北京航空航天大學
BEIHANG UNIVERSITY

Lattice Parton Collaboration

Unpolarized Transverse Momentum Dependent Parton Distributions from Lattice QCD

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Oct. 30, 2022 @ 26th Mini-workshop on the frontier of LHC

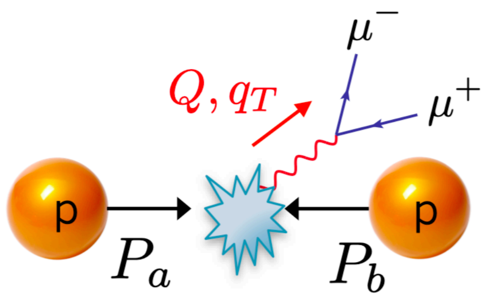
OUTLOOK

- Motivation
  - What's and why TMDs?
  - Progress in the study of TMD PDFs
- TMD PDFs from lattice QCD
  - LaMET formalism
  - Quasi TMD PDF and its renormalization
  - Unpolarized TMD PDF
- Summary and outlook

What's and why TMDs?

TMD processes:

Drell-Yan

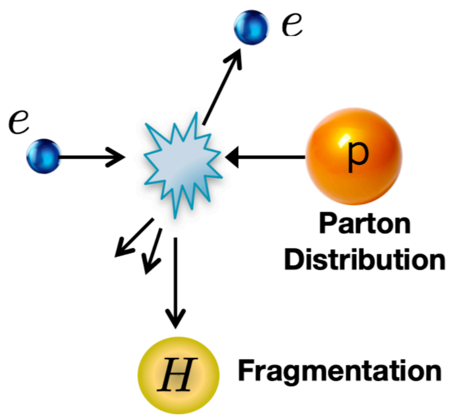


$q_T \ll Q$

LHC, FermiLab, RHIC, ...

$$\sigma \sim f_{q/P}(x, k_T)\, f_{q/P}(x, k_T)$$

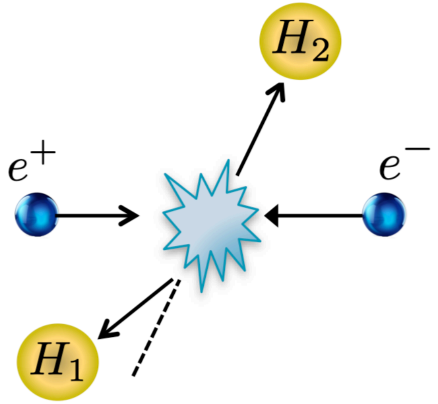
Semi-Inclusive DIS



HERMES, COMPASS, JLab, EIC, ...

$$\sigma \sim f_{q/P}(x, k_T)\, D_{h/q}(x, k_T)$$

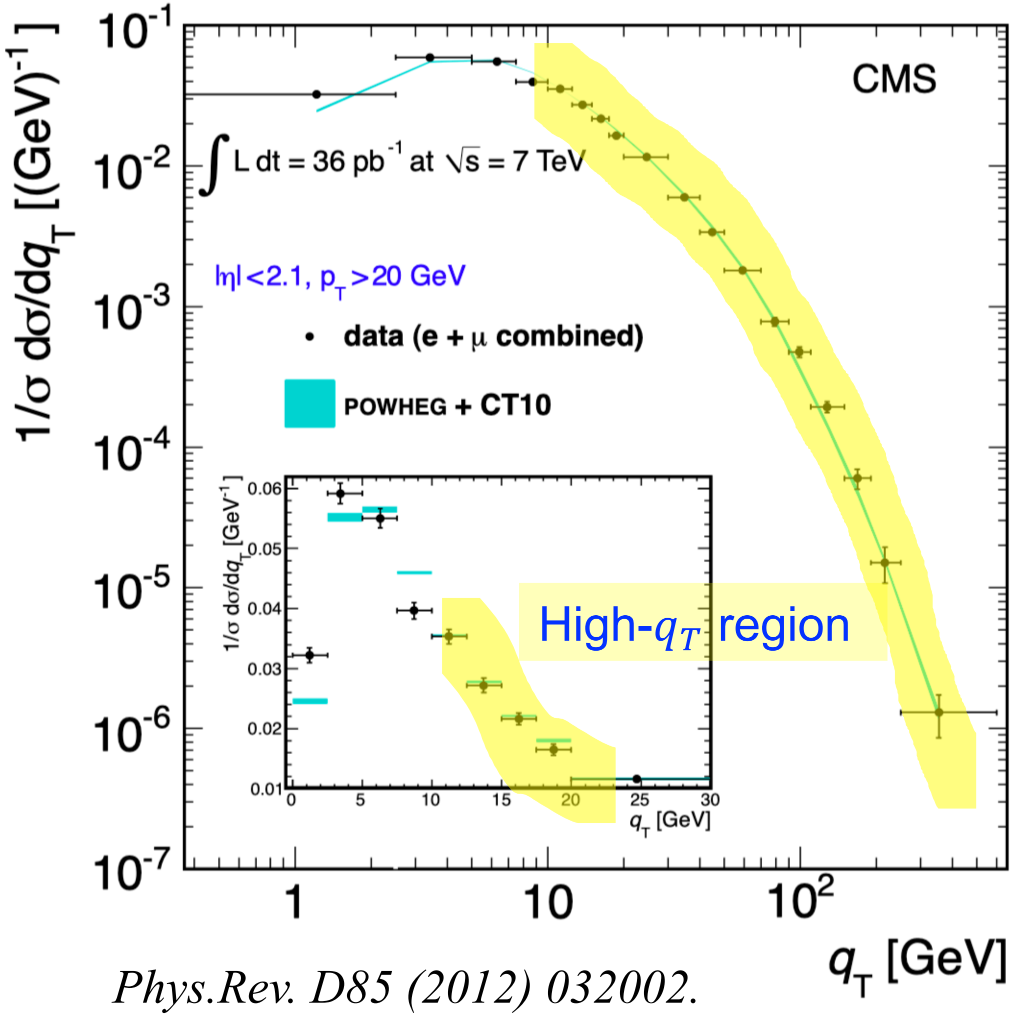
Dihadron in e^+e^-



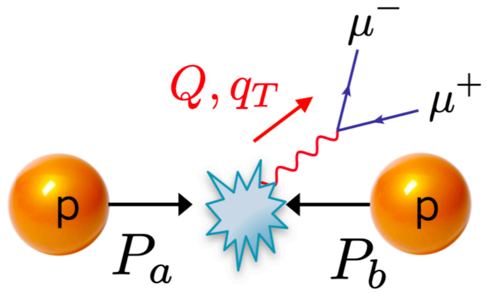
BESIII, Babar, Belle, ...

