The Realistic Scattering of Puffy Dark Matter

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Outline

I Motivation

II Size dark matter-dark matter elastic scattering

III Self-interaction puffy DM

IV Conclusion and discussion

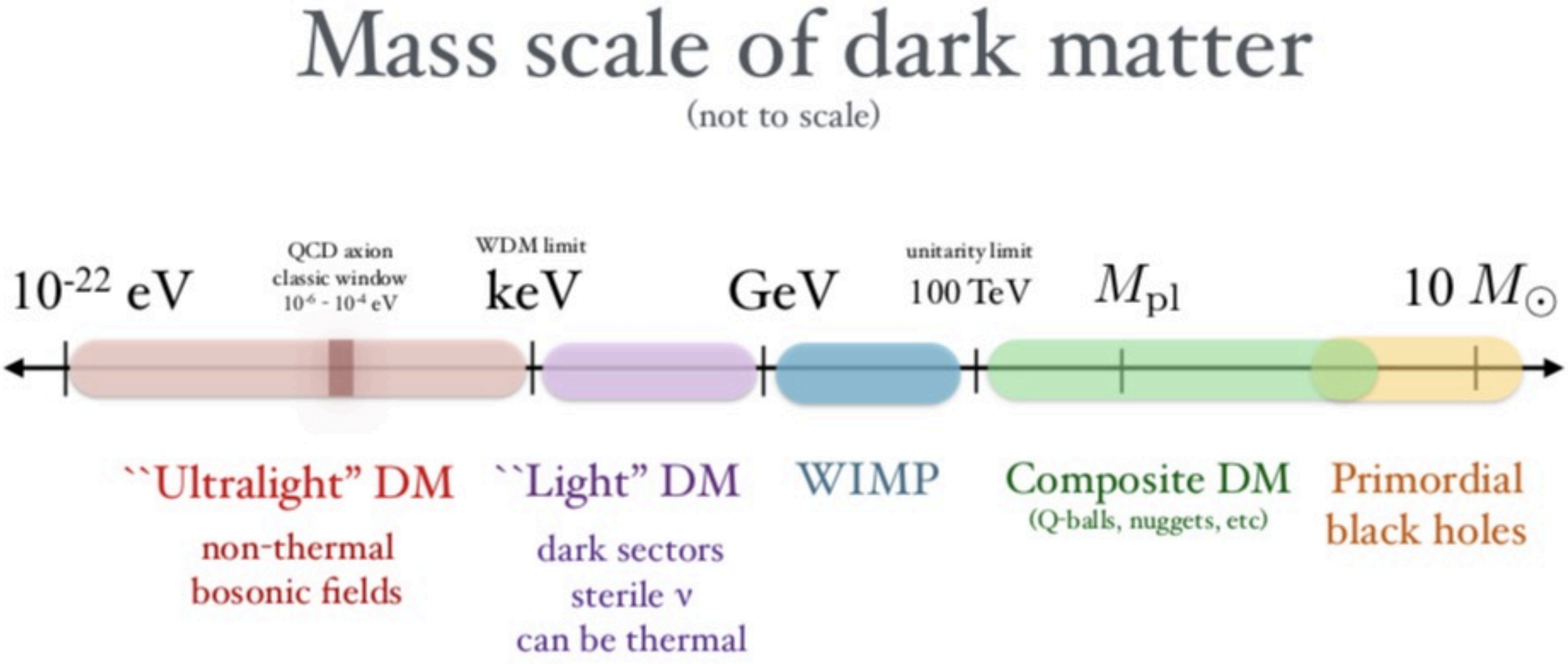
Motivation

Self-interaction DM

Proton-Proton elastic scattering

Puffy DM-DM elastic scattering

Motivation


Mass scale of dark matter
(not to scale)
10^{-22} eV
QCD axion classic window 10^{-6} - 10^{-4} eV
WDM limit
keV
GeV
unitarity limit
100 TeV
$M_{\rm pl}$
10 $M_\odot$
``Ultralight” DM
non-thermal bosonic fields
``Light” DM
dark sectors
sterile ν
can be thermal
WIMP
Composite DM
(Q-balls, nuggets, etc)
Primordial black holes

Motivation

Dark matter evolution

ΛCDM model is further in accordance with the Astronomical Data. It shows that DM is non-relativistic and collisionless in the large scale structure of the universe.

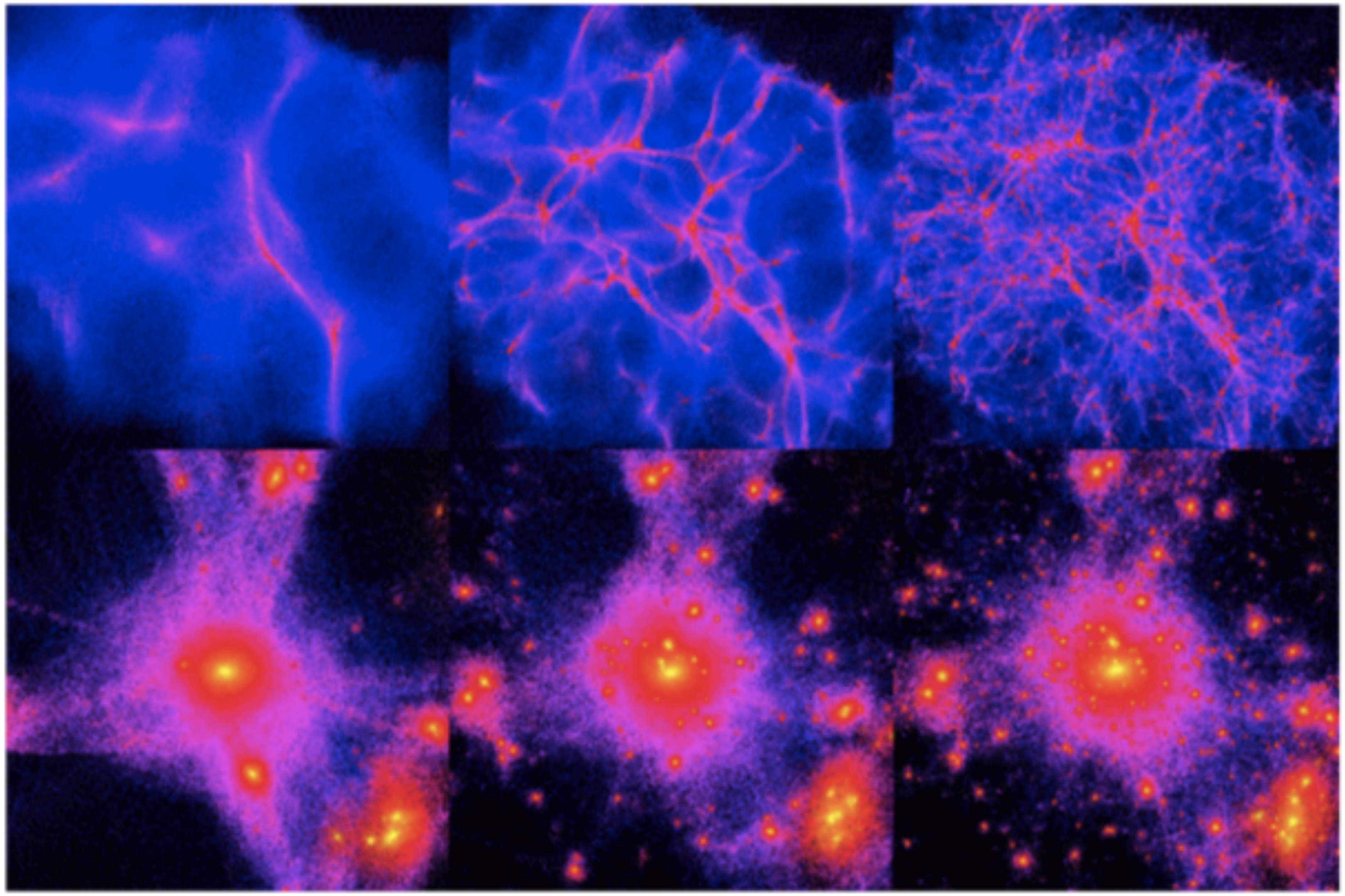


图 2.2.2: 宇宙大尺度结构的数值模拟。上图是冷暗物质模型，下图是热暗物质模型。

Motivation

Self-interaction DM in the small scale

core-vs-cusp problem

The discrepancy is from the steep cusps predicted by the collisionless cold DM(CCDM) and the flat cores for the density profile of the DM holes

