Can sub-GeV dark matter coherently scatter on the electrons in the atom?

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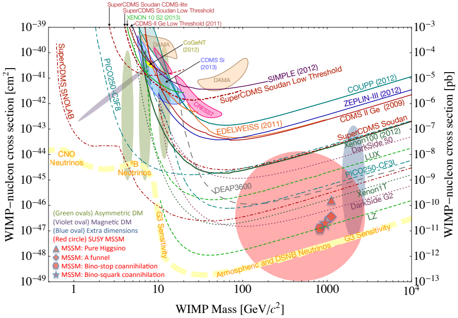
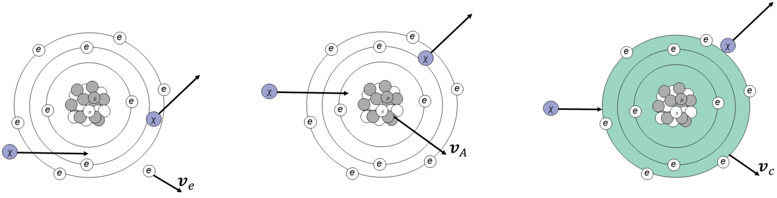
1. Research Status
2. Motivation
3. Model calculation
4. Conclusion

An attractive dark matter candidate is Weakly Interacting Massive Particles (WIMPs), which have been explored in various direct detection, indirect detection and collider experiments. The null results have produced very stringent limits on the WIMP-nucleus scattering cross section heavier than 1 GeV. Thus the hunt for sub-GeV dark matter is a hot topic at the cutting edge of physics research.

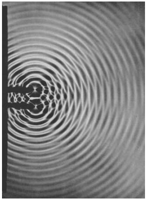




(a) Recoil of Electron (b) Recoil of Nucleus (c) Recoil of Electron Cloud

图 1: The three situations.

Coherent scattering is powerful! But will it happen?
Momentum transfer $q^2 \sim \frac{1}{r^2}$, r is the atom radius?



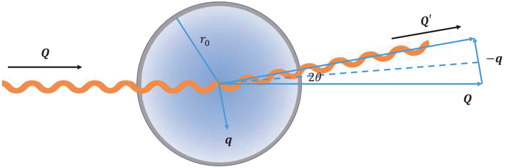


图 2: The X-ray diffraction on the atom. $\frac{\sin\theta}{\lambda} = \frac{q}{4\pi}$.

The summation of all the electrons gives the atomic scattering form factor

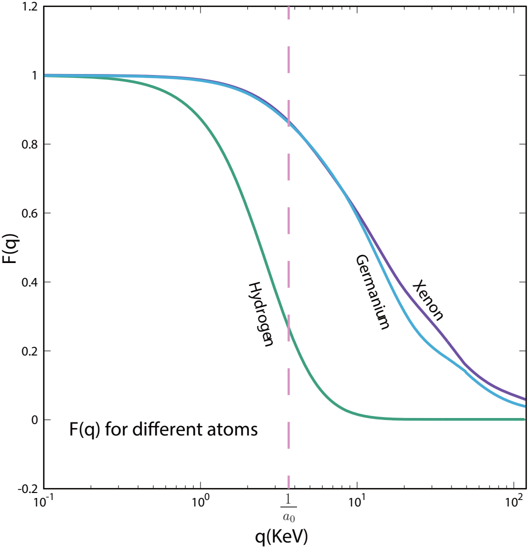
$$F(q) = \sum_j^Z f_j = \sum_j^Z \int \rho(\boldsymbol{r}_j)\exp(-\mathrm{iq}\cdot\mathrm{r_j})\mathrm{dr_j}.$$

$$F(q) \to \rho(r) \quad \rho(r) \to F(q)$$

RHF: Relativistic Hartree-Fock method use wave functions to calculate the form factors in different transfer momentum.

