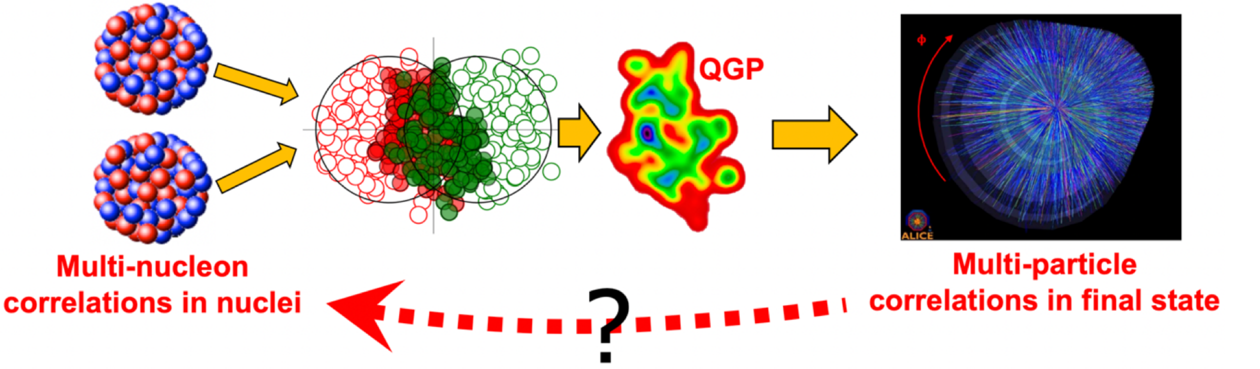
原子核结构与相对论重离子碰撞前沿交叉研讨会

Nuclear shape imaging in heavy ion collisions

Jiangyong Jia



Stony Brook University

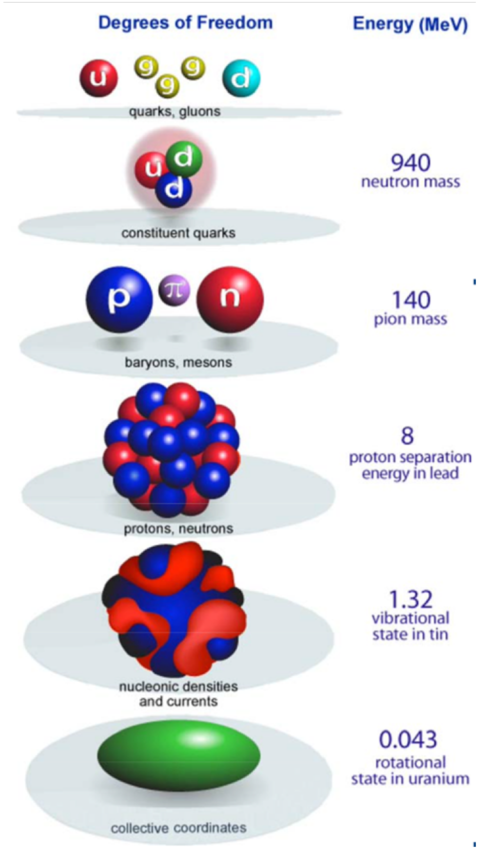
July 31-August 6, 2023

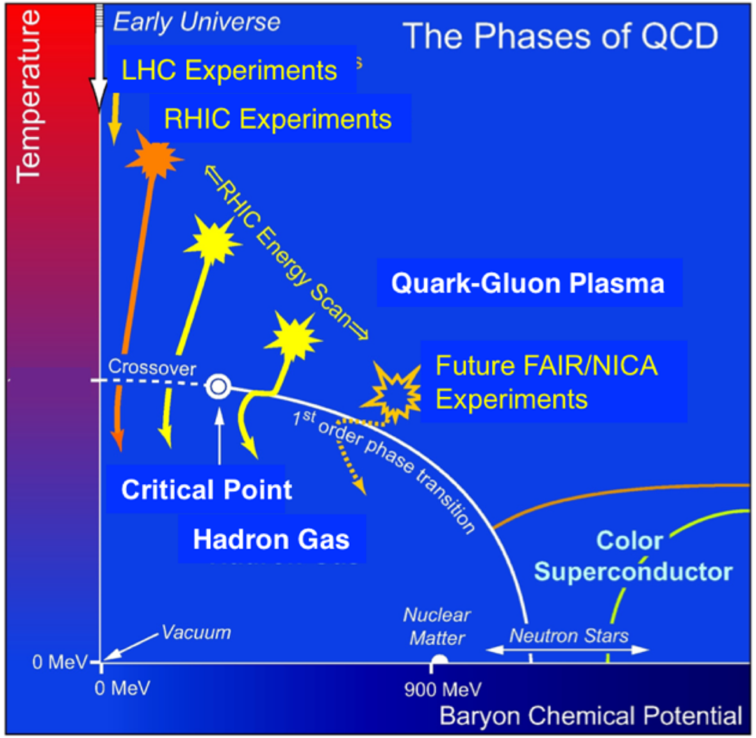
Landscape of nuclear physics

Quark-gluon plasma

hadrons

nuclei

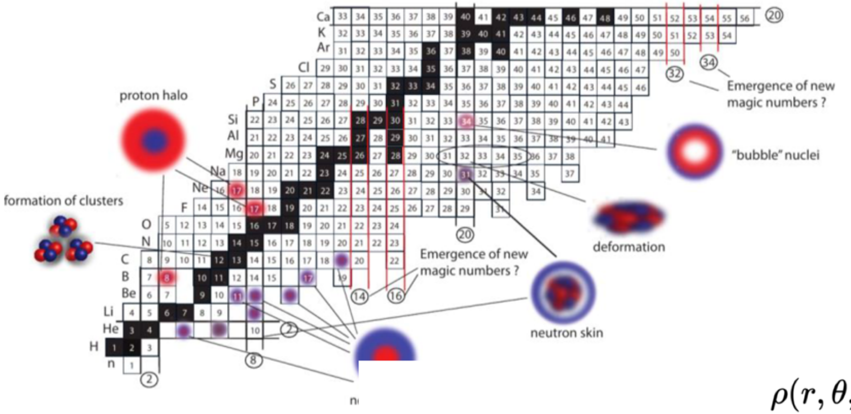




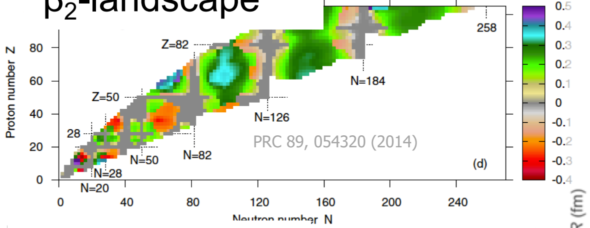
Most nuclear experiments starts with nuclei

Collective structure of atomic nuclei

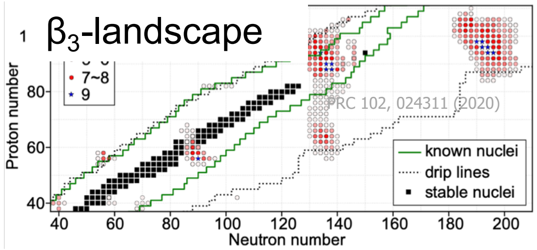
- Emergent phenomena of the many-body quantum system
  - clustering, halo, skin, bubble…
  - quadrupole/octupole/hexdecopole deformations
  - Non-monotonic evolution with N and Z



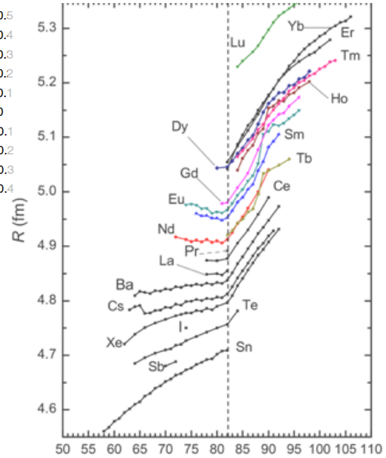
β$_2$-landscape



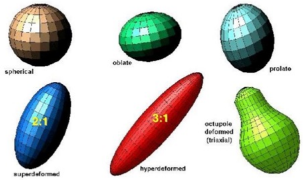
β$_3$-landscape



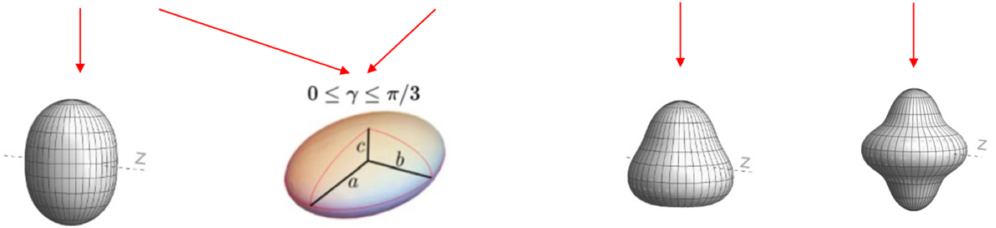
Radii-landscape



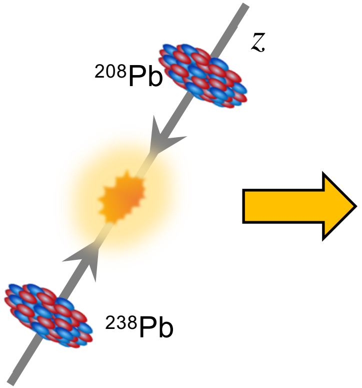
$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a_0}}$$