too-big-to fail problem

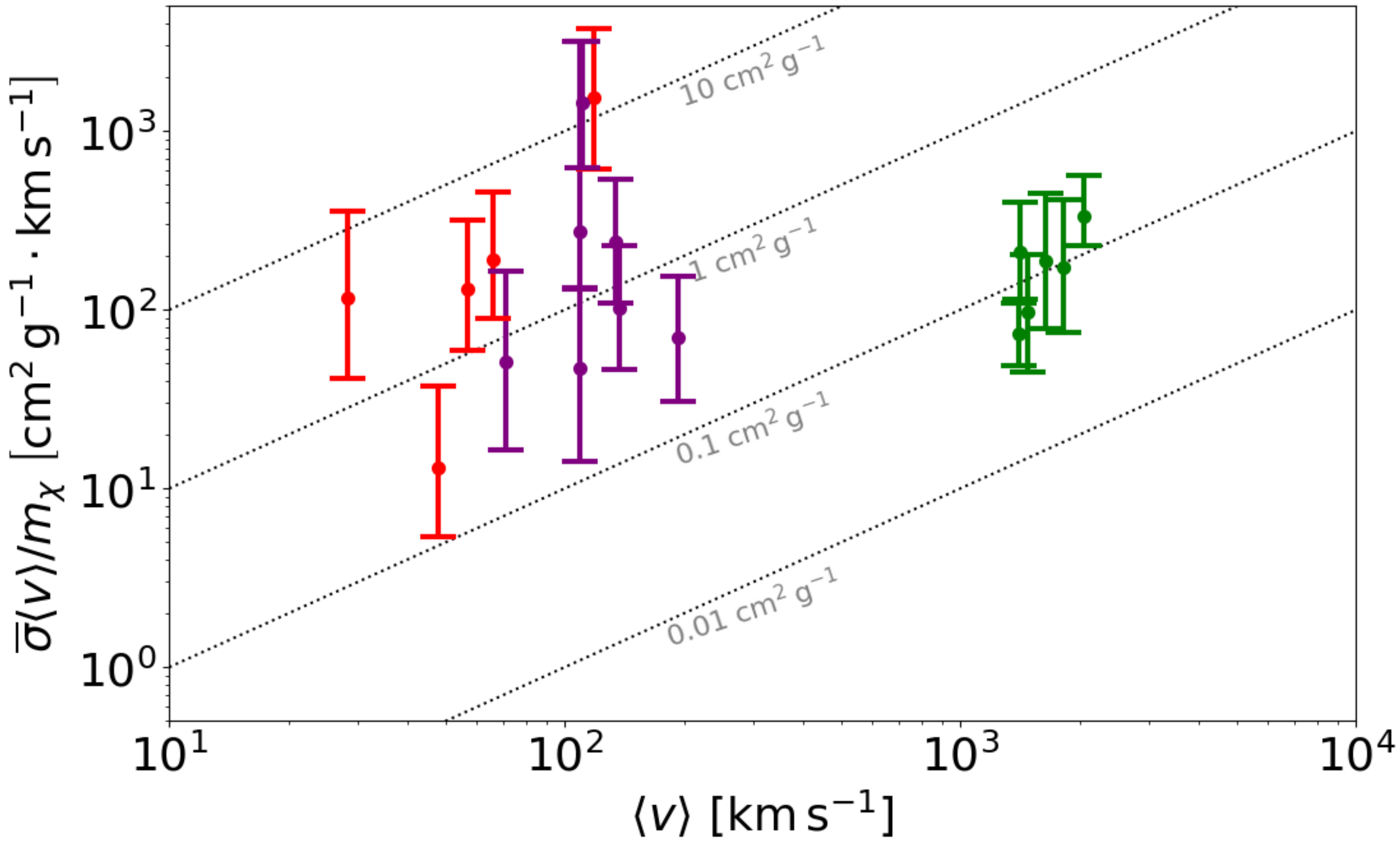
the most massive subhalos predicted by CCDM is more than that observations in the Miky Way(MW) and other dwarf galaxies

Missing satellites problem

Diversity problem

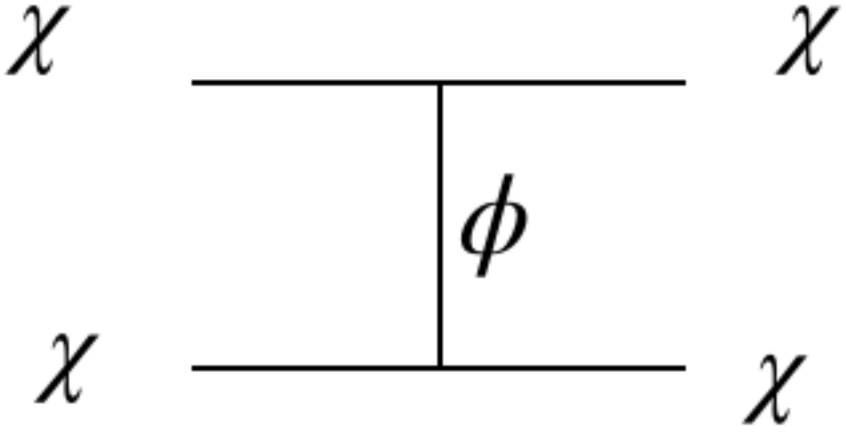
Motivation

Self-interaction DM



$$\frac{d\sigma}{d\Omega} = \frac{4\pi\alpha m^2}{m_\phi^4}\left(\frac{1}{1+q^2 m_\phi^{-2}}\right)^2$$

long-range force



Motivation

Self-interaction DM candidates

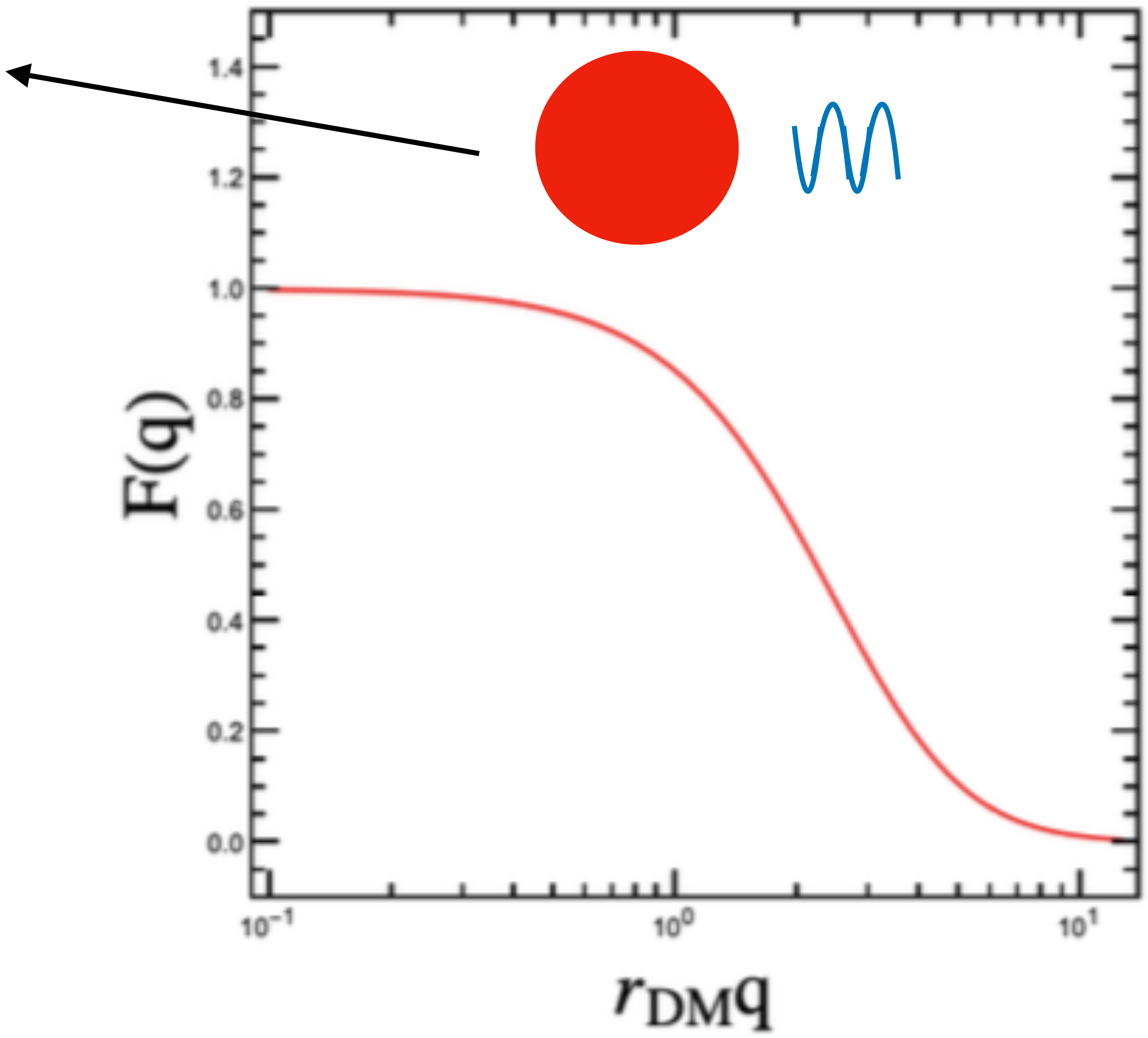
- **Elastic SIDM model**
  - **Light mediator**
    - **model**
      - **Dark sector**
      - **SUSY**
    - **Constraints**
      - **CMB,BBN**
      - **Detection**
      - **Sommerfeld effect**
    - **Solution**
      - **Coupling the neutrinos**
      - **P wave annihilation**
      - **Freeze-in mechanism**
  - **Composite DM**
    - **Atom DM**
    - **meson, nucleus, Bound state etc.**
  - **Resonant SIDM**
    - **Point particle DM**
    - **Dark QCD DM**
- **Inelastic SIDM model**
  - **Two mass splitting dark states**
  - **Excited state**

