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Weak radiative hyperon decays in covariant baryon chiral perturbation theory

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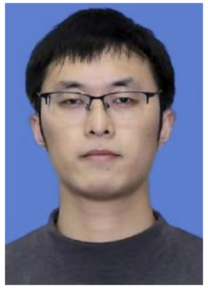

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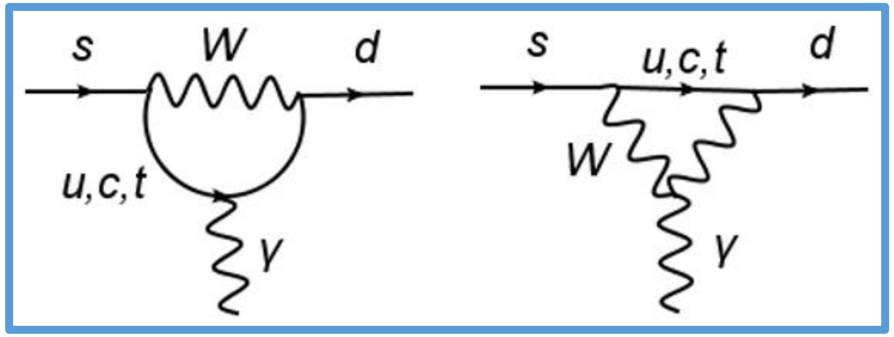
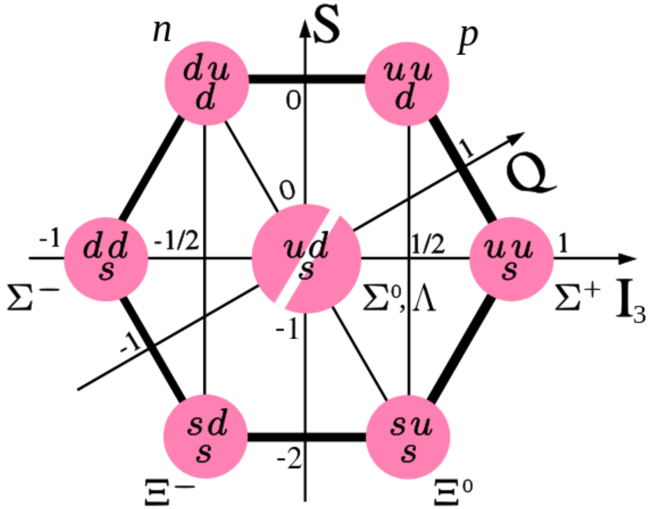
What are weak radiative hyperon decays

- **Weak radiative hyperon decays (WRHDs)** are interesting physical processes **involving the electromagnetic, weak, and strong interactions**
- $s \to d\,\gamma$ transitions in the quark level





- Six WRHDs channels of the ground-state octet baryons

$$\Lambda \to n\gamma \qquad \Sigma^0 \to n\gamma \qquad \Xi^0 \to \Sigma^0\gamma$$
$$\Sigma^+ \to p\gamma \qquad \Xi^0 \to \Lambda\gamma \qquad \Xi^- \to \Sigma^-\gamma$$

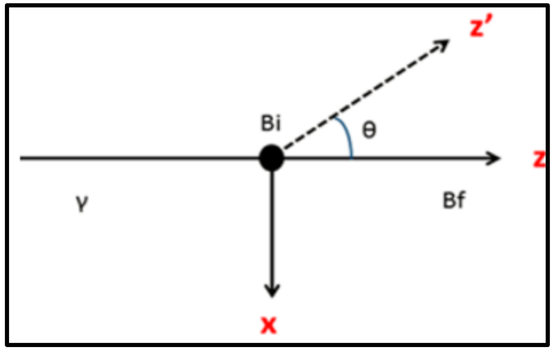
What are weak radiative hyperon decays

- The effective Lagrangian describing the $B_i \to B_f\, \gamma$ WRHDs

$$\mathcal{L} = \frac{eG_F}{2} \bar{B}_f (a + b\gamma_5) \sigma^{\mu\nu} B_i F_{\mu\nu},$$

a: partity-conserving amplitude **b: partity-violating amplitude**

- **Observables** for the WRHDs



$$\frac{d\Gamma}{d\cos\theta} = \frac{e^2 G_F^2}{\pi}(|a|^2 + |b|^2)[1 + \frac{2\mathrm{Re}(ab^*)}{|a|^2 + |b|^2}\cos\theta] \cdot |\vec{k}|^3,$$

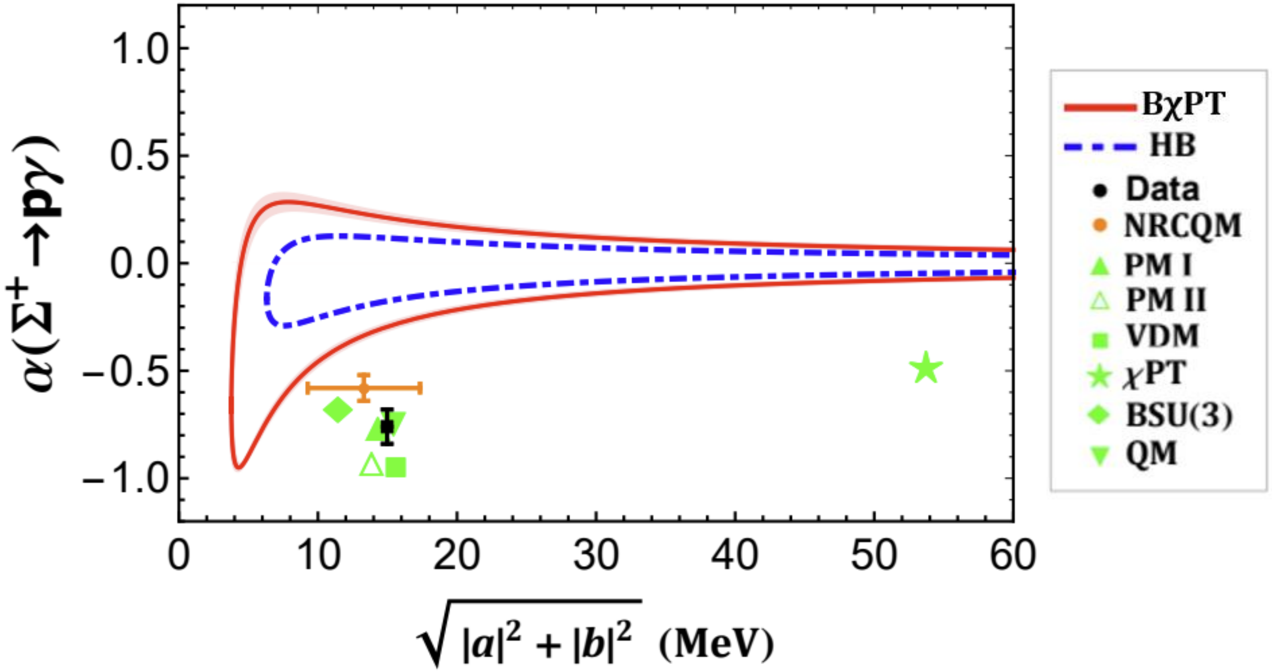
$$\alpha_\gamma = \frac{2\mathrm{Re}(ab^*)}{|a|^2 + |b|^2}, \qquad \Gamma = \frac{e^2 G_F^2}{\pi}(|a|^2 + |b|^2) \cdot |\vec{k}|^3, \qquad |\vec{k}| = \frac{m_i^2 - m_f^2}{2m_i}$$

α_γ: **the asymmetry parameter**

θ: **the angle between the spin of the initial baryon B_i and the 3-momentum $\vec{k}$ of the final baryon B_f**

Why to study WRHDs: the WRHDs puzzle

- The experimental measurement of **a surprisingly large asymmetry for $\Sigma^+ \to p\,\gamma$ decay** [PR188, 2077 (1969)], contradicting **Hara's theorem based on gauge invariance, CP conservation, and U-spin symmetry** [Y. Hara, PRL12, 378 (1964)]

