B 介子非轻衰变和顶夸克稀有衰变中新物理效应的唯象研究

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JHEP 10 (2021) 235, F-M Cai, W-J Deng, X-Q Li, Y-D Yang

hep-ph/2202.08091, F-M Cai, S. Funatsu, X-Q Li, Y-D Yang

2022 年 9 月 16 日

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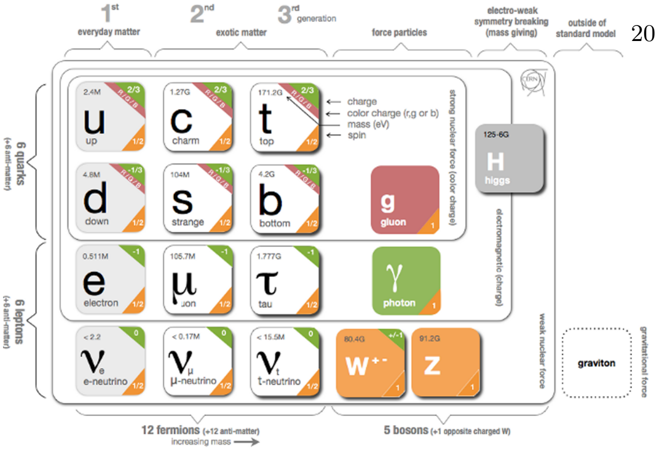
标准模型



2012 年 LHC 发现质量为 125 GeV 的 Higgs

$\Downarrow$

标准模型 (SM) ✓

? 暗物质是由什么组成的

? 为什么宇宙中的正物质比反物质多

? 中微子的质量为什么不为 0 且如此小

. . . . . .

$\Downarrow$

新物理 (NP)

研究方向

新物理的寻找 $\begin{cases} \text{直接寻找：LHC 尚未发现新物理信号} \\ \text{间接寻找：\textbf{味物理过程}} \end{cases}$

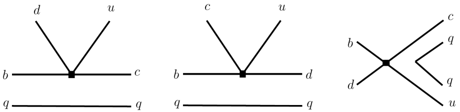
✠ 对比精确的实验测量与理论预言: 通过它们之间的偏差研究潜在的新物理效应；

✓ 在 QCD 因子化框架下研究 B 介子 Class-I 非轻衰变中的新物理效应 $\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^- \; (L = \{\pi, \rho, K^{(*)}\})$

✠ 味改变中性流 (FCNC) 过程: 树图阶没有贡献，圈图阶被 GIM 机制压低 ⇒ 对新物理比较敏感

✓ 在对齐双 Higgs 二重态模型 (A2HDM) 中研究顶夸克 FCNC 衰变 $t \to cg(g)$

研究动机



(a) Class-I (b) Class-II (c) Class-III

- B 介子非轻衰变的类型
  - Class-I 旁观者夸克进入到重的末态介子内 (如：$\bar{B}_0 \to D^+ K^-$)
  - Class-II 旁观者夸克进入到轻的末态介子内 (如：$\bar{B}^0_d \to D^0 \pi^0$)
  - Class-III 两种末态介子都可以作为发射粒子 (如：$\bar{B}^0_d \to \pi^+ \pi^-$)

- B 介子非轻衰变计算方法

  简单因子化; 推广因子化; 微扰 QCD; QCD 因子化

- QCD 因子化下 $B \to M_1 M_2$ QCD 因子化法✓

  M_1轻介子，M_2轻介子：顶角修正图；企鹅图；湮灭图；硬旁观者夸克图

  M_1重介子，M_2轻介子[Class-I]：顶角修正图 (无企鹅图、企鹅算符；其余贡献压低) 理论上干净!!

研究动机

- 实验测量精度的提高 $\begin{cases} \bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^- \text{分支比实验测量的精度提高} \\ \text{CKM 矩阵元及形状因子等参数的精度提高} \end{cases}$

分支比 (HFAG)	$\bar{B}^0 \to D^+\pi^-$	$\bar{B}^0 \to D^+K^-$	$\bar{B}^0_s \to D^+_s\pi^-$	$\bar{B}^0_s \to D^+_s K^-$
更新前2018 $[10^{-3}]$	2.65 ± 0.15	0.219 ± 0.013	3.03 ± 0.25	0.192 ± 0.022
更新后[1,2] $[10^{-3}]$	2.53 ± 0.08	0.208 ± 0.008	3.23 ± 0.18	0.241 ± 0.016

[1]: Belle Collaboration, Phys. Rev. D **105** (2022) 012003 [2]: LHCb Collaboration, Phys. Rev. D **104** (2021) 032005

- 理论与实验之间存在较大的偏差 $\bar{B}^0 \to D^+K^-$, $\bar{B}^0_s \to D^+_s\pi^-$ $(4-5\sigma)$

分支比 $[10^{-3}]$	$\bar{B}^0 \to D^+K^-$	$\bar{B}^0 \to D^{*+}K^-$	$\bar{B}^0_s \to D^+_s\pi^-$	$\bar{B}^0_s \to D^{*+}_s\pi^-$
理论值[3]2016	$0.301^{+0.032}_{-0.031}$	$0.259^{+0.039}_{-0.037}$	$4.39^{+1.36}_{-1.19}$	$2.24^{+0.56}_{-0.50}$
理论值[4]2020	0.326 ± 0.015	$0.327^{+0.039}_{-0.034}$	4.42 ± 0.21	$4.30^{+0.9}_{-0.8}$
HFAG2018	0.219 ± 0.013	0.204 ± 0.047	3.23 ± 0.18	$2.4^{+0.7}_{-0.6}$
偏差 $[\sigma]$	5.4	2.1	3.8	1.3

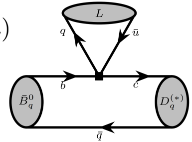
[3]: T. Huber, S. Kränkl and X-Q Li, JHEP **09** (2016) 112 [4]: M. Bordone et al., Eur. Phys. J. C 80 (2020) 951

$\Rightarrow$ **在 QCD 因子化框架下研究 $\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-$ $(L = \{\pi, \rho, K^{(*)}\})$ 中可能存在的新物理效应**

理论基础：弱有效哈密顿量

A. J. Buras, M. Misiak and J. Urban, Nucl. Phys. B **586** (2000) 397
A. J. Buras and J. Girrbach, JHEP **02** (2012) 143

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* \left\{ \sum_i \mathcal{C}_i(\mu) \mathcal{Q}_i^{\rm SM}(\mu) + \sum_{i,j} \left[C_i^{VLL}(\mu) \mathcal{Q}_i^{VLL}(\mu) + C_i^{VLR}(\mu) \mathcal{Q}_i^{VLR}(\mu) + C_j^{SLL}(\mu) \mathcal{Q}_j^{SLL}(\mu) + C_i^{SLR}(\mu) \mathcal{Q}_i^{SLR}(\mu) + (L \leftrightarrow R) \right] \right\} + \text{h.c.}$$



$$\mathcal{Q}_1^{VLL} = \bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\beta \, \bar{q}_\beta \gamma_\mu (1-\gamma_5) u_\alpha \,, \qquad \mathcal{Q}_2^{VLL} = \bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha \, \bar{q}_\beta \gamma_\mu (1-\gamma_5) u_\beta$$

$$\mathcal{Q}_1^{VLR} = \bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\beta \, \bar{q}_\beta \gamma_\mu (1+\gamma_5) u_\alpha \,, \qquad \mathcal{Q}_2^{VLR} = \bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha \, \bar{q}_\beta \gamma_\mu (1+\gamma_5) u_\beta$$

$$\mathcal{Q}_1^{SLL} = \bar{c}_\alpha (1-\gamma_5) b_\beta \, \bar{q}_\beta (1-\gamma_5) u_\alpha \,, \qquad \mathcal{Q}_2^{SLL} = \bar{c}_\alpha (1-\gamma_5) b_\alpha \, \bar{q}_\beta (1-\gamma_5) u_\beta$$

$$\mathcal{Q}_3^{SLL} = \bar{c}_\alpha \sigma^{\mu\nu} (1-\gamma_5) b_\beta \, \bar{q}_\beta \sigma_{\mu\nu} (1-\gamma_5) u_\alpha \,, \qquad \mathcal{Q}_4^{SLL} = \bar{c}_\alpha \sigma^{\mu\nu} (1-\gamma_5) b_\alpha \, \bar{q}_\beta \sigma_{\mu\nu} (1-\gamma_5) u_\beta$$

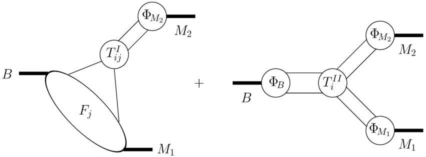
$$\mathcal{Q}_1^{SLR} = \bar{c}_\alpha (1-\gamma_5) b_\beta \, \bar{q}_\beta (1+\gamma_5) u_\alpha \,, \qquad \mathcal{Q}_2^{SLR} = \bar{c}_\alpha (1-\gamma_5) b_\alpha \, \bar{q}_\beta (1+\gamma_5) u_\beta$$

$$\mathcal{A}(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* \sum_i C_i(\mu) \langle M_1 M_2 | \mathcal{Q}_i | \bar{B} \rangle (\mu)$$

