbrace{g''\left(\bar{q}\gamma^\mu T_a q\right) G^a_\mu}_{\text{interactions between quarks and gluons}} + \underbrace{\left(G_1 \bar{L}\phi R + G_2 \bar{L}\phi_c R + h.c.\right)}_{\text{fermion masses and couplings to Higgs}}
\end{aligned}
$$

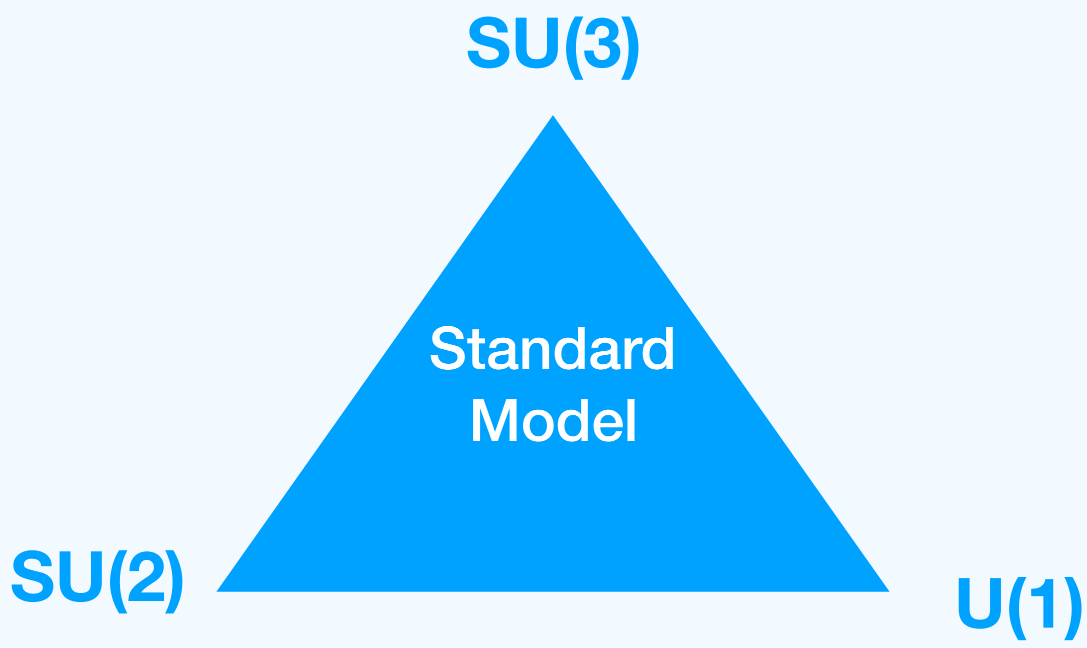
17 elementary particles

19 parameters, all measured but

$$+\,\tilde{\theta} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

Note that $\theta_B B_{\mu\nu}\tilde{B}_{\mu\nu}$ is not physical, while $\theta_W W^k_{\mu\nu}\tilde{W}^k_{\mu\nu}$ can be eliminated by chiral rotation

Symmetry of the SM



Particle(s)	Field(s)	Content	Charge	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Quarks	Q_i	$(u, d)_L$	$(2/3, -1/3)$	1/2	3	2	1/6
(Three generations)	u_{Ri}	u_R	2/3	1/2	$\bar{3}$	1	4/3
	d_{Ri}	d_R	$-1/3$	1/2	$\bar{3}$	1	$-2/3$
Leptons	L_i	$(\nu_e, e)_L$	$(0, -1)$	1/2	1	2	-1
(Three generations)	l_{Ri}	e_R	-1	1/2	1	1	-2
Gluons	G^a_μ	g	0	1	8	1	0
W bosons	$W^{1,2}_\mu$	$W^\pm$	±1	1	1	3	0
Photon, Z boson	W^3_μ, B_μ	γ, Z^0	0	1	1	3, 1	0
Higgs boson	ϕ	H	0	0	1	2	1

SO(3,1)

$$(M_{12}, M_{23}, M_{31}) = (J_3, J_1, J_2)\,, \qquad \tfrac{1}{2}M^{\mu\nu}M_{\mu\nu} = \mathbf{J}^2 - \mathbf{K}^2\,,$$
$$(M_{01}, M_{02}, M_{03}) = (K_1, K_2, K_3)\,, \qquad \tfrac{1}{4}\epsilon^{\mu\nu\sigma\tau}M_{\mu\nu}M_{\sigma\tau} = -\mathbf{J}\cdot\mathbf{K}\,,$$

$$[M_{\lambda\rho}, M_{\mu\nu}] = -i(g_{\lambda\mu}M_{\rho\nu} + g_{\rho\nu}M_{\lambda\mu} - g_{\lambda\nu}M_{\rho\mu} - g_{\rho\mu}M_{\lambda\nu})$$

$$M_i = \tfrac{1}{2}(J_i + iK_i)\,,$$
$$N_i = \tfrac{1}{2}(J_i - iK_i)\,,$$

$$[M_i, M_j] = i\epsilon_{ijk}M_k\,,$$
$$[N_i, N_j] = i\epsilon_{ijk}N_k\,,$$
$$[M_i, N_j] = 0\,.$$

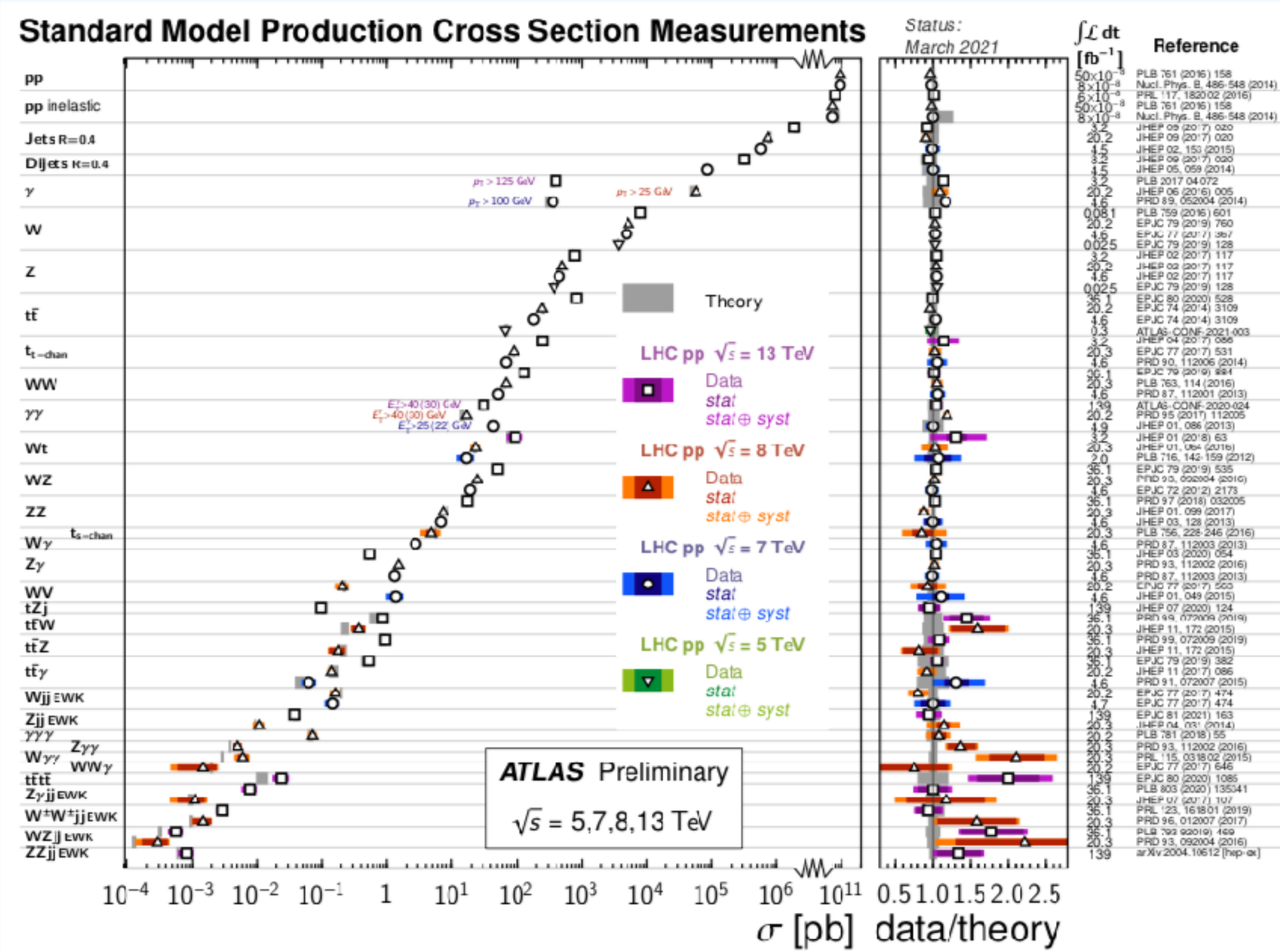
SU(2) x SU(2)

$$H_i \in (0,0)\,, \quad H^{\dagger i} \in (0,0)\,,$$

$$\psi_\alpha \in (1/2, 0), \quad \psi^\dagger_{\dot\alpha} \in (0, 1/2),$$

$$F_{\mathrm{L}\alpha\beta} = \frac{i}{2}F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0), \quad F_{\mathrm{R}\dot\alpha\dot\beta} = -\frac{i}{2}F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \in (0,1).$$

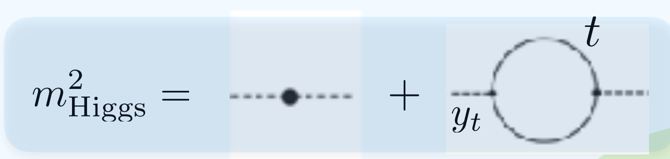
The Standard Model



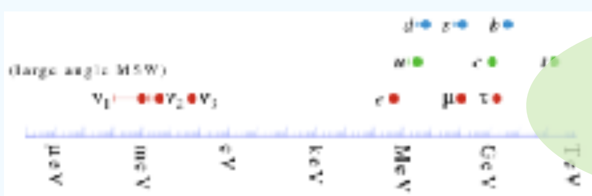
New Physics (NP) Models

theoretical motivation

experimental challenges

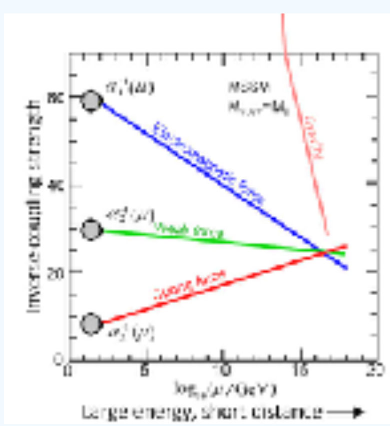


Higgs mass

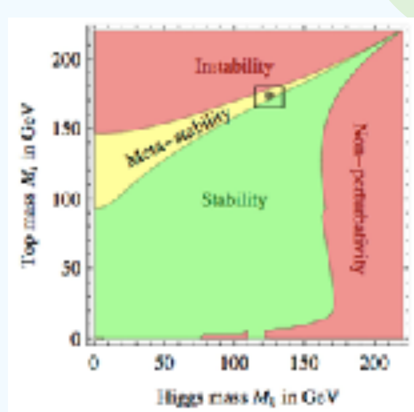


Flavor Hierarchy

Gauge Unification

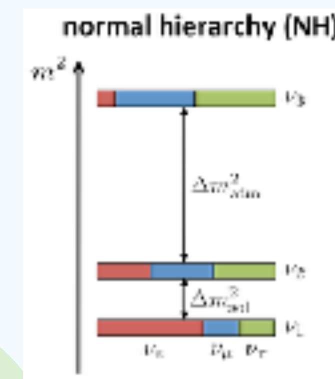


Vacuum Stability

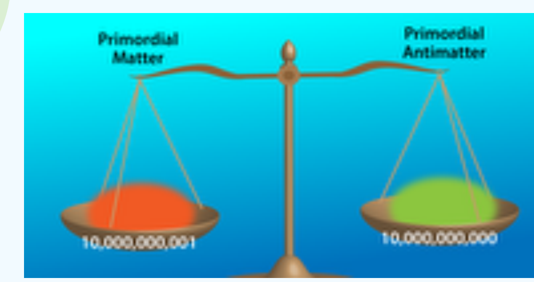


New Physics

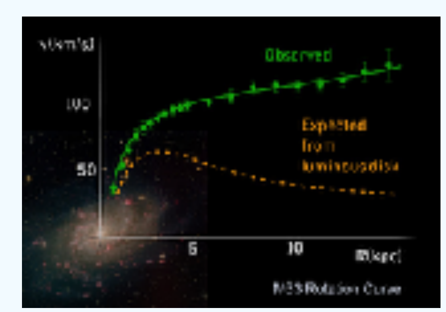
Neutrino



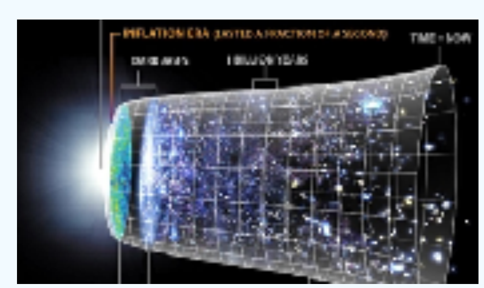
Baryon Asymmetry



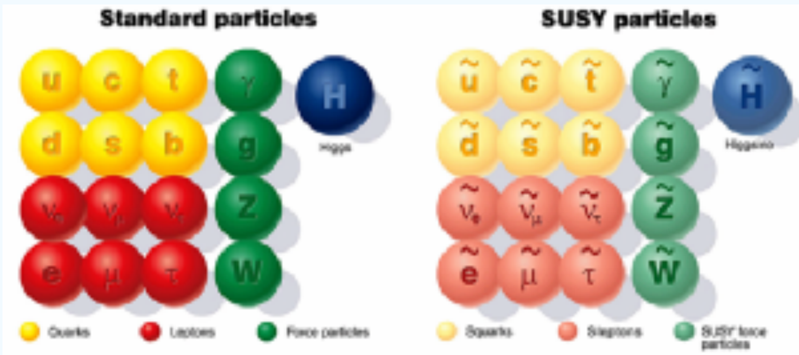
Dark matter



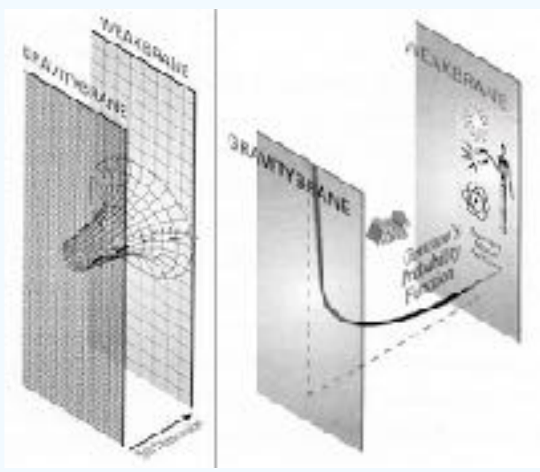
Inflation



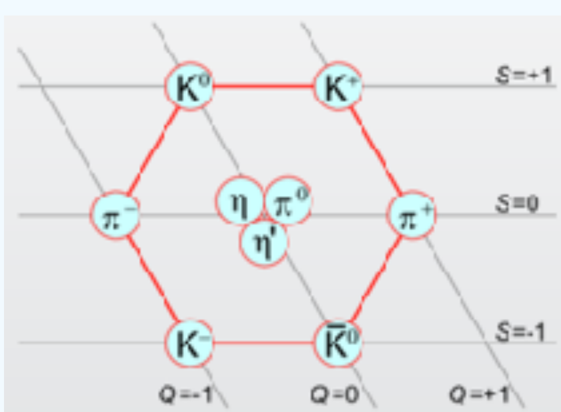
NP Motivated Simplified Model



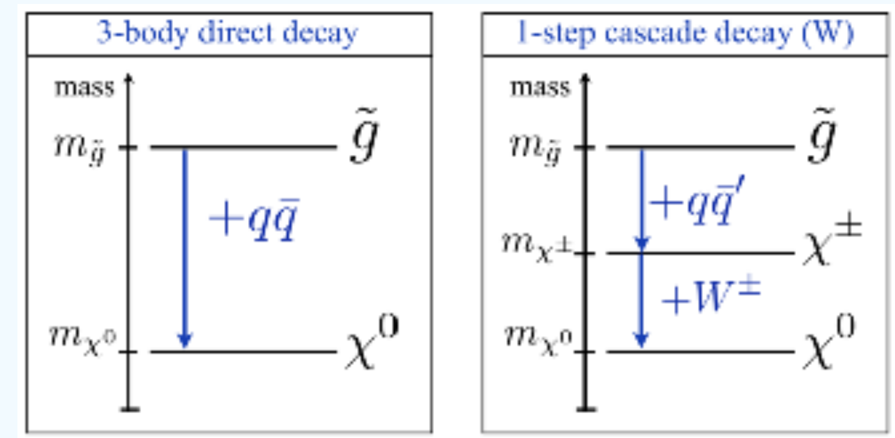
SUSY

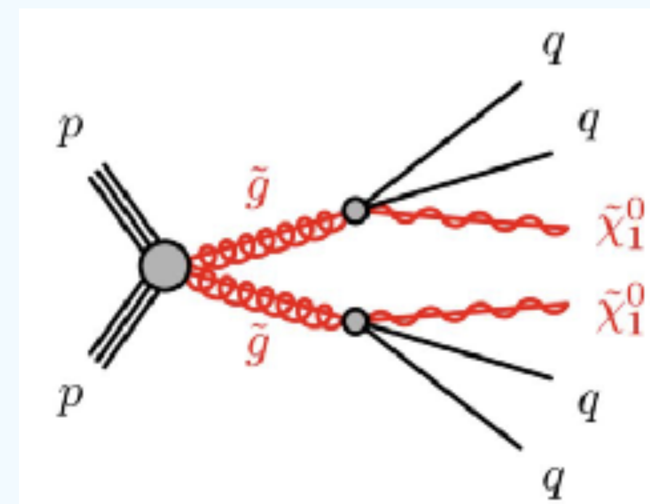


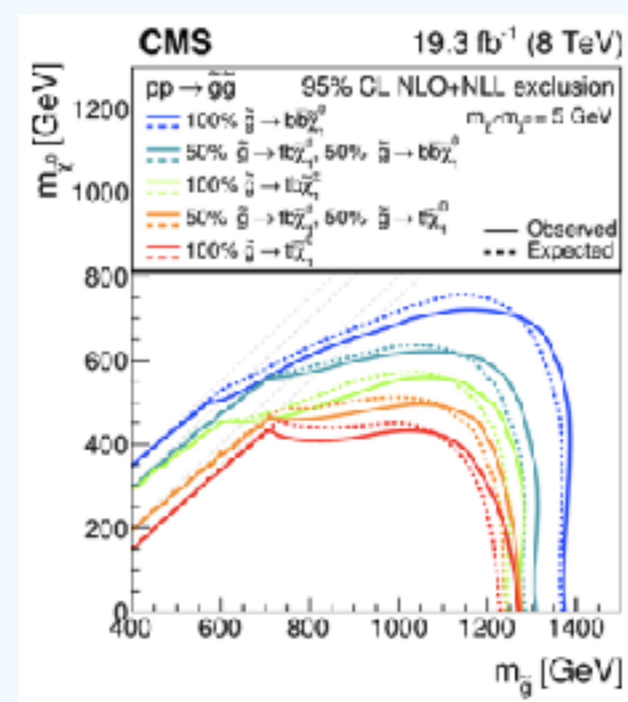
Extra Dimension



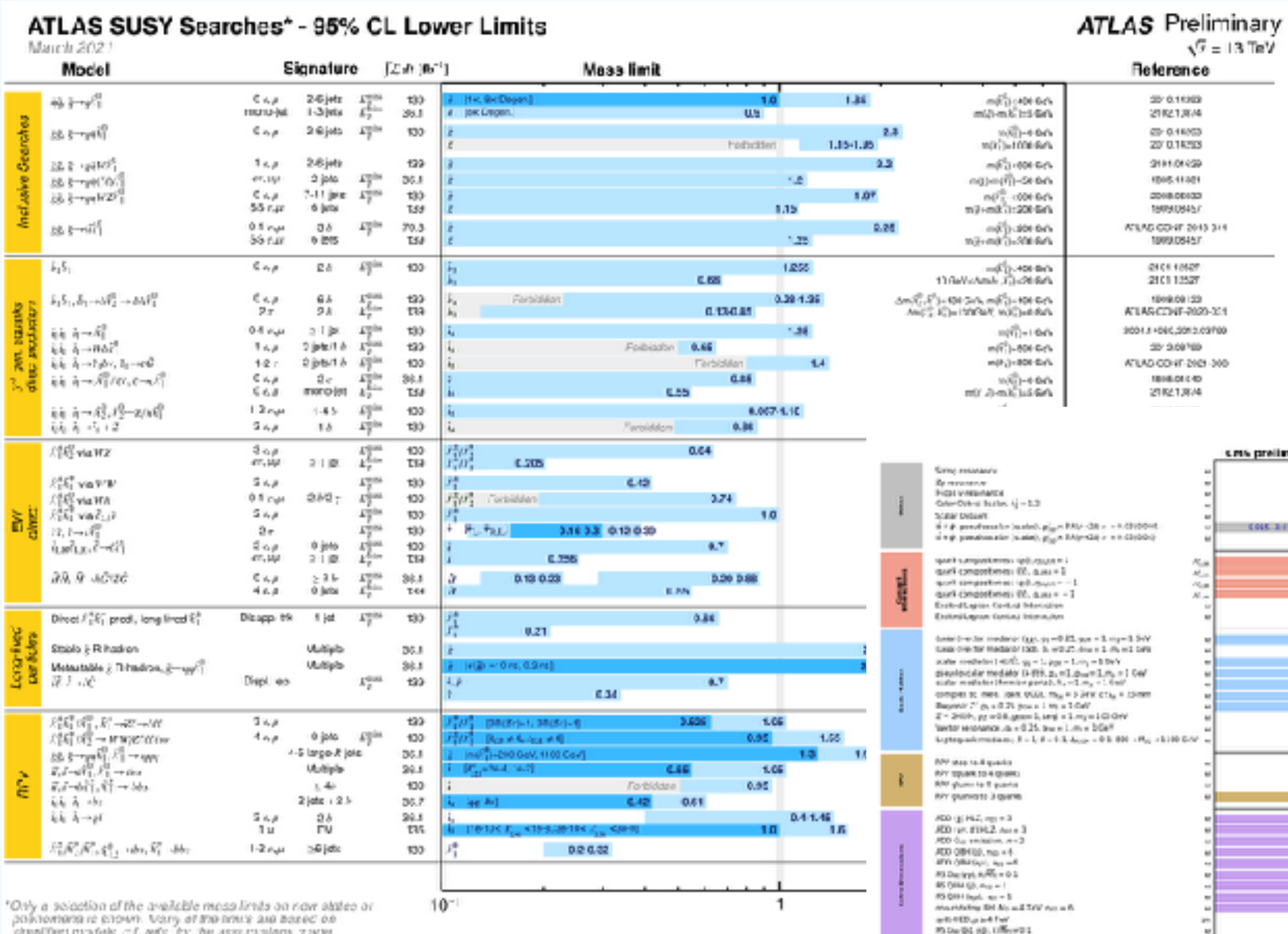
Composite Dynamics

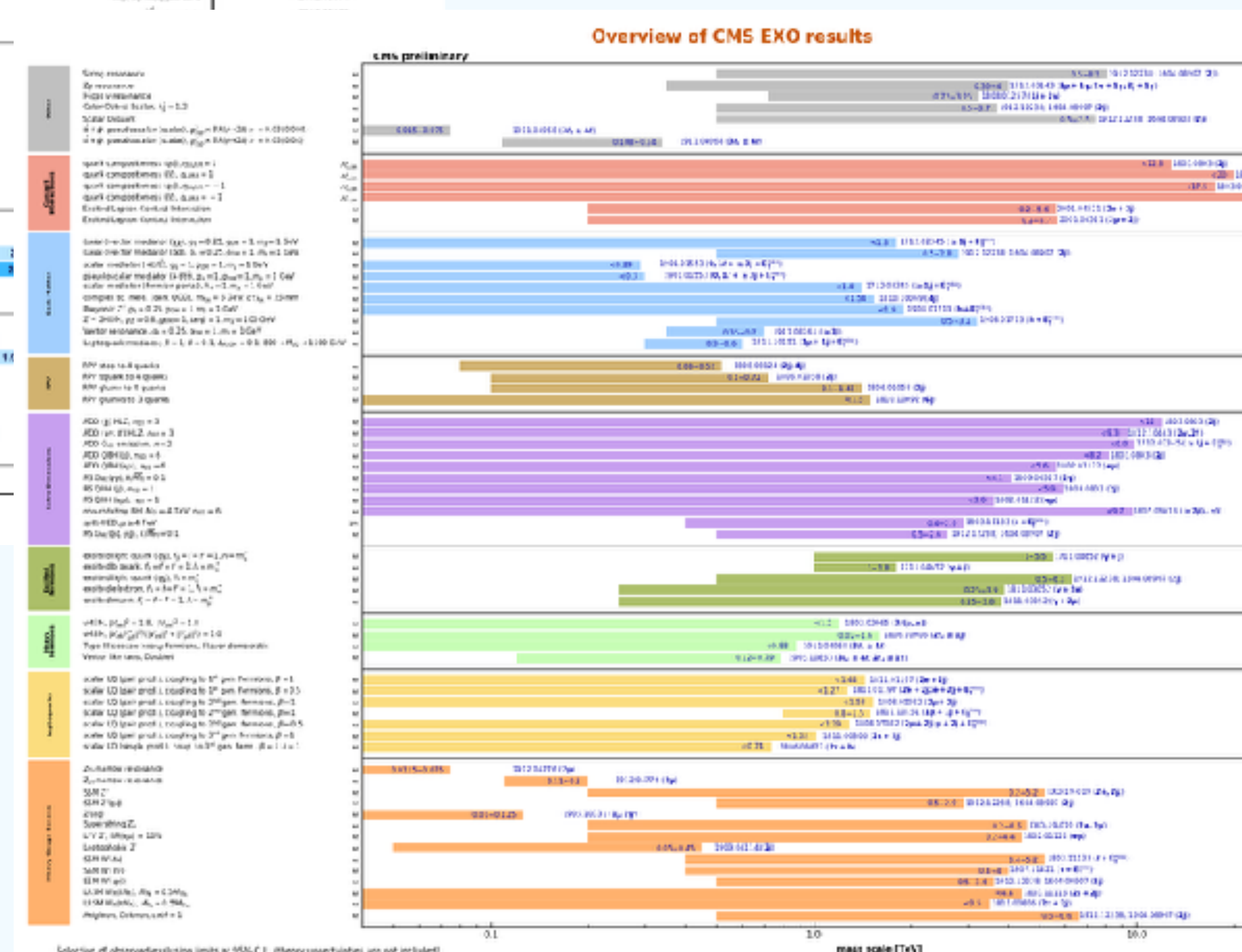






New Physics @ LHC

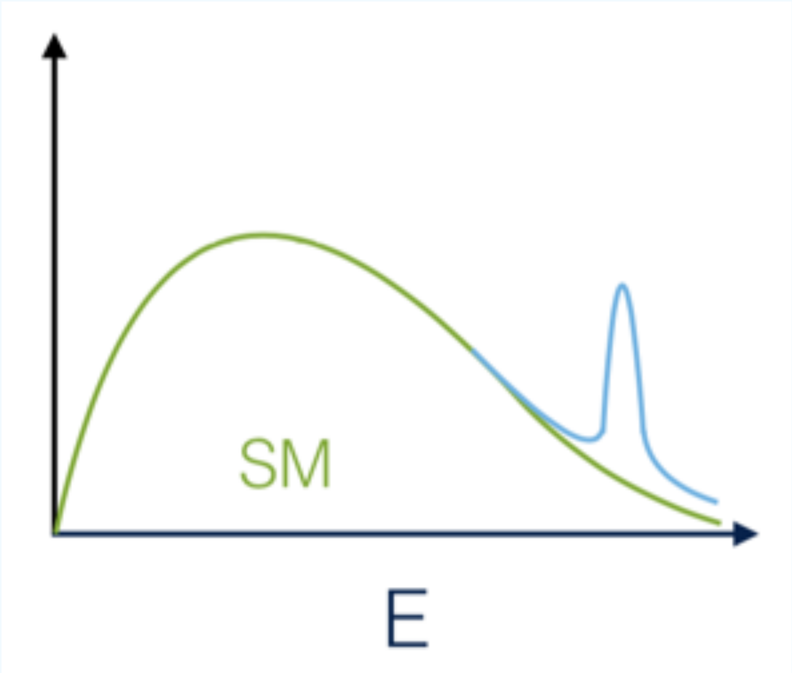




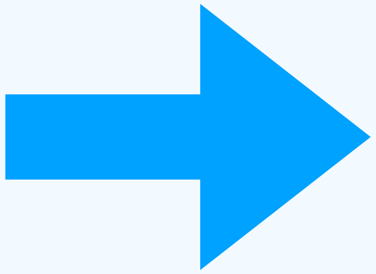
Paradigm Shift

New physics beyond the LHC threshold: paradigm shift for BSM searches

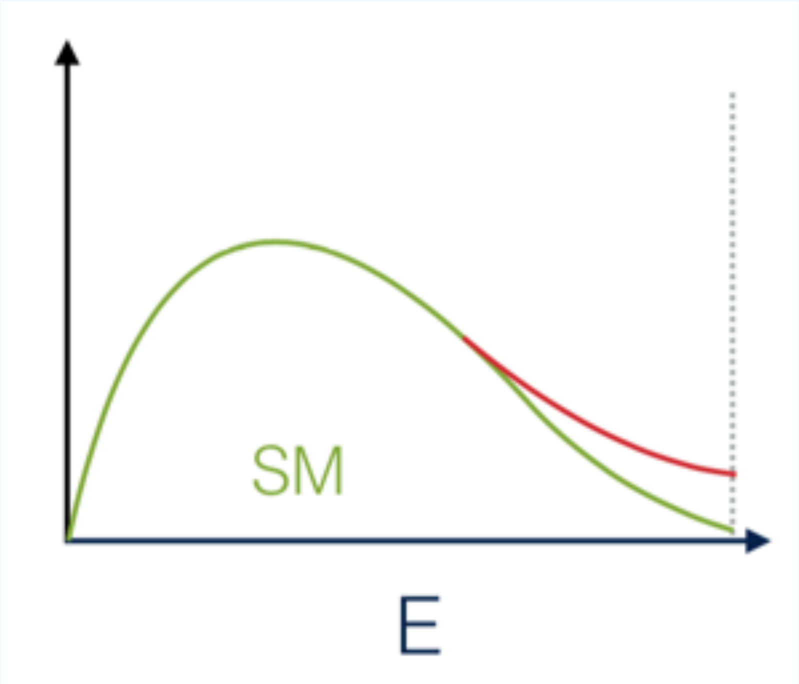
Direct signature



resonance bump hunting at the LHC



Indirect searches



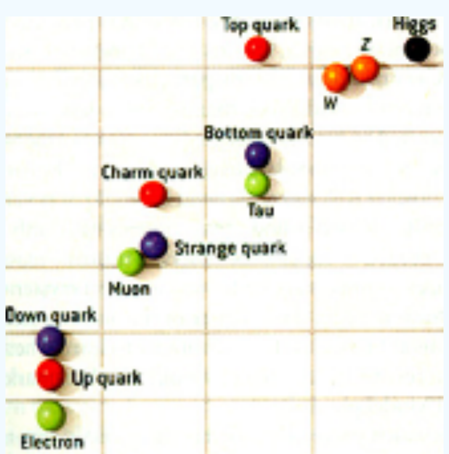
distribution tail deviation at the LHC

Top-Down EFT

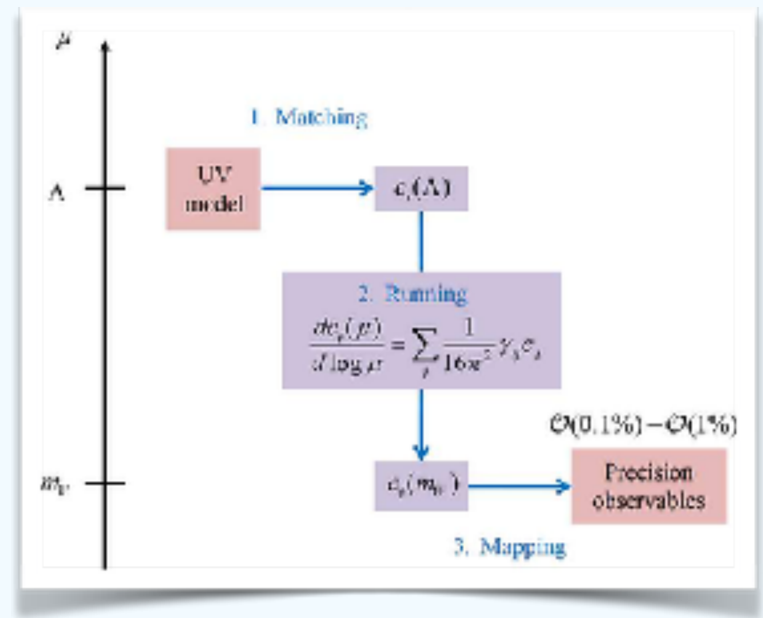
Given new physics models, integrate out heavy particles and match to SMEFT

New Physics Models

Integrate out and matching/running

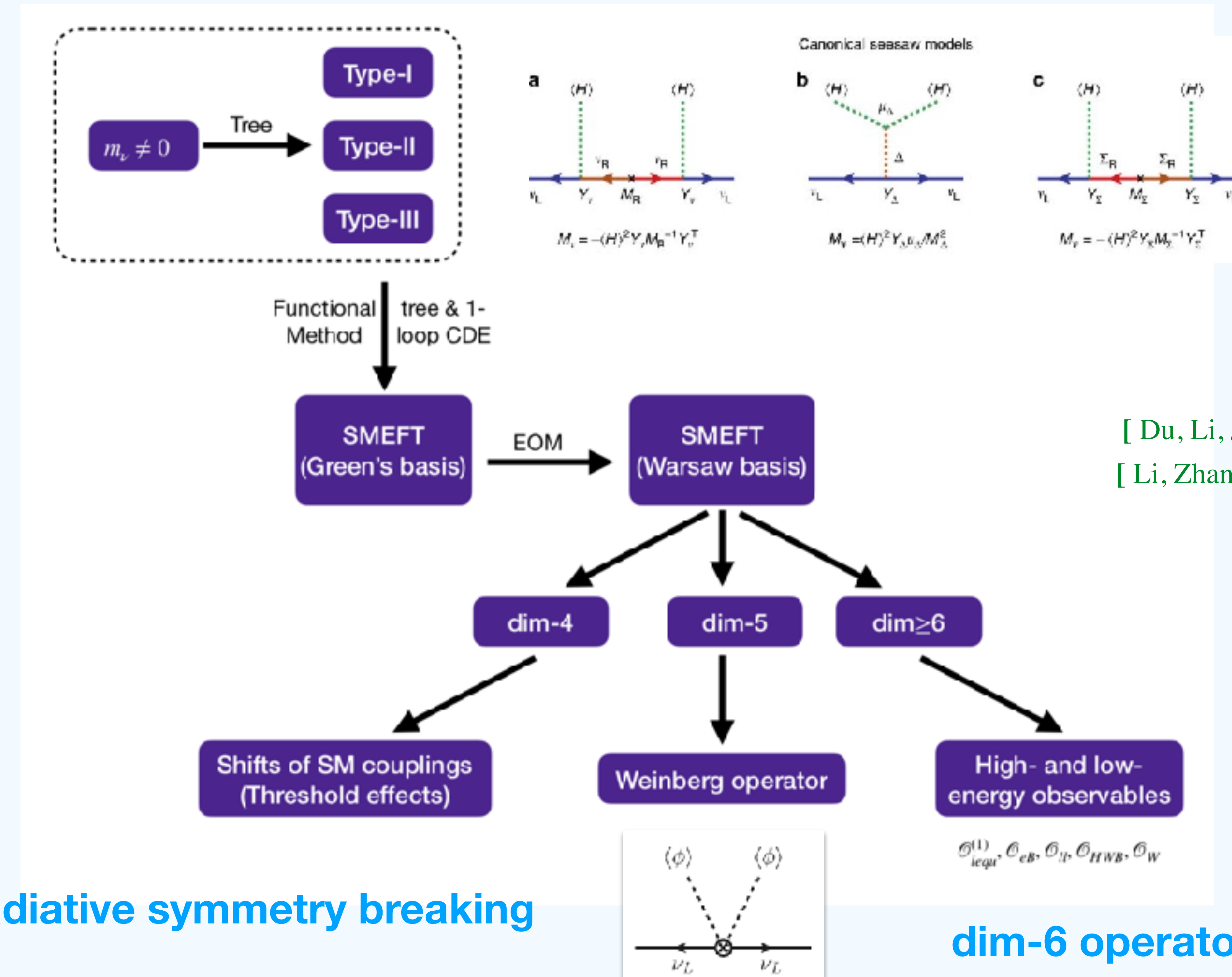


EFT Operators



Decoupling theorem

Canonical Seesaw Models



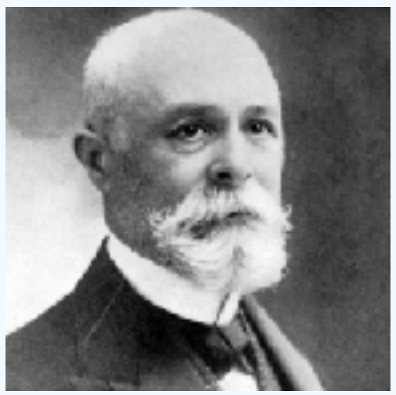
[Du, Li, **JHYu**, 2201.04646]
[Li, Zhang, Zhou, 2021,2022]