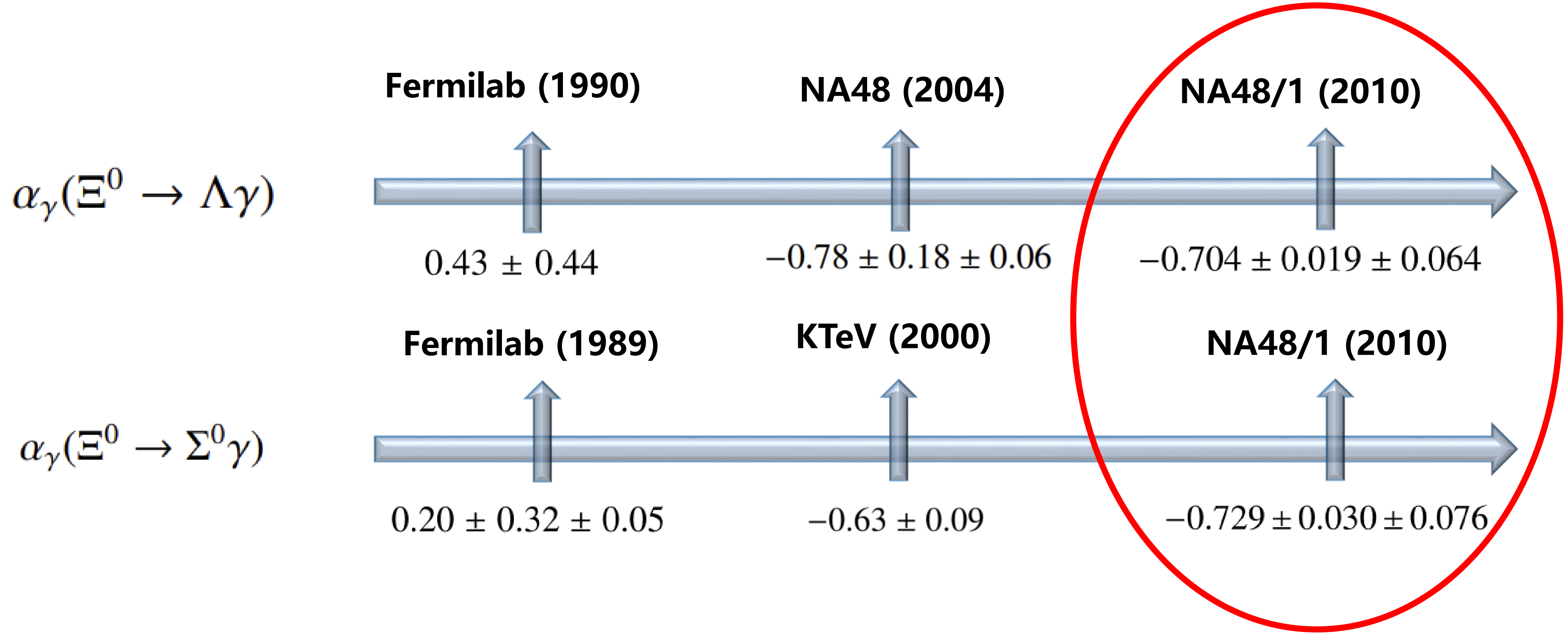


Data: PDG, PTEP 2020, 083C01(2020)
HB: E. E. Jenkins et al, NPB 397, 84 (1993)
BχPT: H. Neufeld, Nucl. Phys. B 402, 166 (1993)
NRCQM: Qiang Zhao et al, CPC45, 013101 (2021)
PM1: M. B. Gavela et al, PLB 101, 417 (1981)
PM2: G. Nardulli, PLB 190, 187 (1987)
VDM: P. Zenczykowski, PRD 44, 1485 (1991)
χPT: B. Borasoy et al, PRD 59, 054019 (1999)
BSU(3): P. Zenczykowski, PRD 73, 076005 (2006)
QM: E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)

- Although some models predictions are in agreement with the measurement of the large asymmetry for the $\Sigma^+ \to p\,\gamma$ decay, **they explain poorly the data of other WRHDs**

Why to study WRHDs: experimentally challenging

- **Significant changes** in the asymmetry parameters of $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^0 \to \Lambda\gamma$



Why to study WRHDs--$\Lambda \to n\gamma$

New BESIII measurement for the $\Lambda \to n\,\gamma$ decay (2206.10791)

Decay Mode	$\Lambda \to n\gamma$	$\bar{\Lambda} \to \bar{n}\gamma$
$N_{\rm ST}$ $(\times 10^3)$	6853.2 ± 2.6	7036.2 ± 2.7
$\varepsilon_{\rm ST}$ (%)	51.13 ± 0.01	52.53 ± 0.01
$N_{\rm DT}$	723 ± 40	498 ± 41
$\varepsilon_{\rm DT}$ (%)	6.58 ± 0.04	4.32 ± 0.03
BF $(\times 10^{-3})$	$0.820\pm0.045\pm0.066$	$0.862\pm0.071\pm0.084$
	$0.832\pm0.038\pm0.054$ (combined)	
α_γ	$-0.13\pm0.13\pm0.03$	$0.21\pm0.15\pm0.06$
	$-0.16\pm0.10\pm0.05$ (combined)	

PDG2022

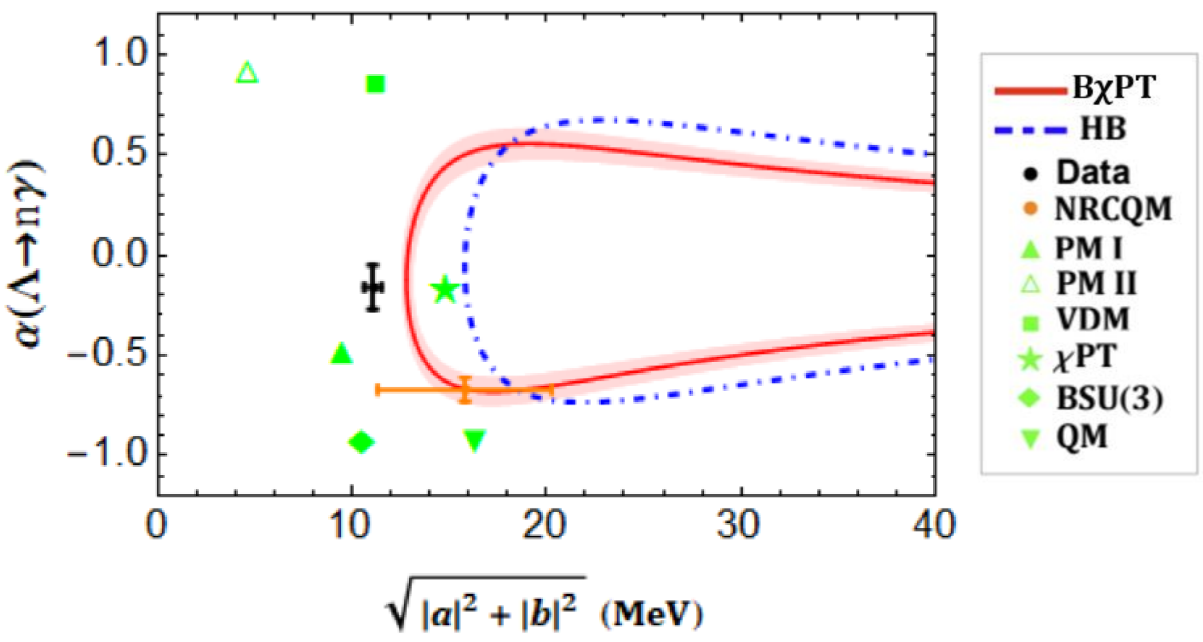
$\Gamma(n\gamma)/\Gamma_{\rm total}$ Γ_3/Γ

VALUE (units 10^{-3})	EVTS	DOCUMENT ID		TECN	COMMENT
1.75±0.15 OUR FIT					
1.75±0.15	1816	LARSON	93	SPEC	$K^- p$ at rest
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$1.78\pm0.24^{+0.14}_{-0.16}$	287	NOBLE	92	SPEC	See LARSON 93

- The branching fraction is only **about one half** of the current PDG average
- The asymmetry parameter α_γ **is determined for the first time**

Why to study WRHDs--$\Lambda \to n\gamma$

- **None of the existing predictions can describe** the new BESIII measurement for the $\Lambda \to n\gamma$ decay