Motivation

Puffy DM

If dark matter has a finite size that is larger than its Compton wavelength, the corresponding self-interaction cross section decreases with the velocity.

Dipole



$$F(\mathbf{q}) = \int \mathbf{d r} e^{i\mathbf{q}\mathbf{r}} \rho(\mathbf{r})$$

$$F(q) = \frac{1}{(1 + 1/24 r_{DM}^2 q^2)^2}$$

- [2] X. Chu, C. Garcia-Cely and H. Murayama, Phys. Rev. Lett. 124, no. 4, 041101 (2020)

Motivation

Puffy DM

$$V(r) = \frac{q}{4\pi\Delta r} = \int \frac{Q\rho(r)}{4\pi|\mathbf{r} - \mathbf{r}'|} d^3 r$$

$$M_{fi} = < \psi_f | | \psi_i > = \int e^{-ip_3 r} V(r) e^{ip_1 r} d^3 r \qquad = \int \int e^{i(p_1 - p_3)(r - r')} \frac{Q\rho(r')}{4\pi|\mathbf{r} - \mathbf{r}'|} e^{iqr'} d^3 r' d^3 r$$

$$= \int e^{iqR} \frac{Q}{4\pi|\mathbf{r} - \mathbf{r}'|} d^3 R \int \rho(r') e^{iqr'} d^3 r' \quad = (M_{fi})_{point} F(q^2)$$

$$V(r)' = \int \frac{\alpha}{r} e^{\frac{-r}{\lambda}} \rho_1(r_1) dr_1^3 \rho_2(r_2) dr_2^3 \qquad M_{fi} = \frac{4\pi\alpha}{\vec{q}^{\,2} + \lambda^{-2}} F_1(q) F_2(-q)$$

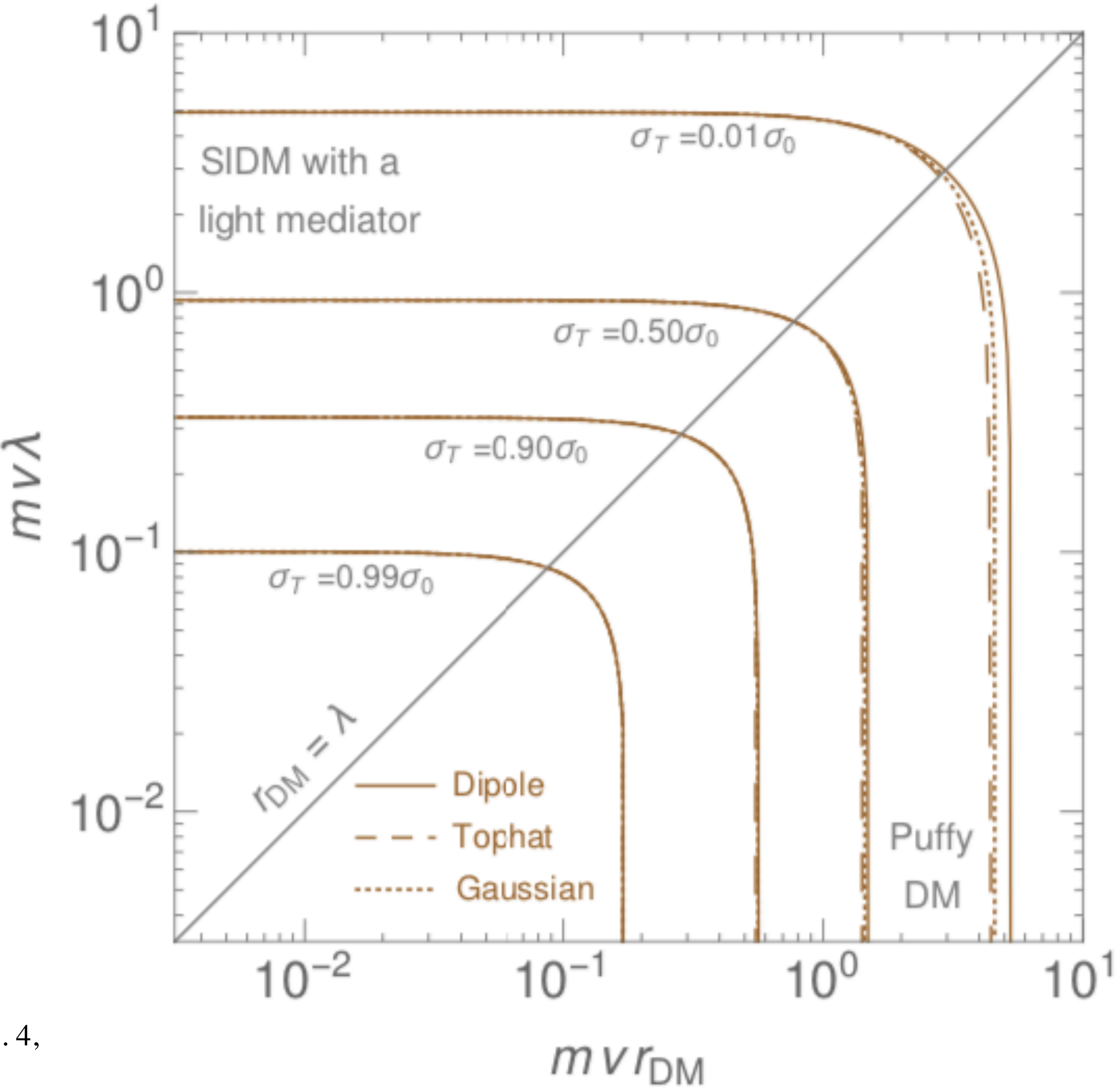
Motivation

Puffy DM

Even in the presence of a light particle mediating self-interactions, the finite-size effect may dominate the velocity dependence.

$$\frac{d\sigma_y}{d\Omega} = \sigma_0 \frac{F^4(q)}{((mv)^2 m_\phi^{-2}\frac{1-\cos\theta}{2}+1)^2}$$

$$F(q) = \frac{1}{(1+r_0^2 m^2 v^2 \frac{1-\cos\theta}{2})^2}$$



- [2] X. Chu, C. Garcia-Cely and H. Murayama, Phys. Rev. Lett. 124, no. 4, 041101 (2020)

Self-interaction puffy DM

If dark matter has a finite size that is larger than its Compton wavelength, the corresponding self-interaction cross section decreases with the velocity.

