LS机制下协变振幅的介绍

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基本介绍

- LS机制下的协变振幅在BES有很广泛的应用，是邹老师回国后引入的。
- 主要的文章是：介子：Eur.Phys.J.A 16 537-547 (2003) Zou, Bugg

  重子：Phys. Rev. C 67, 015204 (2003) Zou, Hussain
- Sayipjamal Dulat（新疆大学）和我在邹老师指导下完成：

Phys. Rev. D 83（2011）094032 $J/\psi \to B^*\bar{B} + B\bar{B}^* \to \gamma B\bar{B}$

模块

- LS机制下的协变振幅：L=> 角动量算符
  S=> 自旋算符
  协变=> 指标要用洛伦兹收缩
- **基本的模块：**
- **表示粒子的场量，介子极化矢量**

$$\boldsymbol{\varepsilon_{\mu_1\mu_2\cdots\mu_S}(p,S,m) = \sum_{m_{S-1},m_S}\langle S-1,m_{S-1};1,m_S|S,m\rangle\varepsilon_{\mu_1\mu_2\cdots\mu_{S-1}}(p,n-1,m_{S-1})\,\varepsilon_{\mu_S}(p,m_S)}$$

  重子旋量

$$u_{\mu_1\mu_2\cdots\mu_S}\left(p,S+\frac{1}{2},m\right)=\sum_{m_S,m_{S+1}}\left\langle S,m_S;\frac{1}{2},m_{S+1}\middle|S+\frac{1}{2},m\right\rangle \boldsymbol{\varepsilon_{\mu_1\mu_2\cdots\mu_S}(p,S,m_S)u(p,m_{S+1})}$$

- **自旋投影算子，**

$$P^{(L)}_{\mu_1\cdots\mu_L;\nu_1\cdots\nu_L}(p)=\sum_m \varepsilon_{\mu_1\mu_2\cdots\mu_L}(p,L,m)\varepsilon^*_{\nu_1\nu_2\cdots\nu_L}(p,L,m)$$

$$P^{\left(L+\frac{1}{2}\right)}_{\mu_1\cdots\mu_L;\nu_1\cdots\nu_L}(p)=(p\cdot\gamma+m)P^{(L)}_{\mu_1\cdots\mu_L;\nu_1\cdots\nu_L}(p)$$

模块

- 基本的模块：
- 表示粒子的场量，介子极化矢量

$$\boldsymbol{\varepsilon}_{\mu_1\mu_2\cdots\mu_S}(\boldsymbol{p},\boldsymbol{S},\boldsymbol{m}) = \sum_{m_{S-1},m_S} \langle \boldsymbol{S-1}, \boldsymbol{m_{S-1}}; \boldsymbol{1}, \boldsymbol{m_S} | \boldsymbol{S}, \boldsymbol{m} \rangle \boldsymbol{\varepsilon}_{\mu_1\mu_2\cdots\mu_{S-1}}(\boldsymbol{p}, \boldsymbol{n-1}, \boldsymbol{m_{S-1}})\, \boldsymbol{\varepsilon}_{\mu_S}(\boldsymbol{p}, \boldsymbol{m_S})$$

  重子旋量

$$u_{\mu_1\mu_2\cdots\mu_S}\left(p, S+\frac{1}{2}, m\right) = \sum_{m_S, m_{S+1}} \left\langle S, m_S; \frac{1}{2}, m_{S+1} \middle| S+\frac{1}{2}, m \right\rangle \boldsymbol{\varepsilon}_{\mu_1\mu_2\cdots\mu_S}(\boldsymbol{p}, \boldsymbol{S}, \boldsymbol{m_S}) \boldsymbol{u}(\boldsymbol{p}, \boldsymbol{m_{S+1}})$$

