河南省科学院
HENAN ACADEMY OF SCIENCES

Explaining The New CDF II W-Boson Mass Data In The Georgi-Machacek Extension Models

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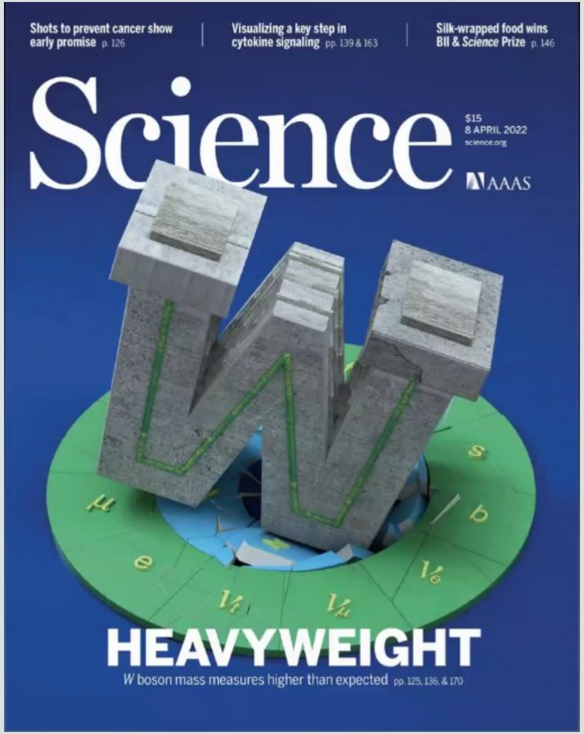
10/29/2022

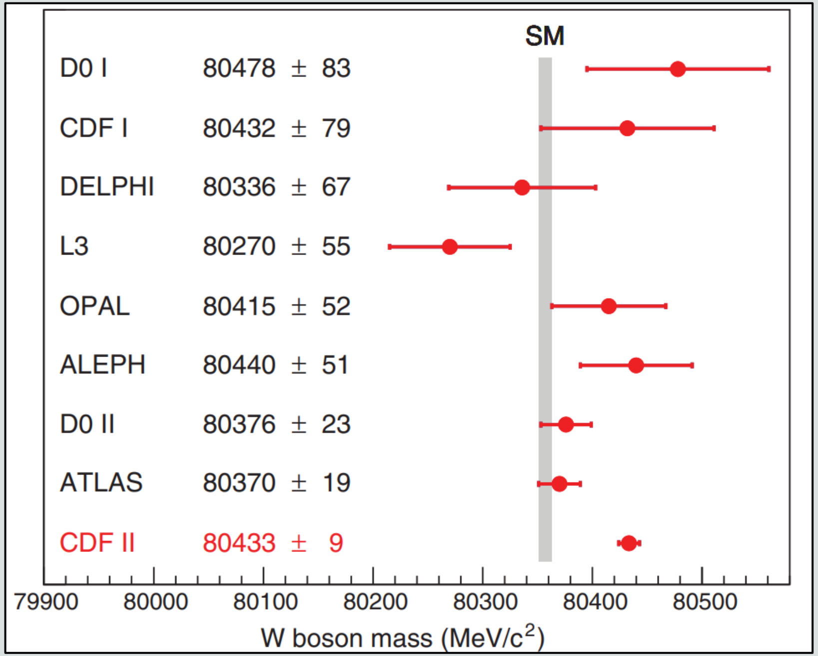
Based on Xk Du, Z Li, Fei Wang, Yk Zhang. 2204.05760

CONTENTS

CDF-II Results on W Boson Mass



Science 376, 170-176 (2022)



SM: $M_W = 80357 \pm 6$ MeV

CDF-II: $M_W = 80433.5 \pm 9.4$ MeV

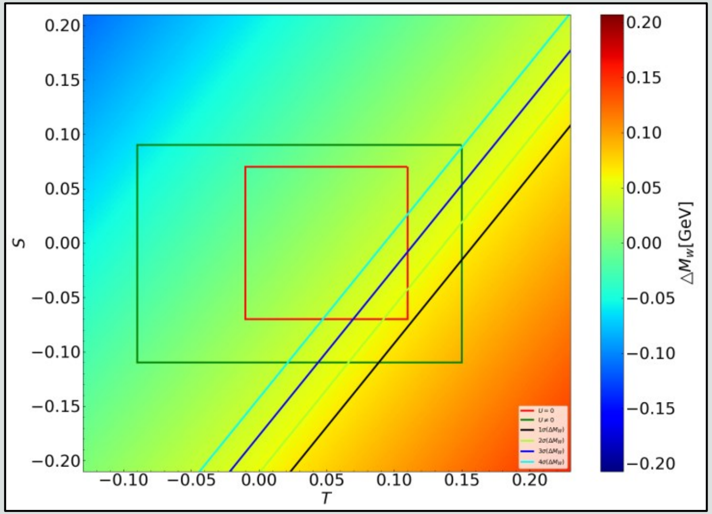
Δm_W ?

