中国科学院高能物理研究所
Institute of High Energy Physics
Chinese Academy of Sciences

Primary Simulation study on injection efficiency of HER

Meng Li

What limits injection efficiency?

- Dynamic aperture of HER
- Beam-beam effects and collimators
- Injection angle and offset
- Beta mismatch between BT and Rings

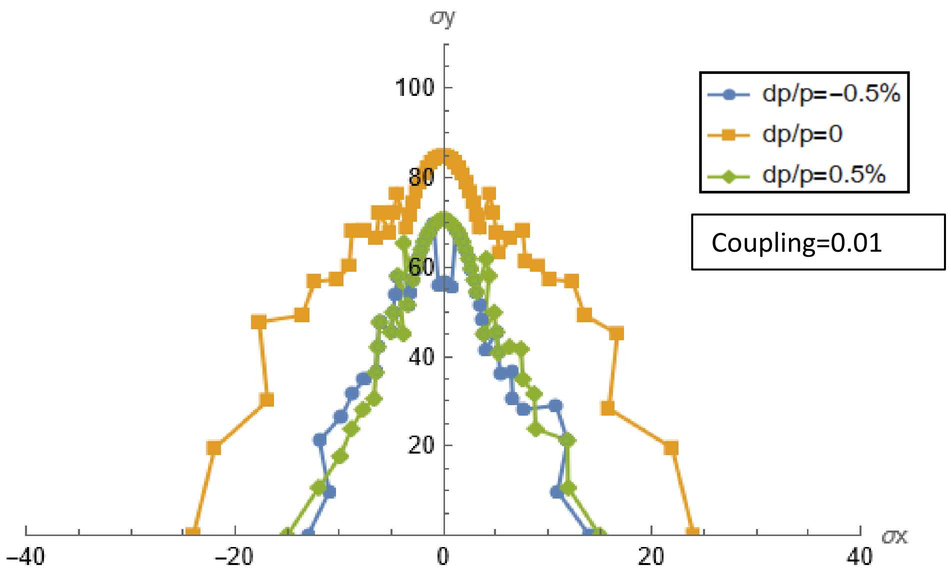
What limits injection efficiency?

- Dynamic aperture of HER
- Beam-beam effects and collimators
- Injection angle and offset
- Beta mismatch between BT and Rings

☐ Dynamic aperture of HER: track 6000 turns(a damping time)

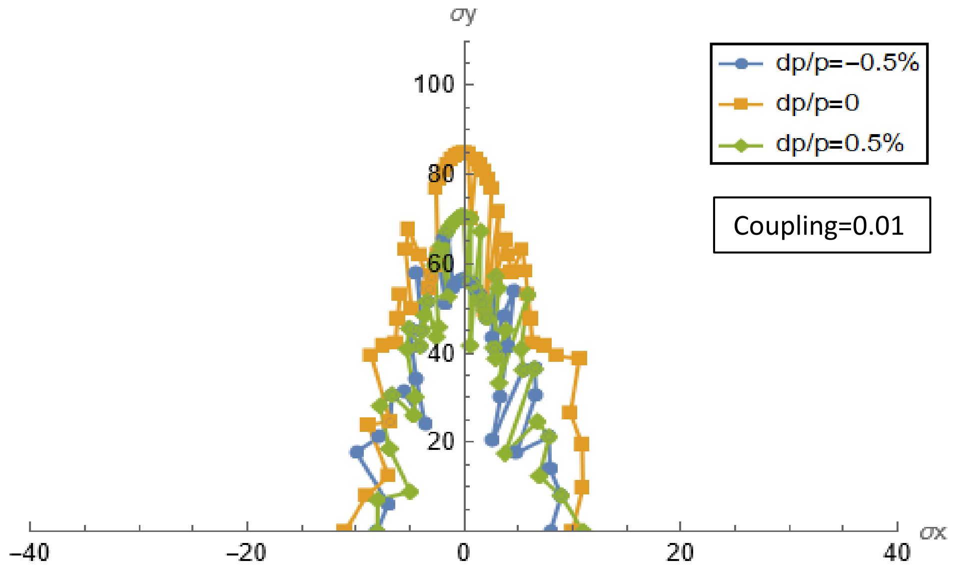
- $design\ momentum = 7.00\ GeV$
- $\beta_x^* = 60mm, \beta_y^* = 1\ mm\ , CW = 40\%$
- $Nx = 45.531, Ny = 43.581$
- $\varepsilon_x = 4.46\ nm, \varepsilon_y = 33.89\ pm$
- $bunch\ length = 5.05\ mm$
- $effective\ voltage = 14.2\ MV$

Without beam-beam



H/V: $24\sigma_x$ /85 σ_y

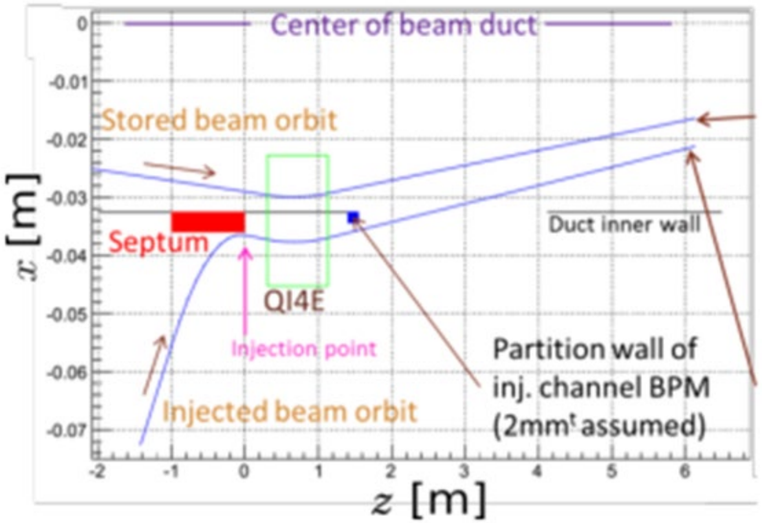
With full beam-beam



H/V: $10\sigma_x$ /85 σ_y

Lattice files SuperKEKB International Task Force Subgroup · KDS (Indico) -- Updated in August 2021

Dynamic aperture required for injection



Parameter	HER	Unit
β_{xR}	100	m
ε_{xR}	4.46	nm
β_{xi}	25.0	m
ε_{xi}	7.3	nm
w_s	6.0	mm
Δx	9.62	mm
$\Delta x'$	-0.76	mrad

- The distance between the injected beam and stored beam at the injection point:

$$\Delta x = w_s + n_I\sigma_I + n_R\sigma_R$$

$$= w_s + n_i\sqrt{\beta_{xi}\varepsilon_{xi} + (D_{xi}\sigma_{\delta i})^2} + n_r\sqrt{\beta_{xR}\varepsilon_{xR} + (D_{xR}\sigma_{\delta R})^2}$$

$$= 6.0 + 3 \times 0.427 + 3.5 \times 0.668 = 9.62\text{mm}$$

- The dynamic aperture required:

$$DA_{required} = \Delta x + n_i\sigma_i = 9.62 + 1.281 = 10.901mm = \mathbf{16.5\sigma_R}$$

What limits injection efficiency?

- Dynamic aperture of HER
- Beam-beam effects and collimators
- Injection angle and offset
- Beta mismatch between BT and Rings

Beam-beam effect

➢ **Define the BEAMBEAM element** (full beam-beam) **and add it at the IP in the lattice sequence.**

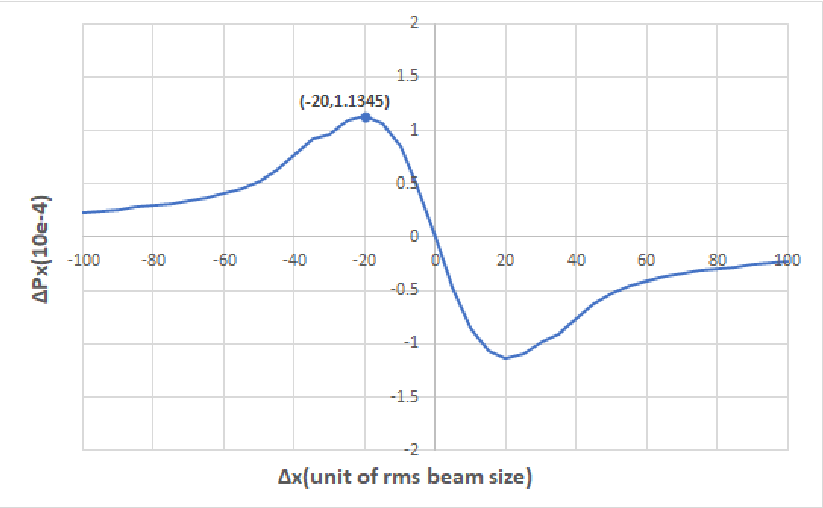
```
BEAMBEAM      FBMBME   =(BX =.032    BY =.00027      XANGLE =.0415
                         EMITX =3.2e-09  EMITY =8.64e-12  DP =.00081
                         SIGZ =.006 SLICE =100 NP =9.04e+10  STURN =10 )

       (IP           FBMBME    FHBBKE    FVBBKE    ESLE0     ECSLE5    ESLE10
        ECSLE15      ESLE20    ECSLE25   ESLE30    ECSLE35   ESLE40    ECSLE45
        ESLE50       ECSLE55   ESLE60    ECSLE65   ESLE70    ECSLE75   ESLE80
        ECSLE85      ESLE90    ECSLE95   ESLE100   ECSLE105  ESLE110   ECSLE115
```

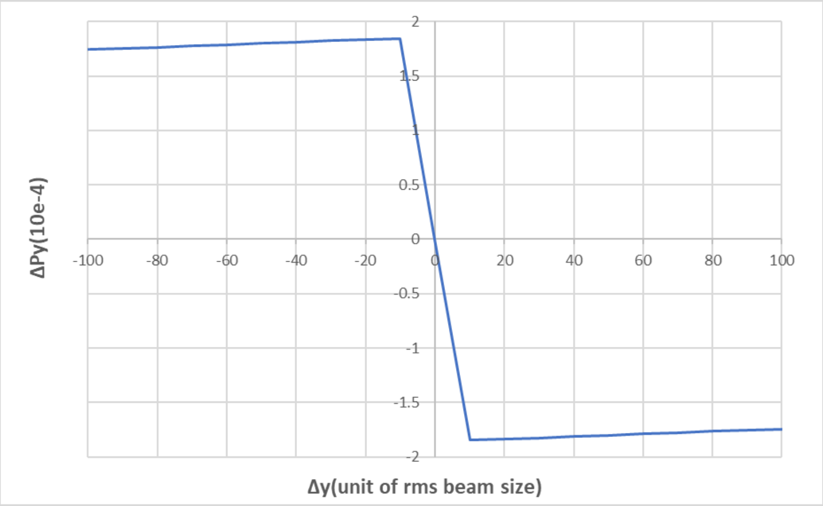
BX: β_x^*　　EMITX: ε_x　　XANGLE: crossing angle　　NP: particle number for each bunch

BY: β_y^*　　EMITY: ε_y　　SIGZ: bunch length　　DP: energy spread

STURN: how many turns do you output information

➢ **Check whether the beam-beam element works:**

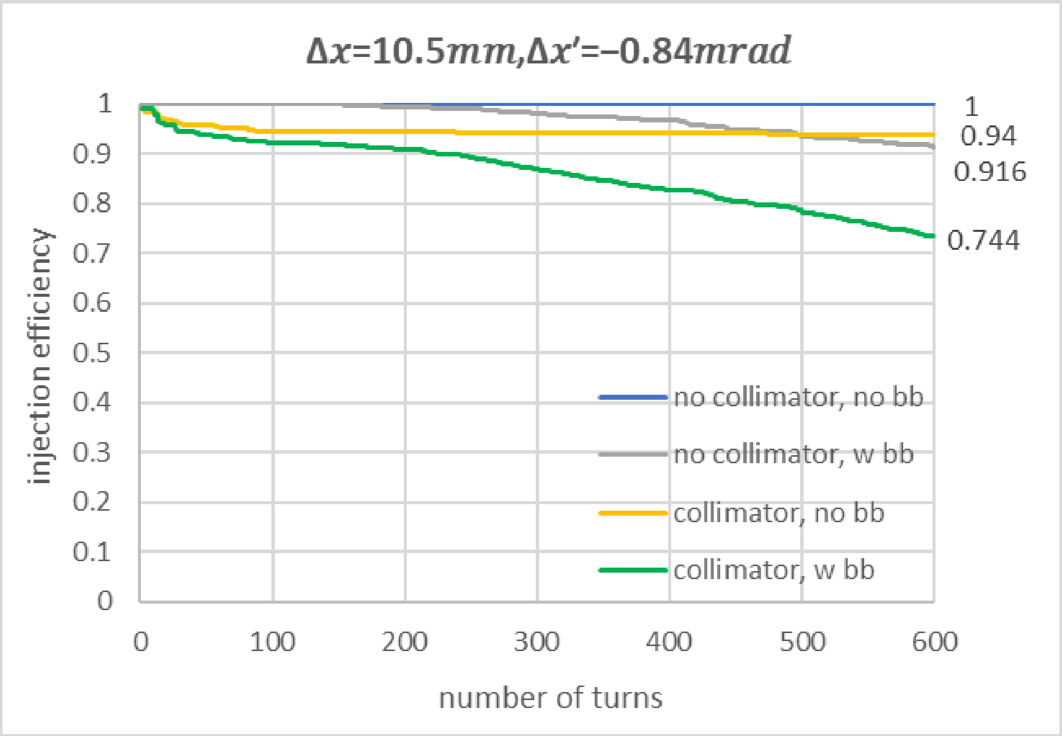




☐ Injection efficiency related to beam-beam and collimator

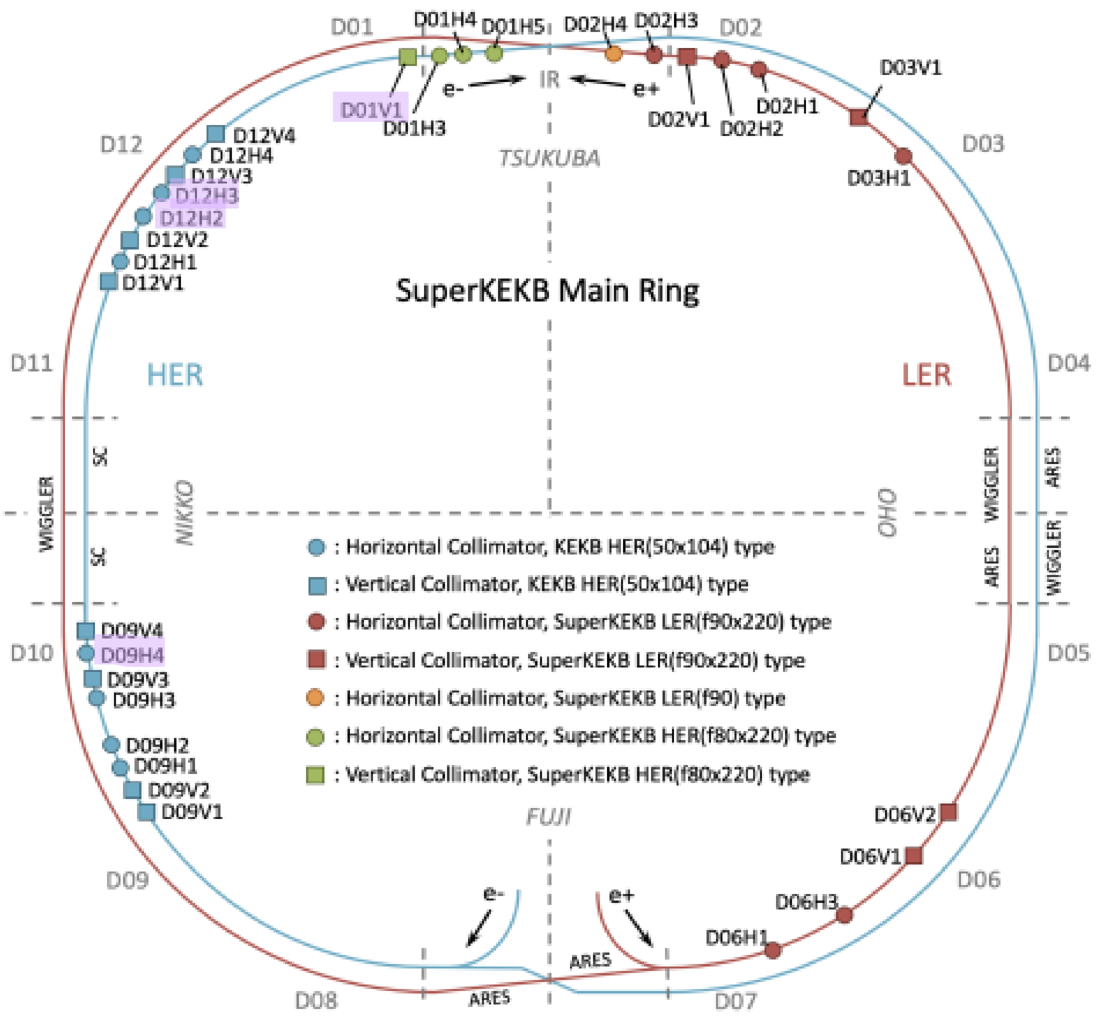
- The efficiency here is defined as the proportion of the beam surviving after serval turns.
- Track 10000 particles 600 turns(6ms) with beam-beam or (and) collimators.

```
APERT  AQC1LC  =(AX =.015     AY =.015  DX =.0007 )
       AQC1LO  =(AX =.015     AY =.015  DX =.0007 )
       AQC2LC  =(AX =.035     AY=.035   DX =.0007 )
       AQC2LO  =(AX =.035     AY =.035  DX =.0007 )
       PMD01H5 =(AX =0.00941   AY =0.045 )
       PMD01H4 =(AX =0.01488   AY =0.045 )
       PMD01H3 =(AX =0.00875   AY =0.045 )
       PMD01V1 =(AX =0.045     AY =0.00178 )
       PMD01H2 =(AX =0.045     AY =0.045 )
       PMD01H1 =(AX =0.045     AY =0.045 )
       PMD12V4 =(AX =0.045     AY =0.00173 )
       PMD12H4 =(AX =0.01542   AY =0.045 )
       PMD12V3 =(AX =0.045     AY =0.00215 )
       PMD12H3 =(AX =0.01000   AY =0.045 )
       PMD12H2 =(AX =0.00877   AY =0.045 )
       PMD12V2 =(AX =0.045     AY =0.00218 )
       PMD12H1 =(AX =0.01206   AY =0.045 )
       PMD12V1 =(AX =0.045     AY =0.00380 )
       PMD09V4 =(AX =0.045     AY =0.00329 )
       PMD09H4 =(AX =0.00886   AY =0.045 )
       PMD09V3 =(AX =0.045     AY =0.00140 )
       PMD09H3 =(AX =0.00982   AY =0.045 )
       PMD09H2 =(AX =0.01585   AY =0.045 )
       PMD09H1 =(AX =0.01108   AY =0.045 )
       PMD09V2 =(AX =0.045     AY =0.00304 )
       PMD09V1 =(AX =0.045     AY =0.00169 )
       AQC2RO  =(AX =.035     AY =.035  DX =-.0007 )
       AQC2RC  =(AX =.035     AY =.035  DX =-.0007 )
       AQC1RO  =(AX =.015     AY =.015  DX =-.0007 )
       AQC1RC  =(AX =.015     AY =.015  DX =-.0007 )
```



- When add the full beam-beam into the simulation, the injection efficiency is reduced to 91%, and then add the collimators also, the injection efficiency is further reduced to 74%.

