Neutrinoless double-eta decay:
The challenge it poses for nuclear structure studies

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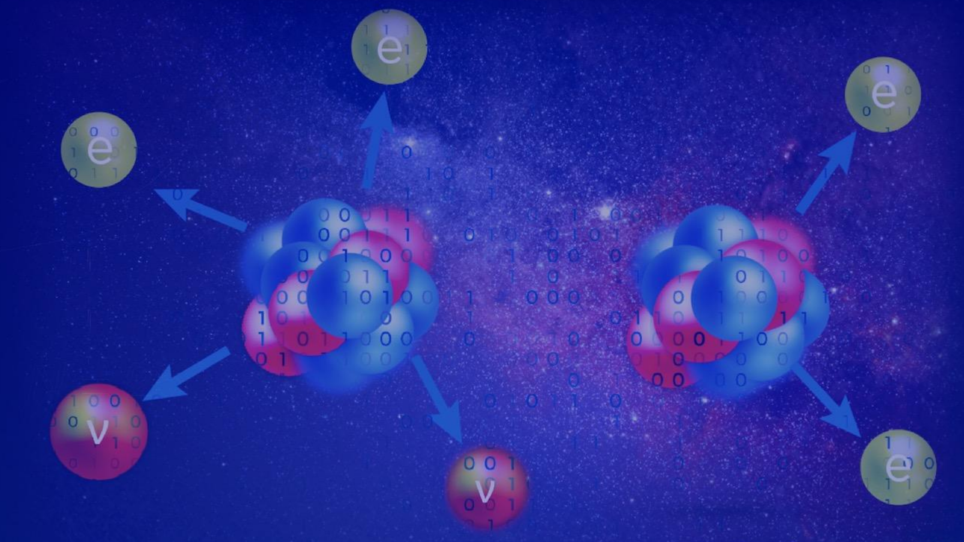
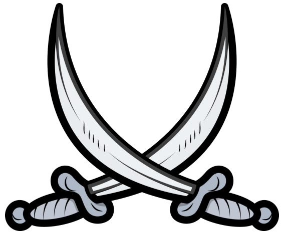


2023年8月4日 原子核结构与相对论重离子碰撞前沿交叉研讨会 @ 大连

Is neutrino a majorana fermion?

中微子振荡实验 → 中微子有质量



无法自然给出左右手粒子场产生的狄拉克质量项 ← 只测到左手性中微子

为了优雅自然地解决中微子质量问题

跷跷板机制: 存在极重的右手中微子

右手越重，左手越轻。

这基于一个假设：中微子是马约拉纳费米子。

If it is true, then

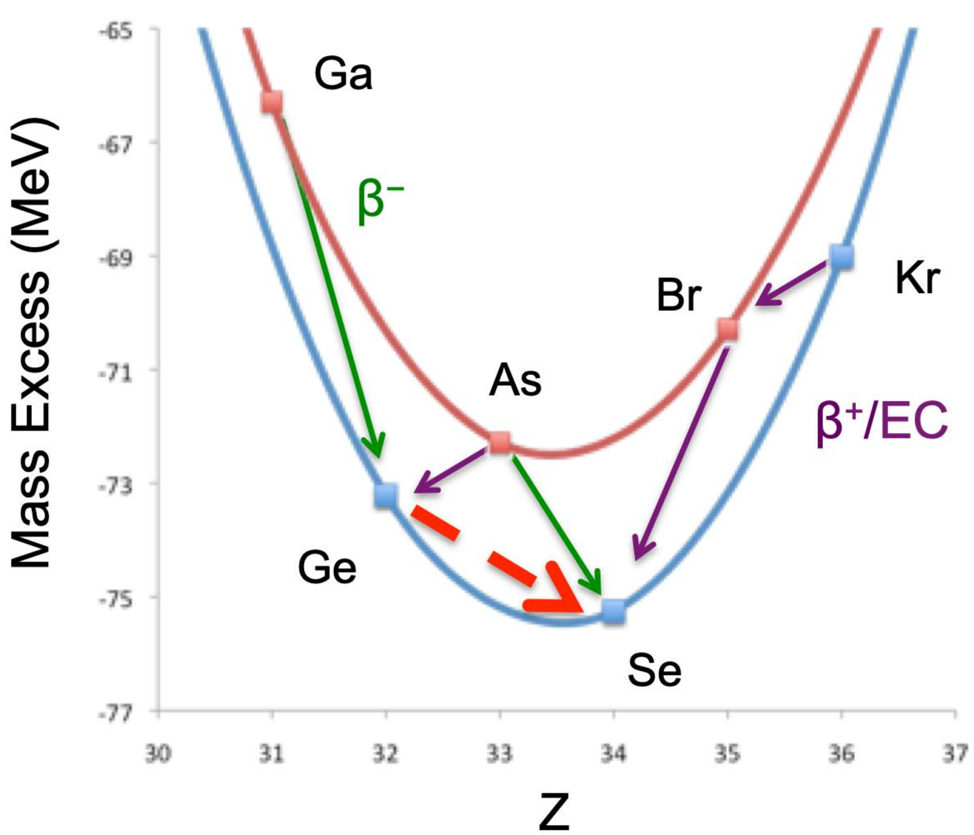
- 解释中微子质量起源
- 解释物质-反物质不对称
- 超越粒子物理标准模型的新物理

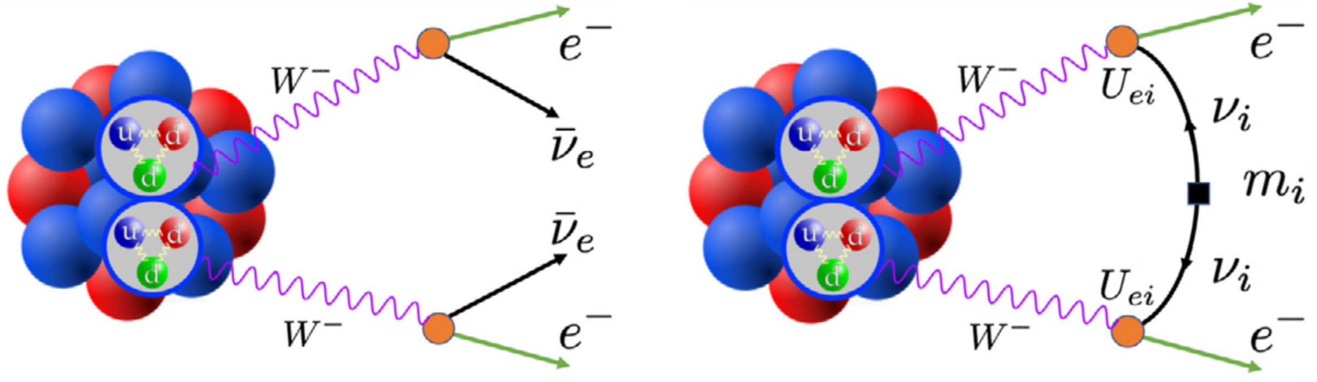
N

ν

Probes: Neutrinoless double-β decay (0vββ decay)

In certain even-even nuclei, β decay is energetically forbidden, because $m(Z, A) < m(Z+1, A)$, while double-β decay, from a nucleus of (Z, A) to $(Z+2, A)$, is allowed.





2vββ: observed **0vββ: not yet**

0vββ decay is interesting since:

- ☐ Lepton-number violation, baryongenesis.
- ☐ May be the only way to determine whether neutrino is a Majorana Fermion.

Probes: Neutrinoless double-β decay (0vββ decay)

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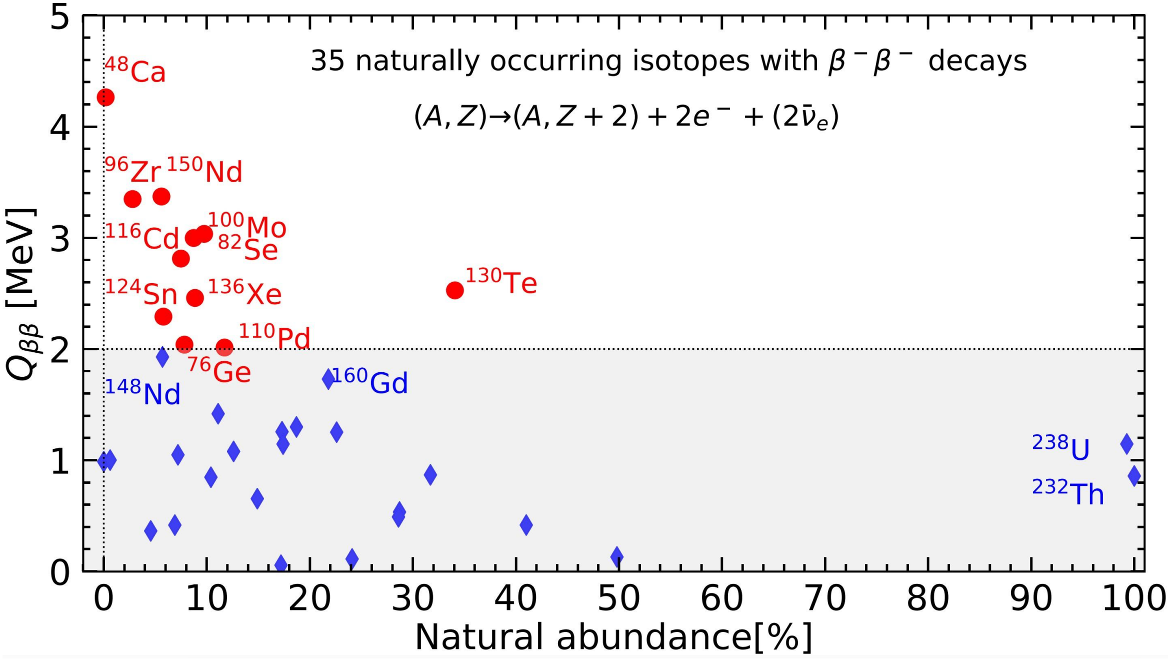
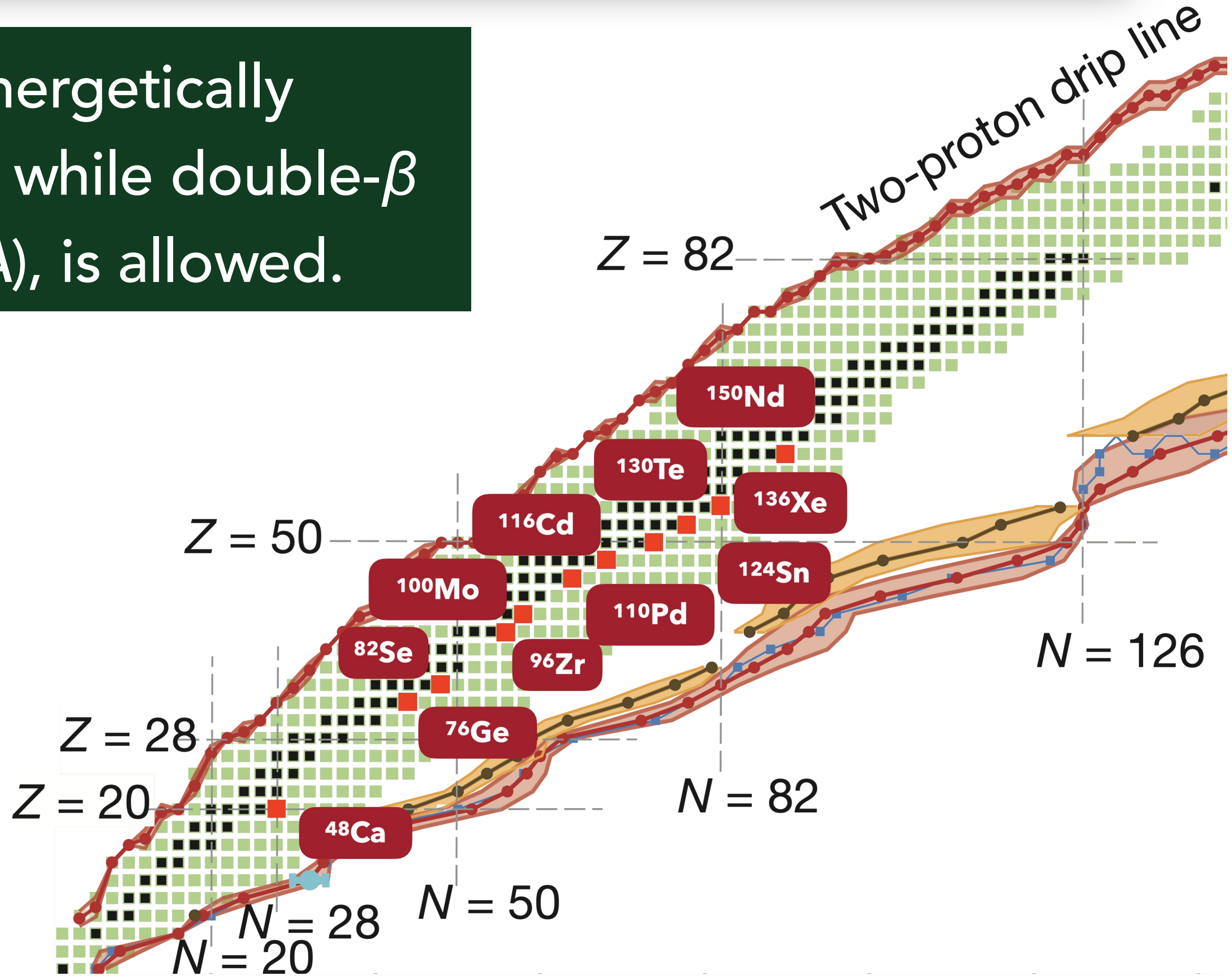
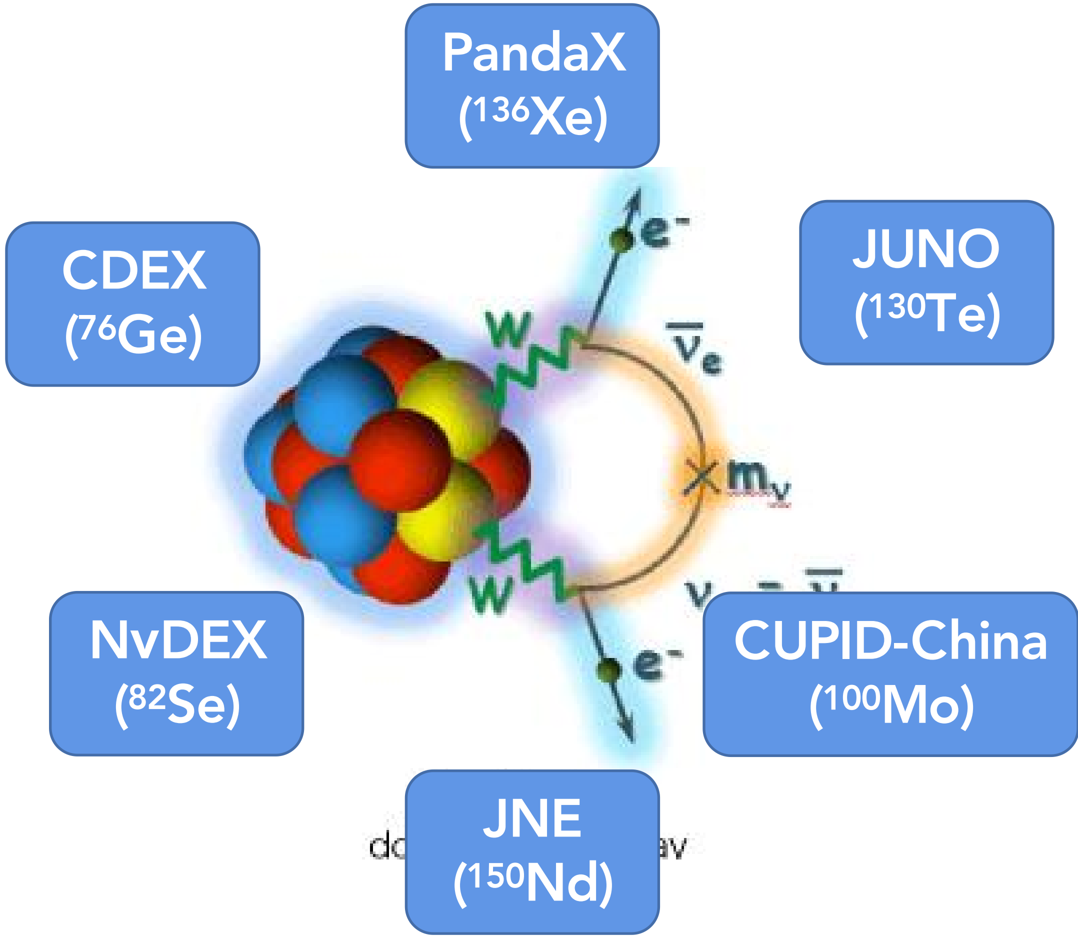




