玻色子代数模型中的核形变及演化

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Outline

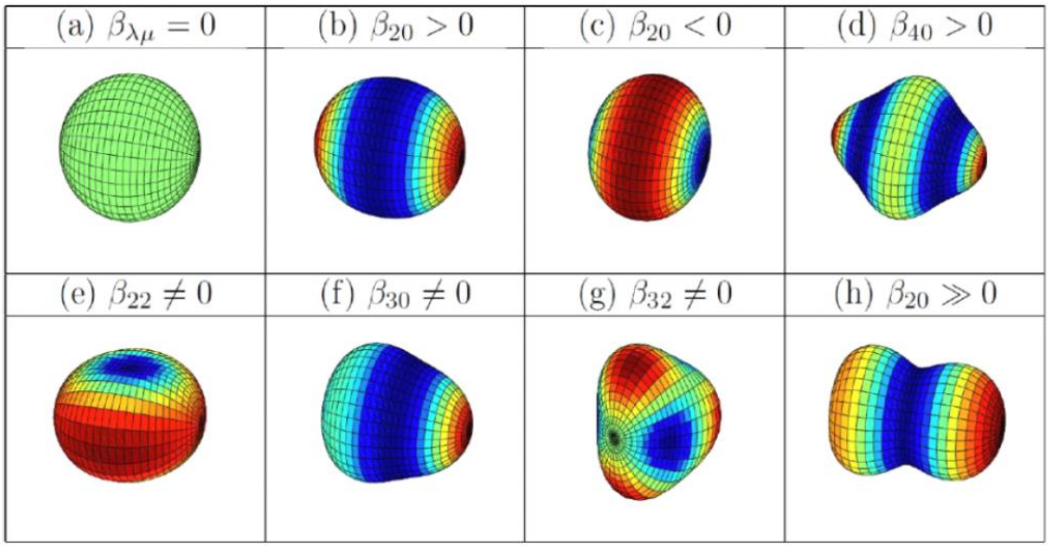
1. Model background

2. Shape relevant stories
   (I) Shape phase transitions
   (I) “Symmetry” description of nuclei without stable shape
   (II) “Anomalous” in triaxial nuclei

3 Summary

Nuclear Shapes (Deformation)

$$R = R_0[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-1}^{\lambda} \beta_{\lambda\mu}^{*} Y_{\lambda\mu}(\theta,\varphi)]$$



A) Collective Model

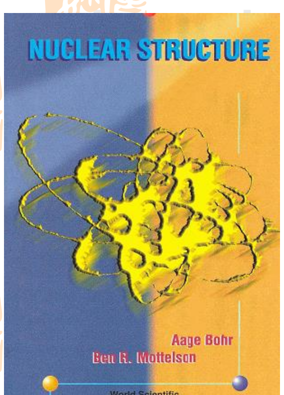
B) Shell Model

C) Algebraic Model (IBM)

D) Mean-field Method

+Projection、Cranking、Mapping, …

Thanks to BN Lv



A common feature of systems that have rotational spectra is the existence of a "deformation", by which is implied a feature of anisotropy that makes it possible to specify an orientation of the system as a whole. In a molecule, as in a solid body, the deformation reflects the highly anisotropic mass distribution, as viewed from the intrinsic coordinate frame defined by the equilibrium positions of the nuclei. In the nucleus, the rotational degrees of freedom are associated with the deformations in the nuclear equilibrium shape that result from the shell structure. (Evidence for

SCI models for low-energy structures of nuclei

Shell Model
(Microscopic,NN interaction)
M.G. Mayer, J.H.D. Jenson
(1963 Nobel Prize)