Output the Loss position



s(m)	APTER	definition	Nloss (bb,no collimator)	Nloss (bb,collimator)	Nloss (no bb,collimator)
61.743688	PMD01V1	PMD01V1 =(AX =0.045 AY =0.00178)	-	1901	-
395.960142	PMD12V4	PMD12V4 =(AX =0.045 AY =0.00173)	-	10	-
475.546941	PMD12H3	PMD12H3 =(AX =0.01000 AY =0.045)	-	49	331
518.518388	PMD12H2	PMD12H2 =(AX =0.00877 AY =0.045)	-	713	213
954.031444	PMD09H4	PMD09H4 =(AX =0.00886 AY =0.045)	-	77	31
3014.71469	AQC1RO	AQC1RO =(AX =.015 AY =.015 DX =-.0007)	640		
3014.90469	AQC1RC	AQC1RC =(AX =.015 AY =.015 DX =-.0007)	76		
1.410001	AQC1LC	AQC1LC =(AX =.015 AY =.015 DX =.0007)	52		
1.600001	AQC1LO	AQC1LO =(AX =.015 AY =.015 DX =.0007)	4		

PMD01V1: $\beta_x = 40.128m$ $\epsilon_x = 4.435nm$ $\sigma_x = \sqrt{\beta_x \epsilon_x} = 0.422mm$
$\beta_y = 46.460m$ $\sigma_y = \sqrt{\beta_y \epsilon_y} = \sqrt{\beta_y * 0.01 * \epsilon_x} = 0.045mm$
$\mathbf{AY = 1.78mm = 39.5\sigma_y}$

PMD12H3: $\beta_x = 39.731m$ $\epsilon_x = 4.435nm$ $\sigma_x = \sqrt{\beta_x \epsilon_x} = 0.42mm$
$\beta_y = 7.703m$ $\sigma_y = \sqrt{\beta_y \epsilon_y} = \sqrt{\beta_y * 0.01 * \epsilon_x} = 0.018mm$
$\mathbf{AX = 10mm = 23.8\sigma_x}$

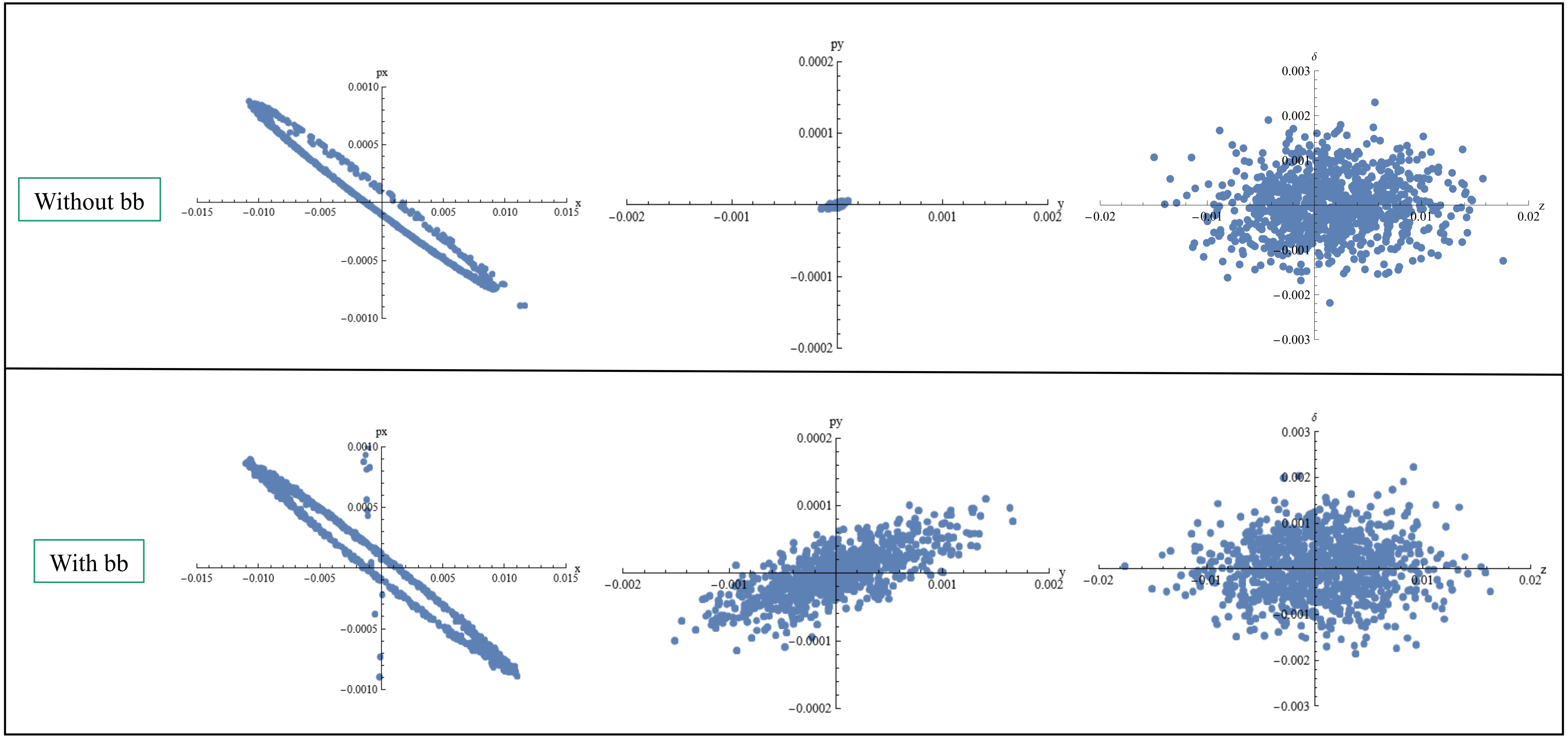
PMD12H2: $\beta_x = 39.728m$ $\epsilon_x = 4.435nm$ $\sigma_x = \sqrt{\beta_x \epsilon_x} = 0.42mm$
$\beta_y = 8.447m$ $\sigma_y = \sqrt{\beta_y \epsilon_y} = \sqrt{\beta_y * 0.01 * \epsilon_x} = 0.019mm$
$\mathbf{AX = 8.77mm = 20.8\sigma_x}$

PMD09H4: $\beta_x = 39.728m$ $\epsilon_x = 4.435nm$ $\sigma_x = \sqrt{\beta_x \epsilon_x} = 0.42mm$
$\beta_y = 7.703m$ $\sigma_y = \sqrt{\beta_y \epsilon_y} = \sqrt{\beta_y * 0.01 * \epsilon_x} = 0.018mm$
$\mathbf{AX = 8.86mm = 21\sigma_x}$

- Although the collimators will reduce the injection efficiency, it can protect the collision area.
- It shows that the beam-beam effect has a great influence on the vertical direction.

Beam distribution in the phase space

- I also output the phase space distribution of particles at the injection point after 600 turns.
- The distribution of particles in $y - p_y$ phase space with full beam-beam is widened by an order of magnitude.

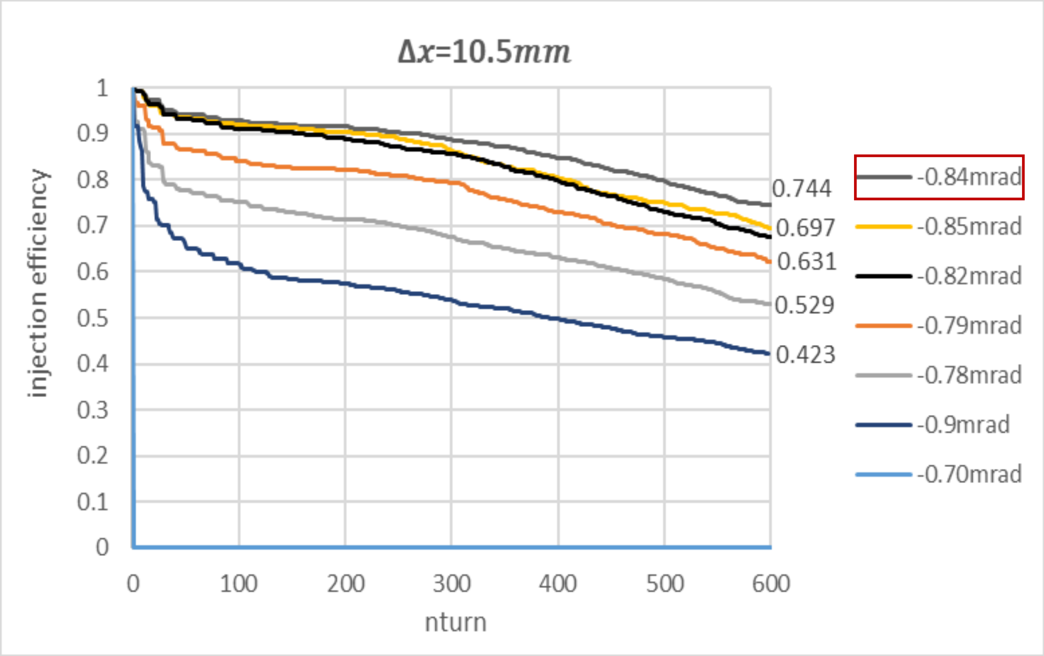


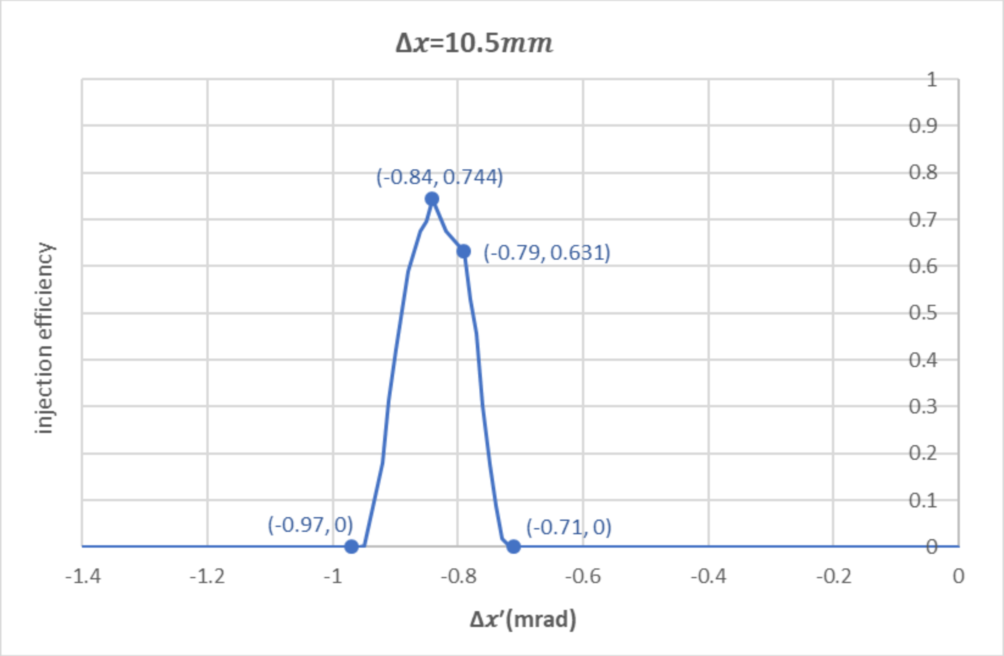
What limits injection efficiency?

- Dynamic aperture of HER
- Beam-beam effects and collimators
- Injection angle and offset
- Beta mismatch between BT and Rings

☐Injection efficiency and injection angle

➢ Track 10000 particles 600 turns with beam-beam, with collimators, optic matched, with same Δx and different $\Delta x'$.



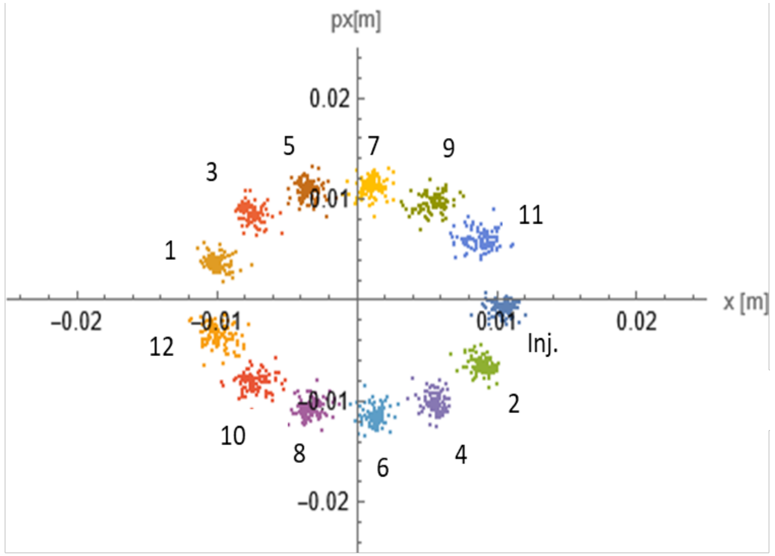


➢ When the offset is given, the injection efficiency is very sensitive to the $\Delta x'$.

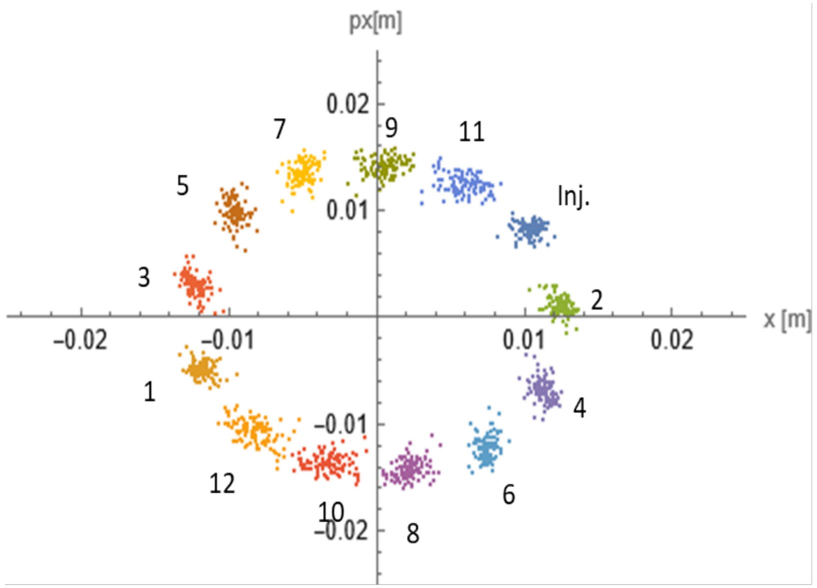
➢ For each Δx, we can find an optimum $\Delta x'$ to make the injection efficiency the highest.

Reason why injection efficiency is very sensitive to $\Delta x'$

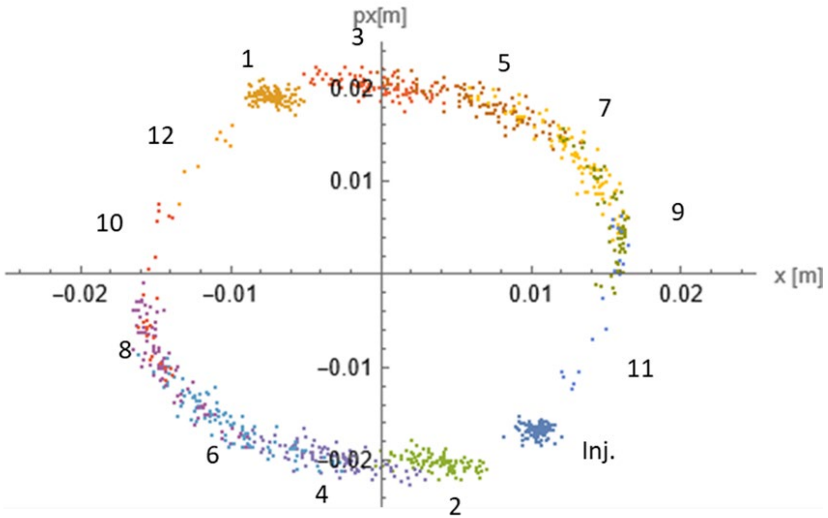
- When the offset is given and set different $\Delta x'$, the amplitude of the residual oscillation of the injected beam is different.
- To achieve high injection efficiency, should minimize the residual oscillation of the injection beam.



$\Delta x = 10.5mm, \Delta x' = \mathbf{-0.84mrad}$
Residual oscillation amplitude: **10.5mm**



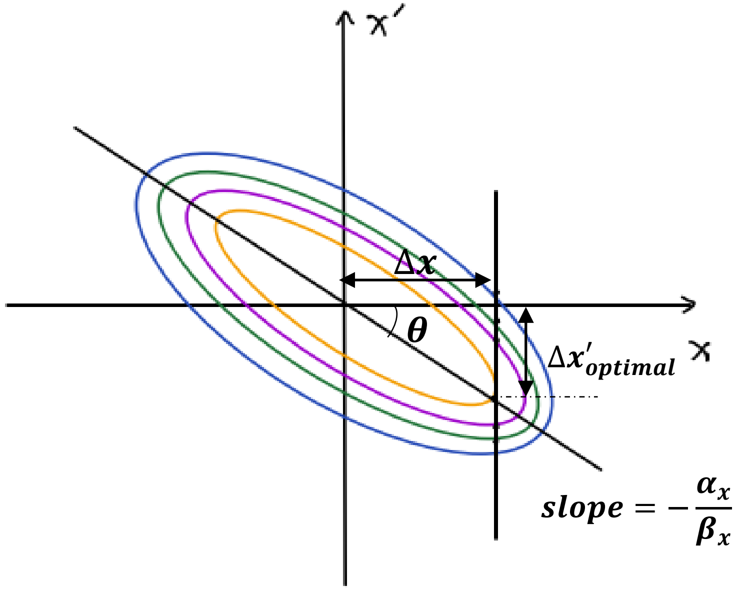
$\Delta x = 10.5mm, \Delta x' = \mathbf{-0.75mrad}$
Residual oscillation amplitude: **13.0mm**



$\Delta x = 10.5mm, \Delta x' = \mathbf{-1.0mrad}$
Residual oscillation amplitude: **16mm**

Theoretical calculation of the optimal $\Delta x'$

- At a certain observation point, the phase ellipses of different particles have the same shape and different sizes.
- **When with same Δx and different $\Delta x'$, the phase ellipses have same shape and different size, and the smallest one tangent to this line x= Δx.**



$$\Delta x'_{optimal} = \Delta x \cdot tan\theta = -\Delta x \frac{\alpha_x}{\beta_x}$$

For example:

$\Delta x = 10.5mm$, $\alpha_x = 7.9254$, $\beta_x = 100m$

$$\theta = -\frac{\alpha_x}{\beta_x} = \frac{\Delta x'}{\Delta x}$$

$$\therefore \Delta x'_{optimum} = -\Delta x \frac{\alpha_x}{\beta_x}$$

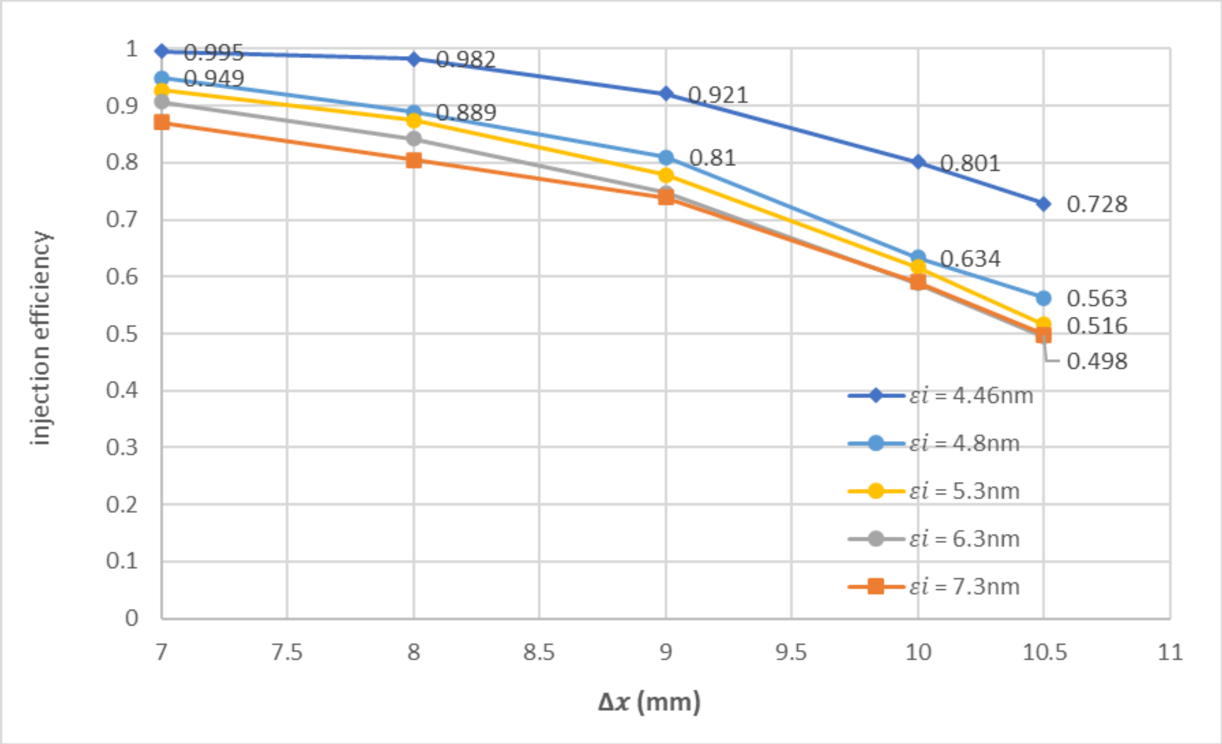
$$= -10.5 \times \frac{7.9254}{100} \text{ mrad}$$

$$= -0.84 \text{ mrad}$$

⇨ The theoretical value is consistent with the simulated value.