$$\Delta m_W = \frac{\alpha M_W}{2(c_W^2 - s_W^2)}\left(-\frac{1}{2}S + c_W^2 T + \frac{c_W^2 - s_W^2}{4 s_W^2} U\right),$$

$$\alpha S = 4 s_W^2 c_W^2 \left[\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W}\Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right],$$

$$\alpha T = \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2},$$

$$\alpha U = 4 s_W^2 \left[\Pi'_{WW}(0) - c_W^2 \Pi'_{ZZ}(0) - 2 s_W c_W \Pi'_{Z\gamma}(0) - s_W^2 \Pi'_{\gamma\gamma}(0)\right]$$



$S = 0.00 \pm 0.07,$
$T = 0.05 \pm 0.06,$
$U = 0$

$S = -0.01 \pm 0.10,$
$T = 0.03 \pm 0.012,$
$U = 0.02 \pm 0.11$

Phys. Rev. D 46 (1992) 381,PTEP 2020 (2020) 8, 083C01(PDG2020)

The Georgi-Machacek Model

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$$

Fields	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$
ϕ	1/2	2	1
χ	1	3	1
ξ	0	3	1

$$\mathcal{L}_{GM} = \mathcal{L}_{kin} + \mathcal{L}_Y + \mathcal{L}_\nu - V_H$$

$$\mathcal{L}_\nu \supset h_{ij} \overline{L_L^{ic}} i\tau_2 \chi L_L^j + h.c.$$

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \Delta = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 \end{pmatrix}$$

$$V(\Phi,\Delta) = \frac{1}{2}\mathrm{m}_\Phi^2 \mathrm{tr}[\Phi^\dagger\Phi] + \frac{1}{2}\mathrm{m}_\Delta^2 \mathrm{tr}[\Delta^\dagger\Delta] + \lambda_1\left(\mathrm{tr}[\Phi^\dagger\Phi]\right)^2 + \lambda_2\left(\mathrm{tr}[\Delta^\dagger\Delta]\right)^2$$
$$+\lambda_3 \mathrm{tr}\left[\left(\Delta^\dagger\Delta\right)^2\right] + \lambda_4 \mathrm{tr}[\Phi^\dagger\Phi]\mathrm{tr}[\Delta^\dagger\Delta] + \lambda_5 \mathrm{tr}\left[\Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2}\right] \mathrm{tr}[\Delta^\dagger T^a \Delta T^b]$$
$$+\mu_1 \mathrm{tr}\left[\Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2}\right] \left(P^\dagger \Delta P\right)_{ab} + +\mu_2 \mathrm{tr}[\Delta^\dagger T^a \Delta T^b]\left(P^\dagger \Delta P\right)_{ab}$$

Nucl.Phys.B 262 (1985) 463-477, JHEP 01 (2013) 026, JHEP 01 (2016) 120

1、CDF-II Results and the Georgi-Machacek Model

$$\boldsymbol{SU(2)_L \times SU(2)_R} \xrightarrow[\langle\chi^0\rangle = \langle\xi^0\rangle = v_\Delta]{\langle\phi^0\rangle = v_\phi/\sqrt{2}} \boldsymbol{SU(2)_c}$$

$$v_{EW}^2 = \sum_i [4T_i(T_i+1) - Y_i^2]\,|v_i|^2 c_i = v_\phi^2 + 4v_\chi^2 + 4v_\xi^2 = v_\phi^2 + 8v_\Delta^2 = \frac{1}{\sqrt{2}G_F} \approx (246\mathrm{GeV})^2$$

$$c_{T,Y} = \begin{cases} 1\,, (T,Y) \in complex\ representation \\ 1/2\,, (T, Y=0) \in real\ representation \end{cases}$$

$$\rho_{tree} \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \sum_i [4T_i(T_i+1) - Y_i^2]\,|v_i|^2 c_i \,/ \sum_i 2Y_i|v_i|^2$$

$$\Delta\rho_{tree} \equiv \rho - 1 = \frac{v_\phi^2 + 4v_\chi^2 + 4v_\xi^2}{v_\phi^2 + 8v_\Delta^2} - 1 \approx \frac{4v_\chi^2 - 4v_\xi^2}{\mathrm{v}_{\mathrm{EW}}^2}$$

$$\tan\theta = 2\sqrt{2}v_\Delta/v_\phi < 0.2$$

1、CDF-II Results and the Georgi-Machacek Model

$$\mathcal{L}_{type-II} \supset h_{ij}\overline{L_L^{ic}} i\tau_2 \chi L_L^j + \mu_1 \mathrm{tr}\left[\Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2}\right] (P^\dagger \Delta P)_{ab}$$