$$\sigma \sim D_{h_1/q}(x, k_T)\, D_{h_2/q}(x, k_T)$$

Z-production q_T spectrum at LHC



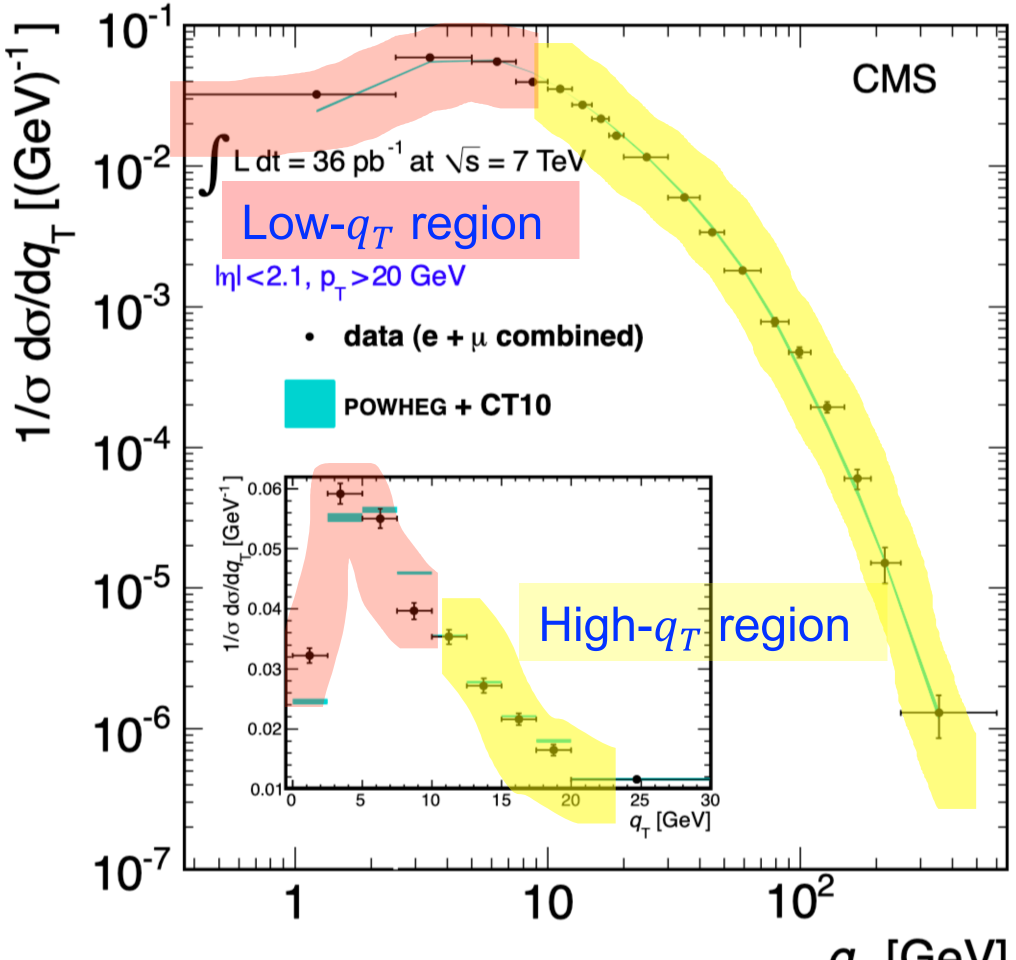
Phys.Rev. D85 (2012) 032002.



- **High-q_T region:**

  Collinear factorization => PDF

Z-production q_T spectrum at LHC



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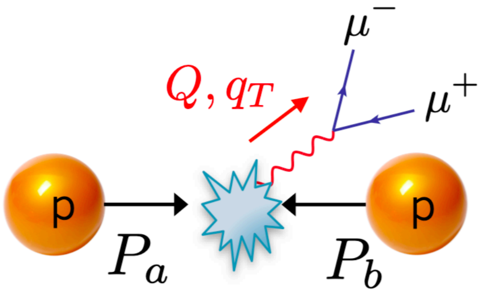
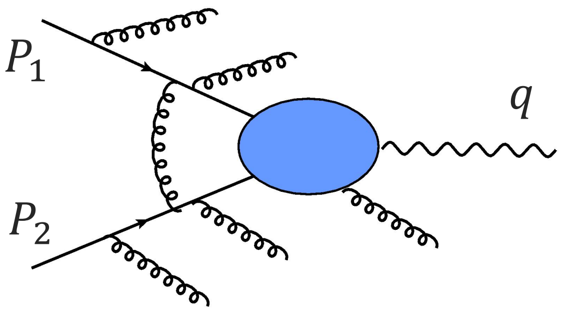


- **High-q_T region:**

  Collinear factorization => PDF

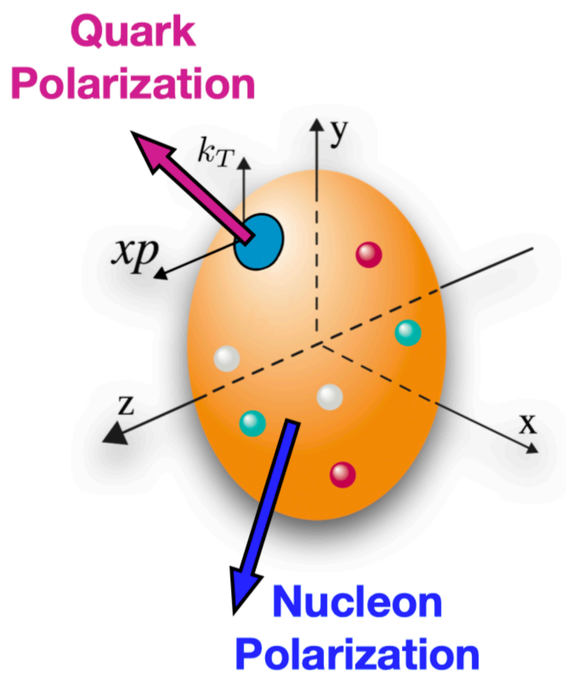
- **Low-q_T region:**



TMD factorization

=> Generalize to TMDPDFs

TMDPDFs: 3D tomography of the proton



Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
	L		g_{1L} = Helicity	$h_{1L}^\perp$ = Worm-gear
	T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity $h_{1T}^\perp$ = Pretzelosity

TMD Handbook, TMD Collaboration, to appear soon

Progress in the study of TMDs

- **Theoretical analysis**
  - **TMD factorization, evolution and resummation:**

    Collins, Foundations of perturbative QCD; ……

- **Phenomenological extractions**
  - **Unpolarized:**

    Bacchetta, JHEP06 (2017); Scimem, JHEP06 (2020);
    Bury, 2201.07114; Bacchetta, 2206.07598; ……

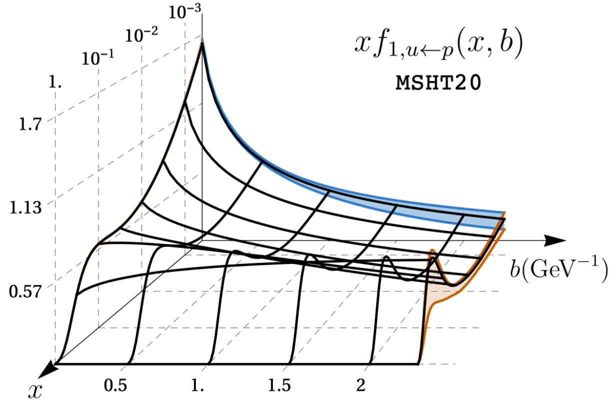
  - **Sivers:**

    Bury, PRL126 (2021), JHEP05 (2021) ; ……

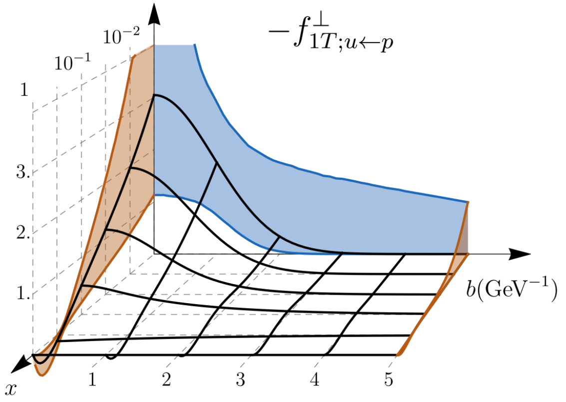
  - **Boer-Mulders:**

    Zhang, PRD77 (2008), Lu, PRD81 (2010) ; ……

  - **Others: worm-gear, gluon TMDs, ……**



u-quark unpolarized TMDPDF, 2201.07114



u-quark Sivers function, PRL126 (2021)

➢ Lattice calculations

- **Lorentz-invariant approach:** ratios of Mellin moments

  Hagler, EPL88(2009); Musch, PRD85(2012); Yoon, 1601.05717, PRD96(2017); ……

- **LaMET formalism:**

  ✓ **Preparation I:** theoretical framework of calculating TMDs and related soft function, Collins-Soper kernel, beam function…

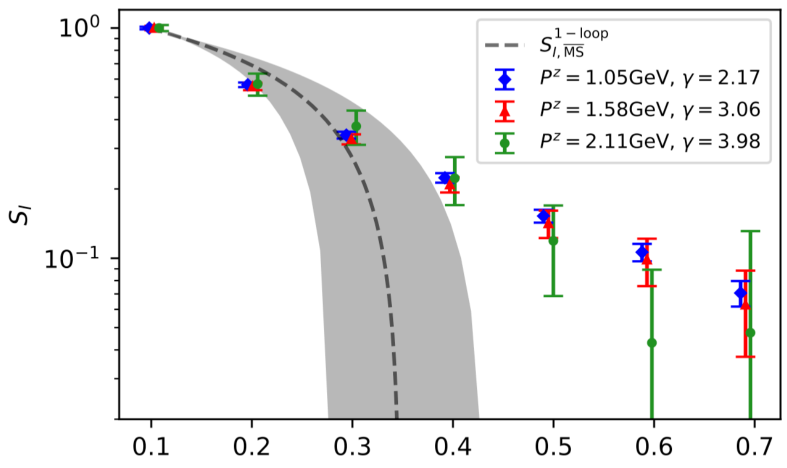
  Ji, RMP93(2021), NPB955(2020), PLB811(2020); Ebert, JHEP04(2022); Deng, JHEP09(2022)……