☐Injection efficiency and injection offset

- Track 10000 particles 600 turns with beam-beam, with collimators, optic matched, with different Δx (match an optimum $\Delta x'$)

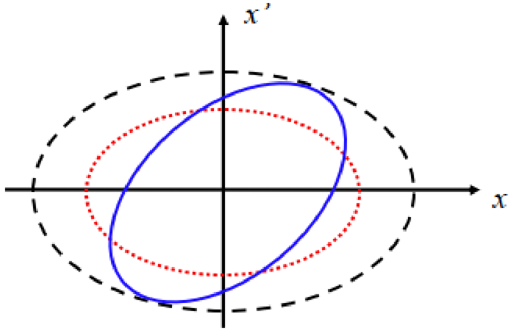


- The injection efficiency decreases with offset.
- Only when the emittance of the injected beam is reduced to almost the same as that of the ring can the injection efficiency be greatly improved.

What limits injection efficiency?

- Dynamic aperture of HER
- Beam-beam effects and collimators
- Injection offset and angle
- Beta mismatch between BT and Rings

Beta mismatch and BMAG



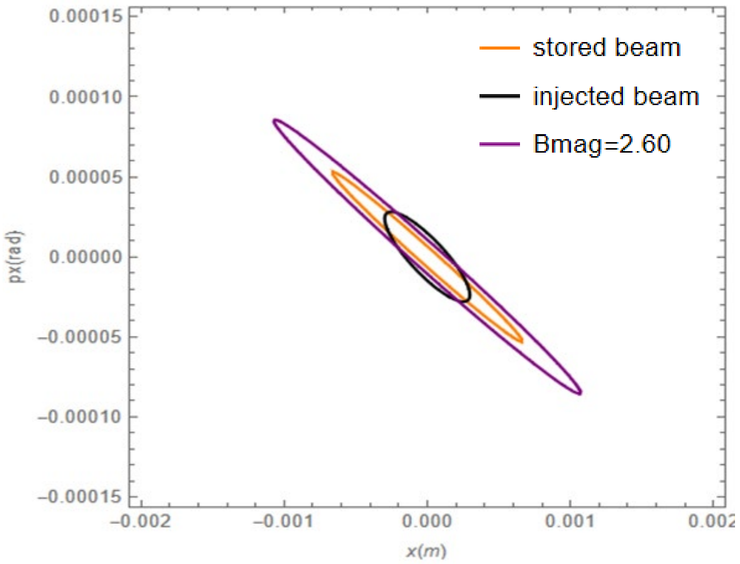
- If the **injected beam** is not matched to the **design lattice**, the **effective emittance** of the injected beam becomes larger and may induce beam loss.
- Beta mismatch usually results in emittance growth, characterized by BMAG.

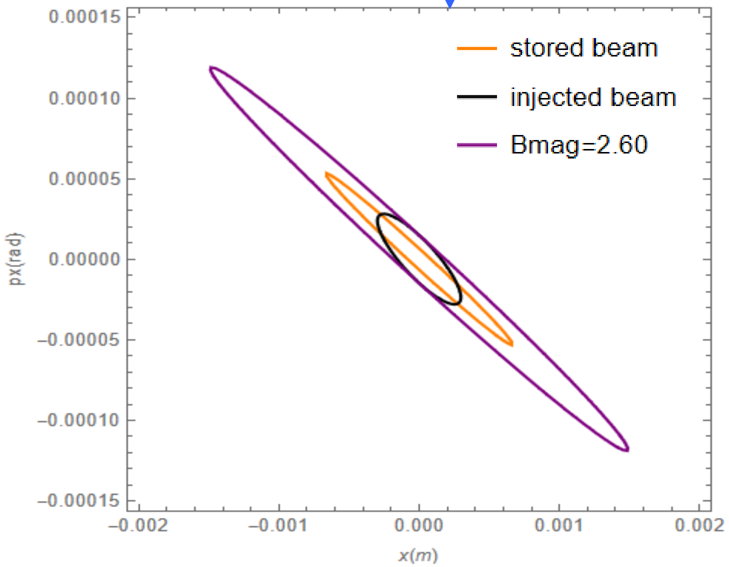
$$BMAG = \frac{1}{2}\left[\frac{\beta_m}{\beta_e} + \frac{\beta_e}{\beta_m} + \beta_b\beta_m\left(\frac{\alpha_b}{\beta_b} - \frac{\alpha_m}{\beta_m}\right)^2\right]$$

* $$\frac{(\gamma\varepsilon)_{final}}{(\gamma\varepsilon)_{initial}} = \frac{\varepsilon_{purple}}{\varepsilon_{orange}} = BMAG$$

⇒

$$\frac{(\gamma\varepsilon)_{final}}{(\gamma\varepsilon)_{initial}} = BMAG + \sqrt{BMAG^2 - 1}$$

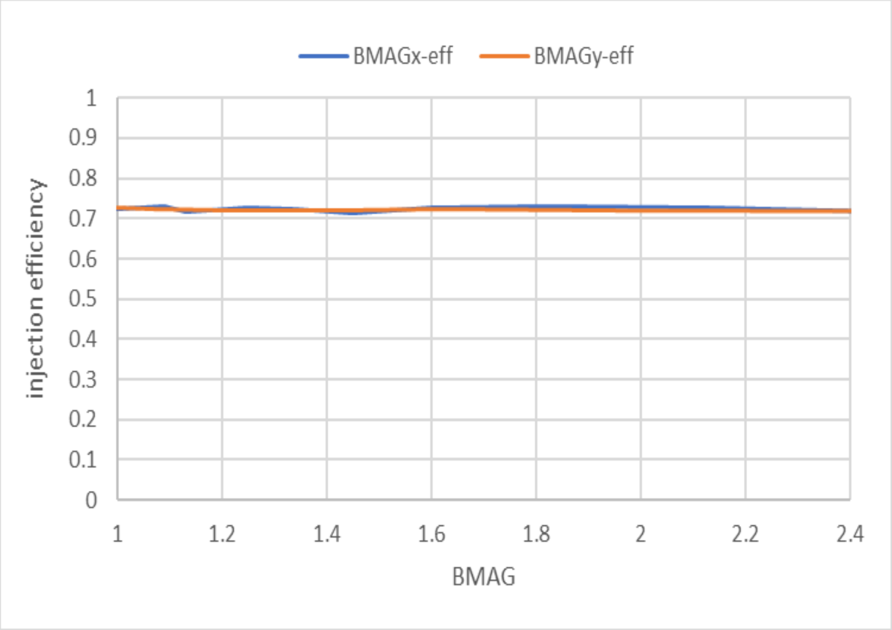


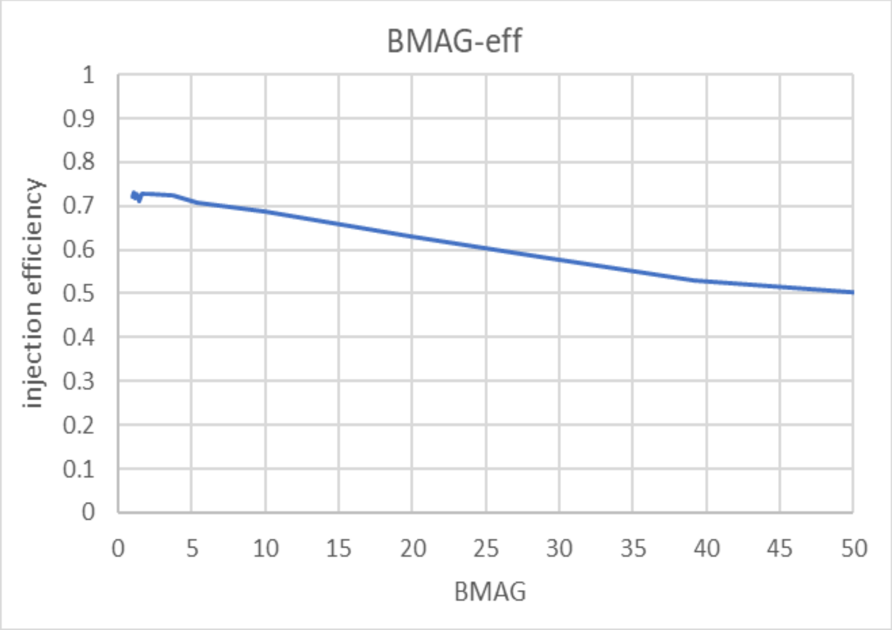


* The Introduction of Trajectory Oscillations to Reduce Emittance Growth in the SLC LINAC(J.T.Seeman,1992)

☐ Injection efficiency and beta mismatch

➢ Track 10000 particles 600 turns with beam-beam, with collimators, with optimum $\Delta x'$, with beta mismatch.

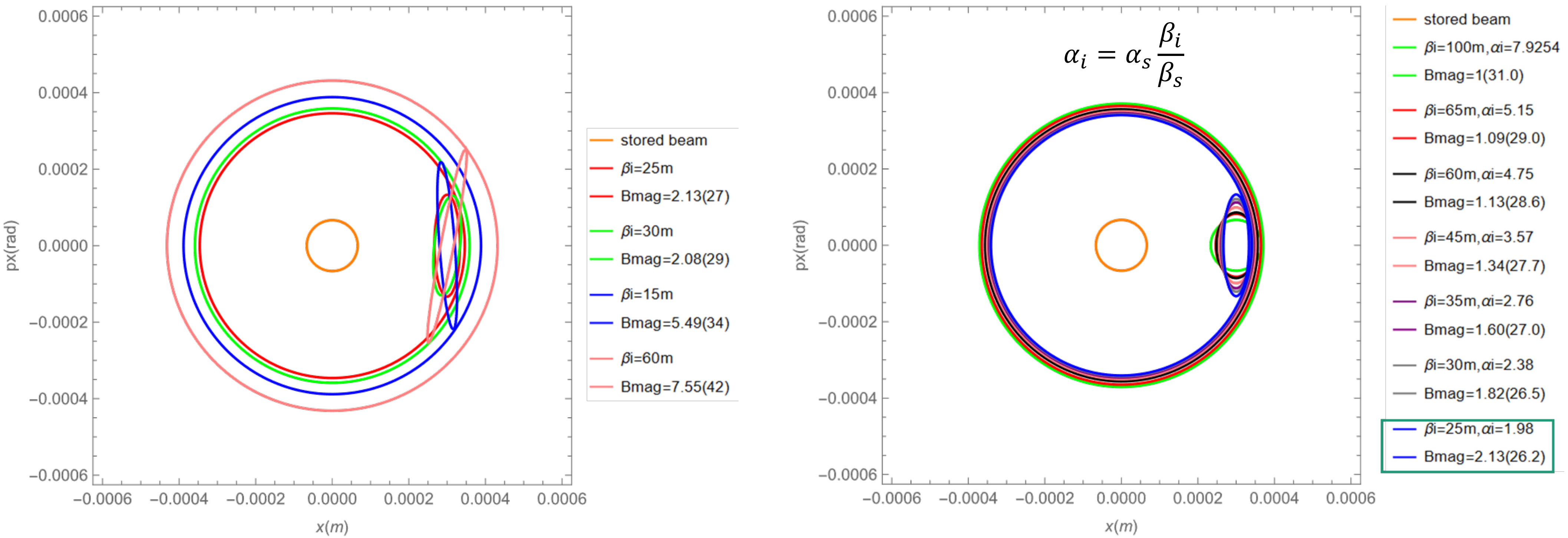




- The influence of beta mismatch on the injection efficiency can be characterized by BMAG.
- When Bmag changes in a small range, the injection efficiency changes little. And when Bmag is larger than 5, the injection efficiency will decrease with BMAG.
- **Since BMAG has little influence on the injection efficiency, how to choose the beta function of the injected beam, which is related to the injection aperture.**

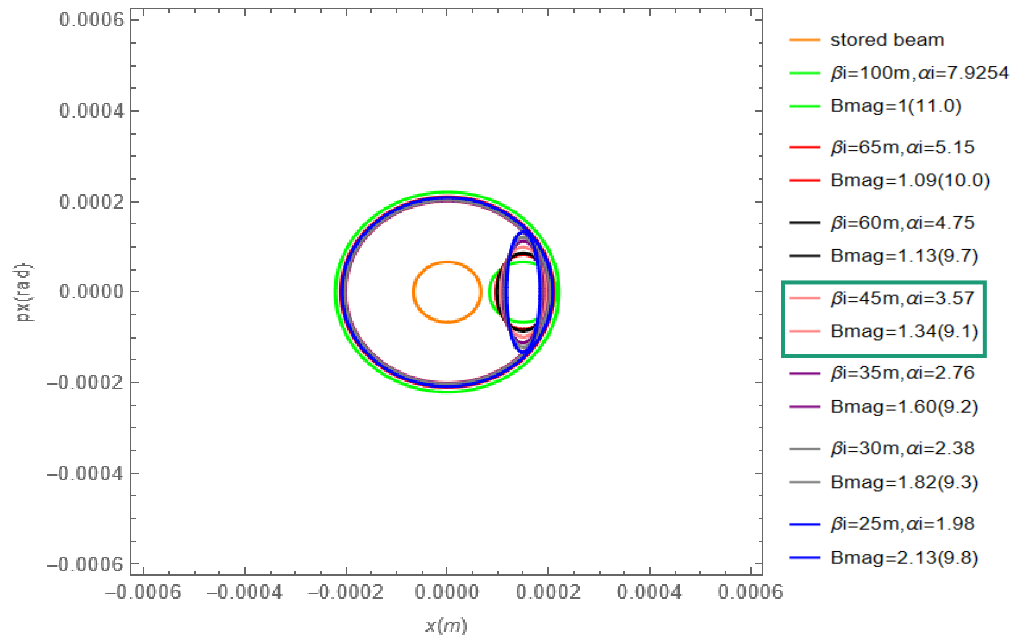
□Injection aperture

- When the rotation angle of injected ellipse is 90 °, the injection aperture required is the smallest, which just corresponds to the case where BMAG takes the minimum value when β_i is a fixed value.
- There are many optimal combinations of β_i and α_i make the rotation angle be 90 °, but the shape is different, and the injection aperture required is also different.
- When the injected beam is matched, the aperture required is the largest, not the smallest, so maybe this is the reason why the BAMG of SuperKEKB is large.

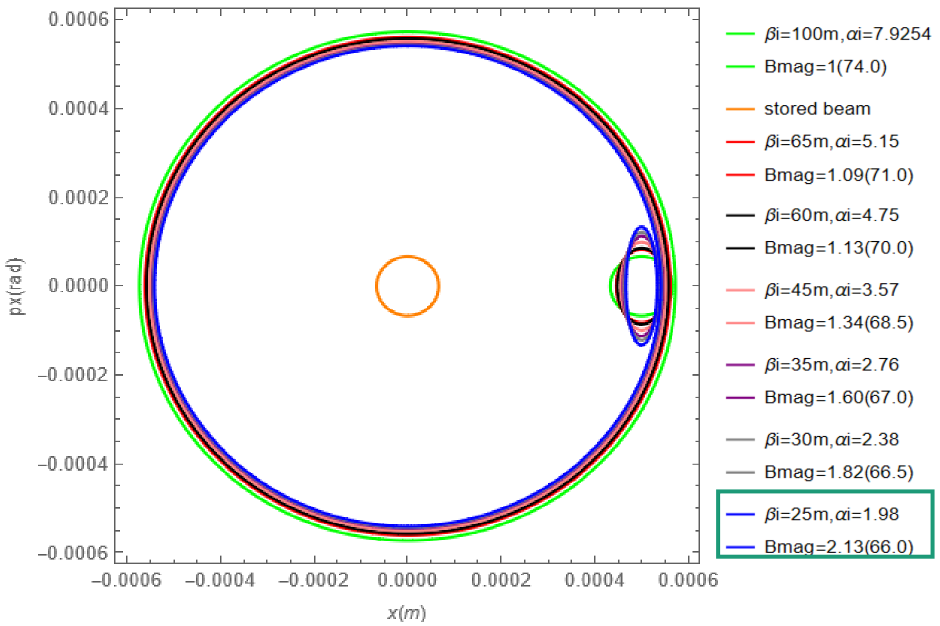


☐ Injection aperture

- The smallest injection aperture required is decided by the w_s and ε_i , that is related to $\Delta \boldsymbol{x}$.