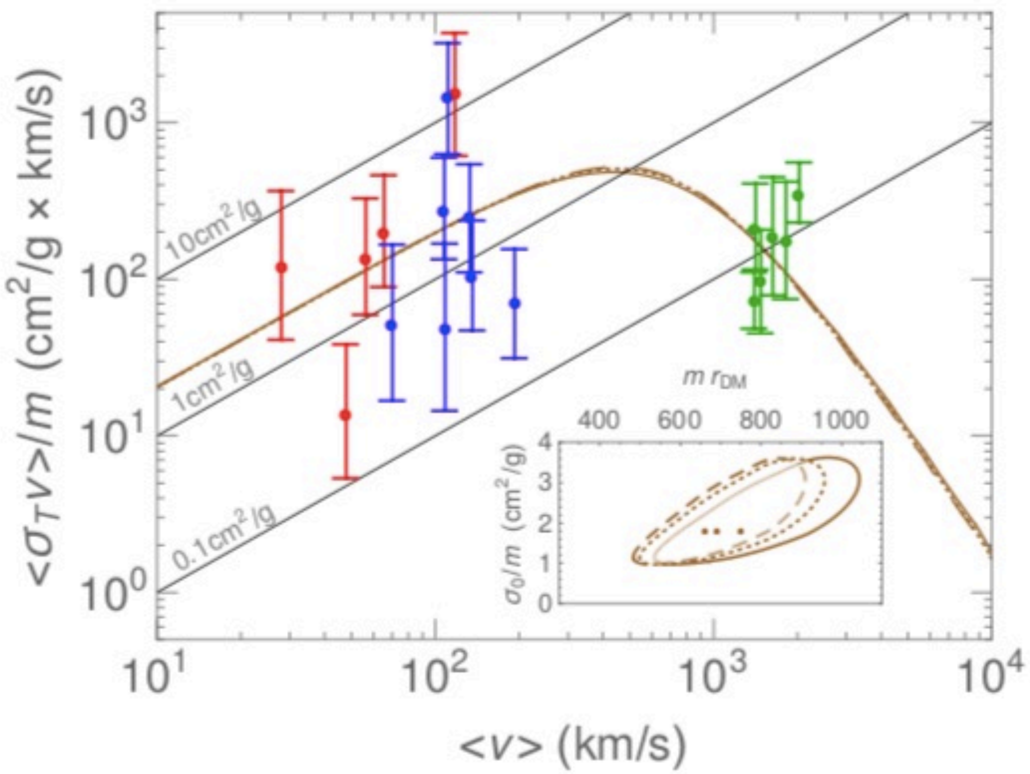
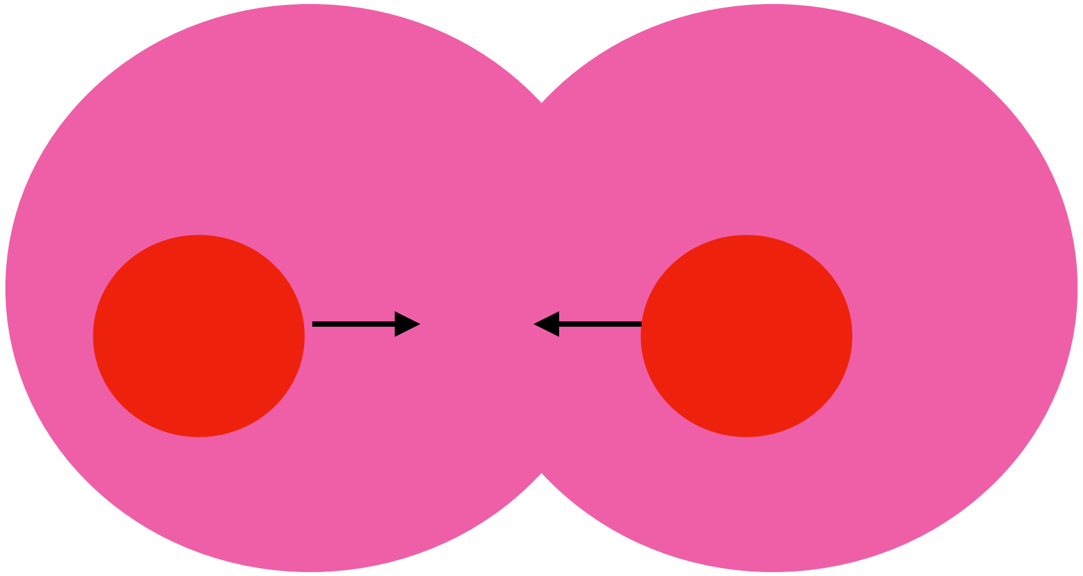


Figure 3: Velocity dependence of the transfer cross section of Puffy DM. Best-fit curves to data [41] for the dipole (solid), tophat (dashed) and the Gaussian (dotted) distributions in Table I. The inset shows the 95% C.L. contours on the parameter σ_0 from Eq. (3) and the DM size together with the corresponding parameter sets plotted in the main figure.



Dark strong interaction?

Motivation

Proton-Proton elastic scattering

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \left| f_{\mathrm{c}}(t)\mathrm{e}^{i\alpha\phi(t)} + f_{\mathrm{n}}(t) \right|^2 = \frac{\mathrm{d}\sigma_{\mathrm{c}}}{\mathrm{d}t} + \frac{\mathrm{d}\sigma_{\mathrm{int}}}{\mathrm{d}t} + \frac{\mathrm{d}\sigma_{\mathrm{n}}}{\mathrm{d}t}, \quad (1)$$

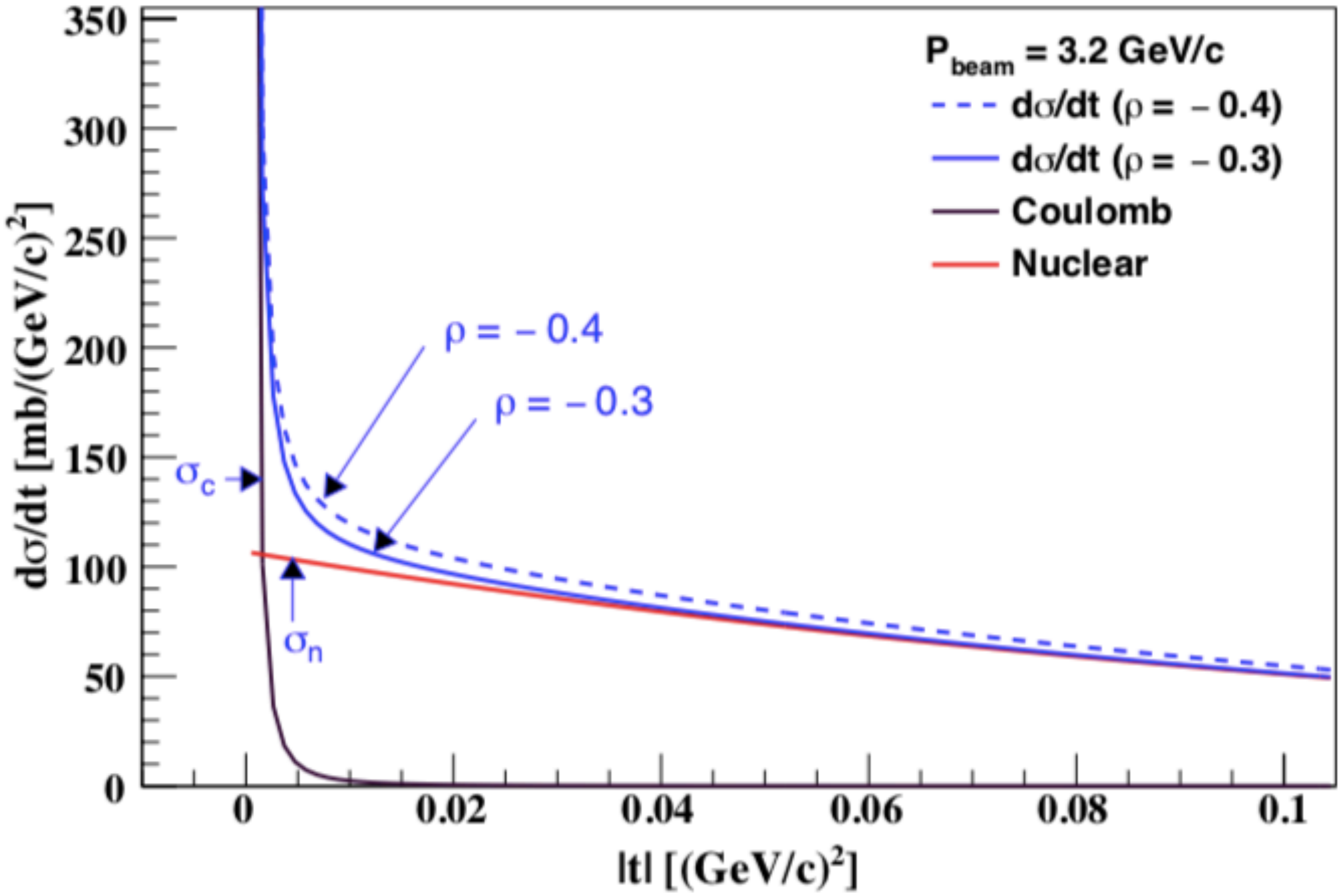
where

$$\frac{\mathrm{d}\sigma_{\mathrm{c}}}{\mathrm{d}t} = \frac{4\pi\alpha^2 G^4(t)(\hbar c)^2}{\beta^2 t^2}, \quad (2)$$