- 自旋投影算子，

$$P^{(L)}_{\mu_1\cdots\mu_L;\nu_1\cdots\nu_L}(p) = \sum_m \varepsilon_{\mu_1\mu_2\cdots\mu_L}(p,L,m)\varepsilon^*_{\nu_1\nu_2\cdots\nu_L}(p,L,m)$$

$$P^{\left(L+\frac{1}{2}\right)}_{\mu_1\cdots\mu_L;\nu_1\cdots\nu_L}(p) = (p\cdot\gamma + m)P^{(L)}_{\mu_1\cdots\mu_L;\nu_1\cdots\nu_L}(p)$$

a Blatt-Weisskopf barrier factor

- 角动量投影算子，$\tilde{t}^{(L)}_{\mu_1\cdots\mu_L}(p) = (-1)^L P^{(L)}_{\mu_1\cdots\mu_L;\nu_1\cdots\nu_L}(p) r^{\nu_1}\cdots r^{\nu_L} \times B_L(Q_{abc})$
- 张量，γ^μ, γ_5, $\epsilon^{\mu\nu\sigma\delta}$, $g^{\mu\nu}$, p^μ

三点振幅的构造

- 振幅的基本单元：三点振幅，$A(J_A) \to B(J_B)C(J_C)$
- LS机制： $J_A = L + S$; $S = J_B + J_C$
- 耦合机制：$J = L + S$, 如果$L + S - J =$ 奇数，引入$\epsilon^{\mu\nu\sigma\delta}$，否则自行收缩。
- J: $\phi^{\mu_1\cdots\mu_J}$; L: $t^{\alpha_1\cdots\alpha_L}$; S: $\psi^{\nu_1\cdots\nu_S}$ $[J = L + S - n]$

$h_{[\mu_1-\mu_J;\alpha_1-\alpha_L;\nu_1-\nu_S]}$

- $M = \phi^{\mu_1\cdots\mu_J} t^{\alpha_1\cdots\alpha_L} \psi^{\nu_1\cdots\nu_S} \times$ ---- n=2k

$$(g_{\alpha_1\nu_1}\cdots g_{\alpha_k\nu_k})(g_{\alpha_{k+1}\mu_1}\cdots g_{\alpha_L\mu_{L-k}})(g_{\nu_{k+1}\mu_{L-k+1}}\cdots g_{\alpha_S\mu_{L+S-2k}})$$

- $M = \phi^{\mu_1\cdots\mu_J} t^{\alpha_1\cdots\alpha_L} \psi^{\nu_1\cdots\nu_S} \times$ ---- n=2k+1 $\sim L \times S \cdot J$

$$\epsilon_{\delta\mu_J\alpha_L\nu_S} P_A^{\delta} (g_{\alpha_1\nu_1}\cdots g_{\alpha_k\nu_k})(g_{\alpha_{k+1}\mu_1}\cdots g_{\alpha_{L-1}\mu_{L-1-k}})(g_{\nu_{k+1}\mu_{L-k}}\cdots g_{\alpha_{S-1}\mu_{L+S-2k-1}})$$

三点振幅的构造

- 振幅的基本单元：三点振幅，$A(J_A) \to B(J_B)C(J_C)$
- LS机制：$J_A = L + S$; $S = J_B + J_C$
- 耦合机制：$J = L + S$, 如果$L + S - J =$ 奇数，引入 $\epsilon^{\mu\nu\sigma\delta}$，否则自行收缩。
- 三点振幅：$\phi_A^{\mu_1\cdots\mu_{J_A}} t^{\alpha_1\cdots\alpha_L} P^{(L)\nu_1\cdots\nu_S;\beta_1\cdots\beta_S} \phi_B^{\delta_1\cdots\delta_{J_B}} \phi_C^{\sigma_1\cdots\sigma_{J_C}} \times$

$$h_{[\mu_1-\mu_{J_A};\alpha_1-\alpha_L;\nu_1-\nu_S]} h_{[\beta_1-\beta_S;\delta_1\cdots\delta_{J_B};\sigma_1\cdots\sigma_{J_C}]}$$

