QCD Matter in Magnetic Field

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为什么考虑电磁场？

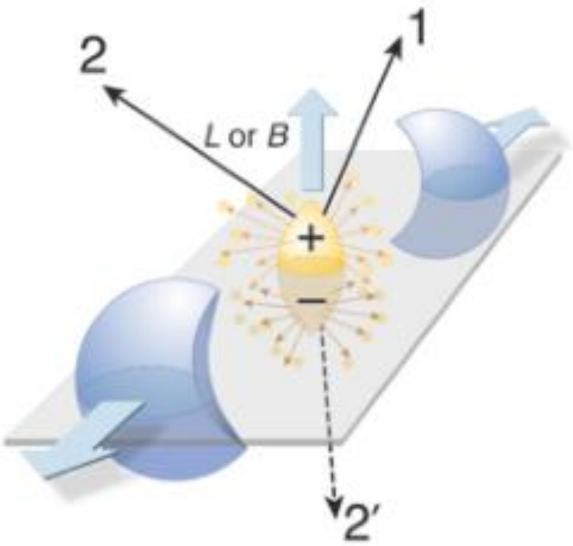
自然界最强的磁场和涡旋场在高能核碰撞产生：

$|eB| \sim 5m_\pi^2$ $(0.1\ GeV^2)$ *at RHIC and* $70m_\pi^2$ $(1\ GeV^2)$ *at LHC,*

$\omega \sim 10^{21}/s$ *at RHIC,*

其强度已经与强相互作用可比拟！

See for instance W.Deng and X.Huang, PRC85,044907 (2012)

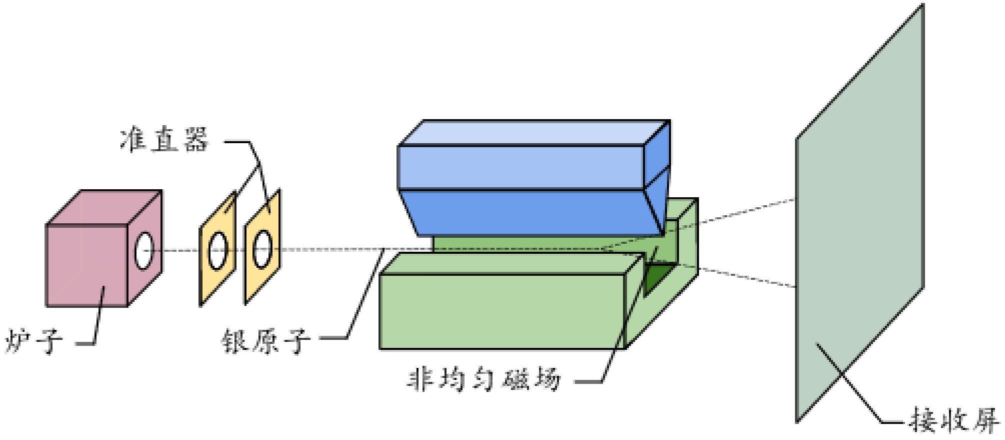


2 Quantum Phenomena in Magnetic Field

See any textbook of Quantum Mechanics, for instance the one by J.J.Sakurai

- ♣ *Spin*

Stern-Gerlach experiment



Spin-magnetic (rotational) field interaction results in CME and CVE.

- ♣ *Landau energy levels*

Fermion energy in magnetic field

$$E = \sqrt{m^2 + \vec{p}^2} \rightarrow \sqrt{m^2 + p_z^2 + 2n|qB|}$$

What is the quantum effect induced by the Landau levels?

What We Want to Discuss

1) Competition between B and T:

$\vec{B}$ *breaks down the translation invariance, but T restores the invariance.*

→ *Is the B field strong enough to be measured in thermal medium with* $T \sim 300-500\ \mathrm{MeV}$ *?*

2) Competition between τ_B and τ_{rel}:

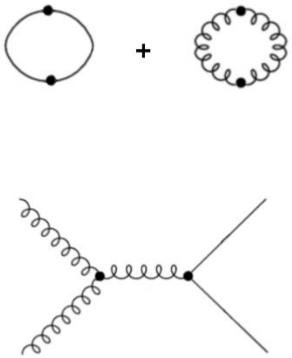
The lifetime of external B field is very short, we need induced B field in QGP .