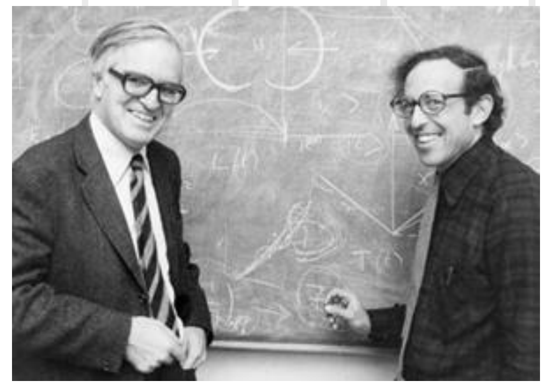
$$H = H_0 - g_0 S^\dagger S - g_2 P^\dagger \cdot P - \kappa Q^{(2)} \cdot Q^{(2)}$$

Mapping

Mean Field

Collective Model
(Global, Geometry)
A. Bohr, B. R. Mottelson,
(1975 Nobel Prize)

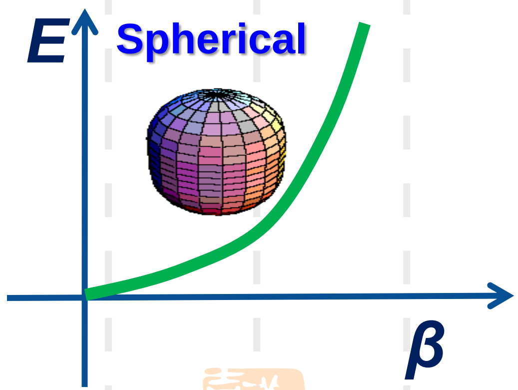
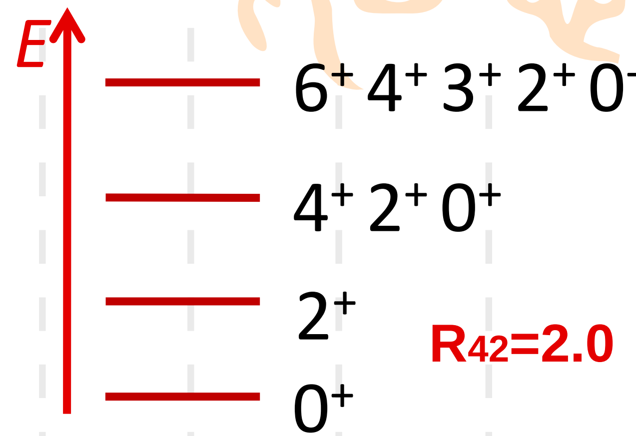
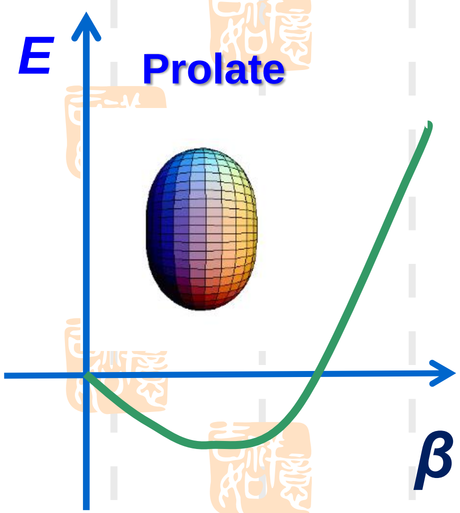
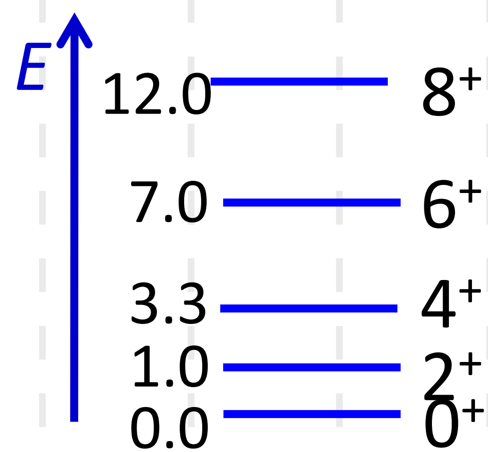
$$H_B = T_{vib} + T_{rot} + V(\beta, \gamma)$$