Radiative symmetry breaking

dim-6 operators

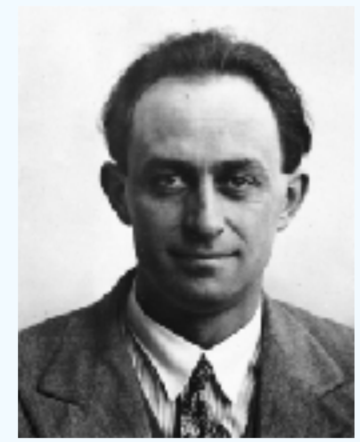
History of Weak Theory

Looking back to the past, we did not know the UV theory ahead



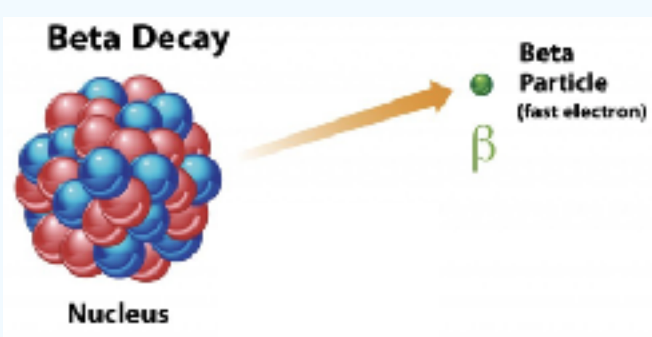
Becquerel
1896

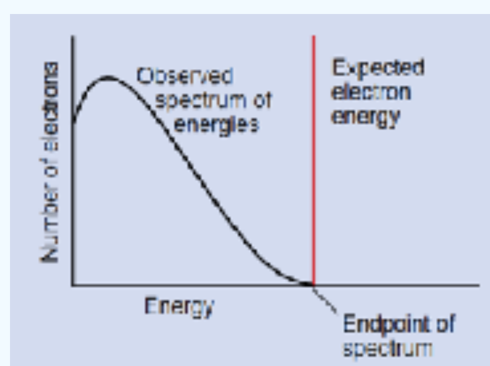
Pauli
1933

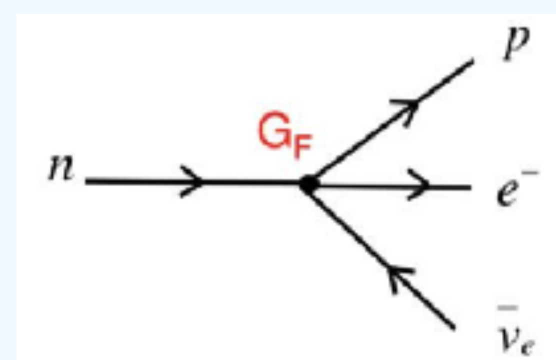


Fermi
1934

Gamov-Teller 1936
Fierz 1937







$$M_{fi} = G_F \left[\overline{\psi_n}\gamma^\mu \psi_p\right] \left[\overline{\psi_e}\gamma^\mu \psi_\nu\right]$$

Four-fermi EFT

$$\mathcal{L}_i = \sum_{i=1}^{5} g_i \left\{\bar{\psi}_1 \mathcal{O}^i \psi_2\right\} \left\{\bar{\psi}_3 \mathcal{O}_i \psi_4\right\}$$

$$\mathcal{O}_i = (\,\mathbf{1},\ \gamma_\mu,\ \sigma_{\mu\nu},\ i\gamma_5\gamma_\mu,\ \text{or}\ \gamma_5\,)$$

vector current to
Fermi(V/S),
GT(A/T), P

Four-Fermi EFT

With parity violation, Lee and Yang wrote the most general 4-fermi operators

Lee-Yang 1956
Wu 1956

$$\vec{\sigma}_{\rm Co} \cdot \vec{p}_e$$

Question of Parity Conservation in Weak Interactions*

T. D. LEE, *Columbia University, New York, New York*

AND

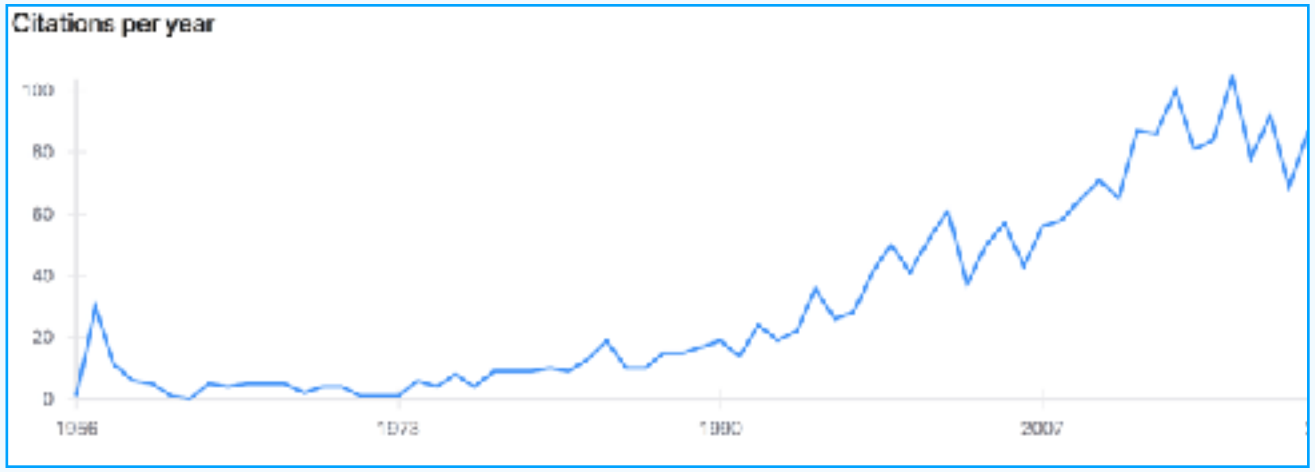
C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

If parity is not conserved in β decay, the most general form of Hamiltonian can be written as

$$\begin{aligned} H_{\rm int} = & (\psi_p^\dagger\gamma_4\psi_n)(C_S\psi_e^\dagger\gamma_4\psi_\nu + C_S'\psi_e^\dagger\gamma_4\gamma_5\psi_\nu) \\ & +(\psi_p^\dagger\gamma_4\gamma_\mu\psi_n)(C_V\psi_e^\dagger\gamma_4\gamma_\mu\psi_\nu + C_V'\psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\psi_\nu) \\ & +\tfrac{1}{2}(\psi_p^\dagger\gamma_4\sigma_{\lambda\mu}\psi_n)(C_T\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}\psi_\nu \\ & +C_T'\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}\gamma_5\psi_\nu) + (\psi_p^\dagger\gamma_4\gamma_\mu\gamma_5\psi_n) \\ & \times(-C_A\psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\psi_\nu - C_A'\psi_e^\dagger\gamma_4\gamma_\mu\psi_\nu) \\ & +(\psi_p^\dagger\gamma_4\gamma_5\psi_n)(C_P\psi_e^\dagger\gamma_4\gamma_5\psi_\nu + C_P'\psi_e^\dagger\gamma_4\psi_\nu), \qquad (A.1) \end{aligned}$$

Complete charge current LEFT operators



Comprehensive analysis of beta decays within and beyond the Standard Model

[Falkowski, et.al 2021]

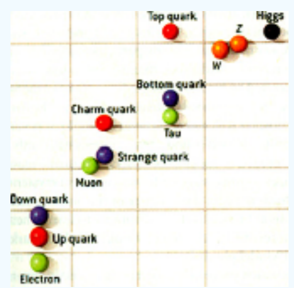
energies. The general EFT Lagrangian describing these interactions at the leading order was written more than 60 years ago by Lee and Yang [6]:

$$\begin{aligned} \mathcal{L}_{\rm Lee-Yang} = & -\bar{p}\gamma^\mu n\left(C_V\bar{e}\gamma_\mu\nu - C_V'\bar{e}\gamma_\mu\gamma_5\nu\right) + \bar{p}\gamma^\mu\gamma_5 n\left(C_A\bar{e}\gamma_\mu\gamma_5\nu - C_A'\bar{e}\gamma_\mu\nu\right) \\ & -\bar{p}n\left(C_S\bar{e}\nu - C_S'\bar{e}\gamma_5\nu\right) - \frac{1}{2}\bar{p}\sigma^{\mu\nu}n\left(C_T\bar{e}\sigma_{\mu\nu}\nu - C_T'\bar{e}\sigma_{\mu\nu}\gamma_5\nu\right) \\ & -\bar{p}\gamma_5 n\left(C_P\bar{e}\gamma_5\nu - C_P'\bar{e}\nu\right) + \text{h.c.} \qquad (1.1) \end{aligned}$$

Bottom-up Approach

PV Lesson: start from the complete bottom-up operators

UV unknown



EFT Operators

[Weinberg 1933 - 2021]

a folk theorem: "if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties."

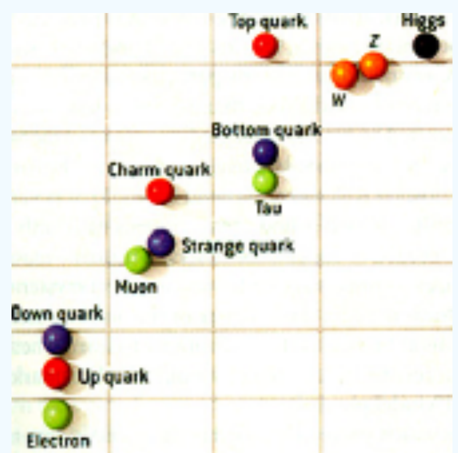
Weinberg's Folk theorem, 1979

Bottom-up: SMEFT

Standard model effective field theory (SMEFT)

UV model unspecified

LHC probing scale



SMEFT

Most general Lagrangian with Lorentz and SM gauge symmetry

$$\mathscr{L}_{\mathrm{EFT}} = \mathscr{L}_{\mathscr{D}\le 4} + \frac{\mathscr{L}_5}{\Lambda} + \frac{\mathscr{L}_6}{\Lambda^2} + \dots$$

power counting: canonical dim.

SMEFT provides systematic parametrization of

… all possible Lorentz inv. new physics!

SMEFT Operators

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

SM Lagrangian

Higher-dimensional $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM

$$\mathscr{L}_{D=2} = \mu_H^2 H^\dagger H$$

$$\mathscr{L}_{D=3} = 0$$

$$\begin{aligned}\mathscr{L}_{D=4} = & -\frac{1}{4}\sum_{V\in B,W^i,G^a} V_{\mu\nu}V^{\mu\nu} + \sum_{f\in q,u,d,L,e} i\bar{f}\gamma^\mu D_\mu f \\ & -\left(\bar{u}Y_u QH + \bar{d}Y_d H^\dagger Q + \bar{e}Y_e H^\dagger L + \mathrm{h.c.}\right) \\ & + D_\mu H^\dagger D^\mu H - \lambda(H^\dagger H)^2 + \bar{\theta}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}\end{aligned}$$

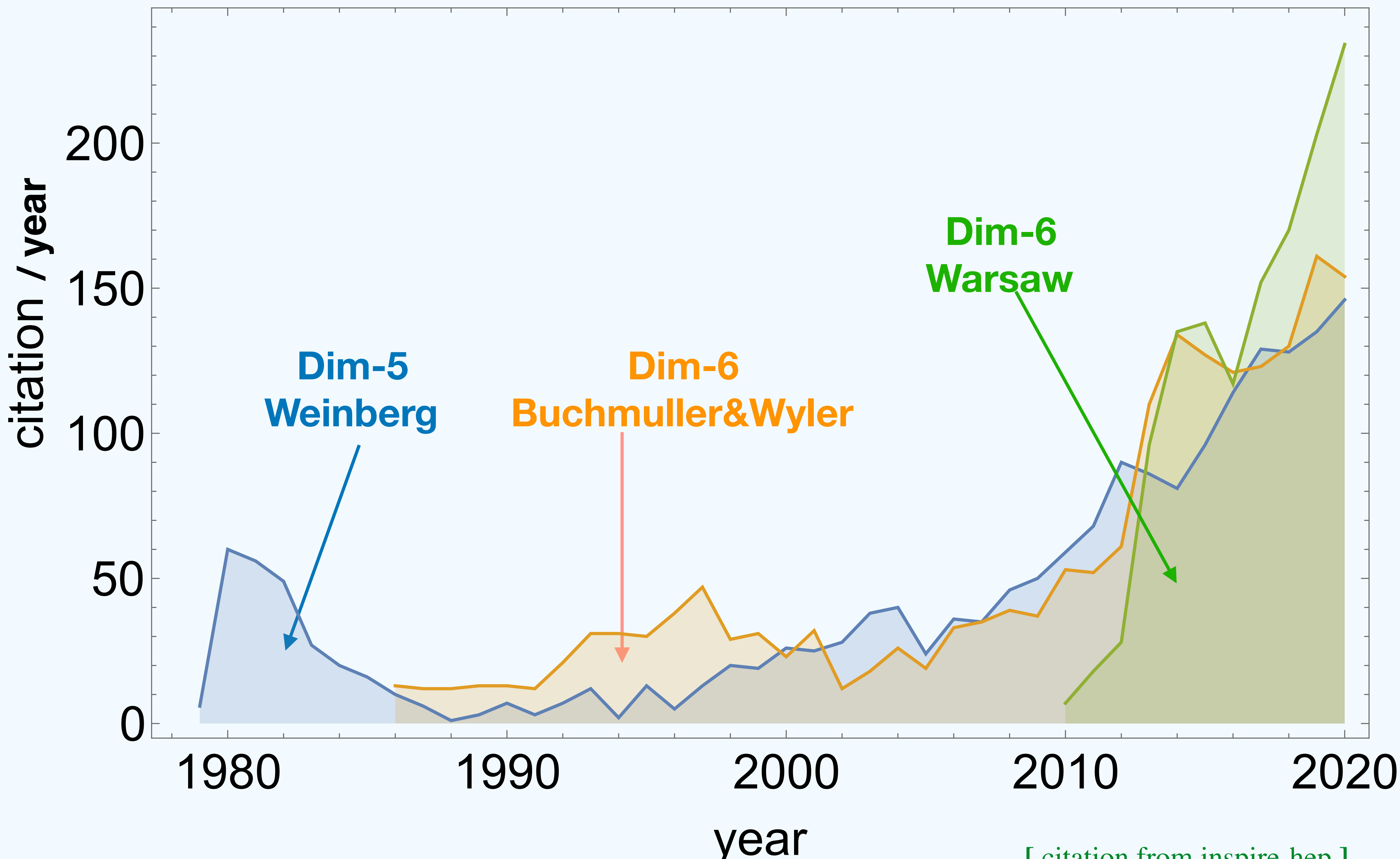
d.o.f: SM fields

symmetry: gauge/Lorentz

power counting: canonical dim.

In the spirit of EFT, each $\mathscr{L}_D$ should include a complete and non-redundant set of interactions

SMEFT Operators



[citation from inspire-hep]

Dim-5 Weinberg Operator

The only operator at the dimension 5

Weinberg (1979)
Phys. Rev. Lett. 43, 1566

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \ldots$$

$$\frac{c_{ij}}{\Lambda}(L_i H)(L_j H) + \text{h.c.} \to c_{ij}\frac{v^2}{\Lambda}\nu_i \nu_j + \text{h.c.}$$

$$H \to \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad L_i \to \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$$

Lepton number violation **Neutrino Majorana mass!**

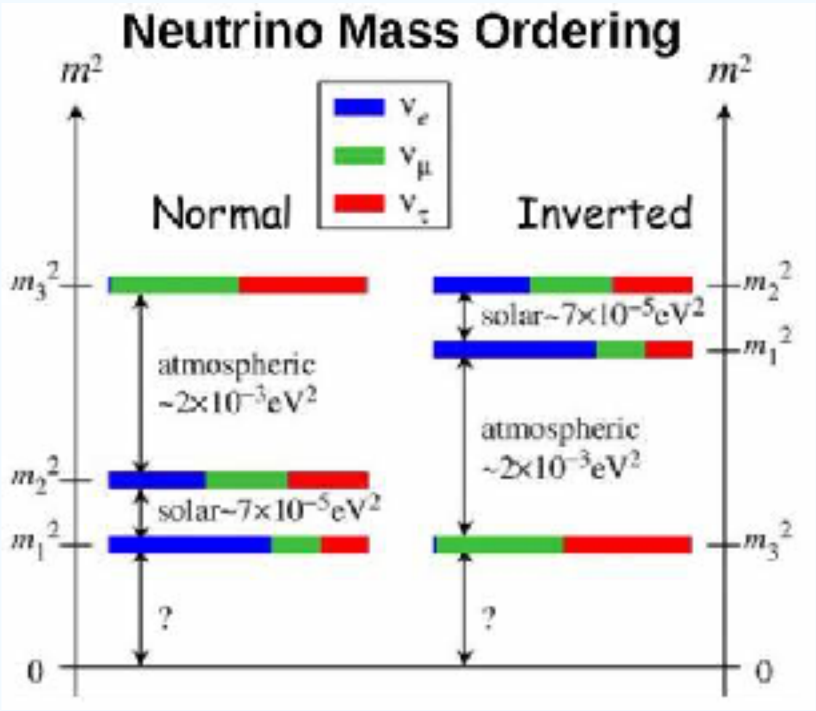
$N+h$ \ $N-h$	1	3	5
1		$\psi^2 D^2,\ F\phi D^2$	$F^2\phi,\ F\psi^2$
3	$\bar\psi^2 D^2,\ \bar F\phi D^2$	$\phi^3 D^2,\ \bar\psi\psi\phi D$	$\psi^2\phi^2$
5	$\bar F^2\phi,\ \bar F\bar\psi^2$	$\bar\psi^2\phi^2$	ϕ^5

Odd power of scalar, and SU(2)L transformation $\bar\psi_L \sigma^{\mu\nu}\psi_R$

Red color: eliminated by equation of motion

Dim-5 Operator

Dim-5 neutrino masses predicted by SMEFT and later observed!



$$\mathscr{L}_{\rm SMEFT} \supset c_{ij}\frac{v^2}{\Lambda}\nu_i\nu_j + \text{h.c.}$$

0.01 eV - 0.1 eV

$$\frac{\Lambda}{c_{ij}} \sim 10^{15}\ \text{GeV}$$

Naively: $\mathscr{L}_{D=5} \sim \frac{1}{\Lambda}$ **and then** $\mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}$, $\mathscr{L}_{D=7} \sim \frac{1}{\Lambda^3}$, **and so on**

It is however possible that Λ is not far from TeV, but instead $c_{ij} \ll 1$

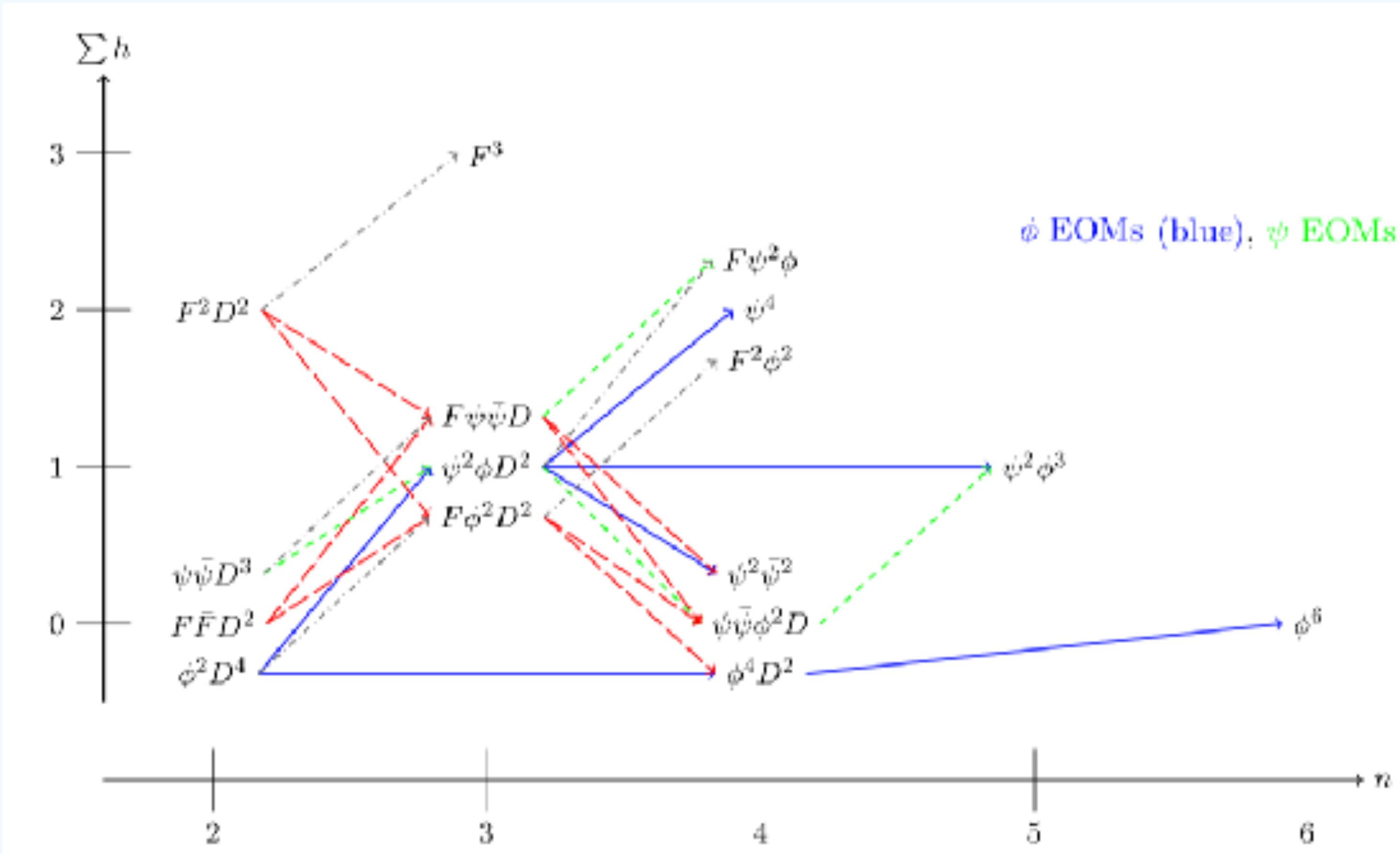
Alternatively, it is possible (and likely) that there is more than one mass scale of new physics

$\mathscr{L}_{D=5} \sim \frac{1}{\Lambda_L}$, $\mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}$, $\mathscr{L}_{D=7} \sim \frac{1}{\Lambda_L^3}$, $\mathscr{L}_{D=8} \sim \frac{1}{\Lambda^4}$, **and so on**

Dim-6 Operators

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

$N+h$ \ $N-h$	0	2	4	6
0			F^2D^2	F^3
2		$\bar\psi\psi D^3$, $\bar F F D^2$, $\phi^2 D^4$	$F\bar\psi\psi D$, $\psi^2\phi D^2$, $F\phi^2 D^2$	$F^2\phi^2$, $F\psi^2\phi$, ψ^4
4	$\bar F^2 D^2$	$\bar F\psi\bar\psi D$, $\bar\psi^2\phi D^2$, $\bar F\phi^2 D^2$	$\bar\psi\psi\phi^2 D$, $\bar\psi^2\psi^2$, $\phi^4 D^2$	$\psi^2\phi^3$
6	$\bar F^3$	$\bar F^2\phi^2$, $\bar F\bar\psi^2\phi$, $\bar\psi^4$	$\bar\psi^2\phi^3$	ϕ^6

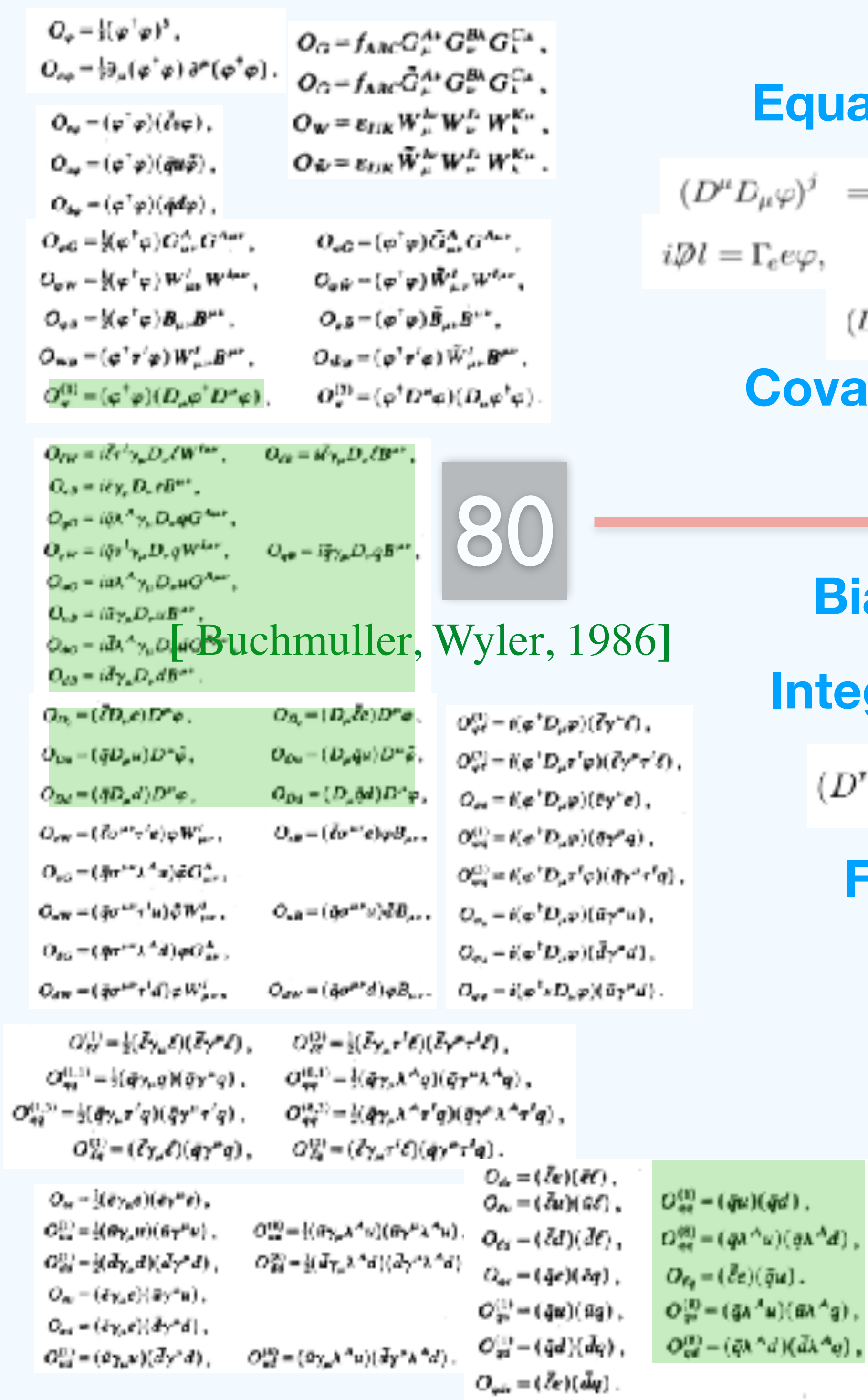


- 59 independent $\mathcal{O}_i^{(6)}$ preserving B and L (for 1 generation)
  Buchmuller-Wyler, Grzadkowski-Iskrzynski-Misiak-Rosiek
- 5 independent $\mathcal{O}_i^{(6)}$ violating B and L (for 1 generation)
  Weinberg, Wilczek-Zee, Abbott-Wise
- 3 generations: 1350 CP-even and 1149 CP-odd operators with D=6
  Alonso-Jenkins-Manohar-Trott

Dim-6 Operators

Why completing dim-6 took more than 25 years?

tedious and prone-to-error



Equation of motion (Field redefinition)

$$(D^\mu D_\mu \varphi)^j = m^2 \varphi^j - \lambda(\varphi^\dagger \varphi)\varphi^j - \bar{e}\Gamma_e^\dagger l^j + \varepsilon_{jk}\bar{q}^k \Gamma_u u - \bar{d}\Gamma_d^\dagger q^j$$

$$i\not{D} l = \Gamma_e e \varphi, \quad i\not{D} e = \Gamma_e^\dagger \varphi^\dagger l, \quad i\not{D} q = \Gamma_u u \tilde{\varphi} + \Gamma_d d \varphi, \quad i\not{D} u = \Gamma_u^\dagger \tilde{\varphi}^\dagger q,$$

$$(D^\rho W_{\rho\mu})^I = \frac{g}{2}\left(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi + \bar{l}\gamma_\mu \tau^I l + \bar{q}\gamma_\mu \tau^I q\right),$$

Covariant derivative commutator

$$[D_\rho, D_\alpha] \sim X_{\rho\alpha}$$

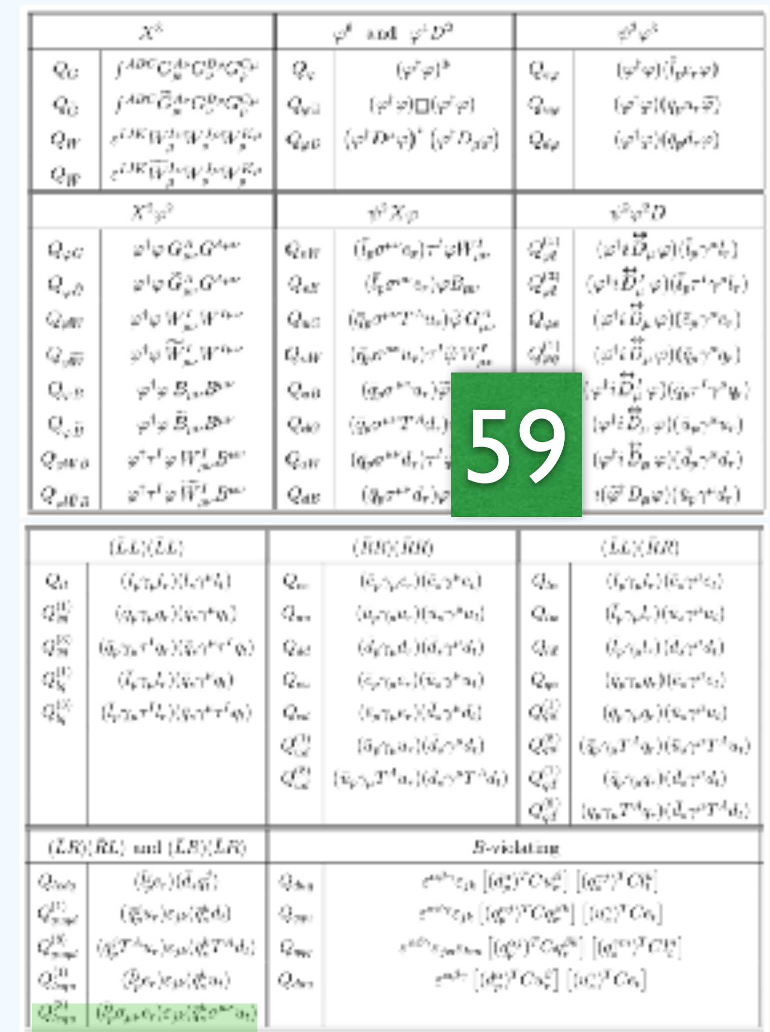
Bianchi identity $D_{[\rho} X_{\mu\nu]} = 0$

Integration by part (total derivatives)

$$(D^n\varphi)^\dagger (D^m\varphi) = -(D^{n+1}\varphi)^\dagger (D^{m-1}\varphi) + \partial\left[(D^n\varphi)^\dagger (D^{m-1}\varphi)\right]$$