Data: *BESIII, 2206.10791*

HB χPT: *E. E. Jenkins et al, NPB 397, 84 (1993)*

BχPT: *H. Neufeld, Nucl. Phys. B 402, 166 (1993)*

NRCQM: *Qiang Zhao et al, CPC45, 013101 (2021)*

PM1: *M. B. Gavela et al, PLB 101, 417 (1981)*

PM2: *G. Nardulli, PLB 190, 187 (1987)*

VDM: *P. Zenczykowski, PRD 44, 1485 (1991)*

χPT: *B. Borasoy et al, PRD 59, 054019 (1999)*

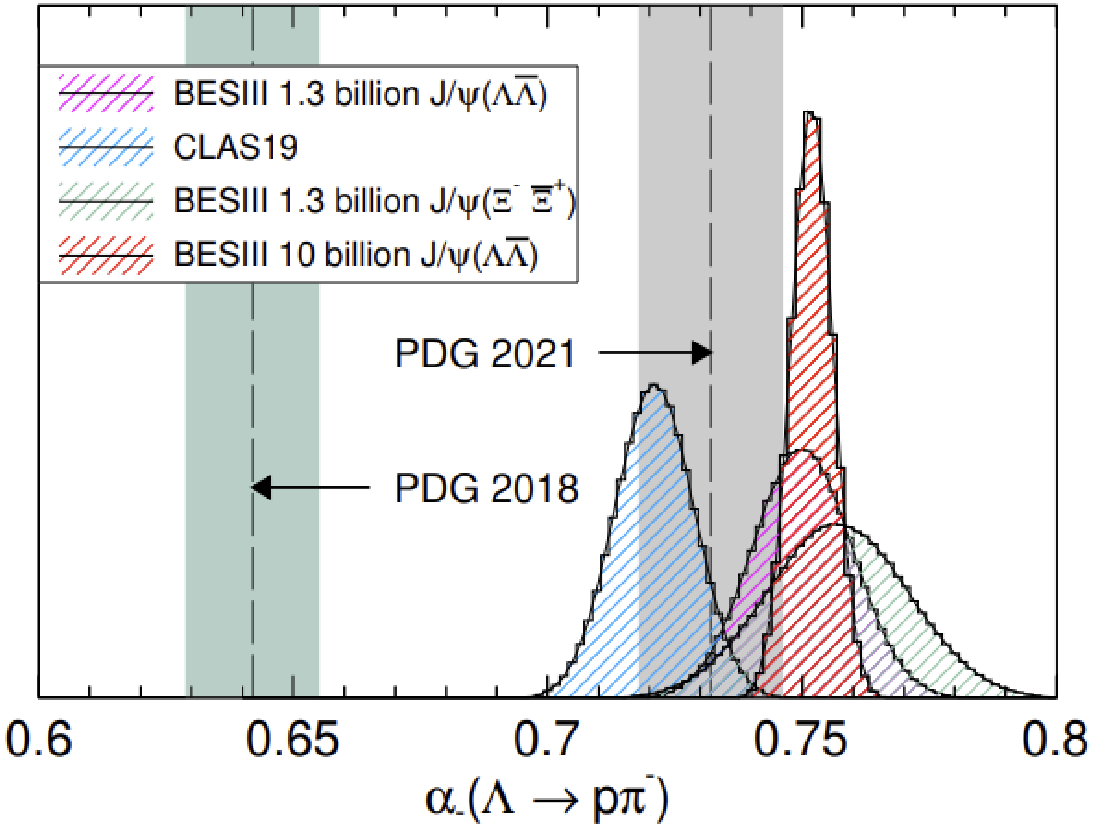
BSU(3): *P. Zenczykowski, PRD 73, 076005 (2006)*

QM: *E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)*

Why to study WRHDs

□ New BESIII and CLAS data for the hyperon non-leptonic decays

BESIII: Nature Phys. 15, 631 (2019) CLAS: PRL123,182301 (2019)
BESIII: Nature 606, 64 (2022) BESIII: PRL129,131801 (2022)



- Definition of decay parameter for the $\Lambda \to p\,\pi^-$ decay

$$\mathcal{M}(B_i \to B_f \pi) = iG_F m_\pi^2 \bar{B}_f (A_S - A_P \gamma_5) B_i$$

$$\alpha_\pi = \frac{2\mathrm{Re}\,(s \cdot p)}{|s|^2 + |p|^2} \qquad s = A_S \qquad p = A_P |\vec{q}|/(E_f + m_f)$$

- Featured by a **larger statistics** and a **small uncertainty** and very different from previous PDG average

- A significant change for the baryon decay parameter of $\Lambda \to p\,\pi^-$ may **greatly affect the values of LECs hD, hF and hyperon non-leptonic decay amplitudes as inputs WRHDs**

Why to study WRHDs—theoretical tools

- Theoretically, **two phenomenological models** are able to explain the current experimental data of WRHDs at least qualitatively **except for the** $\Lambda \to n\gamma$ **decay**

  E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)
  P. Zenczykowski, PRD 73, 076005 (2006)

- **Chiral perturbation theory (χPT)** studies on the WRHDs

  B. Borasoy et al, PRD 59, 054019 (1999) (Tree level)

  E. E. Jenkins et al, NPB397, 84 (1993)
  J. W. Bos et al, PRD 51, 6308 (1995)
  J. W. Bos et al, PRD 54, 3321 (1996)
  J. W. Bos, et al, PRD 57, 4101 (1998)
  (Loop level in the heavy baryon formulation)

  H. Neufeld, NPB 402, 166 (1993) (Loop level in the covariant formulation)

Our purpose

Our goal is to study the WRHDs in covariant baryon chiral perturbation theory (BχPT) with the extended-on-mass-shell (EOMS) renormalization scheme

- **The work in the BχPT** *H. Neufeld, NPB 402, 166 (1993)*

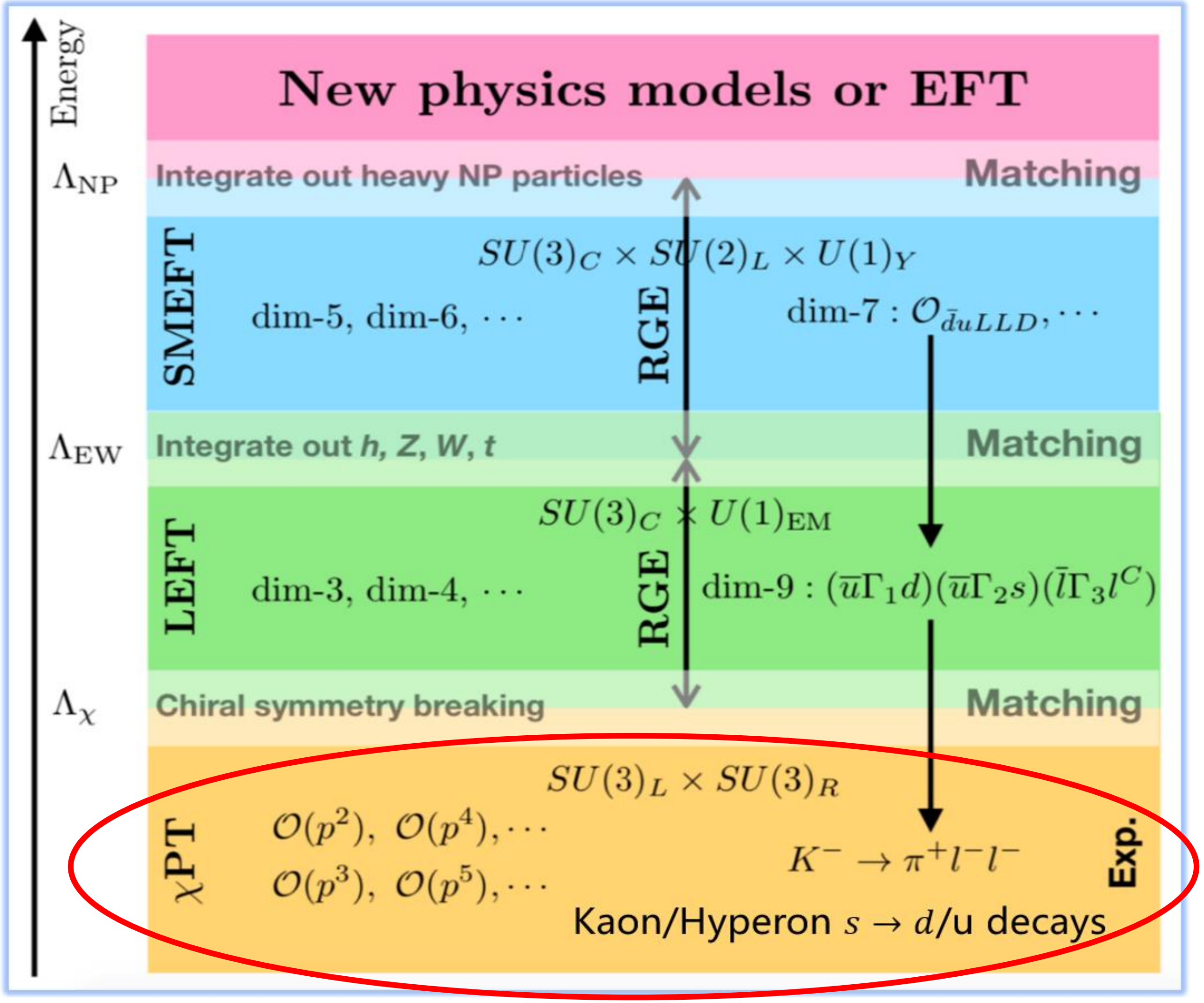
✓ The used low energy constants (LECs) and hyperon non-leptonic decay amplitudes are out of date

✓ No efforts were taken to ensure a consistent power counting

Updating the relevant LECs and hyperon non-leptonic decay amplitudes → Calculating the branching fractions and asymmetry parameters, i.e., **amplitudes a and b**, of the WRHDs order by order → Comparing our predictions with those from other approaches/experimental data

Contents

Chiral perturbation theory : a bottom-up approach



- **Effective theory:** the physics in low energy regions does not depend on the details of the higher energy physics, which has been integrated out

- **Chiral perturbation theory is a powerful tool to study the WRHDs**

Chiral perturbation theory

The effective Lagrangian of the general form

$$\mathcal{L} = \sum_i c_i\,(Q,\Lambda)\,O_i(\{\psi\})$$

Q is the **soft scale**, Λ is the **hard scale**, C_i are **LECs**, O_i refer to **operators containing field ψ**.