  ✓ **Preparation II:** lattice calculation of soft function and Collins-Soper kernel;

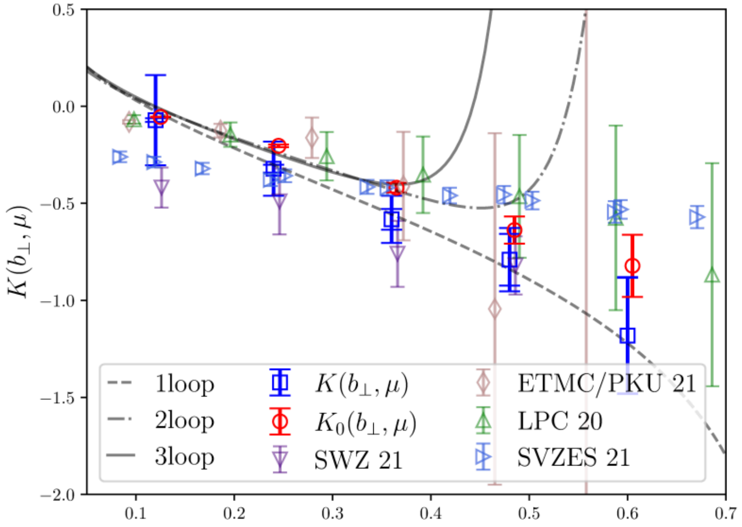
  LPC, PRL125(2020); Li, PRL128(2022); LPC, PRD106(2022); Shanahan, PRD104(2021); Schlemmer, JHEP08(2021); ……

  ✓ **Preparation III:** Renormalization, resummation

  LPC, PRL129(2022); 2209.01236……



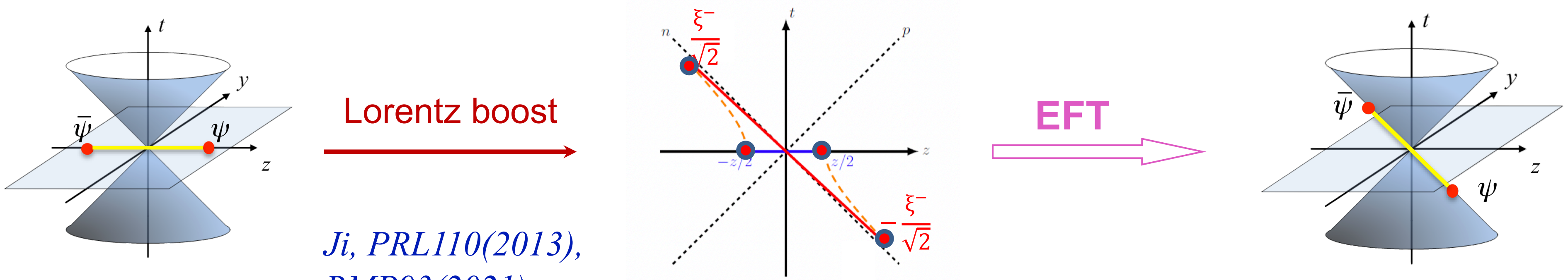
First lattice result of soft function, PRL125(2020)



Lattice calculations of Collins-Soper kernel, PRD106(2022)

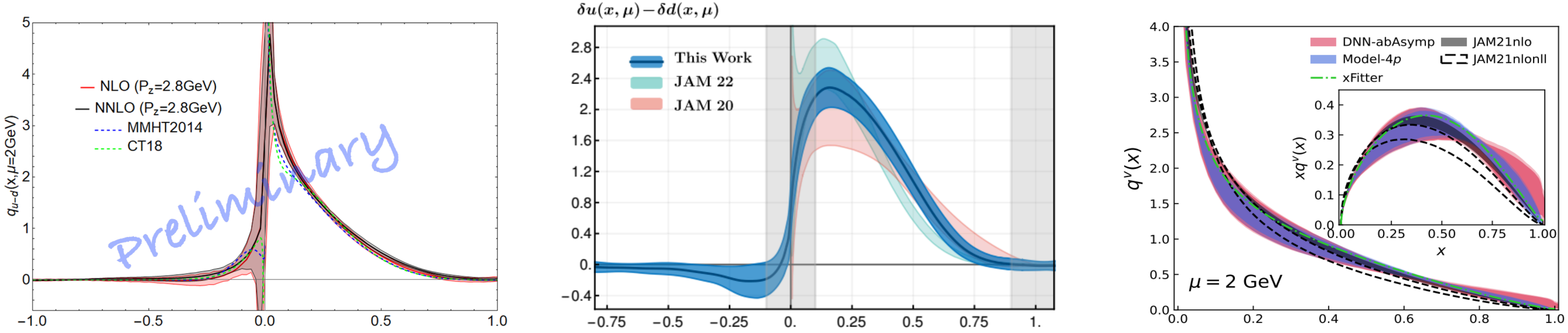
Extracting TMDs in LaMET formalism

- **Large-momentum effective theory: connecting Euclidean lattice and physical observables**



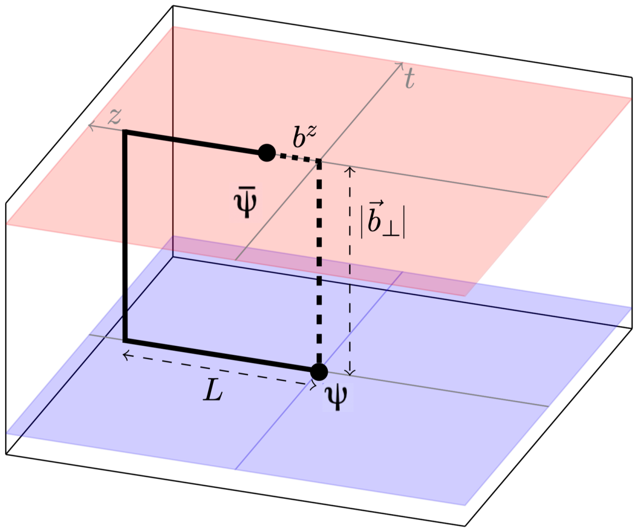
Ji, PRL110(2013), RMP93(2021), …

- **Achieved great success in the studies of PDF:**



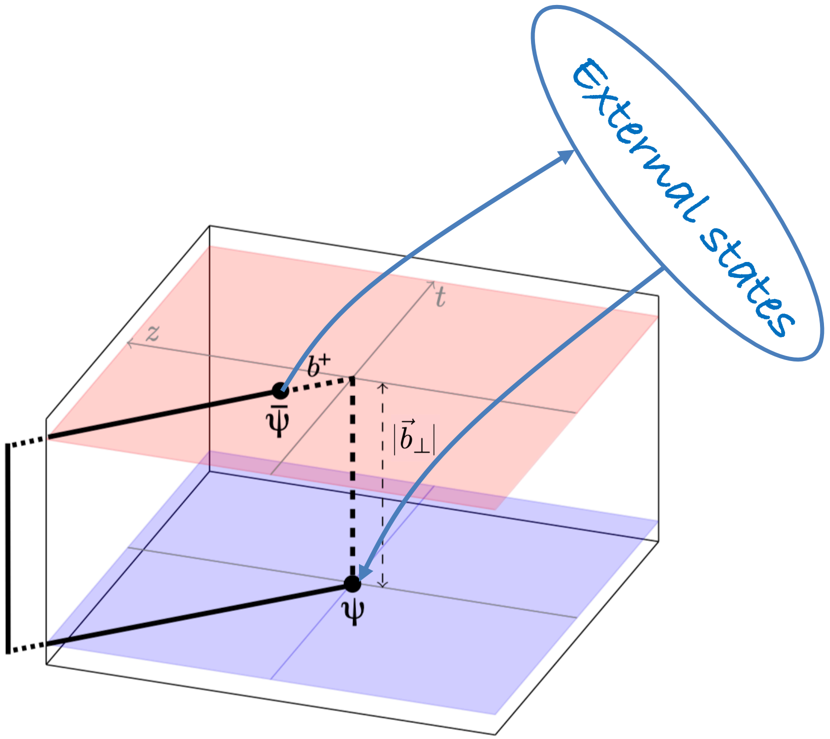
Proton unpolarized PDF, in preparation *Proton transversity PDF, 2208.08008* *Pion valance PDF, 2208.02297*