Fierz identity $T^A_{\alpha\beta} T^A_{\kappa\lambda} = \frac{1}{2}\delta_{\alpha\lambda}\delta_{\kappa\beta} - \frac{1}{6}\delta_{\alpha\beta}\delta_{\kappa\lambda}$

$$\tau^I_{jk}\tau^I_{mn} = 2\delta_{jn}\delta_{mk} - \delta_{jk}\delta_{mn}$$



[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

80-1-16-5+1 = 59

Exercise: EOM Redundancy

$$i\not{D} l = \Gamma_e e \varphi, \quad i\not{D} e = \Gamma_e^\dagger \varphi^\dagger l, \quad i\not{D} q = \Gamma_u u \tilde{\varphi} + \Gamma_d d \varphi, \quad i\not{D} u = \Gamma_u^\dagger \tilde{\varphi}^\dagger q, \quad i\not{D} d = \Gamma_d^\dagger \varphi^\dagger q. \tag{6.2}$$

$$\gamma_\mu \gamma_\nu = g_{\mu\nu} - i\sigma_{\mu\nu}, \qquad \gamma_\mu \gamma_\nu \gamma_\rho = g_{\mu\nu}\gamma_\rho + g_{\nu\rho}\gamma_\mu - g_{\mu\rho}\gamma_\nu - i\varepsilon_{\mu\nu\rho\sigma}\gamma^\sigma \gamma_5 . \tag{6.3}$$

$$\begin{aligned}
(D^\mu D_\mu \varphi)^j &= m^2 \varphi^j - \lambda(\varphi^\dagger \varphi)\varphi^j - \bar{e}\Gamma_e^\dagger l^j + \varepsilon_{jk}\bar{q}^k \Gamma_u u - \bar{d}\Gamma_d^\dagger q^j, \\
(D^\rho G_{\rho\mu})^A &= g_s (\bar{q}\gamma_\mu T^A q + \bar{u}\gamma_\mu T^A u + \bar{d}\gamma_\mu T^A d), \\
(D^\rho W_{\rho\mu})^I &= \frac{g}{2}\left(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi + \bar{l}\gamma_\mu \tau^I l + \bar{q}\gamma_\mu \tau^I q\right), \\
\partial^\rho B_{\rho\mu} &= g' Y_\varphi \varphi^\dagger i \overleftrightarrow{D}_\mu \varphi + g' \sum_{\psi \in \{l,e,q,u,d\}} Y_\psi \bar{\psi}\gamma_\mu \psi.
\end{aligned} \tag{5.1}$$

$$\begin{aligned}
\bar{\psi}\psi D_\mu D^\mu \varphi &\overset{(5.1)}{=} \boxed{\psi^4} + \boxed{\psi^2\varphi^3} + m^2 \boxed{\psi^2\varphi} + \boxed{E}, \\
\varphi \bar{\psi} D_\mu D^\mu \psi &\overset{(6.3)}{=} \varphi \bar{\psi} \not{D}\not{D} \psi + \boxed{\psi^2 X \varphi} \overset{(6.2)}{=} \boxed{\psi^2 X \varphi} + \boxed{\psi^2 \varphi^2 D} + \boxed{E}, \\
(D_\mu \varphi)\bar{\psi}\sigma^{\mu\nu} D_\nu \psi &= \frac{i}{2}(D_\mu \varphi)\bar{\psi}(\gamma^\mu \not{D} - \not{D}\gamma^\mu)\psi = i(D_\mu\varphi)\bar{\psi}\gamma^\mu \not{D}\psi - i(D^\mu\varphi)\bar{\psi}D_\mu\psi \\
&\overset{(6.2)}{=} -i(D^\mu\varphi)\bar{\psi}D_\mu\psi + \boxed{\psi^2\varphi^2 D} + \boxed{E}, \\
2(D^\mu\varphi)\bar{\psi}D_\mu\psi &= (D^\mu\varphi)\bar{\psi}(\gamma_\mu \not{D} + \not{D}\gamma_\mu)\psi \\
&= (D^\mu\varphi)\bar{\psi}\gamma_\mu \not{D}\psi - \bar{\psi}\overleftarrow{\not{D}}\gamma_\mu \psi\, D^\mu\varphi - \bar{\psi}\gamma^\nu\gamma^\mu\psi\, D_\nu D_\mu \varphi + \boxed{T} \\
&\overset{(6.2)}{=} \boxed{\psi^2\varphi^2 D} + \boxed{\psi^4} + \boxed{\psi^2\varphi^3} + m^2\boxed{\psi^2\varphi} + \boxed{\psi^2 X\varphi} + \boxed{E} + \boxed{T},
\end{aligned}$$

$$\begin{aligned}
X^{\mu\nu}\bar{\psi}\gamma_\mu D_\nu \psi &= \frac{1}{2}X^{\mu\nu}\bar{\psi}(\gamma_\mu\gamma_\nu \not{D} + \gamma_\mu \not{D}\gamma_\nu)\psi = \frac{1}{2}X^{\mu\nu}\bar{\psi}(\gamma_\mu\gamma_\nu\not{D} - \not{D}\gamma_\mu\gamma_\nu)\psi + X^{\mu\nu}\bar{\psi}\gamma_\nu D_\mu \psi \\
&\overset{(*)}{=} \frac{1}{4}X^{\mu\nu}\bar{\psi}(\gamma_\mu\gamma_\nu\not{D} - \not{D}\gamma_\mu\gamma_\nu)\psi = \frac{1}{4}X^{\mu\nu}\bar{\psi}\gamma_\mu\gamma_\nu\not{D}\psi + \frac{1}{4}\bar{\psi}\overleftarrow{\not{D}}\gamma_\mu\gamma_\nu\psi\, X^{\mu\nu} \\
&+ \frac{1}{4}\bar{\psi}\gamma_\rho\gamma_\mu\gamma_\nu\psi\, D^\rho X^{\mu\nu} + \boxed{T} \overset{(6.2)}{=} \boxed{\psi^2 X\varphi} + \boxed{\psi^2\varphi^2 D} + \boxed{\psi^4} + \boxed{E} + \boxed{T}.
\end{aligned} \tag{6.6}$$

$$\begin{aligned}
(\varphi^\dagger \tau^I \varphi)\left[(D_\mu\varphi)^\dagger \tau^I (D^\mu\varphi)\right] &\overset{(4.3)}{=} 2(\varphi^\dagger D^\mu \varphi)^*(\varphi^\dagger D_\mu\varphi) - (\varphi^\dagger\varphi)\left[(D_\mu\varphi)^\dagger(D^\mu\varphi)\right], \\
(\varphi^\dagger\varphi)\left[(D_\mu\varphi)^\dagger(D^\mu\varphi)\right] &\overset{(5.1)}{=} \frac{1}{2}(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi) + \boxed{\psi^2\varphi^3} + \boxed{\varphi^6} + m^2\boxed{\varphi^4} + \boxed{E}.
\end{aligned}$$

$$\overset{(\sim)}{X}{}^{\mu\nu} D^\rho D_\rho X_{\mu\nu} = -\overset{(\sim)}{X}{}^{\mu\nu}(D^\rho D_\mu X_{\nu\rho} + D^\rho D_\nu X_{\rho\mu}) = \boxed{X^3} + \boxed{\varphi^2 X D^2} + \boxed{\psi^2 X D} + \boxed{E}.$$

Exercise: Fierz Identity

$$\begin{pmatrix} \delta_{ij}\delta_{kl} \\ (\gamma^\mu)_{ij}(\gamma_\mu)_{kl} \\ \frac{1}{2}(\sigma^{\mu\nu})_{ij}(\sigma_{\mu\nu})_{kl} \\ (\gamma^\mu\gamma_5)_{ij}(\gamma_\mu\gamma_5)_{kl} \\ (\gamma_5)_{ij}(\gamma_5)_{kl} \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & -1/4 & 1/4 \\ 1 & -1/2 & 0 & -1/2 & -1 \\ 3/2 & 0 & -1/2 & 0 & 3/2 \\ -1 & -1/2 & 0 & -1/2 & 1 \\ 1/4 & -1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \begin{pmatrix} \delta_{il}\delta_{kj} \\ (\gamma^\mu)_{il}(\gamma_\mu)_{kj} \\ \frac{1}{2}(\sigma^{\mu\nu})_{il}(\sigma_{\mu\nu})_{kj} \\ (\gamma^\mu\gamma_5)_{il}(\gamma_\mu\gamma_5)_{kj} \\ (\gamma_5)_{il}(\gamma_5)_{kj} \end{pmatrix}$$

$$(\bar\psi_L\gamma_\mu\psi_L)(\bar\chi_L\gamma^\mu\chi_L) = (\bar\psi_L\gamma_\mu\chi_L)(\bar\chi_L\gamma^\mu\psi_L) \tag{4.1}$$

$$\tau^I_{jk}\tau^I_{mn} = 2\delta_{jn}\delta_{mk} - \delta_{jk}\delta_{mn} \tag{4.3}$$

$$T^A_{\alpha\beta}T^A_{\kappa\lambda} = \frac{1}{2}\delta_{\alpha\lambda}\delta_{\kappa\beta} - \frac{1}{6}\delta_{\alpha\beta}\delta_{\kappa\lambda}\,, \tag{7.3}$$

$$(\bar l_p\gamma_\mu\tau^I l_r)(\bar l_s\tau^I\gamma^\mu l_t) = 2(\bar l^j_p\gamma_\mu l^k_r)(\bar l^k_s\gamma^\mu l^j_t) - Q_{ll}^{prst} = 2Q_{ll}^{ptsr} - Q_{ll}^{prst}$$

$$(\bar u_p\gamma_\mu T^A u_r)(\bar u_s T^A\gamma^\mu u_t) \overset{(7.3)}{=} \frac{1}{2}(\bar u^\alpha_p\gamma_\mu u^\beta_r)(\bar u^\beta_s\gamma^\mu u^\alpha_t) - \frac{1}{6}Q_{uu}^{prst} = \frac{1}{2}Q_{uu}^{ptsr} - \frac{1}{6}Q_{uu}^{prst}, \tag{7.4}$$

$$(\bar d_p\gamma_\mu T^A d_r)(\bar d_s T^A\gamma^\mu d_t) \overset{(7.3)}{=} \frac{1}{2}(\bar d^\alpha_p\gamma_\mu d^\beta_r)(\bar d^\beta_s\gamma^\mu d^\alpha_t) - \frac{1}{6}Q_{dd}^{prst} = \frac{1}{2}Q_{dd}^{ptsr} - \frac{1}{6}Q_{dd}^{prst}, \tag{7.5}$$

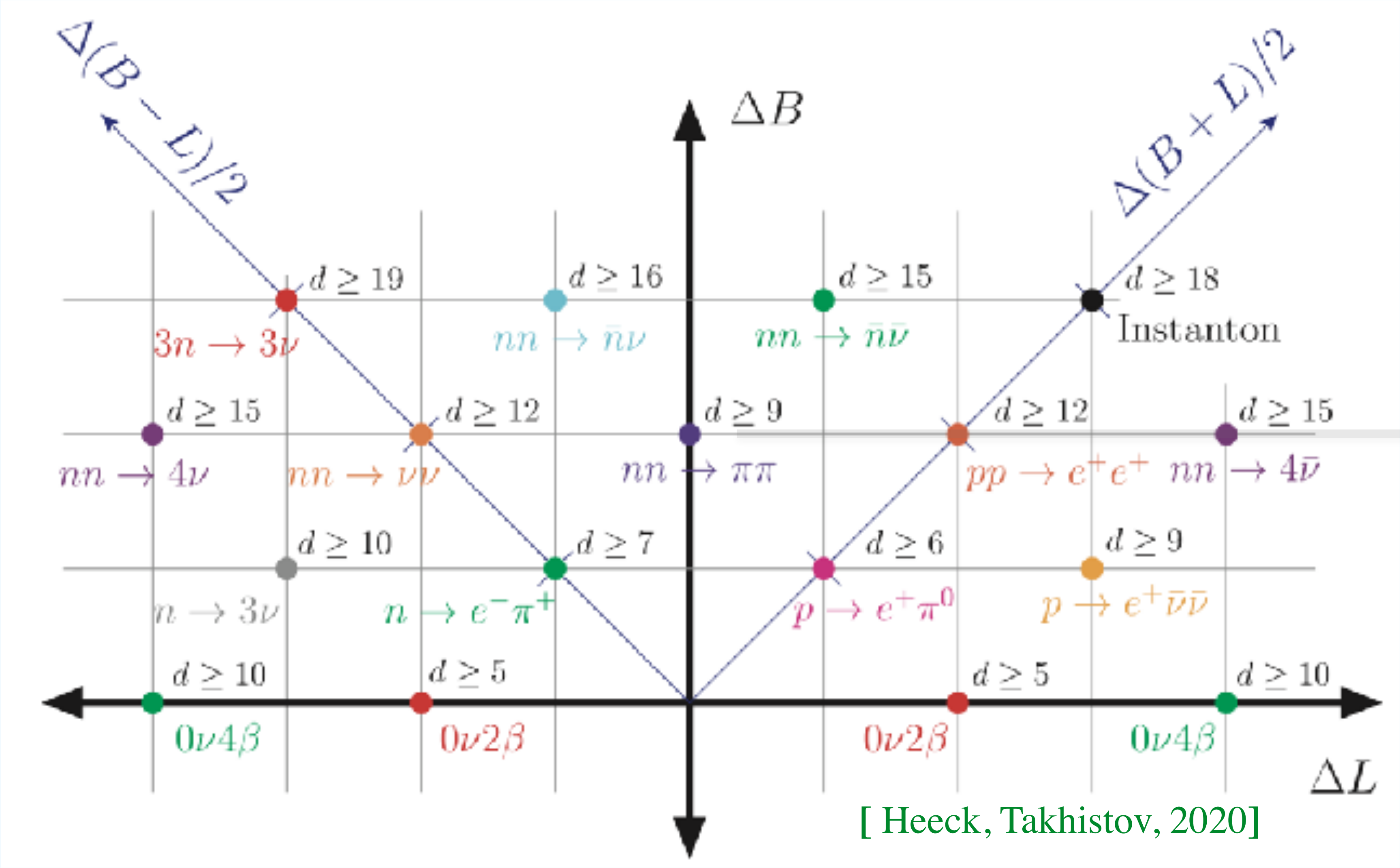
$$\begin{aligned} (\bar q_p\gamma_\mu T^A q_r)(\bar q_s T^A\gamma^\mu q_t) &\overset{(7.3)}{=} \frac{1}{2}(\bar q^{\alpha j}_p\gamma_\mu q^{\beta j}_r)(\bar q^{\beta k}_s\gamma^\mu q^{\alpha k}_t) - \frac{1}{6}Q_{qq}^{(1)prst} \\ &\overset{(4.1)}{=} \frac{1}{2}(\bar q^{\alpha j}_p\gamma_\mu q^{\alpha k}_t)(\bar q^{\beta k}_s\gamma^\mu q^{\beta j}_r) - \frac{1}{6}Q_{qq}^{(1)prst} \\ &\overset{(4.3)}{=} \frac{1}{4}Q_{qq}^{(3)ptsr} + \frac{1}{4}Q_{qq}^{(1)ptsr} - \frac{1}{6}Q_{qq}^{(1)prst}, \end{aligned} \tag{7.6}$$

$$\begin{aligned} (\bar q_p\gamma_\mu T^A\tau^I q_r)(\bar q_s T^A\tau^I\gamma^\mu q_t) &\overset{(7.3)}{=} \frac{1}{2}(\bar q^\alpha_p\gamma_\mu\tau^I q^\beta_r)(\bar q^\beta_s\gamma^\mu\tau^I q^\alpha_t) - \frac{1}{6}Q_{qq}^{(3)prst} \\ &\overset{(4.3)}{=} (\bar q^{\alpha j}_p\gamma_\mu q^{\beta k}_r)(\bar q^{\beta k}_s\gamma^\mu q^{\alpha j}_t) - \frac{1}{2}(\bar q^{\alpha j}_p\gamma_\mu q^{\beta j}_r)(\bar q^{\beta k}_s\gamma^\mu q^{\alpha k}_t) - \frac{1}{6}Q_{qq}^{(3)prst} \\ &\overset{(4.1)}{=} Q_{qq}^{(1)ptsr} - \frac{1}{2}(\bar q^{\alpha j}_p\gamma_\mu q^{\alpha k}_t)(\bar q^{\beta k}_s\gamma^\mu q^{\beta j}_r) - \frac{1}{6}Q_{qq}^{(3)prst} \\ &\overset{(4.3)}{=} -\frac{1}{4}Q_{qq}^{(3)ptsr} + \frac{3}{4}Q_{qq}^{(1)ptsr} - \frac{1}{6}Q_{qq}^{(3)prst}. \end{aligned} \tag{7.7}$$

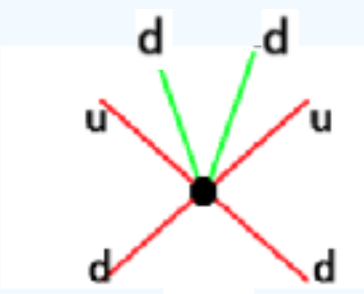
Why High Dim Operator?

new physics without new particle: neutrino masses and baryon asymmetry

B and L violation

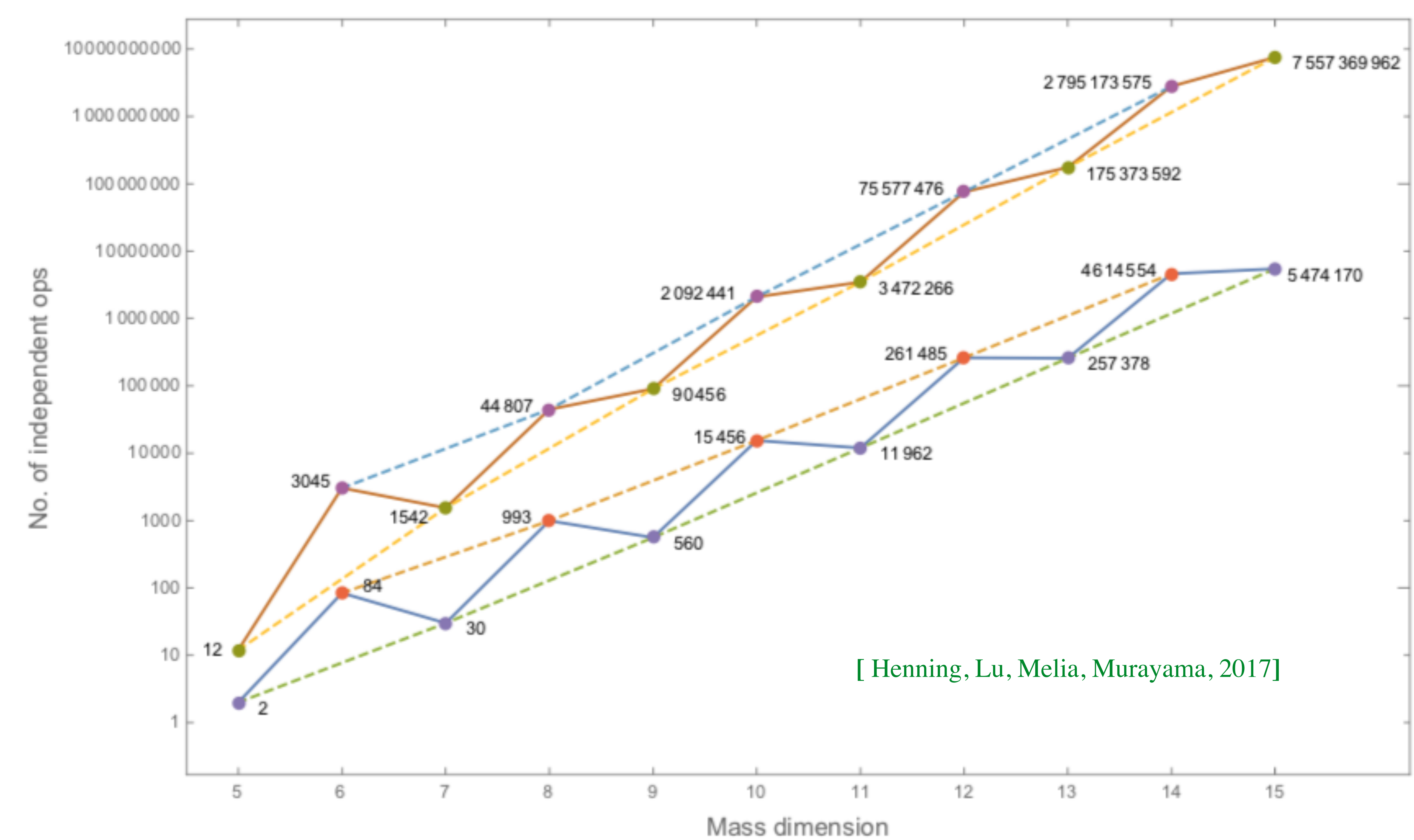


n-nbar oscillation



Dim-9

SMEFT Operators



Main Difficulties

How about higher dimensional operators?

difficult to write down explicit form of operators

Derivatives

$BWHH^\dagger D^2$

2

$$\begin{aligned}
&(D^2 H^\dagger) H B_{L\mu\nu} W_L^{\mu\nu},\ (D^\mu D_\nu H^\dagger) H B_{L\mu\rho} W_L^{\nu\rho},\ (D_\nu D^\mu H^\dagger) H B_{L\mu\rho} W_L^{\nu\rho},\ (D_\mu H^\dagger)(D^\nu H) B_{L\nu\rho} W_L^{\mu\rho},\\
&(D_\mu H^\dagger)(D^\mu H) B_{L\nu\rho} W_L^{\nu\rho},\ (D^\nu H^\dagger)(D_\mu H) B_{L\nu\rho} W_L^{\mu\rho},\ (D_\mu H^\dagger) H (D^\nu B_{L\nu\rho}) W_L^{\mu\rho},\ (D_\mu H^\dagger) H (D^\nu B_{L\nu\rho}) W_L^{\mu\rho},\\
&(D^\nu H^\dagger) H (D_\mu B_{L\nu\rho}) W_L^{\mu\rho},\ (D_\nu H^\dagger) H B_{L\mu\rho} (D^\mu W_L^{\nu\rho}),\ (D_\mu H^\dagger) H B_{L\nu\rho} (D^\nu W_L^{\mu\rho}),\ (D^\nu H^\dagger) H B_{L\nu\rho} (D_\mu W_L^{\mu\rho}),\\
&H^\dagger (D^2 H) B_{L\mu\nu} W_L^{\mu\nu},\ H^\dagger (D^\mu D_\nu H) B_{L\mu\rho} W_L^{\nu\rho},\ H^\dagger (D_\nu D^\mu H) B_{L\mu\rho} W_L^{\nu\rho},\ H^\dagger (D^\mu H)(D_\nu B_{L\mu\rho}) W_L^{\nu\rho},\\
&H^\dagger (D^\nu H)(D_\mu B_{L\nu\rho}) W_L^{\mu\rho},\ H^\dagger (D_\mu H)(D^\nu B_{L\nu\rho}) W_L^{\mu\rho},\ H^\dagger (D^\mu H) B_{L\nu\rho} (D_\mu W_L^{\nu\rho}),\ H^\dagger (D^\nu H) B_{L\nu\rho} (D_\mu W_L^{\mu\rho}),\\
&H^\dagger (D_\nu H) B_{L\mu\rho} (D^\nu W_L^{\mu\rho}),\ H^\dagger H (D^2 B_{L\mu\nu}) W_L^{\mu\nu},\ H^\dagger H (D^\mu D_\nu B_{L\mu\rho}) W_L^{\nu\rho},\ H^\dagger H (D_\nu D^\mu B_{L\mu\rho}) W_L^{\nu\rho},\\
&H^\dagger H (D^\mu B_{L\nu\rho})(D_\mu W_L^{\nu\rho}),\ H^\dagger H (D^\nu B_{L\nu\rho})(D_\mu W_L^{\mu\rho}),\ H^\dagger H (D_\nu B_{L\mu\rho})(D^\mu W_L^{\nu\rho}),\ H^\dagger H B_{L\mu\nu} (D^2 W_L^{\mu\nu}),\\
&H^\dagger H B_{L\mu\rho} (D^\mu D_\nu W_L^{\nu\rho}),\ H^\dagger H B_{L\mu\rho} (D_\nu D^\mu W_L^{\nu\rho}) \qquad (14)
\end{aligned}$$

30

Which 2 should be picked up?

Repeated fields

$QQQL$

57

$$Q^{qqq\ell}_{prst} = C^{prst} \begin{array}{l} \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl}) \\ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}) \end{array} \qquad p,r,s,t = 1,2,3$$

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What flavor relations should be imposed?

Operator as Spinor Tensor

Operator has more symmetries than what we expect

SO(3,1)		SL(2,C) $SU(2)_l \times SU(2)_r$	Spinor-helicity
ϕ		$\phi \in (0,0)$	
ψ		$\psi_\alpha \in (1/2,0)$	λ_α
		$\psi^\dagger_{\dot\alpha} \in (0,1/2)$,	
$F_{\mu\nu}$	→	$F_{L\alpha\beta} = \frac{i}{2} F_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta} \in (1,0)$	$\lambda_\alpha\lambda_\beta$
		$F_{R\dot\alpha\dot\beta} = -\frac{i}{2} F_{\mu\nu}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \in (0,1)$.	
$R_{\mu\nu\rho\sigma}$		$C_{\alpha\beta\gamma\delta} = C_{\mu\nu\rho\sigma}\sigma^{\mu\nu}_{\alpha\beta}\sigma^{\rho\sigma}_{\gamma\delta} \in (2,0)$	$\lambda_\alpha\lambda_\beta\lambda_\gamma\lambda_\delta$
D_μ		$D_{\alpha\dot\alpha} = D_\mu \sigma^\mu_{\alpha\dot\alpha} \in (1/2,1/2)$,	$\lambda_\alpha\tilde\lambda_{\dot\alpha}$

Operator with explicit spinor indices

$$W_{\mu\lambda}\left(e_{cp}\sigma^{\nu\lambda}D^\mu L_r\right)D_\nu H^\dagger \rightarrow$$

$$F_1^{\alpha\beta}\psi_2^{\gamma}(D\psi_3)_{\alpha\beta\dot\alpha}(D\phi_4)_\gamma{}^{\dot\alpha}$$

$$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}F_1^{\alpha_1^2}\psi_2^{\alpha_2}(D\psi_3)^{\alpha_3^2}_{\dot\alpha_3}(D\phi_4)^{\alpha_4}_{\dot\alpha_4}$$

Easier to find more symmetries of the operator with spinor indices

Operator as Spinor Tensor

Modern view: operator as Young tensor and on-shell amplitude, with spin-statistics

[Li, Ren, Shu, Xiao, **Yu**, Zheng, 2005.00008]
[Li, Ren, Xiao, **Yu**, Zheng, 2007.07899]
[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]

Operator

$W_{\mu\lambda}\left(e_{cp}\sigma^{\nu\lambda}D^{\mu}L_r\right)D_{\nu}H^{\dagger}$

↓

Spinor Tensor → **Symmetrize indices**

$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}F_1^{\alpha_1^2}\psi_2^{\alpha_2}\left(D\psi_3\right)^{\alpha_3^2}_{\dot\alpha_3}\left(D\phi_4\right)^{\alpha_4}_{\dot\alpha_4}$

$$(D\psi)_{\alpha\beta\dot\alpha} = -\frac{1}{2}\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot\alpha} + \frac{1}{2}(D\psi)_{(\alpha\beta)\dot\alpha}$$

$$\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left(\frac{1}{2},0\right) = \left(0,\frac{1}{2}\right)\oplus\left(1,\frac{1}{2}\right)$$

$$(D^2\phi)_{\alpha\beta\dot\alpha\dot\beta} = \frac{1}{2}\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta}D^{\mu}D_{\mu}\phi - \frac{i}{4}\epsilon_{\dot\alpha\dot\beta}\sigma^{\mu\nu}_{\alpha\beta}[D_{\mu},D_{\nu}]\phi - \frac{i}{4}\epsilon_{\alpha\beta}\bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta}[D_{\mu},D_{\nu}]\phi + \frac{1}{4}(D^2\phi)_{(\alpha\beta)(\dot\alpha\dot\beta)}$$

$\left(\frac{1}{2},\frac{1}{2}\right)\times\left(\frac{1}{2},\frac{1}{2}\right)$ **(0,0)** **(1,0)** **(0,1)** **(1,1)**

Operator as Spinor Tensor

Modern view: operator as Young tensor and on-shell amplitude, with spin-statistics

Operator

$$W_{\mu\lambda}\left(e_{cp}\sigma^{\nu\lambda}D^{\mu}L_r\right)D_{\nu}H^{\dagger}$$

↓

Spinor Tensor → **Symmetrize indices**

$$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}F_1^{\alpha_1^2}\psi_2^{\alpha_2}\left(D\psi_3\right)^{\alpha_3^2}_{\dot\alpha_3}\left(D\phi_4\right)^{\alpha_4}_{\dot\alpha_4}$$

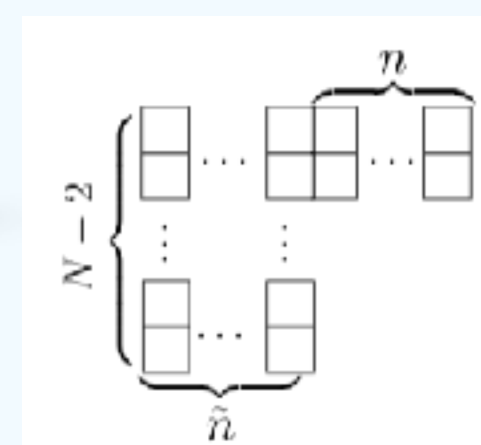
$$(D\psi)_{\alpha\beta\dot\alpha} = -\frac{1}{2}\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot\alpha} + \frac{1}{2}(D\psi)_{(\alpha\beta)\dot\alpha}$$

↓

SL(2,C) x SU(N)

$$\epsilon^{\alpha_i\alpha_j} \to \sum_{k,l}\mathcal{U}^i_k\mathcal{U}^j_l\epsilon^{\alpha_k\alpha_l}, \quad \tilde\epsilon_{\dot\alpha_i\dot\alpha_j} \to \sum_{k,l}\mathcal{U}^{\dagger k}_{\ i}\mathcal{U}^{\dagger l}_{\ j}\tilde\epsilon_{\dot\alpha_k\dot\alpha_l}.$$

$$\mathcal{O} = (\epsilon^{\alpha_i\alpha_j})^n(\tilde\epsilon_{\dot\alpha_i\dot\alpha_j})^{\tilde n}\prod_{i=1}^{N}(D^{r_i-|h_i|}\Phi_i)^{\dot\alpha_i^{r_i+h_i}}_{\alpha_i^{r_i-h_i}}$$



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Operator

$$W_{\mu\lambda}\left(e_{cp}\sigma^{\nu\lambda}D^{\mu}L_r\right)D_{\nu}H^{\dagger}$$

↓

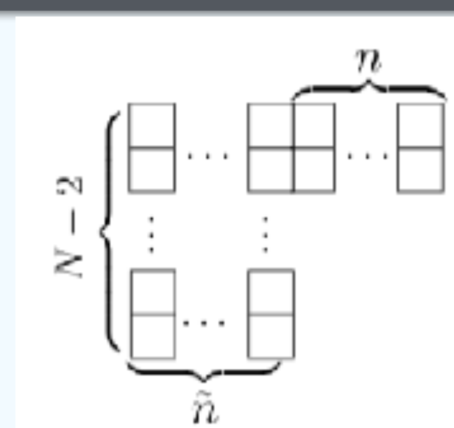
Spinor Tensor → **Symmetrize indices**

$$\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_1\alpha_3}\epsilon_{\alpha_2\alpha_4}\epsilon^{\dot\alpha_3\dot\alpha_4}F_1^{\alpha_1^2}\psi_2^{\alpha_2}\left(D\psi_3\right)^{\alpha_3^2}_{\dot\alpha_3}\left(D\phi_4\right)^{\alpha_4}_{\dot\alpha_4}$$

$$(D\psi)_{\alpha\beta\dot\alpha} = -\frac{1}{2}\epsilon_{\alpha\beta}(\not{D}\psi)_{\dot\alpha} + \frac{1}{2}(D\psi)_{(\alpha\beta)\dot\alpha}$$

↓

SL(2,C) x SU(N)



$$\{\overbrace{1,\ldots,1}^{\#1},\overbrace{2,\ldots,2}^{\#2},\ldots,\overbrace{N,\ldots,N}^{\#N}\}$$

$$\#i = \tilde{n} - 2h_i$$

SSYT = Amplitude

1	1	1	2
2	3	3	4

$\langle 13\rangle\langle 13\rangle\langle 24\rangle[34]$

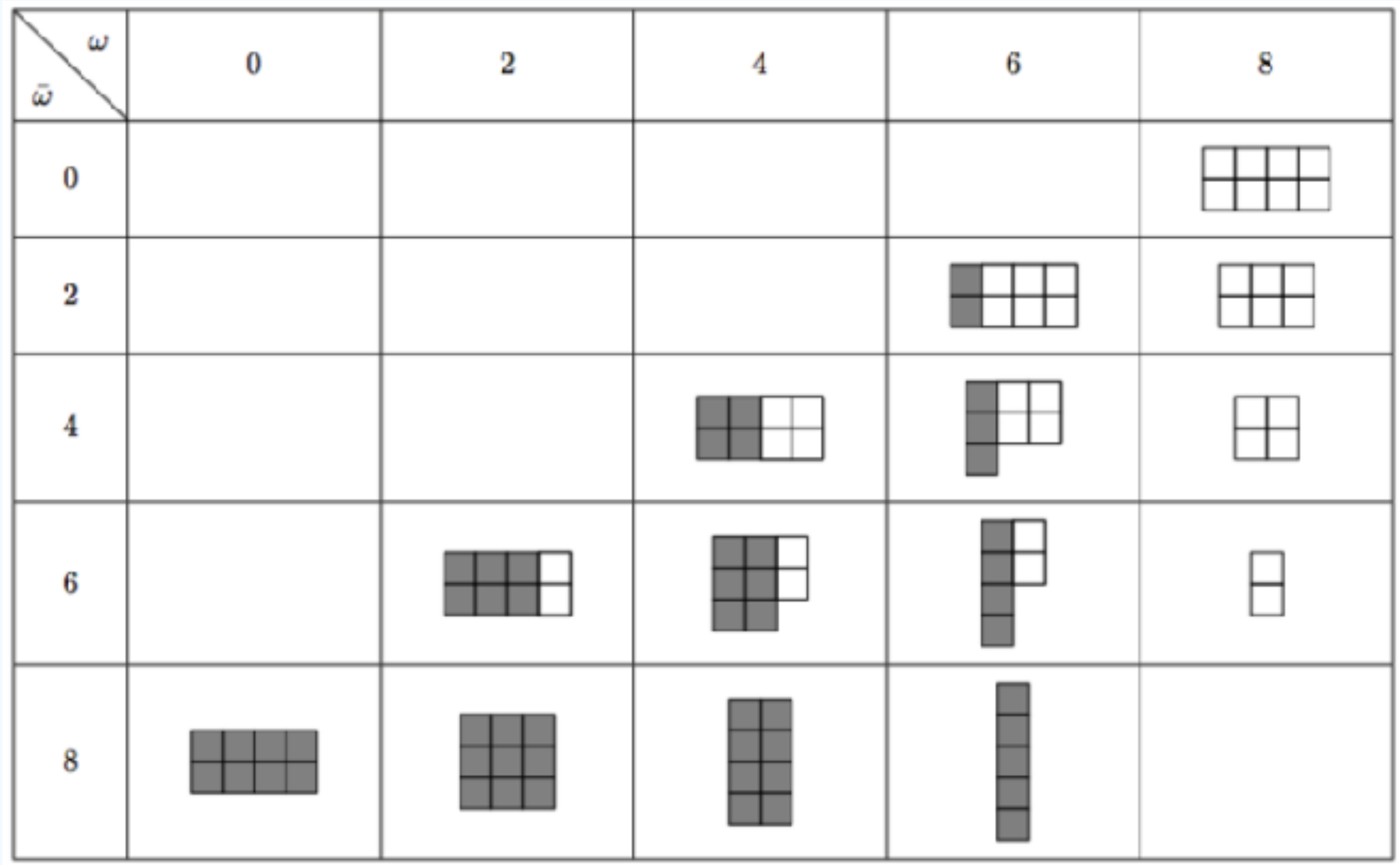
1	1	1	3
2	2	3	4

$\langle 12\rangle\langle 13\rangle\langle 34\rangle[34]$

On-shell

Operator as Spinor Tensor

Dim-8 operators: 993 (44807) operators for 1 (3) generations



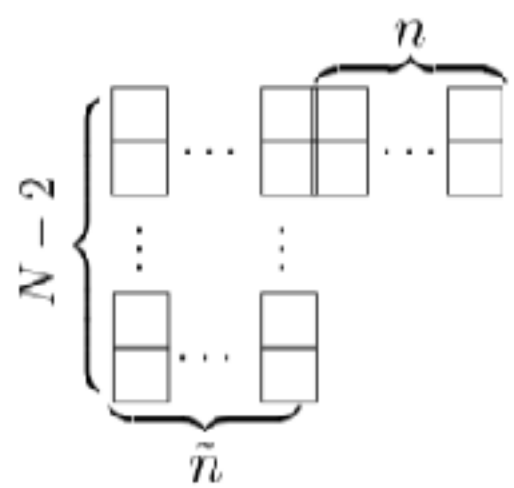
Unified construction of Lorentz & gauge structures by Young Tableau

$$\left(\begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 2 \\ \hline 2 & 3 & 3 & 4 \\ \hline\end{array} + \begin{array}{|c|c|c|c|}\hline 1 & 1 & 1 & 3 \\ \hline 2 & 2 & 3 & 4 \\ \hline\end{array}\right) \times \begin{array}{|c|c|}\hline i & j \\ \hline k & l \\ \hline\end{array} = \boxed{(\tau^I)^i_j W^I_{\mu\nu} (e_{\mathbb{C}p} D^\mu L_{ri}) D^\nu H^{\dagger j}} + \boxed{(\tau^I)^i_j W^I_{\mu\lambda} (e_{\mathbb{C}p} \sigma^{\nu\lambda} L_{ri}) D^\mu D_\nu H^{\dagger j}}$$

Complete Operator Basis

SSYT Filling forms a linear basis, which guarantees all operators found

$Y_{N,n,\tilde{n}} =$



semi-standard Young tableau (SSYT)

$$\{\underbrace{1,\ldots,1}_{\#1},\underbrace{2,\ldots,2}_{\#2},\ldots\} \qquad \#i = \tilde{n} - 2h_i$$

Basis
- **YT method guarantees independence!**
- **Filling all SSYT guarantees completeness!**

Can be cross-checked using the Lorentz/Poincare characters

Schur theorem: orthonormal with Haar measure integral $\int d\mu_G(g)\,\chi_{\mathbf{R}}(g)\,\chi^*_{\mathbf{R}'}(g) = \delta_{\mathbf{R}\mathbf{R}'}.$

$$\mathcal{H}(\phi_{\mathbf{R}},\ldots,\varphi_{\mathbf{R}'}) = \int d\mu_G\,\mathrm{PE}[\phi_{\mathbf{R}},\ldots,\varphi_{\mathbf{R}'}]\,.$$

Molien-Weyl formula

$$\frac{1}{(2\pi\mathrm{i})^2}\oint_{|y_1|=1}\frac{\mathrm{d}y_1}{y_1}(1-y_1^2) \times \oint_{|y_2|=1}\frac{\mathrm{d}y_2}{y_2}(1-y_2^2)$$

$(\frac{1}{2},0)$	$y_1+\frac{1}{y_1}$
$(0,\frac{1}{2})$	$y_2+\frac{1}{y_2}$
$(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	$y_1+\frac{1}{y_1}+y_2+\frac{1}{y_2}$
$(\frac{1}{2},\frac{1}{2})$	$(y_1+\frac{1}{y_1})(y_2+\frac{1}{y_2})$
$(1,0)\oplus(0,1)$	$y_1^2+1+\frac{1}{y_1^2}+(y_1\leftrightarrow y_2)$