Power counting rule

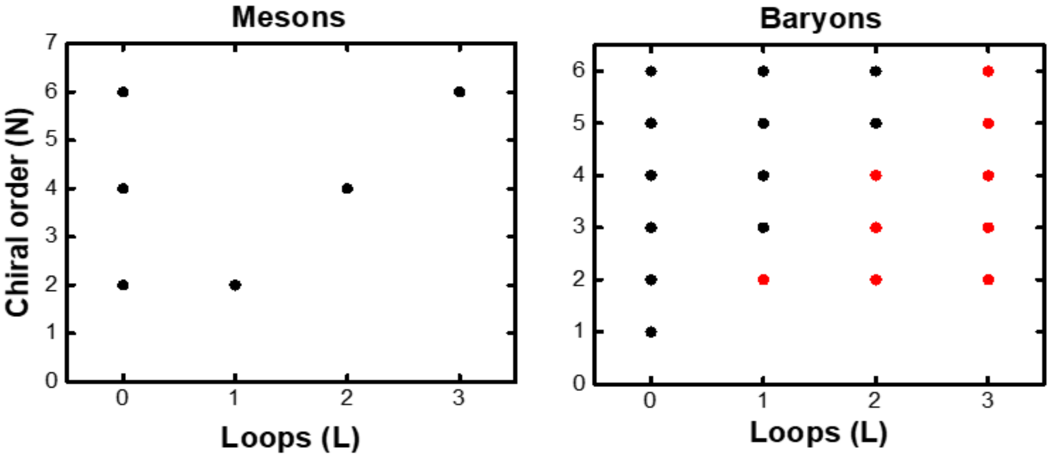
Chiral order: $N = 4L - 2N_M - N_B + \sum k\,V_k$

Power counting:

⇩

$O(Q/\Lambda)^n$

Power counting breaking (PCB) problem



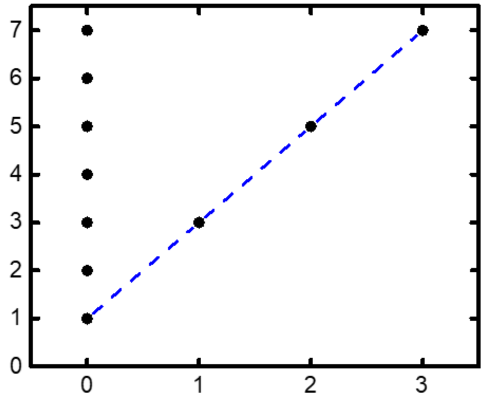
Single baryon /meson system

Red dots: **PCB** terms

Chiral perturbation theory

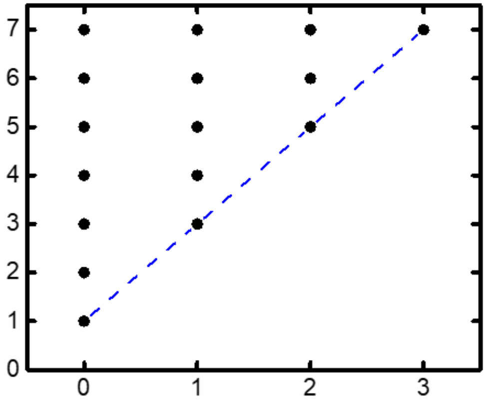
☐Power counting breaking (PCB) solutions

✓ **Nonrelativistic formulation**

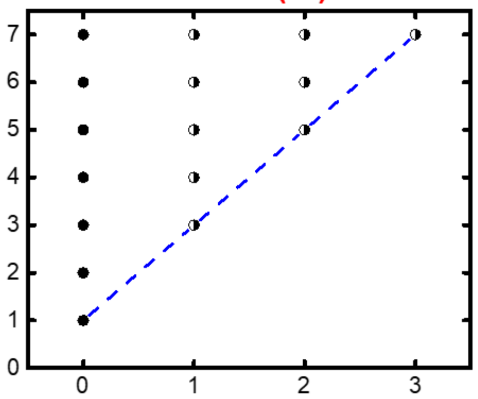


Removing all the PCB and higher order terms

✓ **Covariant formulation (We adopt EOMS)** *Li-Sheng Geng, Front.Phys.(Beijing)* 8 (2013) 328-348



Removing all the PCB terms and remaining all the higher order terms

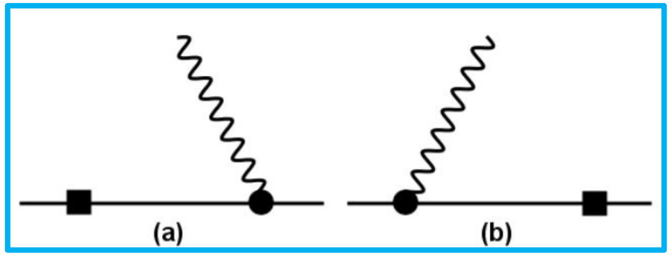


Removing all the PCB terms and remaining partly the higher order terms

WRHDs in the EOMS BχPT

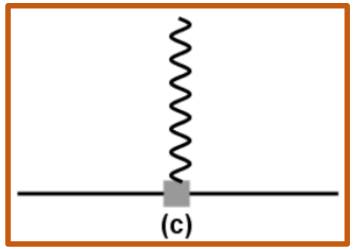
$$a_{B_iB_f} = a^{(1,\text{tree})}_{B_iB_f} + a^{(2,\text{tree})}_{B_iB_f} + a^{(2,\text{loop})}_{B_iB_f}$$
$$b_{B_iB_f} = b^{(2,\text{tree})}_{B_iB_f} + b^{(2,\text{loop})}_{B_iB_f}$$

Feynman diagrams | **Lagrangians** | **Order contributions**



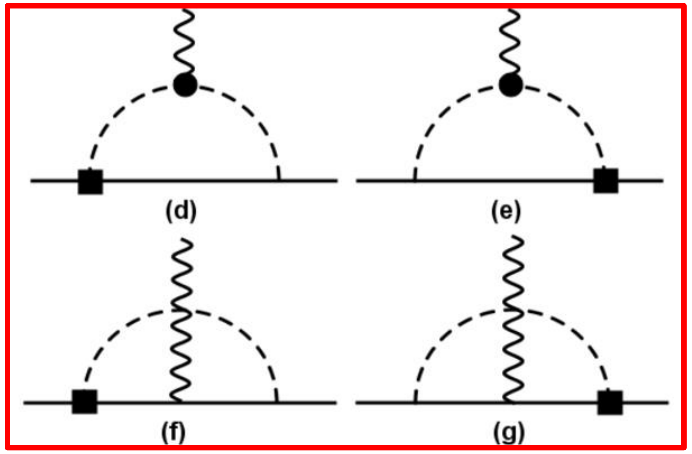
$$\mathcal{L}^{(0)}_{\Delta S=1} = \sqrt{2}G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle,$$
$$\mathcal{L}^{(2)}_{MB} = \frac{b_6^D}{8m_B}\langle \bar{B}\sigma^{\mu\nu}\{F^+_{\mu\nu}, B\}\rangle + \frac{b_6^F}{8m_B}\langle \bar{B}\sigma^{\mu\nu}[F^+_{\mu\nu}, B]\rangle,$$

$a^{(1,\text{tree})}_{B_iB_f}$



$$\mathcal{L}^{(2)}_{\alpha} = C_\alpha \langle \bar{B}\sigma^{\mu\nu} F_{\mu\nu} \lambda Q B\rangle,$$
$$\mathcal{L}^{(2)}_{\beta} = C_\beta \langle \sigma^{\mu\nu} F_{\mu\nu} \bar{B} Q B \lambda\rangle,$$
$$\mathcal{L}^{(2)}_{\gamma} = C_\gamma \langle \bar{B}\sigma^{\mu\nu} F_{\mu\nu} B \lambda Q\rangle,$$
$$\mathcal{L}^{(2)}_{\sigma} = C_\sigma \langle \bar{B}\sigma^{\mu\nu} F_{\mu\nu} \lambda B Q\rangle,$$
$$\mathcal{L}^{(2)}_{\rho} = C_\rho \left(\langle \bar{B}\sigma^{\mu\nu}\gamma_5 F_{\mu\nu} Q\rangle\langle B\lambda\rangle - \langle \bar{B}\sigma^{\mu\nu}\gamma_5 F_{\mu\nu}\lambda\rangle\langle BQ\rangle\right)$$