- **Matching from quasi TMDs to TMDs**



Lorentz boost

$L \to \infty$



Equal-time correlators,
directly calculable on lattice

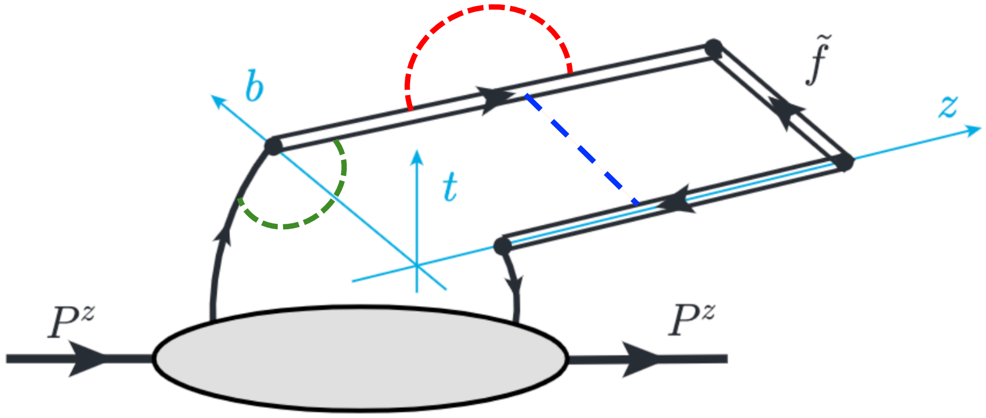
Space-like correlators,
NO effective method for directly calculation

Connected at large-momentum limit

Ji, PLB811(2020); Ebert, JHEP04(2022)

$$\tilde{f}_\Gamma\left(x, b_\perp, \zeta_z, \mu\right) \sqrt{S_I\left(b_\perp, \mu\right)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K\left(b_\perp, \mu\right)} f\left(x, b_\perp, \mu, \zeta\right) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{\zeta_z}, \frac{M^2}{\left(P^z\right)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

Renormalization of quasi TMDs



Linear divergence

from the self-energy of the Wilson line

Pinch-pole singularity

from heavy quark effective potential term

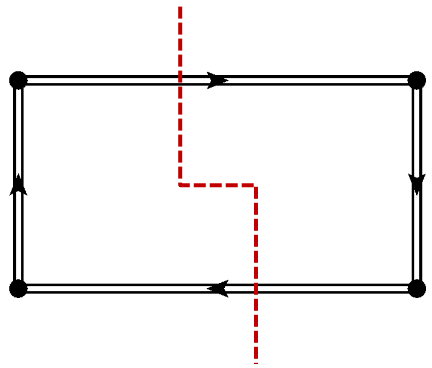
Ji, PRL120(2018),NPB964(2021); Ishikawa, PRD96(2017); Green, PRL121(2018); Shanahan, PRD101(2020); ……

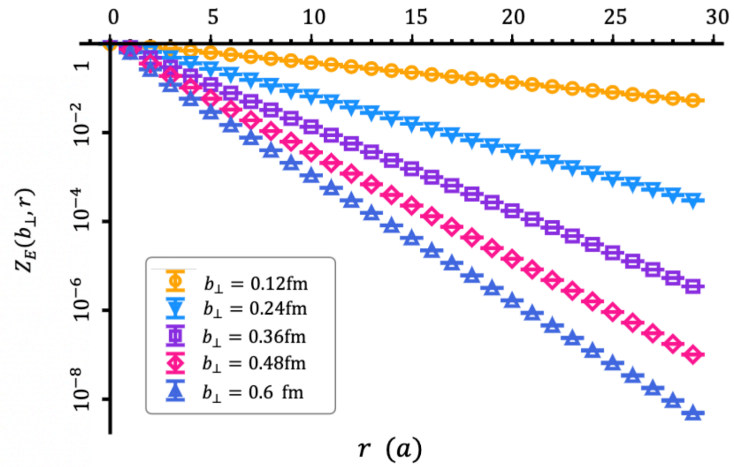
Logarithm divergence

from the vertices correction between Wilson line and light quark

Ji, PLB257(1991); Huo, NPB969(2021); Zhang, PRL129(2022); ……

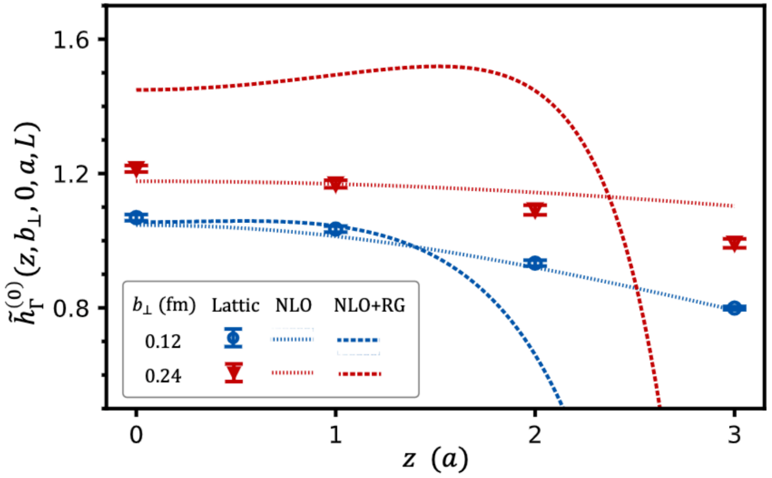
- **Divergences from purely Wilson links can be subtracted by "half" Wilson loop:**

$Z_E =$ 

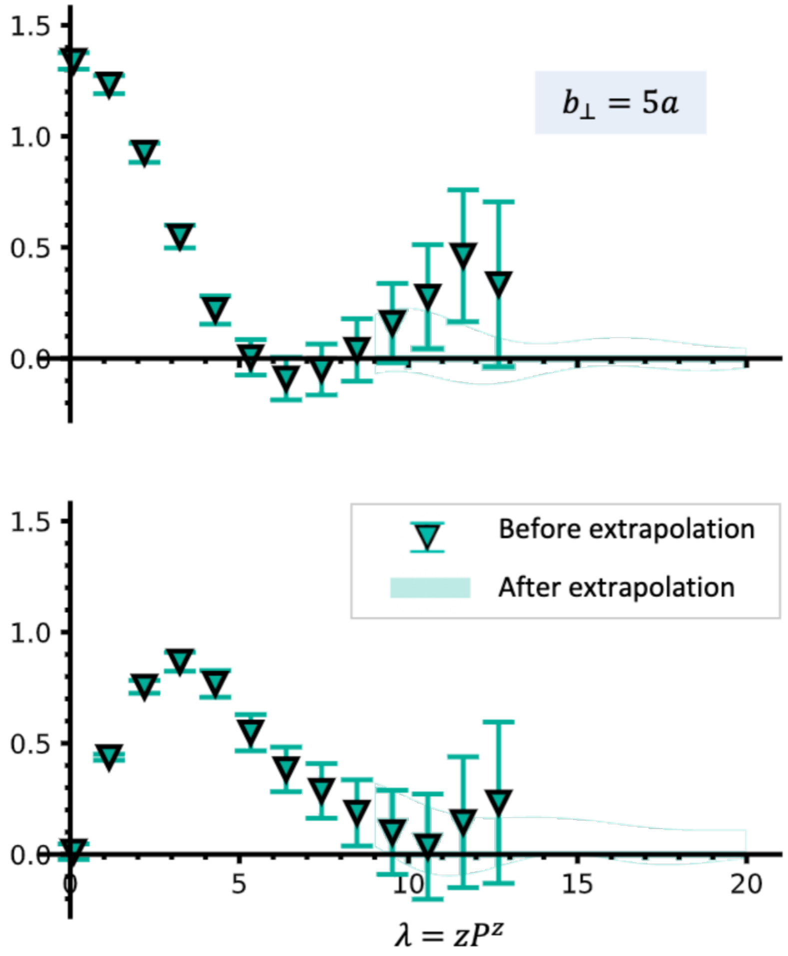


- **Divergence from quark-gauge link vertex correction can be subtracted by renormalization factor Z_O**

$$Z_O(1/a, \mu, \Gamma) = \lim_{L\to\infty} \frac{\tilde{h}^0_\Gamma(z, b_\perp, 0, a, L)}{\sqrt{Z_E(2L+z, b_\perp, a)}\tilde{h}^{\overline{\rm MS}}_\Gamma(z, b_\perp, \mu)}$$