三点振幅的构造

- 振幅的基本单元：三点振幅，$A(3^-) \to B(2^-)C(1^+)$
- LS机制： $(LS) = (03)(21)(22)(23)(41)(42)(43)(63);$
- Example： $(LS) = (42)$
- 三点振幅：$\phi_A^{\mu_1\cdots\mu_{J_A}} t^{\alpha_1\cdots\alpha_L} P^{(L)\nu_1\cdots\nu_S;\beta_1\cdots\beta_S} \phi_B^{\delta_1\cdots\delta_{J_B}} \phi_C^{\sigma_1\cdots\sigma_{J_C}} \times$
$$h_{[\mu_1-\mu_{J_A};\alpha_1-\alpha_L;\nu_1-\nu_S]} h_{[\beta_1-\beta_S;\delta_1\cdots\delta_{J_B};\sigma_1\cdots\sigma_{J_C}]}$$
- $\phi_A^{\mu_1\mu_2\mu_3} t^{\alpha_1\alpha_2\alpha_3\alpha_4} P^{(L)\nu_1\nu_2;\beta_1\beta_2} \phi_B^{\delta_1\delta_2} \phi_C^{\sigma_1} \times$
$$(\epsilon_{\delta\mu_3\alpha_4\nu_2} P_A^{\delta} g_{\alpha_1\mu_1} g_{\alpha_2\mu_2} g_{\alpha_3\nu_1})(\epsilon_{\delta'\beta_2\delta_2\sigma_1} P_A^{\delta'} g_{\beta_1\delta_1})$$

三点振幅的构造

- 振幅的基本单元：三点振幅， $A(J_A) \rightarrow B\left(\frac{1}{2}\right) C\left(J_C + \frac{1}{2}\right)$
- LS机制： $J_A = L + S;\quad S = \frac{1}{2} + J_C + \frac{1}{2}$
- $K^{\nu_1\cdots\nu_S} = \phi_B \Gamma^{\nu_1\cdots\nu_S}_{\sigma_1\cdots\sigma_{J_C}} \phi_C^{\sigma_1\cdots\sigma_{J_C}}$ =>详见**PRC 67, 015204 Zou, Hussain**
- 三点振幅： $\phi_A^{\mu_1\cdots\mu_{J_A}} t^{\alpha_1\cdots\alpha_L} K^{\nu_1\cdots\nu_S} h_{[\mu_1-\mu_{J_A};\alpha_1-\alpha_L;\nu_1-\nu_S]}$
- 举例

$\psi \rightarrow N^*(\frac{1}{2}^-)\bar{N}(1,0,1):\quad \psi^{(0)}\varepsilon_\mu \tilde{t}^{(1)\mu},$

(J_A, S, L)

$(1,1,1):\quad i\Psi^{(1)}_\mu \epsilon^{\mu\nu\lambda\sigma}\varepsilon_\nu \tilde{t}^{(1)}_\lambda \hat{p}_{(\psi)\sigma},$

三点振幅的构造

- 振幅的基本单元：三点振幅，$A(J_A+\frac{1}{2}) \to B\left(\frac{1}{2}\right) C(J_C)$
- LS机制： $J_A+\frac{1}{2}=L+\frac{1}{2}+J_C \to \left(J_A+\frac{1}{2}\right)-\frac{1}{2} \equiv S = L+J_C$
- $K^{\nu_1\cdots\nu_S} = \phi_B \Gamma^{\nu_1\cdots\nu_S}_{\sigma_1\cdots\sigma_{J_A}} \phi_A^{\sigma_1\cdots\sigma_{J_A}}$ =>详见**PRC 67, 015204 Zou, Hussain**
- 三点振幅：$K^{\nu_1\cdots\nu_S} t^{\alpha_1\cdots\alpha_L} \phi_C^{\mu_1\cdots\mu_{J_C}} h_{[\nu_1-\nu_S;\alpha_1-\alpha_L;\,\mu_1-\mu_{J_C}]}$
- 举例

$N^*(\frac{3}{2}^-) \to N\omega(1,1,0)$: $\phi^{(1)}_{\mu}\varepsilon^{*\mu}$,

(Jc, S, L) $(1,1,2)$: $\phi^{(1)}_{\mu}\varepsilon^{*}_{\nu}\tilde{t}^{(2)\mu\nu}$,

$(1,2,2)$: $i\Phi^{(2)}_{\mu\alpha}\epsilon^{\mu\nu\lambda\sigma}\varepsilon^{*}_{\nu}\tilde{t}^{(2)\alpha}_{\lambda}\hat{p}_{*\sigma}$,