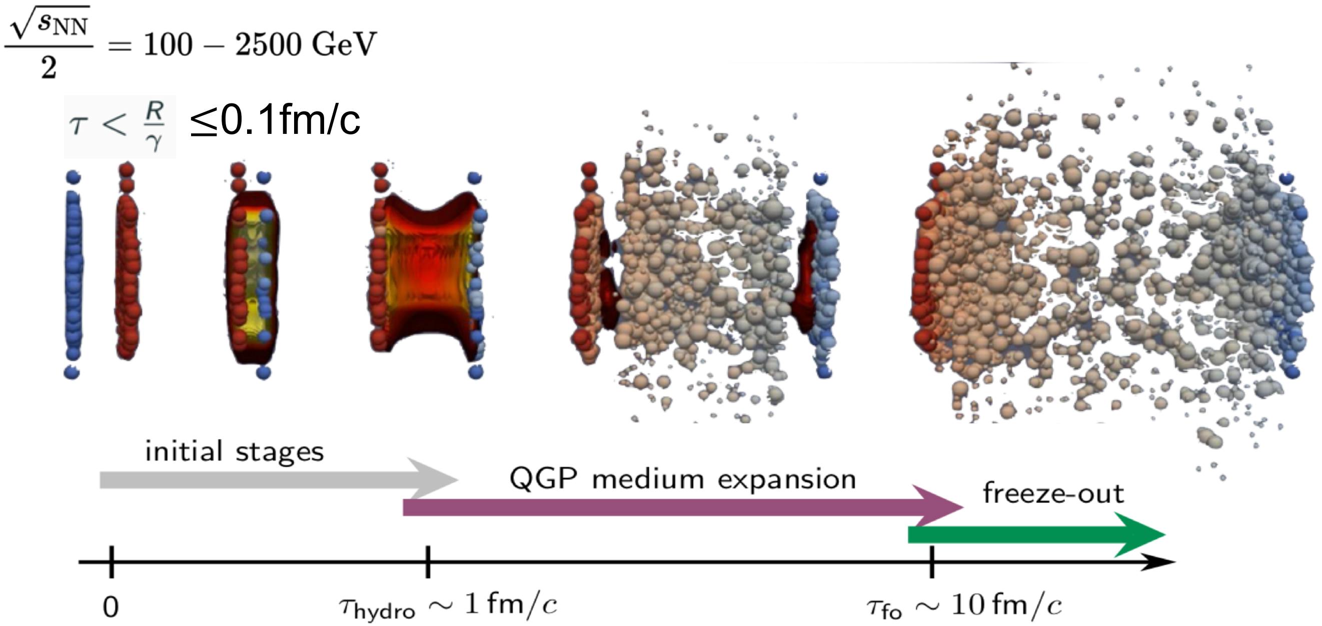


$$R(\theta,\phi) = R_0(1+\beta_2[\cos\gamma Y_{2,0}(\theta,\phi)+\sin\gamma Y_{2,2}(\theta,\phi)]+\beta_3 Y_{3,0}(\theta,\phi)+\beta_4 Y_{4,0}(\theta,\phi))$$



High-energy heavy ion collision





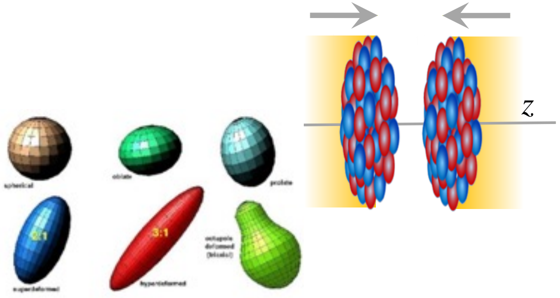
400 nucleons → 30000 hadrons

in 10^{-23} seconds

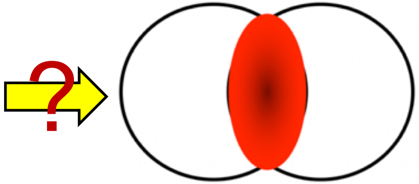
1) Extremely short passing time to take a snap-shot of the nuclear wavefunction in the two nuclei.
2) Large particle production in overlap region means QGP is dense and expand hydrodynamically.

Collective flow assisted nuclear structure imaging

Nuclear structure

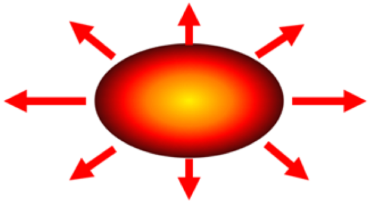


Initial condition



hydrodynamics

Final state



Shape and radial dis.

$\beta_2 \rightarrow$ Quadrupole deformation
$\beta_3 \rightarrow$ Octupole deformation
$a_0 \rightarrow$ Surface diffuseness
$R_0 \rightarrow$ Nuclear size

Volume, size and shape

N_{part}

$R_\perp^2 \propto \langle r_\perp^2 \rangle$

$\mathcal{E}_n \propto \langle r_\perp^n e^{in\phi} \rangle$

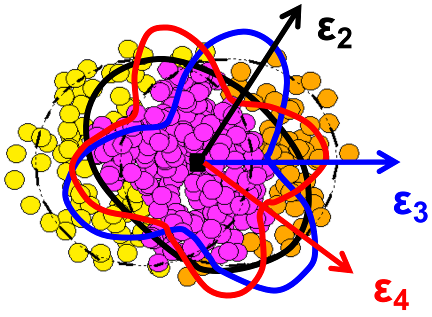
Observables

$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n\, e^{-in\phi}\right)$$

- Constrain the initial condition by comparing nuclei with known structure properties
- Reveal novel properties of nuclei by leveraging known hydrodynamic response.

Infer initial condition from flow correlations

Initial state shape



volume, size and shape

N_{part} $R_\perp^2 \propto \langle r_\perp^2 \rangle$, $\mathcal{E}_2 \propto \left\langle r_\perp^2 e^{i2\phi} \right\rangle$

$\mathcal{E}_3 \propto \left\langle r_\perp^3 e^{i3\phi} \right\rangle$

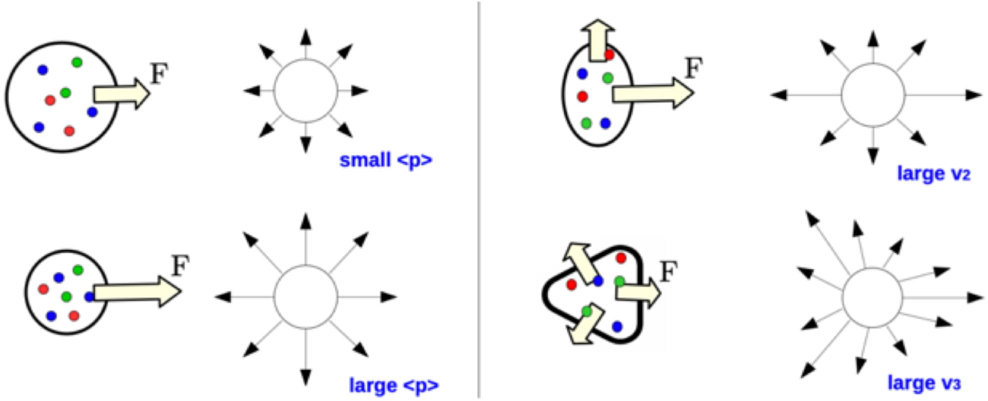
$\mathcal{E}_4 \propto \left\langle r_\perp^4 e^{i4\phi} \right\rangle$

$\ldots$

$F = -\nabla P(\epsilon)$

Hydro-response

Final state flow



Multiplicity Radial Flow Harmonic Flow

$\mathrm{N_{ch}}$

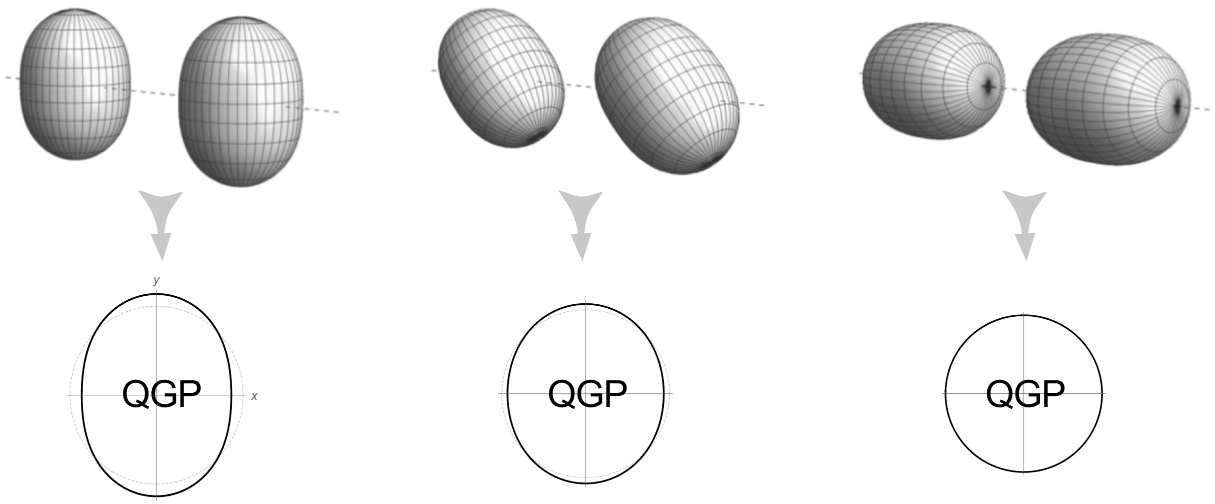
$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n e^{-in\phi}\right)$$