强子矩阵元的计算

- **简单因子化** $\mathcal{A}(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* \sum_i C_i(\mu) \langle M_1 M_2 | \mathcal{Q}_i | \bar{B} \rangle(\mu)$

$$\langle D^+\pi^- | Q | \bar{B}_d \rangle(\mu) \simeq \langle D^+ | (\bar{c}b)_{V-A} | \bar{B}_d \rangle \langle \pi^-(q) | (\bar{d}u)_{V-A} | 0 \rangle$$
$$= i f_\pi (m_B^2 - m_\pi^2) F_0^{B\to D}(m_\pi^2)$$



- **QCD 因子化**

$$\langle M_1 M_2 | \mathcal{Q}_i | \bar{B} \rangle = \overbrace{\sum_j F_j^{B\to M_1}(m_2^2) \int_0^1 du\, T^I_{ij}(u)\, \Phi_{M_2}(u)}^{M_1 = D^{(*)+}_{(s)},\, M_2 = L^-} + (M_1 \leftrightarrow M_2)$$
$$+ \int_0^1 d\xi du dv\, T_i^{II}(\xi, u, v)\, \Phi_B(\xi)\, \Phi_{M_1}(v)\, \Phi_{M_2}(u)$$

$F_j^{B\to M_1}$: $B \to M_1$ 的形状因子（实验提取; 非微扰计算）

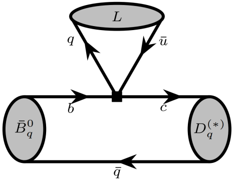
Φ_{M_1}、Φ_{M_2} 和 Φ_B: 光锥分布振幅（实验提取; 非微扰计算）

$T_i^{I,II}$: 算符 $\mathcal{Q}_i$ 的硬散射核（微扰计算 $\mathcal{O}(\alpha_s)$）

D. Fakirov and B. Stech, Nucl. Phys. B 133 (1978) 315
M. Bauer and B. Stech, Phys. Lett. B 152 (1985) 380-384
M. Beneke et al., Nucl. Phys. B **591** (2000) 313
M. Beneke and M. Neubert, Nucl. Phys. B **675** (2003) 333

解析结果：强子矩阵元

- **领头阶 (LO)**



$$\mathcal{Q}_2^{VLL} = \bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha \, \bar{q}_\beta \gamma_\mu (1-\gamma_5) u_\beta, \quad \mathcal{Q}_1^{VLL} = \bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\beta \, \bar{q}_\beta \gamma_\mu (1-\gamma_5) u_\alpha$$

$$\langle D^{(*)+}_{(s)}(p') L^-(q) | \mathcal{Q}_2^{VLL} | \bar{B}^0_{(s)}(p) \rangle (\mu) = \langle D^{(*)+}_{(s)}(p') | \gamma^\mu (1-\gamma_5) | \bar{B}^0_{(s)}(p) \rangle \langle L^-(q) | \gamma^\mu (1-\gamma_5) | 0 \rangle$$

$$\langle D^{(*)+}_{(s)}(p') L^-(q) | \mathcal{Q}_1^{VLL} | \bar{B}^0_{(s)}(p) \rangle (\mu) = \frac{1}{N_c} \langle D^{(*)+}_{(s)}(p') L^-(q) | \mathcal{Q}_2^{VLL} | \bar{B}^0_{(s)}(p) \rangle$$

解析结果：强子矩阵元

次领头阶 (NLO)

- $Q_2^{VLL} = \bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\alpha \, \bar{q}_\beta \gamma_\mu (1-\gamma_5) u_\beta$

$$\langle D^{(*)+}_{(s)}(p') L^-(q) | Q_2^{VLL} | \bar{B}^0_{(s)}(p) \rangle (\mu) = 0$$

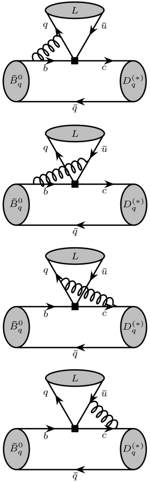
- $Q_1^{VLL} = \bar{c}_\alpha \gamma^\mu (1-\gamma_5) b_\beta \, \bar{q}_\beta \gamma_\mu (1-\gamma_5) u_\alpha$

$$\langle D^{(*)+}_{(s)}(p') L^-(q) | Q_1^{VLL} | \bar{B}^0_{(s)}(p) \rangle (\mu) = \pm i f_L \int_0^1 du \, \Phi_L(u)$$

$$\times \left[\langle D^+_{(s)} | \bar{c} \not{q} b | \bar{B}^0_{(s)} \rangle \cdot T^{VLL}(u,z) - \langle D^{*+}_{(s)} | \bar{c} \not{q} \gamma_5 b | \bar{B}^0_{(s)} \rangle \cdot T^{VLL}(u,-z) \right]$$

$$T^{VLL}(u,z) = \frac{\alpha_s}{4\pi} \frac{C_F}{N_c} \left[-6 \ln \frac{\mu^2}{m_b^2} - 18 + F^{VLL}(u,z) \right]$$

$$T^{VLL}, T^{VLR}, T^{SLL}, T^{SLR}, \cdots$$



解析结果：振幅

$$\mathcal{A}(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-) = A_{D^{(*)+}_{(s)} L^-}\, a_1(D^{(*)+}_{(s)} L^-)$$

$$A_{D^+_{(s)} P^-} = i\, \frac{G_F}{\sqrt{2}}\, V_{cb} V^*_{uq}\, f_{P^-}\, F_0^{B_{(s)} \to D_{(s)}}(m^2_{P^-})\, \left(m^2_{B_{(s)}} - m^2_{D^+_{(s)}}\right)$$

$$A_{D^{*+}_{(s)} P^-} = -i\, \frac{G_F}{\sqrt{2}}\, V_{cb} V^*_{uq}\, f_{P^-}\, A_0^{B_{(s)} \to D^*_{(s)}}(m^2_{P^-})\, 2m_{D^{*+}_{(s)}}\, (\epsilon^* \cdot p)$$

$$A_{D^+_{(s)} V^-} = -i\, \frac{G_F}{\sqrt{2}}\, V_{cb} V^*_{uq}\, f_{V^-}\, F_+^{B_{(s)} \to D_{(s)}}(m^2_{V^-})\, 2m_{V^-}\, (\eta^* \cdot p)$$

$$A_{D^{*+}_{(s)} V^-} = i\, \frac{G_F}{\sqrt{2}}\, V_{cb} V^*_{uq}\, f_{V^-}\, \frac{1}{2m_{D^{*+}_{(s)}}} \Bigg[\left(m^2_{B_{(s)}} - m^2_{D^{*+}_{(s)}} - m^2_{V^-}\right)\left(m_{B_{(s)}} + m_{D^{*+}_{(s)}}\right) \times A_1^{B_{(s)} \to D^*_{(s)}}(m^2_{V^-}) - \frac{4m^2_{B_{(s)}} |\vec{q}|^2}{m_{B_{(s)}} + m_{D^{*+}_{(s)}}}\, A_2^{B_{(s)} \to D^*_{(s)}}(m^2_{V^-}) \Bigg]$$

SM 中： $a_1(D^{*+}_{(s)} L^-) = C_2 + \frac{C_1}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c}\, C_1\, T^{VLL}$

SM 数值结果：$\mathcal{B}(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-)$

[1]: T. Huber, S. Kränkl and X-Q Li, JHEP **09** (2016) 112
[2]: M. Bordone et al., Eur. Phys. J. C 80 (2020) 951 [3]: PDG and HFAG

衰变道	LO	NLO	NNLO	[1]^{2016}	[2]^{2020}	实验值 [3]	偏差 (σ)
$\bar{B}^0 \to D^+\pi^-$	4.20	$4.45^{+0.25}_{-0.40}$	$4.58^{+0.22}_{-0.38}$	$3.93^{+0.43}_{-0.42}$		2.53 ± 0.08	5.7
$\bar{B}^0 \to D^{*+}\pi^-$	3.77	$4.00^{+0.29}_{-0.40}$	$4.13^{+0.27}_{-0.39}$	$3.45^{+0.53}_{-0.50}$		2.58 ± 0.13	4.4
$\bar{B}^0 \to D^+\rho^-$	10.98	$11.64^{+0.88}_{-1.18}$	$11.96^{+0.82}_{-1.15}$	$10.42^{+1.24}_{-1.20}$		7.6 ± 1.2	2.8
$\bar{B}^0 \to D^+K^-$	3.18	$3.37^{+0.17}_{-0.29}$	$3.48^{+0.14}_{-0.28}$	$3.01^{+0.32}_{-0.31}$	3.26 ± 0.15	2.08 ± 0.08	6.2
$\bar{B}^0 \to D^{*+}K^-$	2.82	$3.00^{+0.20}_{-0.29}$	$3.10^{+0.19}_{-0.28}$	$2.59^{+0.39}_{-0.37}$	$3.27^{+0.39}_{-0.34}$	2.04 ± 0.47	2.0
$\bar{B}^0 \to D^+K^{*-}$	5.48	$5.80^{+0.48}_{-0.62}$	$5.94^{+0.46}_{-0.61}$	$5.25^{+0.65}_{-0.63}$		4.6 ± 0.8	1.4
$\bar{B}^0_s \to D^+_s\pi^-$	4.23	$4.49^{+0.27}_{-0.41}$	$4.61^{+0.23}_{-0.39}$	$4.39^{+1.36}_{-1.19}$	4.42 ± 0.21	3.23 ± 0.18	3.8
$\bar{B}^0_s \to D^{*+}_s\pi^-$	3.51	$3.73^{+0.88}_{-0.84}$	$3.84^{+0.90}_{-0.85}$	$2.24^{+0.56}_{-0.50}$	$4.30^{+0.90}_{-0.80}$	$2.4^{+0.7}_{-0.6}$	1.3
$\bar{B}^0_s \to D^+_sK^-$	3.21	$3.41^{+0.18}_{-0.30}$	$3.52^{+0.15}_{-0.29}$	$3.34^{+1.04}_{-0.90}$		2.41 ± 0.16	4.1
$\bar{B}^0_s \to D^{*+}_sK^-$	2.62	$2.79^{+0.65}_{-0.61}$	$2.88^{+0.66}_{-0.63}$	$1.67^{+0.42}_{-0.37}$		1.63 ± 0.50	1.5