$a^{(2,\text{tree})}_{B_iB_f}$ $b^{(2,\text{tree})}_{B_iB_f}$



$$\mathcal{L}^{(0)}_{\Delta S=1} = \sqrt{2}G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle$$
$$\mathcal{L}^{(1)}_{B} = \langle \bar{B} i\gamma^\mu D_\mu B - m_0 \bar{B}B\rangle,$$
$$\mathcal{L}^{(2)}_{M} = \frac{F_\phi^2}{4}\langle u_\mu u^\mu + \chi^+\rangle,$$
$$\mathcal{L}^{(1)}_{MB} = \frac{D}{2}\langle \bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\rangle + \frac{F}{2}\langle \bar{B}\gamma^\mu\gamma_5[u_\mu, B]\rangle,$$

LECs D and F have been determined in Ref. L. S. Geng et al, PRD 90, 054502 (2014)

$a^{(2,\text{loop})}_{B_iB_f}$ $b^{(2,\text{loop})}_{B_iB_f}$

For the amplitude a, weak vertex is γ_5

Contents

LECs hD, hF and hyperon non-leptonic decay amplitudes

- The hyperon non-leptonic decay amplitudes for the octet-to-octet transitions have the following form

$$\mathcal{M}(B_i \to B_f \pi) = i G_F m_\pi^2 \bar{B}_f \left(A_S - A_P \gamma_5\right) B_i$$

Hyperon non-leptonic decay amplitudes: S-wave amplitude A_S and P-wave amplitude A_P

- **Decay width and baryon decay parameters $\boldsymbol{\alpha_\pi}$, $\boldsymbol{\beta_\pi}$ and $\boldsymbol{\gamma_\pi}$** for $B_i \to B_f\, \pi$ decays

$$\Gamma(B_i \to B_f \pi) = \frac{\left(G_F m_\pi^2\right)^2}{8\pi m_i^2} |\vec{q}| \left\{ [(m_i + m_f)^2 - m_\pi^2] |s|^2 + [(m_i - m_f)^2 - m_\pi^2] \left| p \cdot \frac{(E_f + m_f)}{|\vec{q}|} \right|^2 \right\},$$

$$\alpha_\pi = \frac{2\mathrm{Re}\,(s \cdot p)}{|s|^2 + |p|^2}, \quad \beta_\pi = \frac{2\mathrm{Im}\,(s \cdot p)}{|s|^2 + |p|^2}, \quad \gamma_\pi = \frac{|s|^2 + |p|^2}{|s|^2 + |p|^2},$$

with

$$s = A_S \qquad\qquad p = A_P |\vec{q}| / (E_f + m_f)$$

where E_f and $\vec{q}$ are the energy and 3-momentum of the final baryon

LECs hD, hF and hyperon non-leptonic decay amplitudes

Table: By means of isospin symmetry, the Lee-Sugawara relations and the criterion that $A_S(\Lambda \to p\pi^-)$ is conventionally positive, **S - and P-wave hyperon non-leptonic decay amplitudes are uniquely** determined by fitting to the recent data [3,51-53] of branching fraction $\mathcal{B}$, baryon decay parameters α_π and γ_π

Decay modes	$\mathcal{B}$ [3]	α_π [3, 51–53]	ϕ_π (°) [3, 52]	$s = A_S^{(\text{Expt})}$		$p = A_P^{(\text{Expt})}\|\vec{q}\|/(E_f + m_f)$	
				This work	[49]	This work	[49]
$\Sigma^+ \to n\pi^+$	0.4831(30)	0.068(13)	167(20)	0.06(1)	0.06(1)	1.81(1)	1.81(1)
$\Sigma^- \to n\pi^-$	0.99848(5)	−0.068(8)	10(15)	1.88(1)	1.88(1)	−0.06(1)	−0.06(1)
$\Lambda \to p\pi^-$	0.639(5)	0.7462(88)	−6.5(35)	1.38(1)	1.42(1)	0.62(1)	0.52(2)
$\Xi^- \to \Lambda\pi^-$	0.99887(35)	−0.376(8)	0.6(12)	−1.99(1)	−1.98(1)	0.39(1)	0.48(2)
$\Sigma^+ \to p\pi^0$	0.5157(30)	−0.982(14)	36(34)	−1.50(3)	−1.43(5)	1.29(4)	1.17(7)
$\Lambda \to n\pi^0$	0.358(5)	0.74(5)	...	−1.09(2)	−1.04(1)	−0.48(4)	−0.39(4)
$\Xi^0 \to \Lambda\pi^0$	0.99524(12)	−0.356(11)	21(12)	1.62(10)	1.52(2)	−0.30(10)	−0.33(2)

$$\gamma_\pi = \sqrt{1 - \alpha_\pi^2} \cos(\phi_\pi)$$

- Comparing our results with those of Ref. [49]:
  - ✓ P-wave amplitudes, especially for $A_P(\Lambda \to p\pi^-)$ and $A_P(\Xi^- \to \Lambda\pi^-)$, differ a lot, which would affect the imaginary parts of the parity-conserving amplitude a
  - ✓ the experimental S -wave amplitudes are almost unchanged

[3] P. A. Zyla et al. PDG, PTEP 2020, 083C01(2020) [49] E. E. Jenkins, NPB 375, 561 (1992) [51] M. Ablikim et al., BESIII, 2204.11058 (2022) [52] M. Ablikim et al. (BESIII), Nature 606, 64, 2105.11155 (2022) [53] D. G. Ireland et al, PRL 123,182301 (2019)

Non-leptonic decay amplitudes—S/P puzzle

□Amplitudes of hyperon non-leptonic decay

$$\mathcal{M}(B_i \to B_f \pi) = i G_F m_\pi^2 \bar{B}_f \left(A_S - A_P \gamma_5\right) B_i$$

Here, both S-wave amplitude A_S and P-wave amplitude A_P are as functions of LECs hD and hF

□The so-called S/P puzzle: if the two LECs hD and hF can describe well the experimental S-wave amplitudes, they reproduce very poorly the P-wave amplitudes

As a result, we only updated the values of hD and hF by fitting to the experimental S -wave amplitudes for hyperon non-leptonic decays

LECs hD, hF and hyperon non-leptonic decay amplitudes

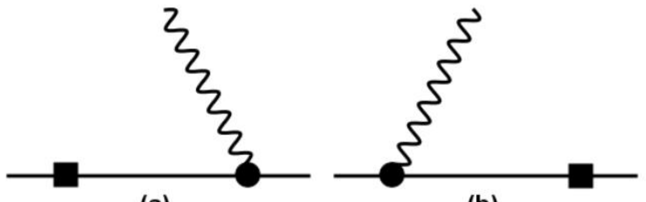
Table: **LECs hD and hF determined by fitting to the S –wave hyperon non-leptonic decay amplitudes.**

Decay modes	$A_S^{\rm th}$	$A_S^{\rm Expt}$
$\Sigma^+ \to n\pi^+$	0	0.06(1)
$\Sigma^- \to n\pi^-$	$-h_D + h_F$	1.88(1)
$\Lambda \to p\pi^-$	$\frac{1}{\sqrt{6}}(h_D + 3h_F)$	1.38(1)
$\Xi^- \to \Lambda\pi^-$	$\frac{1}{\sqrt{6}}(h_D - 3h_F)$	−1.99(1)
$\Sigma^+ \to p\pi^0$	$\frac{1}{\sqrt{2}}(h_D - h_F)$	−1.50(3)
$\Lambda \to n\pi^0$	$-\frac{1}{2\sqrt{3}}(h_D + 3h_F)$	−1.09(2)
$\Xi^0 \to \Lambda\pi^0$	$-\frac{1}{2\sqrt{3}}(h_D - 3h_F)$	1.62(10)
$\chi^2/\text{d.o.f.} = 0.24$	$h_D = -0.61(24)$	$h_F = 1.42(14)$

- In our least-squares fit, an absolute uncertainty of 0.3 is added to each S -wave amplitude in order to match the theoretical predictions with the experimental data at 1σ confidence level
- The tree-level formulae for the S -wave amplitudes derived from the following Lagrangian

$$\mathcal{L}^{(0)}_{\Delta S=1} = \sqrt{2} G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle$$