→ *Can the current be completely induced before the disappearance of the external field with lifetime* $\tau_B \sim 0.1\ \mathrm{fm}$ *?*

3) Competition between B and μ_B:

Is there any quantum effect when the Fermi surface meets Landau energy level?

4) From static properties to dynamical processes:



Feynman Rules in Magnetic Field

A.Kostenko and C.Thompson, Astrophys J. 869, 44(2018), 875, 23(2019).

External lines:

$$[i\gamma^\mu(\partial_\mu + iqA_\mu) - m]\psi = 0$$

$$\psi^\sigma_\mp(x,p) = \begin{cases} e^{-ip\cdot x}u_\sigma(\boldsymbol{x},p) \\ e^{ip\cdot x}v_\sigma(\boldsymbol{x},p) \end{cases}$$

$$u_-(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -ip_z p_n \phi_{n-1} \\ (\epsilon+\epsilon_n)(\epsilon_n+m)\phi_n \\ -ip_n(\epsilon+\epsilon_n)\phi_{n-1} \\ -p_z(\epsilon_n+m)\phi_n \end{bmatrix}, \qquad v_+(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -p_n(\epsilon+\epsilon_n)\phi_{n-1} \\ -ip_z(\epsilon_n+m)\phi_n \\ -p_z p_n \phi_{n-1} \\ i(\epsilon+\epsilon_n)(\epsilon_n+m)\phi_n \end{bmatrix},$$

$$u_+(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} (\epsilon+\epsilon_n)(\epsilon_n+m)\phi_{n-1} \\ -ip_z p_n \phi_n \\ p_z(\epsilon_n+m)\phi_{n-1} \\ ip_n(\epsilon+\epsilon_n)\phi_n \end{bmatrix} \qquad v_-(\boldsymbol{x},p) = \frac{1}{f_n}\begin{bmatrix} -ip_z(\epsilon_n+m)\phi_{n-1} \\ -p_n(\epsilon+\epsilon_n)\phi_n \\ -i(\epsilon+\epsilon_n)(\epsilon_n+m)\phi_{n-1} \\ p_z p_n \phi_n \end{bmatrix}$$

Quark propagator:

$$G(x'-x) = -i\left(\frac{L}{2\pi\lambda}\right)^2 \int dp_z da \sum_{\sigma,n}\left[\theta(t'-t)u_\sigma(\boldsymbol{x}',p)\bar{u}_\sigma(\boldsymbol{x},p)e^{-ip\cdot(x'-x)} - \theta(t-t')v_\sigma(\boldsymbol{x}',p)\bar{v}_\sigma(\boldsymbol{x},p)e^{ip\cdot(x'-x)}\right]$$

$$G(p) = -\int_0^\infty \frac{dv}{|qB|}\left\{[m+(\gamma\cdot p)_\parallel][1-isgn(q)\gamma_1\gamma_2\tanh(v)] - \frac{(\gamma\cdot p)_\perp}{cosh^2(v)}\right\} e^{-\frac{v}{|qB|}\left[m^2-p_\parallel^2+\frac{\tanh(v)}{v}p_\perp^2\right]}$$

Schwinger propagator, 1951

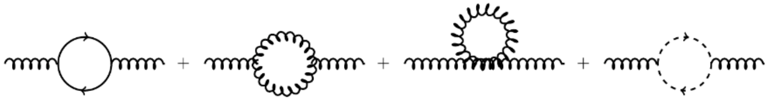
- no more translation invariance.
- the two Schwinger phases for q and $\bar{q}$ are cancelled to each other in loop calculation.