$\boldsymbol{SU(2)_L \times SU(2)_R}$ $\xrightarrow[\langle\chi^0\rangle = \langle\xi^0\rangle = v_\Delta]{\langle\phi^0\rangle = v_\phi/\sqrt{2}}$ $\boldsymbol{SU(2)_c}$

$$m_\nu \approx h_{ij} v_\Delta \sim h_{ij} v_\phi^2 \mu_1 / m_\Delta^2$$

$$\tan\theta = 2\sqrt{2} v_\Delta / v_\phi < 0.2$$

$v_\Delta \sim \mathcal{O}(10\mathrm{GeV})$ → $h_{ij} < 10^{-13}$

Vacuum stability

$$\lambda_1 > 0, \lambda_2 + \lambda_3 > 0, \lambda_2 + \frac{1}{2}\lambda_3 > 0,$$
$$-|\lambda_4| + 2\sqrt{\lambda_1(\lambda_2 + \lambda_3)} > 0,$$
$$\lambda_4 - \frac{1}{4}|\lambda_5| + \sqrt{2\lambda_1(2\lambda_2 + \lambda_3)} > 0.$$

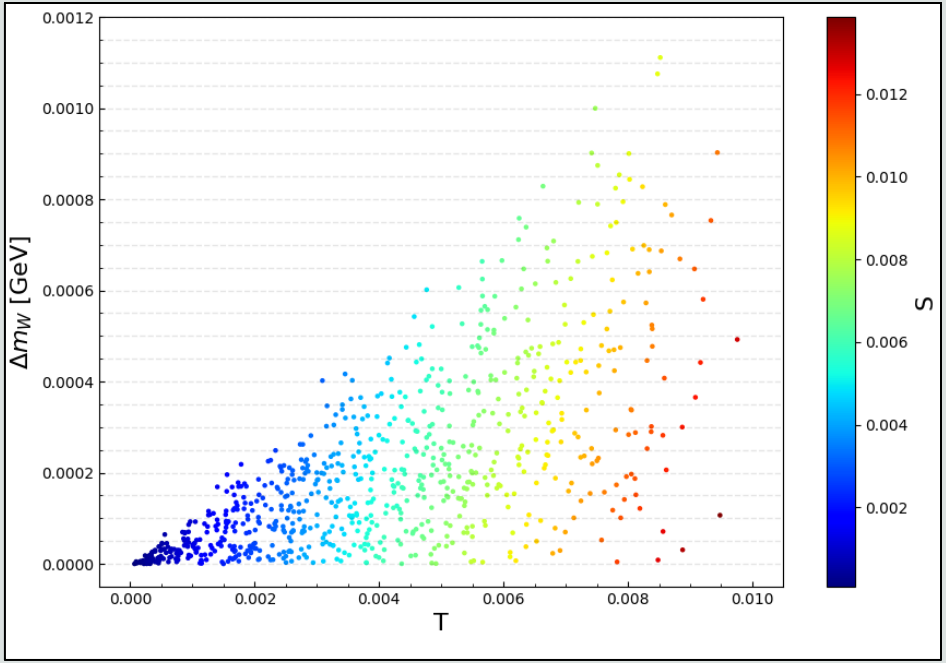
Perturbative unitarity

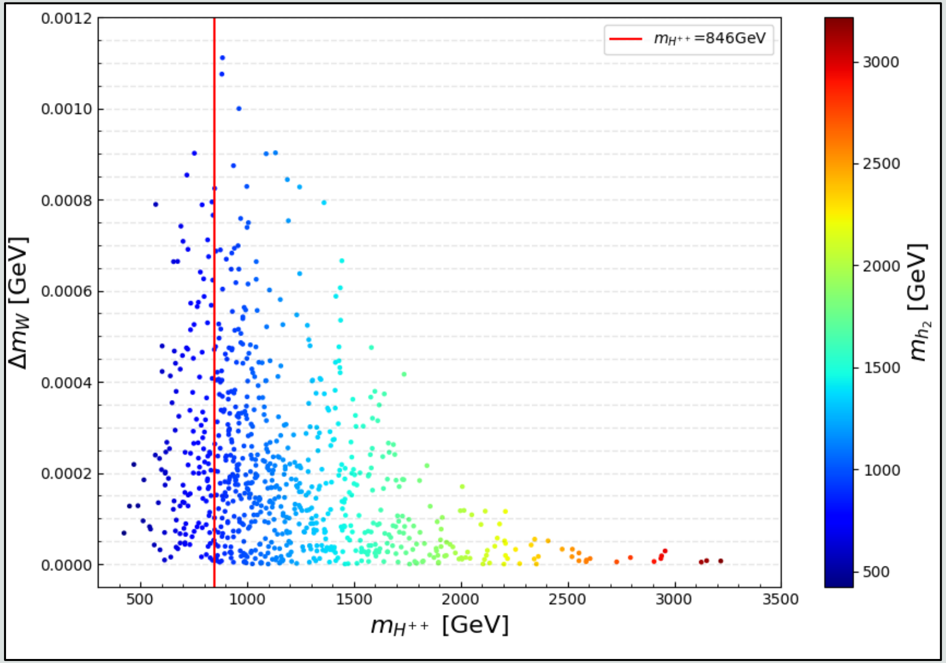
$$|\lambda_4 - \lambda_5| < 2\pi, \qquad |2\lambda_3 + \lambda_2| < \pi,$$
$$|6\lambda_1 + 7\lambda_3 + 11\lambda_2| + \sqrt{(6\lambda_1 - 7\lambda_3 - 11\lambda_2)^2 + 36\lambda_4^2} < 4\pi,$$
$$|2\lambda_1 - \lambda_3 + 2\lambda_2| + \sqrt{(2\lambda_1 + \lambda_3 - 2\lambda_2)^2 + \lambda_5^2} < 4\pi.$$