1206.1905

Flow-assisted imaging relies on linear response: $N_{ch} \propto N_{part}$ $\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp}$ $V_n \propto \mathcal{E}_n$

Connecting initial condition to nuclear shape

Body-Body $\quad \mathcal{E}_2 = \epsilon_2 e^{i2\Phi} \propto \langle \boldsymbol{r}_\perp^2 e^{i2\phi} \rangle \quad$ Tip-Tip



$\varepsilon_2 \sim 0.95\beta_2$ $\quad \varepsilon_2 \sim 0.48\beta_2$ $\quad \varepsilon_2 \sim 0$

$$\boldsymbol{\epsilon}_2 = \underbrace{\boldsymbol{\epsilon}_0}_{\text{undeformed}} + \underbrace{\boldsymbol{p}(\Omega_1, \Omega_2)}_{\text{phase factor}} \beta_2 + \mathcal{O}(\beta_2^2)$$

$$\Rightarrow \quad \langle \epsilon_2^2 \rangle \approx \langle \epsilon_0^2 \rangle + 0.2\beta_2^2$$

Shape depends on Euler angle Ω=φθψ

$\langle v_n^2 \rangle \propto \langle \epsilon_n^2 \rangle$ In intrinsic frame

Impact of nuclear shape on many-body correlations

$$\rho(r,\theta,\phi)=\frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a_0}} \qquad R(\theta,\phi)=R_0(1+\beta_2[\cos\gamma Y_{2,0}(\theta,\phi)+\sin\gamma Y_{2,2}(\theta,\phi)]+\beta_3 Y_{3,0}(\theta,\phi)+\beta_4 Y_{4,0}(\theta,\phi))$$

- In principle, can probe any moments of $p(1/R, \varepsilon_2, \varepsilon_3\ldots)$ via $p([p_T], v_2, v_3\ldots)$…
  - Mean $\langle d_\perp\rangle$ $\quad d_\perp \equiv 1/R_\perp$ $\quad\Leftrightarrow\quad$ $\langle p_T\rangle$
  - Variance: $\langle\varepsilon_n^2\rangle, \left\langle(\delta d_\perp/d_\perp)^2\right\rangle$ $\quad\Leftrightarrow\quad$ $\langle v_n^2\rangle, \left\langle(\delta p_T/p_T)^2\right\rangle$
  - Skewness $\langle\varepsilon_n^2\delta d_\perp/d_\perp\rangle, \left\langle(\delta d_\perp/d_\perp)^3\right\rangle$ $\quad\Leftrightarrow\quad$ $\langle v_n^2\delta p_T/p_T\rangle, \left\langle(\delta p_T/p_T)^3\right\rangle$
  - Kurtosis $\langle\varepsilon_n^4\rangle-2\langle\varepsilon_n^2\rangle^2, \left\langle(\delta d_\perp/d_\perp)^4\right\rangle-3\left\langle(\delta d_\perp/d_\perp)^2\right\rangle^2$ $\quad\Leftrightarrow\quad$ $\langle v_n^4\rangle-2\langle v_n^2\rangle^2, \left\langle(\delta p_T/p_T)^4\right\rangle-3\left\langle(\delta p_T/p_T)^2\right\rangle^2$

  …

Impact of nuclear shape on many-body correlations

$$\rho(r,\theta,\phi)=\frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a_0}} \quad R(\theta,\phi)=R_0(1+\beta_2[\cos\gamma Y_{2,0}(\theta,\phi)+\sin\gamma Y_{2,2}(\theta,\phi)]+\beta_3 Y_{3,0}(\theta,\phi)+\beta_4 Y_{4,0}(\theta,\phi))$$

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  - Skewness $\langle\varepsilon_n^2\delta d_\perp/d_\perp\rangle, \left\langle(\delta d_\perp/d_\perp)^3\right\rangle$ ⟺ $\langle v_n^2\delta p_T/p_T\rangle, \left\langle(\delta p_T/p_T)^3\right\rangle$
  - Kurtosis $\langle\varepsilon_n^4\rangle-2\langle\varepsilon_n^2\rangle^2, \left\langle(\delta d_\perp/d_\perp)^4\right\rangle-3\left\langle(\delta d_\perp/d_\perp)^2\right\rangle^2$ ⟺ $\langle v_n^4\rangle-2\langle v_n^2\rangle^2, \left\langle(\delta p_T/p_T)^4\right\rangle-3\left\langle(\delta p_T/p_T)^2\right\rangle^2$

  …

- All have a simple connection to deformation:
  - Variances

$$\langle\varepsilon_2^2\rangle \sim a_2+b_2\beta_2^2+b_{2,3}\beta_3^2$$
$$\langle\varepsilon_3^2\rangle \sim a_3+b_3\beta_3^2$$
$$\langle\varepsilon_4^2\rangle \sim a_4+b_4\beta_4^2$$
$$\left\langle(\delta d_\perp/d_\perp)^2\right\rangle \sim a_0+b_0\beta_2^2+b_{0,3}\beta_3^2$$

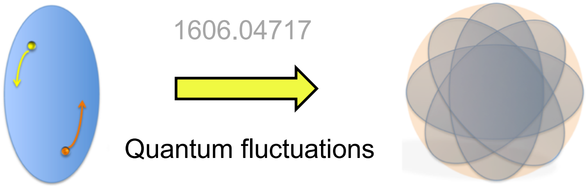
  - Skewness

$$\left\langle\varepsilon_2^2\delta d_\perp/d_\perp\right\rangle \sim a_1-b_1\cos(3\gamma)\beta_2^3$$
$$\left\langle(\delta d_\perp/d_\perp)^3\right\rangle \sim a_2+b_2\cos(3\gamma)\beta_2^3$$

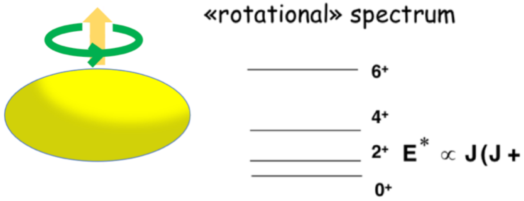
……

Low-energy vs high-energy method

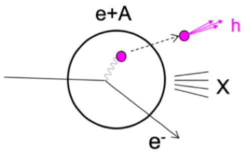
- Intrinsic frame shape not directly visible in lab frame at time scale $\tau > I/\hbar \sim 10^{-21} s$
- Mainly inferred from non-invasive spectroscopy methods.



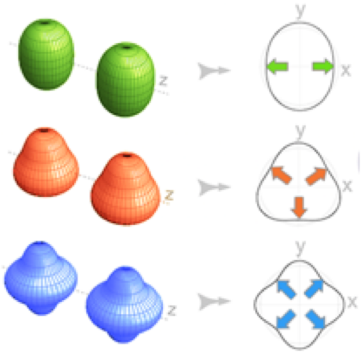
Correlated shape in intrinsic frame   Spherical shape in lab frame



$E^* \propto J(J+$



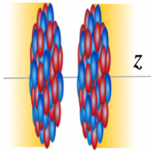
- High-energy collisions destructive imaging: probe entire mass distribution in the intrinsic frame via multi-point correlations. Shape frozen in nuclear crossing (10^{-24}s << rotational time scale 10^{-21}s)

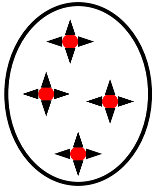


Collective flow assisted imaging

Fast crossing   Large multiplicity

$\tau \approx 2R_0/\gamma \lesssim 0.1\text{fm}/c$





arXiv: 1902.07168

$\mathbf{r} = re^{i\phi}$

$S(\mathbf{r}_1, \mathbf{r}_2) = \langle \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\rangle - \langle \rho(\mathbf{r}_1)\rangle\langle \rho(\mathbf{r}_2)\rangle$ Intrinsic frame

$$\epsilon_2 = \frac{\int_{\mathbf{r}} \mathbf{r}^2 S(\mathbf{r})}{\int_{\mathbf{r}} |\mathbf{r}|^2 \langle S(\mathbf{r})\rangle} \Rightarrow \langle \varepsilon_2^2 \rangle = \frac{\int_{\mathbf{r}_1,\mathbf{r}_2} (\mathbf{r}_1)^2 (\mathbf{r}_2^*)^2 S(\mathbf{r}_1, \mathbf{r}_2)}{\left(\int_{\mathbf{r}} |\mathbf{r}|^2 \langle S(\mathbf{r})\rangle\right)^2}$$

Analogy: Coulomb Explosion Imaging

Instantaneous stripping of electrons (thin foil or x-ray laser), and then let atoms explode under mutual coulomb repulsion

Science 244, 4903 (1989)