Δx = 0.15mm



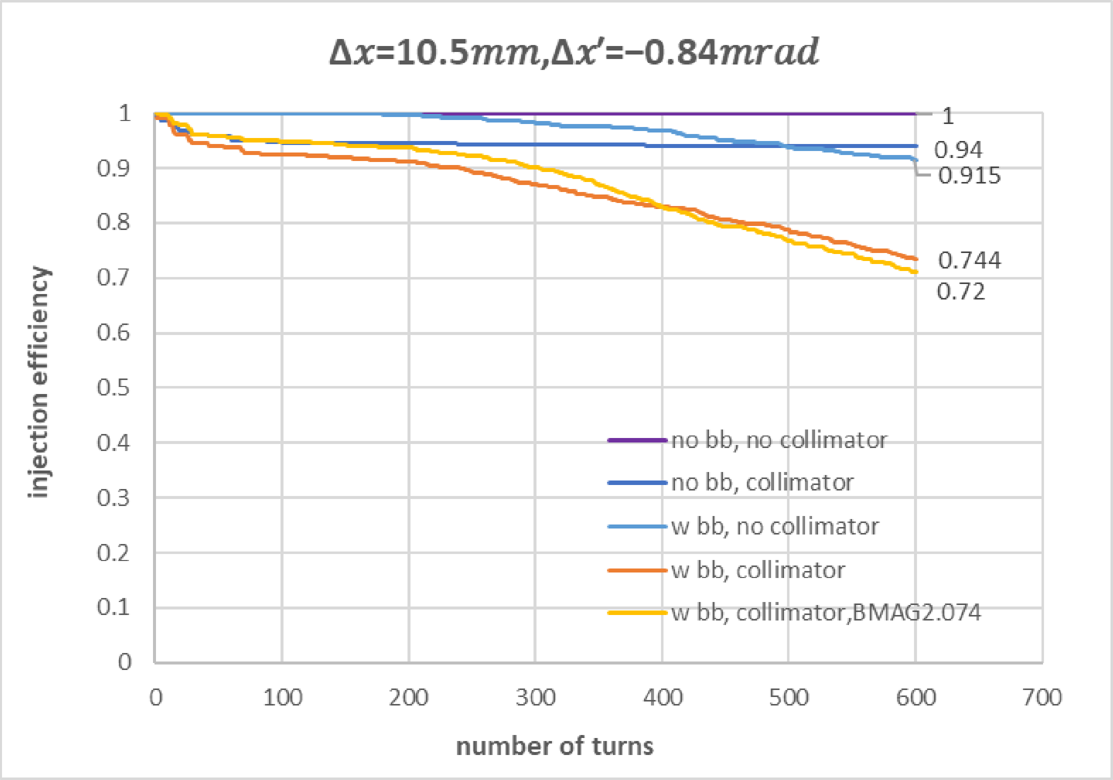
Δx = 0.50mm

$$A = \begin{cases} \frac{A_i}{r} + \frac{1}{1-r^2}\left(u_w + \sqrt{rA_i}\right)^2, & r < r_0 \\ \left(u_w + 2\sqrt{rA_i}\right)^2, & r \geq r_0 \end{cases} \Rightarrow \begin{cases} A_{min} = \min\limits_{r<r_0} \left[\frac{A_i}{r} + \frac{1}{1-r^2}\left(u_w + \sqrt{rA_i}\right)^2\right] \\ r_m = \frac{1}{18}\left(\sqrt{2}h + 2\lambda - \sqrt{8(9+\lambda^2) + 4\sqrt{2}\lambda(27+2\lambda^2)/h - 72/g - 18g}\right) \end{cases}$$

$\Rightarrow$ n_r, σ_r, W_s, β_r → u_w; n_i, ε_i — A_i → $\lambda = \frac{u_w^2}{A_i}$ → r_m → $r = \beta_i / \beta_r$ → β_i → match α_i

Summary of beam loss contribution

- Based on the above study results, summarized the contributions of these factors to beam loss. Among them, the injection angle, offset and the beam-beam contribute more.



Factor	Beam loss
Collimator(w.bb)	6% (17%)
Beam-beam(w.collimator)	9% (20%)
$\Delta x'(\pm 0.05mrad)$	11%
$\Delta x'(\pm 0.1mrad)$	66%
Δx(1mm)	10%-15%
ε_i(1nm)	15%
BMAG	2%

Summary

- **The efficiency is very sensitive to the $\Delta x'$.**
- **The injection efficiency is also sensitive to the beam-beam and Δx.**
- **Only when the emittance of the injected beam is reduced to almost the same as that of the ring can the injection efficiency be greatly improved.**
- **The injection efficiency is insensitive to the beta mismatch when BMAG changes below 5.**
- **Including the above all factors, the final injection efficiency can reach 72% when without any error.**

Discussion of improvements

- The beam-beam added in the current simulation is the full beam-beam computed for the nominal bunch charge and nominal beam size, which is stronger than that in the current condition.

$N_+ = 9.04 \times 10^{10}, \tilde{\sigma}_{x+}^* = \sigma_z sin\theta = 6 \times 10^{-3} \times 0.0415 = 2.49 \times 10^{-4} m, \sigma_{y+}^* = 4.8 \times 10^{-8} m, \quad \beta_{x-}^* = 60mm, \beta_{y-}^* = 1mm$

$$\xi_{x-} = \frac{r_e N_+}{2\pi\gamma_-} \frac{\beta_{x-}^*}{\tilde{\sigma}_{x+}^*(\tilde{\sigma}_{x+}^* + \sigma_{y+}^*)} = 0.0029 \qquad \xi_{y-} = \frac{r_e N_+}{2\pi\gamma_-} \frac{\beta_{y-}^*}{\sigma_{y+}^*(\tilde{\sigma}_{x+}^* + \sigma_{y+}^*)} = 0.2474$$

- The width of the septum is 6.81mm, which may be possible to achieve 2-3mm. Why not reduce the width of the septum to reduce $\Delta \boldsymbol{x}$ and improve the injection efficiency?

- The current simulation uses the Gaussian distribution, how about the effect for modified distributions including non-Gaussian tails from the linac?
- The current simulation is without random errors, it may compensate for the too large beam-beam strength parameter.
- The current simulation uses the emittance of injected beam same as that of the ring, how about larger injection emittance(~7nm)?
- CSR?
- X-Y coupling? …

Back up

Collimator_Nsigma_offset.opi

HER (INJ)

	DIF_POS [mm]	beta_x [m]	nu_x	eta_x [m]	Nsigma (BSC)	Nsigma (beta)	Nsigma (eta)	LM
D09H1	-11.08	39.7	15.98	0.7	17.8	25.9	24.4	-0.07 *
D09H2	-15.85	39.7	15.49	0.7	25.4	37.1	34.9	0.04
D09H3	-9.82	39.7	14.83	0.7	15.7	23.0	21.6	-0.05 *
D09H4	-8.86	39.7	14.34	0.7	14.2	20.7	19.5	-0.13 *
D12H1	-12.06	39.7	8.73	0.7	19.3	28.2	26.5	0.01
D12H2	-8.77	39.7	8.24	0.7	14.1	20.5	19.3	0.00
D12H3	-10.00	39.7	7.58	0.7	16.0	23.4	22.0	0.15 *
D12H4	-15.42	39.7	7.09	0.7	24.7	36.1	33.9	-0.08 *
D01H3OUT	8.75	7.0	0.82	-0.3	30.8	48.6	39.8	0.00
D01H3IN	-9.11	7.0	0.82	-0.3	32.1	50.6	41.4	
D01H4OUT	14.96	16.7	0.52	-0.3	46.0	54.0	88.3	0.00
D01H4IN	-14.88	16.7	0.52	-0.3	45.8	53.7	87.8	
D01H5OUT	9.41	30.9	0.27	-0.1	24.8	24.9	262.4	0.00
D01H5IN	-9.76	30.9	0.27	-0.1	25.7	25.9	272.2	
QC2 (-2.9m)	35.0	415.2	0.24			25.3		

LM: 10=Abort, >7.00, >3.00, >0.70

Dia QCSBW: 12.5 (>50.0, >25.0, >10.0)

D1V1

	DIF_POS [mm]	beta_y [m]	nu_y	Nsigma (beta)	LM
D09V1	-1.69	15.5	18.74	63.3	-0.11 *
D09V2	-3.04	19.4	16.60	101.8	-0.10 *
D09V3	-1.40	15.5	14.29	52.3	-0.09 *
D09V4	-3.29	16.7	13.17	118.7	-0.10 *
D12V1	3.80	16.7	9.91	136.8	-0.01
D12V2	-2.18	15.5	8.79	81.7	0.01
D12V3	2.15	15.5	7.67	80.5	0.92
D12V4	-1.73	19.4	6.65	57.8	-0.00
D01V1TOP	2.88	46.5	1.26	62.2	0.00
D01V1BTM	-1.78	46.5	1.26	38.6	
QC1 (-1.16m)	13.5	1295.2	0.25	55.3	

Dia QCSBW: 12.5

beta_y*[mm]	beta_x* [mm]
1.0	60.0

20200505_1x60CW40_1x80CW60_sher_2020-05

emit_x [nm]	4.60
emit_y [nm]	0.046
emit_y [nm] by XRM	0.028
Sigmay by XRM [um]	26.9
current [mA]	797

Collimator offset

D1V1

offset[mm]	DIF_POS'	Nsigma'
-0.450	2.43	52.5
	-2.23	48.3

LER (INJ)

	DIF_POS [mm]	beta_x [m]	nu_x	eta_x [m]	Nsigma (BSC)	Nsigma (beta)	Nsigma (eta)	LM
D06H1OUT	13.91	24.2	25.01	0.7	24.3	61.6	26.4	0.14
D06H1IN	-13.92	24.2	25.01	0.7	24.3	61.7	26.4	
D06H3OUT	12.26	24.2	26.23	0.7	21.4	54.4	23.3	0.17
D06H3IN	-13.36	24.2	26.23	0.7	23.3	59.2	25.3	
D03H1OUT	11.80	29.0	38.44	0.8	18.6	47.7	20.2	0.00
D03H1IN	-12.17	29.0	38.44	0.8	19.3	49.4	20.9	
D02H1OUT	7.87	20.8	42.26	0.2	31.0	37.6	54.6	0.10
D02H1IN	-8.09	20.8	42.26	0.2	31.8	38.6	56.1	
D02H2OUT	12.07	36.5	42.74	0.6	22.5	43.5	26.2	0.09
D02H2IN	-11.91	36.5	42.74	0.6	22.2	43.0	25.9	
D02H3OUT	13.76	50.8	43.47	-0.9	18.3	42.0	20.3	0.12
D02H3IN	-14.20	50.8	43.47	-0.9	19.0	43.6	21.1	
D02H4OUT	7.90	20.4	44.23	-0.4	20.0	38.1	23.5	0.23
D02H4IN	-8.13	20.4	44.23	-0.4	20.6	39.3	24.2	
QC1(1.18m)	10.5	31.4	44.29			40.9		

LM: 4=Abort, >3.00, >1.50, >0.70

D6V1 neutron: 0.00

Dia QCSFW: 12.0

	DIF_POS [mm]	beta_y [m]	nu_y	Nsigma (beta)	LM
D06V1TOP	3.31	67.3	28.85	88.3	2.36
D06V1BTM	-3.79	67.3	28.85	100.7	
D06V2TOP	3.03	20.6	30.49	145.6	0.00 / 1.69
D06V2BTM	-3.15	20.6	30.49	151.3	
D03V1TOP	7.93	17.0	41.42	420.3	0.00
D03V1BTM	-8.02	17.0	41.42	425.3	
D02V1TOP	0.83	11.9	44.83	52.7	0.16
D02V1BTM	-1.12	11.9	44.83	70.5	
QC1 (1.12m)	13.5	782.2	46.33	105.3	

Dia QCSFW: 12.0

beta_y*[mm]	beta_x* [mm]
1.0	80.0

20211001_1mm_D2V1moved_sler_2021-07-01_

emit_x [nm]	2.10
emit_y [nm]	0.021
emit_y [nm] by XRM	0.066
Sigmay by XRM [um]	67.8
current [mA]	1000

Collimator offset

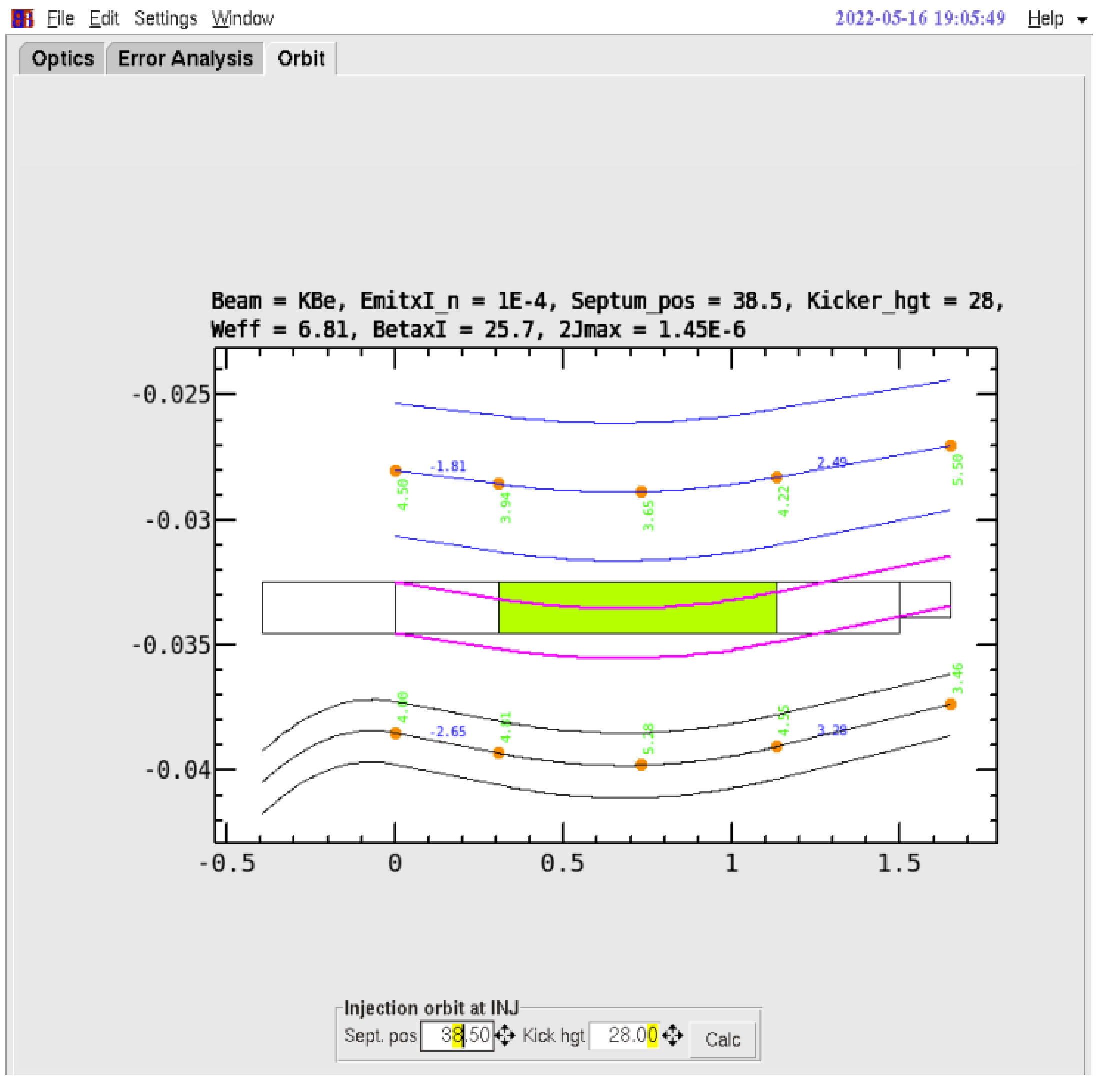
	offset[mm]	DIF_POS'	Nsigma'
D6V1	0.000	3.31	88.3
		-3.79	100.7
D6V2	0.000	3.03	145.6
		-3.15	151.3
D3V1	0.400	8.33	441.5
		-7.62	404.1
D2V1	0.400	1.23	78.0
		-0.72	45.3

- For Nsigma calculation, emittance values are given by hand (1% coupling). XRM measurements are NOT used.
- LER XRM measuremet shows much larger coupling than 1%.
- If nu_x (collimator) - nu_x(QC1) is close to half integer, that collimator can effectively suppress BG.
- Nsigma(collimator) should be similar to Nsigma(QC1) in terms of beam lifetime and BG suppression.
- If Delta(nu) is far from half integer, the collimator need to be further closed to suppress BG but beam life gets shorter.

Collimator movement ongoing: HER LER

2022/04/21 23:49:02.215

☐ A screenshot of the machine's operating parameters



by N.lida report(2022.5.20)

- **The distance between the stored beam and injected beam at the injection point:**

$$\Delta x = x_s - x_k = 38.5 - 28 = 10.5mm$$

$$w_s = 6.81mm$$

$$\left.\begin{aligned} \gamma\varepsilon_{xi} &= 100\mu m \\ \varepsilon_{xi} &= 7.3nm \\ \beta_{xi} &= 25.7m \end{aligned}\right\} \sigma_i = 0.433mm$$

$$\left.\begin{aligned} \varepsilon_{xr} &= 4.6nm \\ \beta_{xr} &= 100m \end{aligned}\right\} \sigma_r = 0.678mm$$

$$\Delta x = 3.5\sigma_r + 3\sigma_i + w_s = 10.5mm$$

- **The relative angle of the injected beam to the stored beam at the injection point:**

$$\Delta x' = \theta_{sp} - \theta_h = -2.65 - (-1.81) = -0.84mrad$$

"Another question: is the crab-waist included in the optics file which you are using, and what level of crab-waist ? Also, specify that this is without random errors in the optics lattice ? Optics errors should make things worse..."

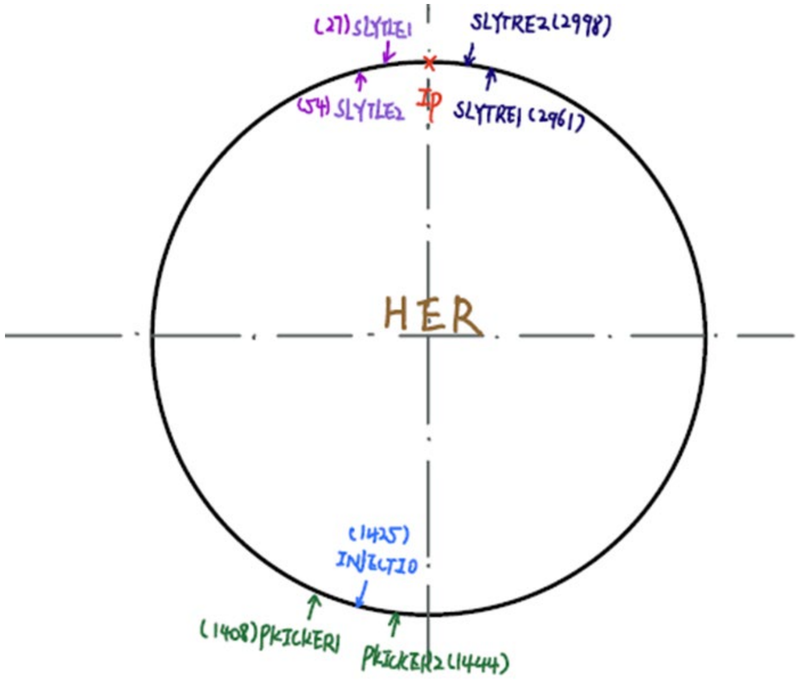
- **The optics of lattce I currently use（found at the TF meeting in August 2021 ）**

```
Matched. (  0.000    ) DP = 0.01000  DP0 = 0.00000  ExponentOfResidual = 2.0  0
ffMomentumWeight =   1.000
$$$        f AX    #######  #  3.372E-12 $$$       f BX    #######  #   .060000
$$$        f NX    #######  #  45.531000
$$$        f AY    #######  #  -5.97E-11 $$$       f BY    #######  #   .001000
$$$        f NY    #######  #  43.581000
$$$        f LENG  #######  #  3016.3147
```

- **With a pair of sextupole on both sides of the collision point**

SLYTRE1 =(L =.6 K2 =-8.759274139113826)
SLYTRE2 =(L =.6 K2 =-9.624333569214944)
SLYTLE1 =(L =.6 K2 =8.36012428756659)
SLYTLE2 =(L =.6 K2 =7.6609691425430535)

Without crab waist, the strength of a pair of sextupole should be the same. In this lattice, the strength of a pair of sextupole is different, so it contains crab waist. The level of crab-waist may be 40%.