SM 数值结果：$|a_1(D_s^+ L^-)|$

[1]: T. Huber, S. Kränkl and X-Q Li, JHEP **09** (2016) 112

[2]: M. Bordone et al., Eur. Phys. J. C 80 (2020) 951 [3]: PDG and HFAG

$\lvert a_1(D^{(*)+}_{(s)} L^-)\rvert$	LO	NLO	NNLO	[1]^{2016}	[2]^{2020}	实验值 [3]
$\lvert a_1(D^+\pi^-)\rvert$	1.028	$1.059^{+0.017}_{-0.019}$	$1.073^{+0.005}_{-0.010}$	$1.073^{+0.012}_{-0.014}$	$1.0727^{+0.0125}_{-0.0140}$	0.86 ± 0.03
$\lvert a_1(D^{*+}\pi^-)\rvert$	1.028	$1.059^{+0.017}_{-0.019}$	$1.075^{+0.006}_{-0.011}$	$1.071^{+0.013}_{-0.014}$	$1.0713^{+0.0128}_{-0.0137}$	0.92 ± 0.04
$\lvert a_1(D^+\rho^-)\rvert$	1.028	$1.059^{+0.017}_{-0.019}$	$1.073^{+0.005}_{-0.010}$	$1.072^{+0.012}_{-0.014}$		0.92 ± 0.08
$\lvert a_1(D^+K^-)\rvert$	1.028	$1.059^{+0.018}_{-0.019}$	$1.075^{+0.007}_{-0.011}$	$1.070^{+0.010}_{-0.013}$	$1.0702^{+0.0101}_{-0.0128}$	0.90 ± 0.03
$\lvert a_1(D^{*+}K^-)\rvert$	1.028	$1.059^{+0.018}_{-0.019}$	$1.078^{+0.009}_{-0.012}$	$1.069^{+0.010}_{-0.013}$	$1.0687^{+0.0103}_{-0.0125}$	0.94 ± 0.11
$\lvert a_1(D^+K^{*-})\rvert$	1.028	$1.058^{+0.017}_{-0.019}$	$1.071^{+0.004}_{-0.009}$	$1.070^{+0.010}_{-0.013}$		1.02 ± 0.10
$\lvert a_1(D_s^+\pi^-)\rvert$	1.028	$1.059^{+0.017}_{-0.019}$	$1.073^{+0.005}_{-0.010}$	$1.073^{+0.012}_{-0.014}$	$1.0727^{+0.0125}_{-0.0140}$	0.90 ± 0.04
$\lvert a_1(D_s^{*+}\pi^-)\rvert$	1.028	$1.059^{+0.017}_{-0.019}$	$1.075^{+0.006}_{-0.011}$	$1.071^{+0.013}_{-0.014}$	$1.0713^{+0.0128}_{-0.0137}$	0.83 ± 0.13
$\lvert a_1(D_s^+K^-)\rvert$	1.028	$1.059^{+0.018}_{-0.019}$	$1.075^{+0.007}_{-0.011}$	$1.070^{+0.010}_{-0.013}$	$1.0702^{+0.0101}_{-0.0128}$	0.89 ± 0.05
$\lvert a_1(D_s^{*+}K^-)\rvert$	1.028	$1.059^{+0.018}_{-0.019}$	$1.078^{+0.009}_{-0.012}$	$1.069^{+0.010}_{-0.013}$	$1.0687^{+0.0103}_{-0.0125}$	0.79 ± 0.14

$$\mathcal{A}(\bar{B}^0 \to D^+ P^-) = i\,\frac{G_F}{\sqrt{2}}\, V_{cb} V_{uq}^*\, f_P\, F_0^{B\to D}(m_P^2)\,(m_B^2 - m_{D^+}^2)\, a_1(D^+ L^-)$$

SM 数值结果：$R^{(*)}_{(s)L}$

[1]: F-M. Cai, et al, JHEP **10** (2021) 235
[2]: T. Huber, S. Krankl and X-Q Li, JHEP **09** (2016) 112
[3]: M. Bordone et al., Eur. Phys. J. C 80 (2020) 951

形状因子	[1][2021]	[2][2016]
$F_0^{B\to D}(m_K^2)$	0.671 ± 0.011	0.670 ± 0.031
$A_0^{B\to D^*}(m_K^2)$	0.664 ± 0.018	0.654 ± 0.068
$F_0^{B_s\to D_s}(m_\pi^2)$	0.666 ± 0.012	0.700 ± 0.100
$A_0^{B_s\to D_s^*}(m_\pi^2)$	0.630 ± 0.069	0.520 ± 0.060

$$|V_{cb}| = \begin{cases} (42.41^{+0.40}_{-1.51}) \times 10^{-3}\ [1] \\ \text{CKMfitter 树图阶} \\ \\ (41.1 \pm 0.5) \times 10^{-3}\ [3] \\ \text{单举遍举求平均} \end{cases}$$

$$R^{(*)}_{(s)L} \equiv \frac{\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-)}{d\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} \ell^- \bar{\nu}_\ell)/dq^2\ |_{q^2=m_L^2}} = 6\pi^2\, |V_{uq}|^2\, f_L^2\, |a_1(D^{(*)+}_{(s)} L^-)|^2\, X_L^{(*)}$$

J. D. Bjorken, Nucl. Phys. B Proc. Suppl. 11 (1989) 325
M. Neubert, and B. Stech, Adv. Ser. Direct. High Energy Phys. 15 (1998) 294
M. Beneke, et al, Nucl.Phys.B 591 (2000) 313

SM 数值结果：$R^{(*)}_{(s)L}$

$R^{(*)}_{(s)L}$	LO	NLO	NNLO	实验值	偏差 (σ)
R_π	1.01	$1.07^{+0.04}_{-0.05}$	$1.10^{+0.03}_{-0.03}$	0.71 ± 0.04	7.8
R^*_π	1.00	$1.06^{+0.04}_{-0.05}$	$1.10^{+0.03}_{-0.04}$	0.80 ± 0.06	4.3
R_ρ	2.77	$2.94^{+0.19}_{-0.19}$	$3.02^{+0.17}_{-0.18}$	2.23 ± 0.37	1.9
R_K	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.59 ± 0.04	6.3
R^*_K	0.72	$0.76^{+0.03}_{-0.03}$	$0.79^{+0.01}_{-0.02}$	0.60 ± 0.14	1.3
R_{K^*}	1.41	$1.49^{+0.11}_{-0.11}$	$1.53^{+0.10}_{-0.10}$	1.38 ± 0.25	0.6
$R_{s\pi}$	1.01	$1.07^{+0.04}_{-0.05}$	$1.10^{+0.03}_{-0.03}$	0.77 ± 0.07	4.3
$R^*_{s\pi}$	1.00	$1.06^{+0.05}_{-0.05}$	$1.10^{+0.03}_{-0.04}$	$0.65^{+0.22}_{-0.19}$	2.2
R_{sK}	0.78	$0.82^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.58 ± 0.06	4.4
R^*_{sK}	0.71	$0.75^{+0.03}_{-0.03}$	$0.78^{+0.02}_{-0.02}$	0.42 ± 0.14	2.5

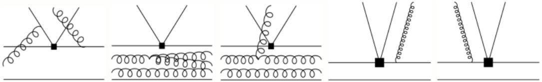
偏差高达 6-8 σ ! !