$$\frac{\mathrm{d}\sigma_{\mathrm{int}}}{\mathrm{d}t} = -\frac{\alpha\sigma_{\mathrm{tot}}}{\beta\,|t|} G^2(t)\mathrm{e}^{\frac{-B|t|}{2}}(\rho\cos(\alpha\phi(t)) + \sin(\alpha\phi(t))), \quad (3)$$

and

$$\frac{\mathrm{d}\sigma_{\mathrm{n}}}{\mathrm{d}t} = \frac{\sigma_{\mathrm{tot}}^2(1+\rho^2)\mathrm{e}^{-B|t|}}{16\pi(\hbar c)^2}. \quad (4)$$



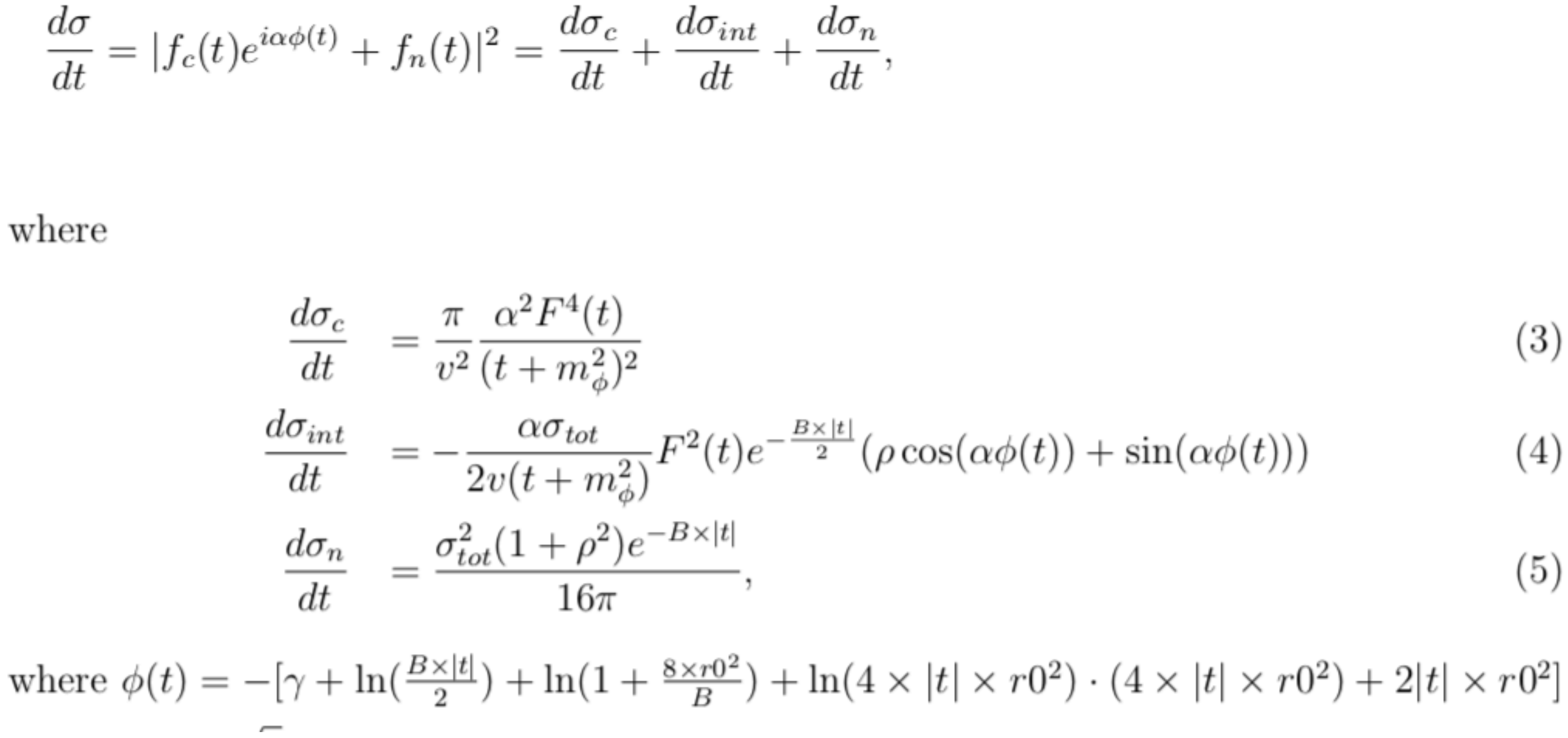
The total cross section σ_{tot}

The relative real amplitude ratio $\rho = Ref_n(0)/Imf_n(0)$

The slope parameter B

- [4] H. Xu, Y. Zhou, U. Bechstedt, J. Bo¨ker, A. Gillitzer, F. Goldenbaum, D. Grzonka, Q. Hu, A. Khoukaz and F. Klehr, et al. Phys. Lett. B 812 (2021), 136022

dark matter-dark matter elastic scattering



$$\frac{d\sigma}{dt} = |f_c(t)e^{i\alpha\phi(t)} + f_n(t)|^2 = \frac{d\sigma_c}{dt} + \frac{d\sigma_{int}}{dt} + \frac{d\sigma_n}{dt},$$

where

$$\frac{d\sigma_c}{dt} = \frac{\pi}{v^2}\frac{\alpha^2 F^4(t)}{(t+m_\phi^2)^2} \tag{3}$$

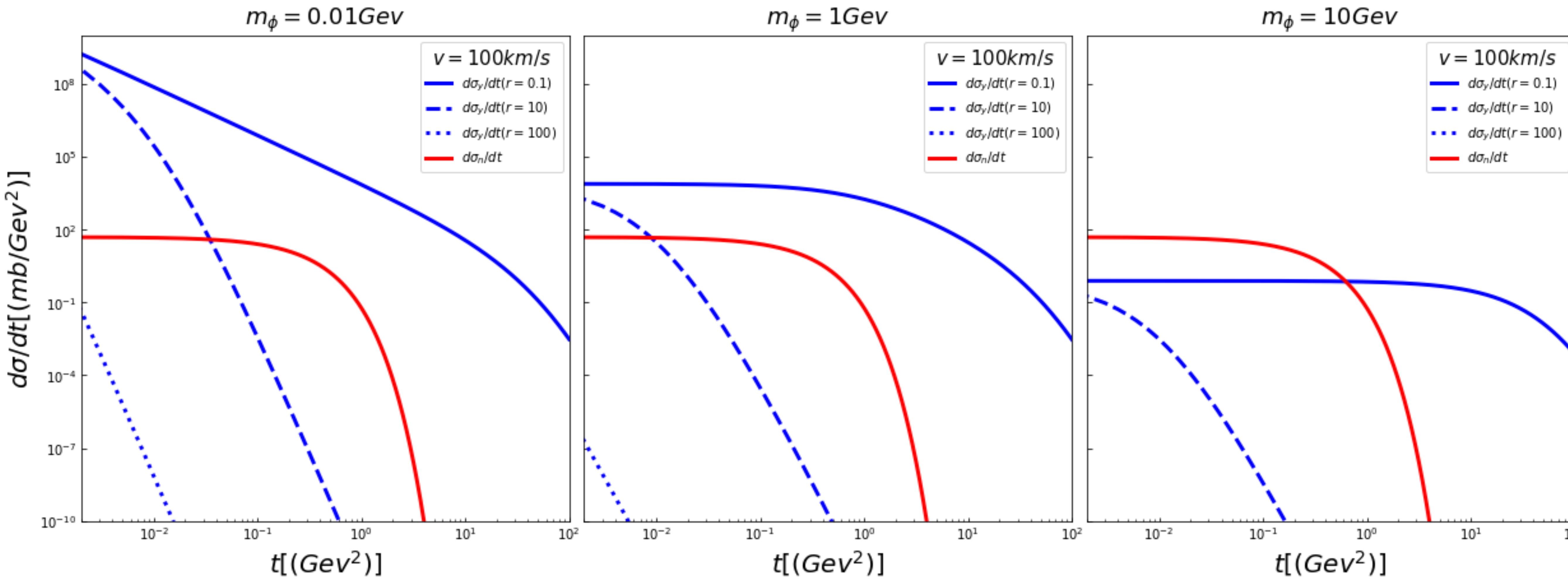
$$\frac{d\sigma_{int}}{dt} = -\frac{\alpha\sigma_{tot}}{2v(t+m_\phi^2)}F^2(t)e^{-\frac{B\times|t|}{2}}(\rho\cos(\alpha\phi(t)) + \sin(\alpha\phi(t))) \tag{4}$$

$$\frac{d\sigma_n}{dt} = \frac{\sigma_{tot}^2(1+\rho^2)e^{-B\times|t|}}{16\pi}, \tag{5}$$

where $\phi(t) = -[\gamma + \ln(\frac{B\times|t|}{2}) + \ln(1+\frac{8\times r0^2}{B}) + \ln(4\times|t|\times r0^2)\cdot(4\times|t|\times r0^2) + 2|t|\times r0^2]$