Lattice files SuperKEKB International Task Force Subgroup · KDS (Indico) -- Updated in August 2021

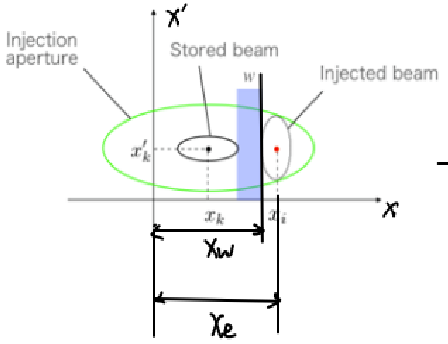
SAD Results@IP

```
$$$       f AX    #######  #  2.493E-12 $$$       f BX    #######  #    .060000 $$$       f NX
    #######  #  45.531000
$$$       f AY    #######  #  -5.95E-11 $$$       f BY    #######  #    .001000 $$$       f NY
 #######  #  43.581000
$$$       f LENG  #######  #  3016.3147
 emit;

    Closed orbit:
                 x      px/p0         y      py/p0         z      dp/p0
  Entrance :  1.19E-13  -3.1E-12  2.25E-14  8.84E-13  -4.39E-5  1.41E-12
      Exit :  1.18E-13  -3.1E-12  3.50E-14  -3.3E-12  -4.39E-5  1.40E-12

    Extended Twiss Parameters:
AX: 2.081E-6 BX:  .060014                            ZX: 8.46E-10  EX: 2.873E-7
           PSIX: -5.7E-17                           ZPX: -3.93E-8 EPX: -7.49E-6
R1: -2.6E-12 R2: 1.40E-12 AY: 1.981E-5 BY:  .001000  ZY: -2.5E-15  EY: -1.2E-10
R3: 5.167E-8 R4: 5.67E-10               PSIY: -3.7E-25 ZPY: -2.0E-13 EPY: 9.76E-10
                                                     AZ:  .017827  BZ: 8.008768
                                                                 PSIZ: 7.85E-17
    Units: B(X,Y,Z), E(X,Y), R2: m | PSI(X,Y,Z): radian | ZP(X,Y), R3: 1/m

Design momentum      P0 = 7.0072900 GeV Revolution freq.     f0 = 99390.311 Hz
Energy loss per turn U0 = 2.4329264 MV  Effective voltage    Vc = 14.200000 MV
Equilibrium position dz = 16.144160 mm  Momentum compact. alpha = 4.5428E-4
Orbit dilation       dl =  .3944769 mm  Effective harmonic #  h = 5120.0000
Bucket height     dV/P0 =  .0203357

  Imag.tune:-0.0000000           0.0000000          0.0000000
  Real tune:-0.4689888          -0.4189835         -0.0271858

    Damping per one revolution:
          X : -1.736358E-04    Y : -1.735835E-04    Z : -3.470266E-04
    Damping time (sec):
          X :  5.794510E-02    Y :  5.796255E-02    Z :  2.899300E-02
    Tune shift due to radiation:
          X : -5.589726E-07    Y : -3.009192E-06    Z : -1.792445E-07
    Damping partition number:
          X :    1.0004        Y :    1.0001        Z :    1.9994

Emittance X            = 4.43553E-9 m   Emittance Y           = 3.3890E-13 m
Emittance Z            = 3.18225E-6 m   Energy spread         = 6.30463E-4
Bunch Length           = 5.04828949 mm  Beam tilt             = 3.51375E-7 rad
Beam size xi           =  .01631556 mm  Beam size eta         = 1.84113E-5 mm
```

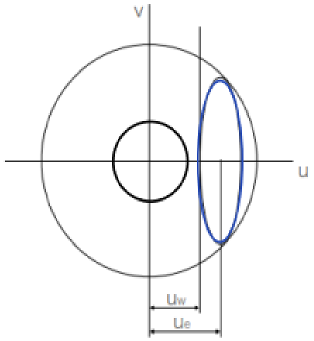
- $design\ momentum = 7.00\ GeV$
- $\beta_x^* = 6\ cm$
- $\beta_y^* = 1\ mm$
- $Nux = 45.531, Nuy = 43.581$

- $\tau_x = 5.79 \times 10^{-2} s$
- $T_0 = 1.006 \times 10^{-5} s$
- $nturn = \tau_x / T_0 = 5755$

☐ Injection aperture:



$$u = \frac{x}{\sqrt{\beta_r}} \longrightarrow V = \frac{\alpha_r x + \beta_r x'}{\sqrt{\beta_r}}$$



The injection aperture should include both stored beam and injected beam. The critical condition is that the ellipse of the injection aperture is tangent to that of injected beam. So we have to establish two equations to find the contact point:

$$\begin{cases} X_w = n_r\sigma_r + w_s \\ X_e = X_w + n_i\sigma_i \end{cases} \rightarrow \begin{cases} u_w = \frac{X_w}{\sqrt{\beta_r}} = \frac{n_r\sigma_r + w_s}{\sqrt{\beta_r}} \\ u_e = \frac{X_e}{\sqrt{\beta_r}} = u_w + \frac{n_i\sigma_i}{\sqrt{\beta_r}} = u_w + \sqrt{rA_i} \end{cases}$$

Ellipse of the injection aperture: $u^2 + v^2 = A$

Ellipse of the injected beam: $\frac{(u-u_e)^2}{b^2} + \frac{v^2}{a^2} = A_i$

where, $b = u_e - u_w = \sqrt{rA_i}$

$a = \sqrt{A_i/r}$

$$\Rightarrow \begin{cases} \frac{(u-u_e)^2}{rA_i} + \frac{v^2}{A_i/r} = 1 \\ \frac{(u-u_e)^2}{r} + rv^2 = A_i \end{cases}$$

$$\frac{(u-u_e)^2}{r} + r(A-u^2) = A_i$$

$$\boxed{(1-r^2)u^2 - 2uu_e + u_e^2 + r^2A - A_i r = 0}$$

And then let $\boxed{\Delta = (-2u_e)^2 - 4(1-r^2)(u_e^2 + r^2A - A_i r) = 0}$

$$\Rightarrow r^2A - A_i r - r^2u_e^2 - r^4A + A_i r^3 = 0$$

$$\Rightarrow \boxed{A = \frac{A_i(1-r^2) + ru_e^2}{r(1-r^2)} = \frac{A_i}{r} + \frac{(u_w+\sqrt{rA_i})^2}{1-r^2}}$$

$$\Rightarrow \boxed{u = \frac{u_e}{(1-r^2)} = \frac{u_w+\sqrt{rA_i}}{1-r^2}}$$

$$\Rightarrow \boxed{V^2 = A - u^2 = \frac{A_i}{r} - \frac{r^2}{(1-r^2)^2}(u_w+\sqrt{rA_i})^2}$$

☐ Injection aperture:

But when r is larger than some critical value of r_0, V^2 becomes negative.

$$V^2(r_0)=0 \Rightarrow r_0^4-\frac{u_w^2}{A_i}r_0^3-2r_0^2+1=0$$

For $r \geq r_0$, the solution is given by: $A=(u_w+2\sqrt{rA_i})^2$

In summary, the solution is,

$$A=\begin{cases}\frac{A_i}{r}+\frac{1}{1-r^2}(u_w+\sqrt{rA_i})^2, & (r<r_0) \quad ①\\ (u_w+2\sqrt{rA_i})^2, & (r\geq r_0) \quad ②\end{cases}$$

where, $r=\beta_i/\beta_r$, $A_i=n_i^2\varepsilon_i$, $u_w=\frac{n_r\sigma_r+w_s}{\sqrt{\beta_r}}$

Since the Eq. ① is a monotonically increasing function of r and $\partial A/\partial r(r_0)>0$, the value r that minimizes A lies in the region $r<r_0$:

$$A_{min}=\min_{r<r_0}\left[\frac{A_i}{r}+\frac{1}{1-r^2}(u_w+\sqrt{rA_i})^2\right]$$

$$\Rightarrow r_m=\frac{1}{18}\left(\sqrt{2}h+2\lambda-\sqrt{8(9+\lambda^2)+4\sqrt{2}\lambda(27+2\lambda^2)/h-72/g-18g}\right)$$

$$g=(8+\lambda^2+\lambda\sqrt{16+\lambda^2})^{1/3}$$

$$h=(18+2\lambda^2+9g+36/g)^{1/2}$$

$$\lambda=u_w^2/A_i$$

$$A_i=n_i^2\varepsilon_i$$

which is given by, approximately, for $0<\lambda<15$

$$r_m=\frac{a_0}{1+b_1\sqrt{\lambda}+b_2\lambda}$$

$a_0=0.577734686$

$b_1=0.44553030$

$b_2=-0.01293990$

- The minimum injection aperture is related to $A_i(=n_i\varepsilon_i^2)$, $\mathbf{r}(=\beta_i/\beta_r)$ *and* $u_w(=(n_r\sigma_r+w_s)/\sqrt{\beta_r})$ and determined by $\mathbf{r}\ (\beta_i)$.
- The optimum r is determined by A_i and u_w.

- Obviously, when the rotation angle of the phase ellipse is 90 °, the required injection aperture is the smallest. At this time, the twiss parameters of the injected beam and the stored beam satisfy this relationship:

Ellispe equation of injected beam in real phase space:

$$\gamma_2 x^2 + 2\alpha_2 x x' + \beta_2 x'^2 = \varepsilon$$

Transforme this ellispe to the normalised phase space defined by ring's parameters.

$$\begin{cases}\bar{x} = \frac{1}{\sqrt{\beta_1}} x \\ \bar{x}' = \frac{\alpha_1 x + \beta_1 x'}{\sqrt{\beta_1}}\end{cases} \Rightarrow \begin{cases} x = \sqrt{\beta_1}\,\bar{x} \\ x' = \frac{\bar{x}' - \alpha_1 \bar{x}}{\sqrt{\beta_1}}\end{cases}$$

The ellispe is defined by:

$$\gamma_2\beta_1\bar{x}^2 + 2\alpha_2\sqrt{\beta_1}\,\bar{x}\,\frac{\bar{x}'-\alpha_1\bar{x}}{\sqrt{\beta_1}} + \beta_2\frac{(\bar{x}'-\alpha_1\bar{x})^2}{\beta_1} = \varepsilon$$

$$\left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2\right]\bar{x}^2 - 2\bar{x}\bar{x}'\left[\frac{\beta_2}{\beta_1}\left(\alpha_1-\alpha_2\frac{\beta_1}{\beta_2}\right)\right] + \frac{\beta_2}{\beta_1}\bar{x}'^2 = \varepsilon$$

which can be characterised by γ_{new}, β_{new} and α_{new}:

$$\begin{cases}\alpha_{new} = -\frac{\beta_2}{\beta_1}\left(\alpha_1-\alpha_2\frac{\beta_1}{\beta_2}\right) \\ \beta_{new} = \frac{\beta_2}{\beta_1} \\ \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1-\alpha_2\frac{\beta_1}{\beta_2}\right)^2\end{cases}$$

and the orientation angle in terms of $\alpha_{new}, \beta_{new}, \gamma_{new}$:

$$\tan 2\theta = \frac{-2\alpha_{new}}{\beta_{new}-\gamma_{new}}$$

If we want the orientation angle is 90°, the twiss parameters of injected beam and stored beam should meet the following relationship:

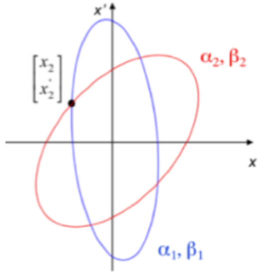
$$\tan 2\theta = \frac{2\frac{\beta_2}{\beta_1}\left(\alpha_1-\alpha_2\frac{\beta_1}{\beta_2}\right)}{\frac{\beta_2}{\beta_1} - \frac{\beta_1}{\beta_2} - \frac{\beta_2}{\beta_1}\left(\alpha_1-\alpha_2\frac{\beta_1}{\beta_2}\right)^2} = 0$$

$$\Rightarrow \alpha_1 = \alpha_2\frac{\beta_1}{\beta_2} \quad (\beta_2 \neq 0)$$

$$\alpha_i = \alpha_s\frac{\beta_i}{\beta_s}$$ (α_i, β_i: injected beam; α_s, β_s: storted beam)

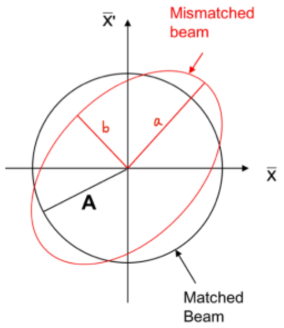
Which just corresponds to the case where, the β_i constant, the α_i is matched to minimize the BMAG.

The definition of Bmag:



real phase space

$$\begin{pmatrix} \overline{x_2} \\ \overline{x_2}' \end{pmatrix} = \sqrt{\frac{1}{\beta_1}} \cdot \begin{pmatrix} 1 & 0 \\ \alpha_1 & \beta_1 \end{pmatrix} \cdot \begin{pmatrix} x_2 \\ x_2' \end{pmatrix}$$



normalised phase space

Ellispe equation of injected beam in real phase space:

$$\gamma_2 x^2 + 2\alpha_2 x x' + \beta_2 x'^2 = \varepsilon$$

Transforme this ellispe to the normalised phase space defined by ring's parameters.

$$\begin{cases} \bar{X} = \frac{1}{\sqrt{\beta_1}} x \\ \bar{X}' = \frac{\alpha_1 x + \beta_1 x'}{\sqrt{\beta_1}} \end{cases} \Rightarrow \begin{cases} x = \sqrt{\beta_1}\,\bar{X} \\ x' = \frac{\bar{X}' - \alpha_1 \bar{X}}{\sqrt{\beta_1}} \end{cases}$$

The ellispe is defined by:

$$\gamma_2 \beta_1 \bar{X}^2 + 2\alpha_2 \sqrt{\beta_1}\,\bar{X}\,\frac{\bar{X}' - \alpha_1 \bar{X}}{\sqrt{\beta_1}} + \beta_2 \frac{(\bar{X}' - \alpha_1 \bar{X})^2}{\beta_1} = \varepsilon$$

$$\left[\frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2}\right)^2\right]\bar{X}^2 - 2\bar{X}\bar{X}'\left[\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2}\right)\right] + \frac{\beta_2}{\beta_1}\bar{X}'^2 = \varepsilon$$

which can be characterised by γ_{new}, β_{new} and α_{new}:

$$\begin{cases} \alpha_{new} = -\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2}\right) \\ \beta_{new} = \frac{\beta_2}{\beta_1} \\ \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2}\right)^2 \end{cases}$$

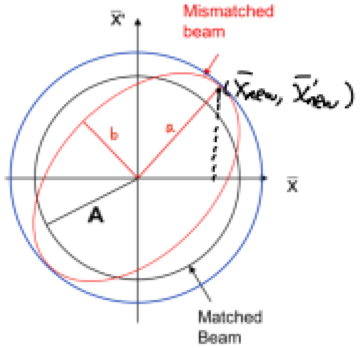
From the general elispe properties, the parameters a, b and λ can be introduced. Where a and b are the half axes of the ellipse which are related to the radius of the matched circle $A = \sqrt{\varepsilon}$:

$$a = \sqrt{\frac{\varepsilon}{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right)$$

$$b = \sqrt{\frac{\varepsilon}{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right)$$

$$H = \frac{1}{2}(\gamma_{new} + \beta_{new}) = \frac{1}{2}\left[\frac{\beta_2}{\beta_1} + \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2 \frac{\beta_1}{\beta_2}\right)\right]$$

After that, the mismatched beam's ellipse will rotate, so its equivalent elispse circumscribes its initial ellipse and similar to the stored beam elispse



- **The definition of Bmag:**

$$\begin{cases} \alpha_{new} = -\frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right) \\ \beta_{new} = \frac{\beta_2}{\beta_1} \\ \gamma_{new} = \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)^2 \end{cases}$$

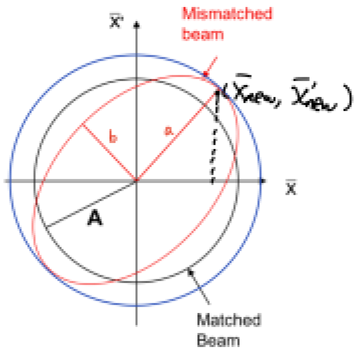
From the general elispe properties, the parameters a, b and λ can be introduced. Where a and b are the half axes of the ellipse which are related to the radius of the matched circle $A=\sqrt{\varepsilon}$:

$$a = \sqrt{\frac{\varepsilon}{2}}\left(\sqrt{H+1} + \sqrt{H-1}\right)$$

$$b = \sqrt{\frac{\varepsilon}{2}}\left(\sqrt{H+1} - \sqrt{H-1}\right)$$

$$H = \frac{1}{2}(\gamma_{new}+\beta_{new}) = \frac{1}{2}\left[\frac{\beta_2}{\beta_1} + \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)\right]$$

After that, the mismatched beam's ellipse will rotate, so its equivalent elispse circumscribes its initial ellipse and similar to the stored beam elispse



The coordinates of the mismatched distribution in normalised phase space at the injection position can be written simply as:

$$\begin{cases} \bar{x}_{new} = a\cos\phi \\ \bar{x}'_{new} = a\sin\phi \end{cases}$$

So the mismatched distribution:

$$\varepsilon_{new} = \bar{x}_{new}^2 + \bar{x}'^2_{new} = a^2$$

$$= \frac{\varepsilon_0}{2}\left(\sqrt{H+1} + \sqrt{H-1}\right)^2$$

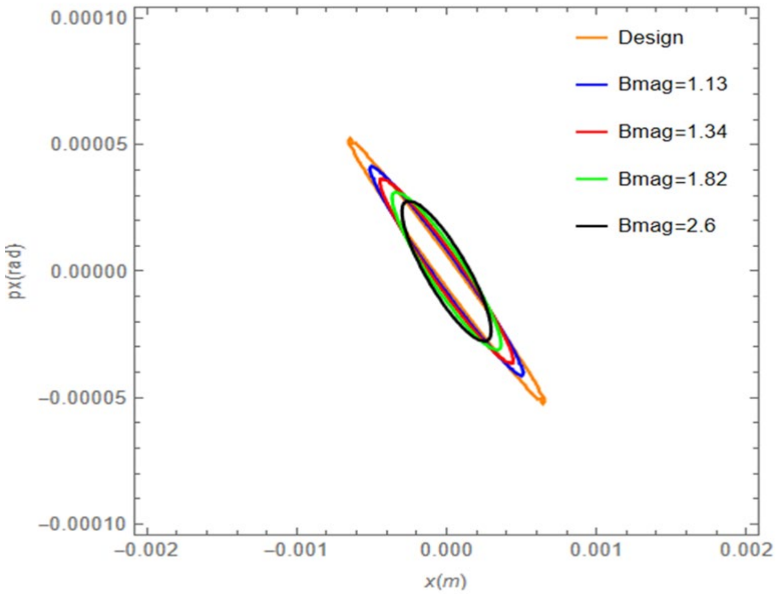
$$= \varepsilon_0\left(H + \sqrt{H^2-1}\right)$$