级联反应的构造

$$A \to B\ C \to B\ (d\ e)$$

$$\Gamma_{\{A\to BC\}} = \tilde{\Gamma}^{\mu_1\cdots\mu_{J_C}}_{\{A\to BC\}} \times \phi_{\mu_1\cdots\mu_{J_C}} \qquad \Gamma_{\{C\to de\}} = \phi^*_{\nu_1\cdots\nu_{J_C}} \times \tilde{\Gamma}^{\nu_1\cdots\nu_{J_C}}_{\{C\to de\}}$$

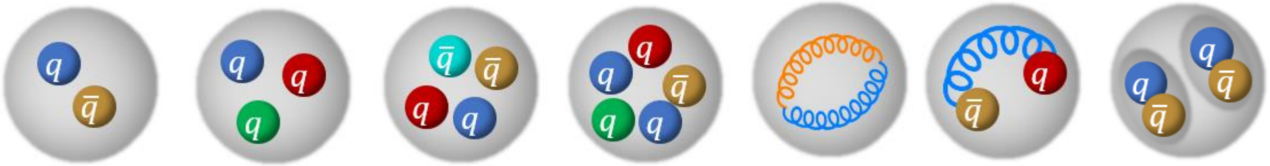
$$\Gamma_{\{A\to B\ C\to B\ (d\ e)\}}$$
$$= \sum_{m_C} \tilde{\Gamma}^{\mu_1\cdots\mu_{J_C}}_{\{A\to BC\}} \times \phi_{\mu_1\cdots\mu_{J_C}}(p_C, m_C)\phi^*_{\nu_1\cdots\nu_{J_C}}(p_C, m_C) \times \tilde{\Gamma}^{\nu_1\cdots\nu_{J_C}}_{\{C\to de\}} \times BW(C)$$
$$= \tilde{\Gamma}^{\mu_1\cdots\mu_{J_C}}_{\{A\to BC\}} \times P^{(J_C)}_{\mu_1\cdots\mu_{J_C};\nu_1\cdots\nu_{J_C}} \times \tilde{\Gamma}^{\nu_1\cdots\nu_{J_C}}_{\{C\to de\}} \times BW(C)$$

强子谱学的研究进展

1. 从几个树图的计算
-> 耦合道计算，即需要解适当的散射方程得到T矩阵

耦合道计算的确更加准确，但是依然没有完整表达非微扰的所有信息。应用于计算的势能已经做了简化；四维散射方程尚无法解，我们通常的散射方程是做了三维约化的。

2. 从简单的组分夸克模型（quenched Quark model）
-> 更多夸克组分的系统。



介子 重子 四夸克态 五夸克态 胶球 混杂态 强子分子态

每一个模型都有自己的参数，最后对于每一个共振态，很难区分是那个模型主导的。紧致多夸克是否真的存在？如何链接这些模型？

3. 格点QCD从遥远的非物理区
-> 实现了对真实强子模型的约束。

格点的数据产生花费还是比较大，能提供的信息还很有限。因而散射振幅的获取还要依赖模型，如何解读格点数据还有待于研究。

总结

- LS机制能够在洛伦兹不变的规则下给出振幅
- 虽然不是严格的LS，但是保证了振幅的独立性
- 最关键的是现在的书写规则比较简单。

谢谢

讨论$\psi \to \gamma f_2$的分波顶点

文献中的分波公式

Zou and Bugg, hep-ph/0211457

We denote the ψ polarization four-vector by $\psi_\mu(m_1)$ and the polarization vector of the photon by $e_\nu(m_2)$. Then the general form for the decay amplitude is

$$A = \psi_\mu(m_1)e^*_\nu(m_2)A^{\mu\nu} = \psi_\mu(m_1)e^*_\nu(m_2)\sum_i \Lambda_i U_i^{\mu\nu}. \tag{100}$$