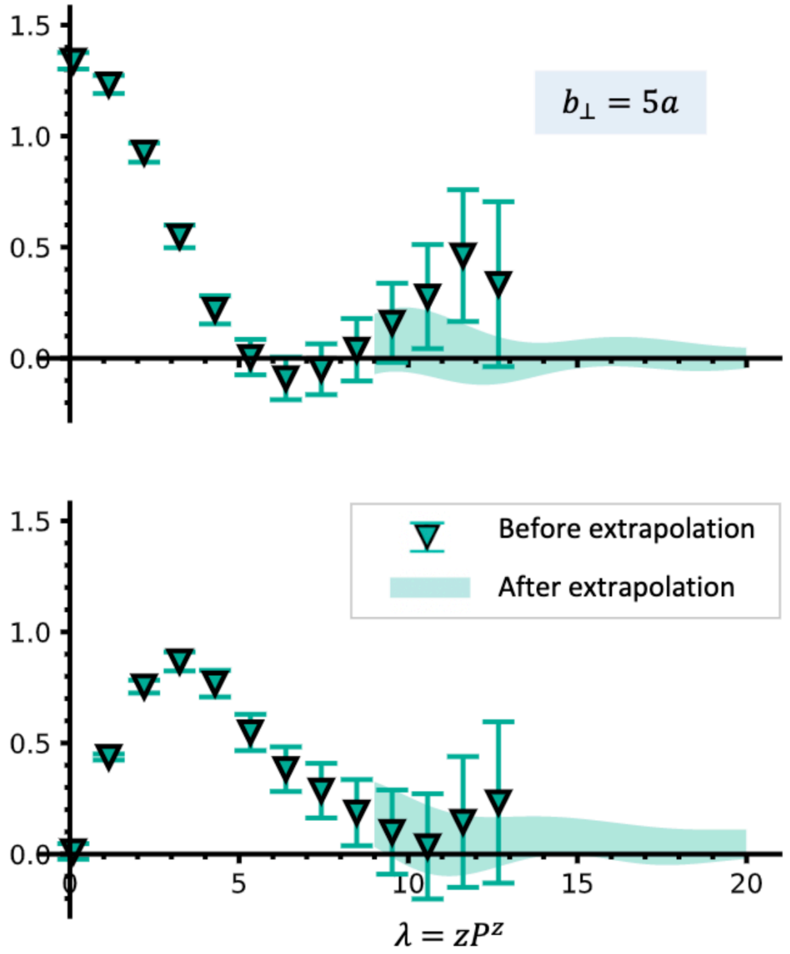


Renormalized quasi TMDPDF



Error grows at large λ region ……

Renormalized quasi TMDPDF



Physical based extrapolation:

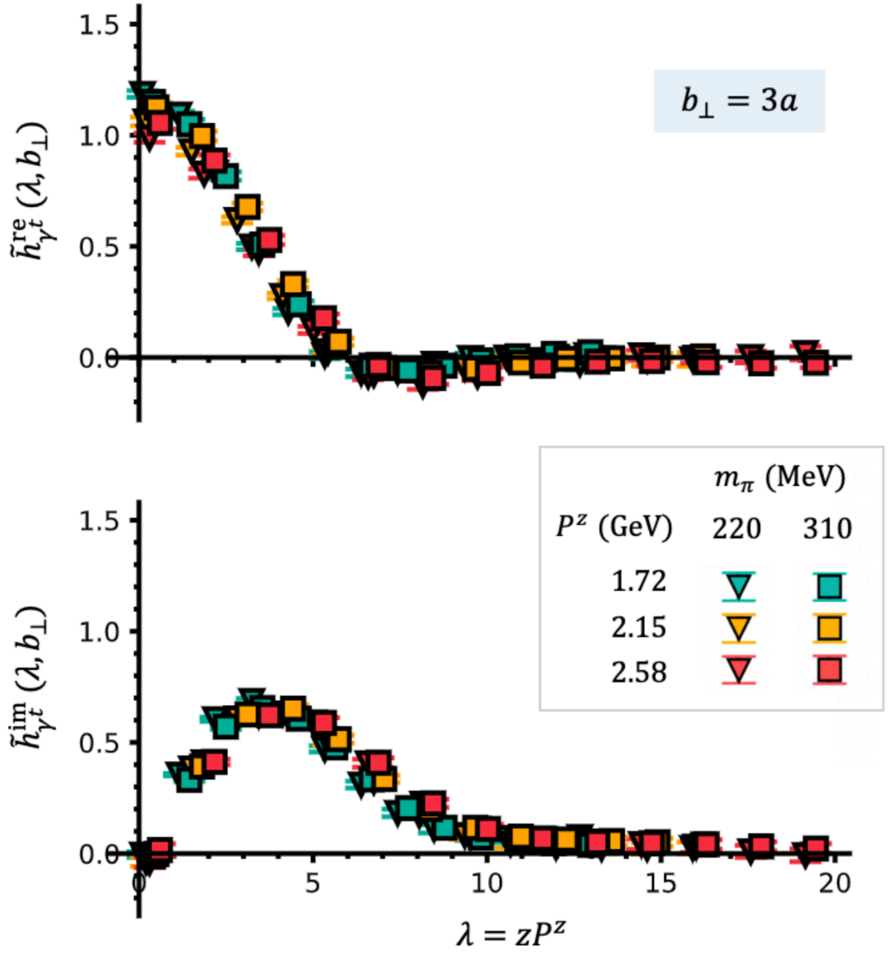
$$\tilde{h}_{\text{extra}}(\lambda) = \left[\frac{c_1}{(-i\lambda)^{n_1}} + e^{i\lambda}\frac{c_2}{(i\lambda)^{n_2}}\right] e^{-\lambda/\lambda_0}$$

- end point power-law behavior $x^a(1-x)^b$;
- correlation function has a finite correlation length λ_0.

😢 **End point region in momentum space will be affected ……**

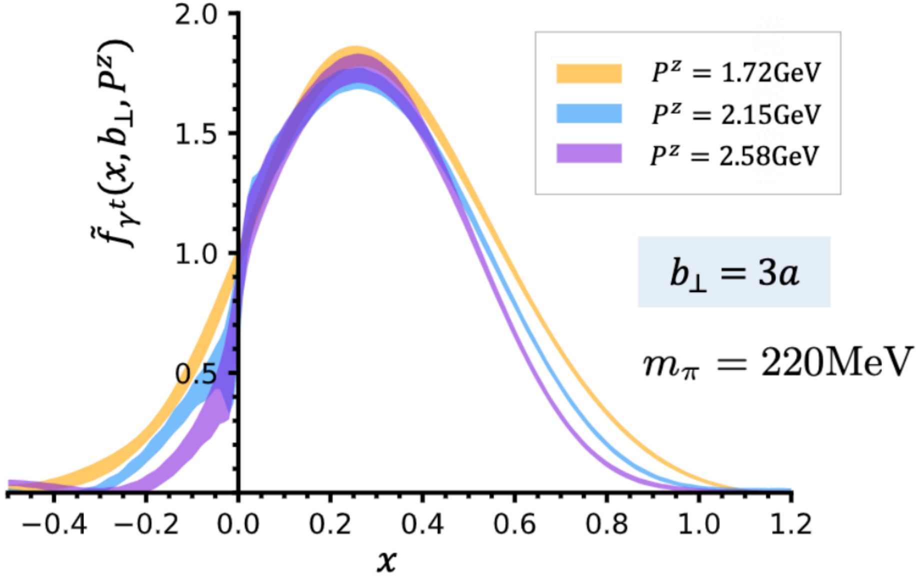
😁 **LaMET is not reliable in that region too ……**