Table 6.1.1.1. *Mean atomic scattering factors for free atoms (cont.)*

Element Z Method $(\sin\theta)/\lambda$ (Å^{-1})	Sb 51 RHF	Te 52 *RHF	I 53 RHF	Xe 54 RHF	Cs 55 RHF	Ba 56 RHF	La 57 *RHF	Ce 58 *RHF	Pr 59 *RHF	Nd 60 *RHF
0.00	51.000	52.000	53.000	54.000	55.000	56.000	57.000	58.000	59.000	60.000
0.01	50.955	51.954	52.955	53.956	54.932	55.925	56.926	57.928	58.929	59.931
0.02	50.819	51.818	52.820	53.821	54.732	55.703	56.708	57.715	58.722	59.728
0.03	50.596	51.594	52.597	53.601	54.417	55.350	56.360	57.375	58.392	59.404
0.04	50.293	51.288	52.292	53.297	54.008	54.888	55.900	56.924	57.956	58.977
0.05	49.915	50.906	51.911	52.917	53.527	54.345	55.351	56.385	57.439	58.468
0.06	49.474	50.458	51.460	52.467	52.996	53.743	54.736	55.779	56.861	57.899
0.07	48.977	49.951	50.950	51.954	52.430	53.106	54.076	55.127	56.242	57.288
0.08	48.434	49.395	50.387	51.388	51.839	52.450	53.388	54.446	55.599	56.651
0.09	47.856	48.800	49.781	50.775	51.229	51.786	52.687	53.750	54.943	56.000
0.10	47.250	48.174	49.142	50.125	50.603	51.122	51.982	53.047	54.281	55.342
0.11	46.625	47.526	48.476	49.447	49.963	50.460	51.278	52.345	53.617	54.680
0.12	45.988	46.863	47.793	48.747	49.309	49.802	50.580	51.646	52.952	54.017
0.13	45.344	46.193	47.099	48.033	48.645	49.146	49.888	50.952	52.288	53.354
0.14	44.699	45.519	46.400	47.311	47.971	48.492	49.202	50.263	51.623	52.689
0.15	44.056	44.848	45.702	46.588	47.291	47.839	48.523	49.579	50.957	52.022
0.16	43.419	44.182	45.008	45.868	46.606	47.186	47.849	48.901	50.289	51.353
0.17	42.789	43.526	44.323	45.155	45.921	46.533	47.182	48.227	49.620	50.682
0.18	42.168	42.879	43.648	44.453	45.237	45.882	46.519	47.557	48.950	50.009
0.19	41.556	42.245	42.987	43.763	44.559	45.232	45.862	46.892	48.280	49.334
0.20	40.955	41.623	42.340	43.088	43.888	44.586	45.212	46.233	47.610	48.660

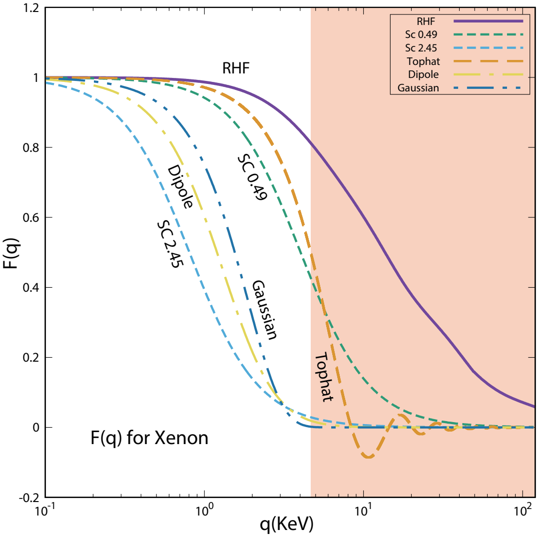

1.2
1
0.8
0.6
0.4
0.2
0
-0.2
F(q)
10^{-1}
10^{0}
$\frac{1}{a_0}$
10^{1}
10^{2}
q(KeV)
Hydrogen
Germanium
Xenon
F(q) for different atoms

The screened Coulomb potential:

$$\phi(r) = \frac{Q}{4\pi r} e^{-\frac{r}{r_0}} \to F(q) = \frac{1}{1 + q^2 r_0^2}.$$

Using Poisson's equation $\nabla^2\phi = -\rho$, we can get the charge density distributions outside the nucleus, and then do the Fourier transform to get $F(q)$.