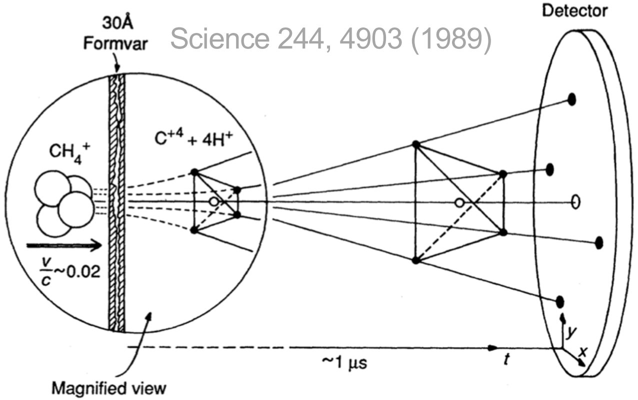
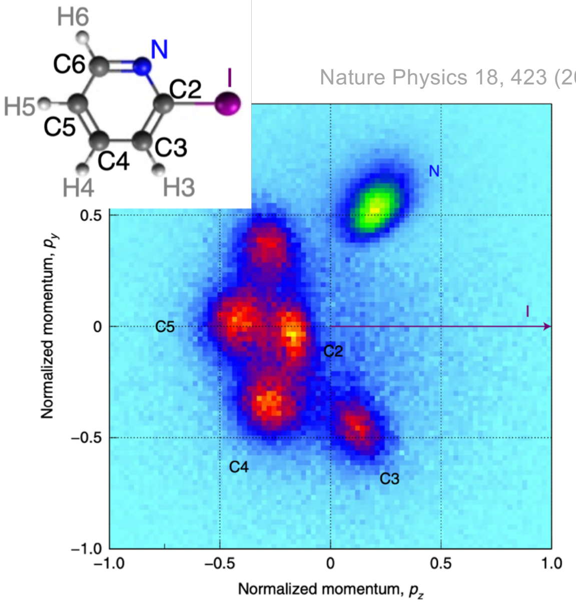
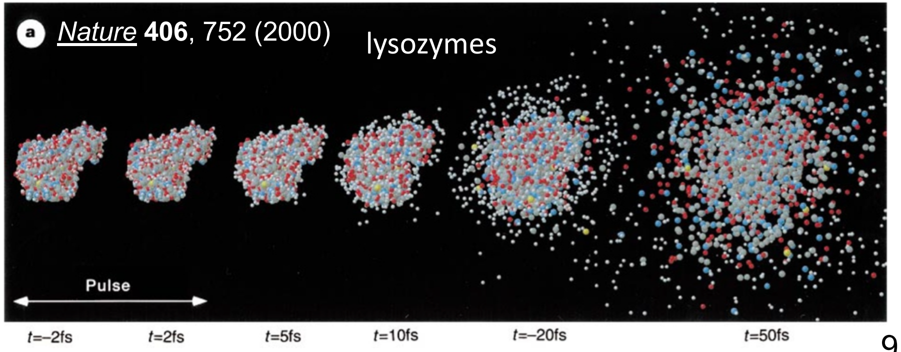


Fig. 1. A schematic view of a Coulomb explosion experiment. When a swift molecule passes through a thin solid film, it loses all of its binding electrons. The remaining positive ions repel each other, thus transforming the microstructure (as seen in the magnified view) into a macrostructure that can be measured precisely with an appropriate detector. The measured traces (x, y, t) of each fragment nucleus for individual molecules are then transformed into the original molecular structure.

Nature Physics 18, 423 (2022)



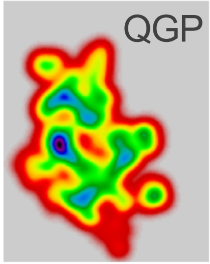


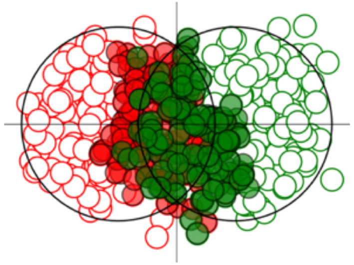
Strategy for nuclear shape imaging

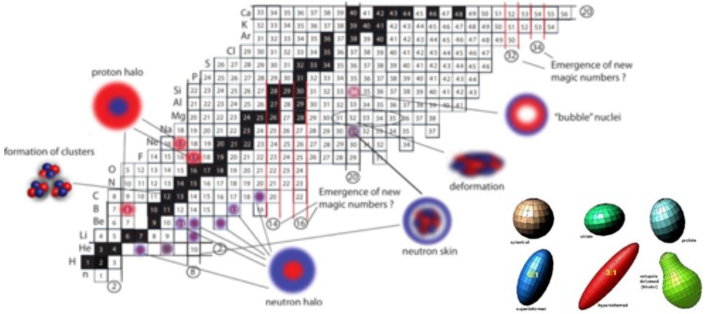
Flow observable = k ⊗ initial condition (structure)

QGP response,
a smooth function of N+Z

Structure of colliding nuclei,
non-monotonic function of N and Z



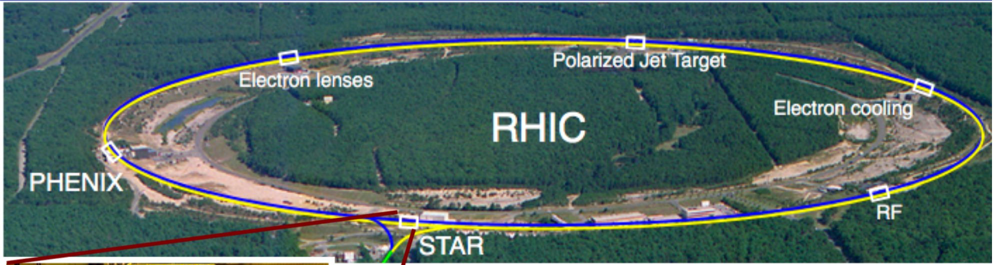


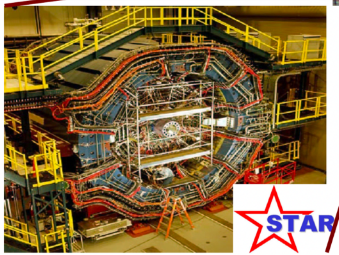


Compare two systems of similar size but different structure

Ions collided at high-energy

Relativistic Heavy-Ion Collider





Most versatile collider machine

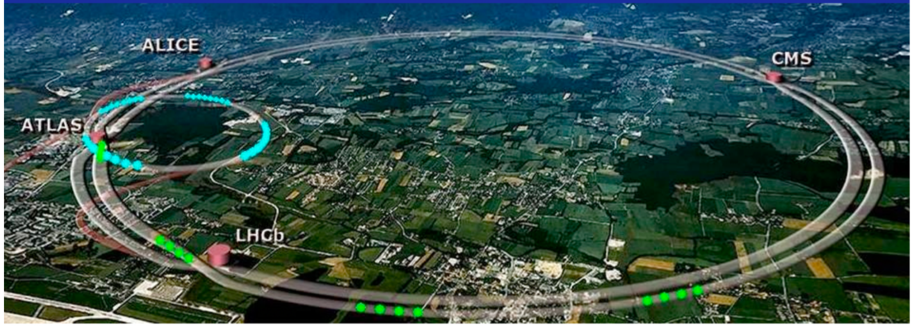
➔Dedicated heavy-ion machine

➔Any pair of nucleus is possible:

p+p,p+Al, p+Cu,p+Au, d+Au, He^3+Au,
O+O, Cu+Cu, Cu+Au,
Zr+Zr, Ru+Ru, Au+Au, U+U

➔Broad energy range 3 - 200 GeV

Large Hadron Collider

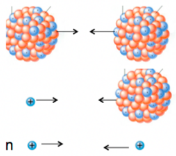


- Collide ions 1 month/year

Pb+Pb

p+Pb

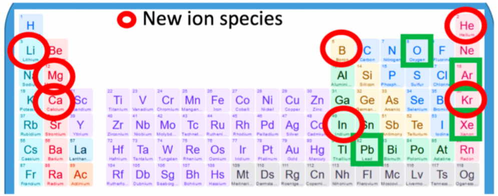
p+p



Extra: Xe+Xe,O+O

- New ions species possible 2028+

R. Alemany Fernandez, INT 2023



Isobar collisions at RHIC

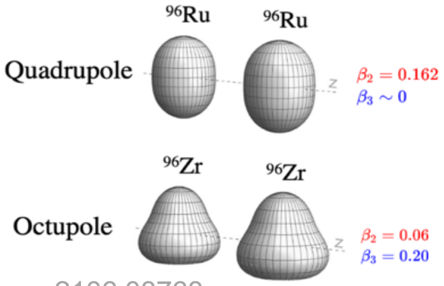
^{96}Ru+^{96}Ru and ^{96}Zr+^{96}Zr at $\sqrt{s_{NN}}$ =200 GeV

- A key question for any HI observable *O*:

$$\frac{O_{^{96}\mathrm{Ru}+^{96}\mathrm{Ru}}}{O_{^{96}\mathrm{Zr}+^{96}\mathrm{Zr}}} \stackrel{?}{=} 1$$

Deviation from 1 must have an origin in the nuclear structure, which impacts the initial state and then survives to the final state.