For the photon polarization four vector e_ν with photon momentum q, there is the usual Lorentz orthogonality condition $e_\nu q^\nu = 0$. This is the same as for a massive vector meson. However, for the photon, there is an additional gauge invariance condition. Here we assume the Coulomb gauge in the ψ rest system, i.e., $e_\nu p^\nu_\psi = 0$. Then we have [13]

$$\sum_m e^*_\mu(m)e_\nu(m) = -g_{\mu\nu} + \frac{q_\mu K_\nu + K_\mu q_\nu}{q\cdot K} - \frac{K\cdot K}{(q\cdot K)^2}q_\mu q_\nu \equiv -g^{(\perp\perp)}_{\mu\nu} \tag{101}$$

with $K = p_\psi - q$ and $e_\nu K^\nu = 0$. The radiative decay cross section is:

$$\begin{aligned}
\frac{d\sigma}{d\Phi_n} &= \frac{1}{2}\sum_{m_1=1}^{2}\sum_{m_2=1}^{2}\psi_\mu(m_1)e^*_\nu(m_2)A^{\mu\nu}\psi^*_{\mu'}(m_1)e_{\nu'}(m_2)A^{*\mu'\nu'} \\
&= -\frac{1}{2}\sum_{\mu=1}^{2}A_{\mu\nu}g^{(\perp\perp)}_{\nu\nu'}A^{*\mu\nu'} \\
&= -\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda_j^*\sum_{\mu=1}^{2}U_i^{\mu\nu}g^{(\perp\perp)}_{\nu\nu'}U_j^{*\mu\nu'}
\end{aligned}$$

因此这里所谓的分波就是写出$U_i^{\mu\nu}$

讨论$\psi \to \gamma f_2$的分波

- J^P量子数上是$1^- \to 1^-\ 2^+$，这里的f_2会再衰变到$\pi\pi$，因此f_2的极化矢量就直接用$\pi\pi$的D波$\tilde{t}^{\mu\nu}_{f_2}$来表示。
- 从LS耦合上看，这里应该有5种分波，(L, S)=(0, 1), (2, 1), (2, 2), (2, 3), (4, 3)。
- 但是由于光子的规范不变性，这些分波并不是完全独立的，最后独立的个数只有3个。
- 方法是

  a. 写出所有这5个分波，每个分波均有一个独立的耦合系数$g_i(i = 1,..,5)$；

  b. 把光子的极化矢量换成光子动量，是的五个分波构成的总振幅为0，由此这5个耦合系数之间会有关系，最终得到真正独立的耦合常数的个数；

  c. 按照独立的耦合常数的写出分波。

讨论$\psi \rightarrow \gamma f_2$的分波

- 方法是

  a. 写出所有这5个分波，每个分波均有一个独立的耦合系数$g_i(i = 1,..,5)$;

(L, S) = (0, 1), (2, 1), (2, 2), (2, 3), (4, 3)　$[\mu \rightarrow \psi,\ \nu \rightarrow \gamma]$

(0, 1)　$U_{01}^{\mu\nu} = \tilde{t}_{f_2}^{\mu\nu}$

(2, 1)　$U_{21}^{\mu\nu} = \tilde{t}_{\alpha}^{(2)\mu} \tilde{t}_{f_2}^{\alpha\nu}$

(2, 2)　$U_{22}^{\mu\nu} = \epsilon^{\mu\alpha\beta\delta} P_{\alpha} \tilde{t}_{\beta}^{(2)\sigma} \hat{P}^{2}{}_{\delta\sigma\,\delta'\sigma'} \epsilon^{\nu\alpha'\beta'\delta'} P_{\alpha'} \tilde{t}_{f_2\,\beta'}^{\sigma'}$

(2, 3)　$U_{23}^{\mu\nu} = \tilde{t}_{\alpha\beta}^{(2)} \hat{P}^{3\,\mu\alpha\beta\,\nu\alpha'\beta'} \tilde{t}_{f_2\,\alpha'\beta'}$

(4, 3)　$U_{43}^{\mu\nu} = \tilde{t}^{(4)\mu\alpha\beta\delta} \hat{P}^{3}{}_{\alpha\beta\delta}{}^{\nu\alpha'\beta'} \tilde{t}_{f_2\,\alpha'\beta'}$

$$U^{\mu\nu} = g_{01} U_{01}^{\mu\nu} + g_{21} U_{21}^{\mu\nu} + g_{22} U_{22}^{\mu\nu} + g_{23} U_{23}^{\mu\nu} + g_{43} U_{43}^{\mu\nu}$$