Gluon Propagator in QCD Matter

Kapusta and Gale, Finite-Temperature Field Theory: Principles and Applications

G.Huang, J.Zhao, PZ, PRD107, 11(2023)

Gluon self-energy in magnetic field:



for quark loop

$$\Pi^{||}_{\mu\mu}(T,B) = g^2 T|qB| \sum_{n p_z n_1} \frac{(2-\delta_{n_1 0})\,(\delta^{||}_{\mu\mu}+g^{||}_{\mu\mu})(-\omega_n^2+p_z^2)}{(m^2+\omega_n^2+p_z^2+2n_1|qB|)^2}$$

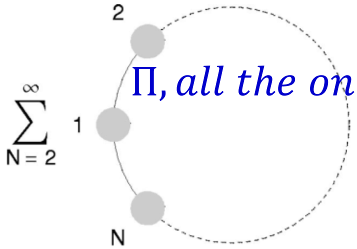
$$\Pi^{\perp}_{\mu\mu}(T,B) = 0$$

Matsubara frequencies $p_0 = i\omega_n = i(2n+1)\pi T$
quark longitudinal momentum p_z
transverse Landau energy $\varepsilon_k = 2n_1|qB|$

for gluon and ghost loops

$$\overline{\Pi}_{\mu\nu}(T,B) = \overline{\Pi}_{\mu\nu}(T,0)$$

To include non-perturbative effect, we take the summation of ring diagrams
→ gluon propagator and thermodynamic potential:

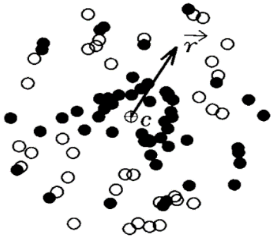


$\Pi, all\ the\ one-loop\ polarization$

Color Screening in QCD Matter

G.Huang, J.Zhao, PZ, PRD107, 11(2023)

Debye screening of a pair of charged particles $q\bar{q}$:



$$\frac{1}{r} \to \frac{1}{r} e^{-m_D r} = \frac{1}{r} e^{-r/r_D}$$

screening mass m_D

screening length r_D

Pole of the propagator at gluon momentum $\left(k_0^2 = 0,\ \vec{k}^2 = -m_D^2\right) \to$ screening mass:

$$m_D^2(T,B) = m_Q^2(T,B) + m_G^2(T),$$

$$m_Q^2(T,B) = -\Pi_{00}^{||}(T,B),$$

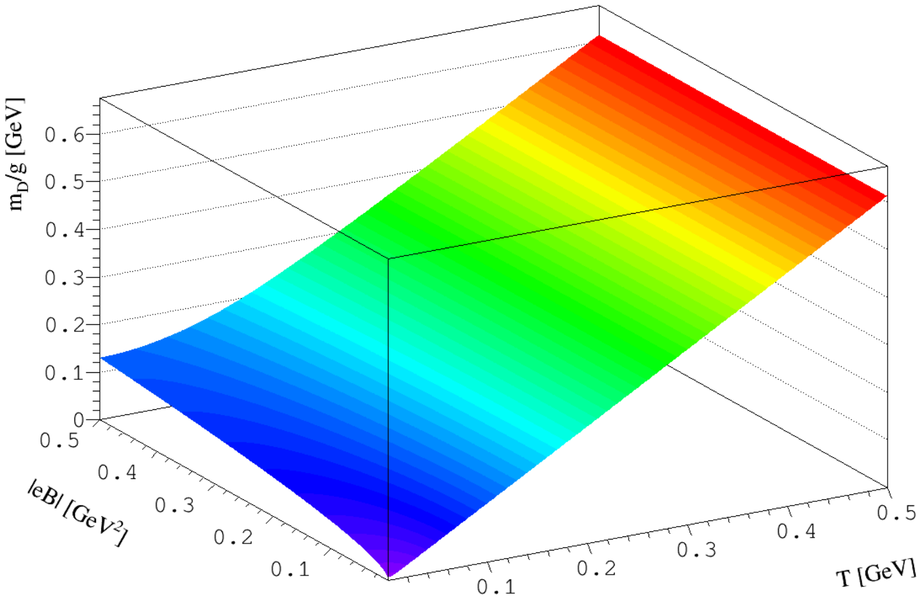
$$m_G^2(T) = -\overline{\Pi}_{00}^{||}(T).$$

$$m_Q^2(T,B) = -g^2 T|qB| \sum_{n p_z n_1} \left[(2-\delta_{n_1,0}) \frac{m^2 - \omega_n^2 + p_z^2 + 2n_1|qB|}{(m^2+\omega_n^2+p_z^2+2n_1|qB|)^2}\right]$$

$$m_G^2(T) = \frac{N_c}{3} g^2 T^2$$

$m_D(T,B)$

G.Huang, J.Zhao, PZ, PRD107, 11(2023)



- m_D/g *is* g *independent in ring diagram summation.*
- $m_D(0,B) = 0.13$ GeV *at* $eB = 25m_\pi^2$.
- *the B effect is gradually washed out by thermal motion.*

Conclusion 1:

While the magnetic field created in HIC is the strongest one in nature, its effect on QCD matter is much weaker in comparison with the fireball temperature.