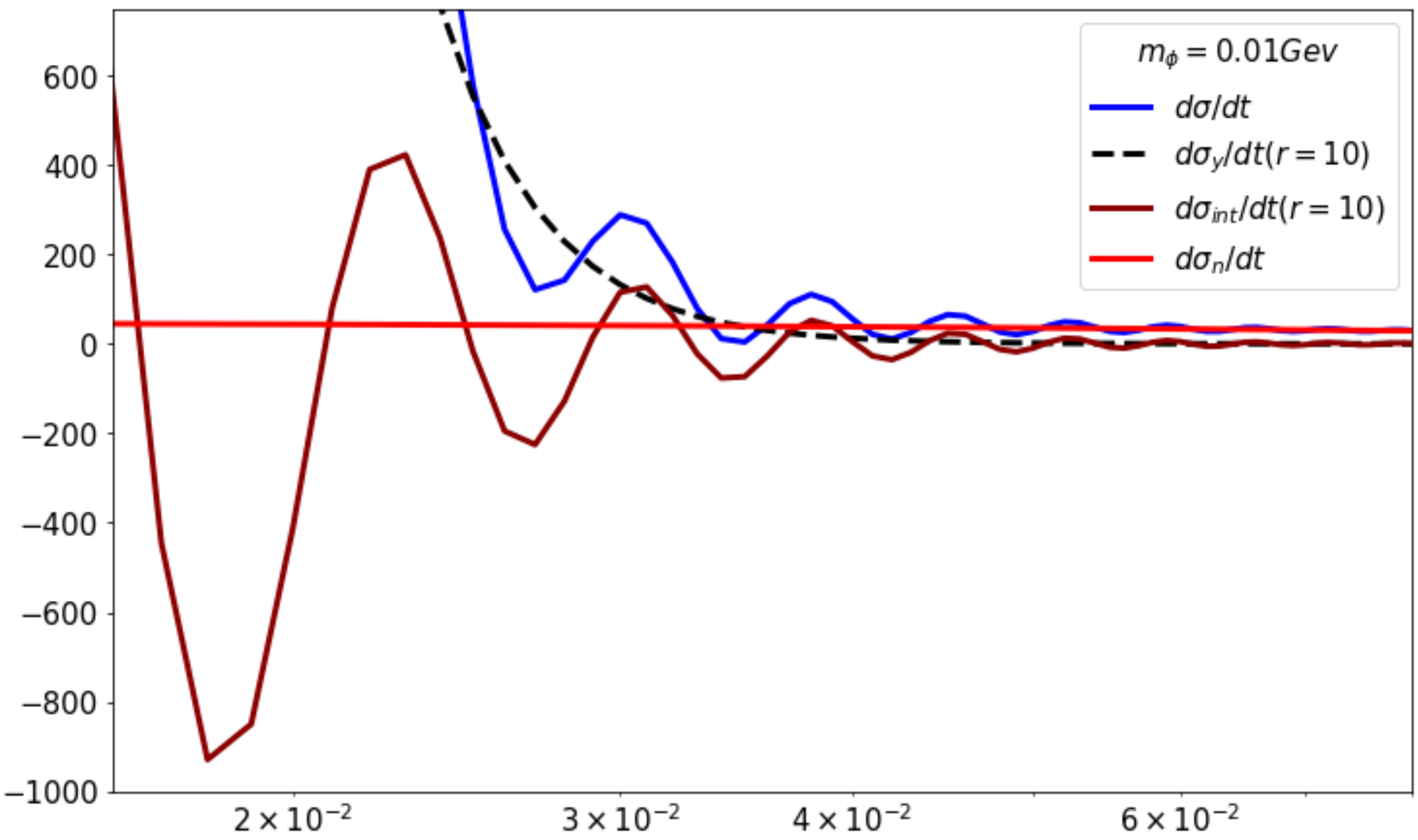
dark matter-dark matter elastic scattering

$$\frac{d\sigma}{dt} = |f_c(t)e^{i\alpha\phi(t)} + f_n(t)|^2 = \frac{d\sigma_c}{dt} + \frac{d\sigma_{int}}{dt} + \frac{d\sigma_n}{dt},$$



dark matter-dark matter elastic scattering

Interference term



dark matter-dark matter elastic scattering

$$\frac{d\sigma}{dt} = |f_c(t)e^{i\alpha\phi(t)} + f_n(t)|^2 = \frac{d\sigma_c}{dt} + \frac{d\sigma_{int}}{dt} + \frac{d\sigma_n}{dt},$$

where

$$\frac{d\sigma_c}{dt} = \frac{\pi}{v^2}\frac{\alpha^2 F^4(t)}{(t+m_\phi^2)^2} \tag{3}$$

$$\frac{d\sigma_{int}}{dt} = -\frac{\alpha\sigma_{tot}}{2v(t+m_\phi^2)}F^2(t)e^{-\frac{B\times|t|}{2}}(\rho\cos(\alpha\phi(t)) + \sin(\alpha\phi(t))) \tag{4}$$

$$\frac{d\sigma_n}{dt} = \frac{\sigma_{tot}^2(1+\rho^2)e^{-B\times|t|}}{16\pi}, \tag{5}$$

where $\phi(t) = -[\gamma + \ln(\frac{B\times|t|}{2}) + \ln(1 + \frac{8\times r0^2}{B}) + \ln(4\times|t|\times r0^2)\cdot(4\times|t|\times r0^2) + 2|t|\times r0^2]$