$0\nu\beta\beta$ decay experiments

Overseas

	Isotope	Technique	mass ($0\nu\beta\beta$ isotope)	Status
CANDLES	Ca-48	305 kg CaF_2 crystals - liq. scint	0.3 kg	Construction
CARVEL	Ca-48	$^{48}CaWO_4$ crystal scint.	~ ton	R&D
GERDA I	Ge-76	Ge diodes in LAr	15 kg	Complete
GERDA II	Ge-76	Point contact Ge in LAr	31	Operating
MAJORANA DEMONSTRATOR	Ge-76	Point contact Ge	25 kg	Operating
LEGEND	Ge-76	Point contact	**~ ton**	R&D
NEMO3	Mo-100 Se-82	Foils with tracking	6.9 kg 0.9 kg	Complete
SuperNEMO Demonstrator	Se-82	Foils with tracking	7 kg	Construction
SuperNEMO	Se-82	Foils with tracking	100 kg	R&D
LUCIFER (CUPID)	Se-82	ZnSe scint. bolometer	18 kg	R&D
AMoRE	Mo-100	$CaMoO_4$ scint. bolometer	1.5 - 200 kg	R&D
LUMINEU (CUPID)	Mo-100	$ZnMoO_4$ / Li_2MoO_4 scint. bolometer	1.5 - 5 kg	R&D
COBRA	Cd-114,116	CdZnTe detectors	10 kg	R&D
CUORICINO, CUORE-0	Te-130	TeO_2 Bolometer	10 kg, 11 kg	Complete
CUORE	Te-130	TeO_2 Bolometer	**206 kg**	Operating
CUPID	Te-130	TeO_2 Bolometer & scint.	**~ ton**	R&D
SNO+	Te-130	0.3% ^{nat}Te suspended in Scint	**160 kg**	Construction
EXO200	Xe-136	Xe liquid TPC	79 kg	Operating
nEXO	Xe-136	Xe liquid TPC	**~ ton**	R&D
KamLAND-Zen (I, II)	Xe-136	2.7% in liquid scint.	**380 kg**	Complete
KamLAND2-Zen	Xe-136	2.7% in liquid scint.	**750 kg**	Upgrade
NEXT-NEW	Xe-136	High pressure Xe TPC	5 kg	Operating
NEXT	Xe-136	High pressure Xe TPC	100 kg - **ton**	R&D
PandaX - 1k	Xe-136	High pressure Xe TPC	~ **ton**	R&D
DCBA	Nd-150	Nd foils & tracking chambers	20 kg	R&D



Domestic

The importance of NME in 0νββ decay

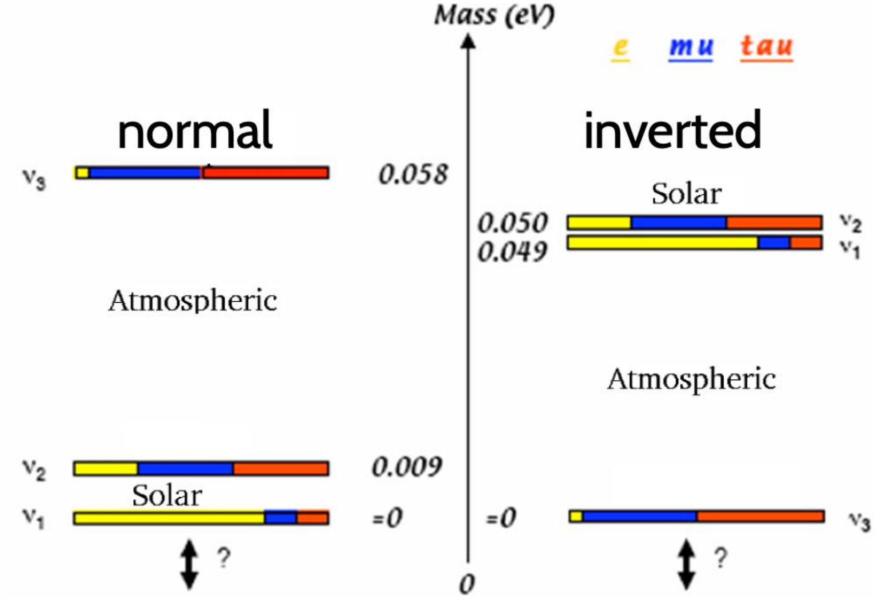
From neutrino oscillations we know

$$\Delta m^2_{\rm sun} \simeq 75\ {\rm meV}^2 \qquad \Delta m^2_{\rm atm} \simeq 2400\ {\rm meV}^2$$

We can get the mixing angles

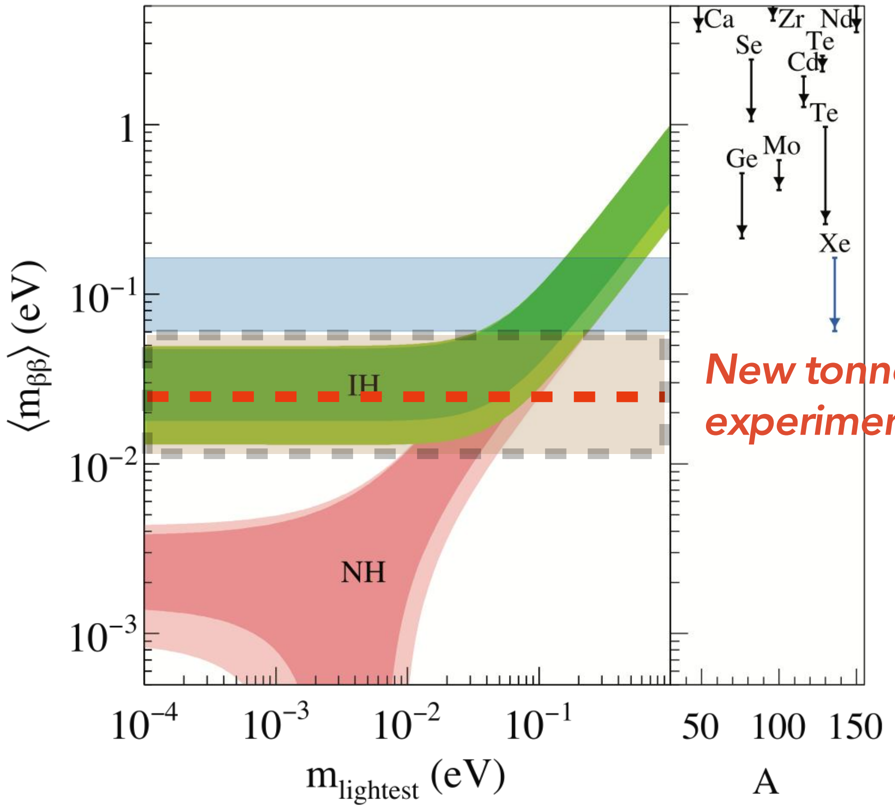
$$m_{\beta\beta} \equiv \left| \sum_k m_k U_{ek}^2 \right|$$ U_{ek} from PMNS matrix

But we *don't* know the absolute mass scale and mass hierarrchy.



0νββ decay can help since

$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu}(Q,Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$



New tonne-scale experiments.

The large uncertainty comes from NMEs.

Probes: Neutrinoless double-β decay (0νββ decay)

$$[T^{0\nu}_{1/2}]^{-1} = G_{0\nu}(Q,Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

$$M^{0\nu} = M^{0\nu}_{\mathrm{GT}} - \frac{g_V^2}{g_A^2} M^{0\nu}_{\mathrm{F}} + M^{0\nu}_{\mathrm{T}}$$

with

$$M^{0\nu}_{\mathrm{GT}} = \frac{2R}{\pi g_A^2}\int_0^\infty |q|d|q|\langle f|\sum_{a,b}\frac{j_0(|q|r_{ab})h_{\mathrm{GT}}(|q|)\vec{\sigma}_a\cdot\vec{\sigma}_b}{|q|+\bar{E}-(E_i+E_f)/2}\tau_a^+\tau_b^+|i\rangle$$

$$M^{0\nu}_{\mathrm{F}} = \frac{2R}{\pi g_A^2}\int_0^\infty |q|d|q|\langle f|\sum_{a,b}\frac{j_0(|q|r_{ab})h_{\mathrm{F}}(|q|)}{|q|+\bar{E}-(E_i+E_f)/2}\tau_a^+\tau_b^+|i\rangle$$

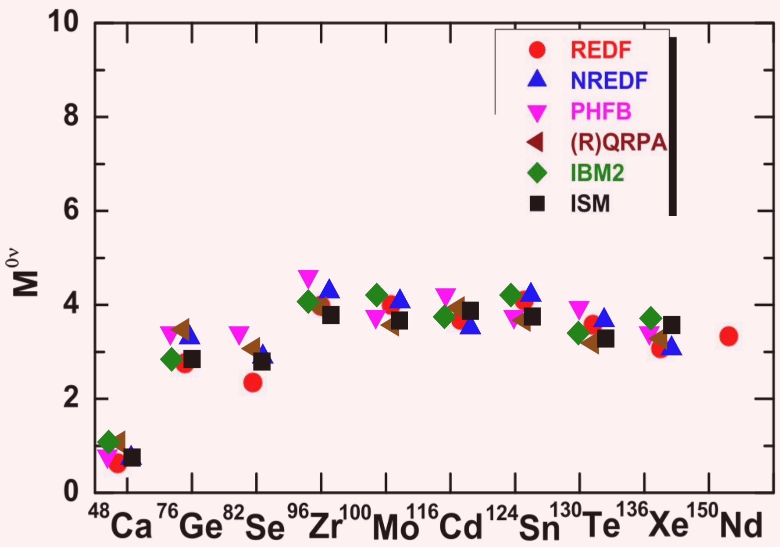
$$M^{0\nu}_{\mathrm{T}} = \frac{2R}{\pi g_A^2}\int_0^\infty |q|d|q|\langle f|\sum_{a,b}\frac{j_2(|q|r_{ab})h_{\mathrm{T}}(|q|)[3\vec{\sigma}_j\cdot\hat{r}_{ab}\vec{\sigma}_k\cdot\hat{r}_{ab}-\vec{\sigma}_a\cdot\vec{\sigma}_b}{|q|+\bar{E}-(E_i+E_f)/2}\tau_a^+\tau_b^+|i\rangle$$

Lines of attack:

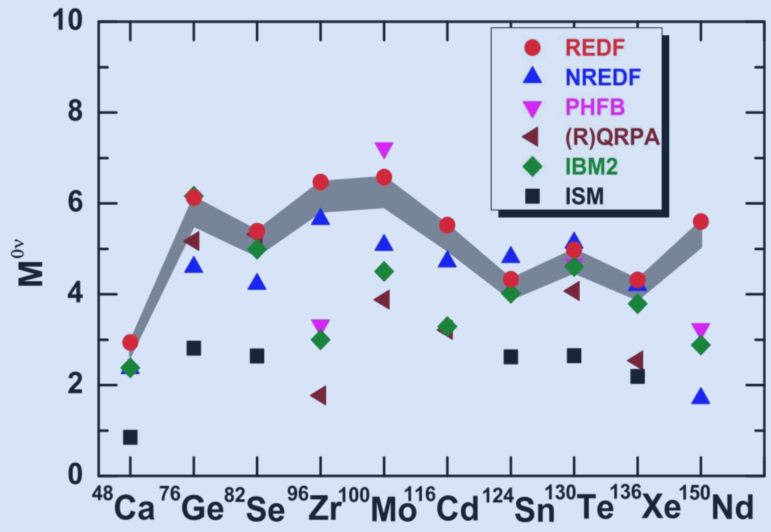
- Construct effective operator.
- *Good initial and final ground-state wave functions: nuclear structure.*

Current status of calculated NMEs in 0νββ decay

What we hope:



What we had got:



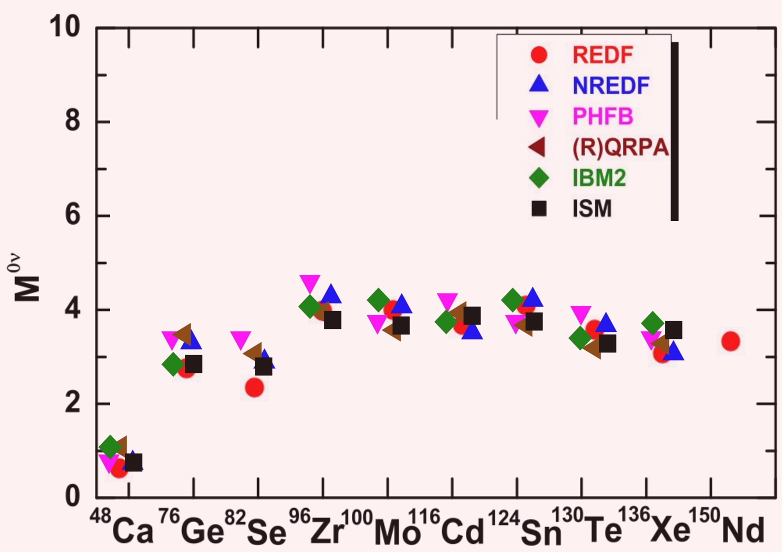
It poses challenges for nuclear structure studies:

- ☐ Some **omits the correlations** underlying nuclear structure aspects.
- ☐ Some limits the correlations in **a small model space.**

Does the discrepancy come from methods, or the interactions they use?