Nucl.Phys.B 262 (1985) 463-477, *JHEP* 01 (2013) 026, *JHEP* 01 (2016) 120

2、Explaining W-Boson Mass In the GM and Extension Models

$$SU(2)_L \times SU(2)_R \xrightarrow[\langle\chi^0\rangle = \langle\xi^0\rangle = v_\Delta]{\langle\phi^0\rangle = v_\phi/\sqrt{2}} SU(2)_c$$



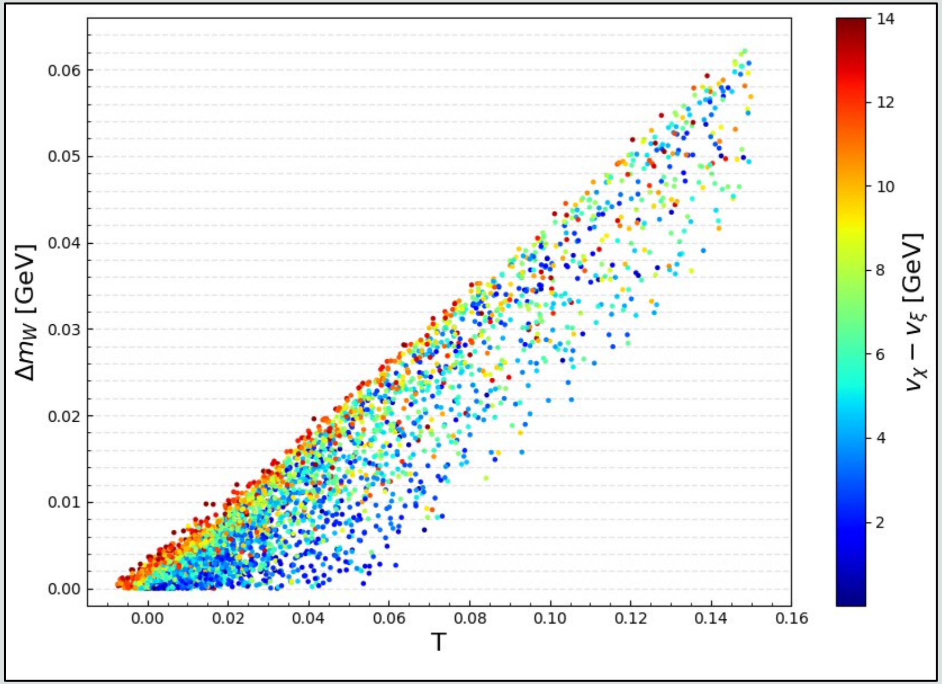


1 $\Delta v = v_\chi - v_\xi > 0$ → ~~$SU(2)_c$~~ → Δm_W ↑

1. hypercharge gauge boson loops as a consequence of $SU(2)_L \times SU(2)_R$ breaking effects in the kinetic term.

2. Loop contributions contain ultra-violet (UV) divergences which cannot be cancelled by counterterms associated with the Vcust part alone.

3. …

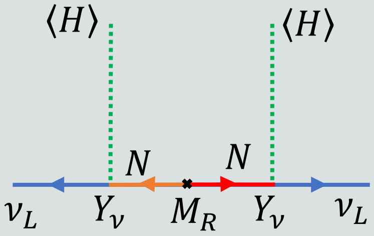


Phys.Rev.D 98 (2018) 1, 013008，Phys.Lett.B 774 (2017) 119-122

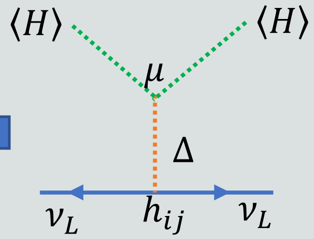
Phys.Rev.D 106 (2022) 5, 055035, arXiv:2204.12898 [hep-ph]