- Expectation



2106.08768

Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm
difference	$\Delta\beta_2^2$	$\Delta\beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

$$\mathcal{O} \approx b_0 + b_1\beta_2^2 + b_2\beta_3^2 + b_3(R_0 - R_{0,\mathrm{ref}}) + b_4(a - a_{\mathrm{ref}})$$

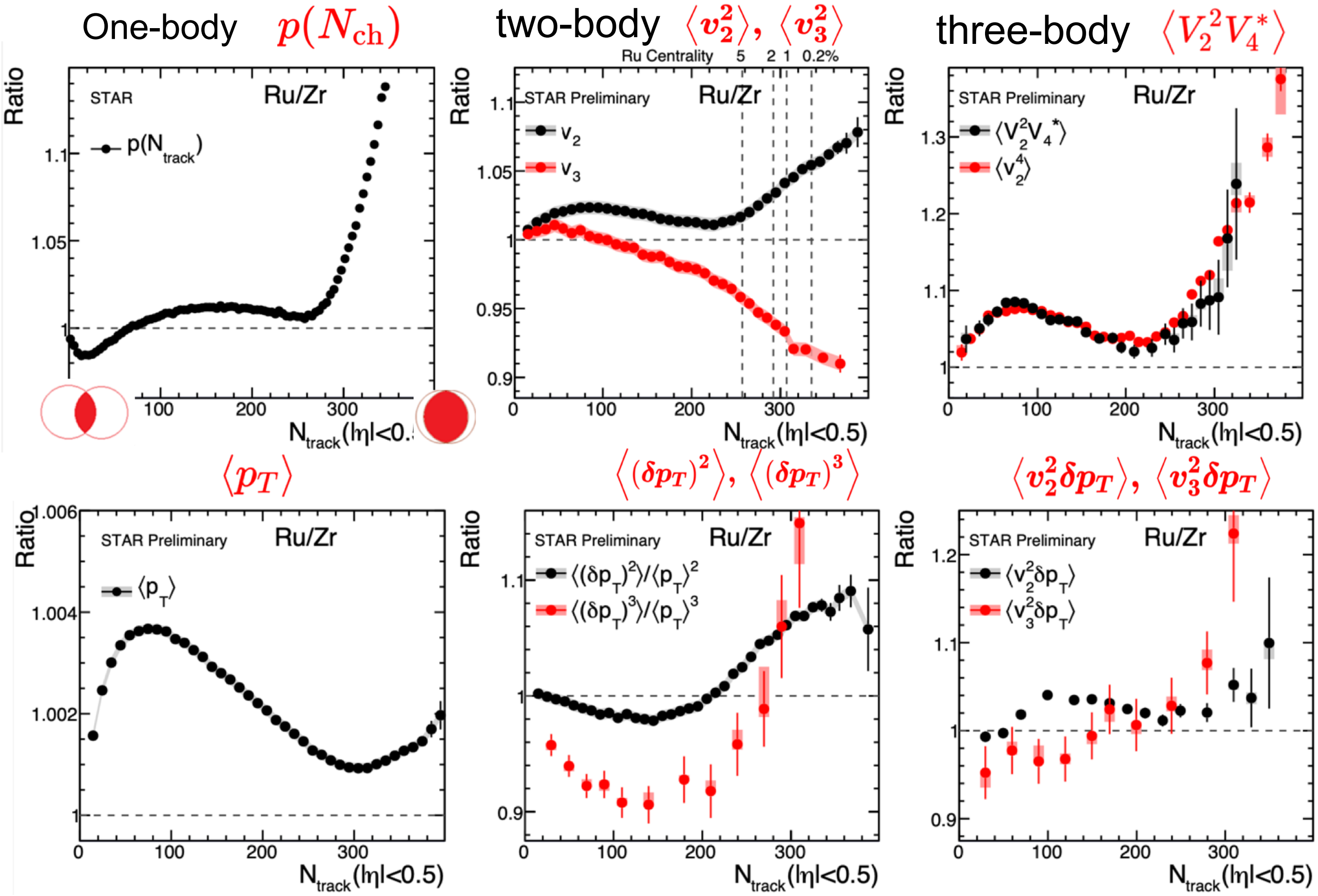
2109.00131

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

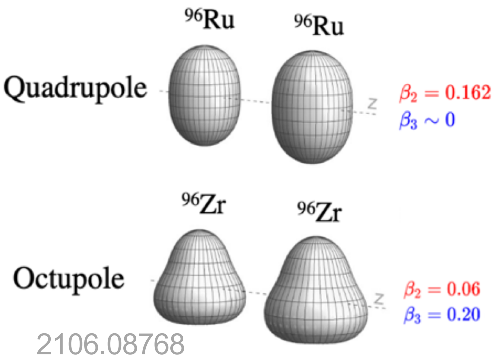
Only probe structure differences

Structure influences everywhere

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\rm Ru}}{\mathcal{O}_{\rm Zr}}$$



Nuclear structure via v_2-ratio and v_3-ratio



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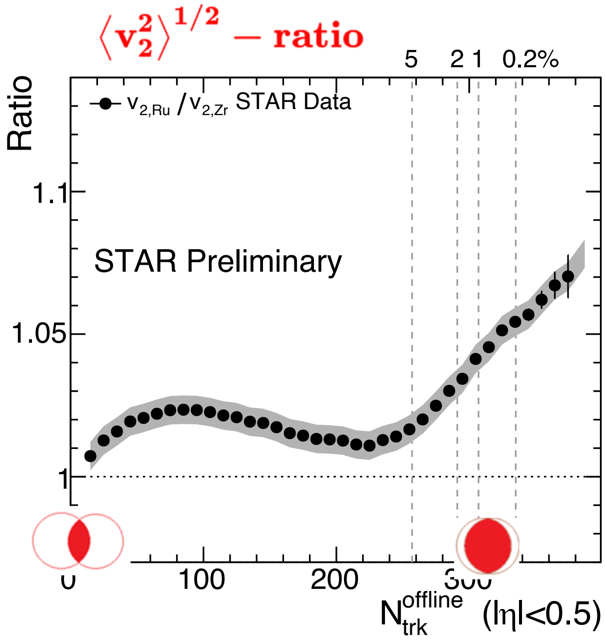
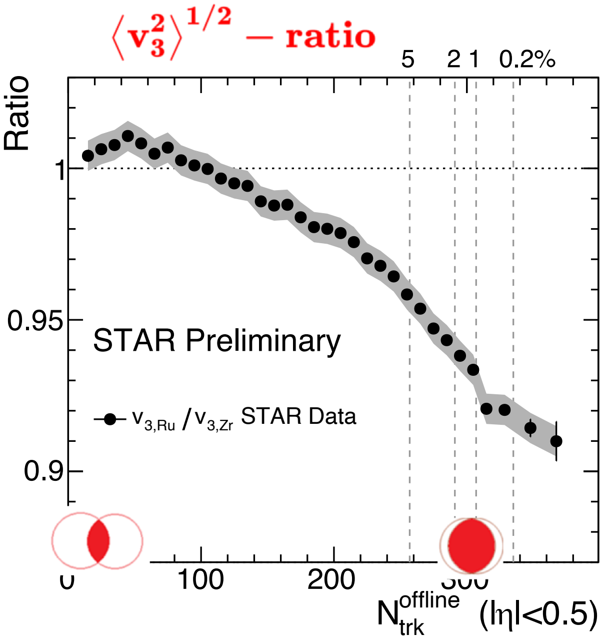
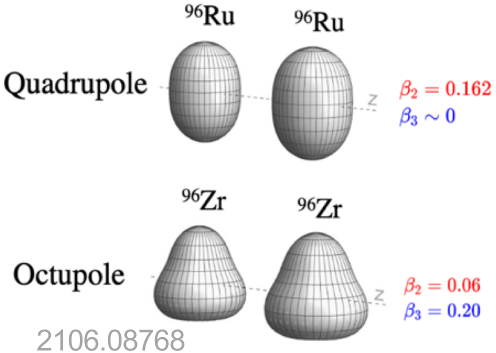




Nuclear structure via v_2-ratio and v_3-ratio



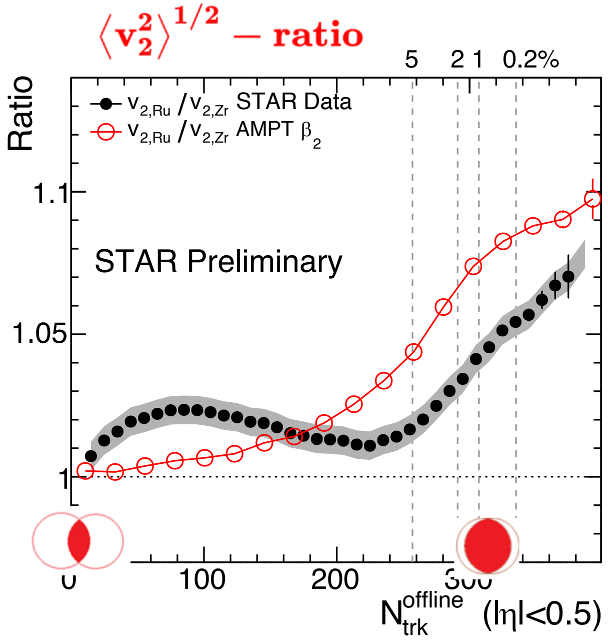
2106.08768

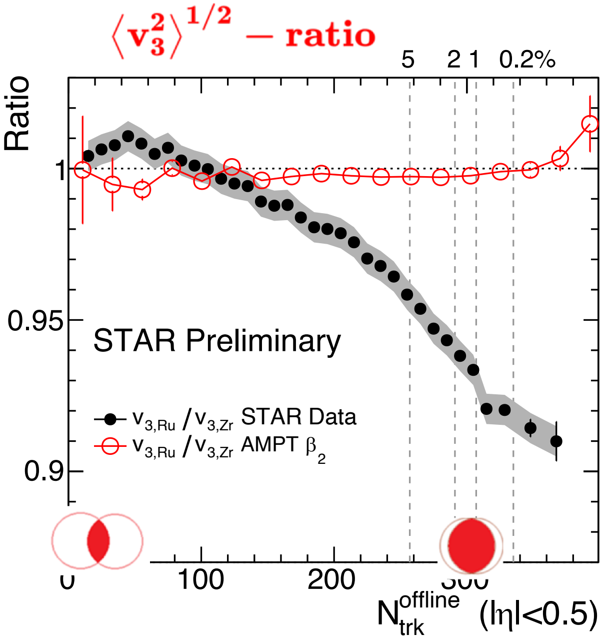
- $\beta_{2Ru} \sim 0.16$ increase v_2, no influence on v_3 ratio