Self-interaction puffy DM

Elkonal approximation

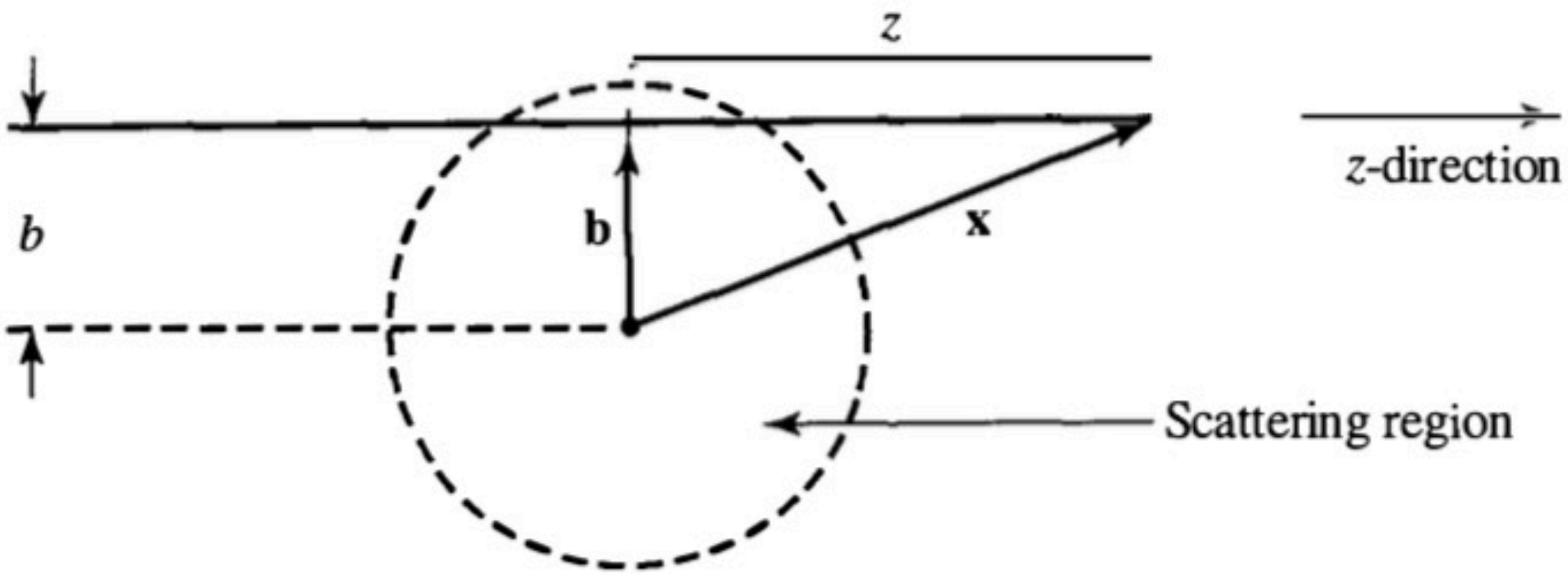


FIGURE 6.10 Schematic diagram of eikonal approximation scattering, where the classical straight-line trajectory is along the z-direction, $|\mathbf{x}| = r$, and $b = |\mathbf{b}|$ is the impact parameter.

eikonal approximation condition： $R \gg \lambda$ **Many particle waves contribute**

$$f_{c.m.}(s,t) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)a_l(k) \qquad a_l(k) = \frac{e^{2i\delta_i}-1}{2i}$$

$$l \to \infty \qquad p_l(\cos\theta) \to J_0[(2l+1)\sin(\theta/2)]$$

Self-interaction puffy DM

The standard partial-wave expansion of the scattering amplitude is

$$f_{c.m.}(s,t) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)a_l(k) \qquad a_l(k) = \frac{e^{2i\delta_i}-1}{2i}$$

$$l \to \infty \qquad p_l(\cos\theta) \to J_0[(2l+1)\sin(\theta/2)]$$

eikonal approximation condition： de Broglie wavelength of the heavy dark matter are much smaller than the size of the dark matter $mvr_{DM} \gg 1$

$$f_n(s,t) = 2k\int bdbJ_0(qb)a(b,s) \qquad bk = l + 1/2 \qquad J_0(t) = \frac{1}{2\pi}\int d\phi exp(-it\cos\phi)$$

$$a_l \to a(b,s) \qquad a(b,s) = \frac{1}{4\pi k}\int d^2bexp(iq\cdot b)f_{c.m.}(s,t)$$

Traditionally, σ_{tot} and the slope parameter B are given as, using the approximation of the impact- parameter representation,

pure imaginary $a(b,s)$ are always adopted for the forward high-energy scattering, there are several profiles such as disk, parabolic form, Gaussian shape and Chou-Yang model etc

$$\sigma_{tot} = \frac{4\pi}{k}Imf_{cm}(s,0) = 4\int d^2bIma(b,s)$$

$$B = \frac{\int d^2bb^2a(b,s)}{2\int d^2ba(b,s)}$$

Self-interaction puffy DM

Chou-Yang model

Chou-Yang model consider that the attenuation of two hadron going through each other is denoted by the evaluating the opaqueness at the impact parameter b [5]. The density of opaqueness can be seen as the charge distribution inside the hadron.

$$a(b,s) = \frac{1}{2i}(e^{2i\delta)-1} = \frac{i}{2}(1 - e^{-\Omega(b)})$$

$$\Omega(b) = A\frac{1}{8}x^3K_3(x) \qquad x = b/r_0$$

$\sigma_A(A, r_0)$ are rewritten as

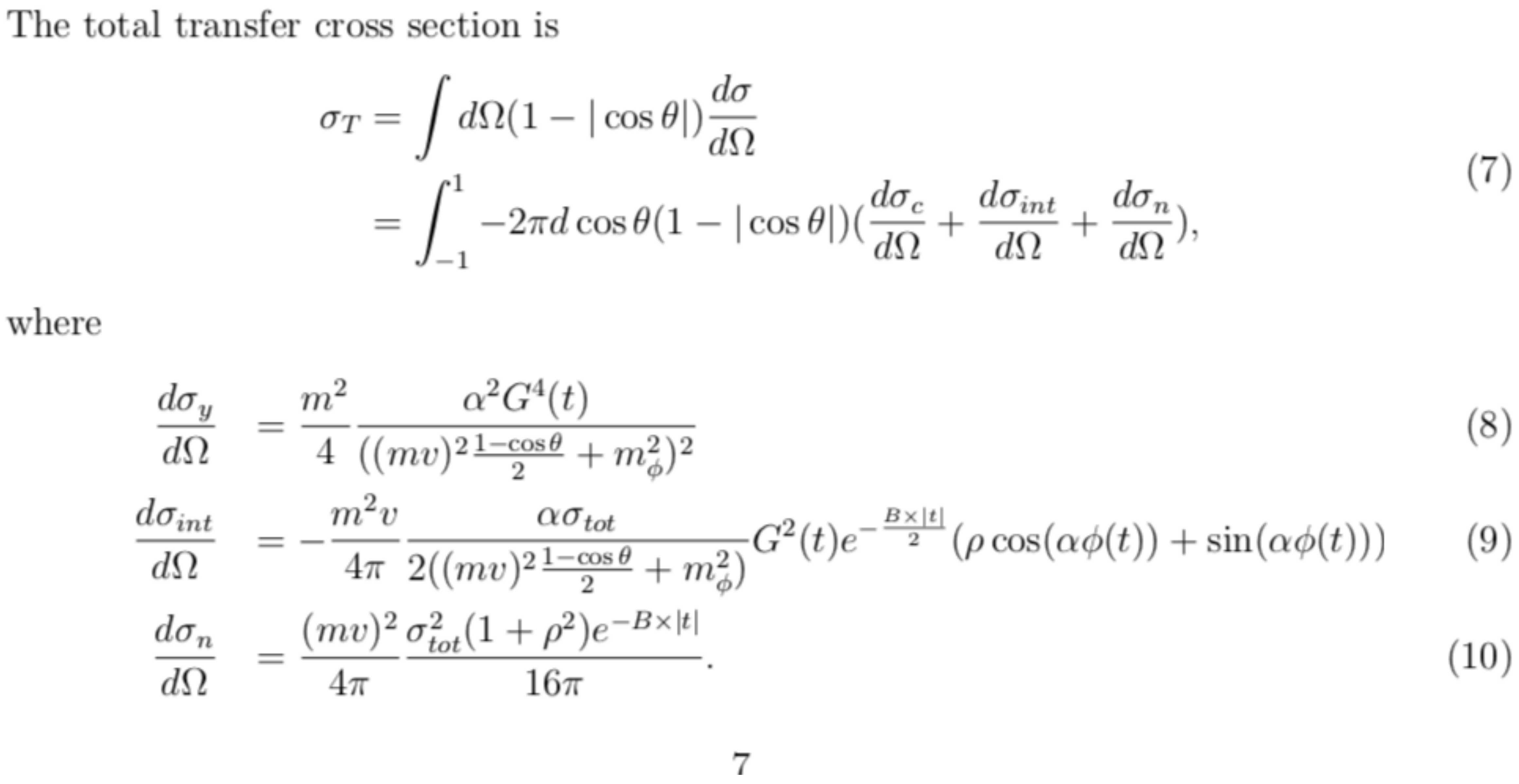
$$\sigma_A = \sigma_0 + \sigma_a$$

$$\sigma_a = r_0^2\sigma_{tot}(\frac{1}{r_0})^2 = 4 \times 2\pi \times r_0^2 \int dx(xa)$$