Operator as Spinor Tensor

Young tensor method (No need EOM&IBP)

[Li, Ren, Xiao, **Yu**, Zheng, 2007.07899]
[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]
[Li, Ren, Shu, Xiao, **Yu**, Zheng, 2005.00008]

$BWHH^\dagger D^2$ $\quad \#1=3, \#2=3, \#3=1, \#4=1$

1	1	1	3
2	2	2	4

1	1	1	2
2	2	3	4

2

$\tilde{\epsilon}_{\dot\alpha_3\dot\alpha_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_3\alpha_4}$ $\qquad$ $\tilde{\epsilon}_{\dot\alpha_3\dot\alpha_4}\epsilon^{\alpha_1\alpha_2}\epsilon^{\alpha_1\alpha_3}\epsilon^{\alpha_2\alpha_4}$

$B_L{}^{\alpha\beta}W_{L\alpha\beta}\left(DH^\dagger\right)^{\gamma}{}_{\dot\alpha}\left(DH\right)_\gamma{}^{\dot\alpha},$ $\qquad$ $B_L{}^{\alpha\beta}W_{L\alpha}{}^{\gamma}\left(DH^\dagger\right)_{\beta\dot\alpha}\left(DH\right)_\gamma{}^{\dot\alpha}$

$\langle 13\rangle\langle 13\rangle\langle 24\rangle[34]$ $\qquad$ $\langle 12\rangle\langle 13\rangle\langle 34\rangle[34]$

All Things EFT...seminar series

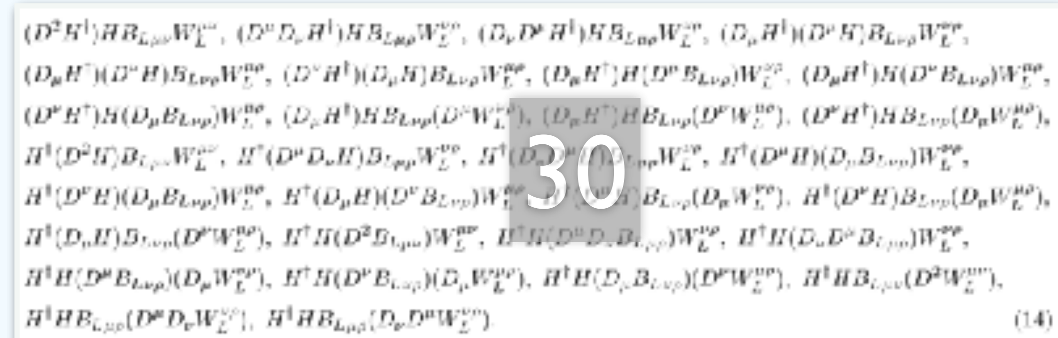
EFT Operator Bases for Standard Model and Beyond
报告时间：2021-06-09
报告人：于江浩

https://www.koushare.com/video/videodetail/12645

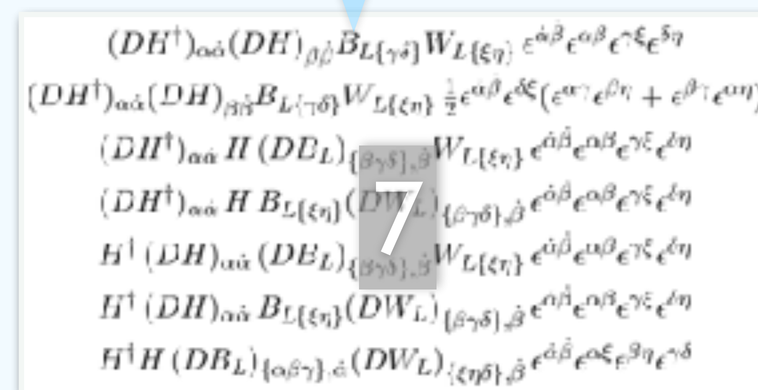
Traditional method

$BWHH^\dagger D^2$

[Hays, Martin, Sanz, Setford, 2018]



EOM


$(DH^\dagger)_{\alpha\dot\alpha}(DH)_{\beta\dot\beta}B_{L\{\gamma\delta\}}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$
$(DH^\dagger)_{\alpha\dot\alpha}(DH)_{\beta\dot\beta}B_{L\{\gamma\delta\}}W_{L\{\xi\eta\}}\,\frac{1}{2}\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}(\epsilon^{\alpha\gamma}\epsilon^{\beta\eta}+\epsilon^{\beta\gamma}\epsilon^{\alpha\eta})$
$(DH^\dagger)_{\alpha\dot\alpha}H\,(DB_L)_{\{\beta\gamma\delta\}\dot\beta}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$
$(DH^\dagger)_{\alpha\dot\alpha}H\,B_{L\{\xi\eta\}}(DW_L)_{\{\beta\gamma\delta\}\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$
$H^\dagger(DH)_{\alpha\dot\alpha}(DB_L)_{\{\beta\gamma\delta\}\dot\beta}W_{L\{\xi\eta\}}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$
$H^\dagger(DH)_{\alpha\dot\alpha}B_{L\{\xi\eta\}}(DW_L)_{\{\beta\gamma\delta\}\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}\epsilon^{\gamma\xi}\epsilon^{\delta\eta}$
$H^\dagger H\,(DB_L)_{\{\alpha\beta\gamma\}\dot\alpha}(DW_L)_{\{\xi\eta\delta\}\dot\beta}\,\epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\xi}\epsilon^{\beta\eta}\epsilon^{\gamma\delta}$

IBP

$B_L{}^{\alpha\beta}W_{L\alpha\beta}\left(DH^\dagger\right)^{\gamma}{}_{\dot\alpha}\left(DH\right)_\gamma{}^{\dot\alpha}$
$B_L{}^{\alpha\beta}W_{L\alpha}{}^{\gamma}\left(DH^\dagger\right)_{\beta\dot\alpha}\left(DH\right)_\gamma{}^{\dot\alpha}$

2

Repeated Fields with Flavor

Another difficulty to write down the independent EFT operators

	B-violating
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

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$$Q_{prst}^{qqq\ell\,(1)} = -(Q_{prst}^{qqq\ell} + Q_{rpst}^{qqq\ell})$$
$$Q_{prst}^{qqq\ell\,(3)} = -(Q_{prst}^{qqq\ell} - Q_{rpst}^{qqq\ell})$$

[Grzadkowski, et.al. v3 2017]

	B-violating
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

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[Alonso, Chang, Jenkins, Manohar, Shotwell 2014]

$$Q_{prst}^{qqq\ell} + Q_{rpst}^{qqq\ell} = Q_{sprt}^{qqq\ell} + Q_{srpt}^{qqq\ell}$$

57

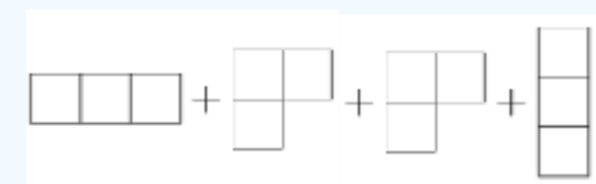
Flavor relations not easy task!

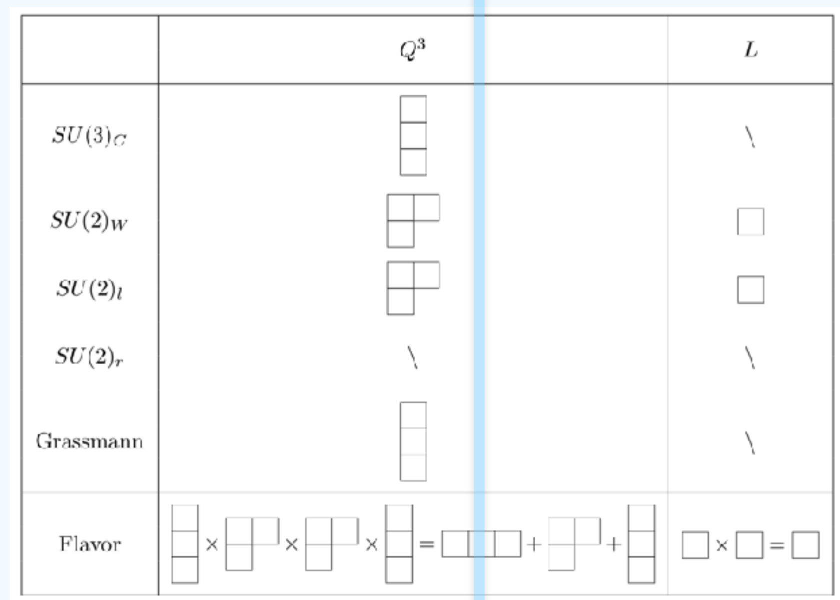
Flavor Symmetry

According to Schur-Weyl theorem, flavor tensor decomposed via S(nf) symmetry

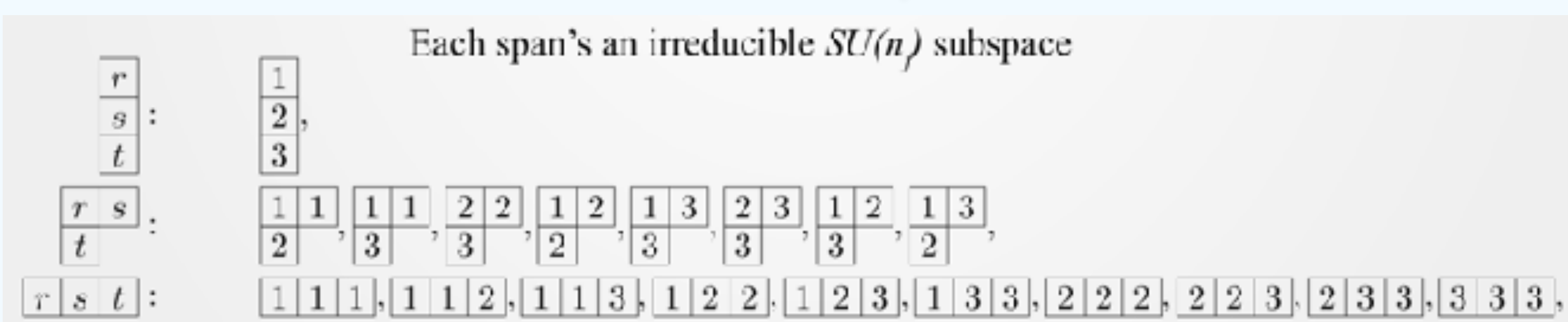
$$O_{qqql}^{p,rst}\epsilon^{abc}\epsilon_{ji}\epsilon_{km}[(q_r^{aj})^T C q_s^{bk}][(q_t^{cm})^T C l_p^i]$$

S_3 :





$SU(n_f)$:



19 x 3 = 57

[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]

Operator: y-basis, f-basis

For QQQL, the Young tableau for Lorentz and gauge structure give the y-basis

$$\left(\begin{array}{|c|c|}\hline 1&2\\\hline 3&4\\\hline\end{array}+\begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline\end{array}\right)\times\begin{array}{|c|}\hline a\\\hline b\\\hline c\\\hline\end{array}\times\left(\begin{array}{|c|c|}\hline i&j\\\hline k&l\\\hline\end{array}+\begin{array}{|c|c|}\hline i&k\\\hline j&l\\\hline\end{array}\right)=$$

$$\mathcal{M}=(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl}),\quad (L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})$$

$$T_G=\epsilon^{abc}\epsilon^{ik}\epsilon^{jl},\ \epsilon^{abc}\epsilon^{ij}\epsilon^{kl}$$

$$\begin{aligned}
O_1&=\epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})\\
O_2&=\epsilon^{abc}\epsilon^{ik}\epsilon^{jl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})\\
O_3&=\epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{sbk})(Q_{raj}Q_{tcl})\\
O_4&=\epsilon^{abc}\epsilon^{ij}\epsilon^{kl}(L_{pi}Q_{raj})(Q_{sbk}Q_{tcl})
\end{aligned}$$

Y-Basis = Young tensor basis

EFT operator should be viewed as flavor tensor in the SU(nf) group

Sn symmetry for repeated field

p-basis $\quad \mathcal{K}^{py}_{ji} \quad$ y-basis

$$\begin{pmatrix} O^{[4],1}\\ O^{[3,1],1}\\ O^{[3,1],2}\\ O^{[2,1,1],1}\end{pmatrix}=\begin{pmatrix}-1&2&2&-1\\2&-1&-1&2\\-1&-1&2&-1\\-1&0&0&1\end{pmatrix}\begin{pmatrix}O_1\\O_2\\O_3\\O_4\end{pmatrix}$$

not SU(nf) symmetry for nf flavor

$O^{[3,1],1}\quad O^{[3,1],2}\quad$ span the same $SU(n_f)$ space.

$$\begin{array}{l|l}
\mathcal{O}^{(p')}_{LQ^3,1} & \mathcal{Y}\left[\begin{array}{|c|c|c|}\hline r&s&t\\\hline\end{array}\right]\epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl})\\
\mathcal{O}^{(p')}_{LQ^3,2} & \mathcal{Y}\left[\begin{array}{|c|c|}\hline r&s\\\hline t\\\cline{1-1}\end{array}\right]\epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl})\\
\mathcal{O}^{(p')}_{LQ^3,3} & \mathcal{Y}\left[\begin{array}{|c|}\hline r\\\hline s\\\hline t\\\hline\end{array}\right]\epsilon^{abc}\epsilon^{ik}\epsilon^{jl}\,(L_{pi}Q_{raj})\,(Q_{sbk}Q_{tcl})
\end{array}$$

Final expression: f-basis!!!

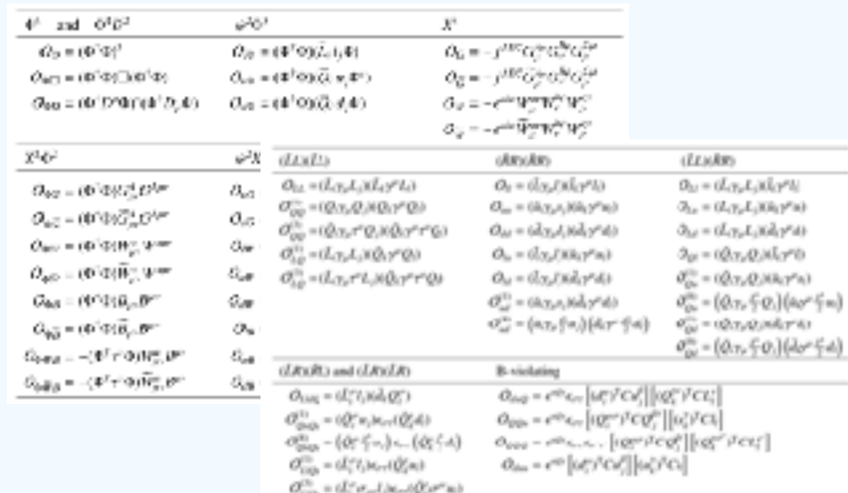
SMEFT Operators

Dimension-5

$\epsilon_{ij}\epsilon_{mn}(L^iCL^m)H^jH^n$

[Weinberg, 1979] **2**

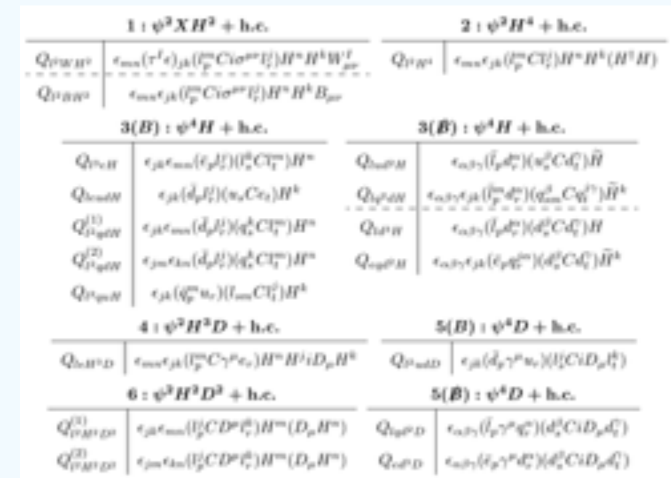
Dimension-6



84

[Buchmuller, Wyler, 1986] [Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

Dimension-7



[Lehman, 2014]
[Henning, Lu, Melia, Murayama, 2015]
[Liao, Ma, 2016]

Dimension-8

[Li, Ren, Shu, Xiao, **Yu**, Zheng, **2020**]

[Murphy, 2020]

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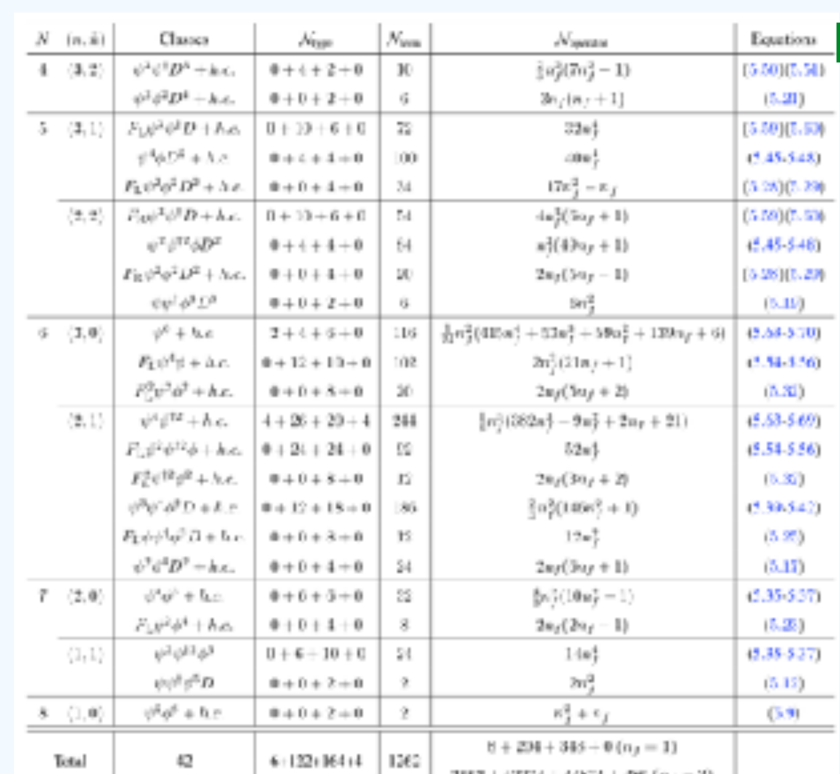
Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, **2020**]



[Liao, Ma, 2020]

LEFT Operators

Dimension-5

			Dim-5 operators
N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$
3	(2,0)	$F_L\psi_L^2 + h.c.$	$10+0+2+0$

10

Dimension-6

		Dim-6 operators		
N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
3	(3,0)	$F_L^3 + h.c.$	$2+0+0+0$	2
4	(2,0)	$\psi_L^4 + h.c.$	$14+12+8+2$	78
	(1,1)	$\psi_L^2\psi_R^2$	$40+20+12+0$	84
Total		5	$56+32+20+2$	164

[Jenkins, Manohar, Stoffer, 2017]

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Dimension-7

		Dim-7 operators		
N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(3,0)	$F_L^2\psi_L^2 + h.c.$	$16+0+4+0$	32
	(2,1)	$F_L^2\psi_R^2 + h.c.$	$16+0+4+0$	24
		$\psi_L^3\psi_R D + h.c.$	$50+32+22+$ [illegible]	[illegible]
Total		6	$82+32+30+$ [illegible]	[illegible]

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[Liao, Ma, Wang, 2020]

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

Dimension-8

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

[Murphy, 2020]

N	$(n,\tilde{n})$	Subclasses	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$	$\mathcal{N}_{\rm operator}$	Equations
4	(4,0)	$F_L^4 + h.c.$	14	26	26	(4.15)
	(3,1)	[illegible]	22	22	$22n_f^2$	(4.21)
		[illegible]	4+4	18+14	[illegible]	(4.75, 4.78, 4.80)
		[illegible]	16	32	$32n_f^2$	(4.44)
		[illegible]	8	12	12	(4.14)
	(2,2)	$F_L^2F_R^2$	14	17	17	(4.15)
		[illegible]	27	35	$35n_f^2$	(4.56, 4.71)
		[illegible]	17+4	54+8	[illegible]	(4.74, 4.79-4.81)
		[illegible]	16	16	$16n_f^2$	(4.44)
		[illegible]	5	4	4	(4.14)
		[illegible]	7	15	$15n_f^2$	(4.51, 4.52)
		[illegible]	1	2	2	(4.3)
5	(3,0)	[illegible]	12+18	46+54	[illegible]	[illegible]
		[illegible]	32	50	$50n_f^2$	(4.47, 4.48)
		[illegible]	4	4	4	(4.16)
	(2,1)	[illegible]	84+24	172+32	[illegible]	[illegible]
		[illegible]	32	45	[illegible]	(4.47, 4.48)
		[illegible]	72+14	140+56	[illegible]	(4.66, 4.69-4.72)
		[illegible]	38	30	$30n_f^2$	(4.36, 4.40)
		[illegible]	6	16	$16n_f^2$	(4.26)
		[illegible]	4	4	4	(4.10)
6	(2,0)	[illegible]	17+4	48+18	[illegible]	[illegible]
		[illegible]	16	22	$22n_f^2$	(4.36)
		[illegible]	8	10	10	(4.12)
	(1,1)	[illegible]	20+10	29+14	[illegible]	[illegible]
		[illegible]	7	15	$13n_f^2$	(4.24, 4.27)
		[illegible]	1	3	3	(4.8)
7	(1,0)	[illegible]	6	6	$6n_f^2$	(4.23)
8	(0,0)	ϕ^8	1	1	1	(4.3)
Total		48	471+70	1070+136	[illegible]	

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Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, 2020]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$	$\mathcal{N}_{\rm operator}$	Equations
4	(3,2)	[illegible]	$0+4+2+0$	10	[illegible]	(5.50)(5.51)
		[illegible]	$0+0+2+0$	6	[illegible]	(5.21)
5	(3,1)	[illegible]	$0+10+6+0$	72	$32n_f^4$	(5.50)(5.53)
		[illegible]	$0+4+4+0$	100	[illegible]	(5.45-5.48)
		[illegible]	$0+0+4+0$	34	$17n_f^2 - n_f$	(5.28)(5.29)
	(2,2)	[illegible]	$0+10+6+0$	54	[illegible]	(5.50)(5.53)
		[illegible]	$0+4+4+0$	64	[illegible]	(5.45-5.48)
		[illegible]	$0+0+4+0$	20	[illegible]	(5.28)(5.29)
		[illegible]	$0+0+2+0$	6	[illegible]	(5.11)
6	(3,0)	[illegible]	$2+4+6+0$	116	[illegible]	(5.64-5.70)
		[illegible]	$0+12+10+0$	102	[illegible]	(5.54-5.56)
		[illegible]	$0+0+8+0$	20	[illegible]	(5.32)
	(2,1)	[illegible]	$4+26+20+4$	244	[illegible]	(5.53-5.69)
		[illegible]	$0+24+24+0$	52	$52n_f^4$	(5.54-5.56)
		[illegible]	$0+0+8+0$	12	[illegible]	(5.35)
		[illegible]	$0+12+18+0$	186	[illegible]	[illegible]
		[illegible]	$0+0+8+0$	16	[illegible]	[illegible]
		[illegible]	$0+0+4+0$	24	[illegible]	(5.13)
7	(2,0)	[illegible]	$0+6+5+0$	22	[illegible]	(5.35-5.37)
		[illegible]	$0+0+4+0$	8	[illegible]	(5.26)
	(1,1)	[illegible]	$0+6+10+0$	24	$14n_f^4$	(5.35-5.37)
		[illegible]	$0+0+2+0$	2	[illegible]	(5.15)
8	(1,0)	[illegible]	$0+0+2+0$	2	[illegible]	(5.9)
Total		42	$6+122+164+4$	1262	$8+234+346+0$ ($n_f=1$) $2862+42234+40874+486$ ($n_f=3$)	

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vSMEFT and vLEFT

Dimension-5

Dim-5 operato

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
3	(2,0)	$F_L\psi^2+h.c.$	$0+0+2+0$	2
4	(1,0)	$\psi^2\phi^2+h.c.$	$0+0+2+0$	2
Total		4	$0+0+4+0$	4

2

Dimension-6

Dim-6 operato

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(2,0)	$\psi^4+h.c.$	$4+2+0+2$	14
		$F_L\psi^2\phi+h.c.$	$4+0+0+0$	4
	(1,1)	$\psi^2\psi^{\dagger2}$	$10+2+0+0$	12
		$\psi\psi^\dagger\phi^2 D$	$3+0+0+0$	3
5	(1,0)	$\psi^2\phi^3+h.c.$	$2+0+0+0$	2
Total		8	$23+4+0+2$	35

29

[Aguila, Bar-Shalom, Soni, Wudka, 2009]

Dimension-7

Dim-7 operato

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(3,0)	$F_L^2\psi^2+h.c.$	$0+0+6+0$	6
	(2,1)	$F_L^2\psi^{\dagger2}+h.c.$	$0+0+6+0$	6
		$\psi^3\psi^\dagger D+h.c.$	$0+4+20+0$	24
		$F_L\psi\psi^\dagger\phi D+h.c.$	$0+0+8+0$	8
		$\psi^2\phi^2D^2+h.c.$	$0+0+4+0$	6
5	(2,0)	$\psi^4\phi+h.c.$	$0+2+10+0$	24
		$F_L\psi^2\phi^2+h.c.$	$0+0+6+0$	6
	(1,1)	$\psi^2\psi^{\dagger2}\phi$	$0+4+22+0$	30
		$\psi\psi^\dagger\phi^3 D$	$0+0+2+0$	4
6	(1,0)	$\psi^2\phi^4+h.c.$	$0+0+2+0$	2
Total		18	$0+10+56+0$	116

[Bhattacharya, Wudka, 2016]

[Liao, Ma, 2017]

Dimension-8

[Li, Ren, Xiao, **Yu**, Zheng, 2021]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(3,1)	$\psi^4D^2+h.c.$	$4+0+2+2$	22
		$F_L\psi^2\phi D^2+h.c.$	$4+0+0+0$	8
	(2,2)	$F_LF_R\psi\psi^\dagger D$	$3+0+0+0$	3
		$\psi^2\psi^{\dagger2}D^2$	$10+2+0+0$	24
		$F_R\psi^2\phi D^2+h.c.$	$4+0+0+0$	4
		$\psi\psi^\dagger\phi^2D^3$	$3+0+0+0$	4
5	(3,0)	$F_L\psi^4+h.c.$	$10+4+0+2$	50
		$F_L^2\psi^2\phi+h.c.$	$8+0+0+0$	12
	(2,1)	$F_L\psi^2\psi^{\dagger2}+h.c.$	$42+12+0+0$	58
		$F_L^2\psi^{\dagger2}\phi+h.c.$	$8+0+0+0$	8
		$\psi^3\psi^\dagger\phi D+h.c.$	$24+6+0+2$	108
		$F_L\psi\psi^\dagger\phi^2D+h.c.$	$12+0+0+0$	16
		$\psi^2\phi^3D^2+h.c.$	$2+0+0+0$	12
6	(2,0)	$\psi^4\phi^2+h.c.$	$8+2+0+2$	30
		$F_L\psi^2\phi^3+h.c.$	$4+0+0+0$	6
	(1,1)	$\psi^2\psi^{\dagger2}\phi^2$	$16+4+0+2$	28
		$\psi\psi^\dagger\phi^4D$	$3+0+0+0$	3
7	(1,0)	$\psi^2\phi^5+h.c.$	$2+0+0+0$	2
Total		31	$167+30+2+10$	398

323

Dimension-9

[Li, Ren, Xiao, **Yu**, Zheng, 2021]

N	$(n,\tilde{n})$	Classes	$\mathcal{N}_{\rm type}$	$\mathcal{N}_{\rm term}$
4	(4,1)	$F_L^2\psi^2D^2+h.c.$	$0+6+0+0$	12
	(3,2)	$F_LF_R\psi\psi^\dagger D^2+h.c.$	$0+6+0+0$	6
		$F_L^2\psi^{\dagger2}D^2+h.c.$	$0+6+0+0$	6
		$\psi^3\psi^\dagger D^3+h.c.$	$4+20+0+3$	45
		$F_L\psi\psi^\dagger\phi D^2+h.c.$	$0+8+0+0$	15
		$\psi^2\phi^2D^4+h.c.$	$0+1+0+0$	8
5	(4,0)	$F_L^3\psi^2+h.c.$	$0+10+0+3$	15
	(3,1)	$F_L^3\psi^{\dagger2}+h.c.$	$0+4+0+0$	4
		$F_L\psi^3\psi^\dagger D+h.c.$	$10+12+8+0$	222
		$F_L^2\psi\psi^\dagger\phi D+h.c.$	$0+10+0+3$	32
		$\psi^4\phi D^2+h.c.$	$9+10+0+3$	130
		$F_L\psi^2\phi^2D^2+h.c.$	$0+8+0+0$	42
	(2,2)	$F_LF_R^2\phi^2+h.c.$	$0+12+0+3$	12
		$F_R\psi^3\psi^\dagger D+h.c.$	$10+12+8+0$	166
		$F_LF_R\psi\psi^\dagger\phi D$	$0+10+0+3$	26
		$\psi^2\psi^{\dagger2}\phi D^2$	$4+22+0+3$	210
		$F_R\psi^2\phi^2D^2+h.c.$	$0+8+0+0$	26
		$\psi\psi^\dagger\phi^3D^3$	$0+2+0+0$	20
6	(3,0)	$\psi^6+h.c.$	$6+10+6+2$	130
		$F_L\psi^4\phi+h.c.$	$6+26+0+3$	110
		$F_L^2\psi^2\phi^2+h.c.$	$0+12+0+3$	18
	(2,1)	$\psi^4\psi^{\dagger2}+h.c.$	[illegible]	474
		$F_L\psi^2\psi^{\dagger2}\phi+h.c.$	$24+106+0+0$	176
		$F_L^2\psi^{\dagger2}\phi^2+h.c.$	$0+10+0+3$	10
		$\psi^3\psi^\dagger\phi^2D+h.c.$	$10+14+8+0$	268
		$F_R\psi\psi^\dagger\phi^3D+h.c.$	$0+8+0+0$	32
		$\psi^2\phi^4D^2+h.c.$	$0+1+0+0$	20
7	(2,0)	$\psi^4\phi^3+h.c.$	$2+12+0+3$	28
		$F_L\psi^2\phi^4+h.c.$	$0+6+0+0$	6
	(1,1)	$\psi^2\psi^{\dagger2}\phi^3$	$4+22+0+3$	34

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Mathematica Code: ABC4EFT

Amplitude Basis Construction for Effective Field Theory

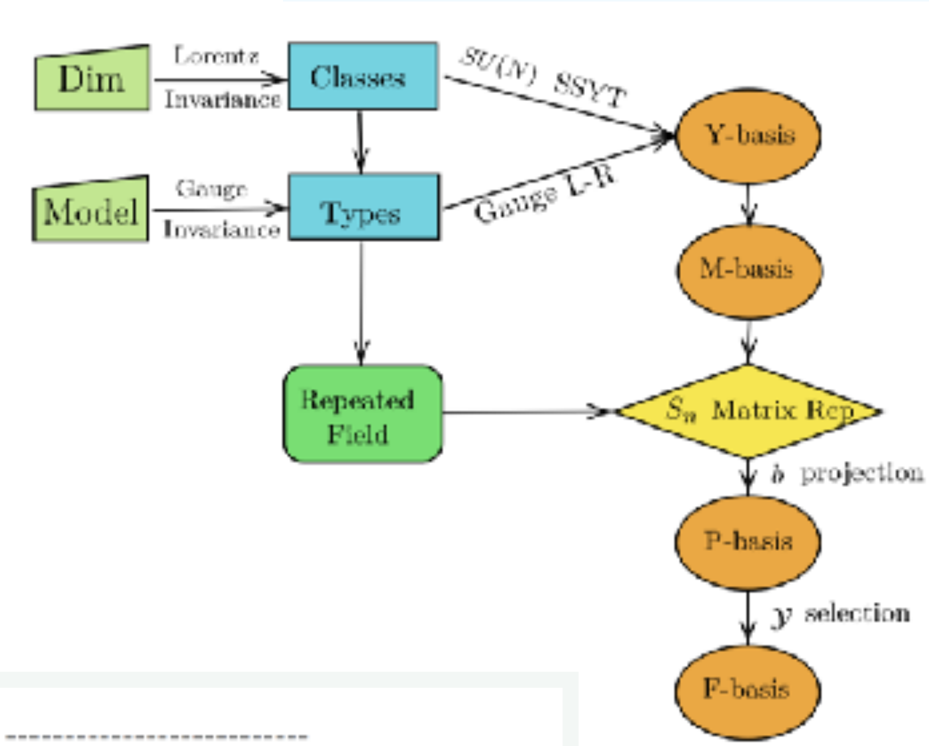
Welcome to the HEPForge Project: ABC4EFT

This is the website for the Mathematica package: Amplitude Basis Construction for Effective Field Thec

Package

This package has the following features:

- It provides a general procedure to construct the independent and complete operator bases for generic Lo invariant effective field theory, given any kind of gauge symmetry and field content, up to any mass dimer
- Various operator bases have been systematically constructed to emphasize different aspects: operator independence (y-basis), flavor relation (p-basis) and conserved quantum number (j-basis).
- It provides a systematic way to convert any operator into our on-shell amplitude basis and the basis conv can be easily done.



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<< ABC4EFT`

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ABC4EFT 1.0.0
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A Mathematica Package for
Amplitude Basis Construction for Effective Field Theories

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The package is available at hepforge
For the latest version, see the GitHub
If you use this package in your research,
Please cite: arXiv: 2201.04639, 2005.00008, 2007.07899
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https://abc4eft.hepforge.org/

[Li, Ren, Xiao, **Yu**, Zheng, 2201.04639]

Dim-6 Operators

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^* \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I\widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$
$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating			
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$		
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$		
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$		
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$		
$Q_{lequ}^{(3)}$	$(\bar l_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k\sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$		

59 independent operators

real parameters (degrees of freedom)

76: flavor universal — All fermion generations have the same coefficient

2499: flavor general — Independent coefficient for all flavor combinations

Dim-6 Operators

Dimension-6 operators of the SMEFT:	Interaction	Impact
$X^3 : \epsilon_{IJK} W^I_{\mu\nu} W^{J,\nu\rho} W^{K,\mu}_{\rho}$	gauge boson self-coupling	diboson
$H^6 : (\varphi^\dagger \varphi)^3$	Higgs potential, self-coupling	di-Higgs
$\psi^2 H^3 : (\varphi^\dagger \varphi)\ (\bar{q}_i\, u_j\, \tilde{\varphi})$	Higgs-fermion (Yukawa)	ttH, H→bb
$\psi^2 H^2 D : (\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{q}_i\, \gamma^\mu\, q_j)$	gauge-fermion (Z,W)	Z,W prod.
$X^2 H^2 : (\varphi^\dagger \varphi)\ G^a_{\mu\nu} G^{\mu\nu}_a$	gauge-Higgs	ggH, H→VV
$H^4 D^2 : (\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D^\mu \varphi)$	Higgs-Z	m_Z (LEP)
$\psi^2 X H : (\bar{q}_i\, \sigma^{\mu\nu}\, u_j\, \tilde{\varphi}) B_{\mu\nu}$	dipole	ffV, ffVH
$\psi^4 : (\bar{q}_i\, \gamma^\mu\, q_j)(\bar{q}_k\, \gamma_\mu\, q_l)$	four fermion	ffff scattering

SM gauge group singlets

Dim-6 RGE

$$\mu \frac{\mathrm{d}C_i}{\mathrm{d}\mu} = \sum_j \frac{\gamma_{ij}}{16\pi^2} C_j \quad \longrightarrow \quad C_i(\mu) = C_i(\Lambda) - \sum_j \frac{\gamma_{ij}}{16\pi^2} C_j(\Lambda) \log\left(\frac{\Lambda}{\mu}\right)$$

$$\gamma \sim O(1, g^2, \lambda, y^2, g^4, g^2\lambda, g^2y^2, \lambda^2, \lambda y^2, y^4, g^6, g^4\lambda, g^6\lambda)$$

Alonso–Jenkins–Manohar–Trott

	g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
g^3X^3	g^2	0	0	1	0	0	0	0
H^6	$g^6\lambda$	λ, g^2, y^2	$g^4, g^2\lambda, \lambda^2$	$g^6, g^4\lambda$	$\lambda y^2, y^4$	0	$\lambda y^2, y^4$	0
H^4D^2	g^6	0	g^2, λ, y^2	g^4	y^2	g^2y^2	g^2, y^2	0
$g^2X^2H^2$	g^4	0	1	g^2, λ, y^2	0	y^2	1	0
$y\psi^2H^3$	g^6	0	g^2, λ, y^2	g^4	g^2, λ, y^2	$g^2\lambda, g^4, g^2y^2$	g^2, λ, y^2	λ, y^2
$gy\psi^2XH$	g^4	0	0	g^2	1	g^2, y^2	1	1
ψ^2H^2D	g^6	0	g^2, y^2	g^4	y^2	g^2y^2	g^2, λ, y^2	y^2
ψ^4	g^6	0	0	0	0	g^2y^2	g^2, y^2	g^2, y^2

4-Fermi EFT: From Beta to NSI

Energy

TeV scale

UV models

many models …..