Shape	$\rho(r)$	r_{Atom}	$F(q)$
Tophat	$\frac{3}{4\pi r_0^3}\theta(r_0 - r)$	$\sqrt{3/5}r_0$	$\frac{3(\sin(r_0 q) - r_0 q \cos(r_0 q))}{r_0^3 q^3}$
Dipole	$\frac{e^{-r/r_0}}{8\pi r_0^3}$	$2\sqrt{3}r_0$	$\frac{1}{\left(1+r_0^2 q^2\right)^2}$
Gaussian	$\frac{1}{8 r_0^3 \pi^{3/2}} e^{-r^2/(4r_0^2)}$	$\sqrt{6}r_0$	$e^{-r_0^2 q^2}$


RHF
Sc 0.49
Sc 2.45
Tophat
Dipole
Gaussian
RHF
Dipole
SC 0.49
SC 2.45
Gaussian
Tophat
F(q) for Xenon
F(q)
q(KeV)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
10^{-1}
10^{0}
10^{1}
10^{2}

The probability of the momentum state $|\boldsymbol{p}'\rangle$ in the final state is proportional to

$$P(\boldsymbol{p}') \propto \left|\langle \boldsymbol{p}'|\psi\rangle\right|^2 \propto \sum_{ij} \left|\left\langle \mathrm{e}^{-\mathrm{i}\Delta \boldsymbol{x}_{ij}\cdot \boldsymbol{q}}\right\rangle\right|^2 \left|\mathcal{M}\left(\boldsymbol{p}, \boldsymbol{p}'\right)\right|^2,$$

where $\Delta \boldsymbol{x}_{ij} = \boldsymbol{x}_i - \boldsymbol{x}_j$.

The model consists a dark matter candidate χ, which scatters off electrons through the exchange of a quite heavy dark photon A'. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{m_{A'}^2}{2}A'_\mu A'^\mu + A'_\mu(g_D J_D^\mu + \epsilon e J_{EM}^\mu)$$

As the total charge of the electron cloud is Z, it is very obvious that the $\mathcal{M}_{\chi C}$ can be reduced a similar interaction as the recoil of a single electron

$$|\mathcal{M}_{\chi C}|^2 = Z^2 |F_C(q)|^2 \, |\mathcal{M}_{\chi e}|^2 \, ,$$

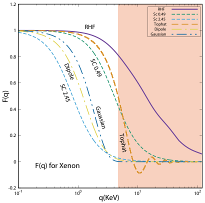
in which $F_C(q)$ is the normalized form factor talked in above section.

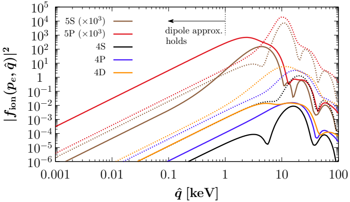
$$\left|f_{\text{ion, e}}(E_e, q)\right|^2 = \frac{k_e^3}{4\pi^3} \times 2 \sum_{n,l,l',m'} \left|\left\langle \psi_{E_e}^f \left| e^{i\boldsymbol{q}\cdot\boldsymbol{x}} \right| \psi_{E_{nl}}^i \right\rangle\right|^2 .$$

$$\left|f_{\text{ion, M}}(E_e, q)\right|^2 = \frac{k_e^3}{4\pi^3} Z^2 \left|F_N(q)\right|^2 \times 2 \sum_{n,l,l',m'} \left|\left\langle \psi_{E_e}^f \left| e^{i\boldsymbol{q_e'}\cdot\boldsymbol{x}} \right| \psi_{E_{nl}}^i \right\rangle\right|^2 .$$

$$\left|f_{\text{ion, C}}(E_e, q)\right|^2 = \frac{k_e^3}{4\pi^3} Z^2 \left|F_C(q)\right|^2 \times 2 \sum_{n,l,l',m'} \left|\left\langle \psi_{E_e}^f \left| e^{i\boldsymbol{q_e}\cdot\boldsymbol{x}} \right| \psi_{E_{nl}}^i \right\rangle\right|^2 .$$

while $\boldsymbol{q_e'} = \frac{m_e}{m_N}\boldsymbol{q}$ and $\boldsymbol{q}_e = \frac{\boldsymbol{q}}{Z}$