$\langle v_2^2 \rangle \sim a_2 + b_2\beta_2^2 + c_2\beta_3^2$

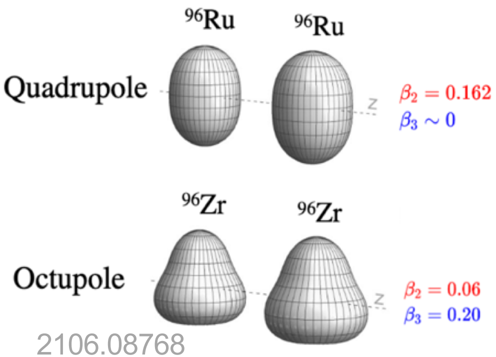
$\langle v_3^2 \rangle \sim a_3 + b_3\beta_3^2$

Compare with AMPT
-- a proxy for hydro





Nuclear structure via v_2-ratio and v_3-ratio



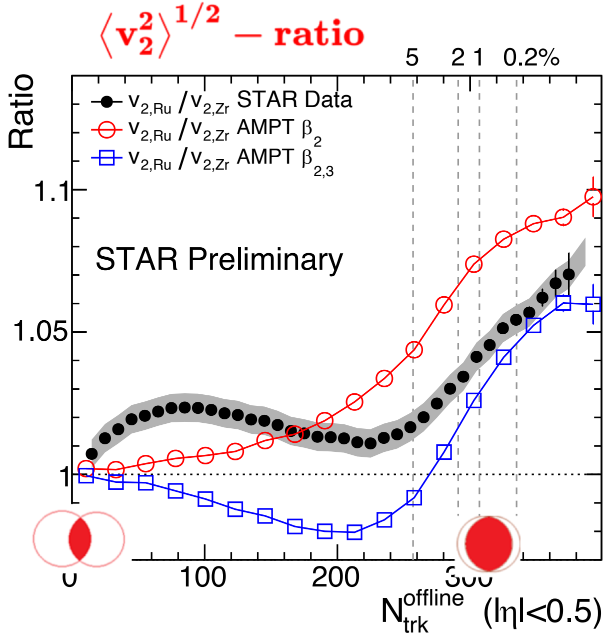
2106.08768

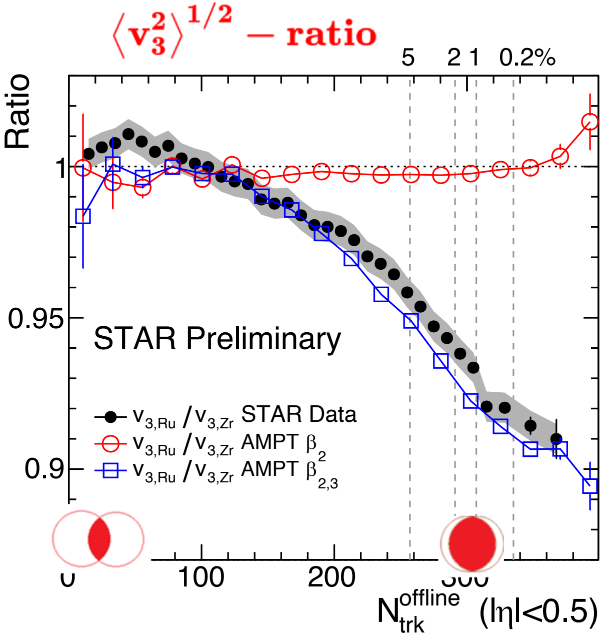
- $\beta_{2Ru} \sim 0.16$ increase v_2, no influence on v_3 ratio
- $\beta_{3Zr} \sim 0.2$ decrease v_2 in mid-central, decrease v_3 ratio

$$\langle v_2^2 \rangle \sim a_2 + b_2\beta_2^2 + c_2\beta_3^2$$

$$\langle v_3^2 \rangle \sim a_3 + b_3\beta_3^2$$

Compare with AMPT
-- a proxy for hydro





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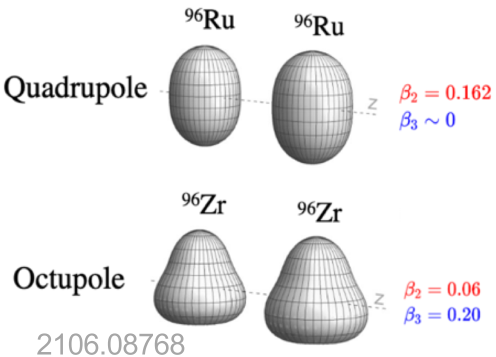

2106.08768

- $\beta_{2Ru} \sim 0.16$ increase v_2, no influence on v_3 ratio
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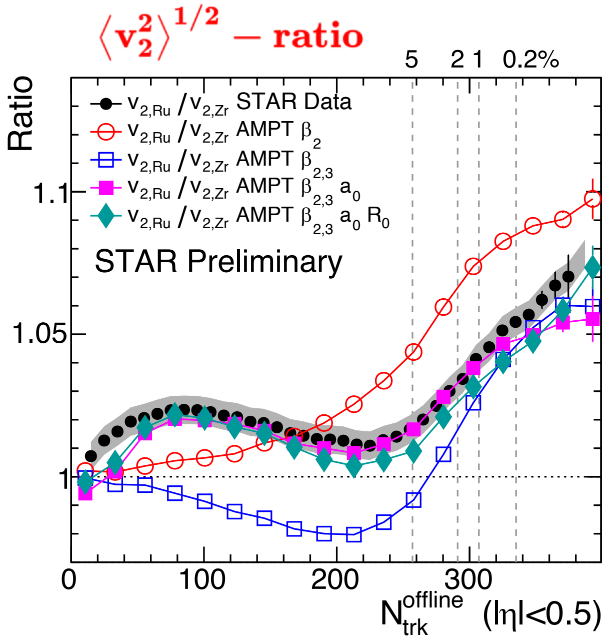
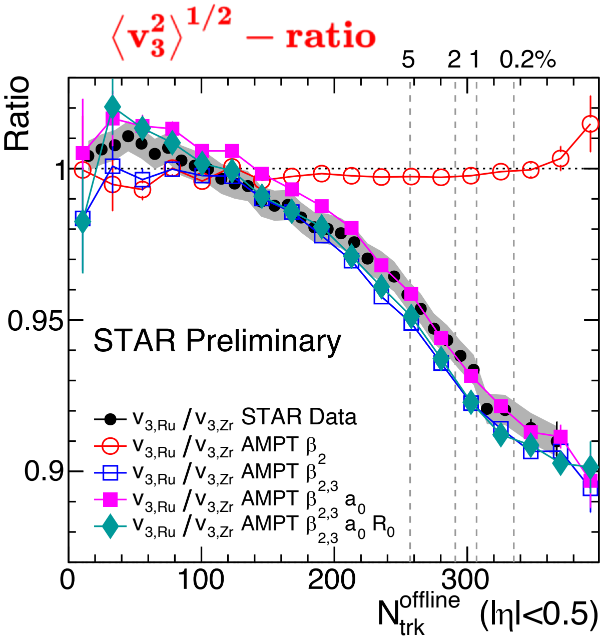
- $\Delta a_0 = -0.06$ fm increase v_2 mid-central, small impact on v_3
- Radius $\Delta R_0 = 0.07$ fm only slightly affects v_2 and v_3 ratio.

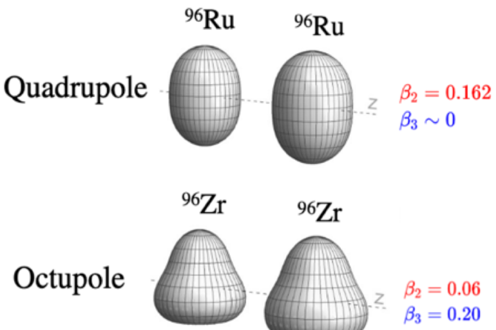
Compare with AMPT
-- a proxy for hydro

Sharper surface enhance v_2
in mid-central collisions
arXiv: 2206.10449





Nuclear structure via v_2-ratio and v_3-ratio



2106.08768

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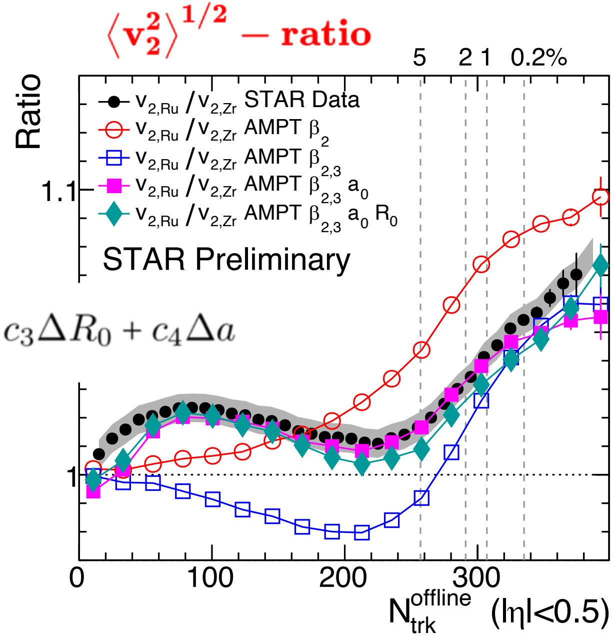
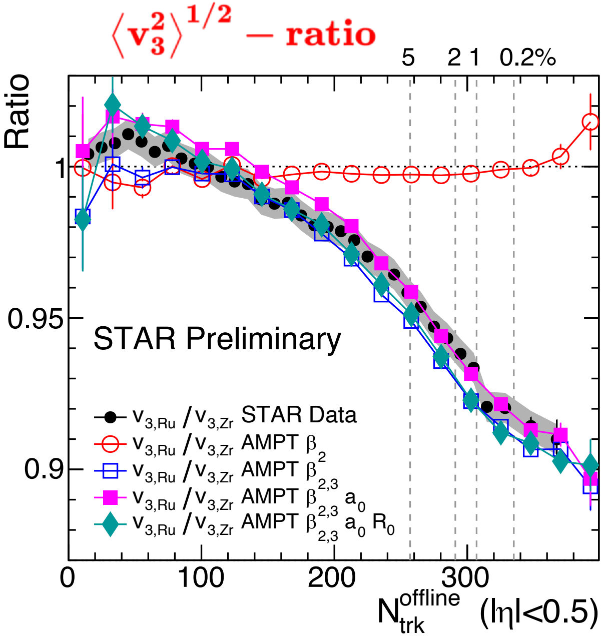
$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1 \Delta\beta_2^2 + c_2 \Delta\beta_3^2 + c_3 \Delta R_0 + c_4 \Delta a$$