Real part of amplitude **a** at $O(p)^1$--tree



$$\mathcal{L}^{(0)}_{\Delta S=1} = \sqrt{2} G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle,$$
$$\mathcal{L}^{(2)}_{MB} = \frac{b_6^D}{8m_B}\langle \bar{B}\sigma^{\mu\nu}\{F^+_{\mu\nu}, B\}\rangle + \frac{b_6^F}{8m_B}\langle \bar{B}\sigma^{\mu\nu}[F^+_{\mu\nu}, B]\rangle,$$

$$a^{(1,\mathrm{tree})}_{\Lambda n} = \frac{m_\pi^2 F_\phi}{2m_B}\left[\frac{1}{\sqrt{3}}(h_D + 3h_F)\frac{\mu_n^{(2)} - \mu_\Lambda^{(2)}}{m_\Lambda - m_n} - (h_D - h_F)\frac{\mu^{(2)}_{\Lambda\Sigma^0}}{m_{\Sigma^0} - m_n}\right],$$

$$a^{(1,\mathrm{tree})}_{\Sigma^+ p} = \frac{m_\pi^2 F_\phi}{2m_B}\left[-\sqrt{2}(h_D - h_F)\frac{\mu_p^{(2)} - \mu_{\Sigma^+}^{(2)}}{m_{\Sigma^+} - m_p}\right],$$

$$a^{(1,\mathrm{tree})}_{\Sigma^0 n} = \frac{m_\pi^2 F_\phi}{2m_B}\left[(h_D - h_F)\frac{\mu_n^{(2)} - \mu_{\Sigma^0}^{(2)}}{m_{\Sigma^0} - m_n} - \frac{1}{\sqrt{3}}(h_D + 3h_F)\frac{\mu^{(2)}_{\Sigma^0\Lambda}}{m_\Lambda - m_n}\right],$$

$$a^{(1,\mathrm{tree})}_{\Xi^0 \Lambda} = \frac{m_\pi^2 F_\phi}{2m_B}\left[\frac{1}{\sqrt{3}}(h_D - 3h_F)\frac{\mu_\Lambda^{(2)} - \mu_{\Xi^0}^{(2)}}{m_{\Xi^0} - m_\Lambda} + (h_D + h_F)\frac{\mu^{(2)}_{\Sigma^0\Lambda}}{m_{\Xi^0} - m_{\Sigma^0}}\right],$$

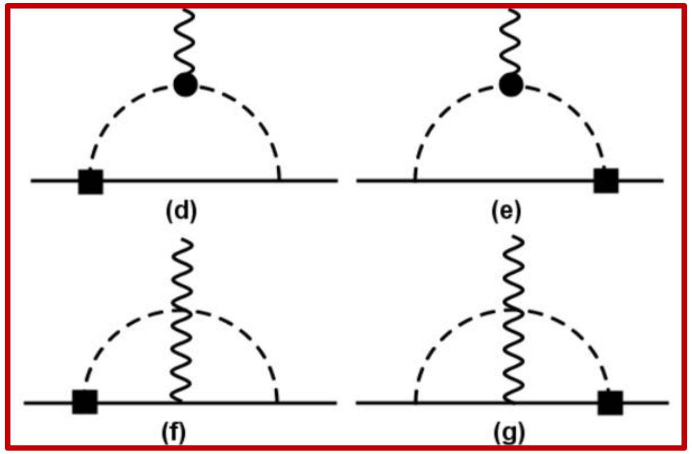
$$a^{(1,\mathrm{tree})}_{\Xi^0 \Sigma^0} = \frac{m_\pi^2 F_\phi}{2m_B}\left[(h_D + h_F)\frac{\mu_{\Sigma^0}^{(2)} - \mu_{\Xi^0}^{(2)}}{m_{\Xi^0} - m_{\Sigma^0}} + \frac{1}{\sqrt{3}}(h_D - 3h_F)\frac{\mu^{(2)}_{\Lambda\Sigma^0}}{m_{\Xi^0} - m_\Lambda}\right],$$

$$a^{(1,\mathrm{tree})}_{\Xi^- \Sigma^-} = \frac{m_\pi^2 F_\phi}{2m_B}\left[\sqrt{2}(h_D + h_F)\frac{\mu_{\Xi^-}^{(2)} - \mu_{\Sigma^-}^{(2)}}{m_{\Xi^-} - m_{\Sigma^-}}\right],$$

- **hD and hF are LECs**
- $\mu_B^{(2)}$ **are the experimental baryon magnetic moments**

$$a_{B_iB_f} = a^{(1,\mathrm{tree})}_{B_iB_f} + a^{(2,\mathrm{tree})}_{B_iB_f} + a^{(2,\mathrm{loop})}_{B_iB_f}$$
$$b_{B_iB_f} = b^{(2,\mathrm{tree})}_{B_iB_f} + b^{(2,\mathrm{loop})}_{B_iB_f}$$

Amplitude b and imaginary part of amplitude a at $O(p^2)$ --loop



$$\mathcal{L}^{(0)}_{\Delta S=1} = \sqrt{2}G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle$$

$$\mathcal{L}^{(1)}_{B} = \langle \bar{B} i\gamma^\mu D_\mu B - m_0 \bar{B}B\rangle,$$

$$\mathcal{L}^{(2)}_{M} = \frac{F_\phi^2}{4}\langle u_\mu u^\mu + \chi^+\rangle,$$

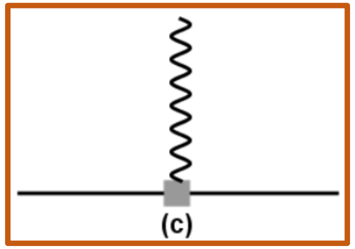
$$\mathcal{L}^{(1)}_{MB} = \frac{D}{2}\langle \bar{B}\gamma^\mu \gamma_5 \{u_\mu, B\}\rangle + \frac{F}{2}\langle \bar{B}\gamma^\mu\gamma_5[u_\mu, B]\rangle,$$

Real part of amplitude a in the loop level cannot be reliably determined due to S/P puzzle in hyperon non-leptonic decays.

$$a_{B_iB_f} = a^{(1,\mathrm{tree})}_{B_iB_f} + a^{(2,\mathrm{tree})}_{B_iB_f} + a^{(2,\mathrm{loop})}_{B_iB_f}$$

$$b_{B_iB_f} = b^{(2,\mathrm{tree})}_{B_iB_f} + b^{(2,\mathrm{loop})}_{B_iB_f}$$

Real part of amplitude a and b at $O(p^2)$ --tree



$$\mathcal{L}_{\alpha}^{(2)} = C_{\alpha}\langle \bar{B}\sigma^{\mu\nu}F_{\mu\nu}\lambda QB\rangle,$$
$$\mathcal{L}_{\beta}^{(2)} = C_{\beta}\langle \sigma^{\mu\nu}F_{\mu\nu}\bar{B}QB\lambda\rangle,$$
$$\mathcal{L}_{\gamma}^{(2)} = C_{\gamma}\langle \bar{B}\sigma^{\mu\nu}F_{\mu\nu}B\lambda Q\rangle,$$
$$\mathcal{L}_{\sigma}^{(2)} = C_{\sigma}\langle \bar{B}\sigma^{\mu\nu}F_{\mu\nu}\lambda BQ\rangle,$$
$$\mathcal{L}_{\rho}^{(2)} = C_{\rho}\left(\langle \bar{B}\sigma^{\mu\nu}\gamma_5F_{\mu\nu}Q\rangle\langle B\lambda\rangle - \langle \bar{B}\sigma^{\mu\nu}\gamma_5F_{\mu\nu}\lambda\rangle\langle BQ\rangle\right)$$

counter-terms

- CPS is CP followed by the SU(3) transformation of $u \to -u$, $d \to s$ and $s \to d$ which exchanges s and d quarks.
- CPS symmetry dictates the existence of five unknown LECs

Table: Contributions to the real parts of amplitudes a and b at tree-level $O(p)^2$. The normalization $2(eG_F)^{-1}$ has been factored out.