[Du, Li, Tang, Vihonen, **Yu, 2021**]

[Du, Li, Tang, Vihonen, **Yu, 2022**]

EW scale

Standard model EFT

$$(\bar{l}^j_\alpha e_\beta)\epsilon_{jk}(\bar{q}^k_s u_t),\quad (\bar{l}^j_\alpha \sigma_{\mu\nu} e_\beta)\epsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t),\quad (\bar{l}^j_\alpha e_\beta)(\bar{d}_s q_{tj}),\quad (\bar{l}_\alpha \gamma^\mu \tau^I l_\beta)(\bar{q}_s \gamma_\mu \tau^I q_t)$$
$$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_\alpha \tau^I \gamma^\mu l_\beta),\quad (H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r),\quad (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu d_r)$$
$$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_2 \tau^I \gamma^\mu l_2),\quad (H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_1 \tau^I \gamma^\mu l_1),\quad (\bar{l}_2 \gamma^\mu l_1)(\bar{l}_1 \gamma_\mu l_2),\quad (\bar{l}_1 \gamma^\mu l_2)(\bar{l}_2 \gamma_\mu l_1)$$

Low energy EFT

$$\{[\epsilon_{L(R)}(\mu)]^{ij}_{\alpha\beta}\,(\bar{u}_i \gamma^\mu P_{L(R)} d_j)\,(\bar{\ell}_\alpha \gamma_\mu P_L \nu_\beta)\,,$$
$$[\epsilon_{S(P)}(\mu)]^{ij}_{\alpha\beta}(\bar{u}_i \mathbf{1}(\gamma_5) d_j)\,(\bar{\ell}_\alpha P_L \nu_\beta)\,,$$
$$[\epsilon_T(\mu)]^{ij}_{\alpha\beta}\,(\bar{u}_i \sigma^{\mu\nu} P_L d_j)\,(\bar{\ell}_\alpha \sigma_{\mu\nu} P_L \nu_\beta)\}$$

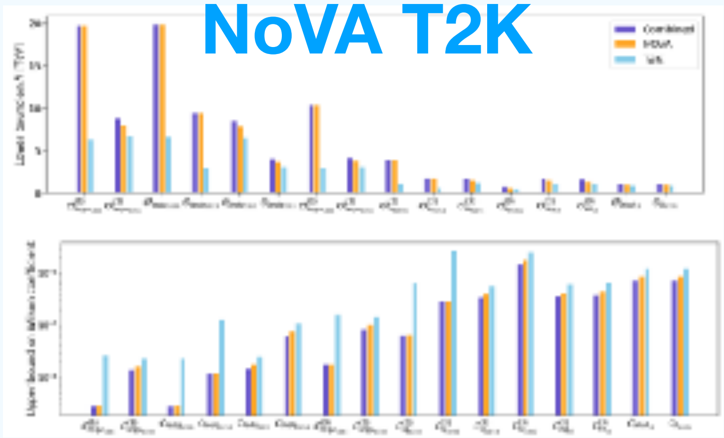
Λ_{QCD}

Chiral Lagrangian

$$[\epsilon_{L(R)}]^{ij}_{\alpha\beta},\,[\epsilon_{S(P)}]^{ij}_{\alpha\beta},\,[\epsilon_T]^{ij}_{\alpha\beta}$$

MeV scale

NoVA T2K



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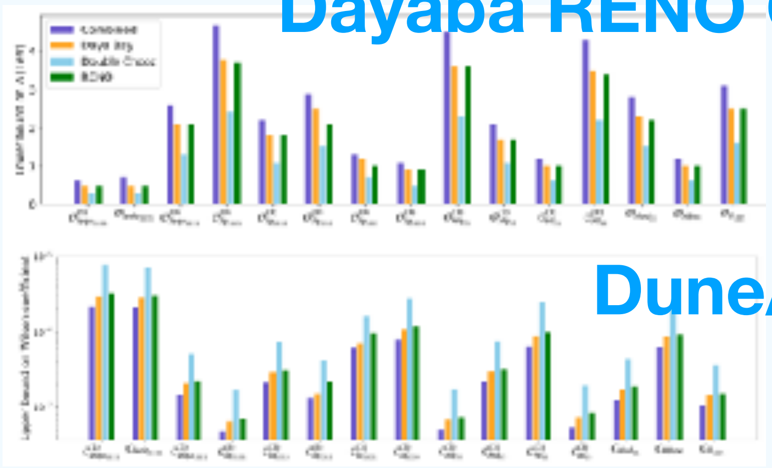


Weak Theory at 1957

S-T?

1953 6He beta decay

V-A?

Bad high energy behavior

$\nu_e + n \to p + e^-$

$\sigma = \frac{G_F^2 s}{\pi}$

$m_w <$ 大约 300 GeV.

[Lee, 1961]

If parity is not conserved in β decay, the most general form of Hamiltonian can be written as

$$
\begin{aligned}
H_{\text{int}} = & (\psi_p^\dagger \gamma_4 \psi_n)(C_S \psi_e^\dagger \gamma_4 \psi_\nu + C_S' \psi_e^\dagger \gamma_4 \gamma_5 \psi_\nu) \\
& + (\psi_p^\dagger \gamma_4 \gamma_\mu \psi_n)(C_V \psi_e^\dagger \gamma_4 \gamma_\mu \psi_\nu + C_V' \psi_e^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_\nu) \\
& + \tfrac{1}{2} (\psi_p^\dagger \gamma_4 \sigma_{\lambda\mu} \psi_n)(C_T \psi_e^\dagger \gamma_4 \sigma_{\lambda\mu} \psi_\nu \\
& + C_T' \psi_e^\dagger \gamma_4 \sigma_{\lambda\mu} \gamma_5 \psi_\nu) + (\psi_p^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_n) \\
& \times (-C_A \psi_e^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_\nu - C_A' \psi_e^\dagger \gamma_4 \gamma_\mu \psi_\nu) \\
& + (\psi_p^\dagger \gamma_4 \gamma_5 \psi_n)(C_P \psi_e^\dagger \gamma_4 \gamma_5 \psi_\nu + C_P' \psi_e^\dagger \gamma_4 \psi_\nu), \quad \text{(A.1)}
\end{aligned}
$$

LEFT

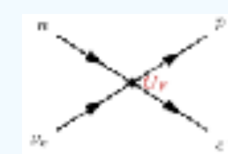
Weak Theory at 1957

Type	Form	Components	"Boson Spin"	
SCALAR	$\overline{\psi}\phi$	1	0	+
PSEUDOSCALAR	$\overline{\psi}\gamma^5\phi$	1	0	-
VECTOR	$\overline{\psi}\gamma^\mu\phi$	4	1	(+,-,-,-)
AXIAL VECTOR	$\overline{\psi}\gamma^\mu\gamma^5\phi$	4	1	(+,+,+,+)
TENSOR	$\overline{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\phi$	6	2	

Need to know complete UV

Bad high energy behavior

$\nu_e + n \rightarrow p + e^-$



$\sigma = \frac{G_F^2 s}{\pi}$

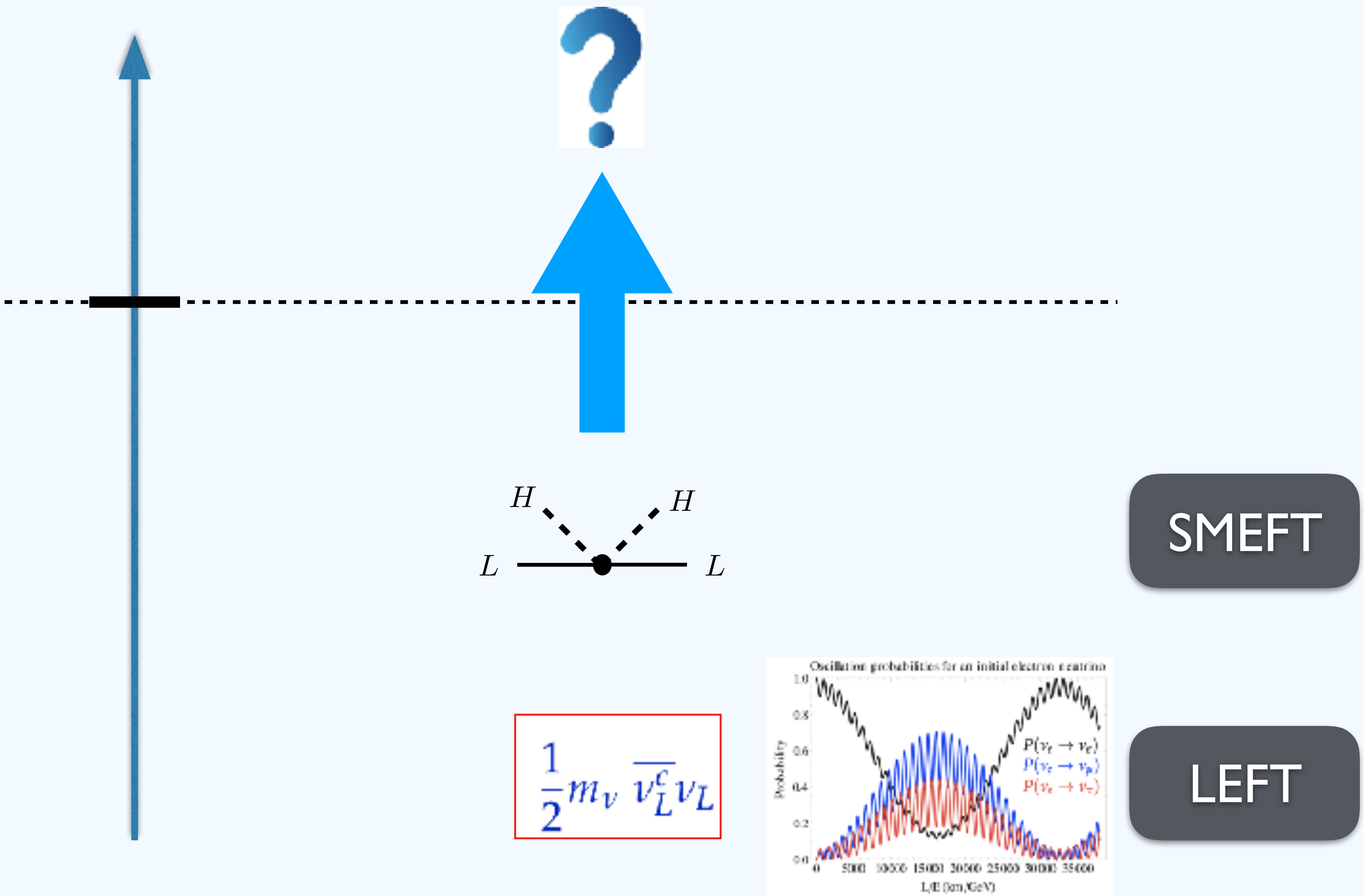
If parity is not conserved in β decay, the most general form of Hamiltonian can be written as

$$
\begin{aligned}
H_{\rm int} = & (\psi_p^\dagger\gamma_4\psi_n)(C_S\psi_e^\dagger\gamma_4\psi_\nu + C_S'\psi_e^\dagger\gamma_4\gamma_5\psi_\nu) \\
& + (\psi_p^\dagger\gamma_4\gamma_\mu\psi_n)(C_V\psi_e^\dagger\gamma_4\gamma_\mu\psi_\nu + C_V'\psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\psi_\nu) \\
& + \tfrac{1}{2}(\psi_p^\dagger\gamma_4\sigma_{\lambda\mu}\psi_n)(C_T\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}\psi_\nu \\
& + C_T'\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}\gamma_5\psi_\nu) + (\psi_p^\dagger\gamma_4\gamma_\mu\gamma_5\psi_n) \\
& \times(-C_A\psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\psi_\nu - C_A'\psi_e^\dagger\gamma_4\gamma_\mu\psi_\nu) \\
& + (\psi_p^\dagger\gamma_4\gamma_5\psi_n)(C_P\psi_e^\dagger\gamma_4\gamma_5\psi_\nu + C_P'\psi_e^\dagger\gamma_4\psi_\nu), \quad \text{(A.1)}
\end{aligned}
$$

LEFT

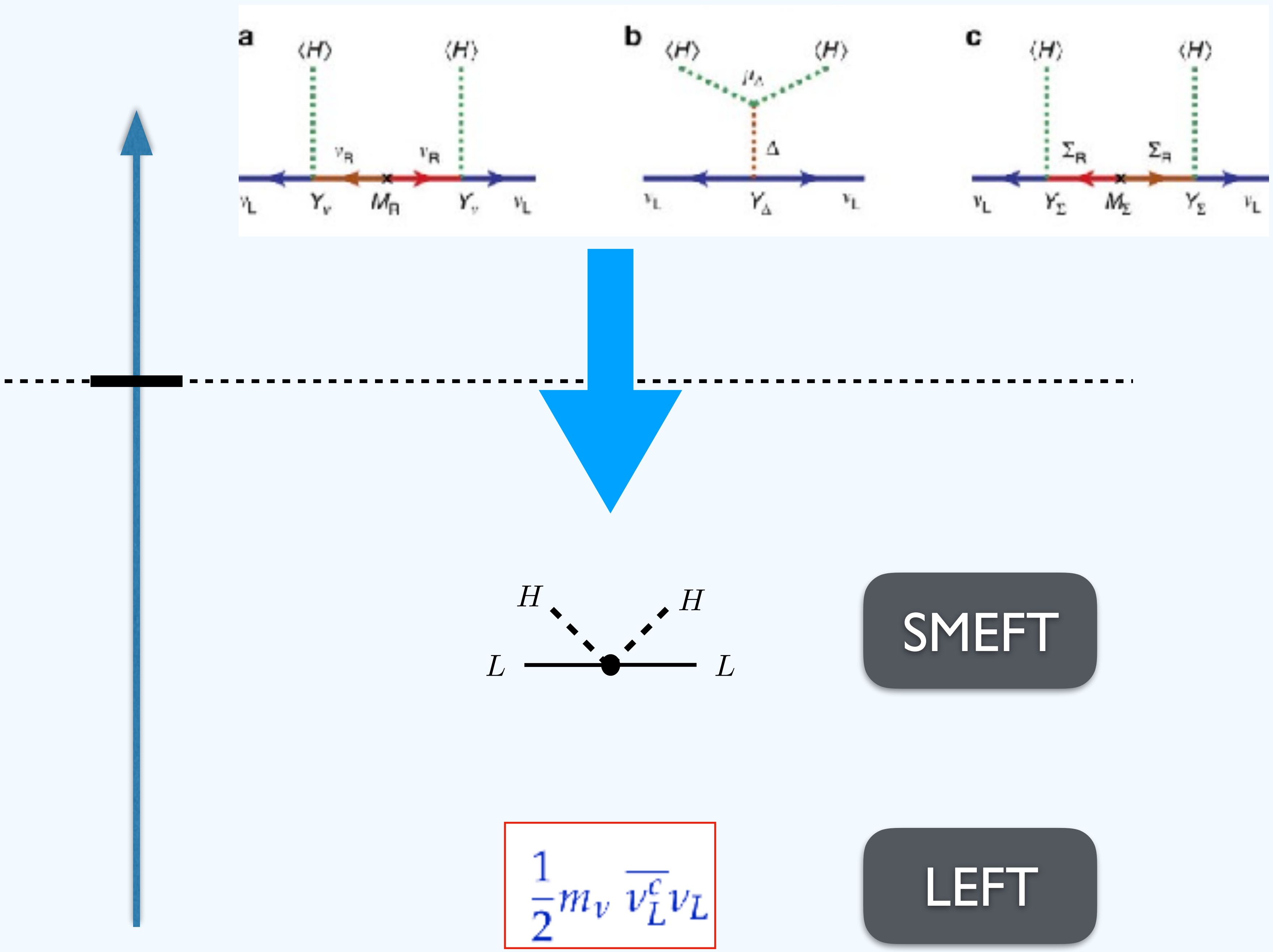
Similar Story: Neutrino Masses

Nowadays, the first evidence of new physics is the neutrino masses



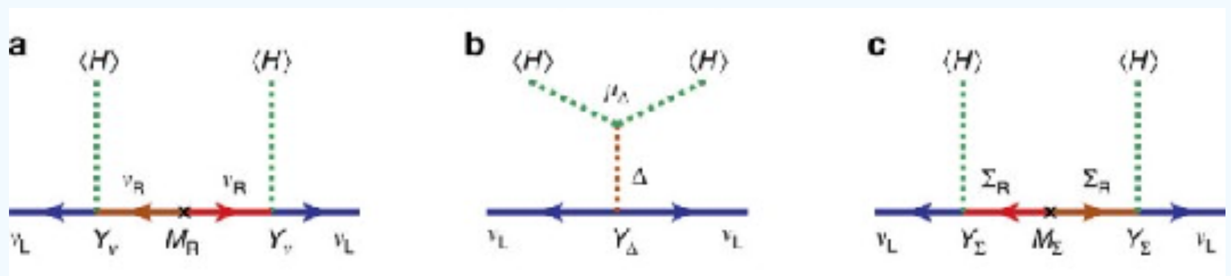
Neutrino Masses

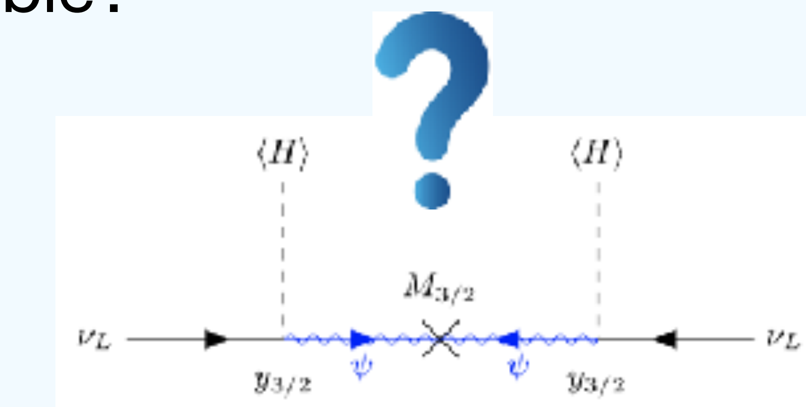
The top-down approach is well-known, how about the bottom-up way?



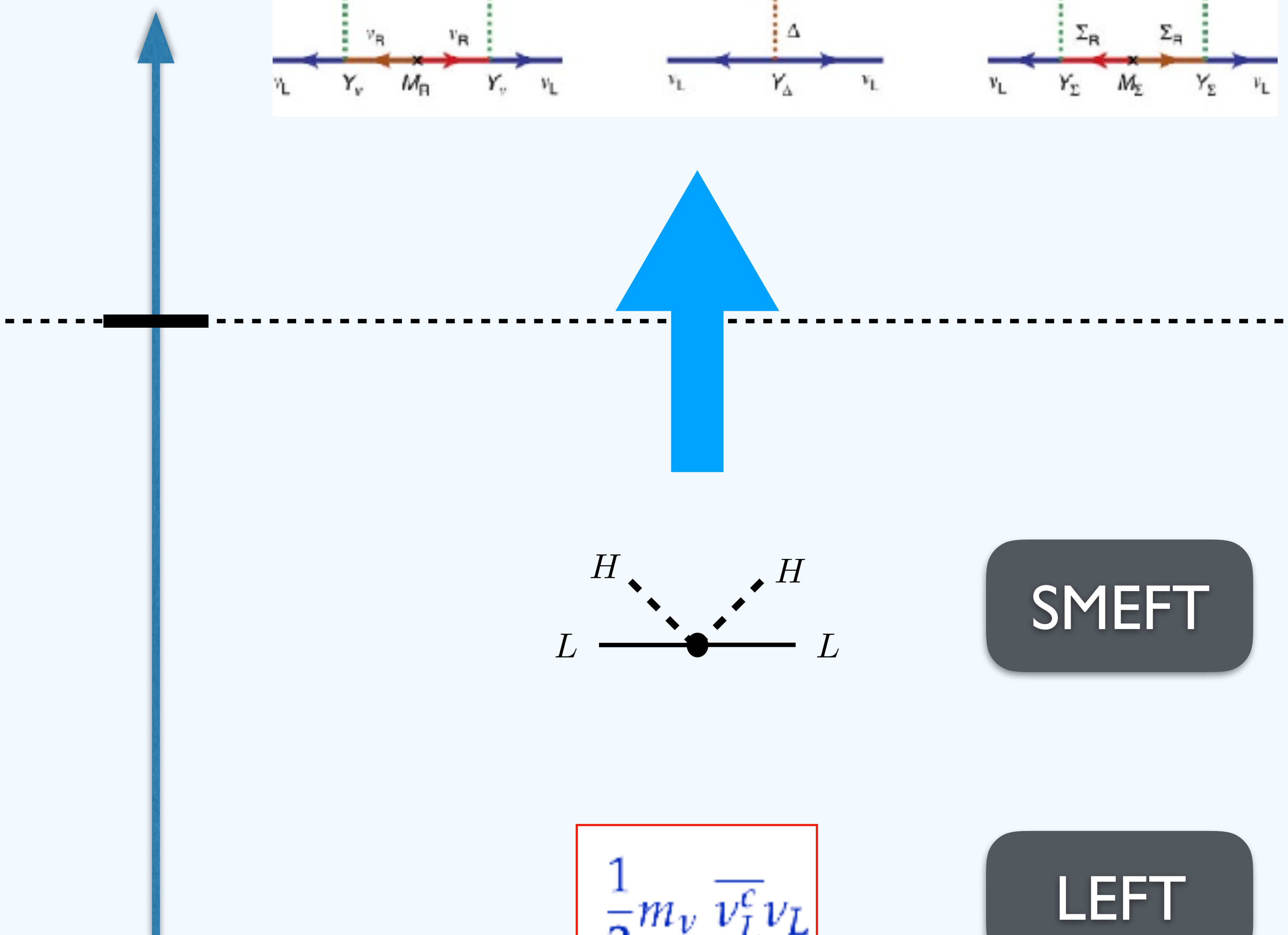
Type-3/2 Seesaw?

Whether additional seesaw (type-3/2 seesaw) is possible?





[Demir, Karahan, Sargm, 2021]

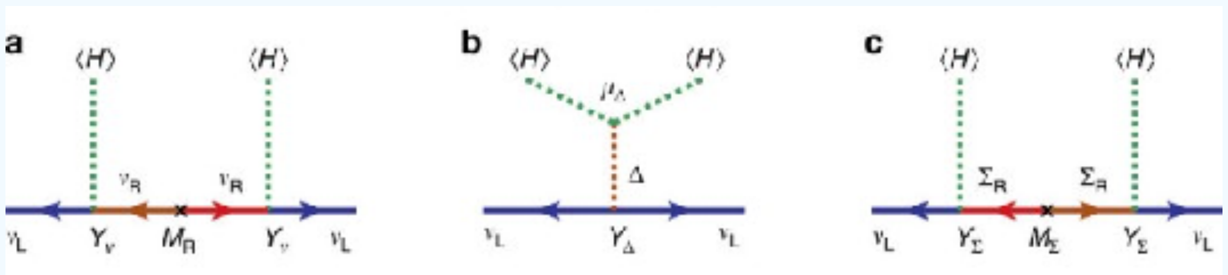


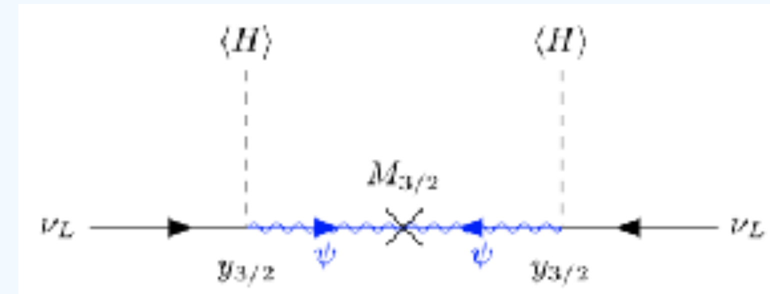
SMEFT

$$\frac{1}{2} m_\nu \overline{\nu_L^c} \nu_L$$

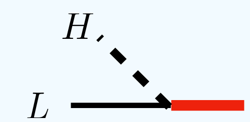
LEFT

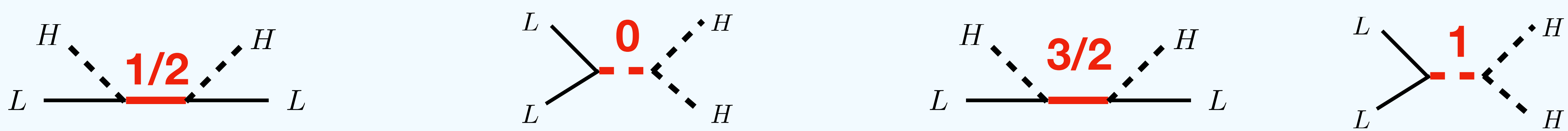
Bottom-Up Approach?



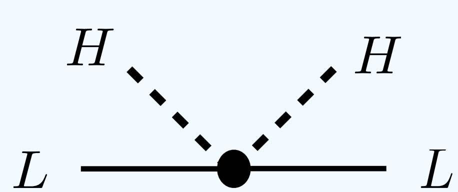


Impose gauge conservation for each vertex





Select UV topology with certain spin

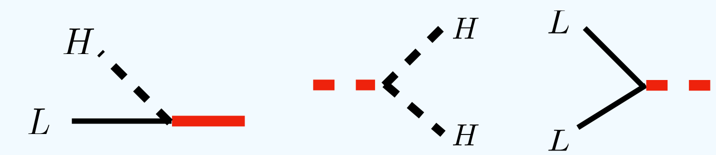


Angular momentum conservation not imposed!

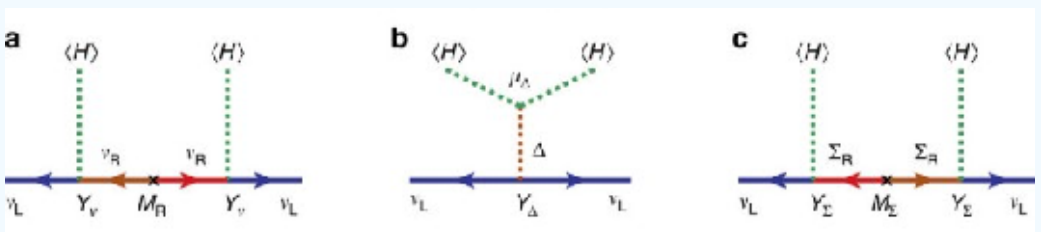
Cannot write 3/2-UV Lag for HHLL

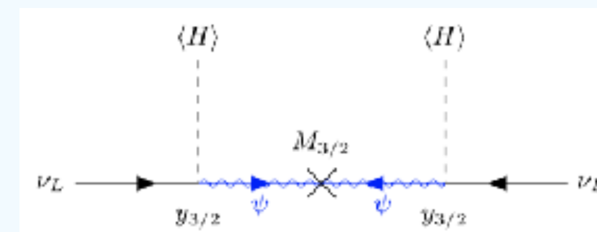
Bottom-Up Approach?

Essentially the top-down approach



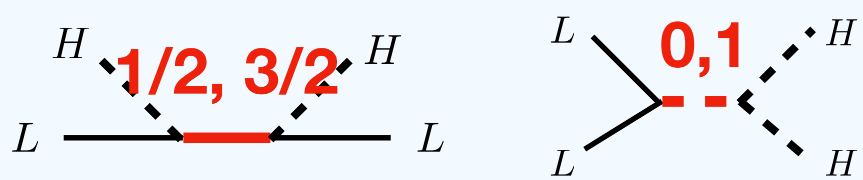
Assume UV spin/gauge/interaction @ **vertex** level



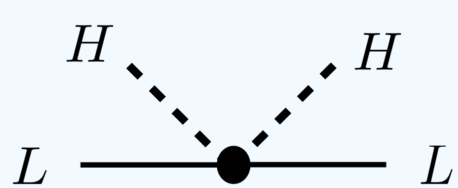


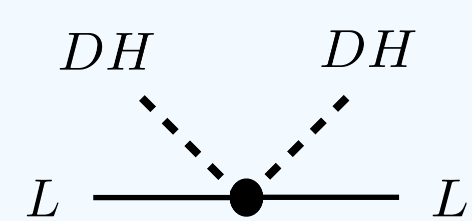
Impose gauge conservation at vertex level

Validation: matching



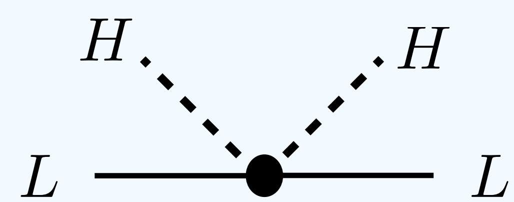
Select UV topology with fixed spin





Jiang-Hao Yu (ITP-CAS)

Poincare Casimir: Spin-1/2&0



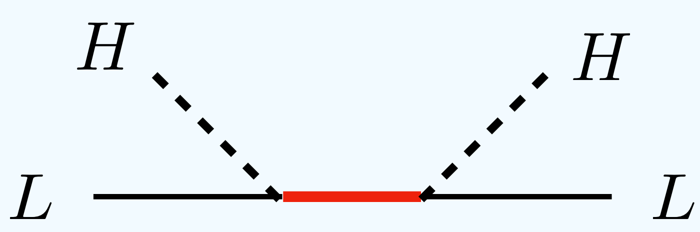
$\mathcal{O}^S = (HL)(HL)$

$\mathcal{B}^y = \langle 12 \rangle$

$$\mathbf{W}^2 \mathcal{B}^J = -sJ(J+1)\mathcal{B}^J$$

$LH \to LH$ channel

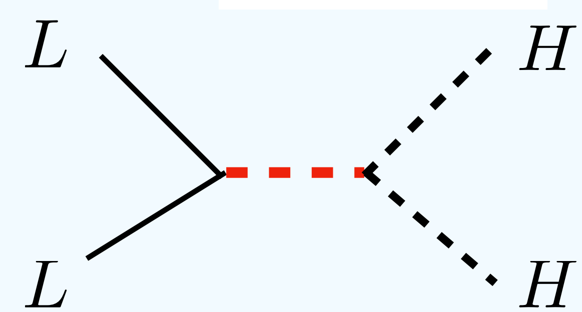
$W^2_{\{1,3\}} \mathcal{B}^y = -\frac{3}{4} s_{13} \langle 12 \rangle$



$J = \frac{1}{2}$

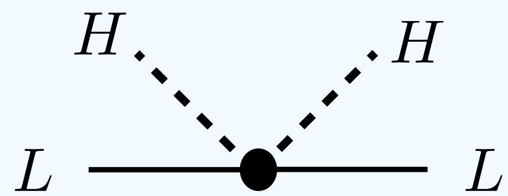
$LL \to HH$ channel

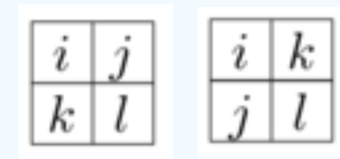
$W^2_{\{1,2\}} \mathcal{B}^y = 0$



$J = 0$

Gauge Casimir: singlet&triplet





$\mathcal{B}_1^R = \epsilon^{ik}\epsilon^{jl}$

$\mathcal{B}_2^R = \epsilon^{ij}\epsilon^{kl}$

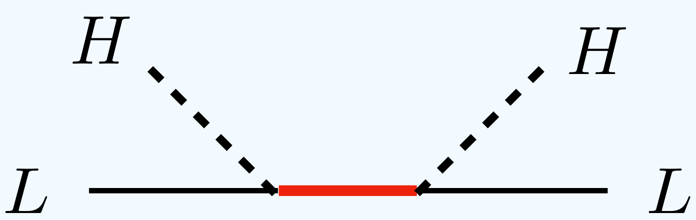
$$\mathbf{C}^2\mathcal{B}^R = r(r+1)\mathcal{B}^R$$

$LH \to LH$ channel

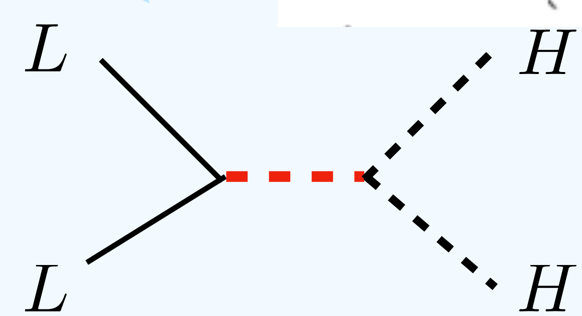
$$C_{21,3}\mathcal{B}^m = \begin{pmatrix} 0 & 0 \\ -1 & 2 \end{pmatrix}\mathcal{B}^m$$

$LL \to HH$ channel

$$C_{21,3}\mathcal{B}^m = \begin{pmatrix} 0 & 0 \\ -1 & 2 \end{pmatrix}\mathcal{B}^m$$



$$\mathcal{B}^{\mathbf{R}} = \begin{cases} \epsilon^{ik}\epsilon^{jl} & \mathbf{R}=1 \\ \epsilon^{ik}\epsilon^{jl} - 2\epsilon^{ij}\epsilon^{kl} & \mathbf{R}=3 \end{cases}$$



Complete Tree Seesaw Proved!