Simultaneously constrain four structure parameters





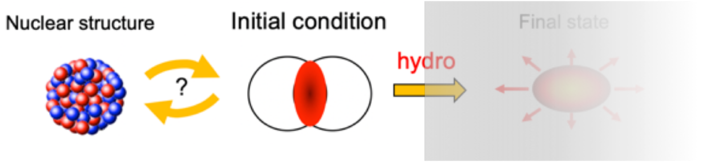
Isobar ratios cancel final state effects

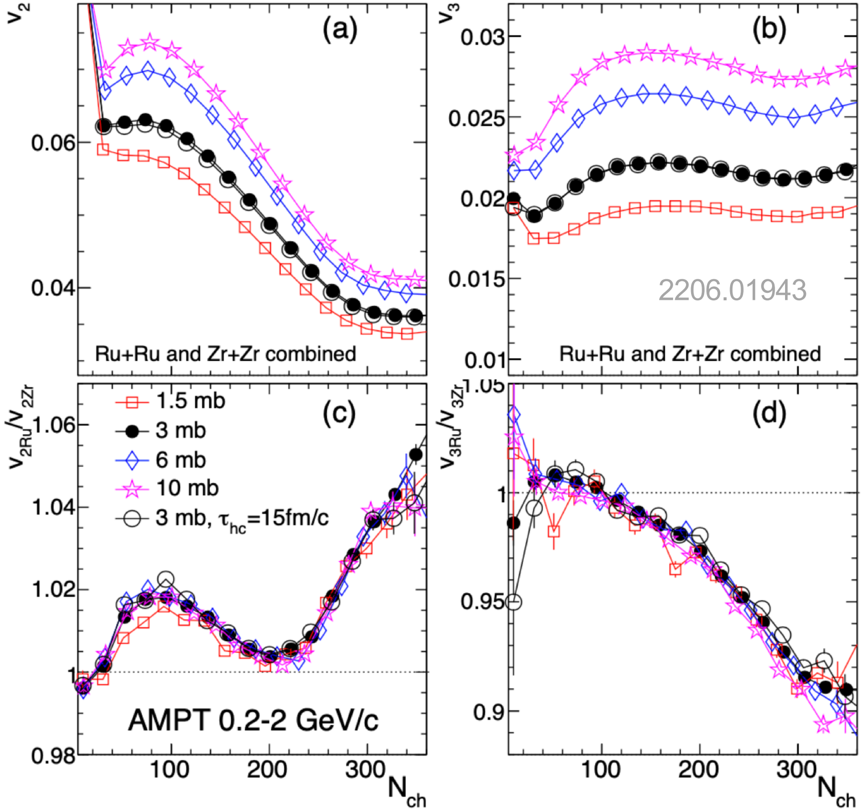
- Vary the shear viscosity by changing partonic cross-section in AMPT
  - Flow signal change by 30-50%, the v_n ratio unchanged.

$$v_n = k_n \varepsilon_n$$

$$\frac{v_{n,\mathrm{Ru}}}{v_{n,\mathrm{Zr}}} \approx \frac{\varepsilon_{n,\mathrm{Ru}}}{\varepsilon_{n,\mathrm{Zr}}}$$

Robust probe of initial state!





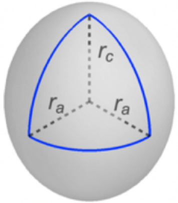
Triaxiality

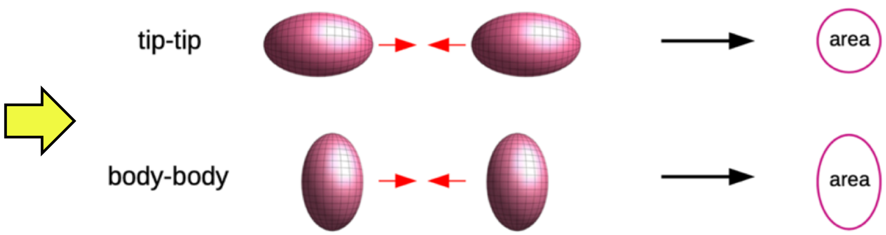
$$R(\theta,\phi) = R_0\left(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}]\right.$$

G. Giuliano et.al 1910.04673, 2004.14463

Prolate

$\beta_2 = 0.25, \cos(3\gamma) = 1$





small v2
small area
large [pt]

$v_2\searrow$ $p_T\nearrow$

large v2
large area
small [pt]

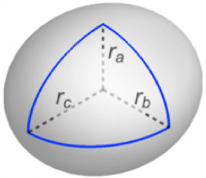
$v_2\nearrow$ $p_T\searrow$

Triaxial

$\beta_2 = 0.25, \cos(3\gamma) = 0$

Oblate

$\beta_2 = 0.25, \cos(3\gamma) = -1$

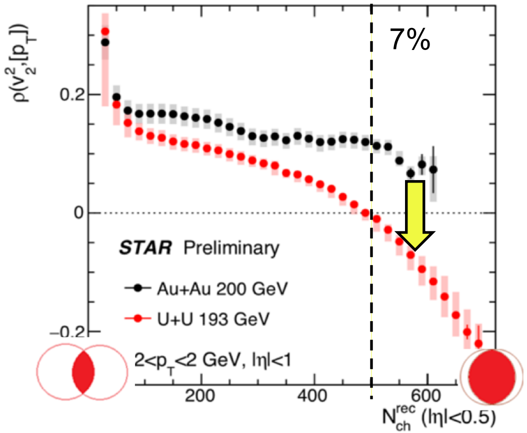


Need 3-point correlators to probe the 3 axes

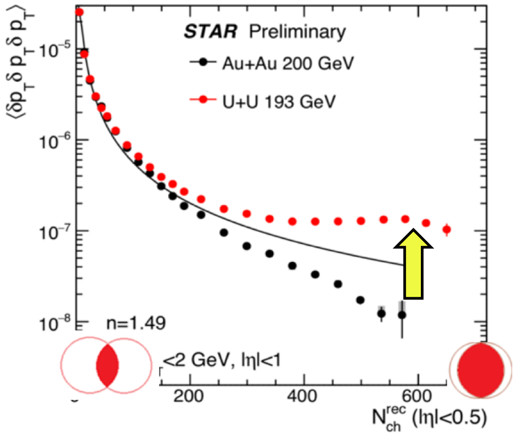
$$\langle v_2^2\delta p_{\rm T}\rangle \sim -\beta_2^3\cos(3\gamma) \qquad \langle(\delta p_{\rm T})^3\rangle \sim \beta_2^3\cos(3\gamma)$$

2109.00604

v_2-$[p_T]$ covariance



$[p_T]$ skewness



Compare U+U vs Au+Au:

β_{2U} ~ 0.28, β_{2Au}~ 0.13:

Extracting the 238U deformation from STAR

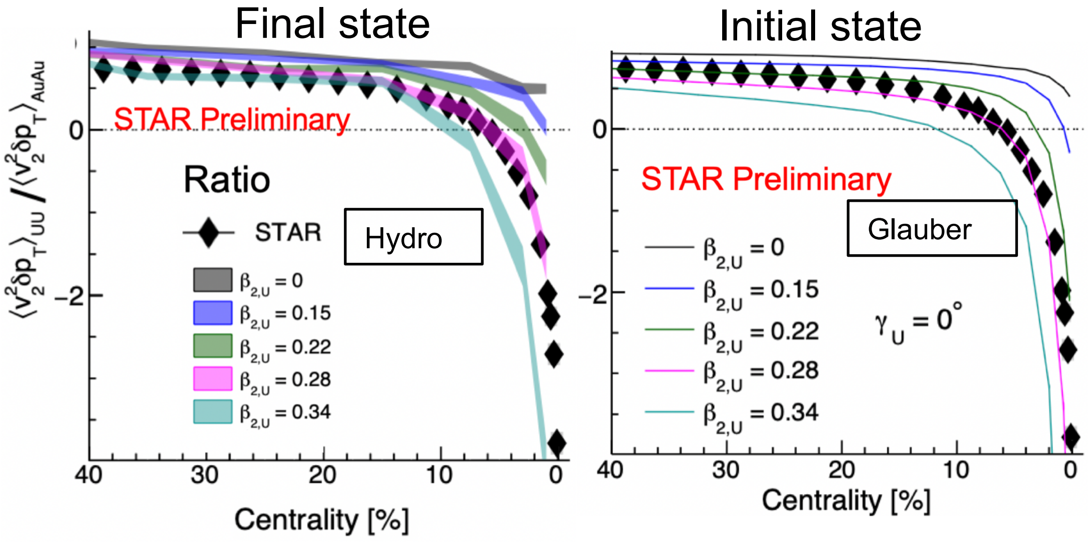
Take ratio of correlations to constrain $\boldsymbol{\beta}_{2U}$

$$\frac{\langle v_2^2 \delta p_T \rangle_{UU}}{\langle v_2^2 \delta p_T \rangle_{AuAu}}$$