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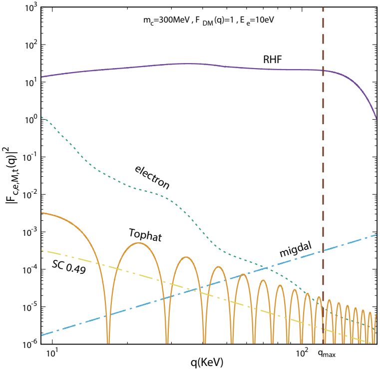

图 3: Comparison with $|f_{\text{ion, e}}|^2$ (green dot), $|f_{\text{ion, M}}|^2$ (sky blue dash dot) and $|f_{\text{ion, C}}|^2$ of the Xenon atom. The maximum q for the REC is shown in brown dash line.

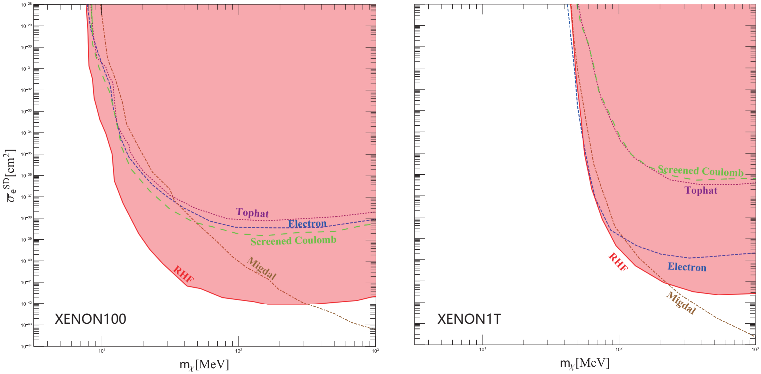

图 4: The limits of the cross sections and dark matter mass from Xenon100 (left panel) and Xenon1T(right panel). The shaded regions show results of RHF form factor.

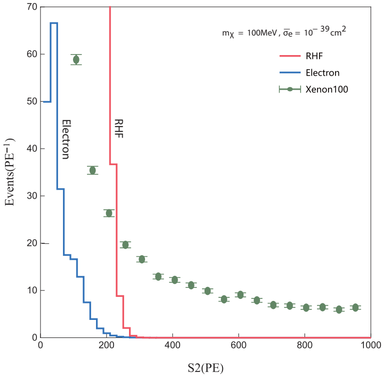

图 5: Observed events versus photonelectrons (PE) from Xenon100 (left panel) and Xenon1T (right panel) data, for Xenon100 $\bar{\sigma}_e = 1 \times 10^{-39}\mathrm{cm}^2$ and $m_\chi = 100\mathrm{MeV}$ and while for Xenon1T, $\bar{\sigma}_e = 1 \times 10^{-40}\mathrm{cm}^2$ and $m_\chi = 100\mathrm{MeV}$.

1. A novel detection are proposed that the electron cloud is boosted by the dark matter and throws away an electron when it is dragged back by the heavy nucleus, namely the coherent scattering of the electron cloud of the atom.
2. The results of the relativistic Hartree-Fock method gives non-trivial shapes of the atom. Having equipped with the RHF form factor and impulse approximation, we proceed to show the detailed calculation of recoil of the electron cloud, the kinetics, the fiducial cross section and the corresponding calculation of detection rate are given analytically.
3. We show the constraints on the cross section from the current experimental measurements.

Thanks!