Current status of calculated NMEs in 0νββ decay

What we hope:



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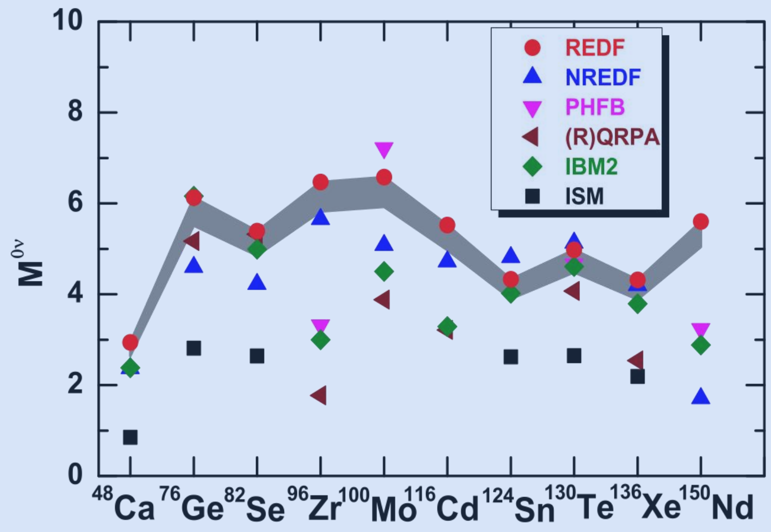


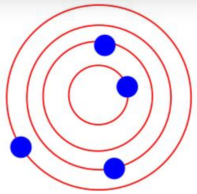
In short, we need:

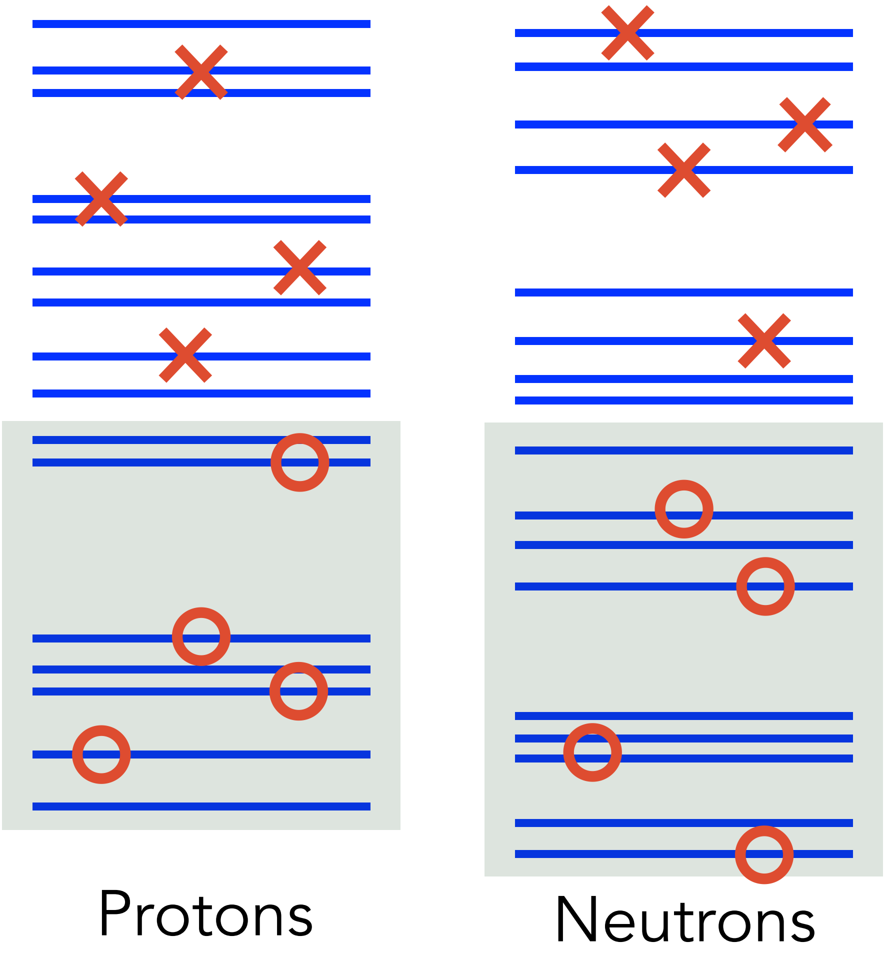
- □ Understanding the effect from collective correlations on NMEs.
- □ Understanding the effect from enlarging the model space.

And a better effective interaction.

Review of Different Nuclear Models

Some models are built on single independent-particle state.





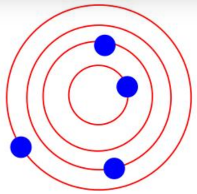
Starting from one Slater determinant, e.g., the HF state $|\psi_0\rangle$, the ground state

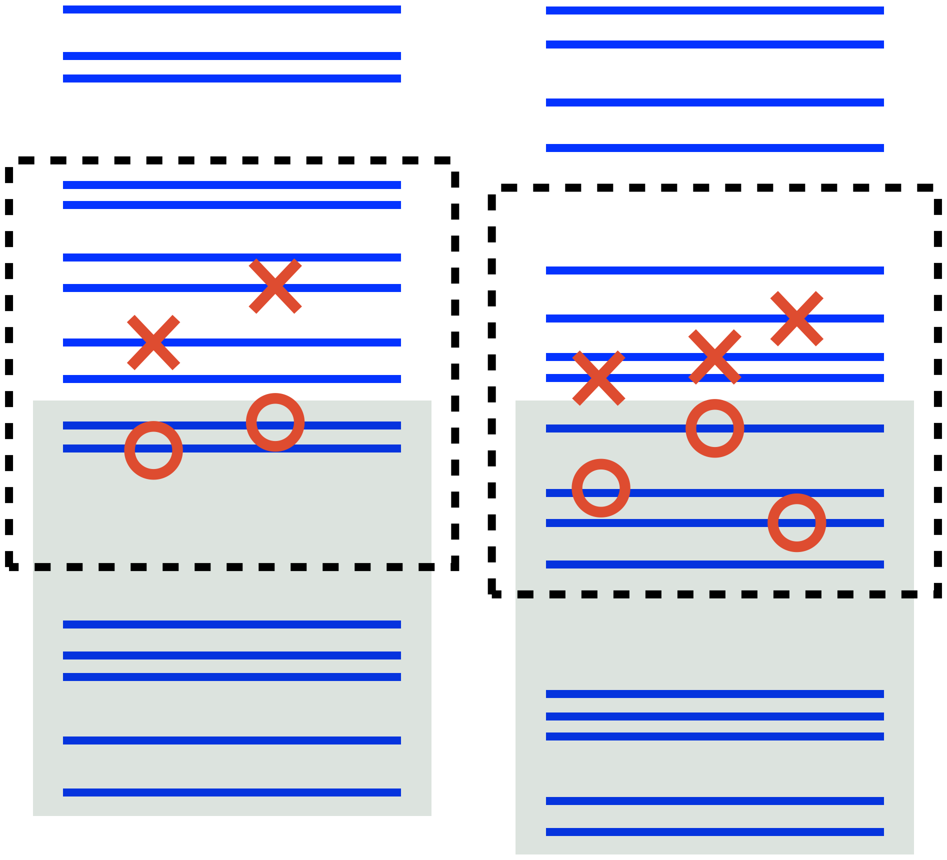
$$|0\rangle = |\psi_0\rangle + \sum_{mi} C^0_{mi} a^\dagger_m a_i |\psi_0\rangle + \frac{1}{4} \sum_{mnij} C^0_{mn,ij} a^\dagger_m a^\dagger_n a_i a_j |\psi_0\rangle + \cdots$$

But exact diagonalization in the complete Hilbert space is not solvable.

Review of Different Nuclear Models

Some models are built on single independent-particle state.





Interacting shell model (ISM)

- Same starting point $|0\rangle$.
- Instead of solving Schrödinger equation in complete Hilbert space, one restricts the dynamics in a configuration space.

$$H|\Phi_i\rangle = E_i|\Phi_i\rangle \rightarrow H_{\text{eff}}|\bar{\Phi}_i\rangle = E_i|\bar{\Phi}_i\rangle$$

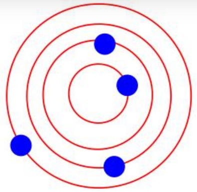
Configuration interaction of orthonormal Slater determinants:

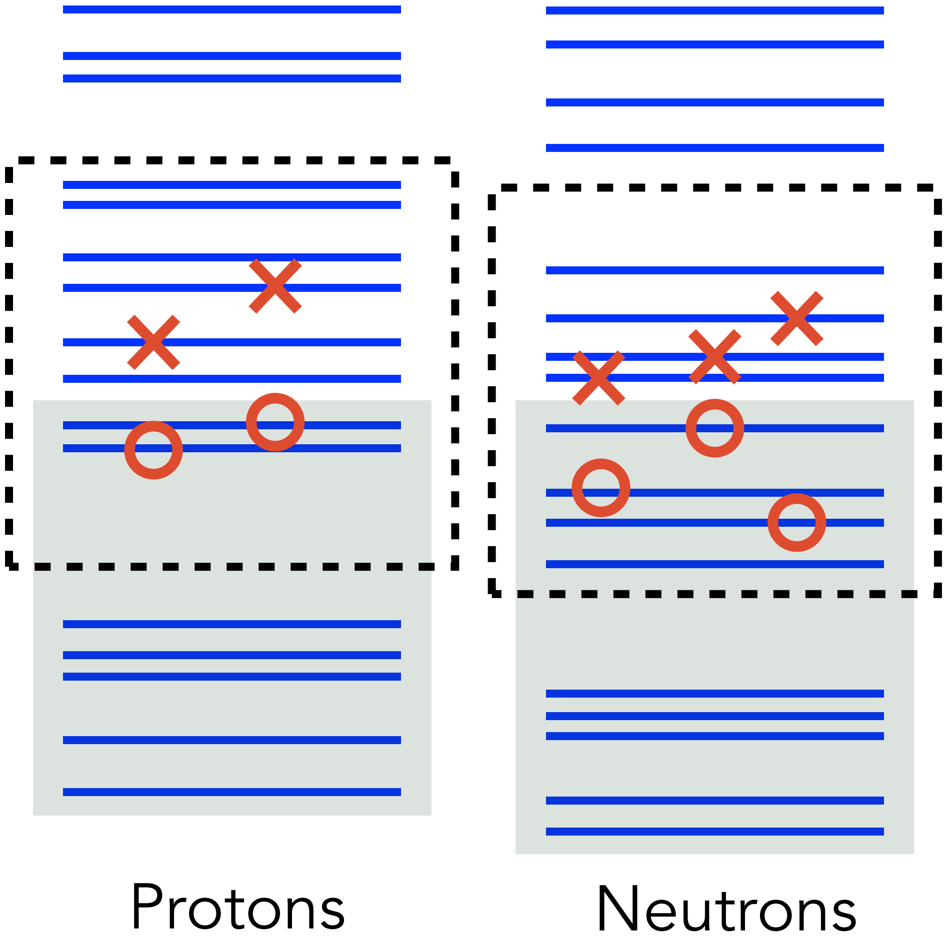
$$|\bar{\Phi}_i\rangle = \sum_j c_{ij}|\psi_j\rangle, \qquad \langle\psi_j|\psi_k\rangle = \delta_{jk}$$

Diagonalizing the H_{eff} in the orthonormal basis.

Review of Different Nuclear Models

Some models are built on single independent-particle state.





Interacting shell model (ISM)

Pros:

- Arbitrarily complex correlations within the model space.

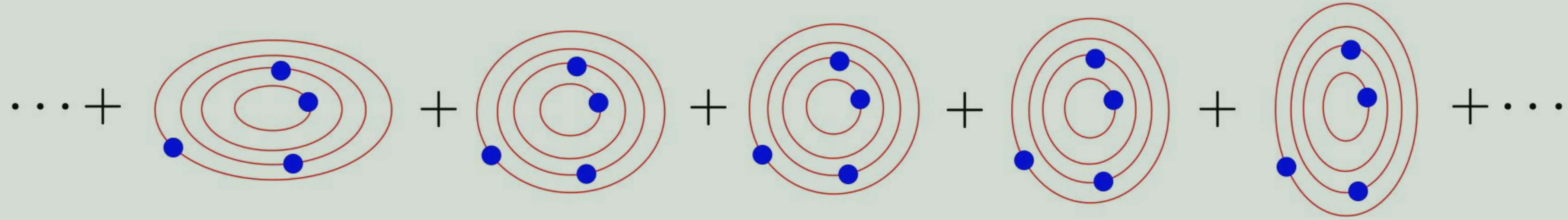
Cons:

- Relatively small configuration spaces.
  - *At present most of the 0νββ decay NME calculations carried out by ISM are limited in one single shell.*

The Other Way Around...

Generator-coordinate method (GCM)

Instead of configuration interaction with orthogonal states, one can diagonalize the Hamiltonian in a set of *non-orthogonal* basis.



$$|\Phi\rangle = \sum_j c_j |\psi_j\rangle, H_{jk} = \langle j|H|k\rangle$$

$$\sum_k H_{jk} c_k = E \sum_k N_{jk} c_k, N_{jk} = \langle j|k\rangle$$

The non-orthogonal states can be generated to give different quantities of many-body correlations as collective coordinates (***fluctuations of deformation, pairing...***).