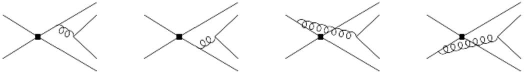
M. Beneke, G. Buchalla and M. Neubert, Nucl. Phys. B **591** (2000) 313
T. Huber, S. Kränkl and X-Q Li, JHEP **09** (2016) 112
M. Bordone et al., Eur. Phys. J. C 80 (2020) 951
M. Beneke et al., JHEP **10** (2021) 223

$$\langle M_1 M_2 | \mathcal{Q}_i | \bar{B} \rangle = \sum_j F_j^{B \to M_1}(m_2^2) \int_0^1 du\, T_{ij}^I(u)\, \Phi_{M_2}(u) \sim m_b^2 \Lambda_{\rm QCD}$$

- 高阶的 QCD 修正不能解释 B 介子非轻衰变中的偏差

- 高阶幂次修正的贡献是压低的 $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$
  - 旁观者硬散射图的贡献 $\sim \alpha_s\, m_b\, \Lambda^2_{\rm QCD}$
  - 软胶子非因子化贡献 $\sim m_b \Lambda^2_{\rm QCD}$
  - 高 twist 修正的贡献 $\sim \alpha_s\, (\Lambda_{\rm QCD}/m_b)^2$





- 湮灭图的贡献: 误差大 (端点发散); 正的贡献

$$\mathcal{A}(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-) = A_{D^{(*)+}_{(s)} L^-} \left[a_1(D^{(*)+}_{(s)} L^-) + b_1(D^{(*)+}_{(s)} L^-) \right]$$

衰变道	$\lvert b_1 \rvert$	$\left\lvert \frac{a_1}{(a_1+b_1)} \right\rvert$	$\left\lvert \frac{a_1}{(a_1+b_1)} \right\rvert^{\rm exp.}$
$\bar{B}^0 \to D^+ \pi^-$	$0.019^{+0.051}_{-0.051}$	0.982 ± 0.056	$1.042^{+0.019}_{-0.019}$
$\bar{B}^0 \to D^{*+} \pi^-$	$0.017^{+0.065}_{-0.064}$	0.984 ± 0.075	$1.016^{+0.031}_{-0.032}$
$\bar{B}^0_s \to D^+_s K^-$	$0.026^{+0.068}_{-0.068}$	0.976 ± 0.072	$1.003^{+0.021}_{-0.020}$
$\bar{B}^0_s \to D^{*+}_s K^-$	$0.025^{+0.095}_{-0.095}$	0.977 ± 0.106	$1.048^{+0.043}_{-0.046}$

$$\Rightarrow \frac{\mathcal{B}(\bar{B}^0 \to D^{(*)+} K^-)}{\mathcal{B}(\bar{B}^0 \to D^{(*)+} \pi^-)}$$

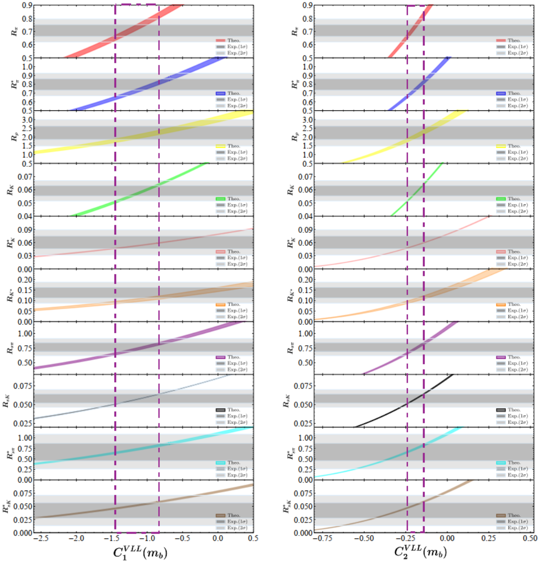
$$\Rightarrow \frac{\mathcal{B}(\bar{B}^0_s \to D^{(*)+}_s \pi^-)}{\mathcal{B}(\bar{B}^0_s \to D^{(*)+}_s K^-)}$$

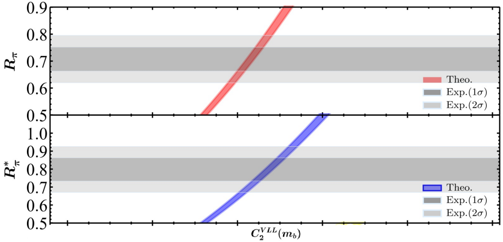
- QED 修正: $< 10^{-2}$

新物理效应!!

NP: 模型无关

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* \left\{ \sum_i \mathcal{C}_i(\mu)\mathcal{Q}_i(\mu) + \sum_{i,j} \left[C_i^{VLL}(\mu)\mathcal{Q}_i^{VLL}(\mu) + C_i^{VLR}(\mu)\mathcal{Q}_i^{VLR}(\mu) + C_j^{SLL}(\mu)\mathcal{Q}_j^{SLL}(\mu) + C_i^{SLR}(\mu)\mathcal{Q}_i^{SLR}(\mu) + (L \leftrightarrow R) \right] \right\} + \text{h.c.}$$





$$1\sigma \begin{cases} C_1^{VLL}(m_b) \in [-1.04, -1.00] \\ C_2^{VLL}(m_b) \in [-0.181, -0.174] \end{cases}$$

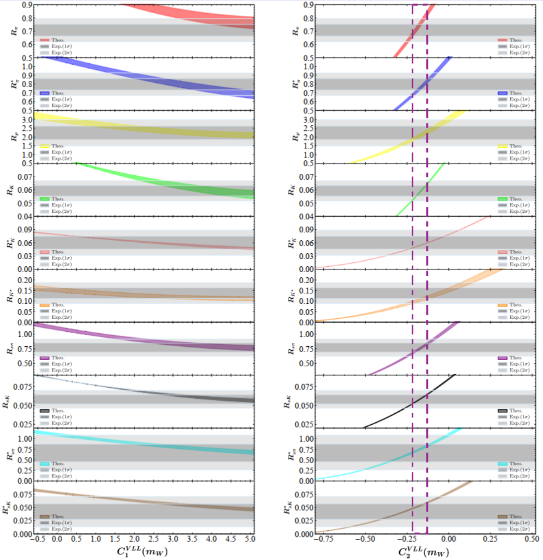
$$\gamma^\mu(1-\gamma_5) \otimes \gamma^\mu(1-\gamma_5) \quad \checkmark$$

$$2\sigma \begin{cases} C_1^{SRL}(m_b) \in [0.488, 1.02] \\ C_2^{SRL}(m_b) \in [0.140, 0.292] \\ C_1^{SRR}(m_b) \in [-0.877, -0.439] \\ C_2^{SRR}(m_b) \in [-0.292, -0.140] \end{cases}$$

$$(1+\gamma_5) \otimes (1 \pm \gamma_5) \quad \checkmark$$

NP: 模型无关

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* \left\{ \sum_i \mathcal{C}_i(\mu)\mathcal{Q}_i(\mu) + \sum_{i,j} \left[C_i^{VLL}(\mu)\mathcal{Q}_i^{VLL}(\mu) + C_i^{VLR}(\mu)\mathcal{Q}_i^{VLR}(\mu) + C_j^{SLL}(\mu)\mathcal{Q}_j^{SLL}(\mu) + C_i^{SLR}(\mu)\mathcal{Q}_i^{SLR}(\mu) + (L \leftrightarrow R) \right] \right\} + \text{h.c.}$$



SM：$\mathcal{C}_1(m_W) = 0$、$\mathcal{C}_2(m_W) = 1$

$$1\sigma \begin{cases} C_1^{VLL}(m_W) \in [3.12, 4.86] & \times \\ C_2^{VLL}(m_W) \in [-0.169, -0.162] \end{cases}$$

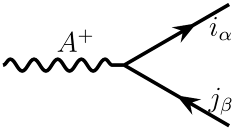
$\gamma^\mu(1-\gamma_5) \otimes \gamma^\mu(1-\gamma_5)$ ✓

$$2\sigma \begin{cases} C_1^{SRL}(m_W) \in [0.222, 0.465] \\ C_2^{SRL}(m_W) \in [0.068, 0.142] \\ C_1^{SRR}(m_W) \in [-0.352, -0.169] \\ C_2^{SRR}(m_W) \in [-0.132, -0.063] \end{cases}$$

$(1+\gamma_5) \otimes (1 \pm \gamma_5)$ ✓

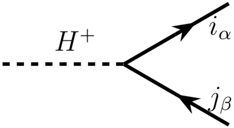
NP: 模型相关 $\mu_{\rm NP} \sim 1$ TeV

A. J. Buras and J. Girrbach, JHEP **02** (2012) 143



- A^+：无色的带电矢量玻色子

$i \frac{g_2}{\sqrt{2}} V_{ij} \gamma^\mu \delta_{\alpha\beta} \left[\Delta^L_{ij}(A) P_L + \Delta^R_{ij}(A) P_R \right]$



- H^+：无色的带电标量粒子

$i \frac{g_2}{\sqrt{2}} V_{ij} \delta_{\alpha\beta} \left[\Delta^L_{ij}(H) P_L + \Delta^R_{ij}(H) P_R \right]$

- 矢量玻色子、标量粒子？

$\gamma^\mu(1-\gamma_5) \otimes \gamma^\mu(1-\gamma_5)$、$(1+\gamma_5) \otimes (1 \pm \gamma_5)$