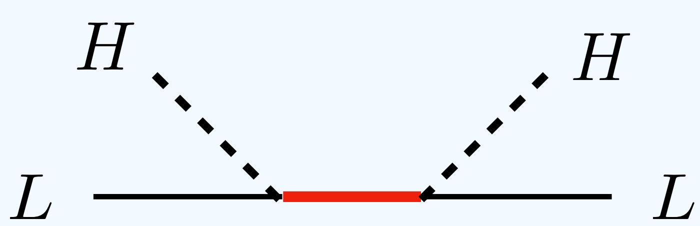
[Li, Ni, Xiao, **Yu**, 2204.03660]

$$\mathcal{O}^S = (HL)(HL)$$

$$\mathbf{W}^2 \mathcal{B}^J = -sJ(J+1)\mathcal{B}^J$$

$LH \to LH$ channel

$$W^2_{\{1,3\}}\mathcal{B}^y = -\frac{3}{4}s_{13}\langle 12\rangle$$



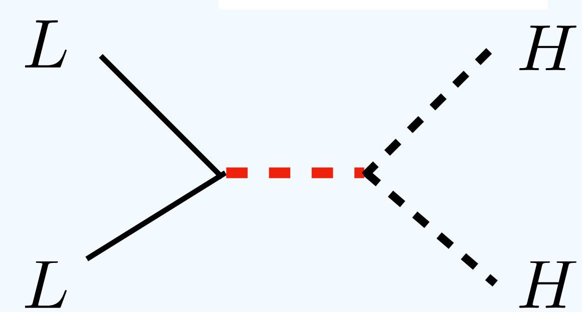
$$J = \frac{1}{2}$$

Type-I and III: SU(2) **single** and **triplet**

j-basis	Model
$\mathcal{O}^{(1/2,\mathbf{1})}_{HL\to HL} = \mathcal{O}^S + \mathcal{O}^A$	type I
$\mathcal{O}^{(1/2,\mathbf{3})}_{HL\to HL} = \mathcal{O}^S - 3\mathcal{O}^A$	type III

$LL \to HH$ channel

$$W^2_{\{1,2\}}\mathcal{B}^y = 0$$



$$J = 0$$

Type-II: SU(2) **triplet**, or singlet (excluded by repeated field)

j-basis	Model
$\mathcal{O}^{(0,\mathbf{1})}_{HH\to LL} = \mathcal{O}^A$	N/A
$\mathcal{O}^{(0,\mathbf{3})}_{HH\to LL} = \mathcal{O}^S$	type II

$$\mathcal{O}^S = (HL)(HL), \quad \mathcal{O}^A = (HH)(LL)$$

LLHHDD UV Resonances

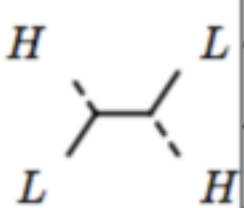
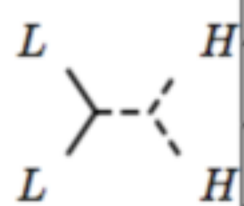
$$\mathbf{W}^2 \mathcal{B}^{\mathrm{y}} = -s\mathcal{W}.\mathcal{B}^{\mathrm{y}} \xrightarrow[\mathcal{K}.\mathcal{W}.\mathcal{K}^{-1} = \mathrm{diag}\{J(J+1)\}]{\mathcal{B}^{\mathrm{j}} = \mathcal{K}.\mathcal{B}^{\mathrm{y}}} \mathbf{W}^2_{\mathrm{initial/final}} \mathcal{B}^J = -sJ(J+1)\mathcal{B}^J$$

$$\mathcal{B}^y_{\psi^2\phi^2D^2} = \begin{pmatrix} s_{34}\langle 12\rangle \\ [34]\langle 13\rangle\langle 24\rangle \end{pmatrix}$$

$$W^2_{\{1,3\}}\mathcal{B}^y = s_{24}\begin{pmatrix} -\frac{15}{4} & 2 \\ 0 & -\frac{3}{4} \end{pmatrix}\mathcal{B}^y$$

$$\Rightarrow \mathcal{B}^j = \begin{cases} 3s_{34}\langle 12\rangle + 2[34]\langle 13\rangle\langle 24\rangle & J = \frac{3}{2} \\ \langle 13\rangle\langle 24\rangle & J = \frac{1}{2} \end{cases}$$

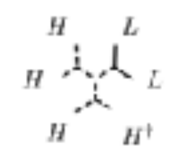
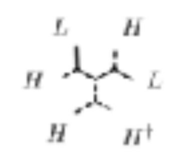
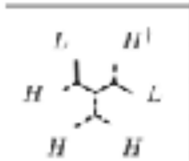
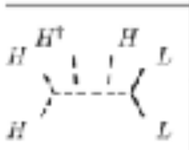
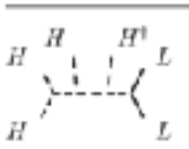
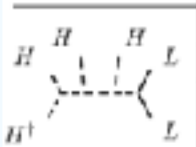
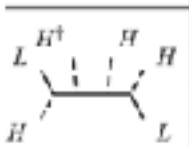
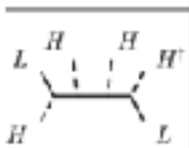
7 UV resonances

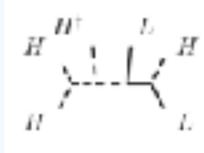
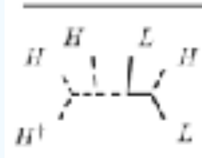
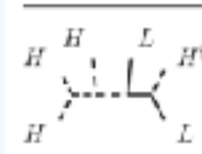
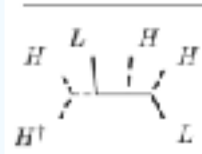
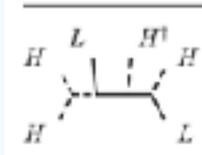
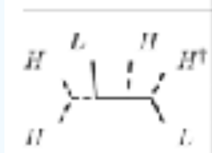
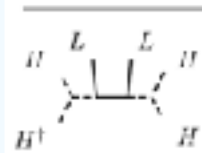
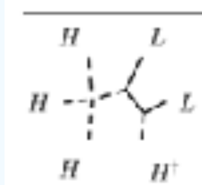
Topology	j-basis	Quantum numbers $\{J, \mathbf{R}, Y\}$
	$\mathcal{B}_{\{13\},1} = 3\mathcal{B}^p_1 + 6\mathcal{B}^p_2 - 9\mathcal{B}^p_3 - 2\mathcal{B}^p_4,$	$\{\frac{3}{2}, 3, 0\}$
	$\mathcal{B}_{\{13\},2} = 3\mathcal{B}^p_2 - \mathcal{B}^p_4,$	$\{\frac{1}{2}, 3, 0\}$
	$\mathcal{B}_{\{13\},3} = -3\mathcal{B}^p_1 + 2\mathcal{B}^p_2 - 3\mathcal{B}^p_3 + 2\mathcal{B}^p_4,$	$\{\frac{3}{2}, 1, 0\}$
	$\mathcal{B}_{\{13\},4} = \mathcal{B}^p_2 + \mathcal{B}^p_4.$	$\{\frac{1}{2}, 1, 0\}$
	$\mathcal{B}_{\{12\},1} = 2\mathcal{B}^p_1 - 4\mathcal{B}^p_4,$	$\{1, 3, -1\}$
	$\mathcal{B}_{\{12\}} = -2\mathcal{B}^p_1,$	$\{0, 3, -1\}$
	$\mathcal{B}_{\{12\}} = 4\mathcal{B}^p_2 - 2\mathcal{B}^p_3,$	$\{1, 1, -1\}$
	$\mathcal{B}_{\{12\}} = 2\mathcal{B}^p_3.$	$\{0, 1, -1\}$ N/A

Genuine dim-7 Seesaw

[Li, Ni, Xiao, **Yu**, 2204.03660]

Tree-level seesaw at dim-7: among 19 topologies, one genuine dim-7 seesaw

Topology	j-basis	Quantum numbers $\{J, \mathrm{R}, Y\}$
	$\mathcal{O}_{\{12\|34\|56\},1} = 2\mathcal{O}^p_1 - 4\mathcal{O}^p_2$.	$\{0,3,-1\},\{0,3,1\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|34\|56\},2} = 2\mathcal{O}^p_2 + 4\mathcal{O}^p_3$.	$\{0,3,-1\},\{0,1,1\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|34\|56\},3} = 12\mathcal{O}^p_4$.	$\{0,1,-1\},\{0,3,1\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|34\|56\},4} = 4\mathcal{O}^p_1 + 4\mathcal{O}^p_2$.	$\{0,3,-1\},\{0,3,1\},\{0,1,0\}$
	$\mathcal{O}_{\{12\|34\|56\},5} = 2\mathcal{O}^p_4 + 4\mathcal{O}^p_5$.	$\{0,1,-1\},\{0,1,1\},\{0,1,0\}$
	$\mathcal{O}_{\{13\|24\|56\},1} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},3,0\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|24\|56\},2} = -\mathcal{O}^p_1 + 3\mathcal{O}^p_2 + 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},1,0\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|24\|56\},3} = -\mathcal{O}^p_1 + \mathcal{O}^p_2 - 2\mathcal{O}^p_3 - 3\mathcal{O}^p_4$.	$\{\frac{1}{2},1,0\},\{\frac{1}{2},3,0\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|24\|56\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_2 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},3,0\},\{0,1,0\}$
	$\mathcal{O}_{\{13\|24\|56\},5} = \mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5$.	$\{\frac{1}{2},1,0\},\{\frac{1}{2},1,0\},\{0,1,0\}$
	$\mathcal{O}_{\{16\|23\|45\},1} = 2\mathcal{O}^p_1 - 2\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 6\mathcal{O}^p_5$.	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},3,0\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|23\|45\},2} = -2\mathcal{O}^p_1 - \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$.	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},1,0\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|23\|45\},3} = 3\mathcal{O}^p_2 + 3\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$.	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},3,0\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|23\|45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$.	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},3,0\},\{0,1,1\}$
	$\mathcal{O}_{\{16\|23\|45\},5} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5$.	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},1,0\},\{0,1,1\}$
	$\mathcal{O}_{\{12\|125\|34\},1} = \mathcal{O}^p_1 + 4\mathcal{O}^p_2$.	$\{0,3,-1\},\{0,4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|125\|34\},2} = -8\mathcal{O}^p_1 + 4\mathcal{O}^p_2$.	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|125\|34\},3} = -12\mathcal{O}^p_4$.	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|125\|34\},4} = -2\mathcal{O}^p_2 - 4\mathcal{O}^p_3$.	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\|125\|34\},5} = -2\mathcal{O}^p_4 - 4\mathcal{O}^p_5$.	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\|126\|34\},1} = 3\mathcal{O}^p_1$.	$\{0,3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|126\|34\},2} = 12\mathcal{O}^p_2$.	$\{0,3,-1\},\{0,2,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|126\|34\},3} = -12\mathcal{O}^p_4$.	$\{0,1,-1\},\{0,2,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{12\|126\|34\},4} = -2\mathcal{O}^p_2 - 4\mathcal{O}^p_3$.	$\{0,3,-1\},\{0,2,-\frac{3}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\|126\|34\},5} = -2\mathcal{O}^p_4 - 4\mathcal{O}^p_5$.	$\{0,1,-1\},\{0,2,-\frac{3}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{12\|124\|36\},1} = -\mathcal{O}^p_1 - 4\mathcal{O}^p_3$.	$\{0,3,-1\},\{0,4,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|124\|36\},2} = 2\mathcal{O}^p_1 + 6\mathcal{O}^p_2 + 2\mathcal{O}^p_5$.	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|124\|36\},3} = -6\mathcal{O}^p_4 - 6\mathcal{O}^p_5$.	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{12\|124\|36\},4} = -2\mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 2\mathcal{O}^p_3$.	$\{0,3,-1\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{12\|124\|36\},5} = -2\mathcal{O}^p_4 + 2\mathcal{O}^p_5$.	$\{0,1,-1\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{13\|135\|24\},1} = \mathcal{O}^p_1 - 2\mathcal{O}^p_3 - 6\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},4,\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{13\|135\|24\},2} = -\mathcal{O}^p_1 - 3\mathcal{O}^p_2 - 4\mathcal{O}^p_3 + 9\mathcal{O}^p_4 + 6\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{13\|135\|24\},3} = \mathcal{O}^p_1 - \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4$.	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{13\|135\|24\},4} = \mathcal{O}^p_1 - 3\mathcal{O}^p_2 - 2\mathcal{O}^p_3 - 3\mathcal{O}^p_4$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},1,0\}$
	$\mathcal{O}_{\{13\|135\|24\},5} = \mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5$.	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{\frac{1}{2},1,0\}$
	$\mathcal{O}_{\{16\|146\|23\},1} = -2\mathcal{O}^p_1 + 4\mathcal{O}^p_3 - 12\mathcal{O}^p_5$.	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},4,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{16\|146\|23\},2} = 2\mathcal{O}^p_1 - 3\mathcal{O}^p_2 - \mathcal{O}^p_3 + 9\mathcal{O}^p_4 + 3\mathcal{O}^p_5$.	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{16\|146\|23\},3} = 3\mathcal{O}^p_2 + 3\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$.	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},3,0\}$
	$\mathcal{O}_{\{16\|146\|23\},4} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 - 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5$.	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},1,0\}$
	$\mathcal{O}_{\{16\|146\|23\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 - \mathcal{O}^p_4 + \mathcal{O}^p_5$.	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{\frac{1}{2},1,0\}$

	$\mathcal{O}_{\{13\|123\|45\},1} = \mathcal{O}^p_1 - 4\mathcal{O}^p_2 - 4\mathcal{O}^p_3$.	$\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|123\|45\},2} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 - 9\mathcal{O}^p_4 - 9\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|123\|45\},3} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$.	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|123\|45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{13\|123\|45\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5$.	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{13\|123\|46\},1} = \mathcal{O}^p_1 - 4\mathcal{O}^p_2 - 4\mathcal{O}^p_3$.	$\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|123\|46\},2} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 9\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|123\|46\},3} = -\mathcal{O}^p_1 - 2\mathcal{O}^p_2 + \mathcal{O}^p_3 - 3\mathcal{O}^p_5$.	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{13\|123\|46\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 3\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{13\|123\|46\},5} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 - 2\mathcal{O}^p_4 - \mathcal{O}^p_5$.	$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{16\|126\|34\},1} = 6\mathcal{O}^p_1$.	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|126\|34\},2} = -3\mathcal{O}^p_2 + 9\mathcal{O}^p_4$.	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|126\|34\},3} = -3\mathcal{O}^p_2 - 3\mathcal{O}^p_4$.	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|126\|34\},4} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5$.	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|126\|34\},5} = \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5$.	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{23\|235\|46\},1} = \mathcal{O}^p_1 - 2\mathcal{O}^p_3 + 6\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},4,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{23\|235\|46\},2} = \mathcal{O}^p_1 - 6\mathcal{O}^p_2 - 5\mathcal{O}^p_3 - 3\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{23\|235\|46\},3} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_5$.	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{23\|235\|46\},4} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 - 6\mathcal{O}^p_4 - 3\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{23\|235\|46\},5} = -\mathcal{O}^p_1 - \mathcal{O}^p_3 + 2\mathcal{O}^p_4 + \mathcal{O}^p_5$.	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{13\|136\|45\},1} = \mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 2\mathcal{O}^p_3 - 6\mathcal{O}^p_4 - 6\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|136\|45\},2} = 2\mathcal{O}^p_1 - 5\mathcal{O}^p_2 - 5\mathcal{O}^p_3 - 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|136\|45\},3} = 2\mathcal{O}^p_1 + \mathcal{O}^p_2 + \mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5$.	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{13\|136\|45\},4} = \mathcal{O}^p_2 - \mathcal{O}^p_3 + 3\mathcal{O}^p_4 - 3\mathcal{O}^p_5$.	$\{\frac{1}{2},3,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{13\|136\|45\},5} = -\mathcal{O}^p_2 + \mathcal{O}^p_3 + \mathcal{O}^p_4 - \mathcal{O}^p_5$.	$\{\frac{1}{2},1,0\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{16\|156\|34\},1} = 2\mathcal{O}^p_1 - 4\mathcal{O}^p_2 + 12\mathcal{O}^p_4$.	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},4,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|156\|34\},2} = 4\mathcal{O}^p_1 + \mathcal{O}^p_2 - 3\mathcal{O}^p_4$.	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|156\|34\},3} = -3\mathcal{O}^p_2 - 3\mathcal{O}^p_4$.	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,3,1\}$
	$\mathcal{O}_{\{16\|156\|34\},4} = -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 + 3\mathcal{O}^p_4 + 6\mathcal{O}^p_5$.	$\{\frac{1}{2},3,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{16\|156\|34\},5} = \mathcal{O}^p_2 + 2\mathcal{O}^p_3 + \mathcal{O}^p_4 + 2\mathcal{O}^p_5$.	$\{\frac{1}{2},1,-1\},\{\frac{1}{2},2,-\frac{1}{2}\},\{0,1,1\}$
	$\mathcal{O}_{\{34\|134\|56\},1} = -\mathcal{O}^p_1 + 2\mathcal{O}^p_2 + 6\mathcal{O}^p_4$.	$\{0,3,1\},\{\frac{1}{2},4,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{34\|134\|56\},2} = 4\mathcal{O}^p_1 - 8\mathcal{O}^p_2 + 12\mathcal{O}^p_4$.	$\{0,3,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{34\|134\|56\},3} = 2\mathcal{O}^p_2 + 4\mathcal{O}^p_3$.	$\{0,1,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,3,0\}$
	$\mathcal{O}_{\{34\|134\|56\},4} = -4\mathcal{O}^p_1 - 4\mathcal{O}^p_2$.	$\{0,3,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{34\|134\|56\},5} = 2\mathcal{O}^p_4 + 4\mathcal{O}^p_5$.	$\{0,1,1\},\{\frac{1}{2},2,\frac{1}{2}\},\{0,1,0\}$
	$\mathcal{O}_{\{16\|126\},1} = 6\mathcal{O}^p_1$.	$\{\frac{1}{2},3,-1\},\{0,4,-\frac{3}{2}\}$
	$\mathcal{O}_{\{16\|126\},2,3} = \begin{pmatrix} \mathcal{O}^p_2 - \mathcal{O}^p_3 - 3\mathcal{O}^p_4 + 3\mathcal{O}^p_5 \\ -2\mathcal{O}^p_2 - \mathcal{O}^p_3 + 6\mathcal{O}^p_4 + 3\mathcal{O}^p_5 \end{pmatrix}$.	$\{\frac{1}{2},3,-1\},\{0,2,-\frac{3}{2}\}$
	$\mathcal{O}_{\{16\|126\},4,5} = \begin{pmatrix} -\mathcal{O}^p_2 - 2\mathcal{O}^p_3 - \mathcal{O}^p_4 - 2\mathcal{O}^p_5 \\ -2\mathcal{O}^p_2 - \mathcal{O}^p_3 - 2\mathcal{O}^p_4 - \mathcal{O}^p_5 \end{pmatrix}$.	$\{\frac{1}{2},1,-1\},\{0,2,-\frac{3}{2}\}$

$\{\frac{1}{2},3,0\},\{0,4,-\frac{1}{2}\}$
$\{\frac{1}{2},3,0\},\{0,2,-\frac{1}{2}\}$
$\{\frac{1}{2},1,0\},\{0,2,-\frac{1}{2}\}$
$\{0,3,-1\},\{0,4,-\frac{1}{2}\}$
$\{0,3,-1\},\{0,2,-\frac{1}{2}\}$
$\{0,1,-1\},\{0,2,-\frac{1}{2}\}$
$\{0,3,-1\},\{0,4,-\frac{3}{2}\}$
$\{0,3,-1\},\{0,2,-\frac{3}{2}\}$
$\{0,1,-1\},\{0,2,-\frac{3}{2}\}$

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[Bonnet, Hernandez, Ota, Winter, 2009]

[Babu, Nandi, Tavartkiladze, 2009]

Complete Dim-6 UV Resonances

[Li, Ni, Xiao, **Yu**, 2204.03660]

Scalar	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$S1\ (\mathbf{1},\mathbf{1},0)$	$B_L{}^2HH^\dagger$ $D^2H^2H^{\dagger 2}$ $d_CHH^{\dagger 2}Q[(F11),(F8)]$ $e_CHH^{\dagger 2}L[(F3),(F2)]$ $G_L{}^2HH^\dagger$ $H^2H^\dagger Qu_C[(S4),(F11),(F9)]$ $HH^\dagger W_L{}^2$ $H^3H^{\dagger 3}[(S6),(S2),(S5),(S4,S6),(S2,S4),(S4,S5),(S4)]$ $e_CHH^{\dagger 2}L$ $d_CHH^{\dagger 2}Q$ $H^2H^\dagger Qu_C$
$S2\ (\mathbf{1},\mathbf{1},1)$	$d_CHH^{\dagger 2}Q[(S4),(F10),(F9)]$ $e_CHH^{\dagger 2}L[(S4),(F4),(F1)]$ $H^2H^\dagger Qu_C[(F8),(F12)]$ $L^2L^{\dagger 2}$ $H^3H^{\dagger 3}[(S4),(S5),(S5,S6),(S1),(S4,S5),(S1,S4),(S5,S6),(S4,S6)]$
$S3\ (\mathbf{1},\mathbf{1},2)$	$e_C{}^2e_C^{\dagger 2}$
$S4\ (\mathbf{1},\mathbf{2},\frac{1}{2})$	$d_C^\dagger e_CLQ^\dagger$ $d_CHH^{\dagger 2}Q[(S6),(S2)]$ $e_CHH^{\dagger 2}L[(S6),(S2)]$ $H^2H^\dagger Qu_C$ $H^2H^\dagger Qu_C[(S5),(S1)]$ $QQ^\dagger u_Cu_C^\dagger$ $H^3H^{\dagger 3}[(S6),(S2),(S5,S6),(S2,S5),(S1,S6),(S1,S2),(S2,S6),(S5),(S1,S5),(S1)]$
$S5\ (\mathbf{1},\mathbf{3},0)$	$B_LHH^\dagger W_L$ $D^2H^2H^{\dagger 2}$ $d_CHH^{\dagger 2}Q[(F11),(F13)]$ $e_CHH^{\dagger 2}L[(F3),(F6)]$ $H^2H^\dagger Qu_C[(S4),(F11),(F14)]$ $HH^\dagger W_L{}^2$ $H^3H^{\dagger 3}[(S7),(S6),(S2,S6),(S1),($ … $e_CHH^{\dagger 2}L$ d_C …
$S6\ (\mathbf{1},\mathbf{3},1)$	$d_CHH^{\dagger 2}Q[(S4),(F10),(F14$ … $H^2H^\dagger Qu_C[(F1$ … $H^3H^{\dagger 3}[(S7),(S4),(S$ … $,(S5,S7),(S4,S5),($ …
$S7\ (\mathbf{1},\mathbf{4},\frac{1}{2})$	$H^3H^{\dagger 3}[(S$ …
$S8\ (\mathbf{1},\mathbf{4},\frac{3}{2})$	H^3 …
$S9\ (\mathbf{3},\mathbf{1},-\frac{4}{3})$	
$S10\ (\mathbf{3},\mathbf{1},-\frac{1}{3})$	$Q^2Q^{\dagger 2}$ e_CLQu …
$S11\ (\mathbf{3},\mathbf{1},\frac{2}{3})$	
$S12\ (\mathbf{3},\mathbf{2},\frac{1}{6})$	d …
$S13\ (\mathbf{3},\mathbf{2},\frac{7}{6})$	L …
$S14\ (\mathbf{3},\mathbf{3},-\frac{1}{3})$	
$S15\ (\mathbf{6},\mathbf{1},-\frac{2}{3})$	
$S16\ (\mathbf{6},\mathbf{1},\frac{1}{3})$	d_CQ^2u …
$S17\ (\mathbf{6},\mathbf{1},\frac{4}{3})$	
$S18\ (\mathbf{6},\mathbf{3},\frac{1}{3})$	
$S19\ (\mathbf{8},\mathbf{2},\frac{1}{2})$	Q …

Vector	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$V1\ (\mathbf{1},\mathbf{1},0)$	$d_C{}^2d_C^{\dagger 2}$ $d_Cd_C^\dagger e_Ce_C^\dagger$ $e_C{}^2e_C^{\dagger 2}$ $Dd_Cd_C^\dagger HH^\dagger$ $De_Ce_C^\dagger HH^\dagger$ $D^2H^2H^{\dagger 2}$ $d_Cd_C^\dagger LL^\dagger$ $e_Ce_C^\dagger LL^\dagger$ $DHH^\dagger LL^\dagger$ $L^2L^{\dagger 2}$ $d_Cd_C^\dagger QQ^\dagger$ $e_Ce_C^\dagger QQ^\dagger$ $DHH^\dagger QQ^\dagger$ $LL^\dagger QQ^\dagger$ $Q^2Q^{\dagger 2}$ $d_Cd_C^\dagger u_Cu_C^\dagger$ $e_Ce_C^\dagger u_Cu_C^\dagger$ $DHH^\dagger u_Cu_C^\dagger$ $LL^\dagger u_Cu_C^\dagger$ $QQ^\dagger u_Cu_C^\dagger$ $d_CHH^{\dagger 2}Q$ $e_CHH^{\dagger 2}L$ $H^2H^\dagger Qu_C$ $e_CHH^{\dagger 2}L$ $d_CHH^{\dagger 2}Q$ $H^2H^\dagger Qu_C$
$V2\ (\mathbf{1},\mathbf{1},1)$	$D^2H^2H^{\dagger 2}$ $Dd_CH^{\dagger 2}u_C^\dagger$ $d_Cd_C^\dagger u_Cu_C^\dagger$ $e_CHH^{\dagger 2}L$ $d_CHH^{\dagger 2}Q$ $H^2H^\dagger Qu_C$ $d_CHH^{\dagger 2}Q$
$V3\ (\mathbf{1},\mathbf{2},\frac{3}{2})$	$e_Ce_C^\dagger LL^\dagger$
$V4\ (\mathbf{1},\mathbf{3},0)$	$D^2H^2H^{\dagger 2}$ $DHH^\dagger LL^\dagger$ $L^2L^{\dagger 2}$ $DHH^\dagger QQ^\dagger$ $LL^\dagger QQ^\dagger$ $Q^2Q^{\dagger 2}$ $e_CHH^{\dagger 2}L$ $d_CHH^{\dagger 2}Q$ $H^2H^\dagger Qu_C$ $e_CHH^{\dagger 2}L$
$V5\ (\mathbf{3},\mathbf{1},\frac{2}{3})$	$d_C^\dagger e_CLQ^\dagger$
$V6\ (\mathbf{3},\mathbf{1},\frac{5}{3})$	$e_Ce_C^\dagger u_Cu_C^\dagger$
$V7\ (\mathbf{3},\mathbf{2},-\frac{5}{6})$	$d_Cd_C^\dagger LL^\dagger$ $d_C^\dagger e_CLQ^\dagger$ $e_Ce_C^\dagger QQ^\dagger$ $d_CL^\dagger Q^\dagger u_C$ $e_CQ^{\dagger 2}u_C$ $QQ^\dagger u_Cu_C^\dagger$
$V8\ (\mathbf{3},\mathbf{2},\frac{1}{6})$	$d_Cd_C^\dagger QQ^\dagger$ $d_CL^\dagger Q^\dagger u_C$ $LL^\dagger u_Cu_C^\dagger$
$V9\ (\mathbf{3},\mathbf{3},\frac{2}{3})$	$LL^\dagger QQ^\dagger$
$V10\ (\mathbf{6},\mathbf{2},-\frac{1}{6})$	$d_Cd_C^\dagger QQ^\dagger$
$V11\ (\mathbf{6},\mathbf{2},\frac{5}{6})$	$QQ^\dagger u_Cu_C^\dagger$
$V12\ (\mathbf{8},\mathbf{1},0)$	$d_C{}^2d_C^{\dagger 2}$ $d_Cd_C^\dagger QQ^\dagger$ $Q^2Q^{\dagger 2}$ $d_Cd_C^\dagger u_Cu_C^\dagger$ $QQ^\dagger u_Cu_C^\dagger$ $u_C{}^2u_C^{\dagger 2}$
$V13\ (\mathbf{8},\mathbf{1},1)$	$d_Cd_C^\dagger u_Cu_C^\dagger$
$V14\ (\mathbf{8},\mathbf{3},0)$	$Q^2Q^{\dagger 2}$

Fermion	
$(SU(3)_c, SU(2)_2, U(1)_y)$	
$F1\ (\mathbf{1},\mathbf{1},0)$	$DHH^\dagger LL^\dagger$ $e_CHH^{\dagger 2}L[(F3),$ …
$F2\ (\mathbf{1},\mathbf{1},1)$	$B_Le_CH^\dagger L$ $DHH^\dagger LL^\dagger$ e_CHH … $e_CHH^{\dagger 2}L$
$F3\ (\mathbf{1},\mathbf{2},\frac{1}{2})$	$B_Le_CH^\dagger L$ $e_CHH^{\dagger 2}L[(F5),(F1),$ …
$F4\ (\mathbf{1},\mathbf{2},\frac{3}{2})$	$De_Ce_C^\dagger HH^\dagger$ $e_CHH^{\dagger 2}L[(F6),(F2),$ …
$F5\ (\mathbf{1},\mathbf{3},0)$	$DHH^\dagger LL^\dagger$ $e_CHH^{\dagger 2}L[(F3),$ …
$F6\ (\mathbf{1},\mathbf{3},1)$	$e_CH^\dagger LW_L$ $e_CHH^{\dagger 2}L[(F$ …
$F8\ (\mathbf{3},\mathbf{1},-\frac{1}{3})$	$B_Ld_CH^\dagger Q$ $d_CG_LH^\dagger Q$ $DHH^\dagger QQ^\dagger$ d_C …
$F9\ (\mathbf{3},\mathbf{1},\frac{2}{3})$	$DHH^\dagger QQ^\dagger$ B_LHQu_C G_LHQu_C $d_CHH^{\dagger 2}Q[(F11),(S2)]$ $H^2H^\dagger Qu_C$
$F10\ (\mathbf{3},\mathbf{2},-\frac{5}{6})$	$Dd_Cd_C^\dagger HH^\dagger$ $d_CHH^{\dagger 2}Q[(F13),(F8),(S6),(S2)]$ $d_CHH^{\dagger 2}Q$
$F11\ (\mathbf{3},\mathbf{2},\frac{1}{6})$	$B_Ld_CH^\dagger Q$ B_LHQu_C G_LHQu_C $DHH^\dagger u_Cu_C^\dagger$ $d_CHH^{\dagger 2}Q[(F14),(F9),(F13),(F8),(S5),(S1)]$ $H^2H^\dagger Qu_C[(F14),(F9),(F13),(F8),(S5),(S1)]$
$F12\ (\mathbf{3},\mathbf{2},\frac{7}{6})$	$DHH^\dagger u_Cu_C^\dagger$ $H^2H^\dagger Qu_C[(F14),(F9),(S6),(S2)]$ $H^2H^\dagger Qu_C$
$F13\ (\mathbf{3},\mathbf{3},-\frac{1}{3})$	$d_CH^\dagger QW_L$ $d_CHH^{\dagger 2}Q[(F10),(F11),(S5)]$ H^2 …
$F14\ (\mathbf{3},\mathbf{3},\frac{2}{3})$	HQu_CW_L $d_CHH^{\dagger 2}Q[(F11),(S6)]$ $H^2H^\dagger Q$ …

[de Blas, Criado, Perez-Victoria, Santiago, 2017]

New LHC searches!

EFT Motivated Simplified Model

Paradigm shift from new physics motivated simplified model

RPV

RPV stop to 4 quarks
RPV squark to 4 quarks
RPV gluino t[illegible] quarks
RPV gluino [illegible] quarks

Extra Dimensions

ADD (jj) HLZ, $n_{ED} =$ [illegible]
ADD ($\gamma\gamma, \ell\ell$) HLZ, $n_{ED} =$ [illegible]
ADD G_{KK} emission, $n = 2$
ADD QBH (jj), [illegible]
ADD QBH ($e\mu$), [illegible]
RS $G_{KK}(\gamma\gamma)$, $k/\overline{M}_{Pl} = 0.1$
RS QBH (jj), $n_{ED} = 1$
RS QBH ($e\mu$), $n_{ED} = 1$
non-rotating BH, M_D = 4 TeV, n_{ED} = 6
split-UED, $\mu \geq 4$ TeV
RS $G_{KK}(q\bar{q}, gg)$, $k/\overline{M}_{Pl} = 0.1$

Contact Interactions

quark compositeness ($q\bar{q}$), $\eta_{LL/RR} = 1$
quark compositeness ($\ell\ell$), $\eta_{LL/RR} = 1$
quark compositeness ($q\bar{q}$), $\eta_{LL/RR} = -1$
quark compositeness ($\ell\ell$), $\eta_{LL/RR} = -1$
Excited Lepton Contact Interaction
Excited Lepton Contact Interaction

Dark Matter

(axial-)vector mediator ($\chi\chi$), $g_q = 0.25$, $g_{DM} = 1$, $m_\chi = 1$ GeV
(axial-)vector mediator ($q\bar{q}$), $g_q = 0.25$, $g_{DM} = 1$, $m_\chi = 1$ GeV
scalar mediator ($+t/t\bar{t}$), $g_q = 1$, $g_{DM} = 1$, $m_\chi = 1$ GeV
pseudoscalar mediator ($+t/t\bar{t}$), $g_q = 1$, $g_{DM} = 1$, $m_\chi = 1$ GeV
scalar mediator (fermion portal), $\lambda_u = 1$, $m_\chi = 1$ GeV
complex sc. med. (dark QCD), $m_{\pi_{DK}} = 5$ GeV, $c\tau_{X_{DK}} = 25$ mm
Baryonic Z′, $g_q = 0.25$, $g_{DM} = 1$, $m_\chi = 1$ GeV
$Z' - 2HDM$, $g_{Z'} = 0.8$, $g_{DM} = 1$, $\tan\beta = 1$, $m_\chi = 100$ GeV
Vector resonance, $g_q = 0.25$, $g_{DM} = 1$, $m_\chi = 1$ GeV
Leptoquark mediator, $\beta = 1$, $B = 0.1$, $\Delta_{X,DM} = 0.1$, $800 < M_{LQ} < 1500$

Excited Fermions

excited light quark ($q\gamma$), $f_S = f = f' = 1, \Lambda = m_q^*$
excited b quark, $f_S = f = f' = 1, \Lambda = m_q^*$
excited light quark (qg), $\Lambda = m_q^*$
excited electron, $f_S = f = f' = 1, \Lambda = m_e^*$
excited muon, $f_S = f = f' = 1, \Lambda = m_\mu^*$

Heavy Fermions

νMSM, $|V_{eN}|^2 = 1.8$, $|V_{\mu N}|^2 = 1.8$
νMSM, $|V_{eN}V_{\mu N}^*|^2/(|V_{eN}|^2 + |V_{\mu N}|^2) = 1.0$
Type-III seesaw heavy fermions, Flavor-democratic
Vector like taus, Doublet

Leptoquarks

scalar LQ (pair prod.), coupling to 1st gen. fermions, $\beta = 1$
scalar LQ (pair prod.), coupling to 1st gen. fermions, $\beta = 0.5$
scalar LQ (pair prod.), coupling to 2nd gen. fermions, $\beta = 1$
scalar LQ (pair prod.), coupling to 2nd gen. fermions, $\beta = 1$
scalar LQ (pair prod.), coupling to 2nd gen. fermions, $\beta = 0.5$
scalar LQ (pair prod.), coupling to 3rd gen. fermions, $\beta = 1$
scalar LQ (single prod.), coup. to 3rd gen. ferm., $\beta = 1, \lambda = 1$

Heavy Gauge Bosons

[illegible], narrow resonance
[illegible] narrow resonance
SSM [illegible]
SSM $Z'(q\bar{q})$
$Z'(q\bar{q})$
[illegible]erstring Z'_ψ
LF[illegible] Z[illegible] BR($e\mu$) = 10[illegible]
Lept[illegible]ob[illegible] Z′
SSM $W'(\ell\nu)$
SSM $W'(\tau\nu)$
SSM $W'(q\bar{q})$
LRSM $W_R(\ell N_R)$, $M_{N_R} = 0.5 M_{W_R}$
LRSM $W_R(\tau N_R)$, $M_{N_R} = 0.5 M_{W_R}$
Axigluon, Coloron, $\cot\theta = 1$

Complete Dim-7 UV Resonances

$(SU(3)_c, SU(2)_L, U(1)_y)$	Scalar
S1 (1,1,0)	$H^3H^\dagger L^2$[(S6),(S2),(F5),(F1),(S4,S6),(S2,S4),(S4,F5),(S4,F1),(F3,F5),(F1,F3),(S6,F3),(S2,F3)]
S2 (1,1,1)	$D^2H^2L^2$ e_CHL^3[(S4),(F4),(F1)] d_CHL^2Q[(S4),(F10),(F9)] $HL^2Q^\dagger u_C^\dagger$[(S4),(F8),(F12)] $De_CH^{\dagger3}L^\dagger$[(F1),(F3),(V3)] $H^3H^\dagger L^2$[(F1,F3),(S5,S6),(S1),(F5,F6),(F1,F2),(S4,S6),(S4),(S5,S6),(S5),(S4,S5),(S1,S4),(S4,F5),(S4,F1),(F3,F5),(S5,F6),(S5,F2),(F3,F6),(F2,F3),(S5,F3),(S1,F3)] $H^2L^2W_L$ $B_LH^2L^2$ e_CHL^3 $HL^2Q^\dagger u_C^\dagger$ d_CHL^2Q $De_C^\dagger H^3L$
S4 (1,2,1/2)	$H^3H^\dagger L^2$[(S6),(S2,S6),(S2),(S5,S6),(S2,S5),(S1,S6),(S1,S2),(S6,F5),(S6,F1),(S2,F5),(S2,F1),(S5,F5),(S5,F1),(S1,F5),(S1,F1)]
S5 (1,3,0)	$H^3H^\dagger L^2$[(S6),(S2,S6),(F5) … (S2,S4),(S7,F5),(S4,F5), … (F1,F3),(S6,F7), …
S6 (1,3,1)	$D^2H^2L^2$ e_CHL^3[(S4),(F4),(F … $HL^2Q^\dagger u_C^\dagger$[(S4 … $De_CH^{\dagger3}L^\dagger$[(F5),(F3),(… $H^3H^\dagger L^2$[(F3,F5),(S5),(S1),(S2, … (S7),(S4),(S2,S4),(S8),(S5),(S2,S5),(S … (S4,F1),(F5,F7),(F1,F3),(S8,F6), … ,(F2,F3),(S5,F7), … $H^2L^2W_L$ B_L … $HL^2Q^\dagger u_C^\dagger$ d_CH …
S7 (1,4,1/2)	$H^3H^\dagger L^2$[(S6),(S5,S …
S8 (1,4,3/2)	$H^3H^\dagger L^2$[(S …
S10 (3,1,−1/3)	$d_C^2HLu_C$[(S12),(F10),(F1)] $d_Ce_C^\dagger HLu_C^\dagger$[(S12),(F10),(F1)]
S11 (3,1,2/3)	$d_C^3H^\dagger L$[(S12),(F11),(F2)] $d_C^2HLu_C$[(F11 …
S12 (3,2,1/6)	$d_C^3H^\dagger L$[(S11),(F11)] d_C^2 … d_CHL^2Q[(S10),(S14),(F5),(F1),(F14)]
S13 (3,2,7/6)	$d_C^2HLu_C$[(S11),(F10)]
S14 (3,3,−1/3)	d_CHL^2Q[(S12),(F10),(F5)]