Hamiltonian-based projected generator-coordinate method

- Using a realistic effective Hamiltonian.
- ***Trying to include all possible correlations.*** (For now, we pick the most important ones)

$$\mathcal{O}_1 = Q_{20}, \quad \mathcal{O}_2 = Q_{22},$$ **quadrupole correlations**

$$\mathcal{O}_3 = \frac{1}{2}(P_0 + P_0^\dagger), \quad \mathcal{O}_4 = \frac{1}{2}(S_0 + S_0^\dagger),$$ **proton-neutron pairing correlations**

- HFB states with multipole constraints

$$\langle H' \rangle = \langle H_{\text{eff}} \rangle - \lambda_Z(\langle N_Z \rangle - Z) - \lambda_N(\langle N_N \rangle - N) - \sum_i \lambda_i(\langle \mathcal{O}_i \rangle - q_i),$$

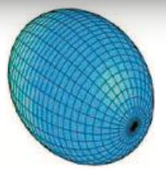
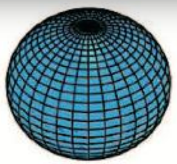
- Angular momentum and particle number projection $|JMK; NZ; q\rangle = \hat{P}^J_{MK} \hat{P^N} \hat{P^Z} |\Phi(q)\rangle$
- Configuration mixing within generator-coordinate method (GCM)

GCM wavefunction: $|\Psi^J_{NZ\sigma}\rangle = \sum_{K,q} f^{JK}_\sigma(q) |JMK; NZ; q\rangle$

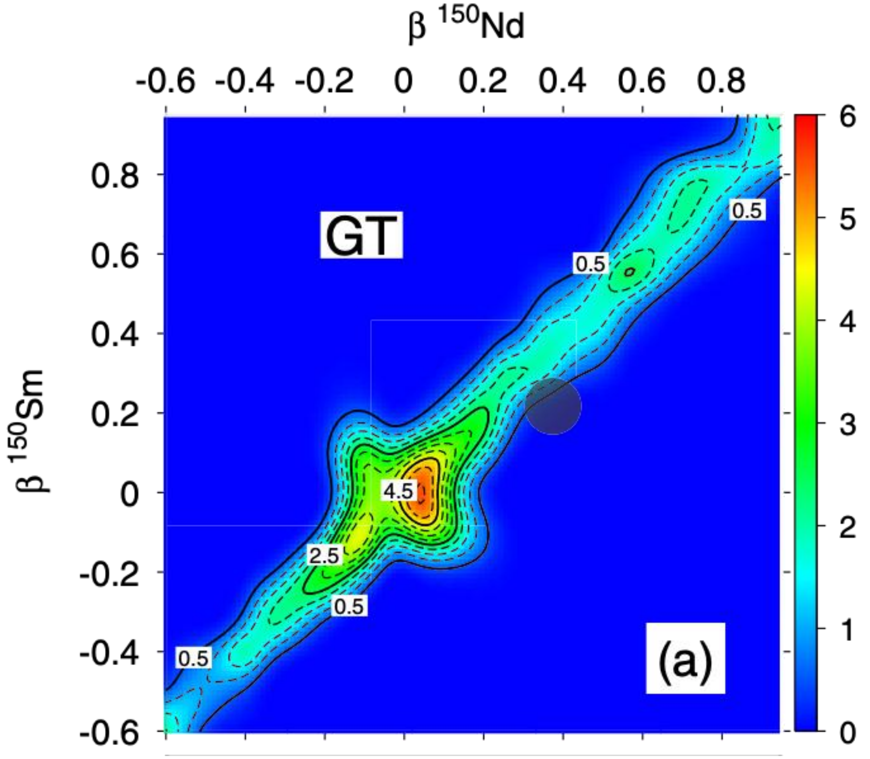
Hill-Wheeler equation: $\sum_{K',q'} \{\mathcal{H}^J_{KK'}(q;q') - E^J_\sigma \mathcal{N}^J_{KK'}(q;q')\} f^{JK'}_\sigma(q') = 0$

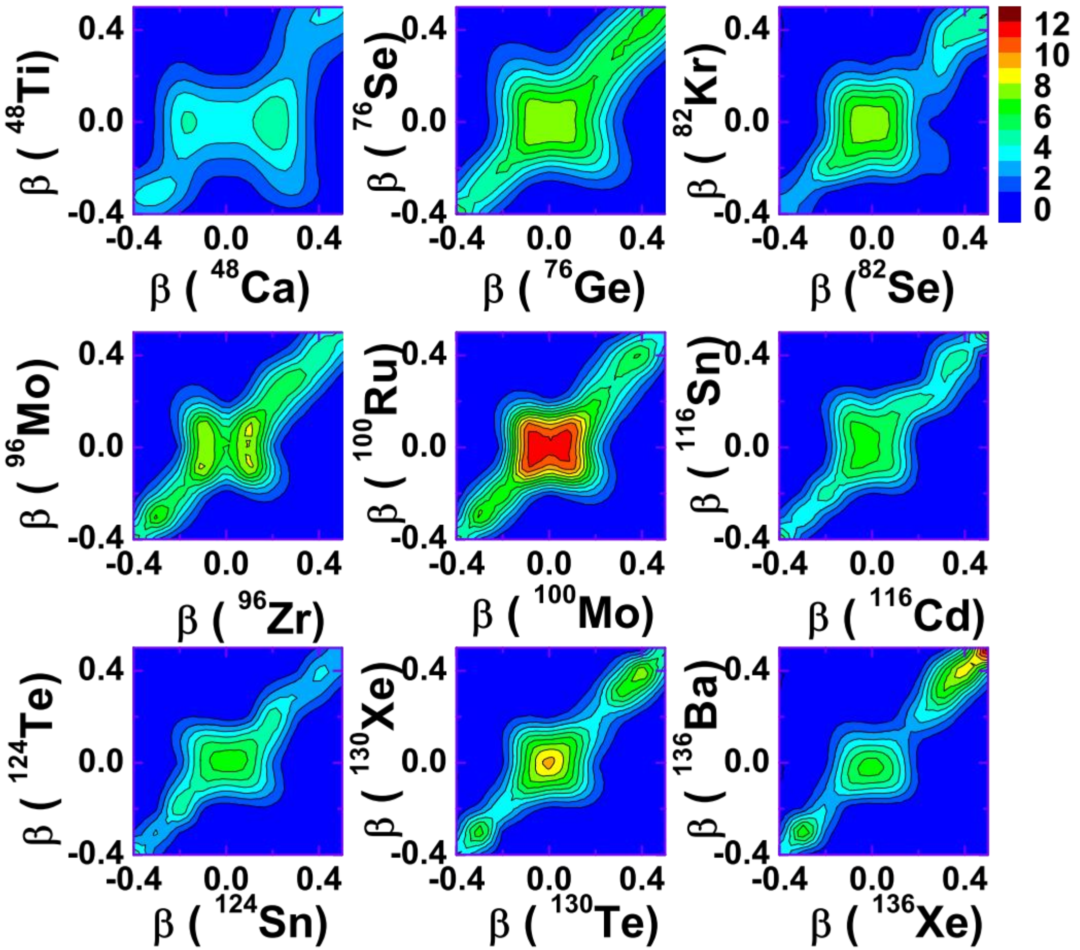
$0\nu\beta\beta$ NME: $M^{0\nu\beta\beta}_\xi = \langle \Psi^{J=0}_{N_f Z_f} | \hat{O}^{0\nu\beta\beta}_\xi | \Psi^{J=0}_{N_i Z_i} \rangle$

Which correlations are the most relevant to NMEs?

Axial deformation prolate   oblate

If parent and daughter nuclei have different axial deformation, **NMEs are suppressed.**

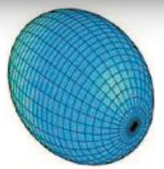
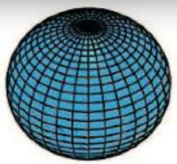




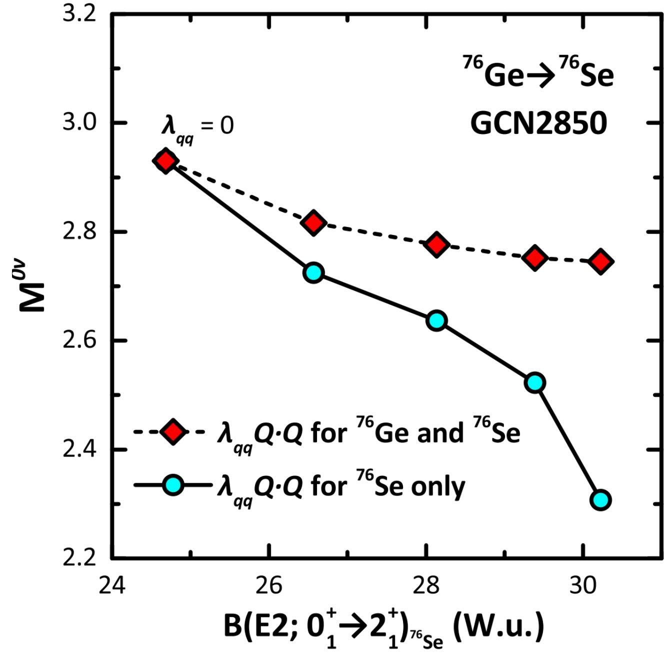
T. Rodriguez *et. al*., PRL **105**, 252503 (2010)

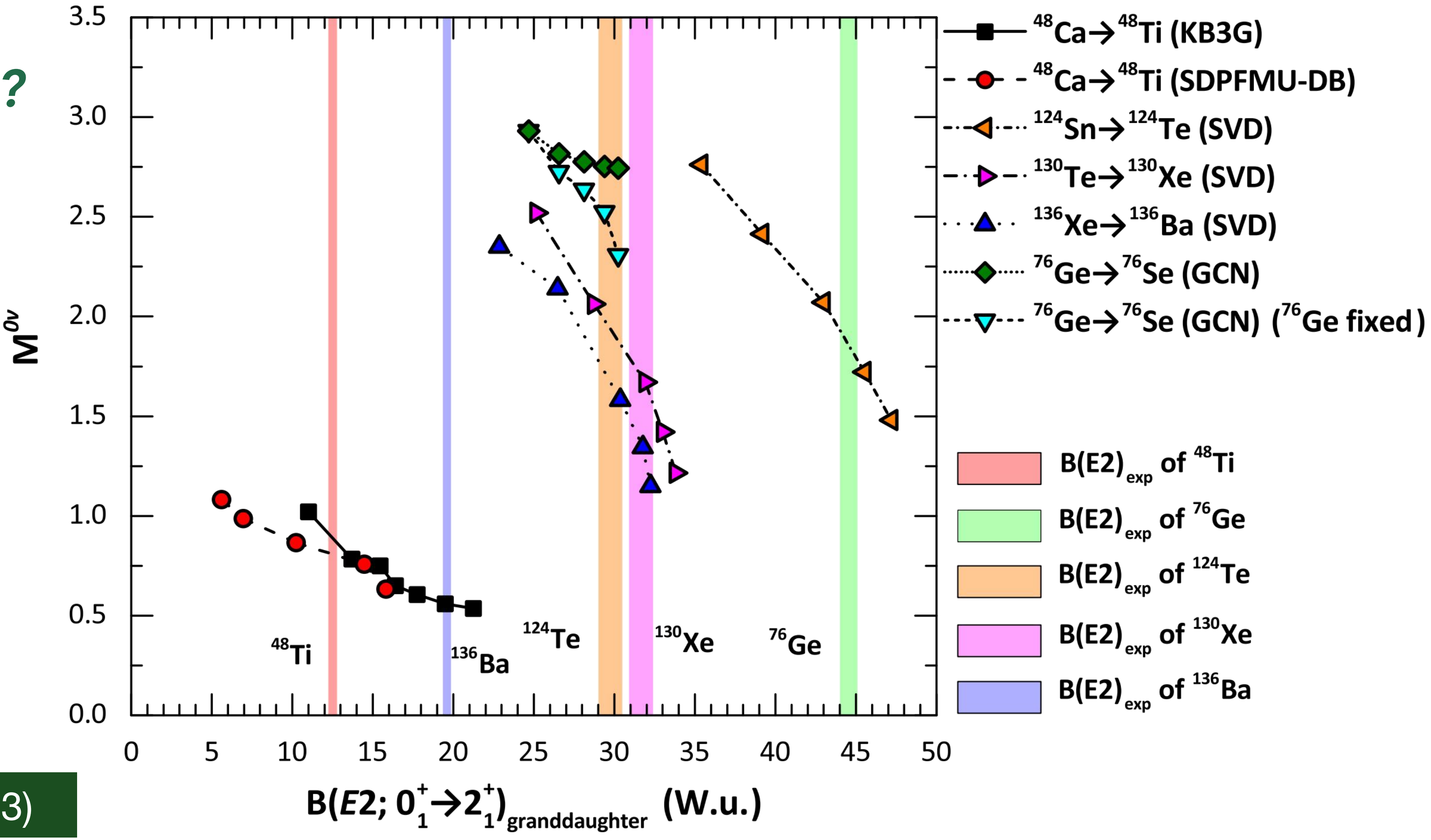
J. Yao, *et. al*., PRC **91**, 024316 (2015).

Which correlations are the most relevant to NMEs?

Axial deformation prolate   oblate ⇨ $E2$ transition probabbility $B(E2)$

$H_{\text{eff}} + \lambda Q\cdot Q$ ⎡ $B(E2)$ / *correlation?* / $0\nu\beta\beta$ NME

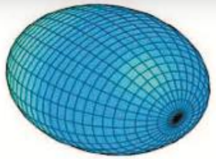




CFJ *et. al.*, Symmetry **15**, 552 (2023)

Which correlations are the most relevant to NMEs?

Triaxial deformation



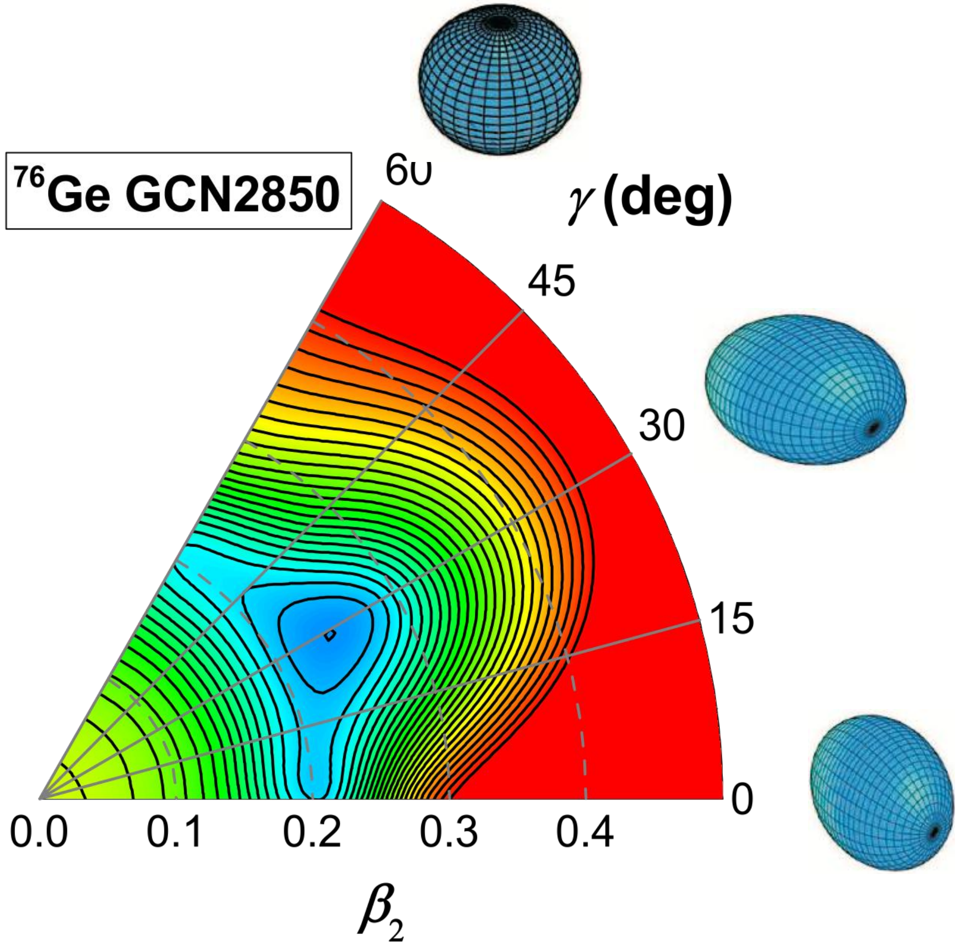
triaxial shape

Both theory and experiment indicate that ^{76}Ge and ^{76}Se are triaxially deformed, but the effect on $0\nu\beta\beta$ NMEs has never been investigated.