2 Type-I + Type-II → h_{ij} ↑ → ~~$SU(2)_c$~~ → Δm_W ↑

$$-\mathcal{L}_\nu \supset y_{ij}^N \bar{L}_{L,i} \phi N_{R,j} + \frac{1}{2}(M_R)_{ij} N_{R.i}^T C N_{R,j} + h_{ij} \overline{L_L^{ic}} i\tau_2 \chi L_L^j + h.c.$$



$$M_\nu = -\langle H\rangle^2 Y_\nu M_R^{-1} Y_\nu^T$$



$$M_\nu = \langle H\rangle^2 h_{ij}\mu / M_\Delta^2$$

$$M_\nu = \begin{pmatrix} h_{ij} v_\Delta & (y_{ij}^N)^T v_\phi \\ y_{ij}^N v_\phi & (M_R)_{ij} \end{pmatrix}$$

$$M_R \gg v_\phi \gg v_\Delta$$

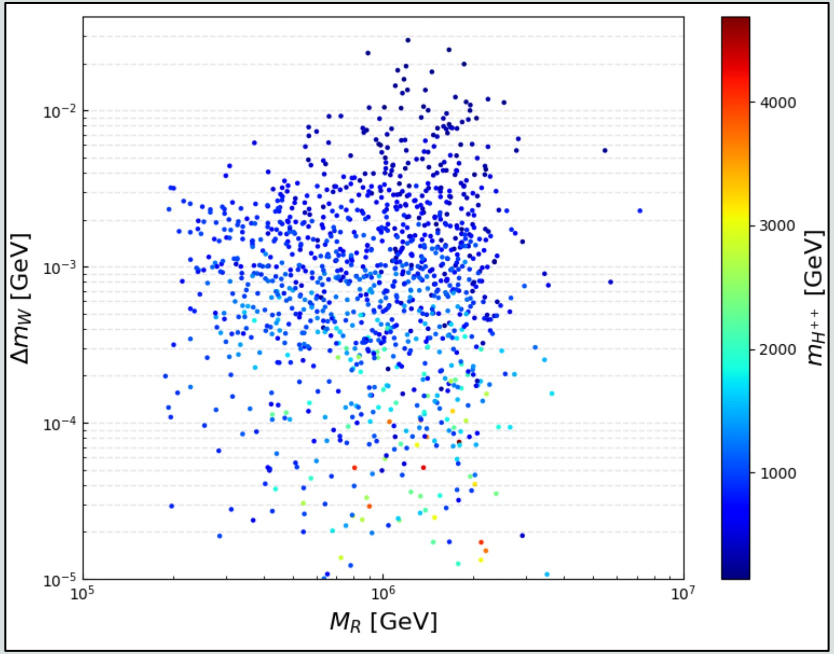
$$m_\nu \approx h_{ij} v_\Delta - v_\phi^2 (y_{ij}^N)^T M_{R,j}^{-1} (y_{ij}^N), \qquad h_{ij} v_\Delta \approx (y^N v_\phi)^2 / M_{R,j}^{-1}$$

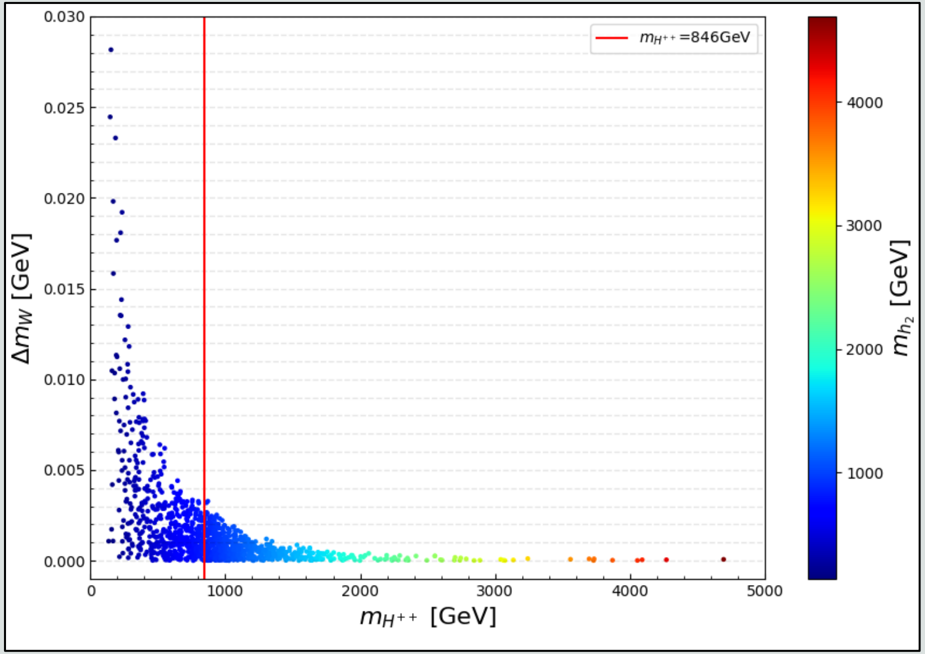
$$h_{ij} = 2\sqrt{2}(V_{PMNS}^T)^{-1} \left(\frac{v(1-s_H^2)}{s_H M_{R,i}}\right) \delta_{ij} (V_{PMNS})^{-1}, \qquad y_{ij}^N = (V_{PMNS})^{-1}$$