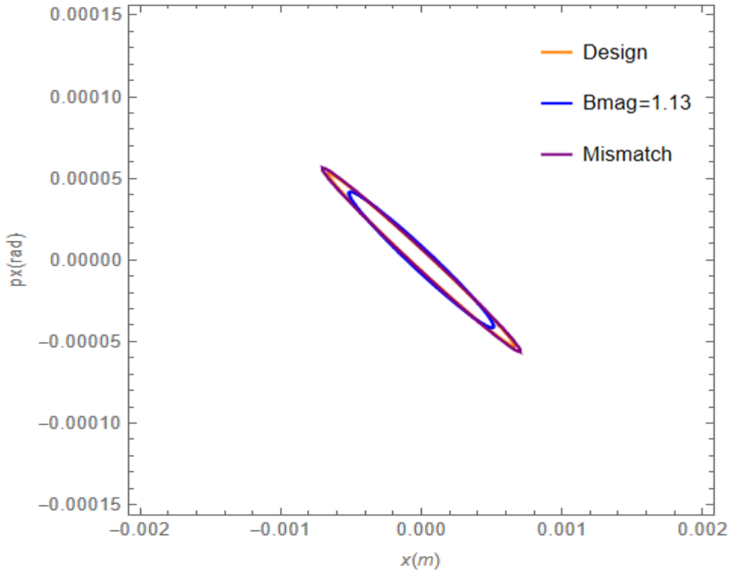
$$\therefore \quad \frac{\varepsilon_{new}}{\varepsilon_0} = H + \sqrt{H^2-1} = B_{mag} + \sqrt{B_{mag}^2 - 1}$$

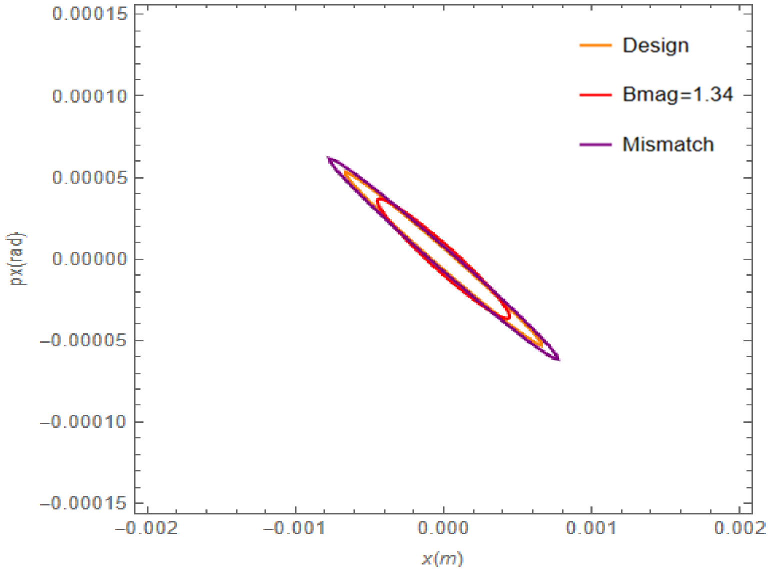
$$\because \quad H = \frac{1}{2}(\gamma_{new}+\beta_{new}) = \frac{1}{2}\left[\frac{\beta_2}{\beta_1} + \frac{\beta_1}{\beta_2} + \frac{\beta_2}{\beta_1}\left(\alpha_1 - \alpha_2\frac{\beta_1}{\beta_2}\right)\right] = B_{mag}$$

$$\therefore \quad \boxed{\frac{\varepsilon_{new}}{\varepsilon_0} = B_{mag} + \sqrt{B_{mag}^2 - 1}}$$

☐ Draw three ellipses in phase space



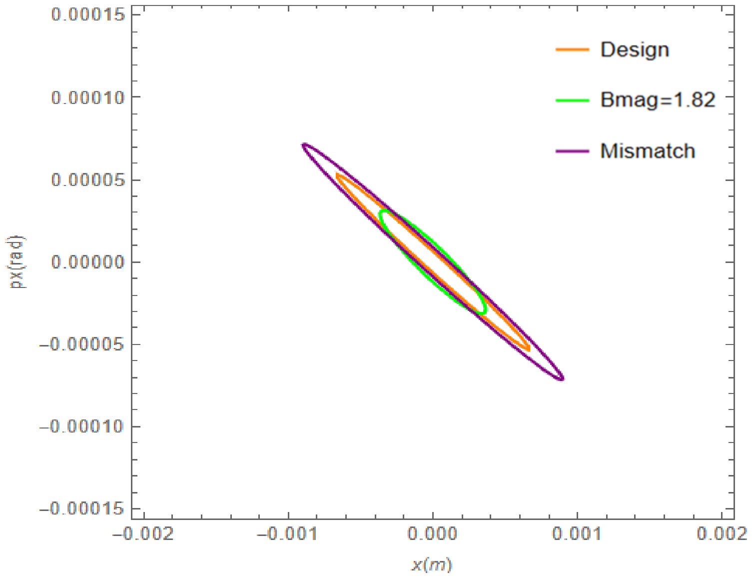


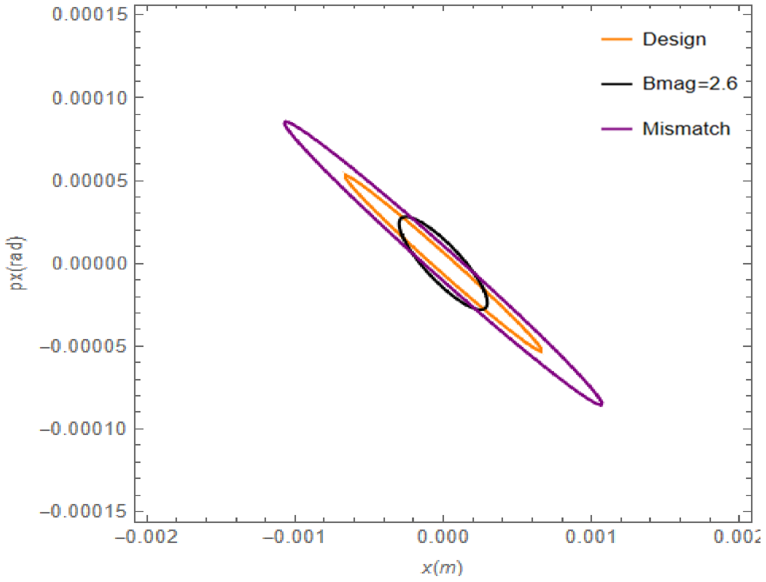


$$(\gamma\varepsilon)_{final} = BMAG \cdot (\gamma\varepsilon)_{initial}$$

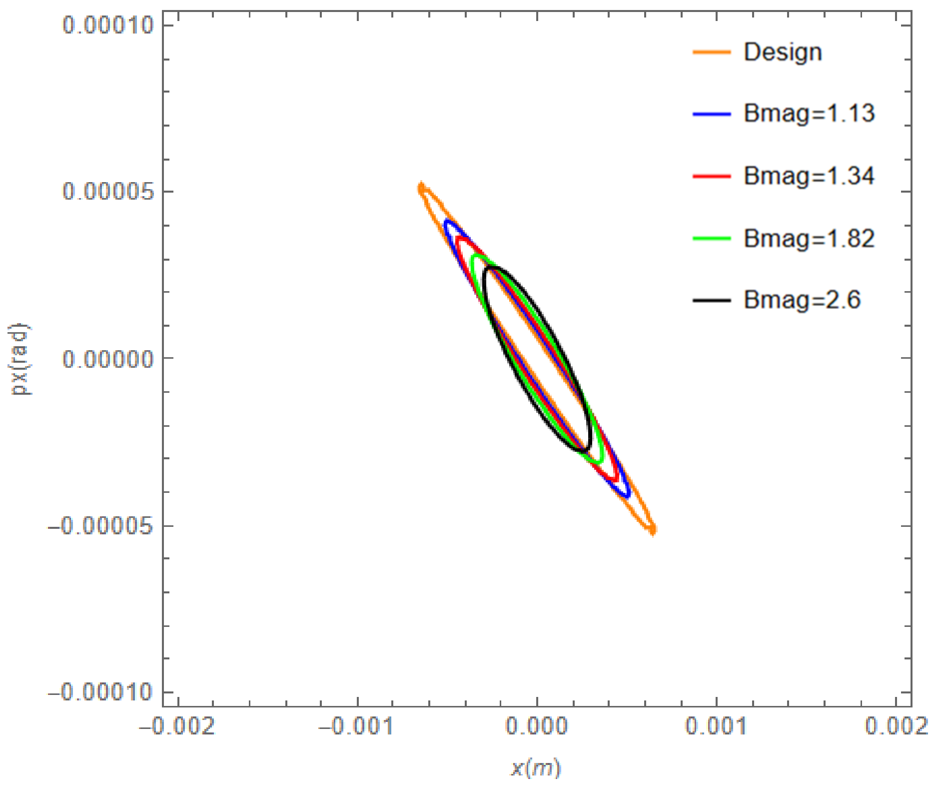
$$BMAG = \frac{1}{2}\left[\frac{\beta_m}{\beta_e} + \frac{\beta_e}{\beta_m} + \beta_b\beta_m\left(\frac{\alpha_b}{\beta_b} - \frac{\alpha_m}{\beta_m}\right)^2\right]$$

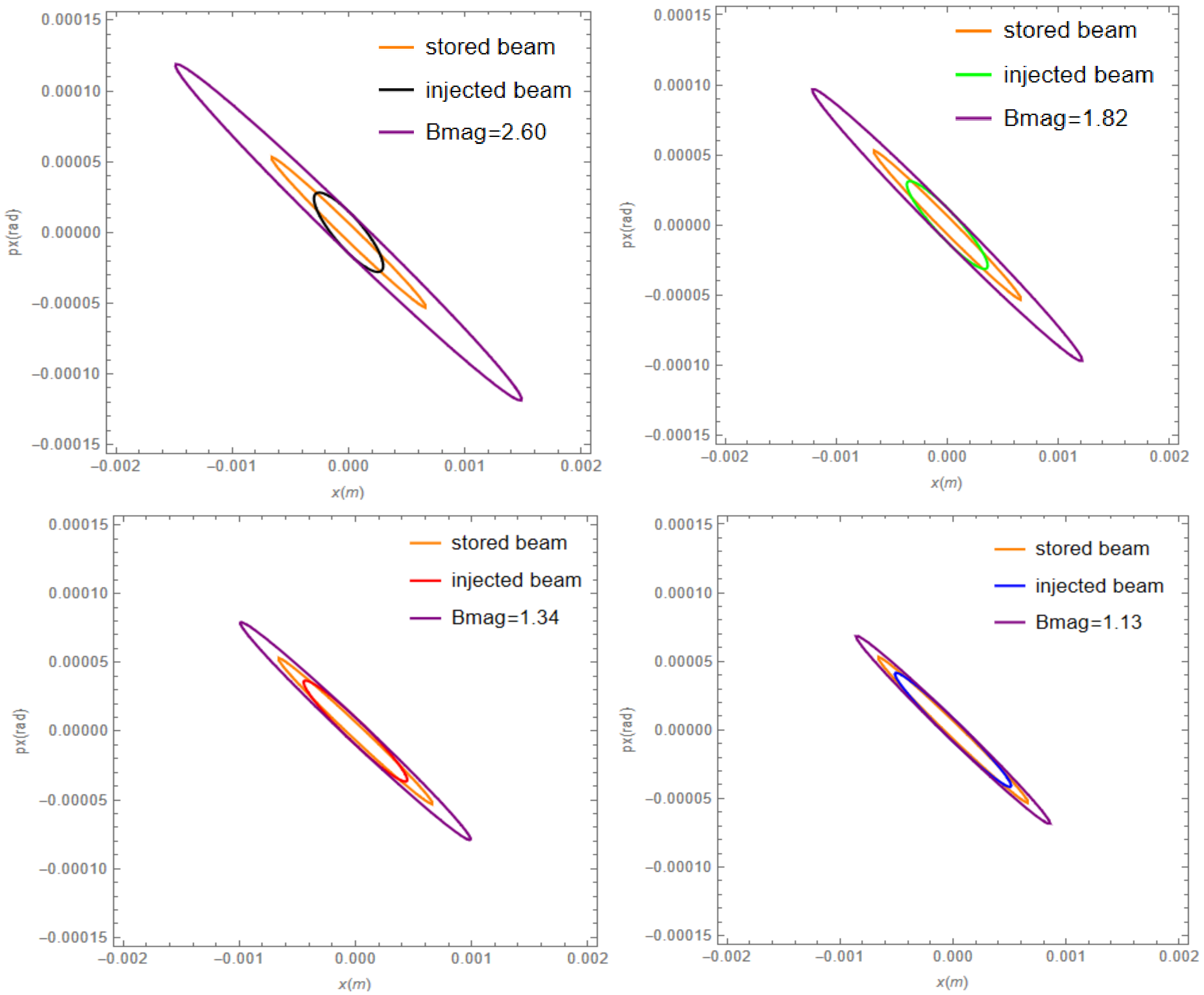
- When BMAG is very close to 1, the third ellipse circumscribes the injected ellipse.
- When BMAG is slightly larger, about larger than 1.2, the third ellipse no longer circumscribes the injected ellipse.





☐ The definition of Bmag:

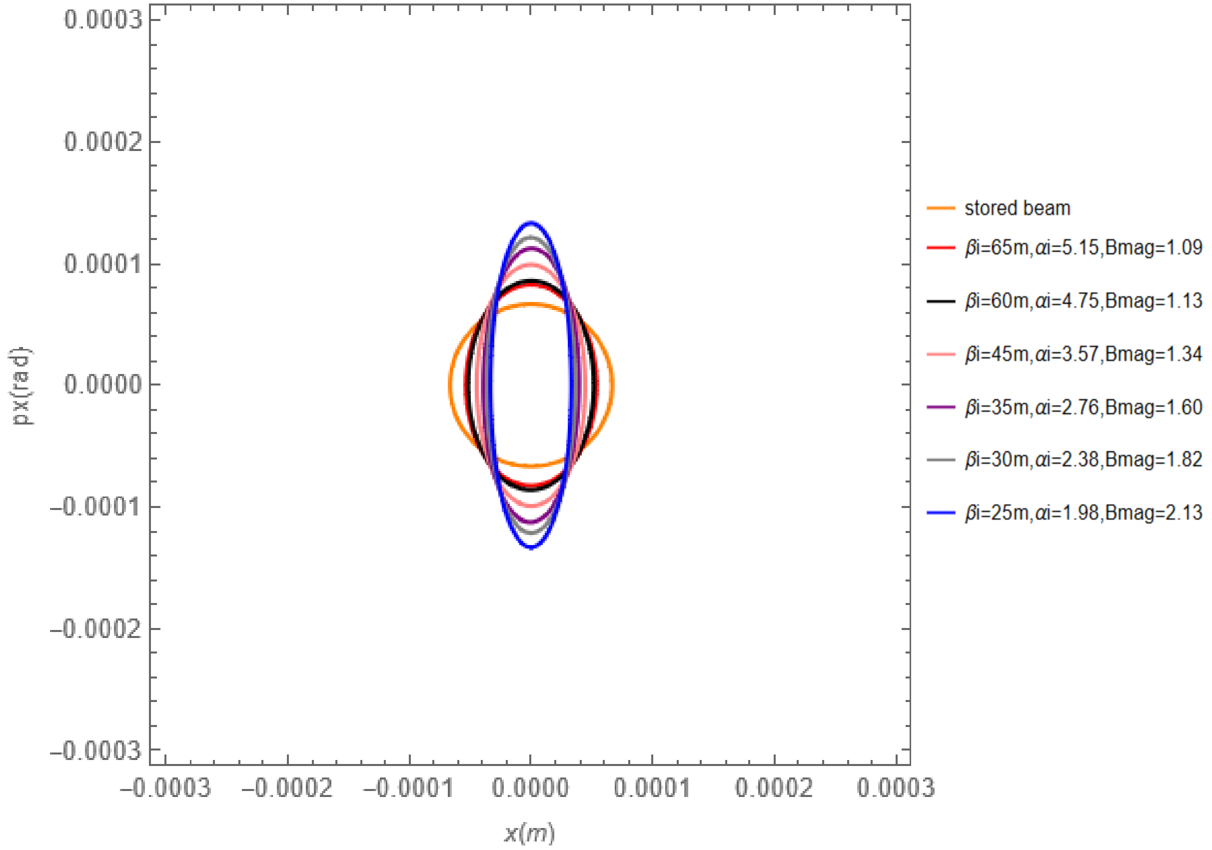


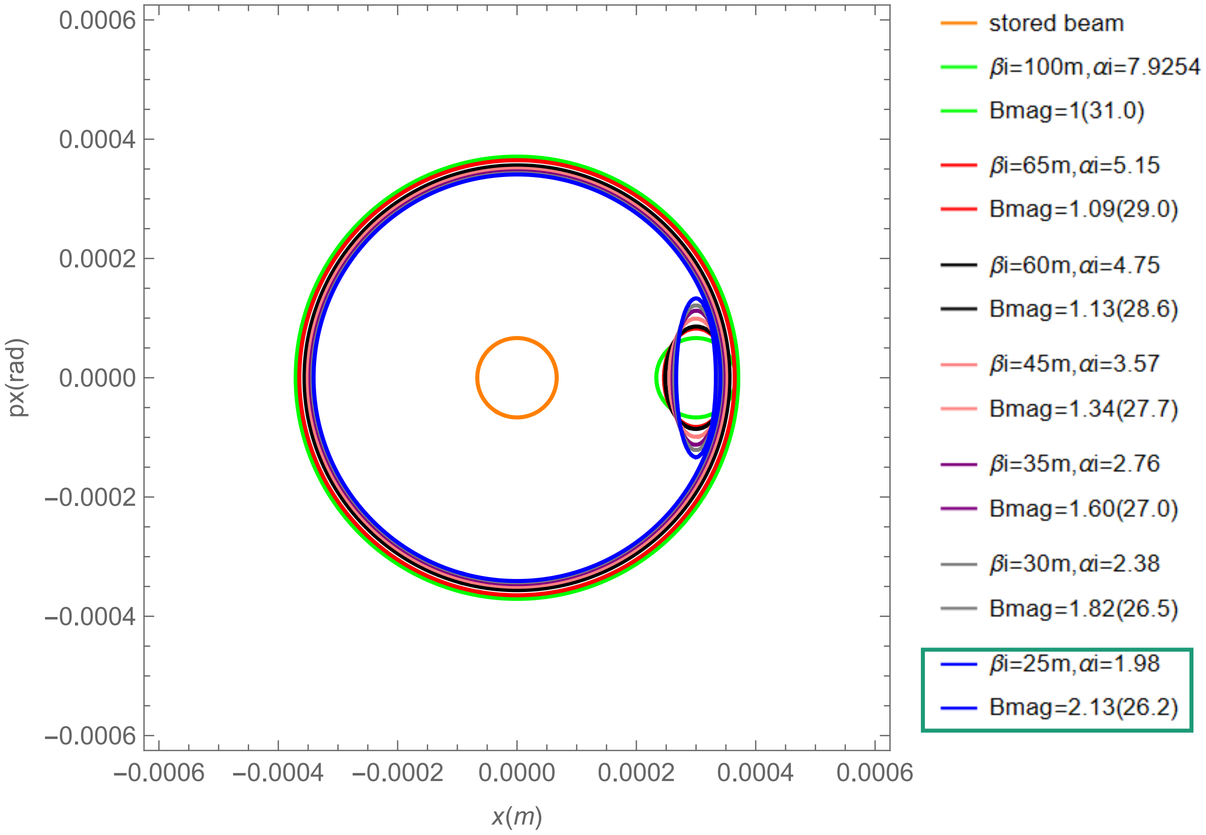


$$\frac{\varepsilon_{final}}{\varepsilon_{initial}} = \frac{\varepsilon_{purple}}{\varepsilon_{orange}} = BMAG + \sqrt{BMAG^2 - 1}$$