TABLE I. Matrix elements $M^{0\nu}$ produced in the GCM by GCN2850 and JUN45 for the decay of ^{76}Ge, with and without triaxial deformation as a generator coordinate, and by those same interactions with exact diagonalization.

	GCN2850	JUN45
Axial GCM	2.93	3.51
Triaxial GCM	2.56	3.16

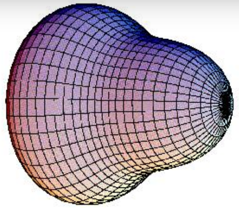
If triaxial deformation is included,
NMEs are slightly suppressed by 10~15%



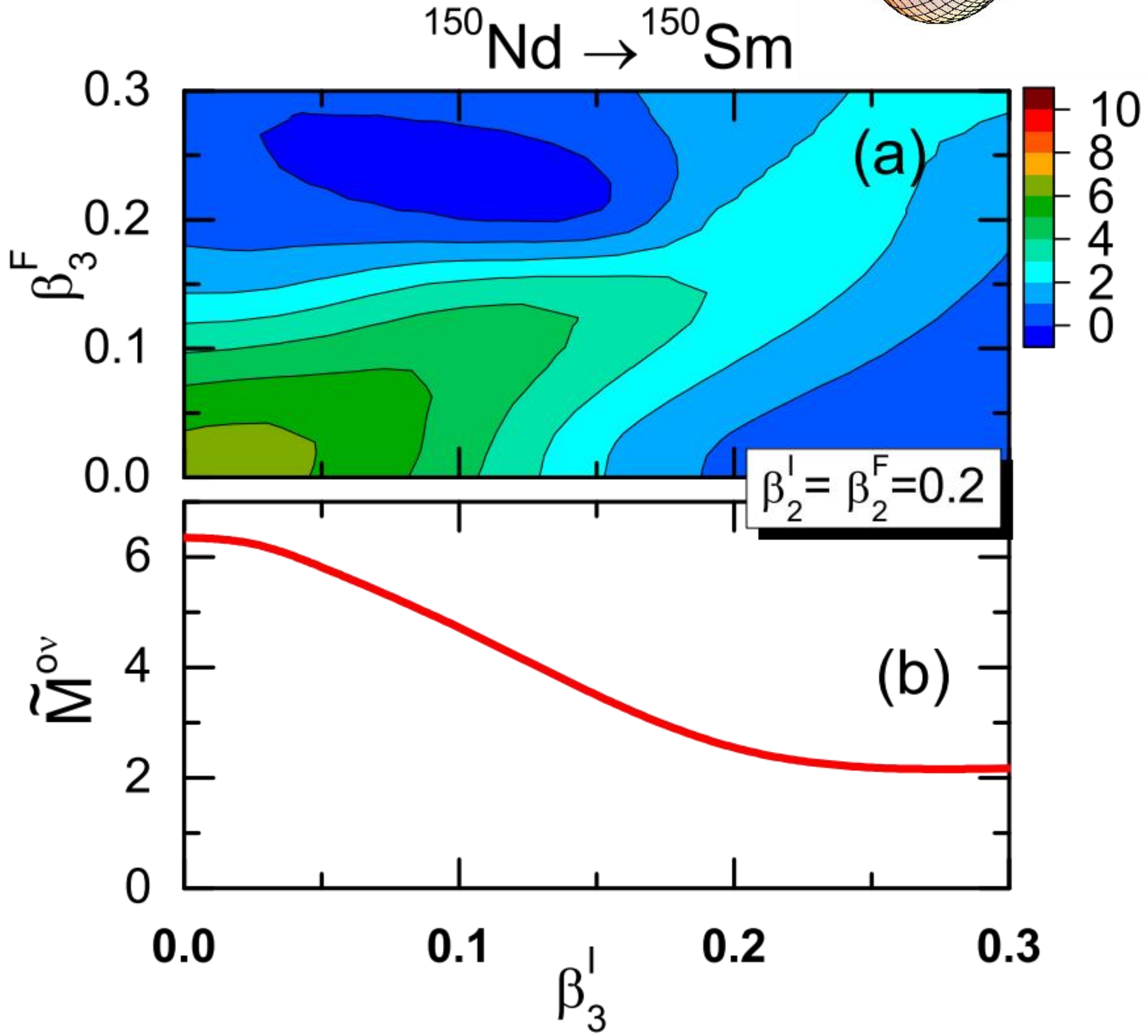
CFJ, J. Engel, J. D. Holt, PRC **96**, 054310 (2017).

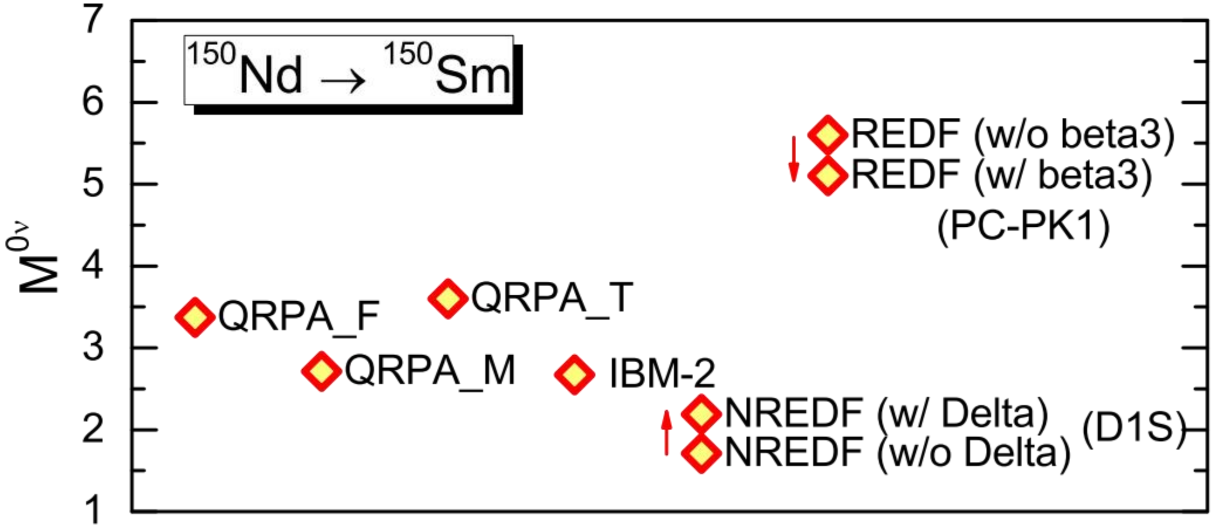
Which correlations are the most relevant to NMEs?

Octupole deformation



octupole "pear-like" deformation



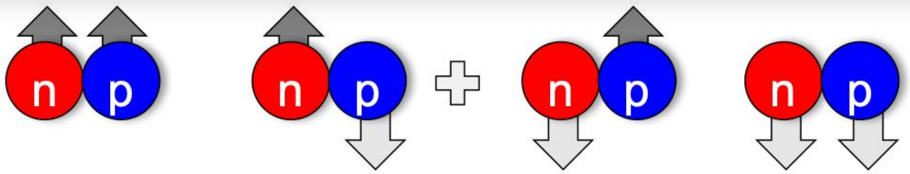


The inclusion of octupole shape fluctuations reduces the NME of ^{150}Nd-^{150}Sm by about 7%.

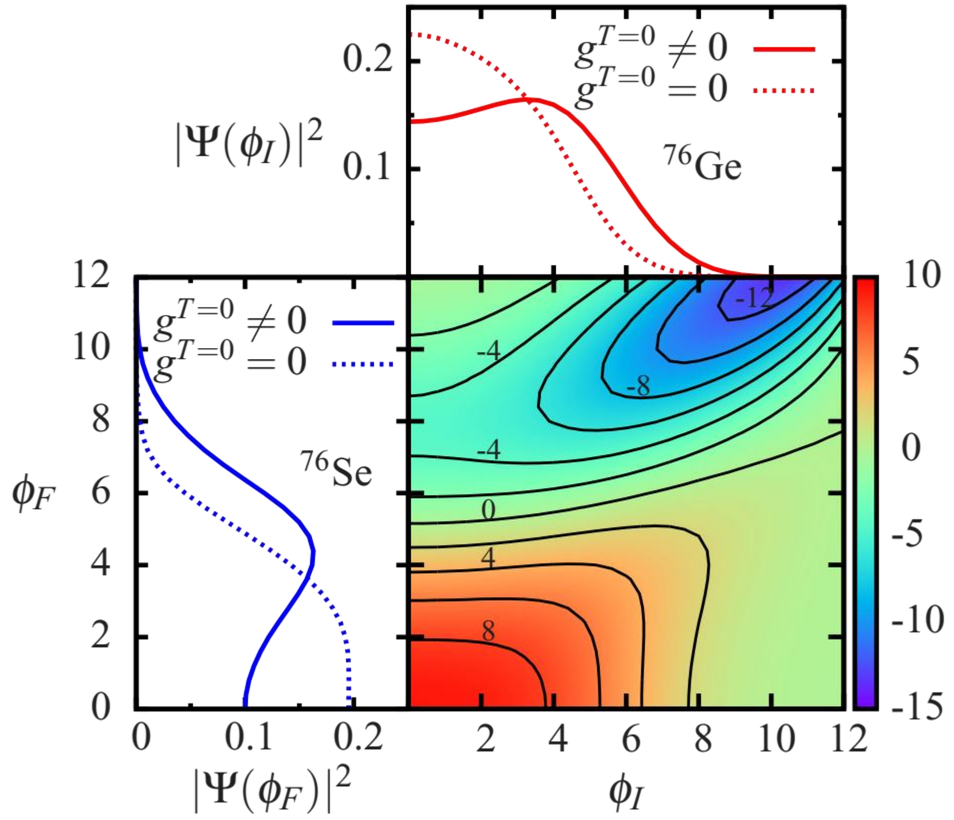
J. Yao, J. Engel, PRC 94, 014306 (2016).

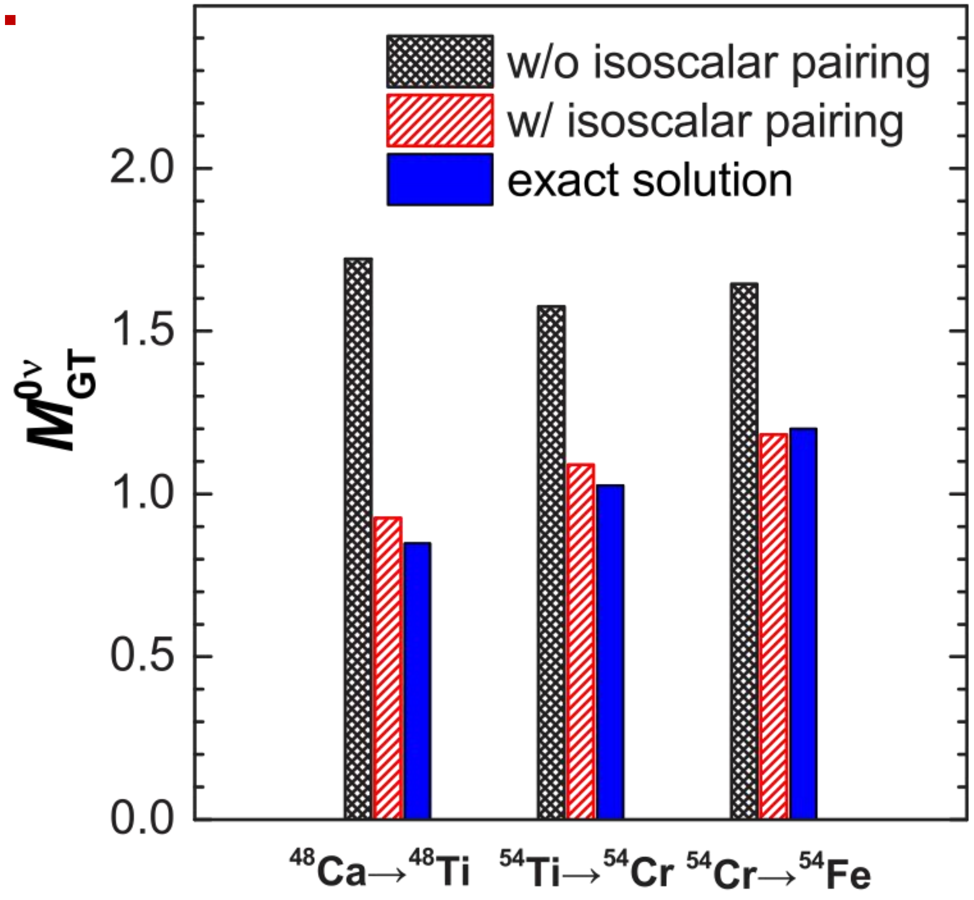
Which correlations are the most relevant to NMEs?