	$\Lambda \to n\gamma$	$\Sigma^+ \to p\gamma$	$\Sigma^0 \to n\gamma$	$\Xi^0 \to \Lambda\gamma$	$\Xi^0 \to \Sigma^0\gamma$	$\Xi^- \to \Sigma^-\gamma$
$a^{(2,\text{tree})}$	$\frac{2C_\alpha - C_\beta - C_\gamma + 2C_\sigma}{3\sqrt{6}}$	$\frac{2C_\beta - C_\gamma}{3}$	$\frac{C_\beta + C_\gamma}{3\sqrt{2}}$	$-\frac{C_\alpha - 2C_\beta - 2C_\gamma + C_\sigma}{3\sqrt{6}}$	$\frac{C_\alpha + C_\sigma}{3\sqrt{2}}$	$\frac{2C_\sigma - C_\alpha}{3}$
$b^{(2,\text{tree})}$	$-\frac{C_\rho}{\sqrt{6}}$	0	$-\frac{C_\rho}{\sqrt{2}}$	$\frac{C_\rho}{\sqrt{6}}$	$\frac{C_\rho}{\sqrt{2}}$	0

$$b^{(2,\text{tree})}_{\Xi^0\Sigma^0} = \sqrt{3}b^{(2,\text{tree})}_{\Xi^0\Lambda}, \quad b^{(2,\text{tree})}_{\Lambda n} = -b^{(2,\text{tree})}_{\Xi^0\Lambda},$$
$$b^{(2,\text{tree})}_{\Sigma^0 n} = -\sqrt{3}b^{(2,\text{tree})}_{\Xi^0\Lambda}, \quad b^{(2,\text{tree})}_{\Sigma^+ p} = 0, \quad b^{(2,\text{tree})}_{\Xi^-\Sigma^-} = 0.$$

Determining the contributions of counter-terms

- Total amplitudes a and b are **a sum of the tree and loop contributions** and read:

$$a_{B_iB_f} = a^{(1,\text{tree})}_{B_iB_f} + a^{(2,\text{tree})}_{B_iB_f} + a^{(2,\text{loop})}_{B_iB_f} = \text{Re}\, a_{B_iB_f} + \text{Im}\, a^{(2,\text{loop})}_{B_iB_f}$$
$$b_{B_iB_f} = b^{(2,\text{tree})}_{B_iB_f} + b^{(2,\text{loop})}_{B_iB_f}$$

- Using $b^{(2,\text{tree})}_{\Xi^0\Sigma^0} = \sqrt{3}b^{(2,\text{tree})}_{\Xi^0\Lambda}$ and fitting to $\mathcal{B}$ and α_γ for $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^0 \to \Lambda\gamma$ decays, we determine **for the first time** the contributions of counter-terms

	Solution I	Solution II
$b^{(2,\text{tree})}_{\Xi^0\Lambda}$	5.62(53)	−8.34(48)
Re $a_{\Xi^0\Lambda}$	−9.56(34)	3.89(45)
Re $a_{\Xi^0\Sigma^0}$	−32.22(64)	32.50(61)
χ^2/d.o.f.	0.04	1.22

- The $\chi^2/d.o.f.$ of Solution I much smaller than that of Solution II.
- Contributions of counter-terms for other WRHDs obtained by the following relations $b^{(2,\text{tree})}_{\Lambda n} = -b^{(2,\text{tree})}_{\Xi^0\Lambda}$, $b^{(2,\text{tree})}_{\Sigma^0 n} = -\sqrt{3}b^{(2,\text{tree})}_{\Xi^0\Lambda}$, $b^{(2,\text{tree})}_{\Sigma^+ p} = 0$, $b^{(2,\text{tree})}_{\Xi^-\Sigma^-} = 0$

$$a_{B_iB_f} = a^{(1,\text{tree})}_{B_iB_f} + a^{(2,\text{tree})}_{B_iB_f} + a^{(2,\text{loop})}_{B_iB_f} = \text{Re}\, a_{B_iB_f} + \text{Im}\, a^{(2,\text{loop})}_{B_iB_f}$$

$$b_{B_iB_f} = b^{(2,\text{tree})}_{B_iB_f} + b^{(2,\text{loop})}_{B_iB_f}$$

Therefore, we take the Re a for each WRHD as a free parameter due to the unknown real parts of amplitudes a in tree and loop levels

Predictions for parity-conserving a and -violating b amplitudes

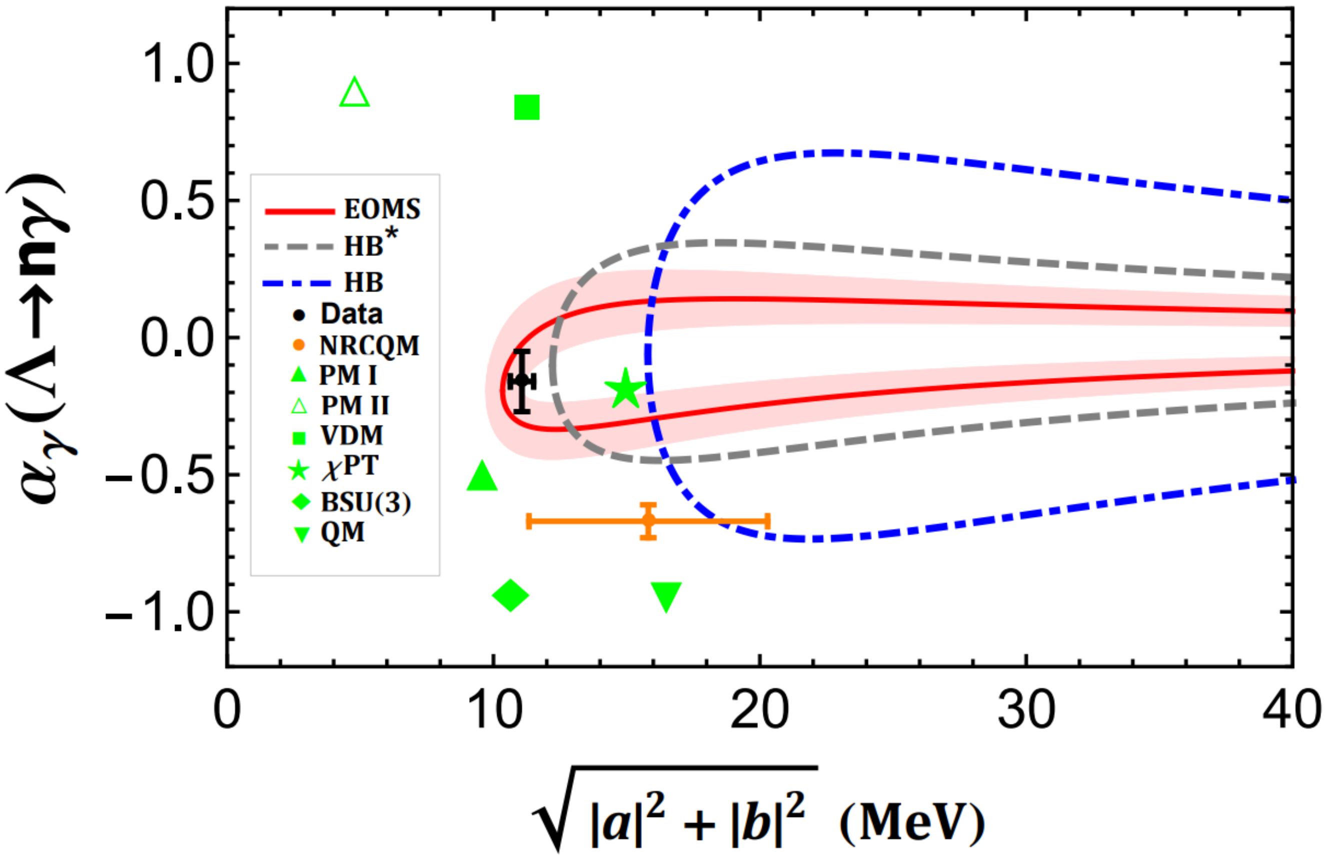
Table: Decomposition of the contributions to the parity-violating amplitudes b (in units of MeV)

Decay modes	EOMS BχPT $b^{(2,\text{tree})}$	$b^{(2,\text{loop})}$	$b^{(2,\text{tot})}$
$\Lambda \to n\gamma$	-5.62(53)	7.87(73) + 10.04(81)i	2.25(90) + 10.04(81)i
$\Sigma^+ \to p\gamma$	0	−1.96(11) – 1.75(12)i	−1.96(11) – 1.75(12)i
$\Sigma^0 \to n\gamma$	−9.73(92)	1.41(11) + 10.09(78)i	−8.32(93) + 10.09(78)i
$\Xi^0 \to \Lambda\gamma$	5.62(53)	−1.60(48)	4.02(72)
$\Xi^0 \to \Sigma^0\gamma$	9.73(92)	2.91(67)	12.64(114)
$\Xi^- \to \Sigma^-\gamma$	0	−3.00(29) – 8.64(54)i	−3.00(29) – 8.64(54)i

Table: Imaginary parts of the loop contributions to the parity-conserving amplitudes a at $O(p^2)$ (in units of MeV)

Decay modes	EOMS BχPT Im $a^{(2,\text{loop})}$
$\Lambda \to n\gamma$	−1.01(2)
$\Sigma^+ \to p\gamma$	2.70(4)
$\Xi^- \to \Sigma^-\gamma$	−0.57(1)

α_γ of the $\Lambda \to n\,\gamma$ decay as a function of $\sqrt{|a|^2 + |b|^2}$



Data: *BESIII, 2206.10791*

HB χPT : *E. E. Jenkins et al, NPB 397, 84 (1993)*

HB* χPT : *E. E. Jenkins et al, NPB 397, 84 (1993) with counter-term contributions*