Interacting Boson Model
(Symmetry)
A. Arima, F. Iachello
(1984,1995, Nobel Prize Nomination)

$$H = E_0 + \sum_{\alpha\beta} \varepsilon_{\alpha\beta} b^\dagger_\alpha b_\beta + \sum_{\alpha\beta\gamma\delta} \tfrac{1}{2} u_{\alpha\beta\gamma\delta} b^\dagger_\alpha b^\dagger_\beta b_\gamma b_\delta + \cdots.$$

Model Language

Collective	Shell model	Symmetry	Spectrum
	Pairing Dominant	U(5)	
	QQ Dominant	SU(3)	

$R_{42}=3.3$

Simple model for complicate nuclear structure

interacting boson model (IBM)

$$\hat{H}(\eta,\chi) = \varepsilon_0\left[(1-\eta)\hat{n}_d - \frac{\eta}{4N}\hat{Q}^\chi \cdot \hat{Q}^\chi\right]$$

U(5)+SU(3)+O(6)

$$\hat{Q}^\chi = (d^\dagger s + s^\dagger \tilde{d})^{(2)} + \chi(d^\dagger \tilde{d})^{(2)}$$

Quadrupole moment

$$\hat{n}_d = d^\dagger \cdot \tilde{d} = N - s^\dagger s$$

S-Pair dominant

$$|N;g\rangle = \frac{1}{\sqrt{N!}}(B_g^\dagger)^N|0\rangle$$

$$B_g^\dagger = \frac{1}{\sqrt{1+\beta^2}}\left[s^\dagger + \beta\cos\gamma\, d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma(d_{-2}^\dagger + d_{+2}^\dagger)\right]$$

ANNALS OF PHYSICS 191, 143–162 (1989)

BCS and IBM

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Received December 5, 1988

Arima and Iachello, 1976

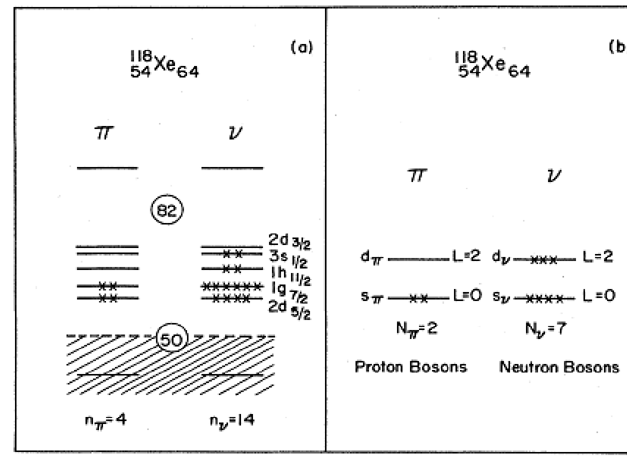


FIG. 2. (a) Schematic representation of the shell-model problem for $^{118}_{54}Xe_{64}$; (b) the boson problem, which replaces the shell-model problem for $^{118}_{54}Xe_{64}$. Both in part (a) and in part (b) the nucleons (a) or bosons (b) can be arranged in all possible ways consistent with the single-particle levels and Fermi (a) or Bose (b) statistics. Of all these possible ways only one is shown in the figure.