Proton-neutron pairing



If *pn* pairing is included, **NMEs are suppressed.**

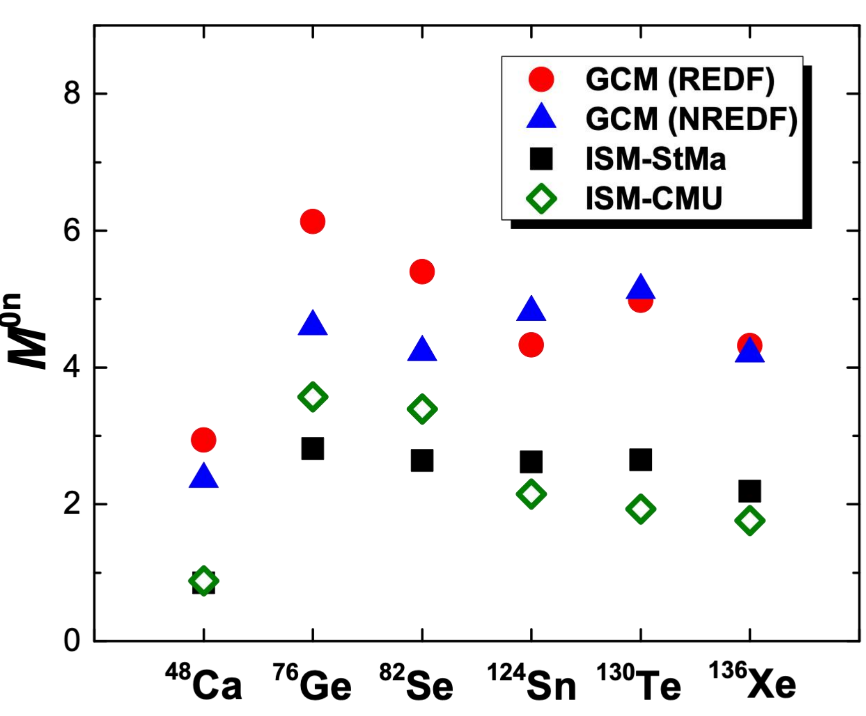




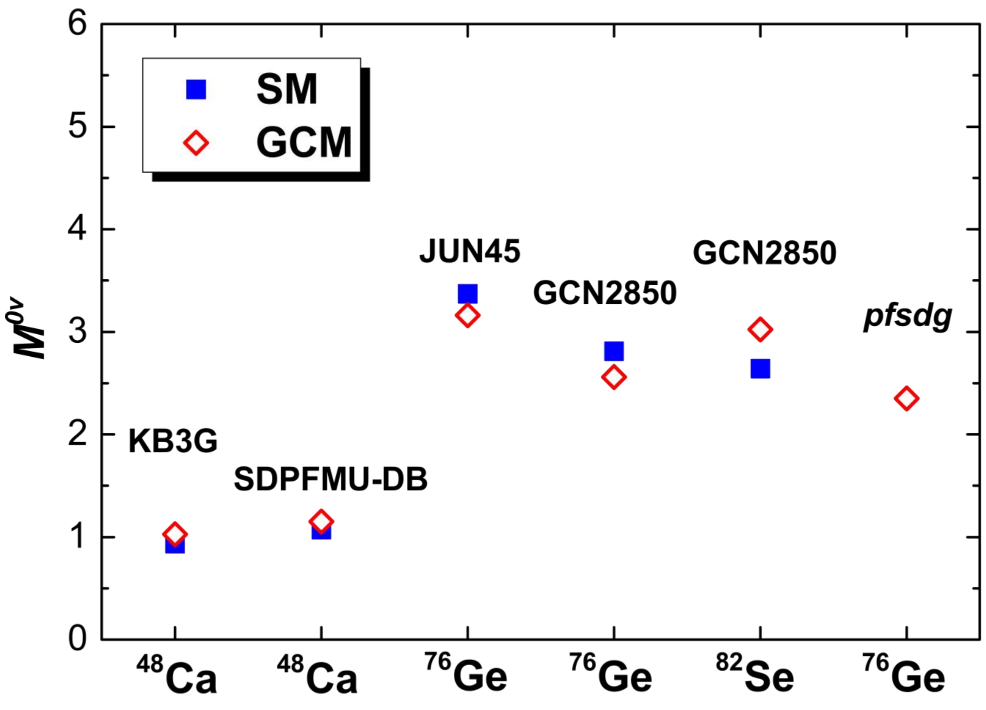
N. Hinohara et. al., PRC 90, 031301(R) (2014)

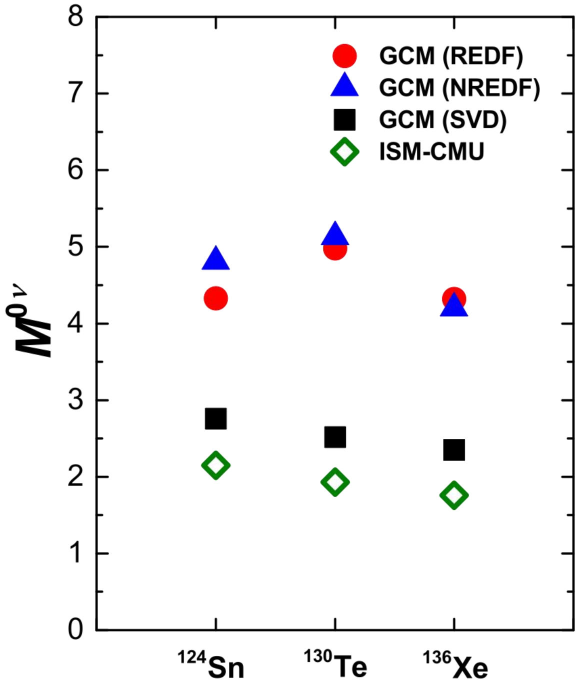
CFJ, J. Engel, J. D. Holt, PRC 96, 054310 (2017).

If we treat these collective correlations correctly…



Axial deformation only





Axial deformation + triaxial deformation + *pn* pairing

Deviation between the GCM and the SM vanishes.

CFJ, J. Engel, J. D. Holt, PRC 96, 054310 (2017).
CFJ, M. Horoi, A. Neacsu, PRC 98, 064324 (2018).

What is the effect from the enlargement of the model space?

We extend the calculation in the full fp-sdg two-shell space, which is unreachable by the shell model.

- There is no *a priori* effective Hamiltonian in this model space.

  We use EKK method of many-body perturbation theory to derive an effective Hamiltonian from the Chiral interaction.

TABLE II. GCM results for the Gamow-Teller ($M^{0\nu}_{\rm GT}$), Fermi ($M^{0\nu}_{\rm F}$), and tensor ($M^{0\nu}_{\rm T}$) $0\nu\beta\beta$ matrix elements for the decay of ^{76}Ge in two shells, without and with triaxial deformation.

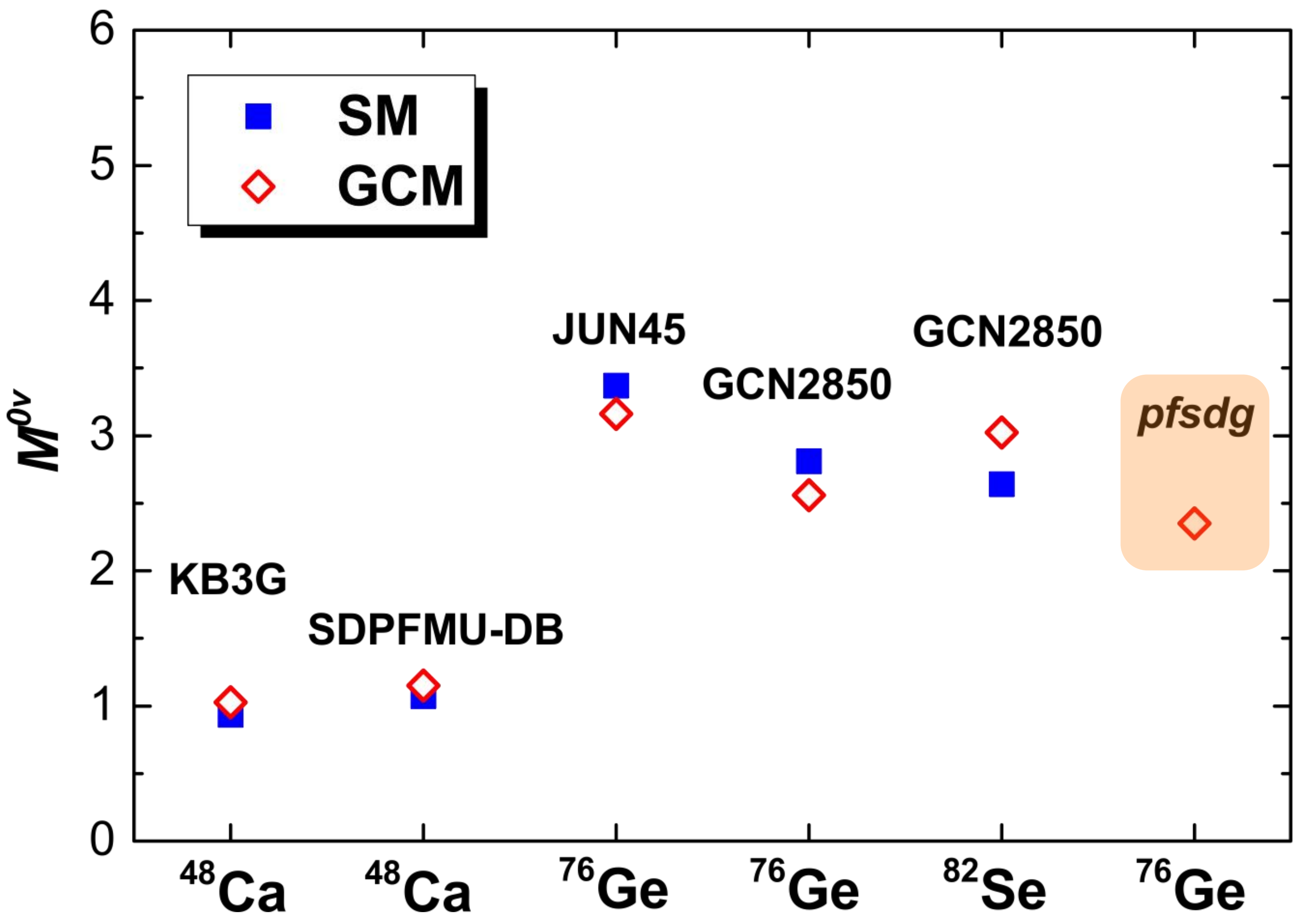
	Axial	Triaxial
$M^{0\nu}_{\rm GT}$	3.18	1.99
$-\frac{g_V^2}{g_A^2} M^{0\nu}_{\rm F}$	0.55	0.38
$M^{0\nu}_{\rm T}$	−0.01	−0.02
Total $M^{0\nu}$	3.72	2.35

Axially-deformed result is enhanced:
Larger space captures more like-particle pairing

Triaxially-deformed result is suppressed:
Larger space captures more effect from triaxial deformation.

CFJ, J. Engel, J. D. Holt, PRC 96, 054310 (2017).

What is the effect from the enlargement of the model space?



The *pfsdg* "two-shell" result is slightly smaller than single-shell result.

Enlarging the space further may not dramatically change NMEs.

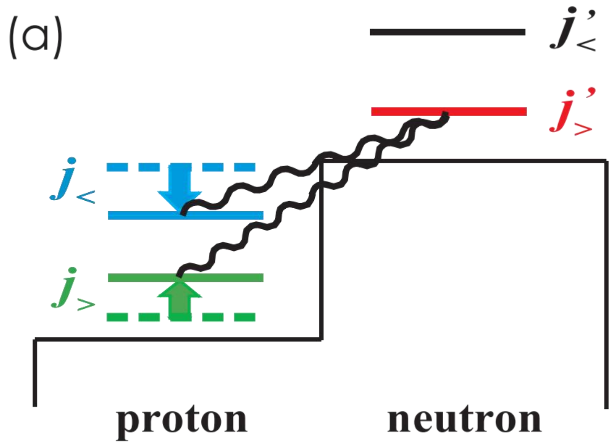
Better effective interactions derived from the Chiral EFT via non-pertubative *ab initio* methods are in progress...

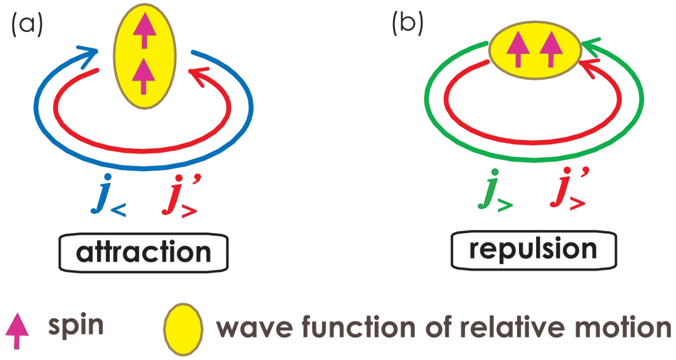
CFJ, J. Engel, J. D. Holt, PRC 96, 054310 (2017).

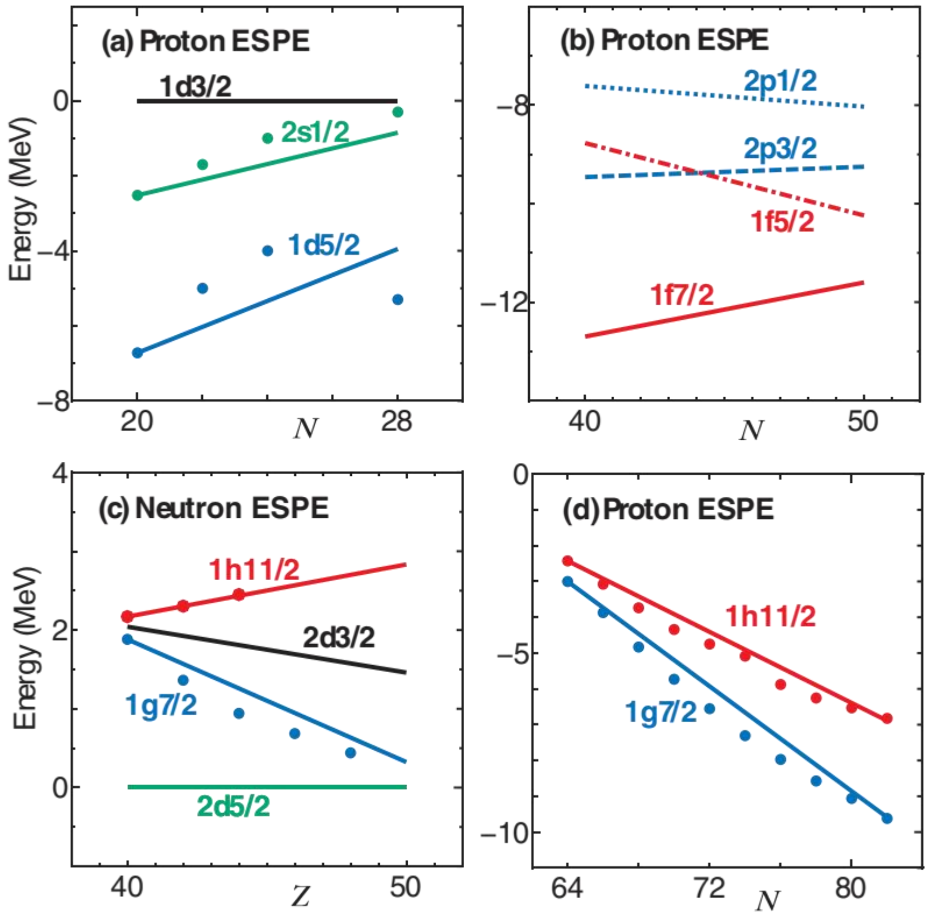
Evaluate the influence of the effective Hamiltonian, e.g., tensor force

The tensor force has a robust effect on the nuclear structure.