讨论$\psi \to \gamma f_2$的分波

- 方法是

b. 把光子的极化矢量换成光子动量，是的五个分波构成的总振幅为0，由此这5个耦合系数之间会有关系，最终得到真正独立的耦合常数的个数；

要求$U^{\mu\nu} q_\nu = 0$

(0, 1) $U_{01}^{\mu\nu} q_\nu = \tilde{t}_{f_2}^{\mu\nu} q_\nu = A_{01} \tilde{t}_{f_2}^{\mu\nu} q_\nu$

(2, 1) $U_{21}^{\mu\nu} q_\nu = \tilde{t}_{\alpha}^{(2)\mu} \tilde{t}_{f_2}^{\alpha\nu} q_\nu = A_{21} \tilde{t}_{f_2}^{\mu\nu} q_\nu + B_{21} q^\mu \tilde{t}_{f_2}^{\alpha\nu} q_\alpha q_\nu \cancel{+ C_{21} P^\mu \tilde{t}_{f_2}^{\alpha\nu}} q_\alpha q_\nu$

(2, 2) $U_{22}^{\mu\nu} q_\nu = \epsilon^{\mu\alpha\beta\delta} P_\alpha \tilde{t}_\beta^{(2)\sigma} \hat{P}^2{}_{\delta\sigma\,\delta'\sigma'}\, \epsilon^{\nu\alpha'\beta'\delta'} P_{\alpha'} \tilde{t}^{\sigma'}_{f_2\,\beta'} q_\nu = A_{22} \tilde{t}_{f_2}^{\mu\nu} q_\nu + B_{22} q^\mu \tilde{t}_{f_2}^{\alpha\nu} q_\alpha q_\nu$

(2, 3) $U_{23}^{\mu\nu} q_\nu = \tilde{t}_{\alpha\beta}^{(2)} \hat{P}^{3\,\mu\alpha\beta\,\nu\alpha'\beta'} \tilde{t}_{f_2\,\alpha'\beta'} q_\nu = A_{23} \tilde{t}_{f_2}^{\mu\nu} q_\nu + B_{23} q^\mu \tilde{t}_{f_2}^{\alpha\nu} q_\alpha q_\nu$

(4, 3) $U_{43}^{\mu\nu} q_\nu = \tilde{t}^{(4)\mu\alpha\beta\delta} \hat{P}^{3\ \nu\alpha'\beta'}_{\alpha\beta\delta} \tilde{t}_{f_2\,\alpha'\beta'} q_\nu = A_{43} \tilde{t}_{f_2}^{\mu\nu} q_\nu + B_{43} q^\mu \tilde{t}_{f_2}^{\alpha\nu} q_\alpha q_\nu$

讨论$\psi \to \gamma f_2$的分波

- 方法是

b. 把光子的极化矢量换成光子动量，是的五个分波构成的总振幅为0，由此这5个耦合系数之间会有关系，最终得到真正独立的耦合常数的个数；

要求$U^{\mu\nu} q_\nu = 0 \ [\alpha = \frac{k_{on}}{m_\psi}]$

(0, 1) $A_{01} = 1$

(2, 1) $A_{21} = -\frac{4}{3} m_\psi^2 \alpha^2; B_{21} = 2$

(2, 2) $A_{22} = -2m_\psi^2 (1-\alpha)^2; B_{22} = -2m_\psi^4 \alpha^2 (1-\alpha)$

(2, 3) $A_{23} = \frac{8}{15} m_\psi^2 \alpha^2 (1-\alpha); B_{23} = \frac{4}{15} \alpha^2$

(4, 3) $A_{43} = -\frac{162}{35} m_\psi^4 \alpha^4 (1-\alpha); B_{43} = \frac{32}{7} \left(4 - 6\alpha + 3\alpha^2\right) + \frac{1}{35} \alpha^2$