重离子碰撞中破缺的欧姆定律

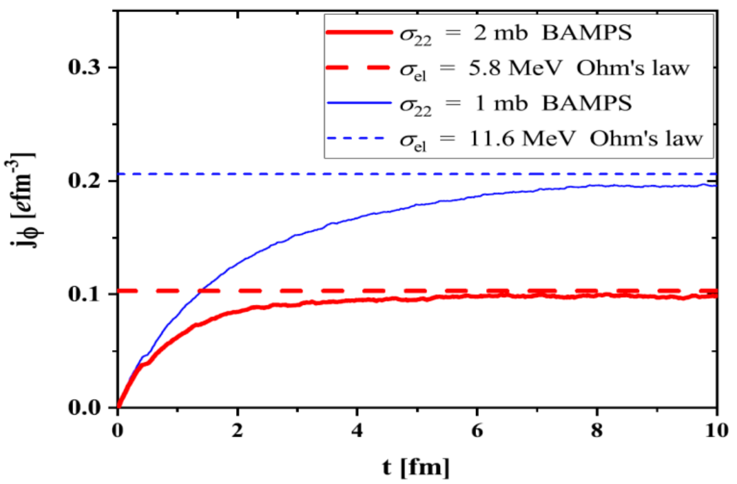
$$\vec{J}_{ohm} \le \sigma_{el}\left(\vec{E}+\vec{v}\times\vec{B}\right)$$

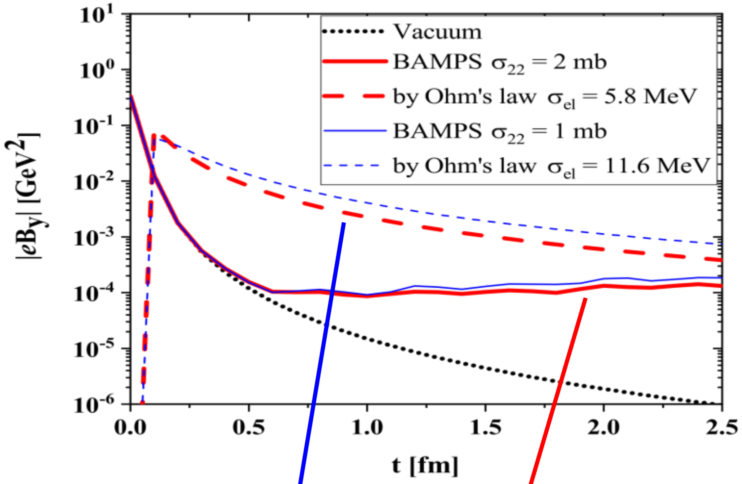
Incomplete electromagnetic response of hot QCD matter

Wang, Zhao, Greiner, Xu and Zhuang, PRC105, L041901(2022), Letter, Featured in Physics

The evolution of the fireball in described by BAMPS:

$$\left(\partial/\partial t+\vec{p}/E\cdot\vec{\nabla}+\vec{F}\cdot\vec{\nabla}_p\right)f = C_{22}+C_{23}, \qquad Lorentz\ force\ \vec{F}=q\left(\vec{p}/E\times\vec{B}+\vec{E}\right)$$





得到黄旭光,王群,*Shovkovy*等人工作的支持。

K.Tuchin, (with Ohm's law)

BAMPS (Without Ohm's law)

Conclusion 2:

The electromagnetic response of the hot QCD matter to the fast decay of the external electromagnetic field is incomplete, which strongly suppresses the induced magnetic field.