Renormalized quasi TMDPDF



$$\tilde{f}_\Gamma\left(x, b_\perp, P^z, \mu\right) \equiv \lim_{L\to\infty} \int \frac{dz}{2\pi} e^{-iz(xP^z)} \frac{\tilde{h}^0_\Gamma\left(z, b_\perp, P^z, L\right)}{\sqrt{Z_E\left(2L+z, b_\perp\right)} Z_O\left(1/a, \mu, \Gamma\right)}$$

FT



1-loop matching and RG resummation

Factorization formula of quasi TMDPDF:

$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right)K(b_\perp,\mu)} f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2\zeta_z}\right)$$

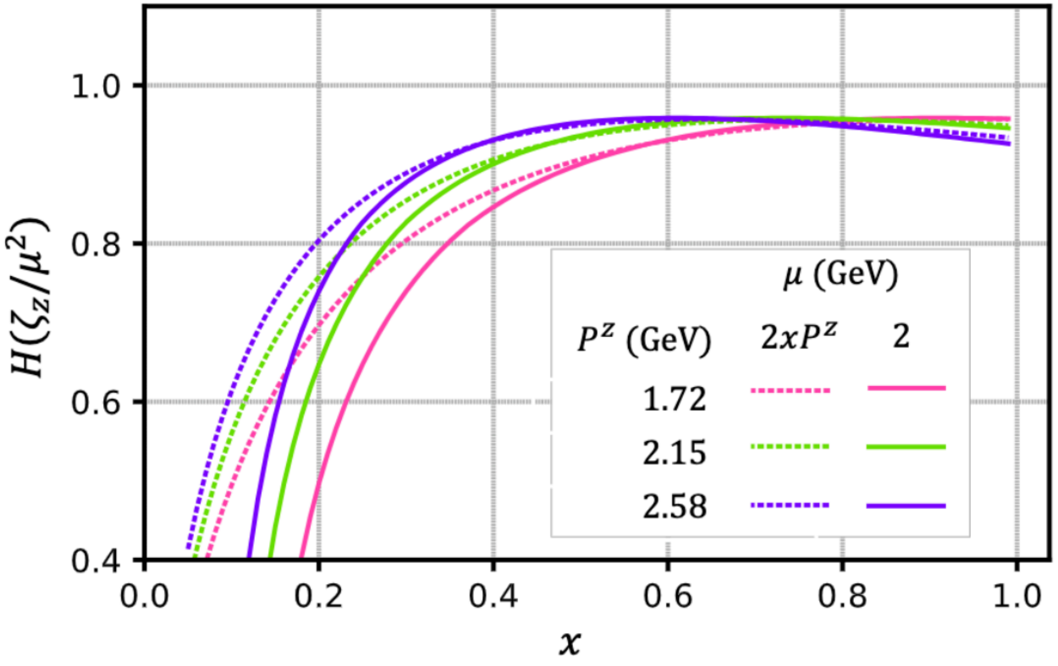
1-loop matching kernel given by *PLB811(2020)* => Fix order

Resum the large logarithms ~ $\log(\mu^2/(2xP^z)^2)$

through the RGE of H:

$$\mu^2\frac{d}{d\mu^2}\ln H\left(\frac{\zeta_z}{\mu^2}\right) = \frac{1}{2}\Gamma_{\mathrm{cusp}}(\alpha_s)\ln\frac{\zeta_z}{\mu^2} + \frac{\gamma_C(\alpha_s)}{2}$$

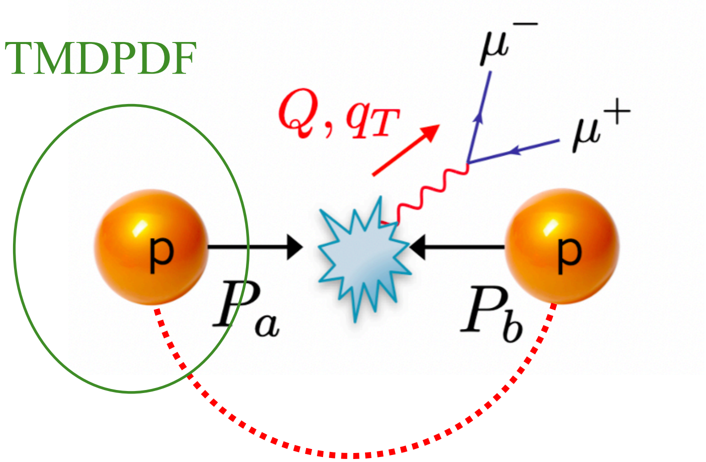
$\mu_0 = 2xP^z$ => 2GeV



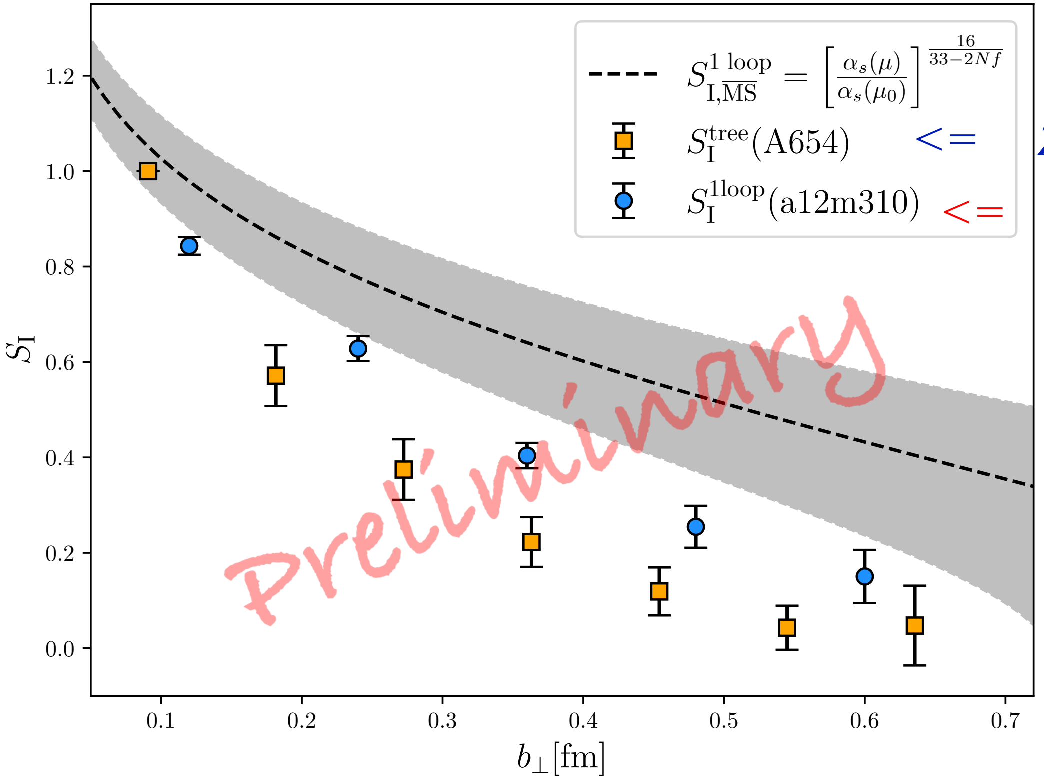
Soft function

- **Rapidity-independent part: intrinsic/reduced soft function**

Ji, NPB955(2020); Zhang, PRL125(2020); Li, PRL128(2022); ……



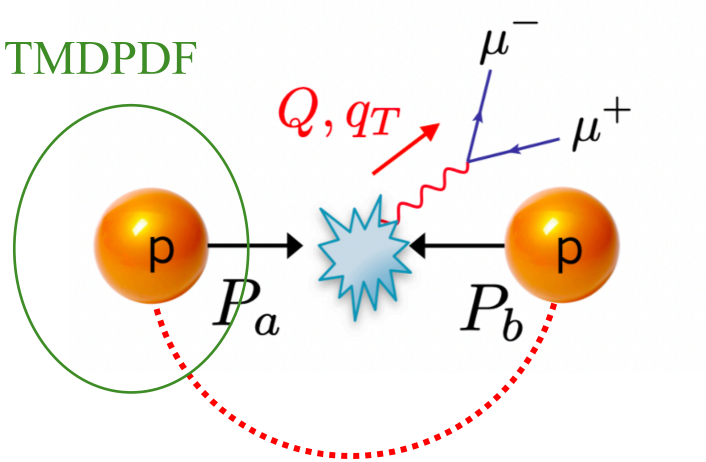
Soft gluon emission => **soft function**