2、Explaining W-Boson Mass In the GM and Extension Models

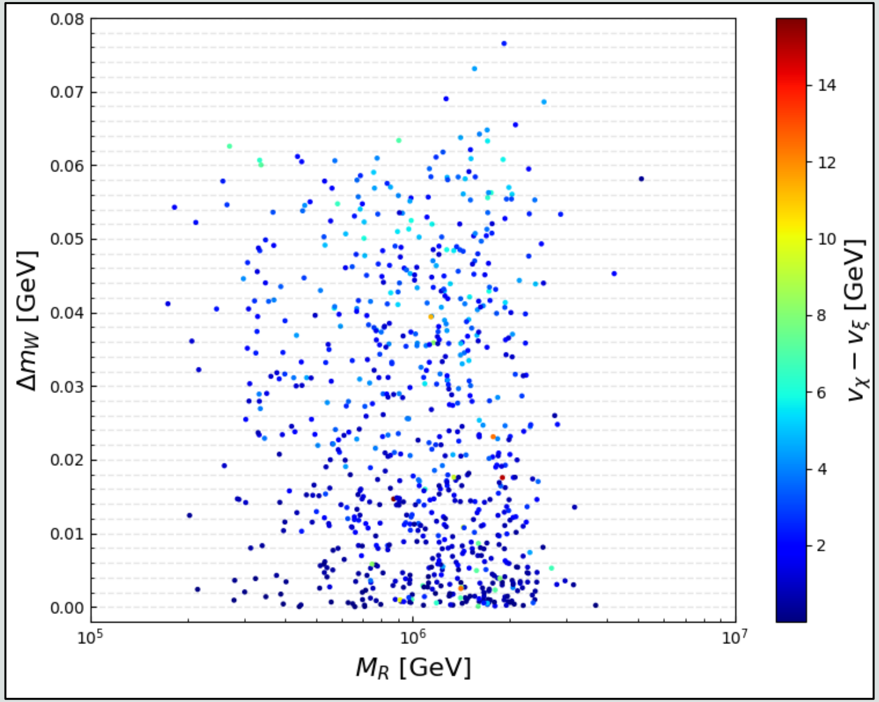
$$BR(\mu \to e\gamma) < 4.2 \times 10^{-13}, BR(\mu \to 3e) < 10^{-12}$$

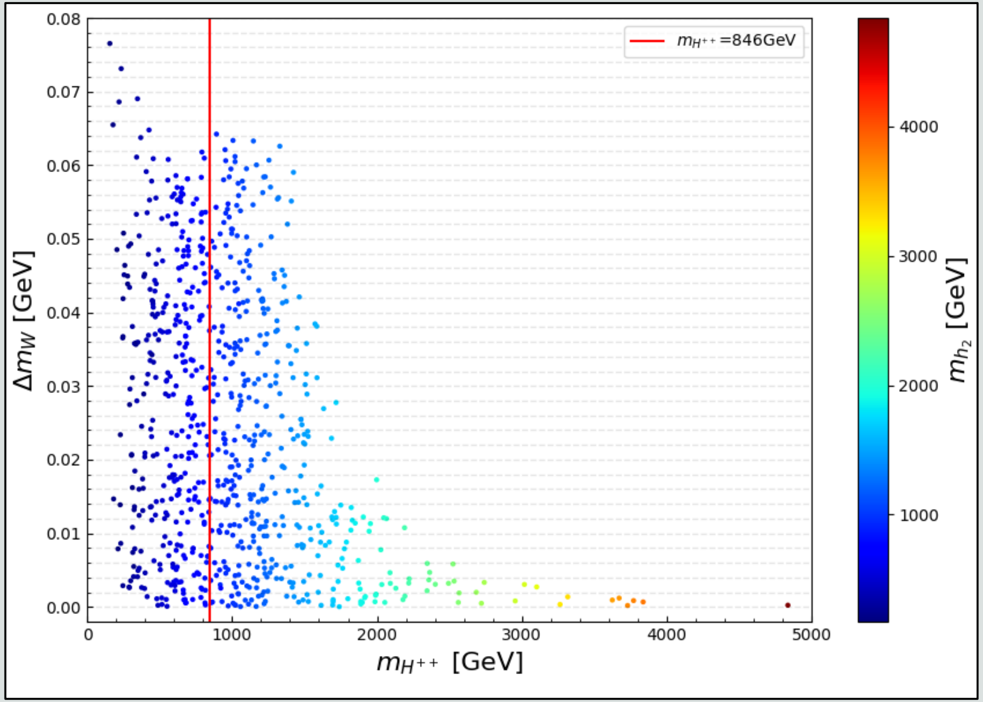
$$BR(\mu \to e\gamma) \sim \frac{\alpha_{EM}}{192\pi} |h_{ij}|^4 \left(\frac{m_W}{M_{H^{++}}}\right)^4 \Rightarrow \quad h_{ij} < 10^{-2}, \quad M_R > 50\text{TeV}$$