- 带电的？

中性粒子产生树图阶的 FCNC

- 无色的？

受 di-jet 共振态寻找的限制

模型相关：A^+ $m_A \sim 1$ TeV

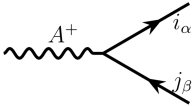
A. J. Buras and J. Girrbach, JHEP **02** (2012) 143

$$\mathcal{H}_{\rm eff}^{A^+} = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* \Big\{ \lambda_{LL}(A) \left[C_1^{VLL}(\mu) Q_1^{VLL}(\mu) + C_2^{VLL}(\mu) Q_2^{VLL}(\mu) \right]$$

$$+ \lambda_{LR}(A) \left[C_1^{VLR}(\mu) Q_1^{VLR}(\mu) + C_2^{VLR}(\mu) Q_2^{VLR}(\mu) \right] + (L \leftrightarrow R) \Big\} + \text{h.c.}$$

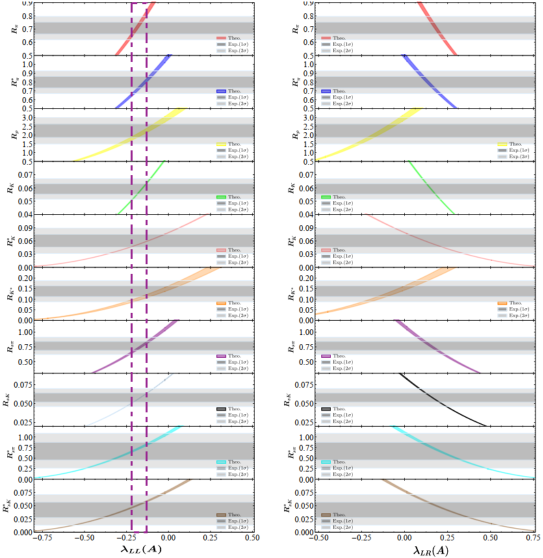
其中： $\lambda_{LL}(A) = \dfrac{m_W^2}{m_A^2} \Delta_{cb}^L(A) \left(\Delta_{uq}^L(A) \right)^*, \qquad \lambda_{LR}(A) = \dfrac{m_W^2}{m_A^2} \Delta_{cb}^L(A) \left(\Delta_{uq}^R(A) \right)^*$

- 方案一：只有一个非零耦合系数
- 方案二：左、右手对称
  $\lambda_{LL}(A) = \lambda_{LR}(A) = \lambda_{RR}(A) = \lambda_{RL}(A)$
- 方案三：左、右手反对称
  $\lambda_{LL}(A) = \lambda_{RR}(A) = -\lambda_{LR}(A) = -\lambda_{RL}(A)$



$$i \frac{g_2}{\sqrt{2}} V_{ij} \gamma^\mu \delta_{\alpha\beta} \left[\Delta_{ij}^L(A) P_L + \Delta_{ij}^R(A) P_R \right]$$

模型相关：$A^+{}_{m_A \sim 1\ \text{TeV}}$



$$\mathcal{H}_{\text{eff}}^{A^+} = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* \Big\{ \lambda_{LL}(A) \left[C_1^{VLL}(\mu) Q_1^{VLL}(\mu) + C_2^{VLL}(\mu) Q_2^{VLL}(\mu) \right]$$
$$+ \lambda_{LR}(A) \left[C_1^{VLR}(\mu) Q_1^{VLR}(\mu) + C_2^{VLR}(\mu) Q_2^{VLR}(\mu) \right] + (L \leftrightarrow R) \Big\} + \text{h.c.}$$

- 方案一

$1\sigma : \lambda_{LL}(A) \in [-0.162, -0.155]$ ✓

- 方案二 ×
- 方案三 ×

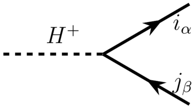
模型相关：H^+ $m_H \sim 1$ TeV

A. J. Buras and J. Girrbach, JHEP **02** (2012) 143

$$\mathcal{H}_{\rm eff}^{\rm H^+} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* \Big\{ \lambda_{LL}(H) \Big[C_1^{SLL}(\mu) Q_1^{SLL}(\mu) + C_2^{SLL}(\mu) Q_2^{SLL}(\mu) + C_3^{SLL}(\mu) Q_3^{SLL}(\mu) + C_4^{SLL}(\mu) Q_4^{SLL}(\mu) \Big] + \lambda_{LR}(H) \Big[C_1^{SLR}(\mu) Q_1^{SLR}(\mu) + C_2^{SLR}(\mu) Q_2^{SLR}(\mu) \Big] + (L \leftrightarrow R) \Big\} + \text{h.c.}$$

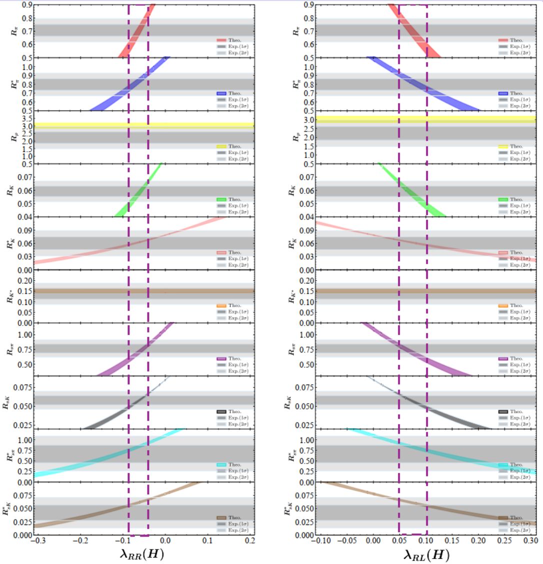
其中： $\lambda_{LL}(H) = \frac{m_W^2}{m_H^2} \Delta_{cb}^L(H) \left(\Delta_{uq}^L(H) \right)^*$ ， $\lambda_{LR}(H) = \frac{m_W^2}{m_H^2} \Delta_{cb}^L(H) \left(\Delta_{uq}^R(H) \right)^*$

- 方案一：只有一个非零耦合系数
- 方案二：左、右手对称
  $\lambda_{LL}(H) = \lambda_{LR}(H) = \lambda_{RR}(H) = \lambda_{RL}(H)$
- 方案三：左、右手反对称
  $\lambda_{LL}(H) = \lambda_{RR}(H) = -\lambda_{LR}(H) = -\lambda_{RL}(H)$



$$i \frac{g_2}{\sqrt{2}} V_{ij} \delta_{\alpha\beta} \left[\Delta_{ij}^L(H) P_L + \Delta_{ij}^R(H) P_R \right]$$

模型相关：H^+ $m_H \sim 1$ TeV



$$\mathcal{H}_{\rm eff}^{H^+} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* \Big\{\lambda_{LL}(H)\left[C_1^{SLL}(\mu)Q_1^{SLL}(\mu) + C_2^{SLL}(\mu)Q_2^{SLL}(\mu) + C_3^{SLL}(\mu)Q_3^{SLL}(\mu) + C_4^{SLL}(\mu)Q_4^{SLL}(\mu)\right] + \lambda_{LR}(H)\left[C_1^{SLR}(\mu)Q_1^{SLR}(\mu) + C_2^{SLR}(\mu)Q_2^{SLR}(\mu)\right] + (L \leftrightarrow R)\Big\} + \text{h.c.}$$

- 方案一

$$2\sigma \begin{cases} \lambda_{RR}(H) \in [-0.086, -0.041] & \checkmark \\ \lambda_{RL}(H) \in [0.047, 0.099] & \checkmark \end{cases}$$

- 方案二 ×
- 方案三 ×

小结

- 更新了 B 介子非轻衰变的分支比，发现它们与实验测量之间的偏差可高达 4-6σ；考虑新物理算符的贡献时，只有流结构为 $(V-A)(V-A)$ 和 $(S+P)(S\pm P)$ 的新物理算符可以在 2σ 内解释这种偏差，且 $(V-A)(V-A)$ 结构的新物理算符甚至可以在 1σ 范围内对理论与实验之间的偏差进行解释；当分别考虑带电矢量玻色子和带电标量粒子诱导的四夸克算符的贡献时，只有 $\lambda_{LL}(A)$ 和 $\lambda_{RR}(H)$ 或 $\lambda_{RL}(H)$ 分别可以在 1σ 和 2σ 的范围内同时解释理论与实验之间的偏差

研究动机

※ $t \to cg(g)$?