Color Screening in Dense QCD Matter

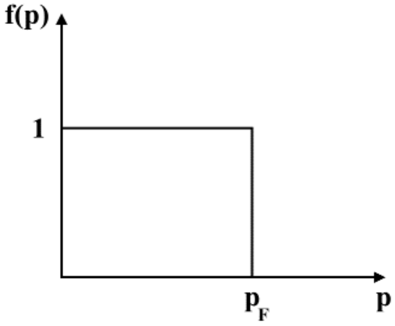
G.Huang, J.Zhao, PZ, arXiv:2307.02608

In the frame of ring diagram summation

$$m_D^2(\mu_f, B) = \frac{g^2}{(2\pi)^2} \sum_{n=0} (2 - \delta_{n0}) \frac{\mu_f |qB|}{\sqrt{\mu_f^2 - 2n|qB|}}$$

For chiral quarks, the distribution

$$f(p) = \theta(\mu_f - p)$$



the Fermi surface is determined by

$$p_z^2 + 2n|qB| = \mu_f^2$$

when μ_f and Landau levels are equal

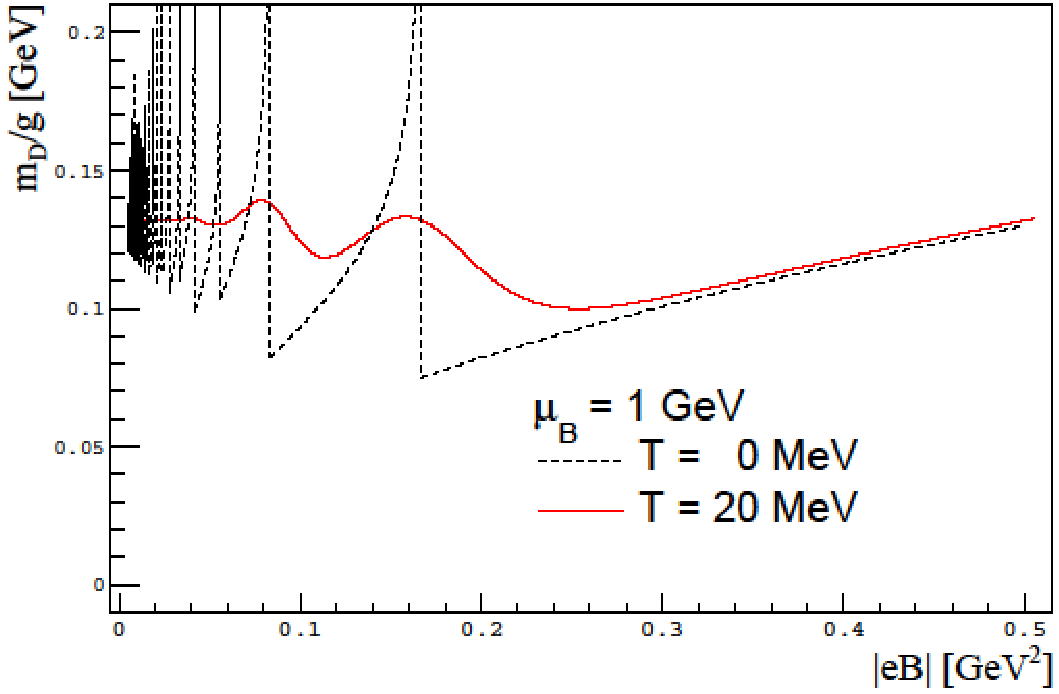
$$\mu_f^2 = 2n|qB|$$

the infrared divergence at the Fermi surface induces a complete screening $m_D \to \infty$, called resonant screening.

The case here is similar to the resonant transmission in Quantum Mechanics.

Resonant Screening

G.Huang, J.Zhao, PZ, arXiv:2307.02608



Conclusion 3:

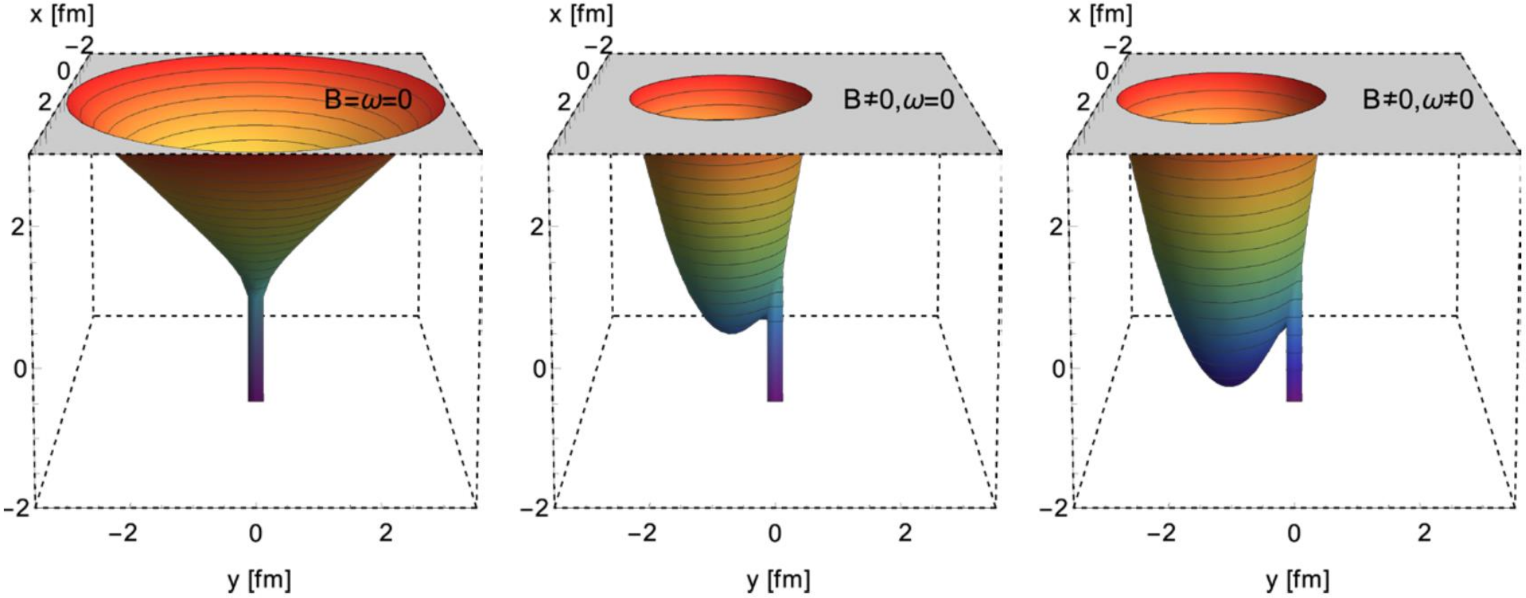
The color interaction between a pair of quarks is completely screened in dense and magnetized QCD matter, when chemical potential matches the Landau levels $\mu_f^2 - 2n|qB| = 0.$

从强束缚态到电磁束缚态的跃迁

Charmonium transition in electromagnetic and rotational fields

Chen, Zhao and Zhuang, PRC103, L031902(2021), Letter

Heavy quarks are produced in the initial stage of HIC, they experience the strongest electromagnetic field and are probably a sensitive signal of the field.



Resonant Dissociation

J.Hu, S.Shi, Z.Xu, J.Zhao, PZ, PRD105,9(2022)

Heavy quarks are produced in the initial stage of HIC, they experience the strongest electromagnetic field and are probably a sensitive signal of the field.

In the frame of QCD multipole expansion:

T.Yan, Phys. Rev. D 22, 1652 (1980),
Y.Kuang and T.Yan, Phys. Rev. D 24, 2874 (1981)
Y.Liu, C.Ko, T.Song, Phys. Rev. C 88, 064902 (2013)
S.Chen, M.He, Phys. Rev. C 96, 034901 (2017)
S.Chen, M.He, Phys. Lett. B 786, 260 (2018).

$$\hat{H} = \hat{H}_0 + \hat{H}_I,$$

$$\hat{H}_0 = \frac{(\hat{\boldsymbol{p}}_1 - q\boldsymbol{A}(x_1))^2}{2m_Q} + \frac{(\hat{\boldsymbol{p}}_2 + q\boldsymbol{A}(x_2))^2}{2m_Q}$$

$$- A_0(x_1) - A_0(x_2) + V_1(|\boldsymbol{r}|) + \sum_{a=1}^{8} \frac{\lambda_a}{2}\frac{\bar{\lambda}_a}{2} V_2(|\boldsymbol{r}|)$$