3 $\Delta v = v_\chi - v_\xi > 0$ + Type-I + Type-II





01 The supersymmetric custodial triplet model (SCTM)

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix} \Rightarrow \Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ \phi^- & \phi^0 \end{pmatrix}, \Delta = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 \end{pmatrix}$$

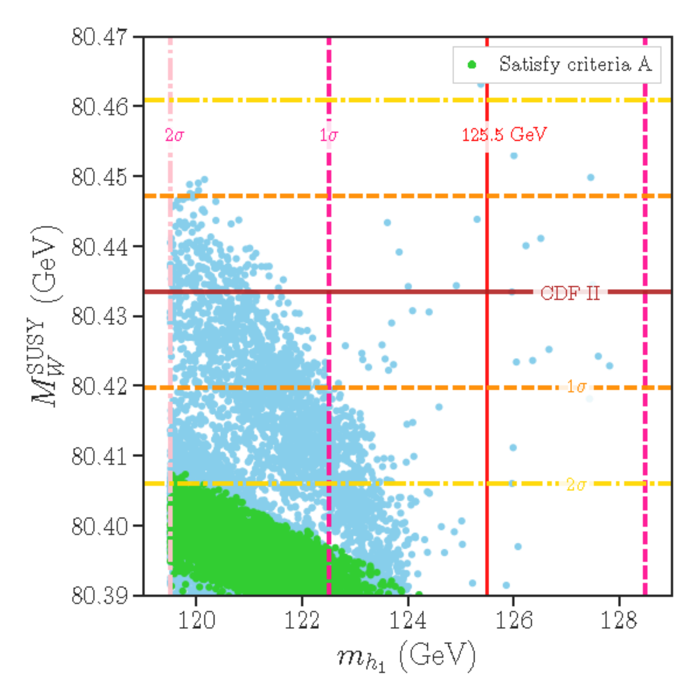
$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix},$$

$$\Sigma_+ = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \Sigma_0 = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}, \Sigma_- = \begin{pmatrix} \psi^0 \\ \psi^- \\ \psi^{--} \end{pmatrix} \Rightarrow \bar{H} = \begin{pmatrix} H_d \\ H_u \end{pmatrix}, \quad \bar{\Delta} = \begin{pmatrix} -\Sigma_0/\sqrt{2} & -\Sigma_{-1} \\ -\Sigma_1 & \Sigma_0/\sqrt{2} \end{pmatrix}$$

$$W_0 \supset \lambda \bar{H}\bar{\Delta}\bar{H} + \frac{\lambda_3}{3}\mathrm{Tr}(\bar{\Delta}^3) + \frac{\mu}{2}\bar{H}\bar{H} + \frac{\mu_\Delta}{2}\mathrm{Tr}(\bar{\Delta}^2)$$

Phys.Rev.D 88 (2013) 7, 075010 , arXiv:1308.4025 [hep-ph]; Phys.Rev.D 91 (2015) 1, 015016 • e-Print: 1409.5737 [hep-ph]; JHEP 03 (2018) 168, arXiv: 1711.05329 [hep-ph]