- 顶夸克作为 SM 中最重的粒子，对理解**电弱对称破缺机制**起着非常重要的作用
- SM 中的 FCNC 过程 {树图阶贡献不存在；圈图阶贡献被 GIM 机制压低} $\Rightarrow$ **对 NP 比较敏感**
-

两体衰变	$t \to cg$	$t \to c\gamma$	$t \to cZ$	$t \to ch$
SM	10^{-12}	10^{-14}	10^{-14}	10^{-15}
A2HDM	—	10^{-10}	10^{-11}	10^{-8}
MSSM	10^{-6}	10^{-6}	10^{-5}	10^{-5}
三体衰变	$t \to cgg$	$t \to c\gamma\gamma$	$t \to cZZ$	$t \to cu\bar{u}$
SM	10^{-9}	10^{-12}	10^{-15}	10^{-15}

实验上：

K. Y. Oyulmaz et al., Phys. Rev. D **99** (2019) 115023
H. Khanpour Nucl. Phys. B **958** (2020) 115141

$\mathcal{B}^{\text{HL-LHC}}(t \to cg) \sim \mathcal{O}(10^{-5})$ (3 ab^{-1})

$\mathcal{B}^{\text{FCC-hh}}(t \to cg) \gtrsim 10^{-8}$ (10 ab^{-1})

NP 中，分支比相对 SM 有不同程度的增强

$\mathcal{B}(t \to cg(g))$ 分支比最大、存在高阶主导效应

G. Abbas et al., JHEP **06** (2015) 005
G. Eilam et al., Phys. Rev. D **73** (2006) 053011
A. Cordero−Cid et al., Phys. Rev. D **73** (2006), 094005

$\Rightarrow$ **A2HDM 中：$t \to cg(g)$ 分支比？高阶主导效应？**

A2HDM: 标量势

※ **标量基** $\phi_a = e^{i\theta_a} \begin{bmatrix} \phi_a^+ \\ \frac{1}{\sqrt{2}}(v_a + \rho_a + i\eta_a) \end{bmatrix} \qquad a = 1, 2, \quad Y = \frac{1}{2}$

S. Davidson and H. E. Haber, Phys. Rev. D **72** (2005) 035004
H. E. Haber, Phys. Rev. D **74** (2006) 015018
H. E. Haber and D. O'Neil, Phys. Rev. D **83** (2011) 055017

$\Downarrow$ SU(2) 变换, Φ_1 获得真空期望值 $\Downarrow$

※ **Higgs 基** $\Phi_1 = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + S_1 + iG^0) \end{bmatrix}, \qquad \Phi_2 = \begin{bmatrix} H^+ \\ \frac{1}{\sqrt{2}}(S_2 + iS_3) \end{bmatrix}$

※ **Higgs 基下的标量势**

$$V = \mu_1\left(\Phi_1^\dagger\Phi_1\right) + \mu_2\left(\Phi_2^\dagger\Phi_2\right) + \left[\mu_3\left(\Phi_1^\dagger\Phi_2\right) + \mu_3^*\left(\Phi_2^\dagger\Phi_1\right)\right] + \lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2$$
$$+ \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) + \left[\left(\lambda_5\Phi_1^\dagger\Phi_2 + \lambda_6\Phi_1^\dagger\Phi_1 + \lambda_7\Phi_2^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right]$$

$\Downarrow$ 电弱对称破缺 $\Downarrow$

5 Higgs: $H^\pm, h, H, A$ $\quad\Rightarrow$ 提供丰富的唯象学研究!

A2HDM: Yukawa 相互作用

※ **Higgs 基下**

A. Pich and P. Tuzon, Phys. Rev. D **80** (2009) 091702
G. C. Branco et al., Phys. Rept **516** (2012) 1-102

$$\mathcal{L}_{Y,\mathrm{Higgs}} = -\frac{\sqrt{2}}{v}\left[\bar{Q}'_L(M'_d\Phi_1 + Y'_d\Phi_2)d'_R + \bar{Q}'_L(M'_u\tilde{\Phi}_1 + Y'_u\tilde{\Phi}_2)u'_R + \bar{L}'_L(M'_\ell\Phi_1 + Y'_\ell\Phi_2)\ell'_R\right] + \mathrm{h.c.}$$

M'_f 与 Y'_f 不能同时对角化: FCNC $\Rightarrow$ **对齐条件: $\boldsymbol{Y_{d,\ell} = \varsigma_{d,\ell} M_{d,\ell}, \quad Y_u = \varsigma_u^* M_u}$**

$\Downarrow$ $\mathcal{Z}_2$ 对称性

$\varsigma_{u,d,\ell}$ 是任意的复数: CP 破坏的新来源!

模型	u	d	ℓ
Type I	ϕ_2	ϕ_2	ϕ_2
Type II	ϕ_2	ϕ_1	ϕ_1
Type X	ϕ_2	ϕ_2	ϕ_1
Type Y	ϕ_2	ϕ_1	ϕ_2

$\Leftrightarrow$

模型	ς_d	ς_u	ς_ℓ
Type I	$\cot\beta$	$\cot\beta$	$\cot\beta$
Type II	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
Type X	$\cot\beta$	$\cot\beta$	$-\tan\beta$
Type Y	$-\tan\beta$	$\cot\beta$	$\cot\beta$

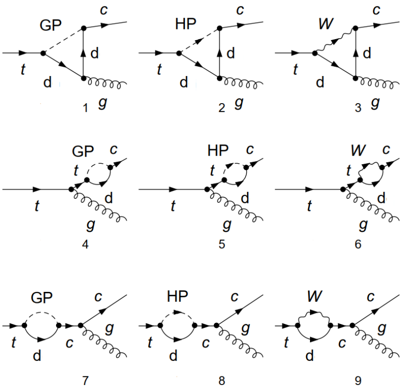
※ **质量基下**

$$\mathcal{L}_{Y,\mathrm{mass}} = -\frac{\sqrt{2}}{v}H^+\left\{\bar{u}\left[\varsigma_d V M_d P_R - \varsigma_u M_u^\dagger V P_L\right]d + \varsigma_\ell \bar{\nu} M_\ell P_R \ell\right\} - \frac{1}{v}\sum_{\varphi_i^0, f} y_f^{\varphi_i^0}\varphi_i^0\left[\bar{f} M_f P_R f\right] + \mathrm{h.c.}$$

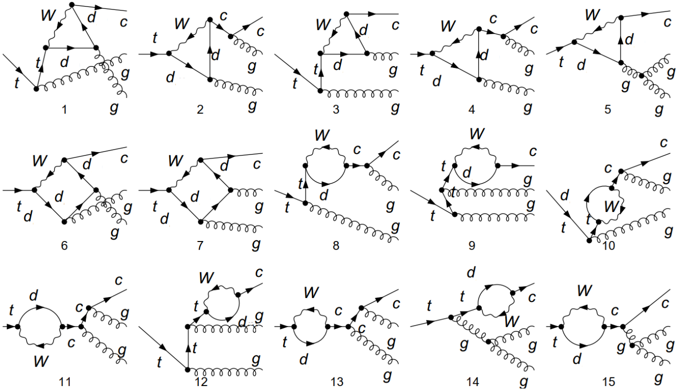
解析计算：费曼图

FeynRules，FeynArts，FormCalc

$t \to c\ g$



$t \to c\ g\ g$



9 diagrams $= 3(W^{\pm}) + 3(G^{\pm}) + 3(H^{\pm})$

45 diagrams $= 15(W^{\pm}) + 15(G^{\pm}) + 15(H^{\pm})$

解析计算：振幅 $(m_c = 0)$

$$A^{\text{vertex}}_{\text{t}\to\text{cgg}} = \frac{\alpha\alpha_s V^*_{tb} V_{cb}}{2m_W^2 \sin^2\theta_W t_{12} s_{23} t R_1 R_2} \Big[T_d t_{12} t R_2 (R_1 R_1^{\text{vertex}} + s_{23} R_2^{\text{vertex}}) + T_e t_{12} s_{23} R_1 (R_2 R_3^{\text{vertex}} + t R_4^{\text{vertex}}) + (T_d - T_e) s_{23} t R_1 R_2 R_5^{\text{vertex}} \Big],$$

$$A^{\text{box}}_{\text{t}\to\text{cgg}} = \frac{\alpha\alpha_s V^*_{tb} V_{cb}}{2m_W^2 \sin^2\theta_W t_{12} s_{23} t R_1 R_2} \Big[m_t (F_1 R_1^{\text{box}} + F_{12} R_5^{\text{box}} + 2F_{13} R_6^{\text{box}} + 2F_{14} R_7^{\text{box}}) - (F_3 R_2^{\text{box}} + F_4 R_3^{\text{box}} + F_5 R_4^{\text{box}} - F_{16} R_8^{\text{box}}) \Big],$$

$$A^{\text{self}}_{\text{t}\to\text{cgg}} = \frac{\alpha\alpha_s V^*_{tb} V_{cb}}{2m_W^2 \sin^2\theta_W t_{12} s_{23} t R_1 R_2} \Big[T_d t_{12} t R_2 (R_1 R_1^{\text{self}} + s_{23} m_b^2 R_2^{\text{self}} + R_6^{\text{self}}) + T_e t_{12} s_{23} R_1 (R_2 R_3^{\text{self}} + t m_b^2 R_4^{\text{self}} + R_7^{\text{self}}) + (T_d - T_e) s_{23} t R_1 R_2 R_5^{\text{self}} \Big].$$