Zhang, PRL125(2020)

Chu, 2211.xxxx

Soft function



Soft gluon emission => **soft function**

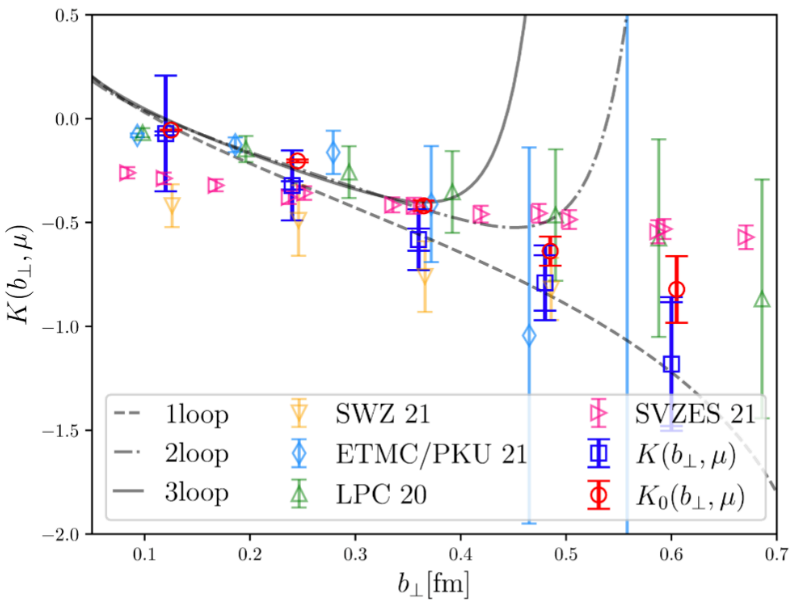
- **Rapidity-dependent part: Collins-Soper kernel**

From quasi WF:

Chu, PRD106(2022); Zhang, PRL125(2020); Li, PRL128(2022); ……

From quasi beam function:

Shanahan, PRD104(2021); Schlemmer, JHEP08(2021); ……

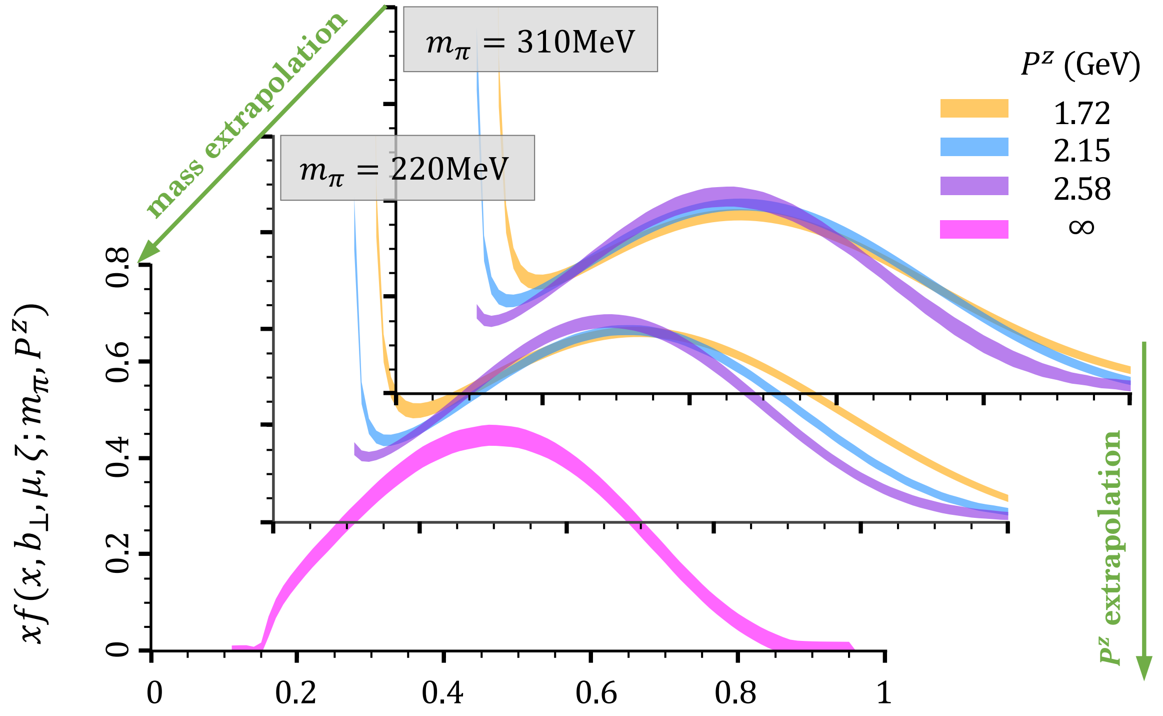


Chu, PRD106(2022)

Physical TMDPDF

- **Physical point & infinite momentum combined extrapolation**

$$f_\Gamma(x, b_\perp, \mu, \zeta; m_\pi, P^z) = f_\Gamma(x, b_\perp, \mu, \zeta)\Big|_{\substack{m_\pi \to m_{\pi,\mathrm{phy}} \\ P^z \to \infty}} \left[1 + c_0\left(m_\pi^2 - m_{\pi,\mathrm{phy}}^2\right) + \frac{c_1}{(P^z)^2}\right]$$



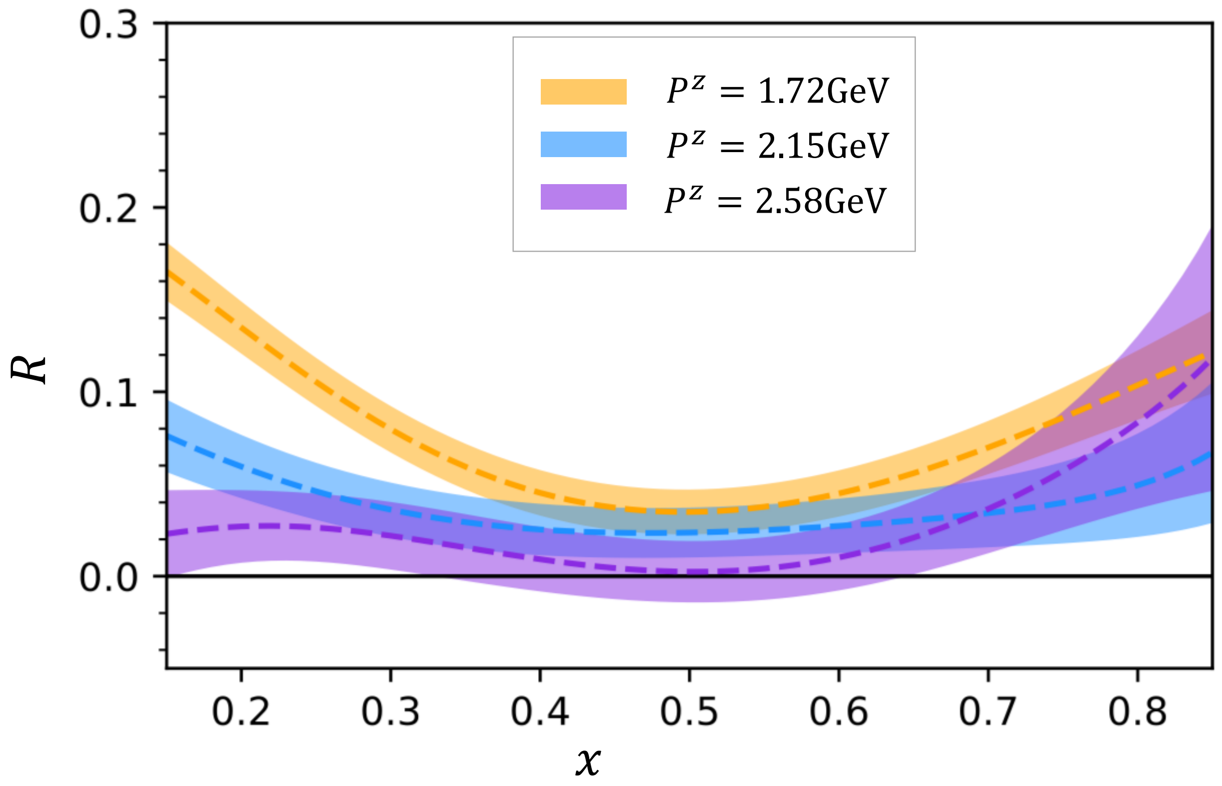
Pion mass effect:

310MeV + 220MeV => 135MeV

Power corrections: ~ $O\left((P^z)^{-2}\right)$

Uncertainties Estimation

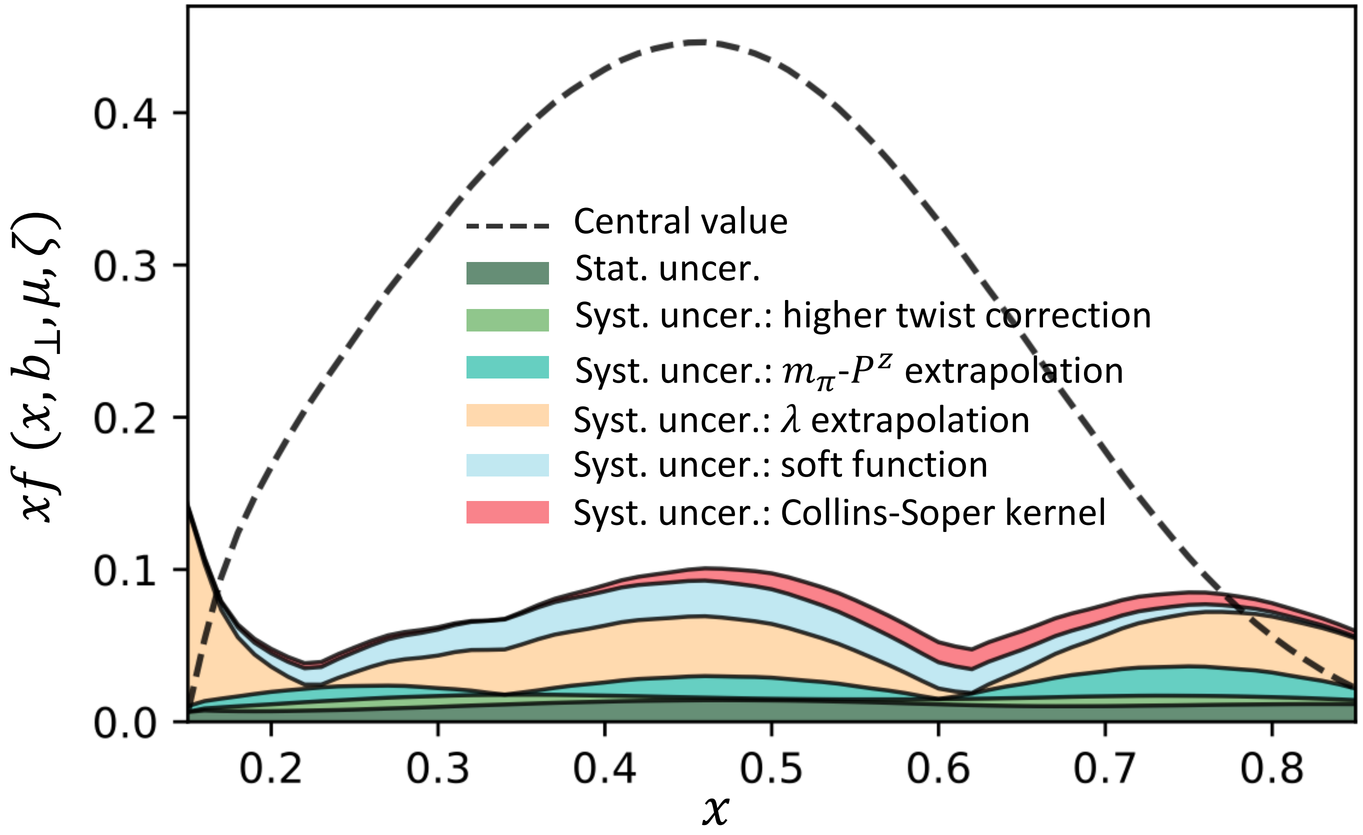
- **Power corrections:**



Difference between highly boosted γ^t and γ^z correlators are start from twist-3 with opposite sign

$$R \equiv \frac{f_{\gamma^t} - f_{\gamma^z}}{f_{\gamma^t} + f_{\gamma^z}}$$

- **Comparison of all uncertainties:**



- **Higher twist:**

  from the R value of largest P^z

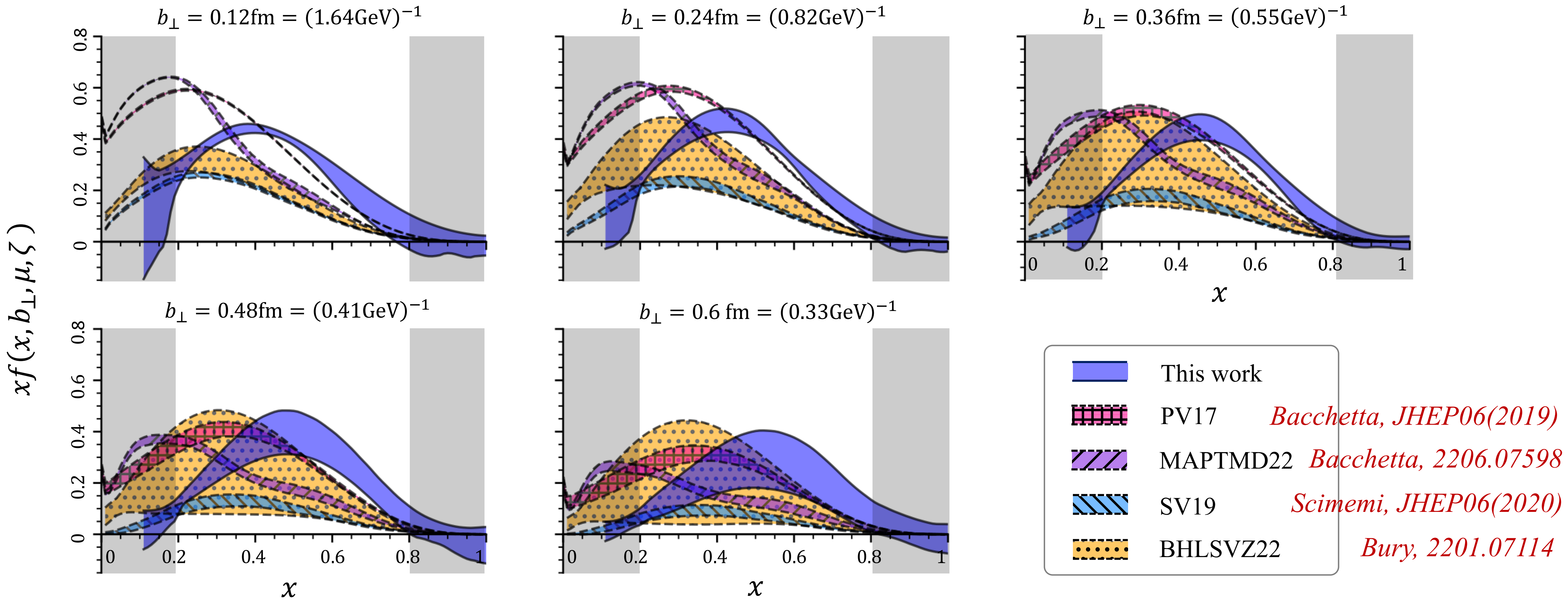
- **m_π-P^z extrapolation:**

  extrapolated vs largest P^z / lightest m_π

- **λ extrapolation:**

  freedom in choosing fit range

Comparison with phenomenological results



Summary and outlook

- **The first lattice calculation of unpolarized TMDPDF under LaMET framework;**

- **The state-of-the-art techniques in analysis:**

  Renormalization, extrapolation, 1-loop matching and RGE, uncertainties estimation, ……

- **A small step for the first-principle calculations of TMDs ……**

  Finer lattice, large P^z and $b_\perp$, more accurate perturbative contributions, ……

Thanks for your attentions!