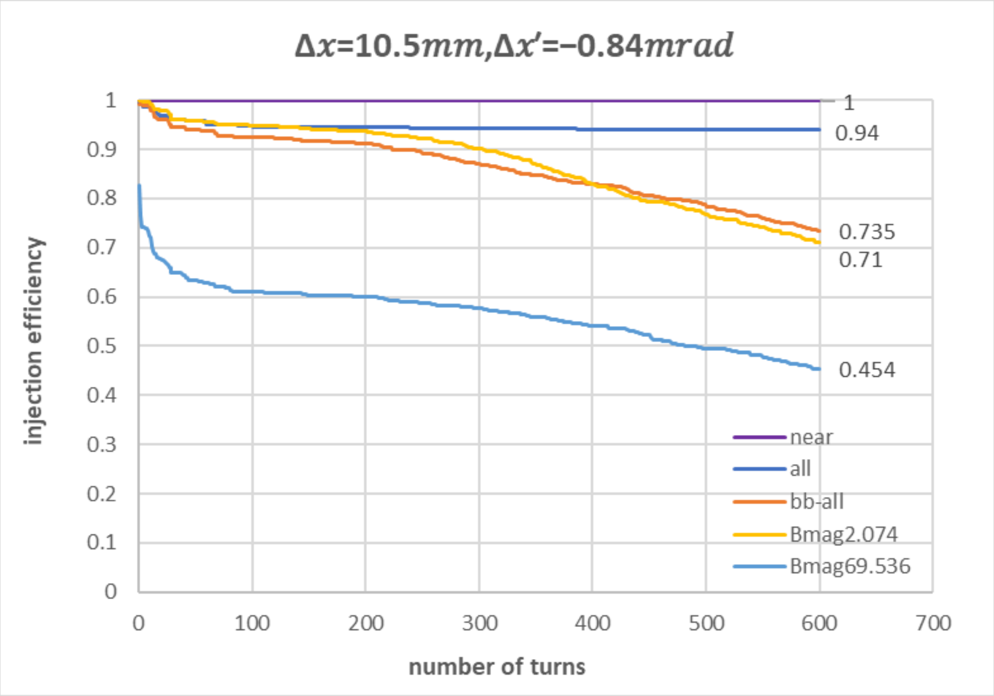
$$BMAG = \frac{1}{2}\left[\frac{\beta_m}{\beta_e} + \frac{\beta_e}{\beta_m} + \beta_b \beta_m \left(\frac{\alpha_b}{\beta_b} - \frac{\alpha_m}{\beta_m}\right)^2\right]$$

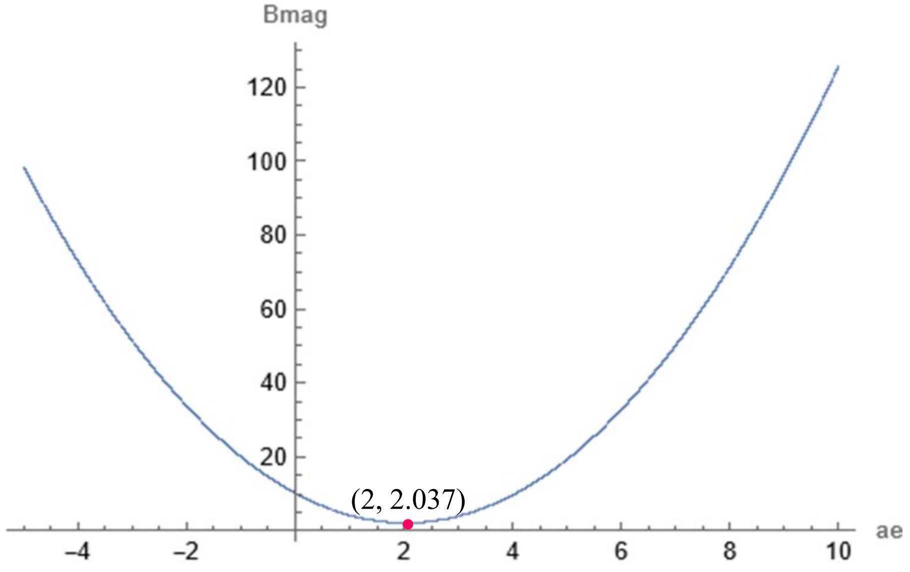
- When the Twiss parameters of the injected beam are set to different optimal combinations of β_{xi} and α_{xi}, the rotation angle is 90 °, but the shape is different, and the injection aperture is different.
- It is not that the larger the BMAG, the larger the injection aperture. On the contrary, the injection aperture is the largest when matched.
- This is also the reason why there is so much difference between the beta function of injected beam and the stored beam in SuperKEKB, the dynamic aperture is too small, so the injection aperture is required to be as small as possible





☐ Add the beta-mismatch into simulation





$\beta_{xi} = 25.7m$

- Add the beam-beam, all the collimators, the injection efficiency is 73.5%.
- Only β_{xi} changes from 100m to 25.7m, α_{xi} stays the same, the BMAG is 70, and the efficiency is reduced to 45%.
- β_{xi} stays the same, α_{xi} changes, and we can find the optimum α_{xi} which minimizes BMAG.

$$\left.\begin{array}{r}\text{injected beam: } \beta_i = 25.7m,\ \alpha_i = 2.037 \\ \text{stored beam: } \beta_s = 100m,\ \alpha_s = 7.9254\end{array}\right\} \Rightarrow \text{BAMG} = \frac{1}{2}\left(\frac{\beta_s}{\beta_i} + \frac{\beta_i}{\beta_s}\right) + \frac{1}{2}\left(\alpha_i\sqrt{\frac{\beta_s}{\beta_i}} - \alpha_s\sqrt{\frac{\beta_i}{\beta_s}}\right)^2 = 2.074$$

Eur. Phys. J. C (2021) 81:972
https://doi.org/10.1140/epjc/s10052-021-09769-3

THE EUROPEAN PHYSICAL JOURNAL C

Check for updates

Special Article - Tools for Experiment and Theory

A time resolved study of injection backgrounds during the first commissioning phase of SuperKEKB

Miroslav Gabriel[1,a], Frank Simon[1,b], Hendrik Windel[1], Yoshihiro Funakoshi[2], Michael Hedges[3,5], Naoko Iida[2], Igal Jaegle[3,6], Christian Kiesling[1], Naomi van der Kolk[1,7], Peter Lewis[3,8], Hiroyuki Nakayama[2], Yukiyoshi Ohnishi[2], Riccardo de Sangro[4], Yusuke Suetsugu[2], Marco Szalay[1,9], Sven Vahsen[3]

[1] Max-Planck-Institut für Physik, 80805 Munich, Germany
[2] High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan
[3] University of Hawaii, Honolulu, HI 96822, USA
[4] INFN Laboratori Nazionali di Frascati, 00044 Frascati, Italy
[5] *Present address*: Purdue University, West Lafayette, IN 47907, USA
[6] *Present address*: Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA
[7] *Present address*: The Hague University of Applied Sciences, The Hague, Netherlands
[8] *Present address*: University of Bonn, 53115 Bonn, Germany
[9] *Present address*: Google LLC., Santa Barbara, CA 93101, USA

Received: 22 December 2020 / Accepted: 25 October 2021 / Published online: 5 November 2021

grounds. We present measurements of various aspects of regular beam background and injection backgrounds which include time structure and decay behavior of injection backgrounds, hit-energy spectra and overall background rates. These measurements show that the elevated background rates following an injection generally last for several milliseconds, with the majority of the background particles typically observed within the first 500 µs. The injection backgrounds exhibit pronounced patterns in time, connected to betatron and synchrotron oscillations in the accelerator rings. The frequencies of these patterns are determined from detector data.

- $\tau_x = 5.79 \times 10^{-2} s$
- $T_0 = 1.006 \times 10^{-5} s$
- $nturn = \tau_x / T_0 = 5755$

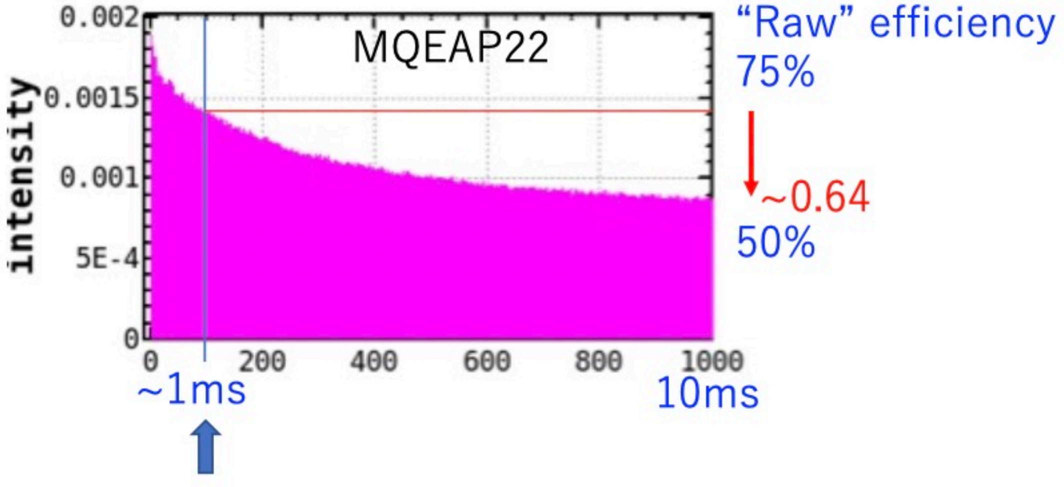
→

- Several milliseconds $\approx 600 turns$
- $500 \mu s \approx 50 turns$

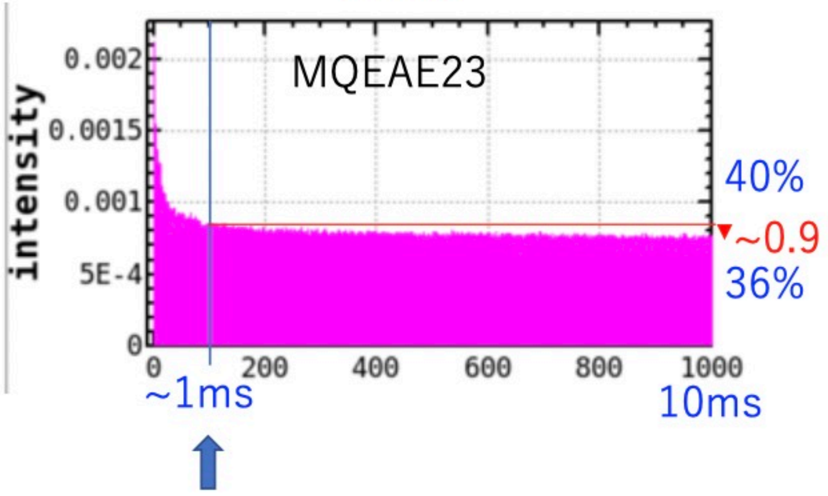
βy*=1mm

4. "Raw" injection efficiency

LER



HER



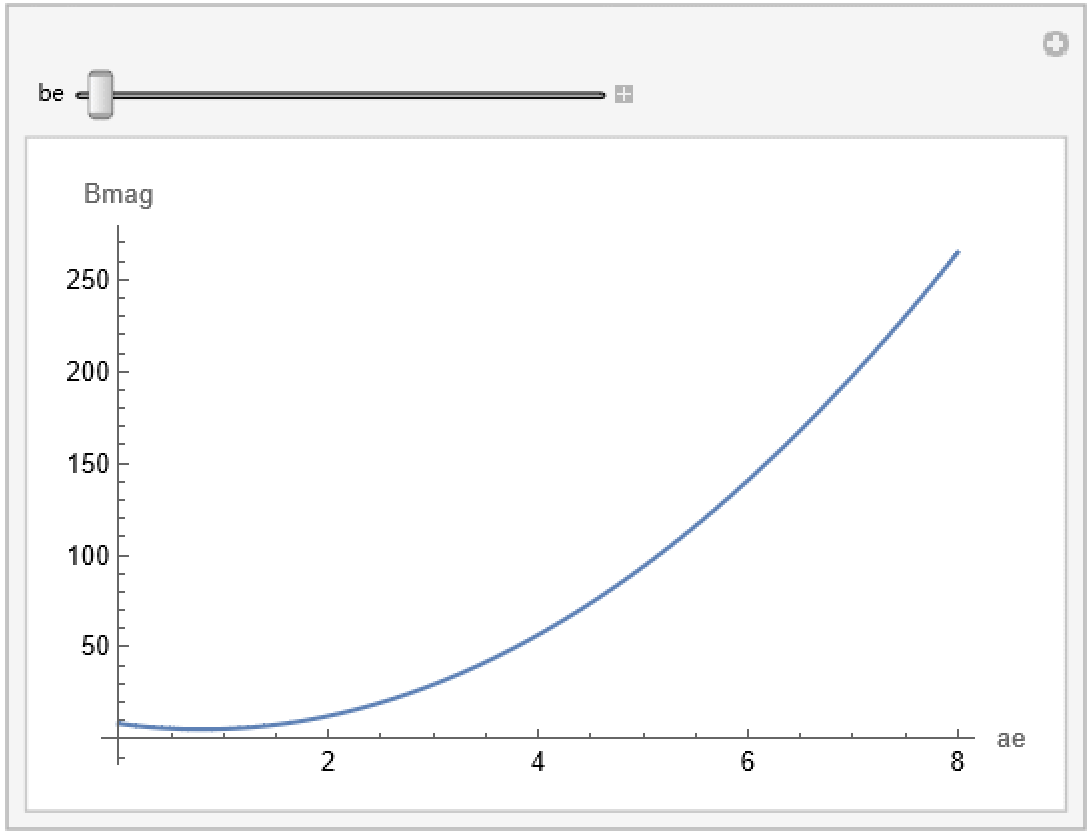
"Raw" injection efficiency

"Raw" injection efficiency uses the value 1ms after injection.
But the decay continues, mainly in LER.
It should be resolved where the beam losses during 1000 turns in LER occur.

Koga-san pointed out that the decay pattern for each ring is similar on the duration.

Horizontal beta-mismatch

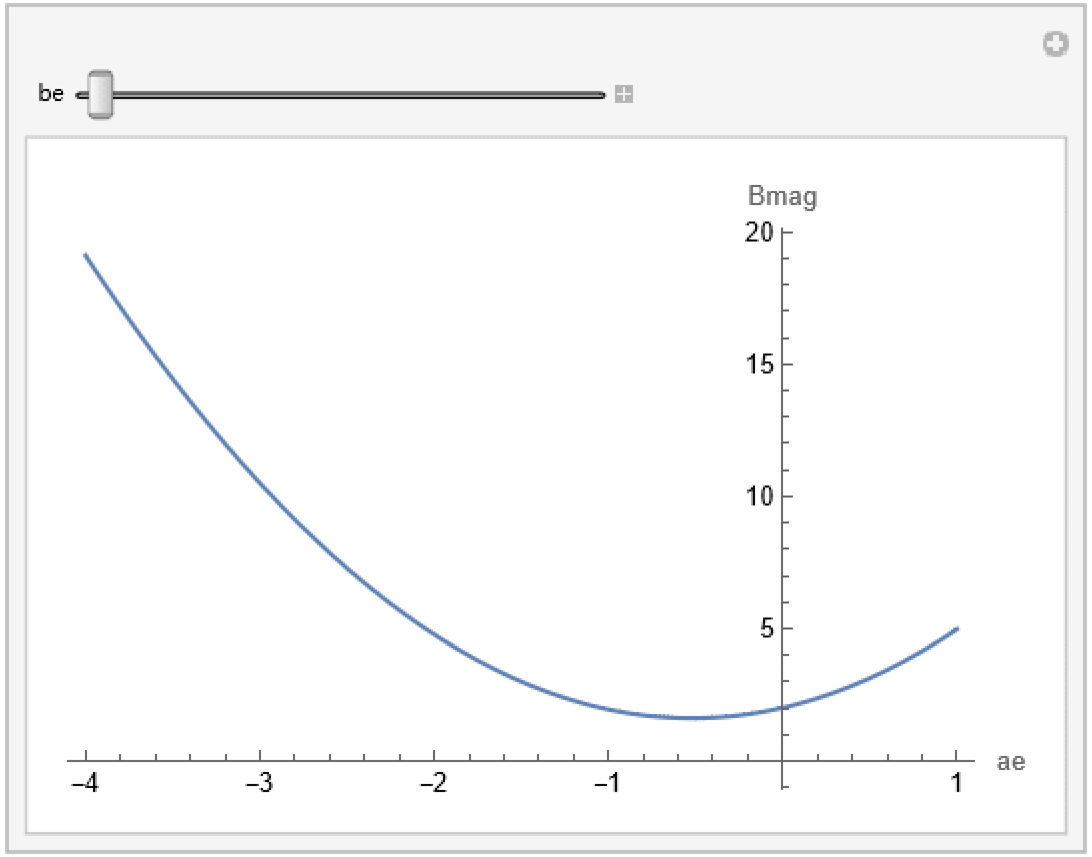
- **The twiss of ring in x direction: $\alpha_{mx} = 7.9254, \beta_{mx} = 100$**
- **Scanning within a large range:** $\alpha_{ex} \in [0,8]$， $\beta_{ex} \in [10,100]$, track 5000 particles for 600 turns.