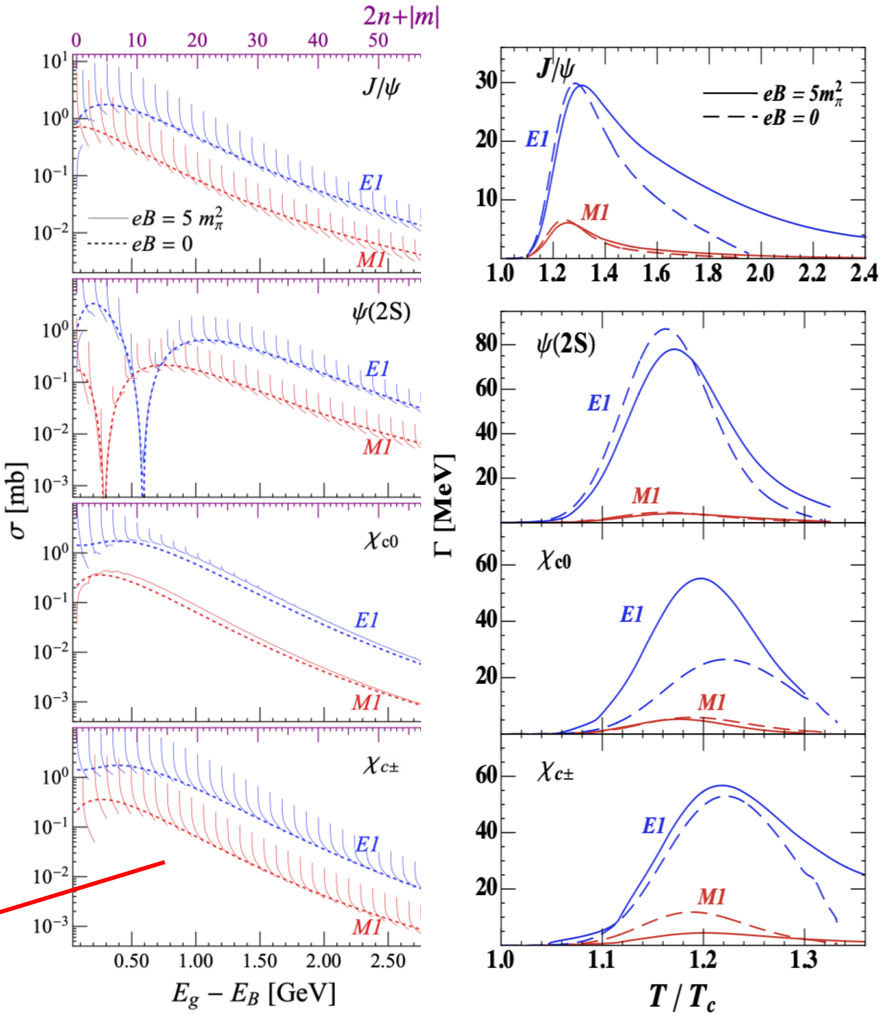
$$\hat{H}_I = q_a A_0^a(X) - \boldsymbol{d}_a \cdot \boldsymbol{\mathcal{E}}^a(X) - \boldsymbol{m}_a \cdot \boldsymbol{\mathcal{B}}^a(X) + \cdots,$$

color monopole, electric dipole and magnetic dipole interactions

Fermi's Golden rule:

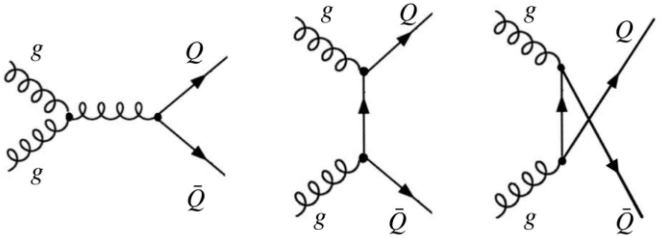
$$\Gamma_{gluon\ dissociation} = 2\pi \left|\langle (c\bar{c})_8 | \hat{H}_I | \Psi \rangle\right|^2 \rho(E_{c\bar{c}})$$

The Landau energy levels in magnetic field leads to a resonant dissociation.