讨论$\psi \to \gamma f_2$的分波

- 方法是

b. 把光子的极化矢量换成光子动量，是的五个分波构成的总振幅为0，由此这5个耦合系数之间会有关系，最终得到真正独立的耦合常数的个数；

要求$U^{\mu\nu}q_\nu = 0$ $[\alpha^2 \equiv \frac{k_{on}^2}{m_\psi^2}]$

For $\tilde{t}_{f_2}^{\mu\nu} q_\nu$, $g_{01}A_{01} + g_{21}A_{21} + g_{22}A_{22} + g_{23}A_{23} + g_{43}A_{43} = 0;$

For $q^\mu \tilde{t}_{f_2}^{\alpha\nu} q_\alpha q_\nu$, $g_{21}B_{21} + g_{22}B_{22} + g_{23}B_{23} + g_{43}B_{43} = 0;$

我们选择g_{01}，g_{21}，g_{23}作为自由的参数，则可以得到

$$g_{22} = \frac{(g_{21}B_{21} + g_{23}B_{23})A_{43} - (g_{01}A_{01} + g_{21}A_{21} + g_{23}A_{23})B_{43}}{A_{22}B_{43} - B_{22}A_{43}}$$

$$g_{43} = \frac{(g_{21}B_{21} + g_{23}B_{23})A_{22} - (g_{01}A_{01} + g_{21}A_{21} + g_{23}A_{23})B_{22}}{-A_{22}B_{43} + B_{22}A_{43}}$$

讨论$\psi \to \gamma f_2$的分波

- 方法是

  c. 按照独立的耦合常数的写出分波。

$$U^{\mu\nu} = g_{01}U_{01}^{\mu\nu} + g_{21}U_{21}^{\mu\nu} + g_{22}U_{22}^{\mu\nu} + g_{23}U_{23}^{\mu\nu} + g_{43}U_{43}^{\mu\nu}$$

$$= g_{01}U_{01}^{\mu\nu} + g_{21}U_{21}^{\mu\nu} + \frac{(g_{21}B_{21} + g_{23}B_{23})A_{43} - (g_{01}A_{01} + g_{21}A_{21} + g_{23}A_{23})B_{43}}{A_{22}B_{43} - B_{22}A_{43}}U_{22}^{\mu\nu} + g_{23}U_{23}^{\mu\nu} + \frac{(g_{21}B_{21} + g_{23}B_{23})A_{22} - (g_{01}A_{01} + g_{21}A_{21} + g_{23}A_{23})B_{22}}{-A_{22}B_{43} + B_{22}A_{43}}U_{43}^{\mu\nu}$$

讨论$\psi \to \gamma f_2$的分波

- 方法是

 c. 按照独立的耦合常数的写出分波。[C=1/$(-A_{22}B_{43} + B_{22}A_{43})$]

$$U^{\mu\nu} = g_{01}\left[U^{\mu\nu}_{01} + B_{43}CU^{\mu\nu}_{22} - B_{22}CU^{\mu\nu}_{43}\right] + g_{21}\left[U^{\mu\nu}_{21} + (A_{21}B_{43} - B_{21}A_{43})CU^{\mu\nu}_{22} + (B_{21}A_{22} - A_{21}B_{22})CU^{\mu\nu}_{43}\right] + g_{23}\left[U^{\mu\nu}_{23} + (A_{23}B_{43} - B_{23}A_{43})CU^{\mu\nu}_{22} + (B_{23}A_{22} - A_{23}B_{22})CU^{\mu\nu}_{43}\right]$$

文献里面的是

For $\psi \to \gamma f_2$ or $\psi \to \gamma f_4$, the e_μ may contract with ψ^μ or with the spin wave function of f_J. Then ψ^μ may contract with e_μ, or q_μ, or the spin wave function of f_J; this gives three independent covariant tensor amplitudes for each f_J:

$$U^{\mu\nu}_{(\gamma f_2)1} = \tilde{t}^{(f_2)\mu\nu} f^{(f_2)}, \tag{106}$$

$$U^{\mu\nu}_{(\gamma f_2)2} = g^{\mu\nu} p^\alpha_\psi p^\beta_\psi \tilde{t}^{(f_2)}_{\alpha\beta} B_2(Q_{\Psi\gamma f_2}) f^{(f_2)}, \tag{107}$$

$$U^{\mu\nu}_{(\gamma f_2)3} = q^\mu \tilde{t}^{(f_2)\nu}_\alpha p^\alpha_\psi B_2(Q_{\psi\gamma f_2}) f^{(f_2)}, \tag{108}$$