The tensor force $V_T = (\vec{\tau}_1 \cdot \vec{\tau}_2)([\vec{s}_1\ \vec{s}_2]^{(2)} \cdot Y^{(2)})f(r)$





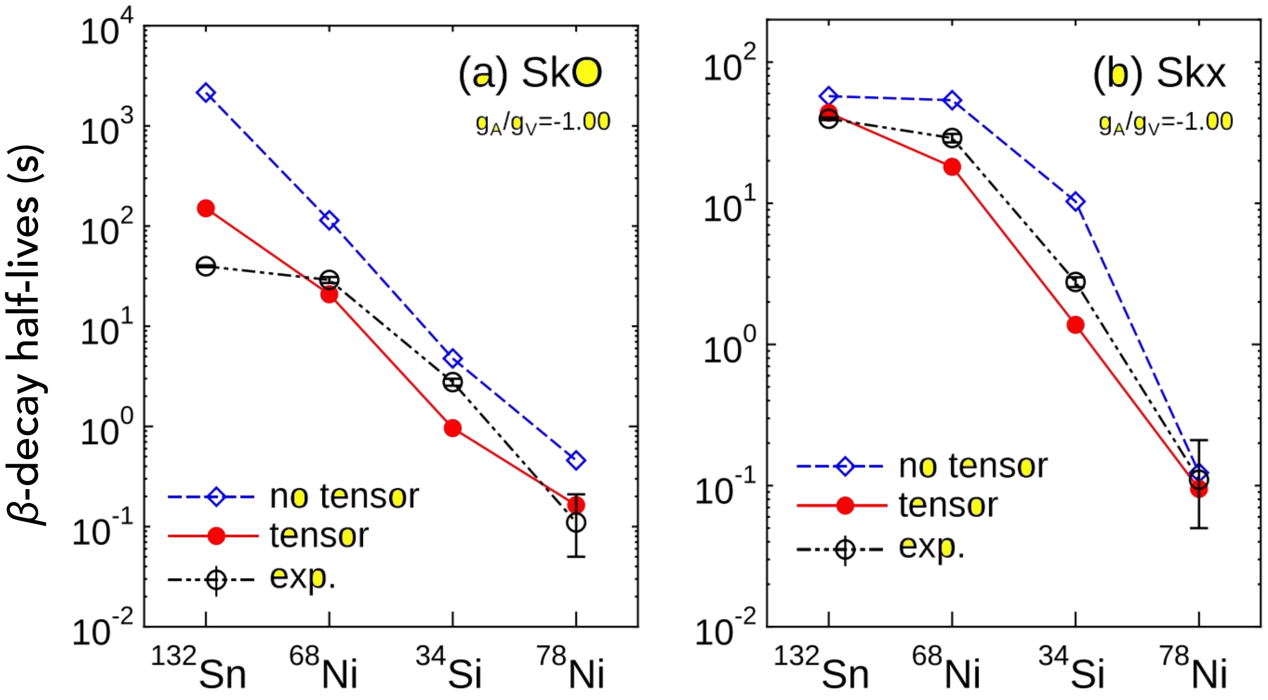


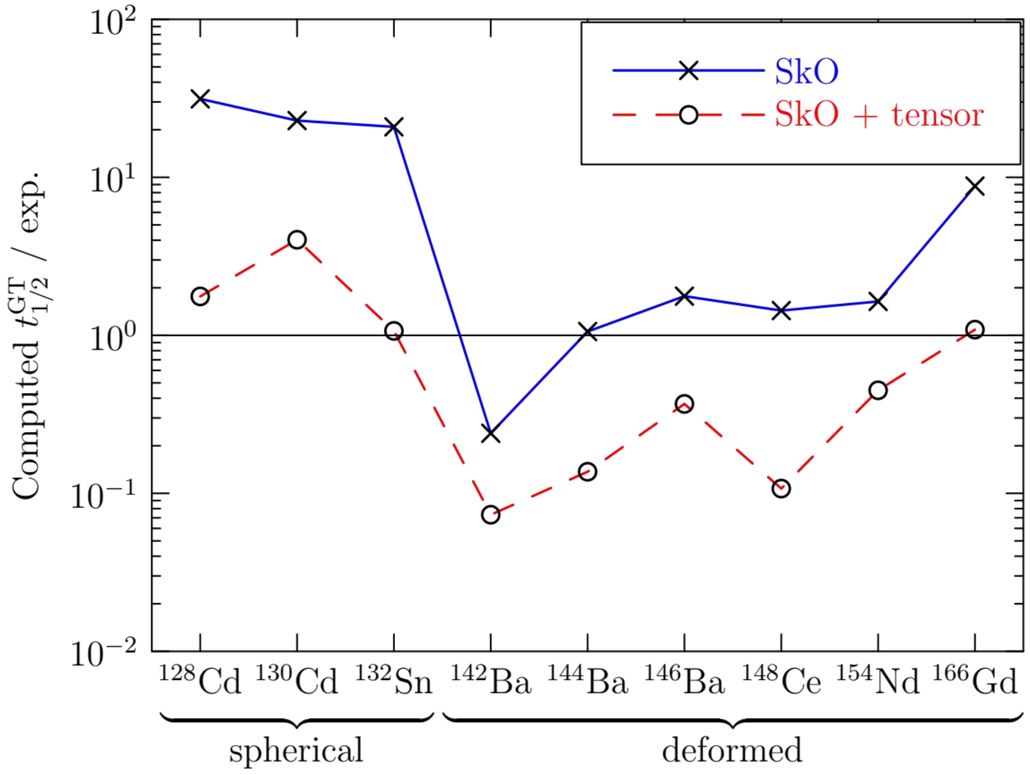
The monopole interaction produced by the tensor force.

T. Otsuka *et al.*, PRL 95, 232502 (2005)
T. Otsuka *et al.*, PRL 105, 012501 (2010)

Evaluate the influence of the effective Hamiltonian, e.g., tensor force

The tensor force has a systematic effect on single-β decay.





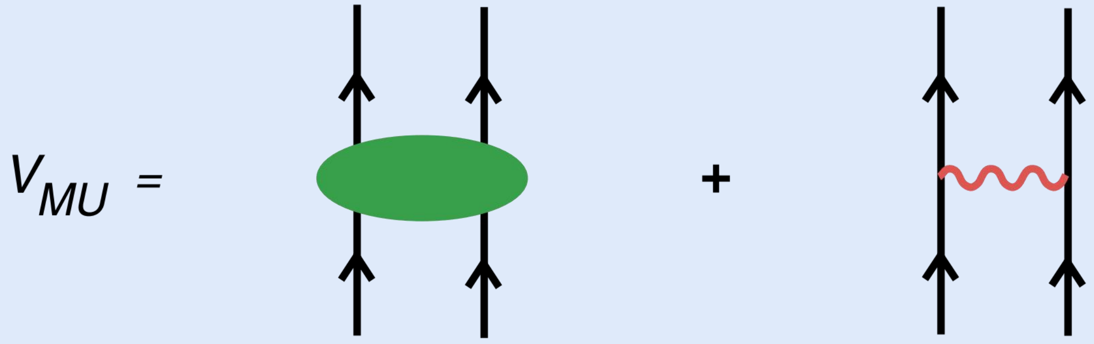
F. Minato and C.L. Bai, PRL 110, 122501 (2013)
F. Minato and C.L. Bai, PRL 116, 089902 (2016)

M. Mustonen *et. al*, PRC 90, 024308 (2014)

Explicit form of the tensor force in effective interactions

(a) central force :
Gaussian
(strongly renormalized)

(b) tensor force :
π + ρ meson
exchange



Diagrams for the V_{MU} interaction

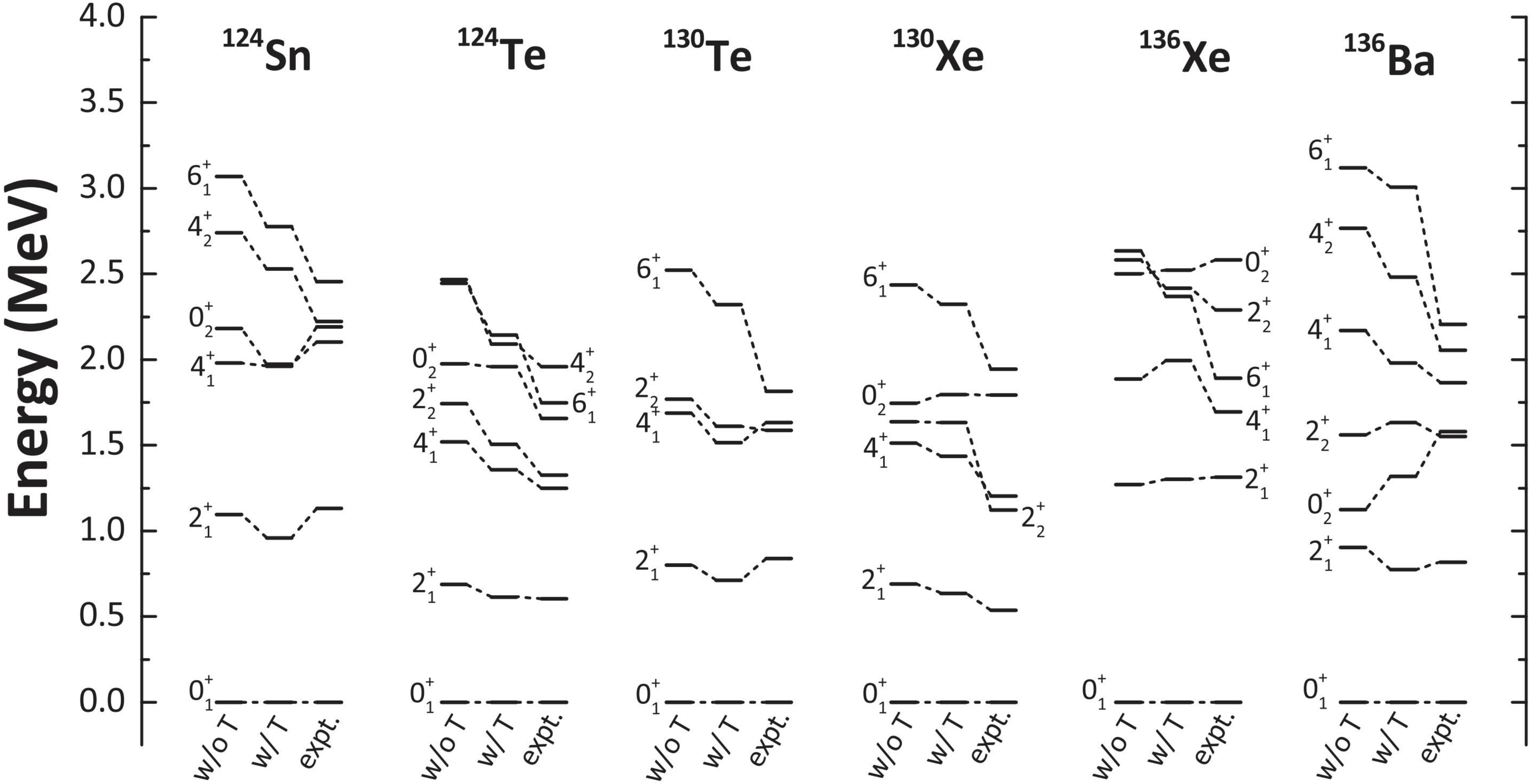
We can investigate the effect by including or excluding the tensor term V_T in V_{MU}

$$V_C(\text{Gauss}) = \sum_{T=0,1.S=0,1} f_{T,S} P_{T,S} exp(-\left(\frac{r}{\mu}\right)^2)$$

$$V_{MU} = V_C(\text{Gauss}) + V_T(\pi + \rho)$$

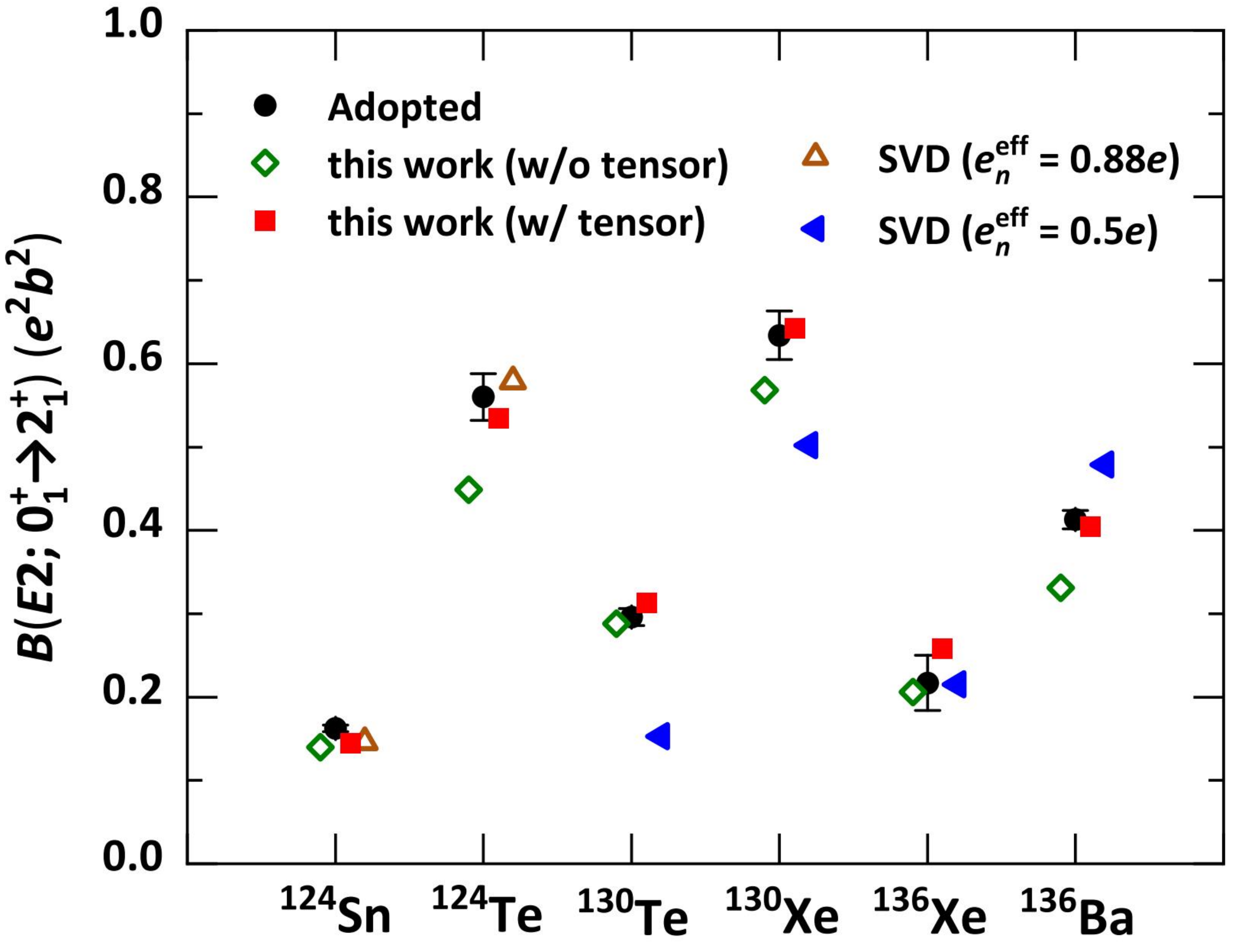
$$V_{MUC} = V_{MU} + V_{LS}(\text{M3Y})$$