Potential

$$V(\beta,\gamma) \equiv \langle N;g|\hat{H}(\eta,\chi)|N;g\rangle$$

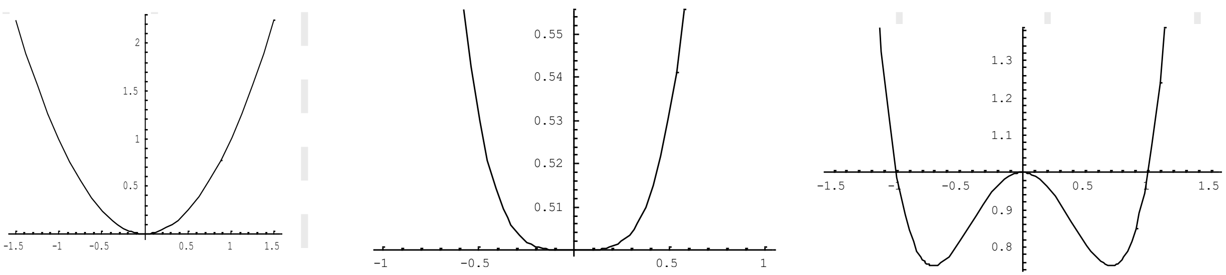
$$= \frac{\varepsilon_0 N\beta^2}{1+\beta^2}\left[(1-\eta) - (\chi^2+1)\frac{\eta}{4N}\right] - \frac{5\varepsilon_0\eta}{4(1+\beta^2)} - \frac{\varepsilon_0\eta(N-1)}{4(1+\beta^2)^2}\left[4\beta^2 - 4\sqrt{\frac{2}{7}}\chi\beta^3\cos3\gamma + \frac{2}{7}\chi^2\beta^4\right]$$