01 The supersymmetric custodial triplet model (SCTM)

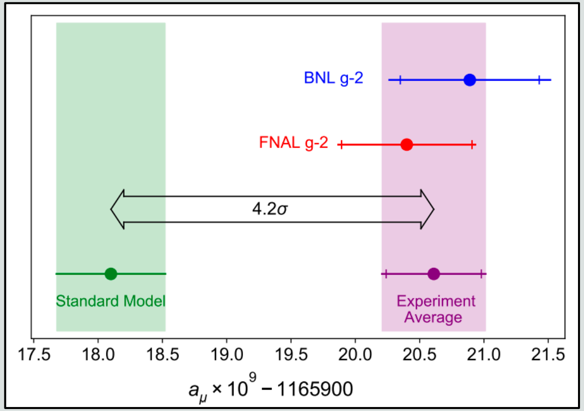


$$\mathbf{M} = \begin{pmatrix} M_1 & 0 & -\frac{1}{\sqrt{2}}g' c_\beta v_H & \frac{\sqrt{2}}{2}g' s_\beta v_H & 0 & -g' v_\Delta & g' v_\Delta \\ 0 & M_2 & \frac{\sqrt{2}}{2}g_2 c_\beta v_H & -\frac{1}{\sqrt{2}}g_2 s_\beta v_H & 0 & g_2 v_\Delta & -g_2 v_\Delta \\ -\frac{1}{\sqrt{2}}g' c_\beta v_H & \frac{1}{\sqrt{2}}g_2 c_\beta v_H & -\sqrt{2}\lambda v_\Delta & -\frac{1}{\sqrt{2}}\lambda v_\Delta - \mu & -\lambda s_\beta v_H & 0 & -2\lambda c_\beta v_H \\ \frac{\sqrt{2}}{2}g' s_\beta v_H & -\frac{1}{\sqrt{2}}g_2 s_\beta v_H & -\frac{1}{\sqrt{2}}\lambda v_\Delta - \mu & -\sqrt{2}\lambda v_\Delta & -\lambda c_\beta v_H & -2\lambda s_\beta v_H & 0 \\ 0 & 0 & -\lambda s_\beta v_H & \lambda c_\beta v_H & \mu_\Delta & -\frac{1}{\sqrt{2}}\lambda_3 v_\Delta & -\frac{1}{\sqrt{2}}\lambda_3 v_\Delta \\ g' v_\Delta & g_2 v_\Delta & 0 & -2\lambda s_\beta v_H & -\frac{1}{\sqrt{2}}\lambda_3 v_\Delta & 0 & \mu_\Delta - \frac{1}{\sqrt{2}}\lambda v_\Delta \\ g' v_\Delta & -g_2 v_\Delta & -2\lambda c_\beta v_H & 0 & -\frac{1}{\sqrt{2}}\lambda_3 v_\Delta & \mu_\Delta - \frac{1}{\sqrt{2}}\lambda_3 v_\Delta & 0 \end{pmatrix}$$

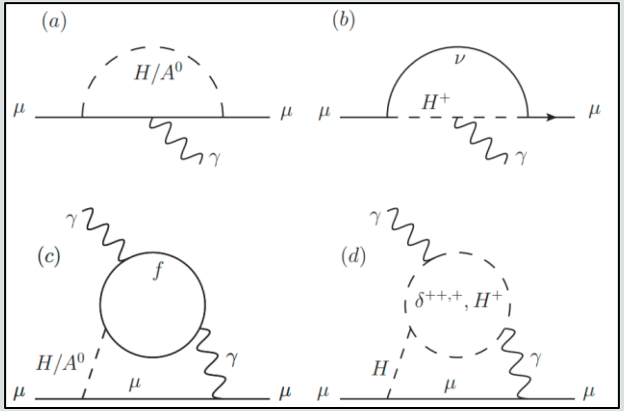
$$m_h^2 = \begin{pmatrix} m_{H_d^{0r}H_d^{0r}} & m_{H_u^{0r}H_d^{0r}} & m_{\chi^{0r}H_d^{0r}} & m_{\phi^{0r}H_d^{0r}} & m_{\psi^{0r}H_d^{0r}} \\ m_{H_d^{0r}H_u^{0r}} & m_{H_u^{0r}H_u^{0r}} & m_{\chi^{0r}H_u^{0r}} & m_{\phi^{0r}H_u^{0r}} & m_{\psi^{0r}H_u^{0r}} \\ m_{H_d^{0r}\chi^{0r}} & m_{H_u^{0r}\chi^{0r}} & m_{\chi^{0r}\chi^{0r}} & m_{\phi^{0r}\chi^{0r}} & m_{\psi^{0r}\chi^{0r}} \\ m_{H_d^{0r}\phi^{0r}} & m_{H_u^{0r}\phi^{0r}} & m_{\chi^{0r}\phi^{0r}} & m_{\phi^{0r}\phi^{0r}} & m_{\psi^{0r}\phi^{0r}} \\ m_{H_d^{0r}\psi^{0r}} & m_{H_u^{0r}\psi^{0r}} & m_{\chi^{0r}\psi^{0r}} & m_{\phi^{0r}\psi^{0r}} & m_{\psi^{0r}\psi^{0r}} \end{pmatrix}$$

Jinmin Yang, Yang Zhang, Sci.Bull. 67 (2022) 14, 1430-1436 , arXiv:2204.04202 [hep-ph]

02 Muon $g-2$ in the GM Model



Phys.Rev.Lett. 126 (2021) 14, 141801



Phys.Rev.D 104 (2021) 5, 055011

03 Leptongenesis in the GM Model

To resolve BAU by the Leptongenesis → Heavy singlet neutrinos with hierarchical mass spectrum → lower bound of about **$10^8 \sim 10^9$GeV**

Leptonic CP Asymmetry → Mass differences of heavy majorana neutrinos comparable to their decay width / Resonant Effect → lower bound of about ~ **TeV**

Phys.Rev.D 56 (1997) 5431-5451

1. Taken a general discussion about the contribution to W boson mass in the original GM Model;
2. Explaining CDF-II results in the GM Extension Models. Misalignment among the triplet VEVs and large h_{ij} couplings allowed with RH neutrino sector;
3. Works what to do right now. To explain W mass shift in SCTM, to investigate $g_\mu - 2$ and BAU by leptongenesis in GM model with right hand neutrino extension.

THANKS

Q&A