Threshold Enhancement

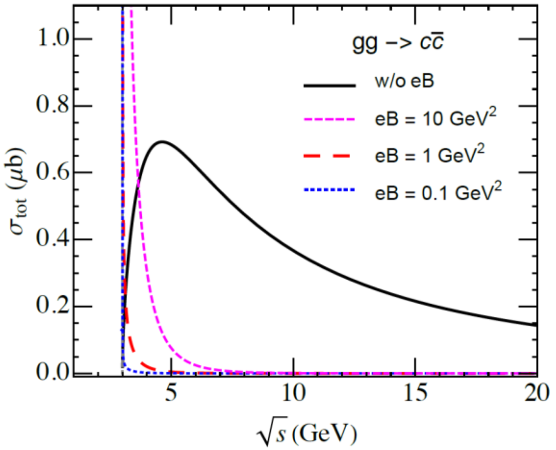
*S.Chen, J.Zhao and PZ, ArXiv: 2307.******



$$\sigma(s,B,\theta) = \frac{\pi m^2\alpha_s^2|qB|}{s^3\chi}\left\{\frac{3}{2}\cos^2\theta\left[1+\frac{\sin^2\theta}{1+\sqrt{4m^2/s}}\frac{1+\cos^2\theta-4\chi^2}{\sin^4\theta+16m^2/s\cos^2\theta}e^{-\frac{s\sin^2\theta}{8|qB|}}\right]\right. \tag{30}$$

$$\left. + \frac{2}{3}\sin^4\theta\left[\left(\frac{\cos\theta+2\chi}{(\chi+\cos\theta)^2+4m^2/s}\right)^2+\left(\frac{-\cos\theta+2\chi}{(\chi-\cos\theta)^2+4m^2/s}\right)^2-\frac{1}{4}\frac{4\chi^2-\cos^2\theta}{\sin^4\theta+16m^2/s\cos^2\theta}\right]e^{-\frac{s\sin^2\theta}{4|qB|}}\right\},$$

$$\chi = \sqrt{1-\frac{4m^2}{s}}$$



Conclusion 4:

Landau level induced enhancement around the threshold !

Summary

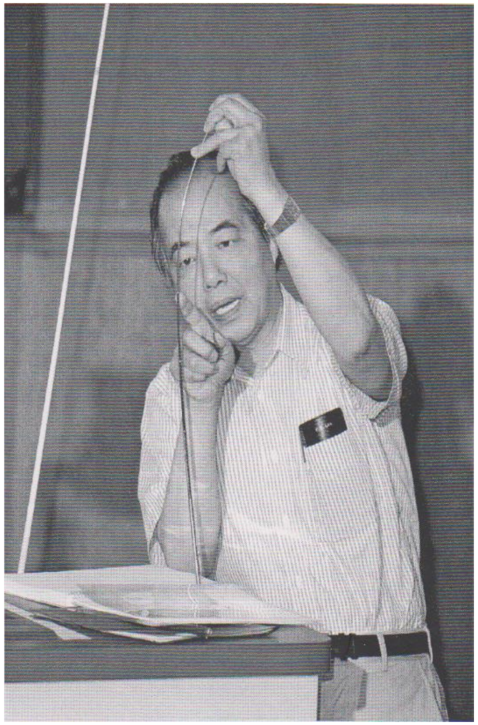
♣ 电磁场的强度和寿命都会被热*QCD*物质强烈压低。

♣ 但自旋和*Landau*能级等量子效应使得在重离子碰撞中观测**量子**现象成为可能，例如*CME*，*CVE*和本报告讨论的色相互作用的共振屏蔽和重味产生的阈值增强等等现象。

Thank you for your attention!

要想认识最小的，需要知道最大的

李政道, 1996



Large things are made of small
And even smaller.
To know the smallest
We need also the largest

All lie in vacuum
Everywhen and everywhere.
How can the micro
Be separate from the macro?

Let vacuum be a condensate
Violating harmony
We can then penetrate
Through asymmetry into symmetry

大事物由小事物组成
甚至是更小的。
要想认识最小的
我们也需要知道最大的。
一切都取决于真空
无论何时何地。
微观的事物怎能
与宏观相分离？
真空其实是一种凝聚
破坏了和谐。
如此我们方可洞穿
不对称中的对称。

【杨振伟翻译】