Compare with hydro and Glauber model
→clearly reflects initial state geometry

Derive Woods-Saxon parameters

$$\beta_{2U,WS} \approx 0.28 \quad , \quad \gamma_U \lesssim 17^\circ$$



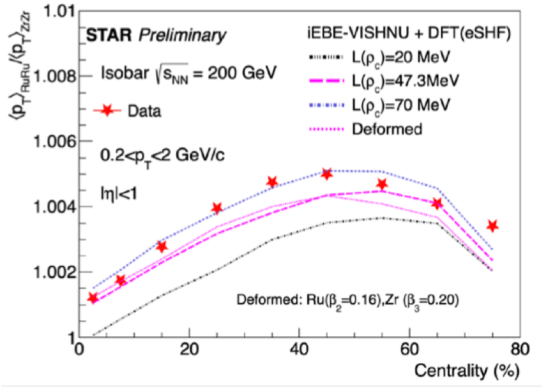
Low-energy estimate with rigid rotor assumption from B(E2) data $\beta_{2,LD} = \frac{4\pi}{5R_0^2 Z}\sqrt{\frac{B(E2)}{e^2}}$

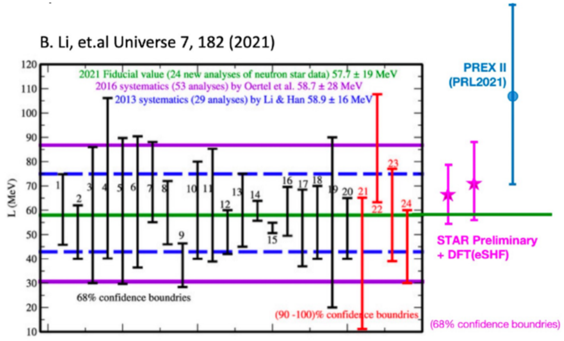
$\beta_{2U,LD} = 0.287 \pm 0.007 \quad \gamma_{U,LD} = 6^\circ - 8^\circ$ 1312.5975 *PRC54, 2356 (1996)*

Imaging the radial structures

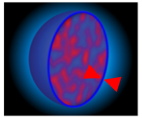
See talks by Haojie and Chunjian

- Radial parameters R_0, a_0 are properties of one-body distribution, constrained by $\langle p_T \rangle$, $\langle N_{ch} \rangle$, σ_{tot}, , $v_2^{RP} \sim v_2\{4\}$



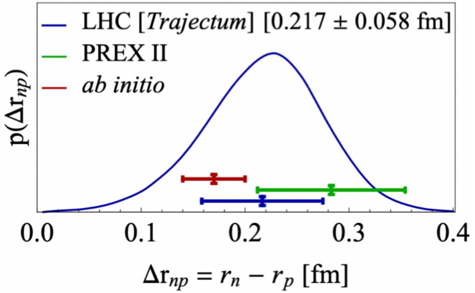


Constrain neutron skin and symmetry energy

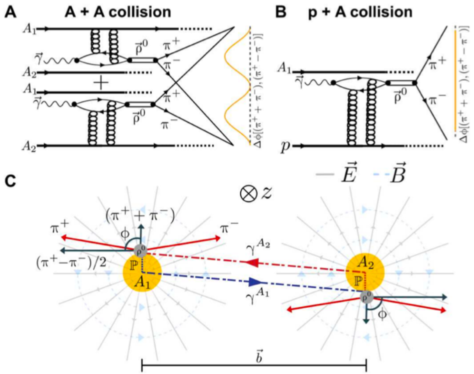


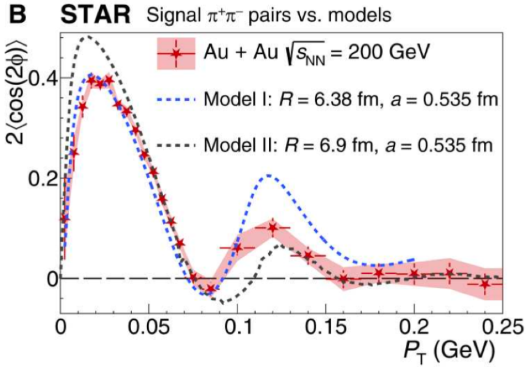
R_n-R_p

2305.00015



- Also accessed via photo-nuclear diffractive process in UPC or e+A



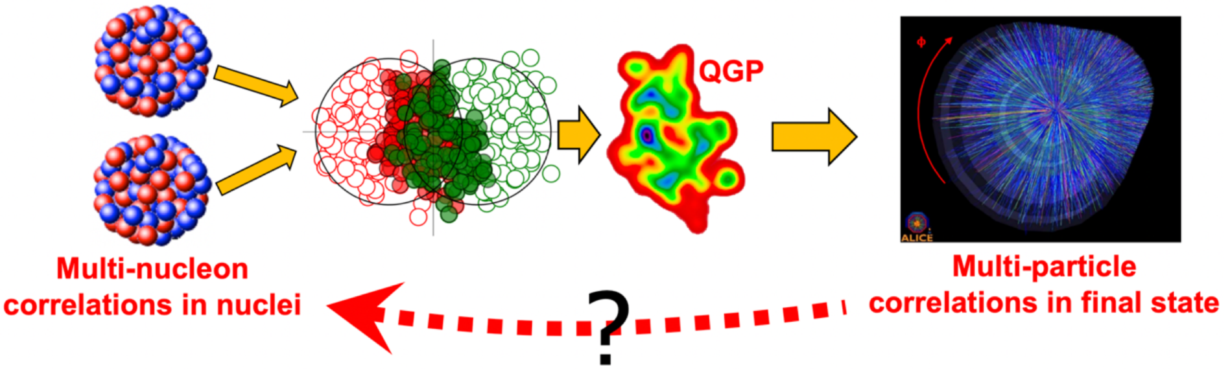


$\Delta R_{Au} = R_n - R_p =$
0.17 ± 0.03 (stat.) ± 0.08 (sys.) fm

$\Delta R_{U} = R_n - R_p =$
0.44 ± 0.05 (stat.) ± 0.08 (sys.) fm

Science Advance 9, 3903 (2023)

Opportunities at the intersection of nuclear structure and hot QCD



Many examples in https://arxiv.org/abs/2209.11042, but here is my list

- Probe octupole and hexadecapole deformations via v3 and v4 in central collisions.
- Gauge shape of odd-mass nuclei by comparing with neighboring even-even nuclei.
- Separate average shape from shape fluctuations via multi-particle correlations
- Constrain the radial structure of nuclei, including the neutron skin
- Structure in small systems including alpha clustering (e.g. $^{16}O+^{16}O$ vs $^{20}Ne+^{20}Ne$)

…… See recent INT program 23-1A

Summary and outlook

- High-energy collisions image nuclear shape at ultra-short time scale of 10^{-24}s; Large particle multiplicity enables many-particle correlation event-by-event to probe many-nucleon correlations in nuclei.
- Collisions of carefully-selected isobar species (at LHC) can reveal the many-body nucleon correlations & constrain the heavy ion initial condition from small to large nuclei

2102.08158

A	isobars	A	isobars	A	isobars	A	isobars	A	isobars	A	isobars
36	Ar, S	80	Se, Kr	106	Pd, Cd	124	Sn, Te, Xe	148	Nd, Sm	174	Yb, Hf
40	Ca, Ar	84	Kr, Sr, Mo	108	Pd, Cd	126	Te, Xe	150	Nd, Sm	176	Yb, Lu, Hf
46	Ca, Ti	86	Kr, Sr	110	Pd, Cd	128	Te, Xe	152	Sm, Gd	180	Hf, W
48	Ca, Ti	87	Rb, Sr	112	Cd, Sn	130	Te, Xe, Ba	154	Sm, Gd	184	W, Os
50	Ti, V, Cr	92	Zr, Nb, Mo	113	Cd, In	132	Xe, Ba	156	Gd,Dy	186	W, Os
54	Cr, Fe	94	Zr, Mo	114	Cd, Sn	134	Xe, Ba	158	Gd,Dy	187	Re, Os
64	Ni, Zn	96	Zr, Mo, Ru	115	In, Sn	136	Xe, Ba, Ce	160	Gd,Dy	190	Os, Pt
70	Zn, Ge	98	Mo, Ru	116	Cd, Sn	138	Ba, La, Ce	162	Dy,Er	192	Os, Pt
74	Ge, Se	100	Mo, Ru	120	Sn, Te	142	Ce, Nd	164	Dy,Er	196	Pt, Hg
76	Ge, Se	102	Ru, Pd	122	Sn, Te	144	Nd, Sm	168	Er,Yb	198	Pt, Hg
78	Se, Kr	104	Ru, Pd	123	Sb, Te	146	Nd, Sm	170	Er,Yb	204	Hg, Pb