解析计算：振幅

$$
\begin{aligned}
F_1 &= \bar{c}(k_2,0)\,P_R\,t(k_1,m_t), & F_2 &= \bar{c}(k_2,0)\,P_L\,t(k_1,m_t),\\
F_3 &= \bar{c}(k_2,0)\,P_R\not{\epsilon}^*(k_3)\,t(k_1,m_t), & F_4 &= \bar{c}(k_2,0)\,P_R\not{\epsilon}^*(k_4)\,t(k_1,m_t),\\
F_5 &= \bar{c}(k_2,0)\,P_R\not{k}_3\,t(k_1,m_t), & F_6 &= \bar{c}(k_2,0)\,P_L\not{\epsilon}^*(k_3)\,t(k_1,m_t),\\
F_7 &= \bar{c}(k_2,0)\,P_L\not{\epsilon}^*(k_4)\,t(k_1,m_t), & F_8 &= \bar{c}(k_2,0)\,P_L\not{k}_3\,t(k_1,m_t),\\
F_9 &= \bar{c}(k_2,0)\,P_L\not{\epsilon}^*(k_3)\not{\epsilon}^*(k_4)\,t(k_1,m_t) & F_{10} &= \bar{c}(k_2,0)\,P_L\not{\epsilon}^*(k_3)\not{k}_3\,t(k_1,m_t),\\
F_{11} &= \bar{c}(k_2,0)\,P_L\not{\epsilon}^*(k_4)\not{k}_3\,t(k_1,m_t), & F_{12} &= \bar{c}(k_2,0)\,P_R\not{\epsilon}^*(k_3)\not{\epsilon}^*(k_4)\,t(k_1,m_t),\\
F_{13} &= \bar{c}(k_2,0)\,P_R\not{\epsilon}^*(k_3)\not{k}_3\,t(k_1,m_t), & F_{14} &= \bar{c}(k_2,0)\,P_R\not{\epsilon}^*(k_4)\not{k}_3\,t(k_1,m_t),\\
F_{15} &= \bar{c}(k_2,0)\,P_L\not{\epsilon}^*(k_3)\not{\epsilon}^*(k_4)\not{k}_3\,t(k_1,m_t), & F_{16} &= \bar{c}(k_2,0)\,P_R\not{\epsilon}^*(k_3)\not{\epsilon}^*(k_4)\not{k}_3\,t(k_1,m_t).
\end{aligned}
$$

解析计算：分支比

FeynRules，FeynArts，FormCalc，LoopTools

※ **分支比** $\mathcal{B}(t \to cg(g)) \simeq \dfrac{\Gamma(t \to cg(g))}{\Gamma(t \to bW^+)}$ $\Gamma(t \to bW^+) = 1.35(1.47)\text{GeV}$

G. Eilam et al., Phys. Rev. D **73** (2006) 053011

※ **极化矢量求和**
- $\sum_\lambda \varepsilon^*_\mu(k,\lambda)\varepsilon_\nu(k,\lambda) = -g_{\mu\nu} +$ 引入鬼场的贡献 (9 个包含鬼场贡献的费曼图)
- $\sum_\lambda \varepsilon^*_\mu(k,\lambda)\varepsilon_\nu(k,\lambda) = -g_{\mu\nu} - \frac{\eta^2 k_\mu k_\nu}{(\eta\cdot k)^2} + \frac{\eta_\mu k_\nu + \eta_\nu k_\mu}{\eta\cdot k}$ ✓

※ **衰变宽度**

$$\Gamma(t \to cg) = \frac{\sqrt{\lambda(m_t, m_c, 0)}}{16\pi m_t^3} \sum_{\text{polarizations}} \frac{1}{3} \sum_{\text{colors}} \frac{1}{2} \sum_{\text{spins}} |A_{t\to cg}|^2$$

$$k_3^0 = \left[2Cm_t,\ \tfrac{m_t}{2}(1-2C)\right],$$
$$k_2^0 = \left[\tfrac{m_t}{2} - k_3^0 + Cm_t,\ \tfrac{m_t}{2}(1-2C)\right]$$

$$\Gamma(t \to cgg) = \frac{1}{64\pi^3 m_t} \sum_{\text{polarizations}} \frac{1}{3} \sum_{\text{colors}} \frac{1}{2} \sum_{\text{spins}} \int dk_3^0 \int dk_2^0 \times \frac{1}{2} |A_{t\to cgg}|^2$$

数值结果：参数空间 $\{m_{H^\pm}, \varsigma_u, \varsigma_d\}$

※ **对$m_{H^\pm}$**

$m_{H^\pm} < m_t \simeq 173\text{GeV}$

$\Rightarrow t \to bH^+$ 会被打开

$m_{H^\pm} > m_t \simeq 173$ GeV

$\Rightarrow m_{H^\pm} = [200, 600]$ GeV

※ **对ς_u，ς_d** (CP 守恒)

微扰范围: $\sqrt{2}\varsigma_f m_f / v \leq 1$

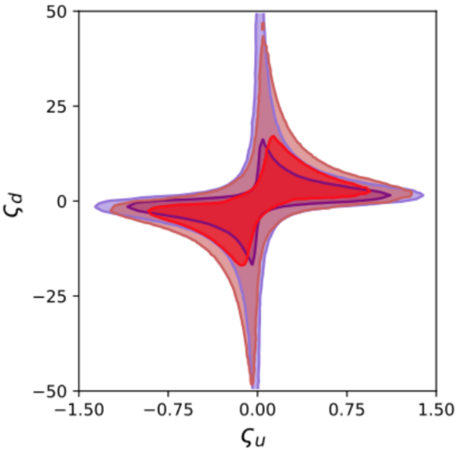
$\Rightarrow \varsigma_u \simeq [-1.5, 1.5]$, $\varsigma_d \simeq [-50, 50]$

全局拟合的限制:

味物理过程、电弱精确测量、高能对撞物理 $\cdots$

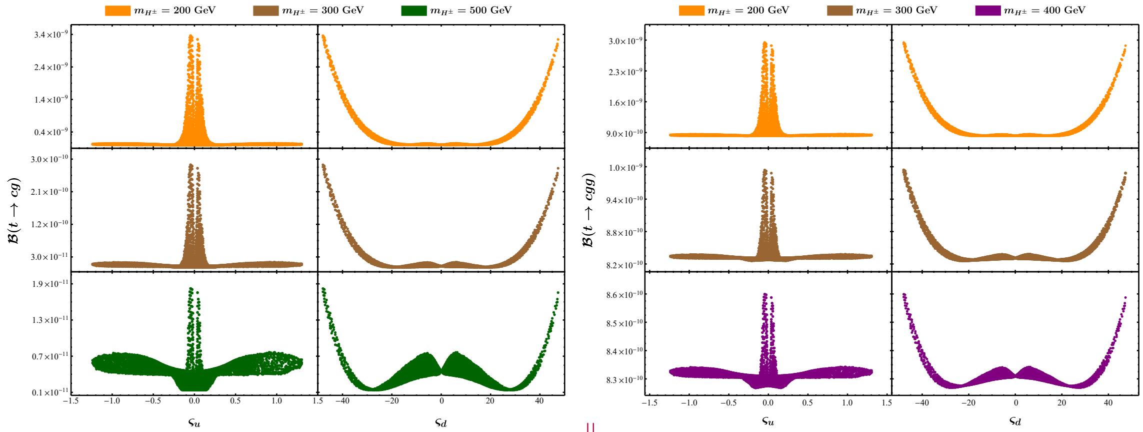
$\Rightarrow$ 95.5% C.L. 下对 ς_u 和 ς_d 的限制 (淡红色区域)

O. Eberhardt et al., JHEP **05** (2021) 005



数值结果：$\mathcal{B}(t \to cg(g))[\varsigma_{u,d}]$

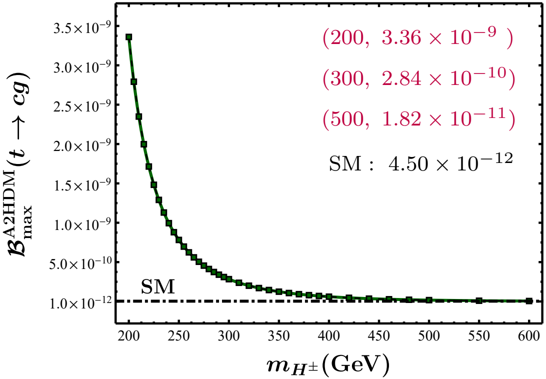
$(\varsigma_u, \varsigma_d) \approx (-0.047, -48)$ with $C = 0.01$

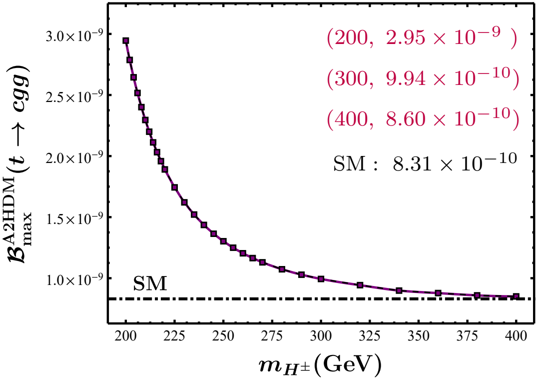


⇓

在 $(\varsigma_u, \varsigma_d) \approx (-0.047, -48)$ 处达到最大值

数值结果：$\mathcal{B}(t \to cg(g))[m_{H^\pm}]$ $(\varsigma_u, \varsigma_d) \approx (-0.047, -48)$ with $C = 0.01$