SIDM in Astrophysical halos

When the DM-DM scatting is proceeded in the Astrophysical halos, the transfer cross section

$$\sigma_T = \int d\Omega (1 - cos\theta) \frac{d\sigma}{d\Omega}$$



The total transfer cross section is

$$\begin{aligned} \sigma_T &= \int d\Omega (1 - |\cos\theta|) \frac{d\sigma}{d\Omega} \\ &= \int_{-1}^{1} -2\pi d\cos\theta (1 - |\cos\theta|)(\frac{d\sigma_c}{d\Omega} + \frac{d\sigma_{int}}{d\Omega} + \frac{d\sigma_n}{d\Omega}), \end{aligned} \tag{7}$$

where

$$\frac{d\sigma_y}{d\Omega} = \frac{m^2}{4} \frac{\alpha^2 G^4(t)}{((mv)^2 \frac{1-\cos\theta}{2} + m_\phi^2)^2} \tag{8}$$

$$\frac{d\sigma_{int}}{d\Omega} = -\frac{m^2 v}{4\pi} \frac{\alpha \sigma_{tot}}{2((mv)^2 \frac{1-\cos\theta}{2} + m_\phi^2)} G^2(t) e^{-\frac{B\times|t|}{2}} (\rho\cos(\alpha\phi(t)) + \sin(\alpha\phi(t))) \tag{9}$$

$$\frac{d\sigma_n}{d\Omega} = \frac{(mv)^2}{4\pi} \frac{\sigma_{tot}^2 (1+\rho^2) e^{-B\times|t|}}{16\pi}. \tag{10}$$

SIDM in Astrophysical halos

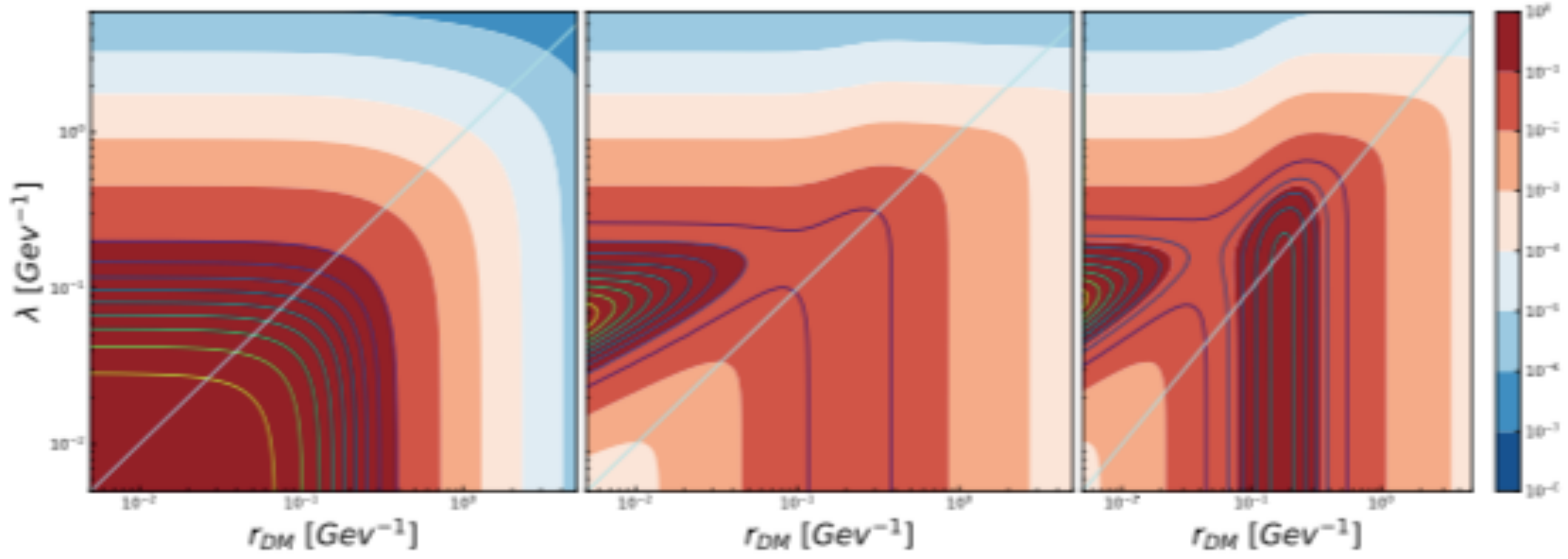


FIG. 1: Ratio σ_T/σ_A in r_{DM} and λ space with different absorption parameter $A = 0, 1, 10$, respectively. The other parameters are the same which is $m_{\mathrm{DM}} = 100\mathrm{GeV}$, $\alpha = 0.01$ and $v/c = 0.1$.

SIDM in Astrophysical halos

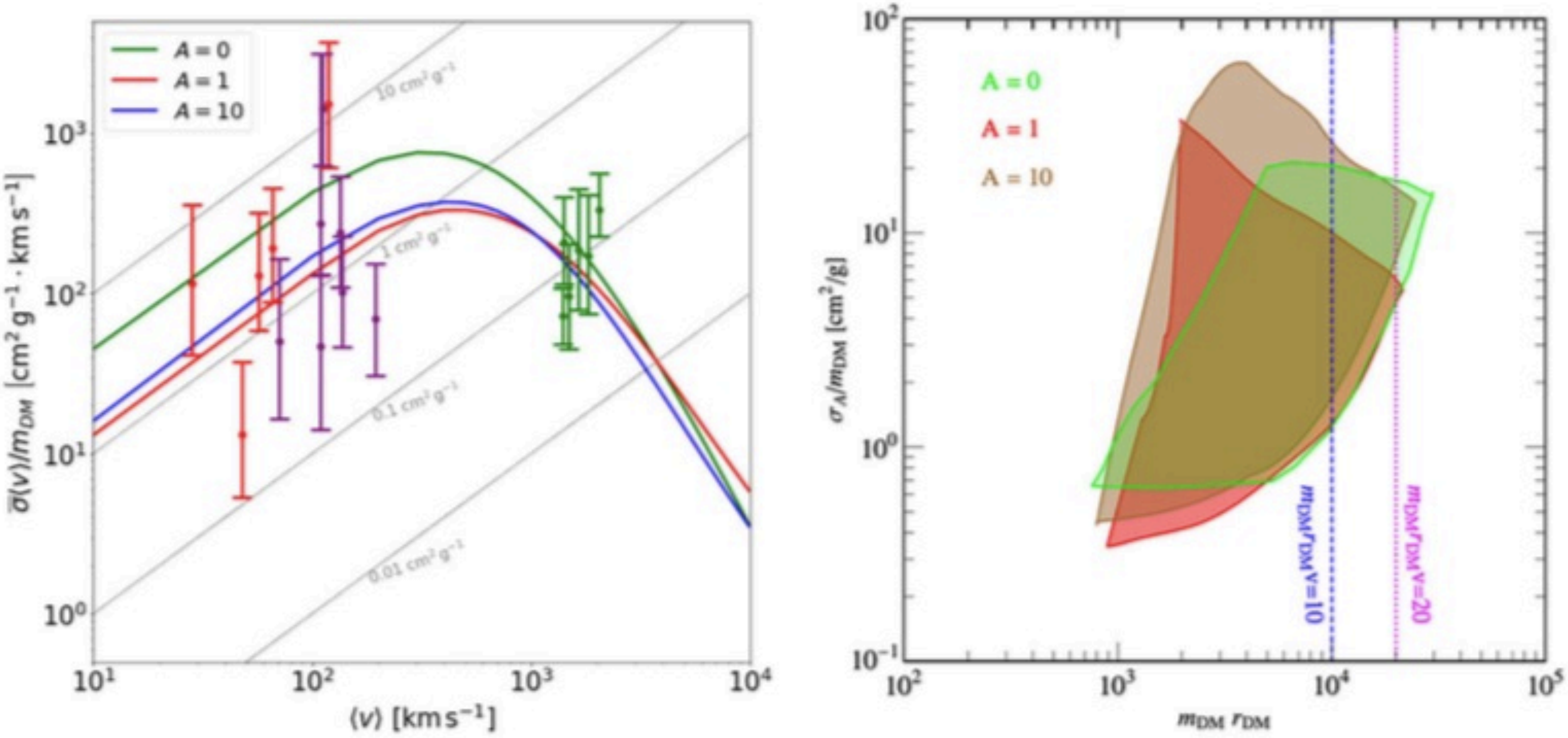


FIG. 2: Left panel: The best fit point for the velocity dependence the the puffy dark matter, Right panel: the fitted $\sigma_A/m_{\rm DM}$ versus $m_{\rm DM} r_{\rm DM}$ in the $\lambda < r_{\rm DM}$ case.

Conclusion

The sketch map of the calculation of the cross-section and a more realistic realization of the matter and charge distribution, Chou-Yang model, are shown in this work.

With the participation of the strong interaction, the space of the ratio for the cross-section the mass which is needed in the simulation can be enlarged, giving us a more flexible parameter space to other processes related to dark matter.

Discussion

Puffy dark matter’s production mechanism.

The detection of puffy DM.