$(SU(3)_c, SU(2)_L, U(1)_y)$	Vector
V2 (1,1,1)	$Dd_CL^2u_C^\dagger$ $D^2H^2L^2$ $De_CH^{\dagger3}L^\dagger$[(F1),(V3),(F3)] $H^2L^2W_L$ $B_LH^2L^2$ e_CHL^3 $HL^2Q^\dagger u_C^\dagger$ d_CHL^2Q $De_C^\dagger H^3L$ $HL^2Q^\dagger u_C^\dagger$ d_CHL^2Q
V3 (1,2,3/2)	$De_CH^{\dagger3}L^\dagger$ $d_Ce_C^\dagger HLu_C^\dagger$[(F10),(F12)] $De_CH^{\dagger3}L^\dagger$[(V2),(V5),(S6),(S2)]
V5 (1,3,1)	$D^2H^2L^2$ $De_CH^{\dagger3}L^\dagger$[(F3),(V3),(F5)] $H^2L^2W_L$ $B_LH^2L^2$ e_CHL^3 $HL^2Q^\dagger u_C^\dagger$ d_CHL^2Q $De_C^\dagger H^3L$
V5 (3,1,2/3)	$Dd_C^2e_C^\dagger$ $HL^2Q^\dagger u_C^\dagger$[(F1),(V8),(F12)] $d_Ce_C^\dagger HLu_C^\dagger$[(F1),(V8),(F12)] $d_CHLQ^{\dagger2}$[(V8),(F10),(F8)]
V8 (3,2,1/6)	$Dd_CL^2u_C^\dagger$ $Dd_C^2LQ^\dagger$ $HL^2Q^\dagger u_C^\dagger$[(F5),(F1),(V9),(V5),(F13),(F8)] $d_CHLQ^{\dagger2}$[(F5),(F1),(V9),(V5),(F13),(F8)] $d_Ce_C^\dagger HLu_C^\dagger$[(V5),(F10),(F3)] $d_C^2e_C^\dagger HQ^\dagger$[(V5),(F10),(F3)] $d_CHLQ^{\dagger2}$ $d_C^2e_C^\dagger HQ^\dagger$ $d_C^2HLu_C$ $HL^2Q^\dagger u_C^\dagger$ d_CHL^2Q
V9 (3,3,2/3)	$HL^2Q^\dagger u_C^\dagger$[(F5),(V8),(F12)] $d_CHLQ^{\dagger2}$[(V8),(F10),(F13)]

$(SU(3)_c, SU(2)_L, U(1)_y)$	Fermion
F1 (1,1,0)	$D^2H^2L^2$ e_CHL^3[(S4),(S2)] d_CHL^2Q[(S4),(S10),(S12)] $HL^2Q^\dagger u_C^\dagger$[(S4),(V5),(V8)] $De_CH^{\dagger3}L^\dagger$[(S2),(F3),(V2)] $d_C^2HLu_C$[(S11),(S10)] $d_Ce_C^\dagger HLu_C^\dagger$[(S10),(V5)] $d_CHLQ^{\dagger2}$[(S10),(V8)] $H^2L^2W_L$[(F5)] $H^3H^\dagger L^2$[(S2,F3),(S5,F5),(S1),(S6,F6),(S2,F2),(F5,F5),(F3),(S4,S6),(S2,S1),(S5,F3),(S4,S5),(S1,S4),(F3,F6),(F2,F3),(S5,F3),(S1,F3)] $H^2L^2W_L$ $B_LH^2L^2$ e_CHL^3 $HL^2Q^\dagger u_C^\dagger$ d_CHL^2Q $De_C^\dagger H^3L$
F2 (1,1,1)	$d_C^3H^\dagger L$[(S11)] $H^3H^\dagger L^2$[(S6,F5),(S2,F1),(F3,F5),(F1,F3),(S5,S6),(S2,S5),(S6,F3),(S2,F3)]
F3 (1,2,1/2)	$De_CH^{\dagger3}L^\dagger$[(F5),(F1),(S6),(S2),(V2),(V5)] $d_Ce_C^\dagger HLu_C^\dagger$[(S12),(V8)] $d_C^2e_C^\dagger HQ^\dagger$[(V8),(S11)]
F4 (1,2,3/2)	e_CHL^3[(S6),(S2)]
F5 (1,3,0)	e_CHL^3[(S4),(S8)] d_CHL^2Q[(S4),(S12),(S14)] $HL^2Q^\dagger u_C^\dagger$[(S4),(V9),(V8)] $D^2H^2L^2$ $De_CH^{\dagger3}L^\dagger$[(S6),(F3),(V5)] $d_CHLQ^{\dagger2}$[(S14),(V8)] $H^2L^2W_L$[(F7),(F1)] $H^2L^2W_L$ $B_LH^2L^2$ e_CHL^3 $HL^2Q^\dagger u_C^\dagger$ d_CHL^2Q $De_C^\dagger H^3L$
F6 (1,3,1)	$H^3H^\dagger L^2$[(S5,F5),(S6,F1),(S2,F5),(F5,F7),(F3,F5),(F1,F3),(S6,S8),(S5,S6),(S2,S5),(S5,F7),(S6,F3),(S2,F3)]
F7 (1,4,1/2)	$H^2L^2W_L$[(F5),(S6)] $H^3H^\dagger L^2$[(F5),(S8,F5),(F5,F6),(S6,F6),(S5,F5),(S5,S6)]
F8 (3,1,−1/3)	$HL^2Q^\dagger u_C^\dagger$[(S2),(V8)] $d_CHLQ^{\dagger2}$[(V8),(S12),(V5)] $d_C^2e_C^\dagger HQ^\dagger$[(V5),(S11)]
F9 (3,1,2/3)	d_CHL^2Q[(S12),(S2)]
F10 (3,2,−5/6)	$d_C^2HLu_C$[(S12),(S10),(S13)] d_CHL^2Q[(S10),(S6),(S … $d_Ce_C^\dagger HLu_C^\dagger$[(S10),(V3),(V8)] $d_CHLQ^{\dagger2}$[(S10),(S14), …
F11 (3,2,1/6)	$d_C^3H^\dagger L$[(S11),(S12)] $d_C^2HLu_C$[(S11),(S12) …
F12 (3,2,7/6)	$HL^2Q^\dagger u_C^\dagger$[(S6),(S2),(V9),(V5)] $d_Ce_C^\dagger HLu_C^\dagger$[(V5),(S12),(V3)]
F13 (3,3,−1/3)	$HL^2Q^\dagger u_C^\dagger$[(S6),(V9)] $d_CHLQ^{\dagger2}$[(V8),(S12),(V9)]
F14 (3,3,2/3)	d_CHL^2Q[(S12),(S6)]

[Li, Ni, Xiao, **Yu**, 2204.03660]

More LHC searches!

Complete Dim-8 UV Resonances

[Li, Ni, Xiao, **Yu**, in preparation **]**

Type: $D^4H^2H^{\dagger 2}$

$$\mathcal{O}_1^f = \tfrac{1}{4}\mathcal{Y}[\square\square_H]\mathcal{Y}[\square\square_{H^\dagger}] H_i H_j (D_\mu D_\nu H^{\dagger i})(D^\mu D^\nu H^{\dagger j}),$$
$$\mathcal{O}_2^f = \tfrac{1}{4}\mathcal{Y}[\square\square_H]\mathcal{Y}[\square\square_{H^\dagger}] H^{\dagger i} H_i (D_\mu D_\nu H_j)(D^\mu D^\nu H^{\dagger j}),$$
$$\mathcal{O}_3^f = \tfrac{1}{4}\mathcal{Y}[\square\square_H]\mathcal{Y}[\square\square_{H^\dagger}] H_i (D_\mu H_j)(D_\nu H^{\dagger i})(D^\mu D^\nu H^{\dagger j}).$$

	group: (Spin, $SU(3)_c$, $SU(2)_w$, $U(1)_y$)	
	$\{H_1, H_2\}, \{H^\dagger{}_3, H^\dagger{}_4\}$	
*	$(2,1,3,1)$	$-8\mathcal{O}_1^f - 48\mathcal{O}_2^f - 48\mathcal{O}_3^f$
	$(0,1,3,1)$	$8\mathcal{O}_1^f$
	$(1,1,1,1)$	$8\mathcal{O}_1^f + 16\mathcal{O}_3^f$
	$\{H_1, H^\dagger{}_3\}, \{H_2, H^\dagger{}_4\}$	
*	$(2,1,3,0)$	$16\mathcal{O}_1^f - 4\mathcal{O}_2^f + 56\mathcal{O}_3^f$
	$(1,1,3,0)$	$8\mathcal{O}_1^f - 4\mathcal{O}_2^f + 8\mathcal{O}_3^f$
	$(0,1,3,0)$	$8\mathcal{O}_1^f + 4\mathcal{O}_2^f + 16\mathcal{O}_3^f$
*	$(2,1,1,0)$	$-24\mathcal{O}_1^f - 4\mathcal{O}_2^f - 24\mathcal{O}_3^f$
	$(1,1,1,0)$	$-4\mathcal{O}_2^f - 8\mathcal{O}_3^f$
	$(0,1,1,0)$	$4\mathcal{O}_2^f$

In the forward limit, a twice-subtracted dispersion relation

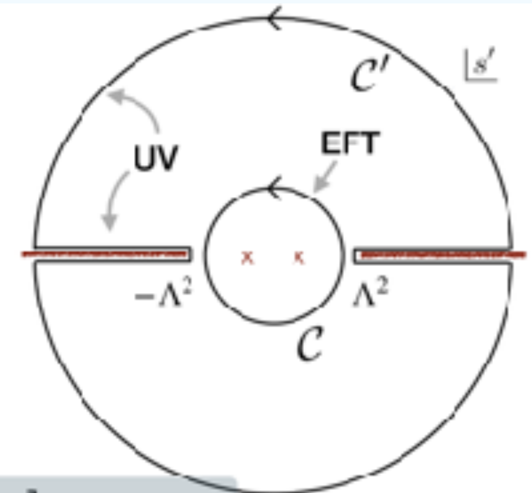
$$\mathcal{M}^{ijkl} = \frac{1}{2\pi}\int_{(c\Lambda)^2}^{\infty} \frac{ds}{s^3} \sum_X \left[\mathbf{M}_{ij\to X}\mathbf{M}^*_{kl\to X} + (j \leftrightarrow l)\right]$$

Particle	Spin	Charge/irrep	Interaction	ER	$\vec{c}$	$\vec{c}^{(6)}$
$\mathcal{B}_1$	1	1_1	$g\mathcal{B}_1^{\mu\dagger}(H^T\epsilon \overleftrightarrow{D}_\mu H) + h.c.$	✓	$8(1,0,-1)$	$2(-1,2)$
Ξ_1	0	3_1	$gM\Xi_1^{I\dagger}(H^T\epsilon\tau^I H) + h.c.$	✗	$8(0,1,0)$	$2(1,2)$
$\mathcal{S}$	0	$1_0(S)$	$gM\mathcal{S}(H^\dagger H)$	✓	$2(0,0,1)$	$-\frac{1}{2}(1,0)$
$\mathcal{B}$	1	$1_0(A)$	$g\mathcal{B}^\mu(H^\dagger \overleftrightarrow{D}_\mu H)$	✓	$2(-1,1,0)$	$-\frac{1}{2}(1,4)$
Ξ_0	0	$3_0(S)$	$gM\Xi_0^I(H^\dagger\tau^I H)$	✗	$2(2,0,-1)$	$\frac{1}{2}(1,-4)$
$\mathcal{W}$	1	$3_0(A)$	$g\mathcal{W}^{\mu I}(H^\dagger\tau^I \overleftrightarrow{D}_\mu H)$	✗	$2(1,1,-2)$	$-\frac{3}{2}(1,0)$

Analyticity in complex s plane (fixed t)

$$A(s,t) = \frac{1}{2\pi i}\oint_{\mathcal{C}} ds' \frac{A(s',t)}{s'-s}$$

Cauchy's integral formula



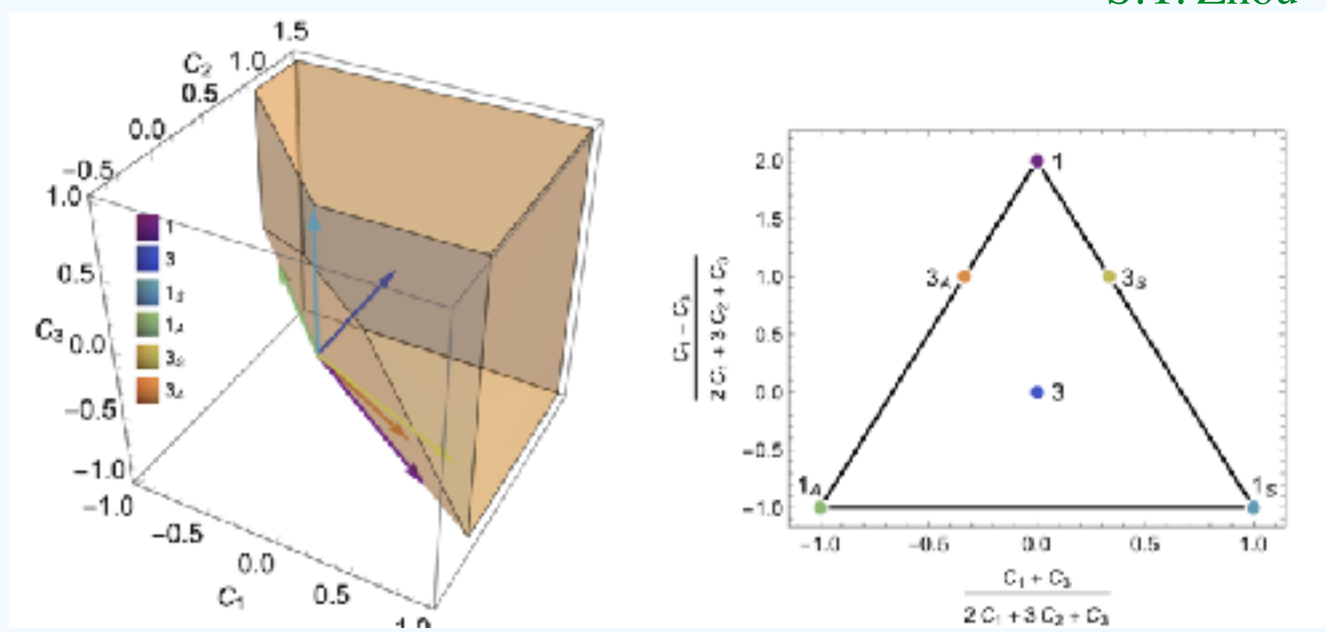
Fixed t dispersion relation

$$A(s,t) \sim \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi\mu^2}\left[\frac{s^2}{\mu - s} + \frac{u^2}{\mu - u}\right] \text{Im}\, A(\mu,t) \qquad \mu > \Lambda^2$$

EFT amplitude — **IR ~ UV connection** — **UV full amplitude**

$$\text{Disc}\, A_{ij\to kl}(s) = A_{ij\to kl}(s) - A_{kl\to ij}(s)^* = i\sum_X M_{ij\to X}(s) M_{kl\to X}(s)^*$$

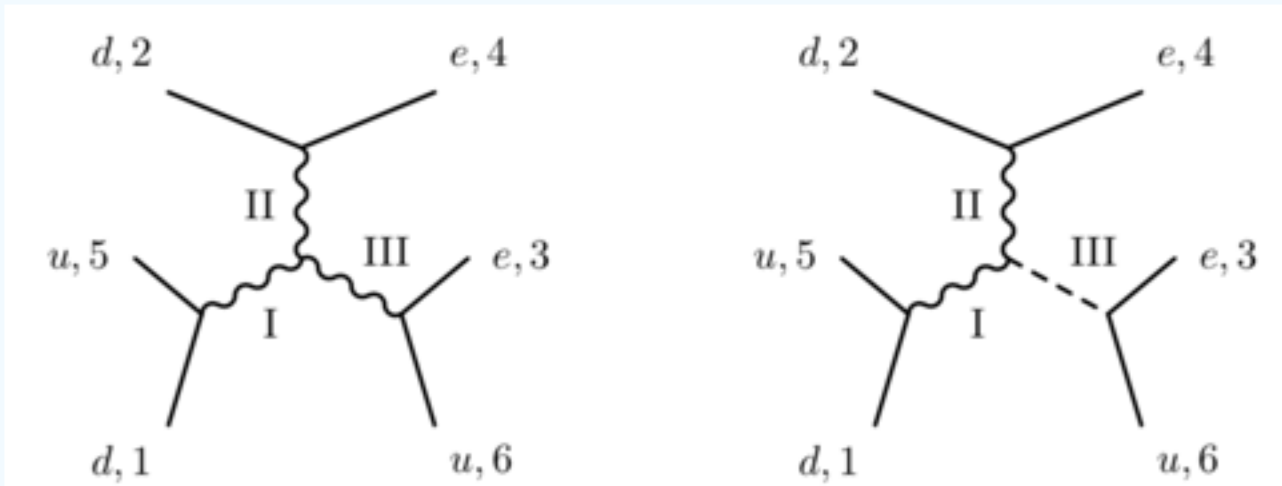
S.Y. Zhou

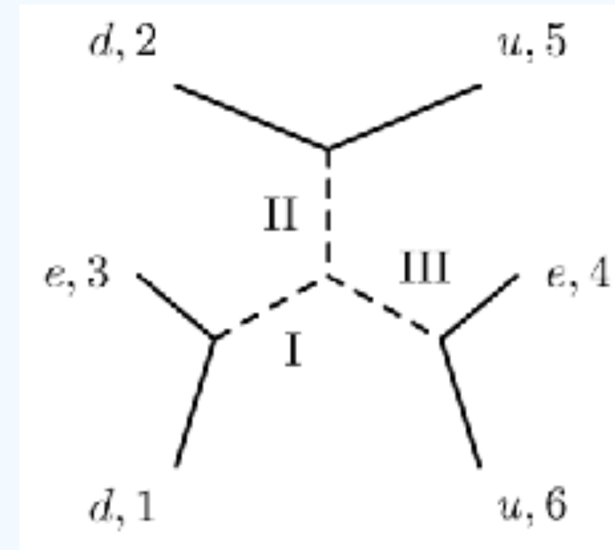


[Cen Zhang, 2021]

Dim-9: 0vbb

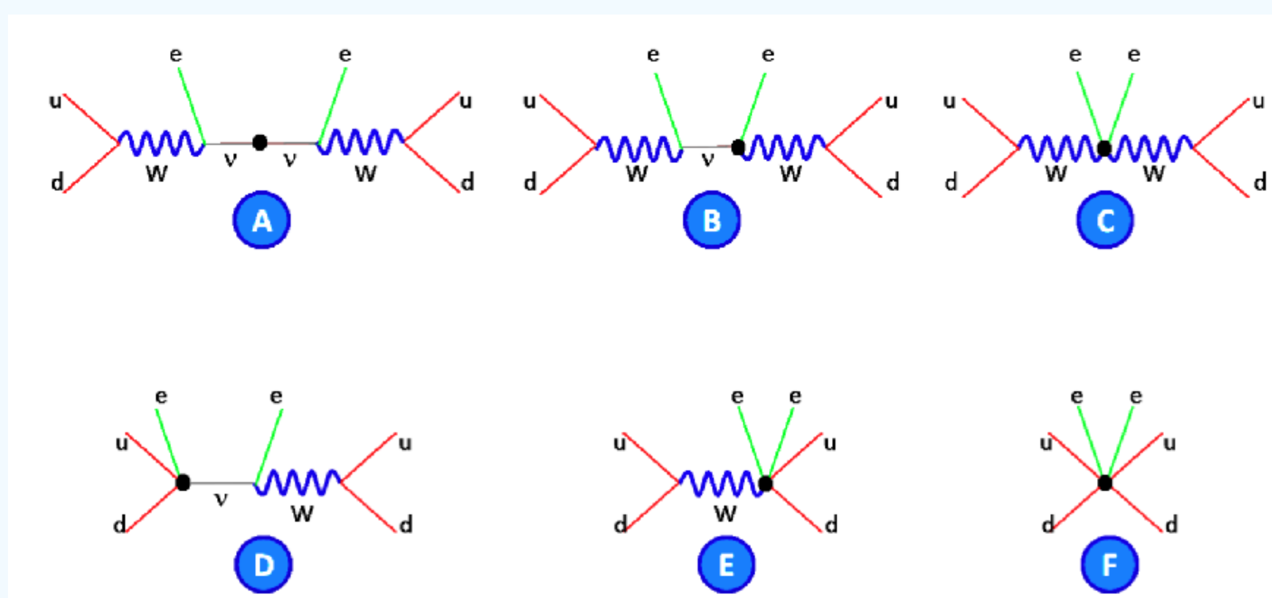
[Li, Ni, Xiao, **Yu**, in preparation]





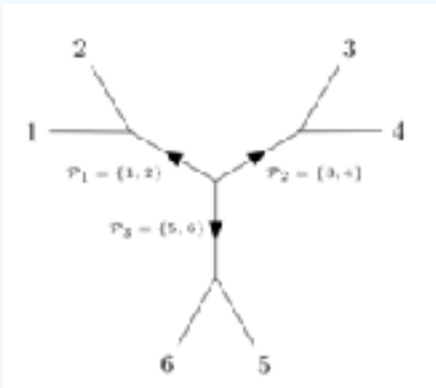
[Bonnet, Hirsch, Ota, Winter, 2012]

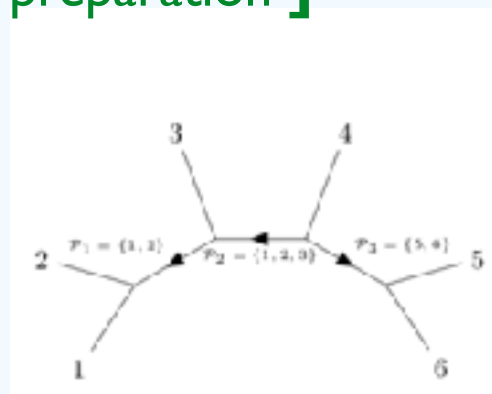
$$\mathbf{W}^2\mathcal{B}^J = -sJ(J+1)\mathcal{B}^J$$

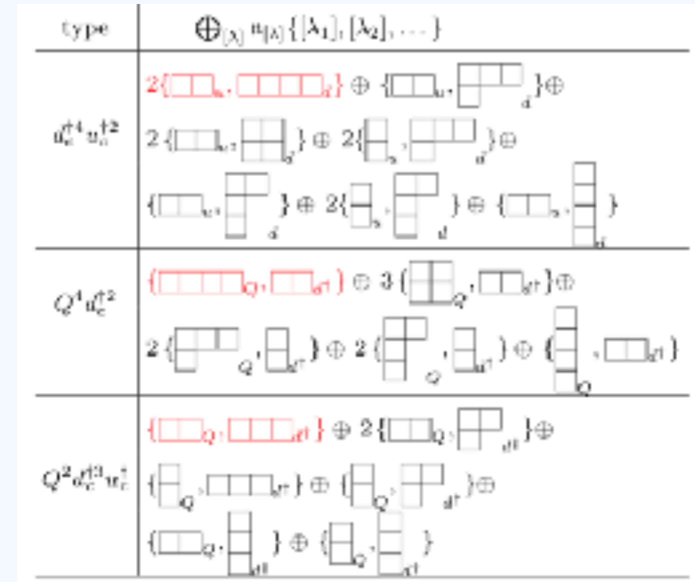


Dim-9: n-nbar oscillation

[Li, Ni, Xiao, **Yu**, in preparation]







$(\mathbf{r}_i, J_i)$	$(1,1,1)$	$(0,1,1)$	$(1,0,1)$	$(1,1,0)$	$(0,0,0)$
$(\mathbf{6},\mathbf{6},\mathbf{6})$	0	$3\mathcal{O}_1+8\mathcal{O}_2$	0	0	$\mathcal{O}_1-8\mathcal{O}_2$
$(\mathbf{6},\bar{\mathbf{3}},\bar{\mathbf{3}})$	0	$\mathcal{O}_2$	0	0	$\mathcal{O}_2$
$(\bar{\mathbf{3}},\mathbf{6},\bar{\mathbf{3}})$	$3\mathcal{O}_1+8\mathcal{O}_2$	0	$\mathcal{O}_1-8\mathcal{O}_2$	$3\mathcal{O}_1+8\mathcal{O}_2$	0
$(\bar{\mathbf{3}},\bar{\mathbf{3}},\mathbf{6})$	$3\mathcal{O}_1+8\mathcal{O}_2$	0	$3\mathcal{O}_1+8\mathcal{O}_2$	$\mathcal{O}_1-8\mathcal{O}_2$	0
$(\bar{\mathbf{3}},\bar{\mathbf{3}},\bar{\mathbf{3}})$	$-3\mathcal{O}_1+8\mathcal{O}_2$	0	$3\mathcal{O}_1+8\mathcal{O}_2$	$3\mathcal{O}_1+8\mathcal{O}_2$	0

[Babu, Mohapatra, Nasri, 2006]

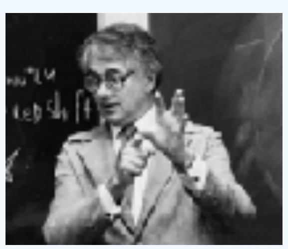
$$\mathcal{O}_1^{\rm P} = \epsilon^{ace}\epsilon^{bdf}(d_{Ra}d_{Rb})(d_{Rc}d_{Rd})(u_{Re}u_{Rf}),$$
$$\mathcal{O}_2^{\rm P} = \epsilon^{acd}\epsilon^{bef}(d_{Ra}d_{Rb})(d_{Rc}u_{Re})(d_{Rd}u_{Rf}).$$

Electroweak Standard Model

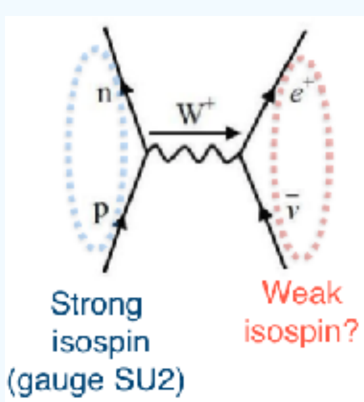
Which UV should be picked out? Lesson from history of SM

V-A Theory
1958

Lee-Yang
1960

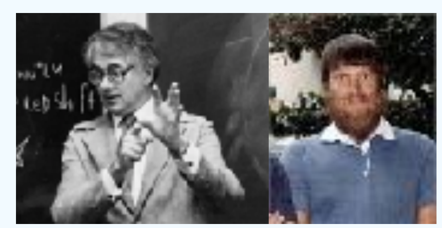


Glashow
1961

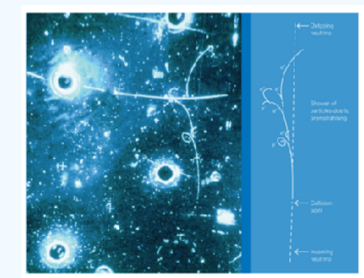


Englert-Brout-Higgs
Guralnik-Hagen
-Kibble
1964

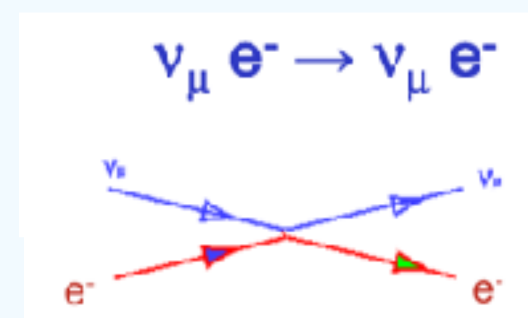
Weinberg-Salam
1967



Georgi-Glashow
1972



CERN Bubble Chamber
1973



Nobel prize
1979

Neutral current EFT operator

No resonance observed yet

Remark: A Bigger Picture

Effective operator has more symmetries than what we expected

Conformal symmetry
Spinor representation

$$[D, P_\mu] = -iP_\mu,$$
$$[D, K_\mu] = iK_\mu,$$
$$[K_\mu, P_\nu] = 2i(\eta_{\mu\nu} D + M_{\mu\nu}),$$
$$[M_{\mu\nu}, K_\rho] = i(\eta_{\mu\rho} K_\nu - \eta_{\nu\rho} K_\mu),$$

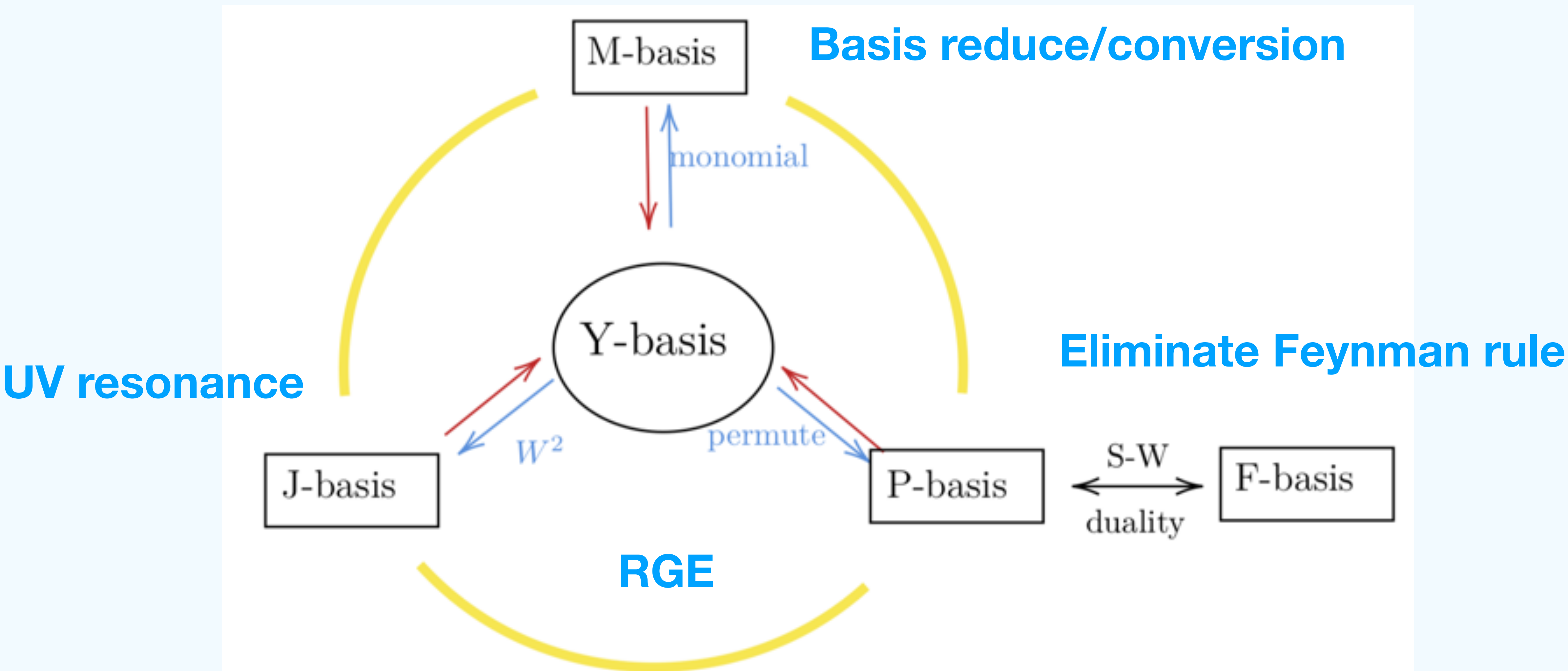
$$P^{\alpha\dot\alpha} = \sum_i \lambda_i^\alpha \tilde\lambda_i^{\dot\alpha} \qquad -iD = n + \frac{1}{2}\sum_i \left(\lambda_i^\alpha \partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha} \tilde\partial_{i\dot\alpha}\right),$$
$$K_{\alpha\dot\alpha} = -4 \sum \partial_{i\alpha} \tilde\partial_{i\dot\alpha} \qquad -iM_{\alpha\beta} = \sum_i \lambda_{i\alpha}\partial_{i\beta} + \lambda_{i\beta}\partial_{i\alpha},$$
$$W_{\alpha\dot\alpha} = \frac{i}{2}\left(P_{\alpha\dot\beta} \bar M^{\dot\beta}{}_{\dot\alpha} - M_\alpha{}^\beta P_{\beta\dot\alpha}\right) \qquad -i\bar M_{\dot\alpha\dot\beta} = \sum_i \tilde\lambda_{i\dot\alpha}\tilde\partial_{i\dot\beta} + \tilde\lambda_{i\dot\beta}\tilde\partial_{i\dot\alpha}.$$

Special conformal K	Pauli-Lubanski W	Dilatation D
Amplitude-basis	UV resonances	Anomalous dim

(Global) symmetry determines interaction (operator)!

Why Young Tensor Basis?