NRCQM: *Qiang Zhao et al, CPC45, 013101 (2021)*

PM1: *M. B. Gavela et al, PLB 101, 417 (1981)*

PM2: *G. Nardulli, PLB 190, 187 (1987)*

VDM: *P. Zenczykowski, PRD 44, 1485 (1991)*

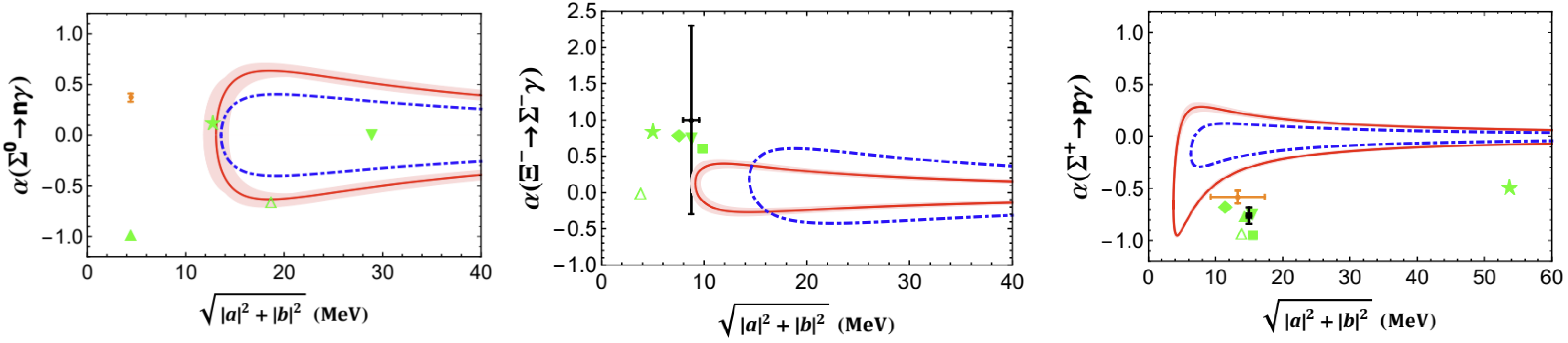
χPT: *B. Borasoy et al, PRD 59, 054019 (1999)*

BSU(3): *P. Zenczykowski, PRD 73, 076005 (2006)*

QM: *E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)*

- Interestingly, only **EOMS BχPT agrees with** the latest BESIII measurement
- The prediction in the HB χPT **with counter-term contributions** is very close to the BESIII data
- The vector dominance model (VDM) and the pole model (PM II) **are disfavored** by the BESIII data

α_γ of the other WRHDs as a function of $\sqrt{|a|^2+|b|^2}$



- For the $\Sigma^0 \to n\,\gamma$ decay, not yet measured, **our result contradicts** the predictions of PM I and NRCQM
- For the $\Xi^- \to \Sigma^-\gamma$ decay, **our prediction agrees better** with the experimental measurement, and the current PDG data disfavor the results of PM II and tree-level χPT
- **For the $\Sigma^+ \to p\,\gamma$ decay, the results predicted in all the χPT deviate from the PDG average but our prediction is closer**

Hara's theorem: α_γ for $\Xi^- \to \Sigma^-\gamma$ and $\boldsymbol{\Sigma^+ \to p\,\gamma}$ should not be too large.

Contents

Summary and outlook

- We **first updated** the two relevant low energy constants hD, hF and hyperon non-leptonic decay amplitudes determined by fitting to **the latest experimental data on the $B_i \to B_f \, \pi$ decays**
- We **determined the $O(p^2)$ counter-term contributions** determined by fitting to $\Xi^0 \to \Sigma^0 \, \gamma$ and $\Xi^0 \to \Lambda \, \gamma$ **for the first time**
- We **showed** that the latest precise measurement of the branching fraction and asymmetry parameter of $\Lambda \to n \, \gamma$ by the BESIII Collaboration **can be well explained** in covariant baryon chiral perturbation theory with the EOMS renormalization scheme (**The four WRHDs channels with observed results can also be described well**)
- A more precise measurement of $\boldsymbol{\alpha_\gamma(\Xi^- \to \Sigma^-\gamma)}$ is highly desirable in order to test Hara's theorem and confirm the present experimental result
- In addition, this work provides the essential SM inputs for studying new physics in the rare hyperon semi-leptonic decay $B_i \to B_f \gamma^* \to B_f l \, l$.

LHCb: JHEP 05 (2019) 048
LHCb:CERN Yellow Rep.Monogr. 7 (2019) 867-1158

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Thanks for your attention !

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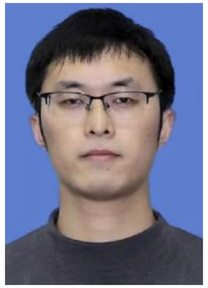

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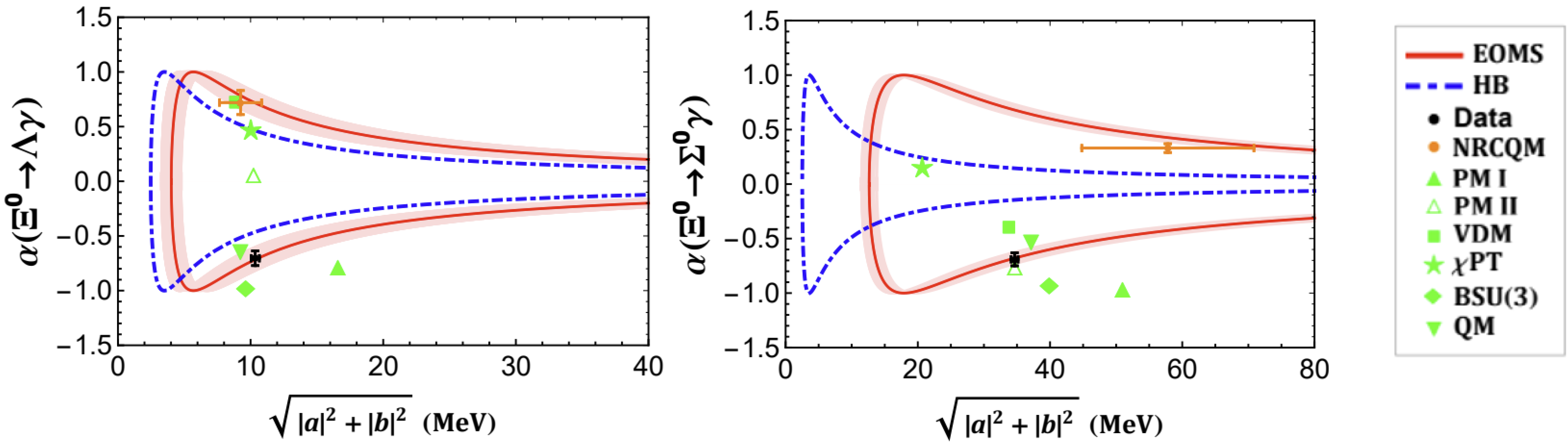
α_γ of $\Xi^0 \to \Sigma^0\,\gamma$ and $\Xi^0 \to \Lambda\,\gamma$ as a function of $\sqrt{|a|^2+|b|^2}$



Hara's theorem

- Based on gauge invariance, CP conservation, and U-spin symmetry

- Hara's theorem dictates that the WRHDs $B \to B'\gamma$ and $B' \to B\gamma$ must be identical under the U-spin transformation $s \leftrightarrow d$

$$\mathcal{L}_{B\to B'\gamma} \propto \bar{B}'(a + b\gamma_5)\sigma^{\mu\nu}$$

$$\mathcal{L}_{B'\to B\gamma} \propto \bar{B}(a - b\gamma_5)\sigma^{\mu\nu} B' F_{\mu\nu}$$

leads to

$b = 0$, where B and B' refer to (Σ^+, Ξ^-) and (p, Σ^-) respectively.

Branching fractions $\mathcal{B}$ and asymmetry parameters α_γ data for WRHDs

Decay modes	$\mathcal{B} \times 10^{-3}$	α_γ
$\Lambda \to n\gamma$	0.832(38)(54)	−0.16(10)(50)
$\Sigma^+ \to p\gamma$	1.23(5)	−0.76(8)
$\Sigma^0 \to n\gamma$	⋯	⋯
$\Xi^0 \to \Lambda\gamma$	1.17(7)	−0.704(19)(64)
$\Xi^0 \to \Sigma^0\gamma$	3.33(10)	−0.69(6)
$\Xi^- \to \Sigma^-\gamma$	0.127(23)	1.0(13)