讨论$\psi \to \gamma f_2$的分波

For the photon polarization four vector e_ν with photon momentum q, there is the usual Lorentz orthogonality condition $e_\nu q^\nu = 0$. This is the same as for a massive vector meson. However, for the photon, there is an additional gauge invariance condition. Here we assume the Coulomb gauge in the ψ rest system, i.e., $e_\nu p_\psi^\nu = 0$. Then we have [13]

$$\sum_m e^*_\mu(m) e_\nu(m) = -g_{\mu\nu} + \frac{q_\mu K_\nu + K_\mu q_\nu}{q\cdot K} - \frac{K\cdot K}{(q\cdot K)^2} q_\mu q_\nu \equiv -g^{(\perp\perp)}_{\mu\nu} \qquad (101)$$

- 方法是

  a. 写出所有这5个分波，每个分波均有一个独立的耦合系数$g_i (i = 1, .., 5)$

(L, S) = (0, 1), (2, 1), (2, 2), (2, 3), (4, 3) $[\mu \to \psi,\ \nu \to \gamma]$

(0, 1) $U^{\mu\nu}_{01} e_\nu = \tilde{t}^{\mu\nu}_{f_2} e_\nu$

(2, 1) $U^{\mu\nu}_{21} e_\nu = \tilde{t}^{(2)\mu}_{\alpha} \tilde{t}^{\alpha\nu}_{f_2} e_\nu = A_{21} \tilde{t}^{\mu\nu}_{f_2} e_\nu + B_{21} q^\mu q_\alpha \tilde{t}^{\alpha\nu}_{f_2} e_\nu$

(2, 2) $U^{\mu\nu}_{22} e_\nu = \epsilon^{\mu\alpha\beta\delta} P_\alpha \tilde{t}^{(2)\sigma}_{\beta} \hat{P}^2{}_{\delta\sigma\,\delta'\sigma'}\, \epsilon^{\nu\alpha'\beta'\delta'} P_{\alpha'} \tilde{t}^{\sigma'}_{f_2\,\beta'} e_\nu$

$= A_{22}\, e_\nu g^{\mu\nu} q_\alpha \tilde{t}^{\alpha\beta}_{f_2} q_\beta$

(2, 3) $U^{\mu\nu}_{23} e_\nu = \tilde{t}^{(2)}_{\alpha\beta} \hat{P}^{3\ \mu\alpha\beta\ \nu\alpha'\beta'} \tilde{t}_{f_2\,\alpha'\beta'} e_\nu = A_{23}\, e_\nu g^{\mu\nu} q_\alpha \tilde{t}^{\alpha\beta}_{f_2} q_\beta$

(4, 3) $U^{\mu\nu}_{43} = \tilde{t}^{(4)\mu\alpha\beta\delta} \hat{P}^{3\ \nu\alpha'\beta'}_{\alpha\beta\delta} \tilde{t}_{f_2\,\alpha'\beta'} = A_{43}\, e_\nu g^{\mu\nu} q_\alpha \tilde{t}^{\alpha\beta}_{f_2} q_\beta$

在该规范下，光子的极化矢量和任意一个粒子的动量点乘都是0。
这样一来e^ν只能和ψ的极化矢量，f_2的极化矢量收缩。

$$U^{\mu\nu}_{(\gamma f_2)1} = \tilde{t}^{(f_2)\mu\nu} f^{(f_2)},$$
$$U^{\mu\nu}_{(\gamma f_2)2} = g^{\mu\nu} p^\alpha_\psi p^\beta_\psi \tilde{t}^{(f_2)}_{\alpha\beta} B_2(Q_{\Psi\gamma f_2}) f^{(f_2)},$$
$$U^{\mu\nu}_{(\gamma f_2)3} = q^\mu \tilde{t}^{(f_2)\nu}_{\alpha} p^\alpha_\psi B_2(Q_{\psi\gamma f_2}) f^{(f_2)},$$