Young Tensor basis exhibits the space-time symmetry of underlying S-matrix



example of “(global) symmetry determines interaction”

Short Summary

1. paradigm shift in new physics

Bottom-up EFT provides a clear pathway to new physics step by step

2. symmetry determines interaction

From Warsaw dim-6 to any-dim Young tensor basis and UVs systematically

Warsaw basis → Young Tensor Basis → Complete UVs

Any operator to any mass dimension

Complete UV resonances

@ LHC

Chiral Lagrangian

Bottom-up

For QCD and Electroweak

EFT Ladder

Energy

UV models — many models .....

$$\mathcal{L} = \mathcal{L}^{\text{eff}}_{\text{Gravity}} + \underbrace{\mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{EW}}}_{\mathcal{L}_{SM}} + \mathcal{L}^{NP}_{\text{heavy}}$$

TeV scale

SMEFT

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \frac{C_i^{(5)}}{\Lambda_{NP}} Q_i^{(5)} + \frac{C_i^{(5)}}{\Lambda_{NP}^2} Q_i^{(6)} + \cdots$$

EW scale

HEFT

m_W

Low energy EFT (QCD+QED)

$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}_{\text{QED+QCD}} + \frac{C_i^{(5)}}{M_W} O_i^{(5)} + \frac{C_i^{(6)}}{M_W^2} O_i^{(6)} + \cdots$$

Λ_{QCD}

Chiral Lagrangian + Heavy B EFT

$$\mathcal{L}_\chi = \mathcal{L}_\pi(s, p, l_\mu, r_\mu) + \mathcal{L}_{\pi N}(s, p, l_\mu, r_\mu) + \cdots$$

MeV scale

Nuclear EFT (nuclear matrix elements)

The Sigma Model

$\mathcal{L}_\sigma = \frac{1}{2}\,\partial_\mu \mathbf{\Phi}^T \partial^\mu \mathbf{\Phi} \;-\; \frac{\lambda}{4}\,\left(\mathbf{\Phi}^T\mathbf{\Phi} - v^2\right)^2$

$\mathbf{\Phi}^T = (\phi_1, \cdots, \phi_N)$

Global Symmetry: $O(4) \sim SU(2) \otimes SU(2)$

SSB: $O(4) \to O(3)$ $\left[\frac{4\times 3}{2} - \frac{3\times 2}{2} = 3 \text{ broken generators}\right]$

$$\mathcal{L}_\sigma = \frac{1}{2}\left\{\partial_\mu\hat{\sigma}\,\partial^\mu\hat{\sigma} + \partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi} - M^2\hat{\sigma}^2\right\} - \frac{M^2}{2v}\,\hat{\sigma}\left(\hat{\sigma}^2 + \vec{\pi}^2\right) - \frac{M^2}{8v^2}\left(\hat{\sigma}^2 + \vec{\pi}^2\right)^2$$

1) $\mathbf{\Sigma}(x) \equiv \sigma(x)\,\mathbf{I}_2 + i\,\vec{\tau}\,\vec{\pi}(x)$; $\langle \mathbf{A}\rangle \equiv \mathrm{Tr}\,(\mathbf{A})$

$$\mathcal{L}_\sigma = \frac{1}{4}\,\langle\partial_\mu\mathbf{\Sigma}^\dagger\,\partial^\mu\mathbf{\Sigma}\rangle - \frac{\lambda}{16}\left(\langle\mathbf{\Sigma}^\dagger\mathbf{\Sigma}\rangle - 2\,v^2\right)^2$$

$O(4) \sim SU(2)_L \otimes SU(2)_R$ Symmetry: $\mathbf{\Sigma} \to g_R\,\mathbf{\Sigma}\,g_L^\dagger$; $g_{L,R} \in SU(2)_{L,R}$

2) $\mathbf{\Sigma}(x) \equiv [v + S(x)]\;\mathbf{U}(x)$; $\mathbf{U} \equiv \exp\left\{\frac{i}{v}\,\vec{\tau}\,\vec{\phi}\right\} \to g_R\,\mathbf{U}\,g_L^\dagger$

$$\mathcal{L}_\sigma = \frac{v^2}{4}\left(1 + \frac{S}{v}\right)^2\langle\partial_\mu\mathbf{U}^\dagger\partial^\mu\mathbf{U}\rangle + \frac{1}{2}\left(\partial_\mu S\,\partial^\mu S - M^2 S^2\right) - \frac{M^2}{2v}\,S^3 - \frac{M^2}{8v^2}\,S^4$$

$E \ll M \sim v$: $\boxed{\mathcal{L}_\sigma \approx \frac{v^2}{4}\,\langle\partial_\mu\mathbf{U}^\dagger\,\partial^\mu\mathbf{U}\rangle}$

Symmetry Breaking

Symmetry **G** $\{T_a\}$ ⟷ Conserved charges $\mathcal{Q}_a$

Noether Theorem: $\partial_\mu j_a^\mu = 0 \quad ; \quad \mathcal{Q}_a = \int d^3x \, j_a^0(x) \quad ; \quad \frac{d}{dt}\mathcal{Q}_a = 0$

Wigner–Weyl	Nambu–Goldstone
$\mathcal{Q}_a \vert 0 \rangle = 0$	$\mathcal{Q}_a \vert 0 \rangle \neq 0$
• Exact Symmetry	• Spontaneously Broken Symmetry
• Degenerate Multiplets	• Massless Goldstone Bosons
• Linear Representation	• Non-Linear Representation

$$\mathcal{L}_\sigma = \frac{1}{2}\,\partial_\mu \mathbf{\Phi}^T \partial^\mu \mathbf{\Phi} - \frac{\lambda}{4}\left(\mathbf{\Phi}^T\mathbf{\Phi} - v^2\right)^2$$

$$\langle 0|\sigma|0\rangle = v$$

$$m_\Phi^2 = \lambda\, v^2$$

$$M^2 = 2\,\lambda\, v^2 \qquad m_\pi = 0$$

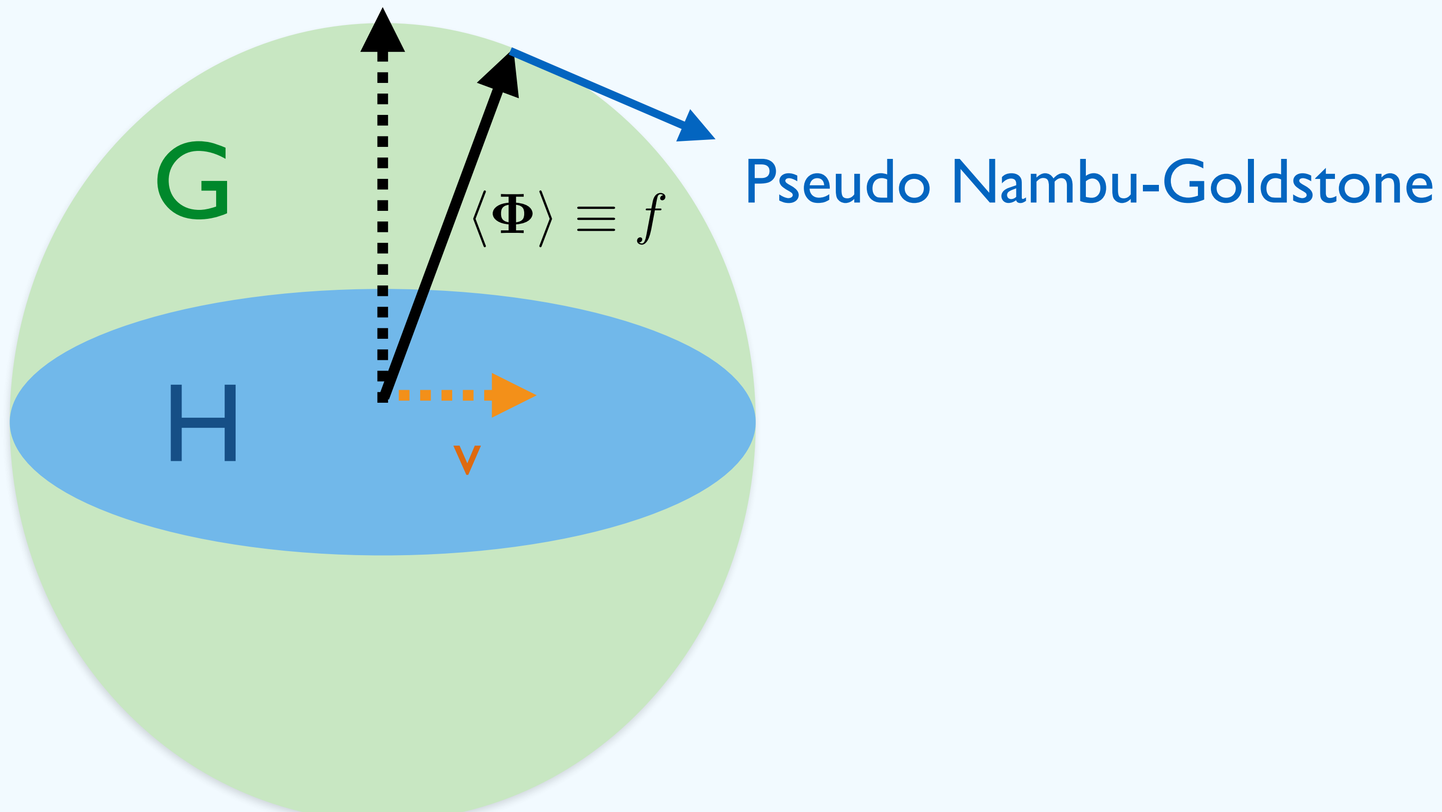
$E \ll M \sim v$: $\quad \mathcal{L}_\sigma \approx \frac{v^2}{4}\langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U}\rangle$

Symmetry Breaking

Chiral Lagrangian description

G → H

$$\mathbf{\Phi}(x) \equiv \exp\left(\frac{i}{f}\pi^{\hat{a}}(x)T^{\hat{a}}\right)\langle\mathbf{\Phi}\rangle$$



Chiral Symmetry

$$\mathcal{L}_{QCD}^{0} = -\frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} + \bar{\mathbf{q}}_L i\gamma^\mu D_\mu \mathbf{q}_L + \bar{\mathbf{q}}_R i\gamma^\mu D_\mu \mathbf{q}_R$$

$$\mathbf{q}^T \equiv (u\,,\, d\,,\, s)$$

$$q'_L = V_L \cdot q_L$$
$$q'_R = V_R \cdot q_R$$

- $\mathcal{L}_{QCD}^{0}$ **invariant under** $\mathbf{G} \equiv \mathbf{SU(3)_L} \otimes \mathbf{SU(3)_R}$**:**

$$\bar{\mathbf{q}}_L \to g_L \bar{\mathbf{q}}_L \quad ; \quad \bar{\mathbf{q}}_R \to g_R \bar{\mathbf{q}}_R \quad ; \quad (g_L, g_R) \in \mathbf{G}$$

- **Only** $\mathbf{SU(3)_V}$ **in the hadronic spectrum:** $(\pi, K, \eta)_{0^-}\,;\,(\rho, K^*, \omega)_{1^-}\,;\,\cdots$

$$M_{0^-} < M_{0^+} \quad ; \quad M_{1^-} < M_{1^+}$$

- **The** $\mathbf{0^-}$ **octet is nearly massless:** $\mathbf{m_\pi \approx 0}$

The vacuum is not invariant (SSB): $\langle 0|\,(\bar{\mathbf{q}}_L\mathbf{q}_R + \bar{\mathbf{q}}_R\mathbf{q}_L)\,|0\rangle \neq 0$

$$G \equiv SU(3)_L \otimes SU(3)_R \xrightarrow{SCSB} H \equiv SU(3)_V \quad \Rightarrow \quad \mathbf{U}_{ij}(\phi) = \left\{\exp\left(i\sqrt{2}\,\mathbf{\Phi}/f\right)\right\}_{ij}$$

$$\mathbf{U} \longrightarrow g_R\, \mathbf{U}\, g_L^\dagger$$

$$\mathbf{\Phi} = \frac{\vec{\lambda}}{\sqrt{2}}\,\vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\,\eta \end{pmatrix}$$

Goldstone Theorem

Noether QCD Currents: $G \equiv SU(3)_L \otimes SU(3)_R$

$$J_X^{a\mu} = \bar{\mathbf{q}}_X \gamma^\mu \frac{\lambda^a}{2} \mathbf{q}_X \quad ; \quad \mathcal{Q}_X^a = \int d^3x \, J_X^{a0}(x) \qquad (a = 1, \cdots, 8 \; ; \; X = L, R)$$

Current Algebra ('60) : $\left[\mathcal{Q}_X^a , \mathcal{Q}_Y^b\right] = i \, \delta_{XY} \, f^{abc} \, \mathcal{Q}_X^c$

"Chiral symmetry" of massless QCD $[Q_i^V, H_0] = 0 \qquad [Q_i^A, H_0] = 0$

Vafa and Witten 1984: $Q_i^V |0\rangle = 0$

Axial charges ? $Q_i^A |0\rangle = ?$

- $\mathcal{Q}_A^a = \mathcal{Q}_R - \mathcal{Q}_L \quad ; \quad \mathcal{O}^b = \bar{\mathbf{q}} \gamma_5 \lambda^b \mathbf{q}$

$$\langle 0| \left[\mathcal{Q}_A^a , \mathcal{O}^b\right] |0\rangle = -\frac{1}{2} \langle 0| \bar{\mathbf{q}} \{\lambda^a , \lambda^b\} \mathbf{q} |0\rangle = -\frac{2}{3} \langle 0| \bar{\mathbf{q}} \mathbf{q} |0\rangle$$

$Q_i^A \|0\rangle = 0$	$Q_i^A \|0\rangle \neq 0$
Wigner-Weyl realization of G ground state is symmetric $\langle 0\| \bar{q}_R q_L \|0\rangle = 0$ ordinary symmetry spectrum contains parity partners degenerate multiplets of G	Nambu-Goldstone realization of G ground state is asymmetric $\langle 0\| \bar{q}_R q_L \|0\rangle \neq 0$ "order parameter" spontaneously broken symmetry spectrum contains Goldstone bosons degenerate multiplets of $SU(3)_V \subset G$

$\langle 0| \bar{u} u |0\rangle = \langle 0| \bar{d} d |0\rangle = \langle 0| \bar{s} s |0\rangle \neq 0$

Goldstone Theorem

$H_0 \, Q_i^A |0\rangle = Q_i^A \, H_0 |0\rangle = 0$

spectrum must contain 8 states

$Q_1^A |0\rangle, \ldots , Q_8^A |0\rangle \quad$ with $\quad E = 0,$

spin 0, negative parity, octet of $SU(3)_V$

Goldstone bosons

CCWZ

Define Goldstone boson matrix, which transform nonlinearly under G

$$U(\Pi) \equiv e^{i\Pi(x)} \to g\, U(\Pi)\, h^\dagger(\Pi(x), g))$$

CCWZ construction

[Callan, Coleman, Wess, Zumino, 1969]

$$-i\, U^\dagger D_\mu U = d_\mu^{\hat{a}} T^{\hat{a}} + E_\mu^a T^a \equiv d_\mu + E_\mu \,.$$

Building block

$$d_\mu(\Pi), \quad E_{\mu\nu}(\Pi)$$

$$f_{\mu\nu} \equiv U^\dagger F_{\mu\nu} U = f^+ T^a + f^- T^{\hat{a}}$$

$$[X^a, X^b] = i f^{abc} X^c + i f^{abi} T^i$$

Symmetric coset

$$\bar{\Sigma} \equiv U(\Pi)^2 = \exp(2i\Pi(x))$$

Transform linearly under G

$$\bar{\Sigma} \to g\, \bar{\Sigma}\, R(g)^\dagger$$

Building block

$$\overline{\mathbf{V}}_\mu = (\mathbf{D}_\mu \boldsymbol{\Sigma})\, \boldsymbol{\Sigma}^{-1} \qquad \overline{\mathbf{T}} \equiv \boldsymbol{\Sigma} Q_Y \boldsymbol{\Sigma}^{-1}$$

$$\overline{\mathrm{F}}_{\mu\nu}, \quad \boldsymbol{\Sigma} \overline{\mathrm{F}}^R_{\mu\nu} \boldsymbol{\Sigma}^{-1}$$

Chiral Lagrangian

- **$SU(3)_L \otimes SU(3)_R$ invariant**

$\mathbf{U}$ → $g_R\, \mathbf{U}\, g_L^\dagger$; $g_{L,R} \in SU(3)_{L,R}$ → $\mathcal{L}_2 = \frac{f^2}{4} \langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle$ **Derivative Coupling**

$$\mathcal{L}_2 = \frac{f^2}{4} \langle \partial_\mu \mathbf{U}^\dagger \partial^\mu \mathbf{U} \rangle = \partial_\mu \pi^- \partial^\mu \pi^+ + \frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi^0 + \cdots$$
$$+ \frac{1}{6f^2} \left\{ \left(\pi^+ \overleftrightarrow{\partial}_\mu \pi^-\right) \left(\pi^+ \overleftrightarrow{\partial}^\mu \pi^-\right) + 2 \left(\pi^0 \overleftrightarrow{\partial}_\mu \pi^+\right) \left(\pi^- \overleftrightarrow{\partial}^\mu \pi^0\right) + \cdots \right\}$$
$$+ O\left(\pi^6/f^4\right)$$

- **Expansion in powers of momenta ↔ derivatives**

Parity → even dimension ; $\mathbf{U}\,\mathbf{U}^\dagger = 1$ → $2n \geq 2$

$$\mathcal{L}_4 = L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle + L_3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U \rangle$$

Explicit Symmetry Breaking

$$\mathcal{L}_{QCD} \equiv \mathcal{L}^0_{QCD} + \bar{\mathbf{q}}\,(\not{\mathbf{v}} + \not{\mathbf{a}}\,\gamma_5)\,\mathbf{q} - \bar{\mathbf{q}}\,(\mathbf{s} - i\,\gamma_5\,\mathbf{p})\,\mathbf{q}$$

$$= \mathcal{L}^0_{QCD} + \bar{\mathbf{q}}_L\,\not{\mathbf{l}}\,\mathbf{q}_L + \bar{\mathbf{q}}_R\,\not{\mathbf{r}}\,\mathbf{q}_R - \bar{\mathbf{q}}_R\,(\mathbf{s} + i\,\mathbf{p})\,\mathbf{q}_L - \bar{\mathbf{q}}_L\,(\mathbf{s} - i\,\mathbf{p})\,\mathbf{q}_R$$

$$\mathbf{l}_\mu \equiv \mathbf{v}_\mu - \mathbf{a}_\mu = e\,\mathcal{Q}\,A_\mu + \cdots$$

$$\mathbf{r}_\mu \equiv \mathbf{v}_\mu + \mathbf{a}_\mu = e\,\mathcal{Q}\,A_\mu + \cdots \qquad \mathcal{Q} \equiv \tfrac{1}{3}\,\mathrm{diag}(2,-1,-1)$$

$$\mathbf{s} = \mathcal{M} + \cdots \quad ; \quad \mathcal{M} \equiv \mathrm{diag}(m_u, m_d, m_s)$$

Local $SU(3)_L \otimes SU(3)_R$ Symmetry:

$$\mathbf{q}_L \to g_L\,\mathbf{q}_L \qquad \mathbf{l}_\mu \to g_L\,\mathbf{l}_\mu\,g_L^\dagger + i\,g_L\,\partial_\mu g_L^\dagger$$

$$\mathbf{q}_R \to g_R\,\mathbf{q}_R \qquad \mathbf{r}_\mu \to g_R\,\mathbf{r}_\mu\,g_R^\dagger + i\,g_R\,\partial_\mu g_R^\dagger$$

$$(\mathbf{s} + i\,\mathbf{p}) \to g_R\,(\mathbf{s} + i\,\mathbf{p})\,g_L^\dagger$$

$$\mathcal{L} = \frac{f^2}{4}\,\langle D_\mu \mathbf{U}\, D^\mu \mathbf{U}^\dagger + \chi\,\mathbf{U}^\dagger + \mathbf{U}\,\chi^\dagger \rangle$$

$$D_\mu \mathbf{U} = \partial_\mu \mathbf{U} - i\,\mathbf{r}_\mu\,\mathbf{U} + i\,\mathbf{U}\,\mathbf{l}_\mu$$

$$\chi \equiv 2\,B_0\,(\mathbf{s} + i\,\mathbf{p})$$

Pseudo-Goldstone Boson

$$\mathcal{L} = \frac{f^2}{4}\langle D_\mu \mathbf{U}\, D^\mu \mathbf{U}^\dagger + \chi\, \mathbf{U}^\dagger + \mathbf{U}\, \chi^\dagger \rangle$$

$$D_\mu \mathbf{U} = \partial_\mu \mathbf{U} - i\, \mathbf{r}_\mu\, \mathbf{U} + i\, \mathbf{U}\, \mathbf{l}_\mu$$

$$\chi \equiv 2\, B_0\, (\mathbf{s} + i\, \mathbf{p})$$

Currents:

$$\mathbf{J}^\mu_L = \frac{\partial}{\partial \mathbf{l}_\mu}\mathcal{L}_2 = \frac{i}{2} f^2\, D^\mu \mathbf{U}^\dagger\, \mathbf{U} = \frac{f}{\sqrt{2}}\, D^\mu \mathbf{\Phi} + \cdots$$

$$\mathbf{J}^\mu_R = \frac{\partial}{\partial \mathbf{r}_\mu}\mathcal{L}_2 = \frac{i}{2} f^2\, D^\mu \mathbf{U}\, \mathbf{U}^\dagger = -\frac{f}{\sqrt{2}}\, D^\mu \mathbf{\Phi} + \cdots$$

$$\langle 0|\, (J^\mu_A)_{12}\, |\pi^+(p)\rangle = i\sqrt{2}\, f\, p^\mu \quad \rightarrow \quad f = f_\pi \approx 92.2\ \mathrm{MeV}$$

$(\pi^+ \to \mu^+ \nu_\mu)$

$$\bar{\mathbf{q}}^j_L\, \mathbf{q}^i_R = -\frac{\partial \mathcal{L}_2}{\partial(\mathbf{s} - i\,\mathbf{p})^{ji}} = -\frac{f^2}{2} B_0\, \mathbf{U}^{ij} \quad \rightarrow \quad \langle 0|\bar{\mathbf{q}}^j \mathbf{q}^i|0\rangle = -f^2\, B_0\, \delta_{ij}$$

$$\frac{f^2}{4}\langle \chi\, \mathbf{U}^\dagger + \mathbf{U}\, \chi^\dagger \rangle \quad \rightarrow \quad \mathcal{L}_m = -B_0\, \langle \mathcal{M}\, \Phi^2 \rangle \quad + \frac{B_0}{6 f^2}\langle \mathcal{M}\, \Phi^4 \rangle + \cdots$$

$$= B_0 \frac{f^2}{2}\langle s(U^\dagger + U)\rangle$$

$$\mathbf{U}_{ij}(\phi) = \left\{ \exp\left(i\sqrt{2}\, \mathbf{\Phi}/f \right) \right\}_{ij}$$

$$= -B_0 \Big\{ (m_u + m_d) \left[\pi^+\pi^- + \frac{1}{2}\pi^0\pi^0 \right] + (m_u + m_s)\, K^+ K^-$$

$$+ (m_d + m_s)\, K^0 \bar{K}^0 + \frac{1}{6}(m_u + m_d + 4\, m_s)\, \eta^2 + \frac{1}{\sqrt{3}}(m_u - m_d)\, \pi^0 \eta \Big\}$$

Gell-Mann–Okubo: $4\, M_K^2 = M_\pi^2 + 3\, M_\eta^2$

Dashen Theorem: $(M^2_{K^0} - M^2_{K^\pm})_{em} = (M^2_{\pi^0} - M^2_{\pi^\pm})_{em} + \mathcal{O}(e^2 p^2)$

Chiral Lagrangian at p4

$\mathbf{U}$	$\mathcal{O}(p^0)$
$D_\mu \mathbf{U}\,,\ \mathbf{l}_\mu\,,\ \mathbf{r}_\mu$	$\mathcal{O}(p^1)$
$\chi\,,\ \mathbf{F}^{\mu\nu}_{L,R}$	$\mathcal{O}(p^2)$

$$\mathbf{F}^{\mu\nu}_L \equiv \partial^\mu \mathbf{l}^\nu - \partial^\nu \mathbf{l}^\mu - i\,[\mathbf{l}^\mu, \mathbf{l}^\nu]$$

$$\mathbf{F}^{\mu\nu}_R \equiv \partial^\mu \mathbf{r}^\nu - \partial^\nu \mathbf{r}^\mu - i\,[\mathbf{r}^\mu, \mathbf{r}^\nu]$$

General connected diagram with N_d vertices of $\mathcal{O}(p^d)$ and L loops:

$$D = 2L + 2 + \sum_d N_d\,(d-2)$$

Weinberg

$$\begin{aligned}
\mathcal{L}_4 = {} & L_1 \langle D_\mu U^\dagger D^\mu U\rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U\rangle \langle D^\mu U^\dagger D^\nu U\rangle \\
& + L_3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U\rangle + L_4 \langle D_\mu U^\dagger D^\mu U\rangle \langle U^\dagger \chi + \chi \dagger U\rangle \\
& + L_5 \langle D_\mu U^\dagger D^\mu U \left(U^\dagger \chi + \chi^\dagger U\right)\rangle + L_6 \langle U^\dagger \chi + \chi^\dagger U\rangle^2 \\
& + L_7 \langle U^\dagger \chi - \chi^\dagger U\rangle^2 + L_8 \langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi\rangle \\
& - i\, L_9 \langle F^{\mu\nu}_R D_\mu U D_\nu U^\dagger + F^{\mu\nu}_L D_\mu U^\dagger D_\nu U\rangle + L_{10} \langle U^\dagger F^{\mu\nu}_R U F_{L\mu\nu}\rangle
\end{aligned}$$

Custodial Symmetry

Consider the Higgs sector (gauge-less limit)

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad \Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_\Phi &= (D_\mu\Phi)^\dagger D^\mu\Phi - \lambda\left(|\Phi|^2 - \frac{v^2}{2}\right)^2 \\ &= \frac{1}{2}\mathrm{Tr}\left[(D^\mu\Sigma)^\dagger D_\mu\Sigma\right] - \frac{\lambda}{4}\left(\mathrm{Tr}\left[\Sigma^\dagger\Sigma\right] - v^2\right)^2 \end{aligned}$$

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\Sigma \to g_L\, \Sigma\, g_R^\dagger$

EW chiral Lagrangian

$$\Phi = \frac{1}{\sqrt{2}}(v+H)\, U(\vec{\varphi}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \Sigma \equiv (\Phi^c, \Phi) \equiv \frac{1}{\sqrt{2}}(v+H)\, U(\vec{\varphi})$$

$$\mathcal{L}_{\mathrm{Higgs}} = \frac{(v+H)^2}{4}\mathrm{Tr}\left[(D^\mu U)^\dagger D_\mu U\right] - \frac{\lambda}{4}(H^2 - v^2)^2$$

EW Chiral Lagrangian

Same Goldstone Lagrangian as QCD pions:

$$f_\pi \to v \quad , \quad \vec{\pi} \to \vec{\varphi} \to W_L^{\pm}, Z_L$$

- **Goldstone Bosons**

$\langle 0 | \bar{q}^j_R q^i_L | 0 \rangle$ **(QCD)**, Φ **(SM)** ➡ $U_{ij}(\phi) = \{\exp(i\vec{\sigma}\cdot\vec{\varphi}/v)\}_{ij}$

- **Expansion in powers of momenta** ⬌ **derivatives**

Parity ➡ even dimension ; $U\,U^\dagger = 1$ ➡ $2n \geq 2$

- **$SU(2)_L \otimes SU(2)_R$ invariant**

U ➡ $g_L\, U\, g_R^\dagger$; $g_{L,R} \in SU(2)_{L,R}$

$$\mathcal{L}_2 = \frac{v^2}{4} \mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)$$

Derivative Coupling

Higgs EFT

$$\Delta\mathcal{L}_2^{\text{Bosonic}} = \frac{1}{2}\partial_\mu h\,\partial^\mu h - \frac{1}{2}m_h^2 h^2 - V(h/v) + \frac{v^2}{4}\mathcal{F}_u(h/v)\,\langle (D^\mu U)^\dagger D_\mu U\rangle$$

Assumptions:

- **Spontaneous Symmetry Breaking:** $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$
- **h(125)** **is an** $\mathrm{SU(2)}_{\mathrm{L+R}}$ **scalar singlet**

All Higgsless operators can be multiplied by an arbitrary function of **h**:

$$\mathcal{O}_X \quad \Rightarrow \quad \tilde{\mathcal{O}}_X \equiv \mathcal{F}_X(h/v)\,\mathcal{O}_X$$

$$\mathcal{F}_X(h/v) = \sum_{n=0} c_n^{(X)} \left(\frac{h}{v}\right)^n$$

In addition, the LO Lagrangian should include the scalar potential:

$$V(h/v) = v^4 \sum_{n=2} c_n^{(V)} \left(\frac{h}{v}\right)^n$$

HEFT Lagrangian

$$\mathcal{L}_{\rm EW}^{(2)} = -\frac{1}{2g^2}\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle - \frac{1}{2g'^2}\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\langle D^\mu U^\dagger D_\mu U\rangle$$

$$U(\varphi) = \exp\left\{\frac{i\sqrt{2}}{v}\Phi\right\} \quad , \quad \Phi \equiv \frac{1}{\sqrt{2}}\vec{\sigma}\cdot\vec{\varphi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\varphi^0 & \varphi^+ \\ \varphi^- & -\frac{1}{\sqrt{2}}\varphi^0 \end{pmatrix}$$

$$D^\mu U = \partial^\mu U - i\,\hat{W}^\mu U + i\,U\hat{B}^\mu \quad , \quad D^\mu U^\dagger = \partial^\mu U^\dagger + i\,U^\dagger \hat{W}^\mu - i\,\hat{B}^\mu U^\dagger \quad , \quad \langle A\rangle \equiv {\rm Tr}(A)$$

$$\hat{W}^{\mu\nu} = \partial^\mu \hat{W}^\nu - \partial^\nu \hat{W}^\mu - i\,[\hat{W}^\mu, \hat{W}^\nu] \quad , \quad \hat{B}^{\mu\nu} = \partial^\mu \hat{B}^\nu - \partial^\nu \hat{B}^\mu - i\,[\hat{B}^\mu, \hat{B}^\nu]$$

$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger$

$$\hat{W}^\mu \to g_L\,\hat{W}^\mu g_L^\dagger + i\,g_L\,\partial^\mu g_L^\dagger \quad , \quad \hat{B}^\mu \to g_R\,\hat{B}^\mu g_R^\dagger + i\,g_R\,\partial^\mu g_R^\dagger$$

SM Symmetry Breaking: $\hat{W}^\mu = -\frac{g}{2}\vec{\sigma}\cdot\vec{W}^\mu \quad , \quad \hat{B}^\mu = -\frac{g'}{2}\sigma_3 B^\mu$

$$\mathcal{L}_2 = \frac{v^2}{4}{\rm Tr}\left(D_\mu U^\dagger D^\mu U\right) \xrightarrow{U=1} \mathcal{L}_2 = M_W^2 W_\mu^\dagger W^\mu + \frac{1}{2}M_Z^2 Z_\mu Z^\mu$$

$$M_W = M_Z \cos\theta_W = \frac{1}{2} g\, v$$

- **EW Goldstones are responsible for** $M_{W,Z}$ **(not the Higgs!)**

Yukawa Sector

$$\Delta\mathcal{L}_2^{\rm Ferm.} = -v \left\{ \bar{Q}_L\, U(\varphi) \left[\hat{\mathrm{Y}}_{\rm u}\, \mathcal{P}_+ + \hat{\mathrm{Y}}_{\rm d}\, \mathcal{P}_- \right] Q_R \,+\, \bar{L}_L\, U(\varphi)\, \hat{\mathrm{Y}}_{\ell}\, \mathcal{P}_+\, L_R \,+\, \mathrm{h.c.} \right\}$$

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}$$

$$U(\varphi) \to g_L\, U(\varphi)\, g_R^\dagger \quad , \quad Q_L \to g_L\, Q_L \quad , \quad Q_R \to g_R\, Q_R \quad , \quad \mathcal{P}_\pm \to g_R\, \mathcal{P}_\pm\, g_R^\dagger$$

- **Symmetry Breaking:** $\mathcal{P}_\pm = \frac{1}{2}\,(\mathrm{I}_2 \pm \sigma_3)$
- **Flavour Structure:** $\hat{\mathrm{Y}}_{\rm u,d,\ell}$ 3×3 matrices in flavour space
- **Higgs field:** $\hat{\mathrm{Y}}_{\rm u,d,\ell}(h/v) = \sum_{n=0} \hat{\mathrm{Y}}^{(n)}_{\rm u,d,\ell} \left(\frac{h}{v}\right)^n$

Higgs EFT

EW chiral Lagrangian | 2012 | EWChL with light Higgs | 2022 | Higgs EFT

LO and NLO boson

LO Lagrangian

NLO Bosonic Lagrangian

[Appelquist, Bernard, 1980]

[Longhitano, 1980, 1981]

[Feruglio, 1993]

[Herrero, Morales, 1993]

NLO fermion sector

NLO Fermionic Lagrangian

[Buchalla, Cata, 2013]

[Buchalla, Cata, Krause, 2014]

[Brivio, Gonzalez-Fraile/Garcia, Merlo, 2016]

[Pich, Rosell, Santos, Sanz-Cillero, 2015, 2018]

Full NLO and NNLO

Complete NLO Lagrangian

237 (8595) operators for one (three)

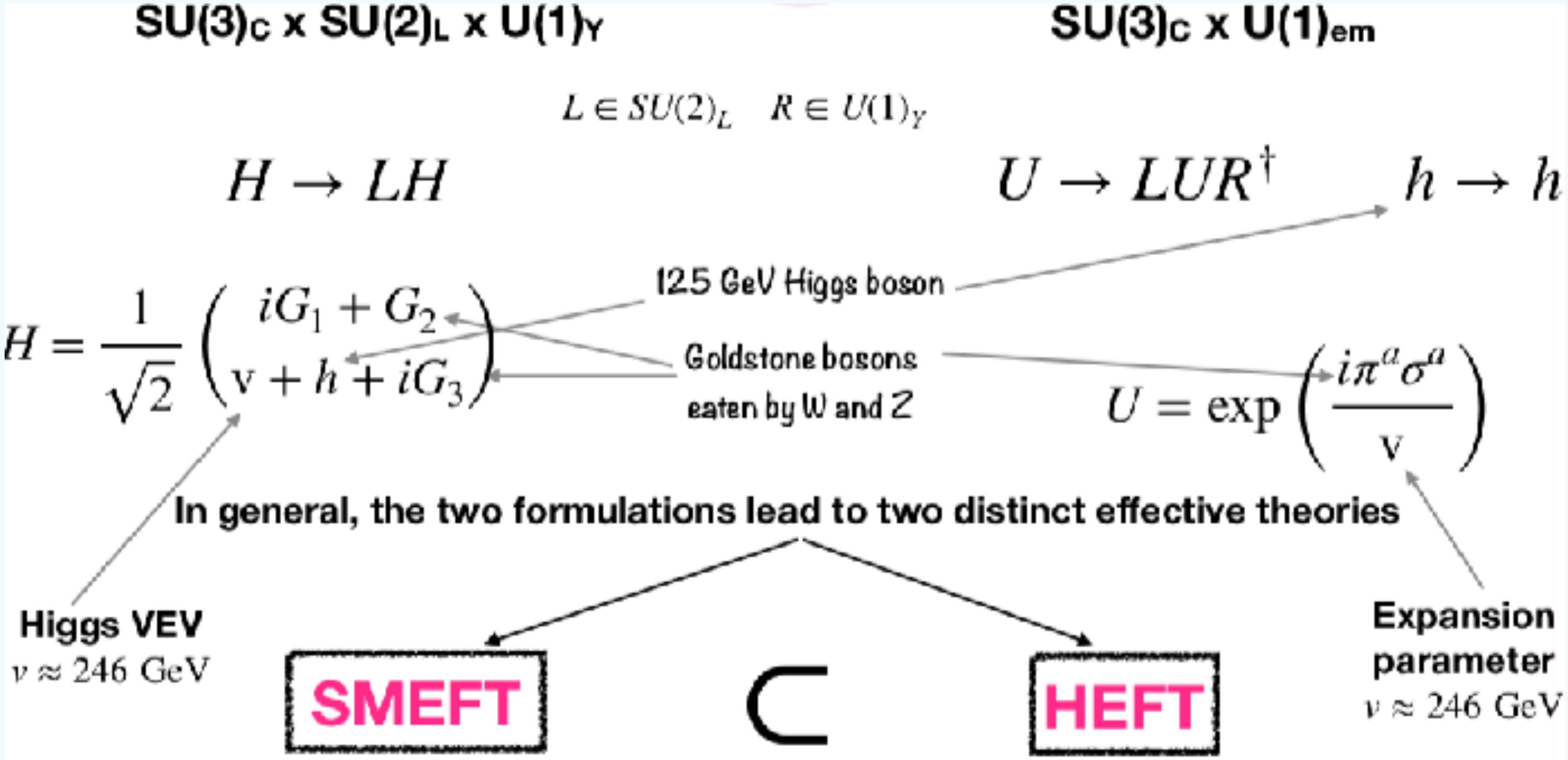
[Sun, Xiao, **Yu**, 2206.07722]

Complete NNLO Lagrangian

11506(1927574) NNLO operators with flavor number 1(3).

[Sun, Xiao, **Yu**, in preparation]

SMEFT vs HEFT



$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{\rm SM} - \frac{c_6}{\Lambda^2}|H|^6 + \mathcal{O}(\Lambda^{-4})$$

$$\mathscr{L}_{\rm SMEFT} \supset -\frac{m_h^2}{2v}(1+\delta\lambda_3)h^3 - \frac{m_h^2}{8v^2}(1+\delta\lambda_4)h^4 - \frac{\lambda_5}{v}h^5 - \frac{\lambda_6}{v^2}h^6$$

$$\delta\lambda_3 = \frac{2c_6 v^4}{m_h^2\Lambda^2},\ \delta\lambda_4 = \frac{12c_6 v^4}{m_h^2\Lambda^2},\ \lambda_5 = \frac{3c_6 v^2}{4\Lambda^2},\ \lambda_6 = \frac{c_6 v^2}{8\Lambda^2}$$

SMEFT: Predicts correlations between self-couplings as long as Λ >> v

$$\mathscr{L}_{\rm HEFT} \supset -c_3\frac{m_h^2}{2v}h^3 - c_4\frac{m_h^2}{8v^2}h^4 - \frac{c_5}{v}h^5 - \frac{c_6}{v^2}h^6 + \dots$$

HEFT: no correlations between self-couplings

Short Summary: SMEFT vs HEFT

- EFT with non-linearly realized electroweak symmetry (aka HEFT) is equivalent to EFT with linearly realized electroweak symmetry but whose Lagrangian is a non-polynomial function of the Higgs field that is non-analytic at H=0

- This non-analyticity leads to explosion of multi-Higgs amplitudes at the scale $4 \pi v$. For this reason, the validity regime of HEFT is limited below the scale of order $4 \pi v \sim 3$ TeV

- HEFT is useful to approximate BSM theories where new particles' masses vanish in the limit $v \to 0$, e.g. SM + a 4th generation of chiral fermions or when most of the new particle mass comes from EW symmetry breaking

- On the other hand, an EFT with linearly realized electroweak symmetry and the Lagrangian polynomial in the Higgs field (aka SMEFT) is useful to approximate BSM theories where new particles' masses do not vanish in the limit $v \to 0$, and are parametrically larger than the electroweak scale, e.g. SM + vector-like fermions

Summary

Take home message 1:

Core of EFTs: d.o.f separation, symmetry, power counting, decoupling

Take home message 2:

All QFTs are EFT, and EFT is renormalizable and predictive

Take home message 3:

EFT would reproduce full theory results with matching and running

Take home message 4:

SMEFT (NP), Chiral Lagrangian (QCD), EW Chiral Lagrangian

Thanks for your attention!