$\Downarrow$

$m_{H^\pm} > 600$ GeV 时, $\mathcal{B}(t \to cg) \simeq \mathcal{B}^{\rm SM}(t \to cg)$

$\Downarrow$

$m_{H^\pm} > 400$ GeV 时, $\mathcal{B}(t \to cgg) \simeq \mathcal{B}^{\rm SM}(t \to cgg)$

数值分析

$$\mathcal{B}^{\rm SM}(t \to cg) = 4.50 \times 10^{-12} \qquad \mathcal{B}^{\rm A2HDM}(t \to cg) = 3.39 \times 10^{-9}$$

$$\mathcal{B}^{\rm SM}(t \to cgg) = 8.31 \times 10^{-10} \qquad \mathcal{B}^{\rm A2HDM}(t \to cgg) = 2.95 \times 10^{-9}$$

※ SM 中，$\frac{\mathcal{B}(t\to cgg)}{\mathcal{B}(t\to cg)} \simeq 2 \times 10^2 \quad \Rightarrow$ 高阶主导效应

※ A2HDM 中，$\frac{\mathcal{B}(t\to cgg)}{\mathcal{B}(t\to cg)} \simeq 1 \quad \Rightarrow$ 高阶主导效应消失!

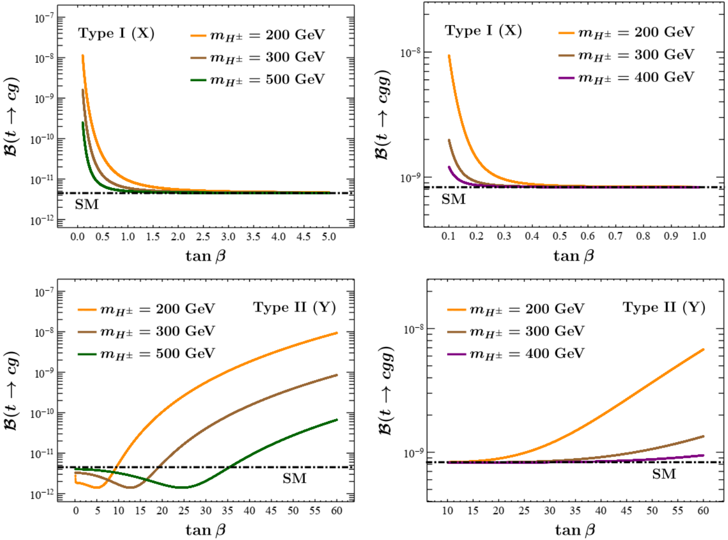
☺ 对顶点 t-c-$g^*(k)(m_c = 0)$ 而言:

$$\Gamma_\mu = F_1(k^2)(k^2\gamma_\mu - k_\mu \not{k})P_L - iF_2(k^2)m_t\sigma_{\mu\nu}k_\nu P_R$$

W.-S. Hou, Nucl. Phys. B **308** (1988) 561

实胶子时 $\begin{cases} t \to cg : F_2(k^2) \\ t \to cgg : F_1(k^2),\ F_2(k^2) \end{cases} \Rightarrow \begin{cases} F_1(k^2) > F_2(k^2) : \mathcal{B}^{\rm SM}(t \to cgg) > \mathcal{B}^{\rm SM}(t \to cg) \\ F_2(k^2) > F_1(k^2) : \mathcal{B}^{\rm A2HDM}(t \to cgg) \simeq \mathcal{B}^{\rm A2HDM}(t \to cg) \end{cases}$

2HDM 数值结果：$\mathcal{B}(t \to cg(g))[\tan\beta]$



$\Leftarrow$ Type I (X): $\varsigma_d = \varsigma_u = 1/\tan\beta$

实验上:

$\mathcal{B}^{\text{HL-LHC}}(t \to cg) \sim \mathcal{O}(10^{-5})$ (3 ab^{-1})

$\mathcal{B}^{\text{FCC-hh}}(t \to cg) \gtrsim 10^{-8}$ (10 ab^{-1})

$\mathcal{B}(t \to cg(g)) < 10^{-8}$: 达不到实验灵敏度

$\Leftarrow$ Type II (Y): $\varsigma_d = -1/\varsigma_u = -\tan\beta$

2HDM 数值结果：考虑对参数的限制

※ 对 **Type I (X)**: $\varsigma_d = \varsigma_u = 1/\tan\beta$

$(m_{H^\pm}, \tan\beta)$	(200, 2.5)	(300, 2)	(500, 1.5)
$\mathcal{B}(t \to cg)$	5.75×10^{-12}	5.13×10^{-12}	4.85×10^{-12}
$\mathcal{B}(t \to cgg)$	8.32×10^{-10}	8.31×10^{-10}	8.31×10^{-10}

$\mathcal{B}^{\text{Type I (X)}}(t \to cg(g)) \simeq \mathcal{B}^{\text{SM}}(t \to cg(g))$: 无明显增强

实验上:

$\mathcal{B}^{\text{HL-LHC}}(t \to cg) \sim \mathcal{O}(10^{-5})$ (3 ab^{-1})

$\mathcal{B}^{\text{FCC-hh}}(t \to cg) \gtrsim 10^{-8}$ (10 ab^{-1})

※ 对 **Type II (Y)**: $\varsigma_d = -1/\varsigma_u = -\tan\beta$

$\varsigma_u\varsigma_d < 0$ 被严格限制，且 $m_{H^\pm} \lesssim 600$ GeV 被实验排除

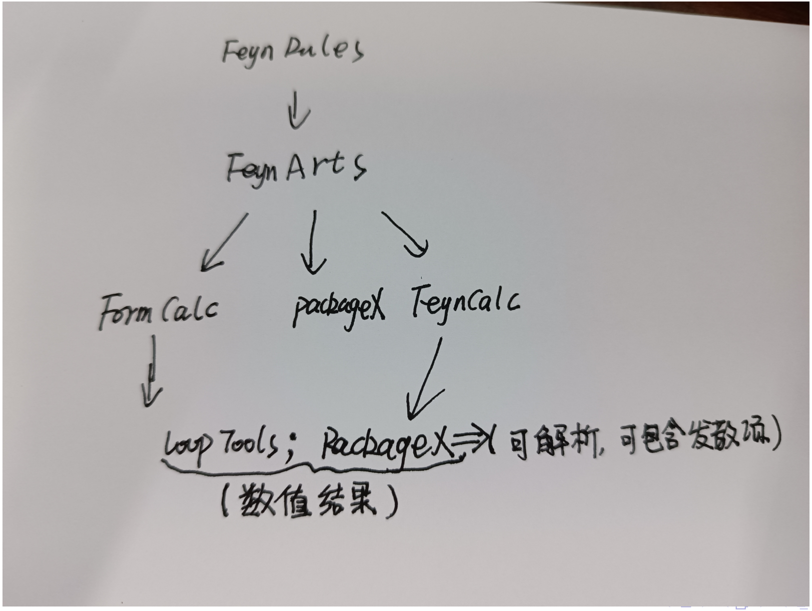
A. Arbey et al., Eur. Phys. J. C 78 (2018), 182
J. Haller et al., Eur. Phys. J. C 78 (2018), 675
A. G. Akeroyd et al., Eur. Phys. J. C 77 (2017), 276
M. Jung, A. Pich and P. Tuzon, JHEP **11** (2010) 003

小结

※ 计算了 SM 和 (A)2HDM 中 $t \to cg(g)$ 的分支比，发现 SM 中 $\mathcal{B}(t \to cgg)$ 比 $\mathcal{B}(t \to cg)$ 高两个数量级，验证了其中存在的高阶主导效应；而 A2HDM 中 $\mathcal{B}(t \to cg)$ 和 $\mathcal{B}(t \to cgg)$ 处在同一数量级，高阶主导效应消失；并且 (A)2HDM 中 $t \to cg(g)$ 的分支比都不能达到将来实验的灵敏度

附录

- ※ *FeynRules*: A Mathematica package to calculate Feynman rules. [https://feynrules.irmp.ucl.ac.be]
- ※ *FeynArts*: A Mathematica package for the generation and visualization of Feynman diagrams and amplitudes. [https://feynarts.de]
- ※ *FeynCalc*: A Mathematica package for symbolic evaluation of Feynman diagrams and algebraic calculations in quantum field theory and elementary particle physics. [https://feyncalc.github.io]
- ※ *FormCalc*: A Mathematica package for the calculation of tree-level and one-loop Feynman diagrams. [https://feynarts.de/formcalc]
- ※ *PackageX*: A Mathematica package for the analytic calculation and symbolic manipulation of one-loop Feynman integrals.
- ※ *LoopTools*: A package for evaluation of scalar and tensor one-loop integrals. [https://feynarts.de/looptools]


Feyn Rules
Feyn Arts
Form Calc
packageX
FeynCalc
LoopTools; PackageX ⇒ (可解析，可包含发散项)
(数值结果)

谢　谢　！