β_x	α_x	Bmagx	efficiency
20	1.6	2.6	71.4%
25	1.98	2.13	72.6%
30	2.38	1.82	72.9%
35	2.76	1.60	72.7%
40	3.16	1.45	71.2%
45	3.57	1.34	72.4%
50	3.96	1.25	72.7%
60	4.75	1.13	71.6%
65	5.15	1.09	73.2%
100	7.93	1.00	72.2%

Vertical beta-mismatch

- **The twiss of ring in y direction: $\alpha_{my} = -1.5111, \beta_{my} = 28.9281$**
- **Scanning within a large range:** $\alpha_{ey} \in [-4,1]$， $\beta_{ey} \in [10,40]$



β_y	α_y	Bmag	efficiency
5	-0.3	2.98	71.6%
8	-0.4	1.95	72.0%
10	-0.52	1.62	72.3%
12	-0.70	1.42	72.0%
15	-0.78	1.22	72.1%
20	-1.04	1.07	72.3%
29	-1.51	1.00	72.7%

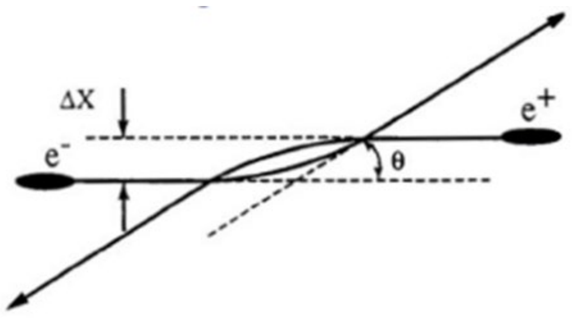
Beam-beam

➢ **Define the BEAMBEAM element and add it after the IP in the lattice sequence.**

```
BEAMBEAM        FBMBME   =(BX =.032   BY =.00027    XANGLE =.0415
                          EMITX =3.2e-09  EMITY =8.64e-12  DP =.00081
                          SIGZ =.006 SLICE =100 NP =9.04e+10  STURN =10 )
```

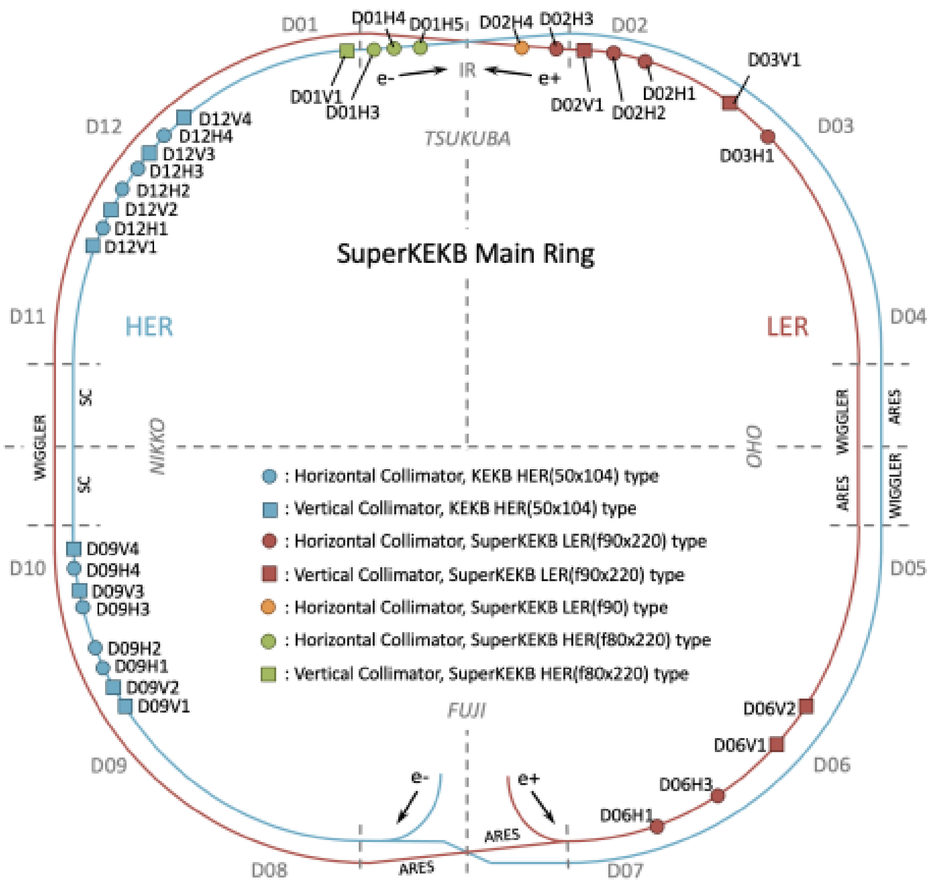
```
LINE ASCE=
     (IP          FBMBME    FHBBKE    FVBBKE    ESLE0     ECSLE5    ESLE10
      ECSLE15     ESLE20    ECSLE25   ESLE30    ECSLE35   ESLE40    ECSLE45
      ESLE50      ECSLE55   ESLE60    ECSLE65   ESLE70    ECSLE75   ESLE80
      ECSLE85     ESLE90    ECSLE95   ESLE100   ECSLE105  ESLE110   ECSLE115
```

BX: β_x^* BY: β_y^*
EMITX: ε_x EMITY: ε_y
SIGZ: bunch length
XANGLE: half crossing angle
DP: energy spread
NP: particle number for each bunch
STURN: how many turns do you output information

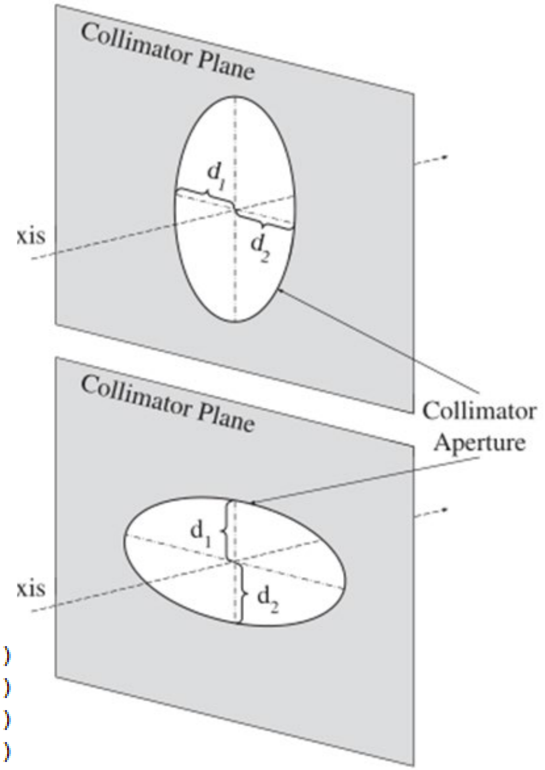


☐ Set physical aperture to judge whether particles are lost

➢ Aperture set——collimator




```
APERT  AQC1LC   =(AX =.015      AY =.015   DX =.0007 )
       AQC1LO   =(AX =.015      AY =.015   DX =.0007 )
       AQC2LC   =(AX =.035      AY=.035    DX =.0007 )
       AQC2LO   =(AX =.035      AY =.035   DX =.0007 )
       PMD01H5  =(AX =0.00941    AY =0.045 )
       PMD01H4  =(AX =0.01488    AY =0.045 )
       PMD01H3  =(AX =0.00875    AY =0.045 )
       PMD01V1  =(AX =0.045      AY =0.00178 )
       PMD01H2  =(AX =0.045      AY =0.045 )
       PMD01H1  =(AX =0.045      AY =0.045 )
       PMD12V4  =(AX =0.045      AY =0.00173 )
       PMD12H4  =(AX =0.01542    AY =0.045 )
       PMD12V3  =(AX =0.045      AY =0.00215 )
       PMD12H3  =(AX =0.01000    AY =0.045 )
       PMD12H2  =(AX =0.00877    AY =0.045 )
       PMD12V2  =(AX =0.045      AY =0.00218 )
       PMD12H1  =(AX =0.01206    AY =0.045 )
       PMD12V1  =(AX =0.045      AY =0.00380 )
       PMD09V4  =(AX =0.045      AY =0.00329 )
       PMD09H4  =(AX =0.00886    AY =0.045 )
       PMD09V3  =(AX =0.045      AY =0.00140 )
       PMD09H3  =(AX =0.00982    AY =0.045 )
       PMD09H2  =(AX =0.01585    AY =0.045 )
       PMD09H1  =(AX =0.01108    AY =0.045 )
       PMD09V2  =(AX =0.045      AY =0.00304 )
       PMD09V1  =(AX =0.045      AY =0.00169 )
       AQC2RO   =(AX =.035      AY =.035   DX =-.0007 )
       AQC2RC   =(AX =.035      AY =.035   DX =-.0007 )
       AQC1RO   =(AX =.015      AY =.015   DX =-.0007 )
       AQC1RC   =(AX =.015      AY =.015   DX =-.0007 )
```



- **The collimator aperture is defined as the distance from the beam core to the collimator tip.**

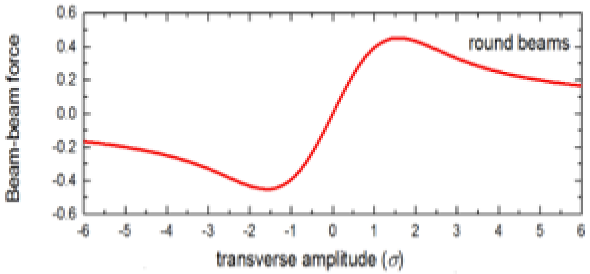
☐ Beam-beam

➢ Check beam-beam element—— Theoretical calculation

The more general beam-beam effects are quantified by the so-called disruption parameter $D_{x,y}$, defined as

$$D_{x,y} = \frac{2r_e N_b \sigma_z}{\gamma \sigma_{x,y}(\sigma_x + \sigma_y)} \approx \frac{\sigma_z}{f_{beam}}$$

This force changes the momentum of each bunch. When the oscillation amplitude of the particle is small, the force can be considered as linear, and its effect is equivalent to a Quadrupole, that is, the force is proportional to its transverse offset.



In the thin lens approximation, the transfer matrix is: $M = \begin{bmatrix} 1 & 0 \\ -1/f_{beam} & 1 \end{bmatrix}$, and the focus length is:

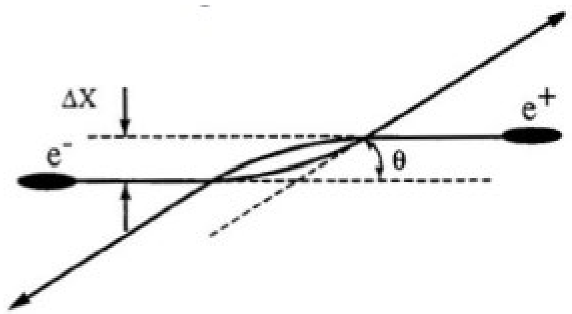
$$f_{beam} = f_{x,y} = \frac{\sigma_z}{D_{x,y}} = \frac{2r_e N_b}{\gamma \sigma_{x,y}(\sigma_x + \sigma_y)}$$

Beam-beam

- **Check beam-beam element—— Theoretical calculation**

when a particle with a horizontal offset x passes through it, the coordinates change to

$$\begin{bmatrix} x_1 \\ x_1' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f_{beam} & 1 \end{bmatrix} \begin{bmatrix} \Delta x \\ x_0' \end{bmatrix} \rightarrow \begin{cases} x_1 = \Delta x \\ x_1' = x_0' - x_0/f_{beam} \end{cases}$$



Therefore, the scattering angle produced by beam-beam effect is:

$$\Delta x' = -\frac{\Delta x}{f_x} = -\frac{2r_e N_b}{\gamma\sigma_x(\sigma_x + \sigma_y)}\Delta x \approx -\frac{2r_e N_b}{\gamma\sigma_x} \times \frac{\Delta x}{\sigma_x}$$

$$\Delta y' = -\frac{\Delta y}{f_y} = -\frac{2r_e N_b}{\gamma\sigma_y(\sigma_x + \sigma_y)}\Delta y \approx \frac{2r_e N_b}{\gamma\sigma_x} \times \frac{\Delta y}{\sigma_y}$$

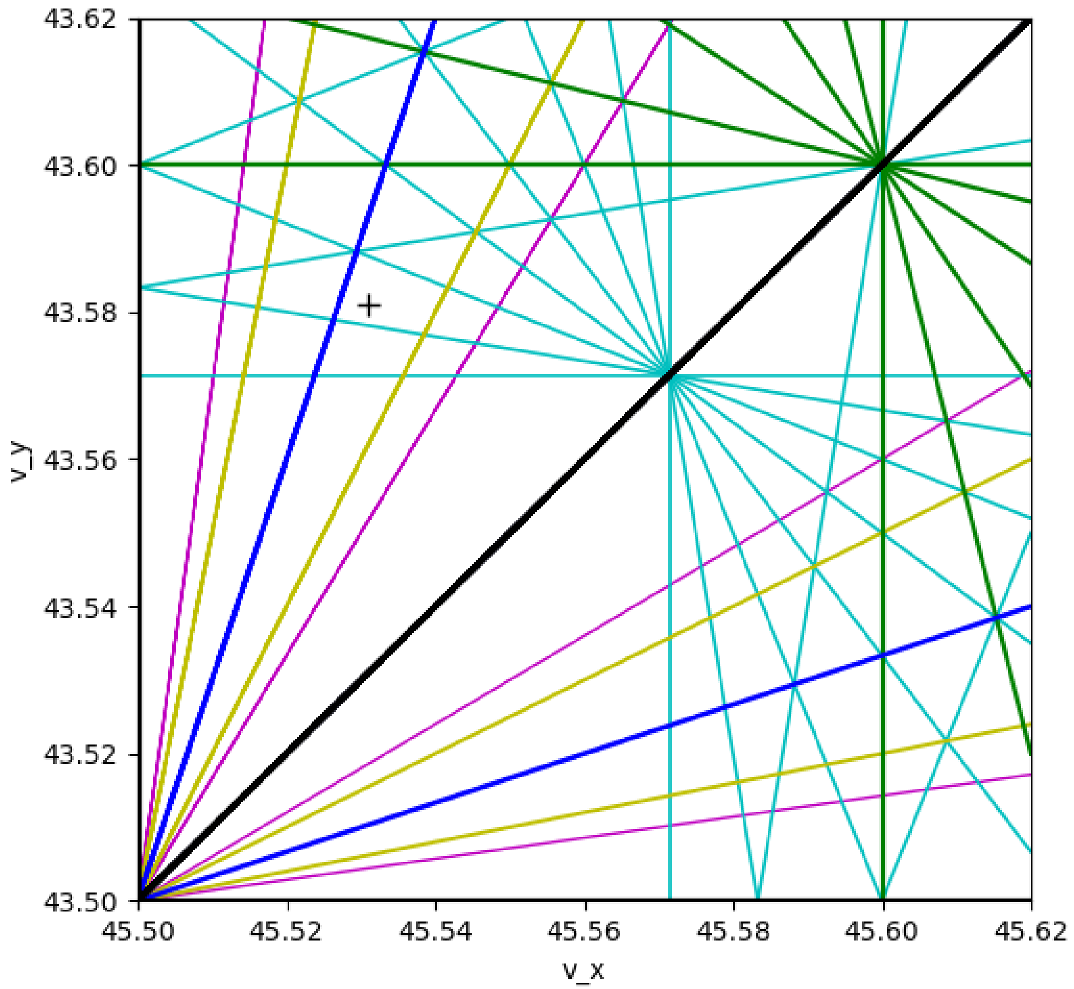
For HER(electron ring) and in the thin lens approximation, the beam-beam deflections is:

$$\Delta \boldsymbol{x}' = \frac{\boldsymbol{dx}}{\boldsymbol{ds}} = -\frac{\boldsymbol{4\pi}}{\boldsymbol{\beta}^*_{\boldsymbol{x}-}} \boldsymbol{\xi}_{\boldsymbol{x}-} \Delta \boldsymbol{x} = -\frac{\boldsymbol{2r_e N_+}}{\boldsymbol{\gamma_- \tilde{\sigma}^*_{x+}(\tilde{\sigma}^*_{x+} + \sigma^*_{y+})}} \Delta \boldsymbol{x}$$

$$\Delta \boldsymbol{y}' = \frac{\boldsymbol{dy}}{\boldsymbol{ds}} = -\frac{\boldsymbol{4\pi}}{\boldsymbol{\beta}^*_{\boldsymbol{y}-}} \boldsymbol{\xi}_{\boldsymbol{y}-} \Delta \boldsymbol{y} = -\frac{\boldsymbol{2r_e N_+}}{\boldsymbol{\gamma_- \sigma^*_{y+}(\tilde{\sigma}^*_{x+} + \sigma^*_{y+})}} \Delta \boldsymbol{y}$$

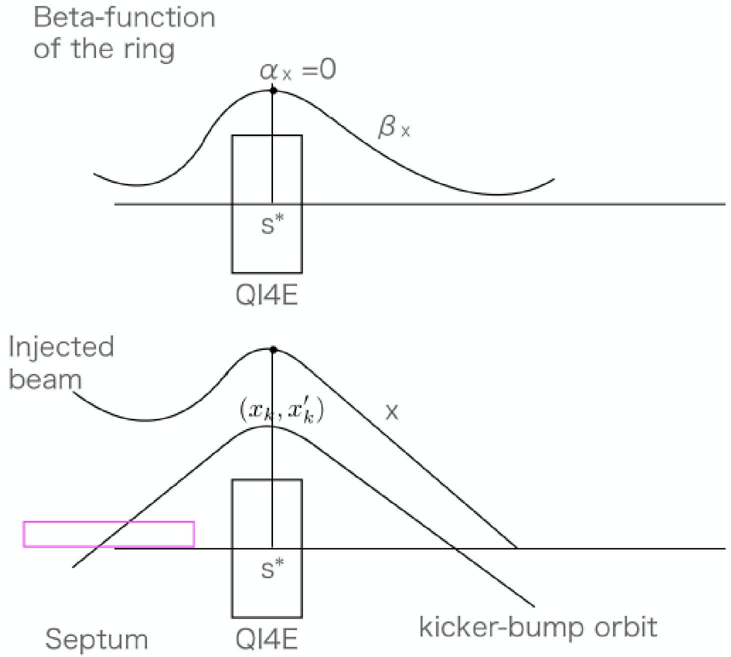
☐ Resonance lines:

color = ['k', 'k', 'r', 'b', 'g', 'y', 'c', 'm']

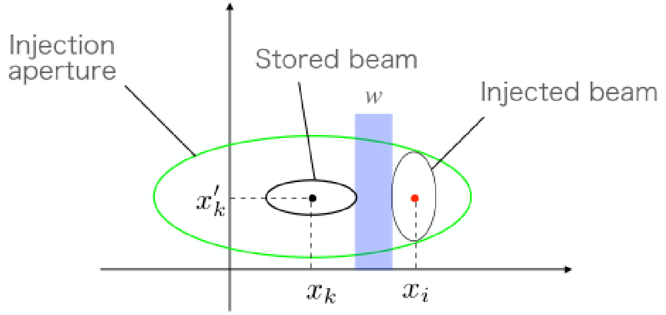


给定注入束的发射度，注入束的beta function决定注入束的尺寸，进而会影响注入所需的孔径。

■ 入射アパーチャの最適化



「入射点」の定義として都合上，セプタム直後でかつ $\alpha_x = 0$ となるような場所を選ぶことにする．そのような場所は通常，入射付近のQの中にある．すると入射点 s* における位相空間は下図のようになる．



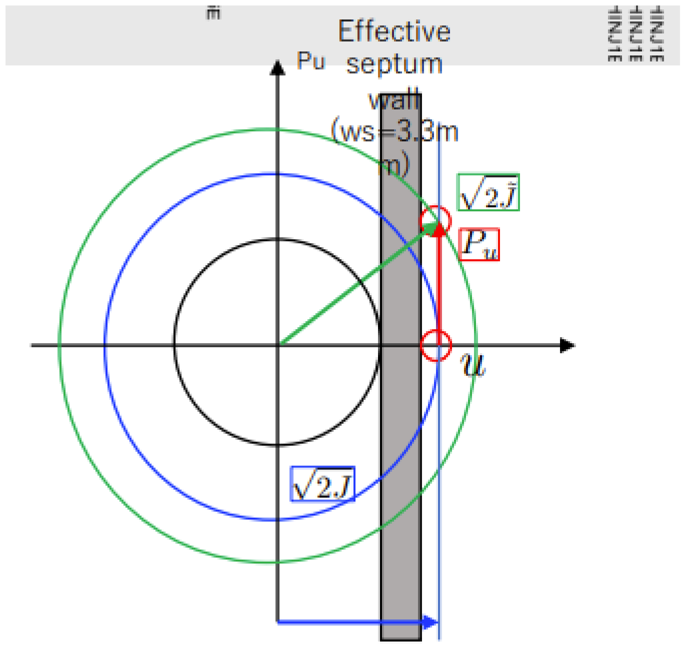
入射点における位相空間

入射ビーム中心の位置および傾きを (x_i, x'_i) とすると，入射ビームのマッチング条件から，

$$x'_i = x'_k$$

でなければならない．x_i はセプタム厚さ，蓄積ビームおよび入射ビームの大きさで決まる*)．

*) 必ずしもそうならない



$X = 3\sigma_r + w_s + 2.5\sigma_i$ =6.3 mm