Low-lying spectra given by PGCM



CFJ and C. X. Yuan, in revision

Nuclear structure properties and calculated 0νββ NMEs



V_{MU} provides a better description of nuclear structure properties of ^{124}Sn/^{124}Te, ^{130}Te/^{130}Xe, and ^{136}Xe/^{136}Ba

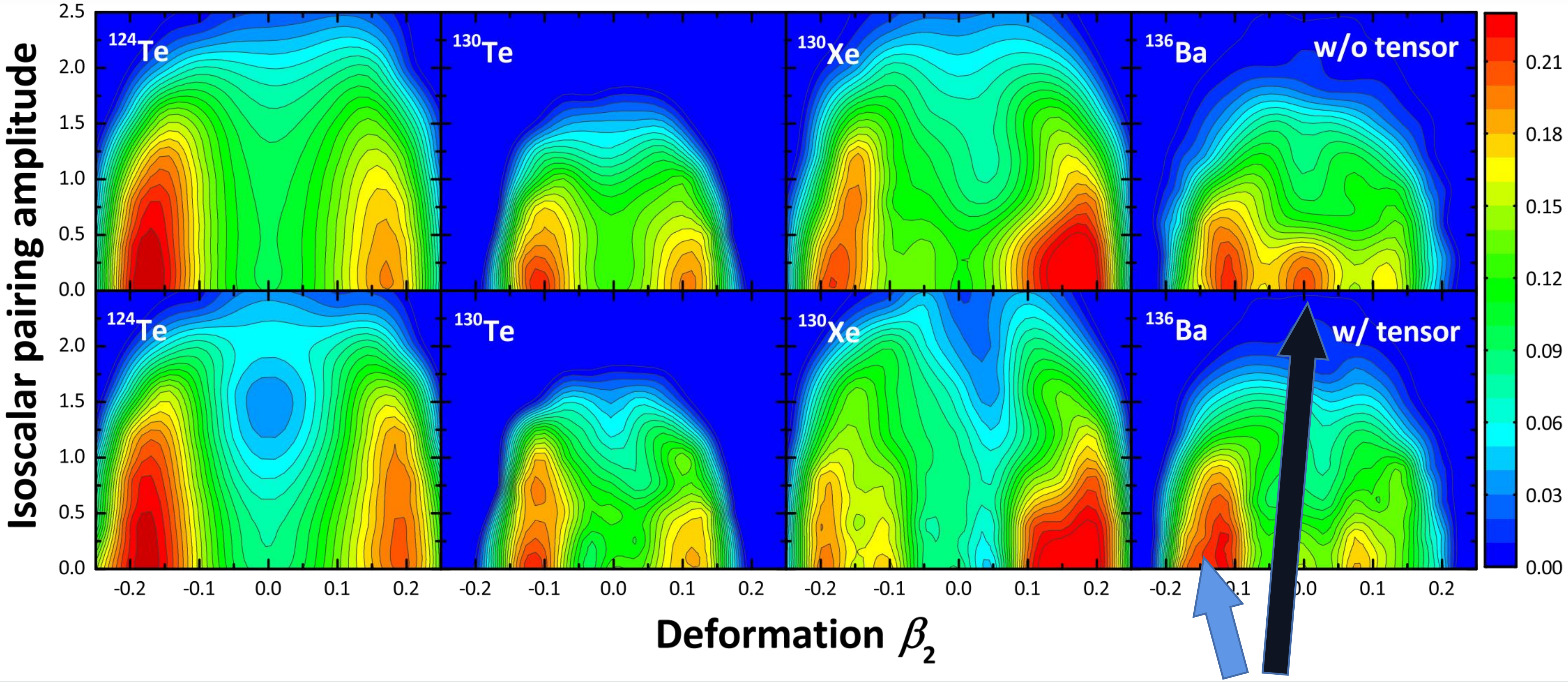
How about 0νββ NMEs?

		$M_{GT}^{0\nu}$	$M_{F}^{0\nu}$	$M_{T}^{0\nu}$	$M^{0\nu}$
^{124}Sn	w/o tensor	3.56	−0.64	−0.061	3.91
	w/ tensor	2.65	−0.64	−0.020	3.04
^{130}Te	w/o tensor	4.29	−0.75	−0.064	4.70
	w/ tensor	3.33	−0.65	−0.015	3.73
^{136}Xe	w/o tensor	3.26	−0.44	−0.046	3.49
	w/ tensor	2.17	−0.50	−0.009	2.48

CFJ and C. X. Yuan, in revision

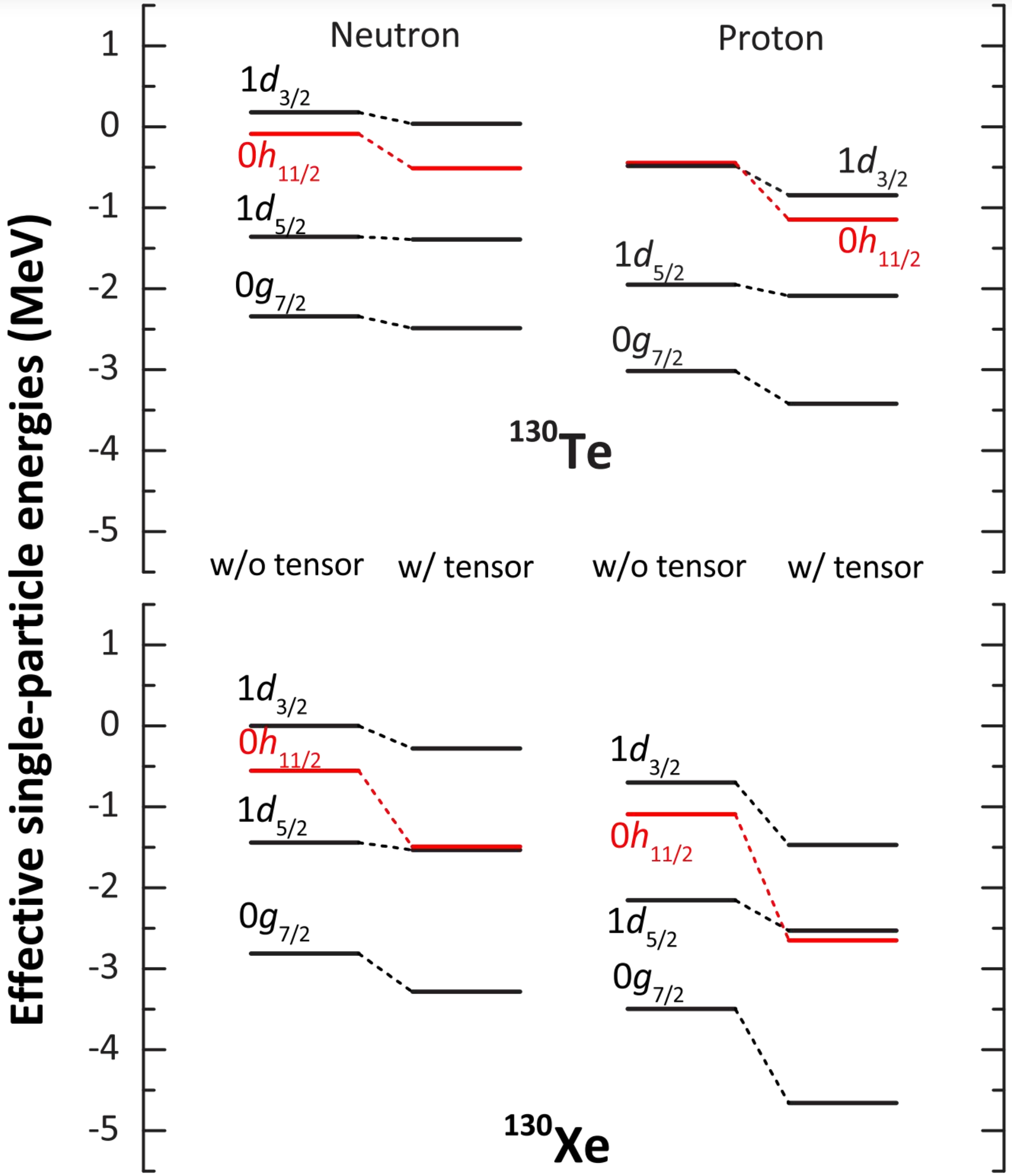
NMEs are suppressed, ***why?***

Effect from tensor force on axial deformation



- Enhanced quadrupole deformation, especially in daughter nuclei.
- Enhanced isoscalar pairing: suppression of NMEs.

Effective single-particle energies: change of shell structure



- The neutron and proton $0h_{11/2}$ orbits are shifted most significantly.
- Suppressions are more drastically in daughter nuclei.
  - Attraction between $\pi 0g_{7/2}$ and $\nu 0h_{11/2}$

    More $0g_{7/2}$ protons in daughter nuclei.
  - Repulsion between $\pi 0h_{11/2}$ and $\nu 0h_{11/2}$
    Repulsion between $\pi 0h_{11/2}$ and $\nu 1d_{5/2}$

    Less $1d_{5/2}$ and $0gh_{11/2}$ neutrons in daughter nuclei.
- Both proton and neutron Fermi surface get close to $0h_{11/2}$, **more deformation-driving effects occur in daughter nuclei.**

Change of the g.s. nucleon occupancies: *a constraint on the NMEs*

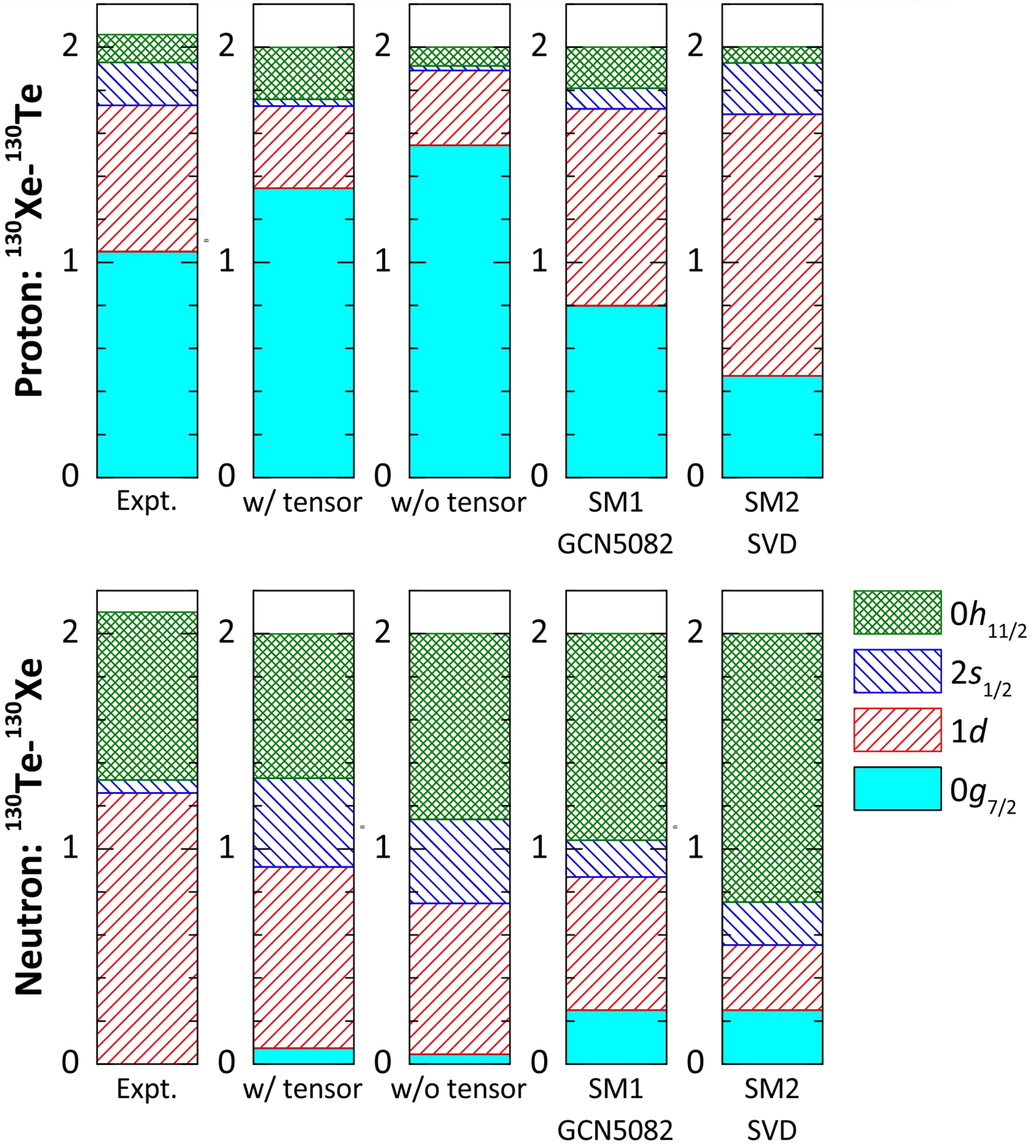
Why?

It directly determines which neutrons decay and which protons are created in the decay, and how their configurations are re-arranged.

Our calculation reproduces qualitatively the two most important contributions.

Inclusion of tensor force improves the description of the change of the nucleon occupancies.

CFJ and C. X. Yuan, in revision



Summary

- $0\nu\beta\beta$ decay is crucial for determining whether neutrinos are Majorana fermion.
- Hamiltonian-based GCM enables treatment of systems currently **unreachable** by other methods. It can be used to evaluate the effect from aspects of nuclear structure on $0\nu\beta\beta$ NME calculations.
- The tensor force may change the shell structure, enhancing the deformation difference between parent and daughter nuclei and isoscalar pairing, and hence suppress the $0\nu\beta\beta$ NMEs.

Next Steps from Here…

- Improvement of GCM: more correlations, QRPA-evolved basis.
- Effective Hamiltonian in a larger space from *ab initio* non-perturbative method.
  - **Target nuclei: ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{150}Nd…**

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Thanks for your attention!