Modeling nuclear deformation and its evolution

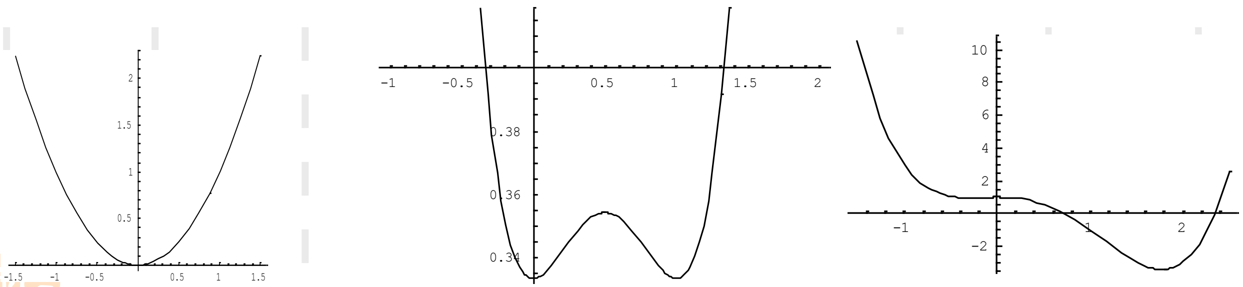
Story I

Shape Phase Transition

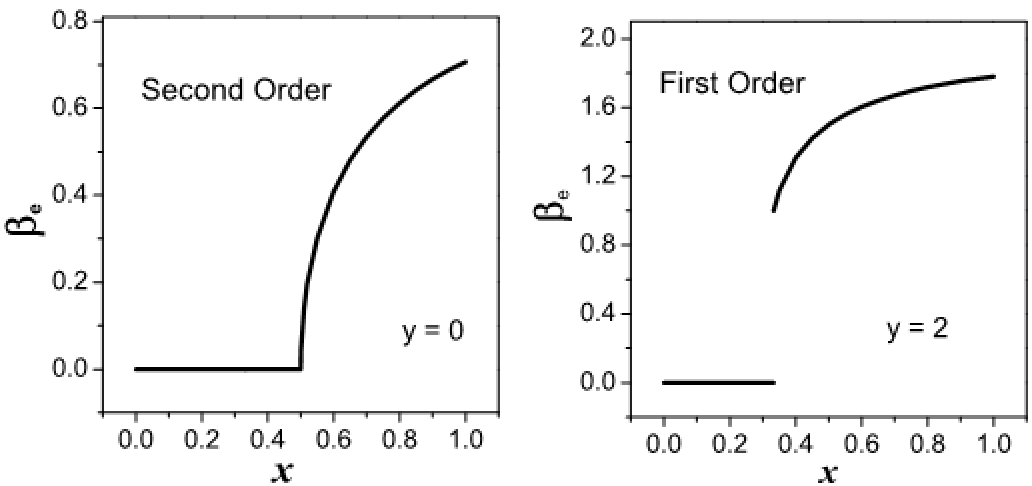
-A way of nuclear shape evolution



$y = 0$



$y = 2$



$$V(\beta) = \beta^2 + x[(1-\beta^2)^2 - y\beta^3]$$

$$E_g = V(\beta)_{\min}$$

Shape phase diagram in IBM

VOLUME 89, NUMBER 18 PHYSICAL REVIEW LETTERS 28 OCTOBER 2002

Triple Point of Nuclear Deformations

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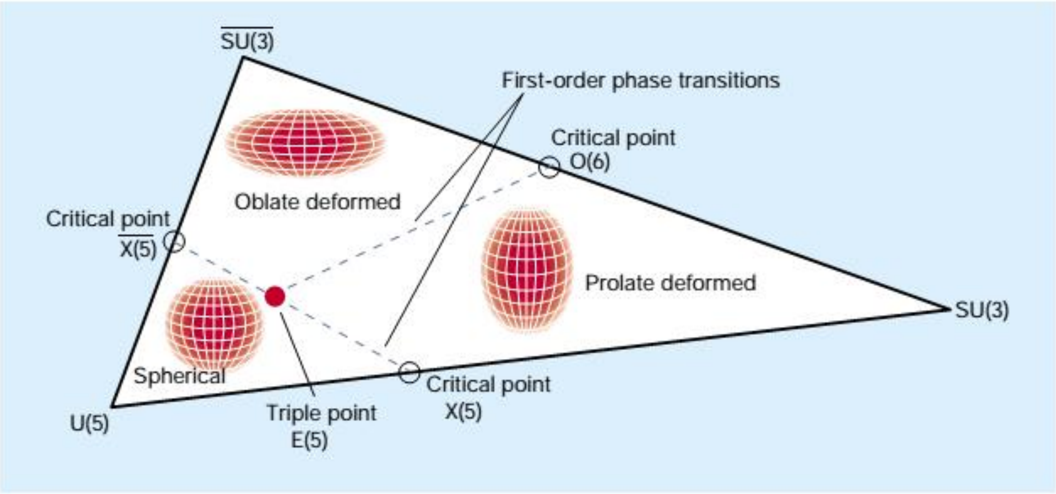
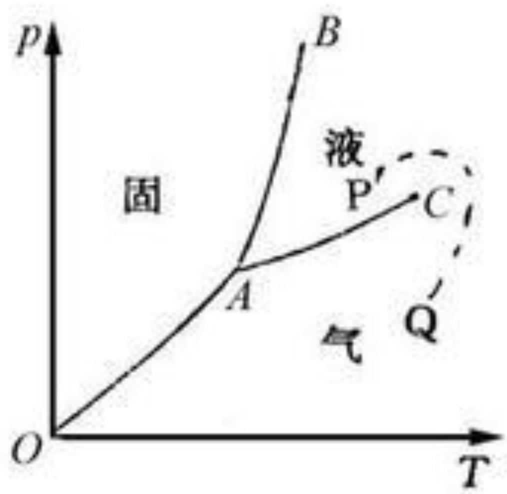




Warner, Nature, 420, (2002)

10th International workshop on
Quantum Phase Transitions in Nuclei and Many-Body Systems
Dubrovnik, Croatia, 29 June - 4 July 2020

REVIEWS OF MODERN PHYSICS, VOLUME 82, JULY–SEPTEMBER 2010

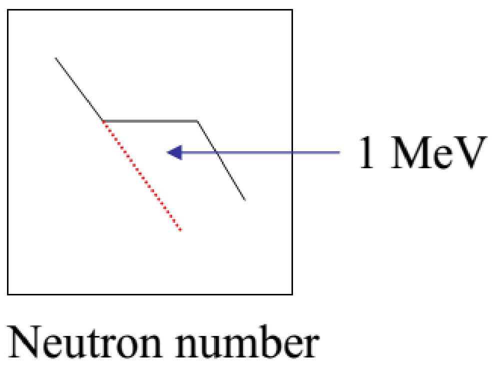
Quantum phase transitions in the shapes of atomic nuclei

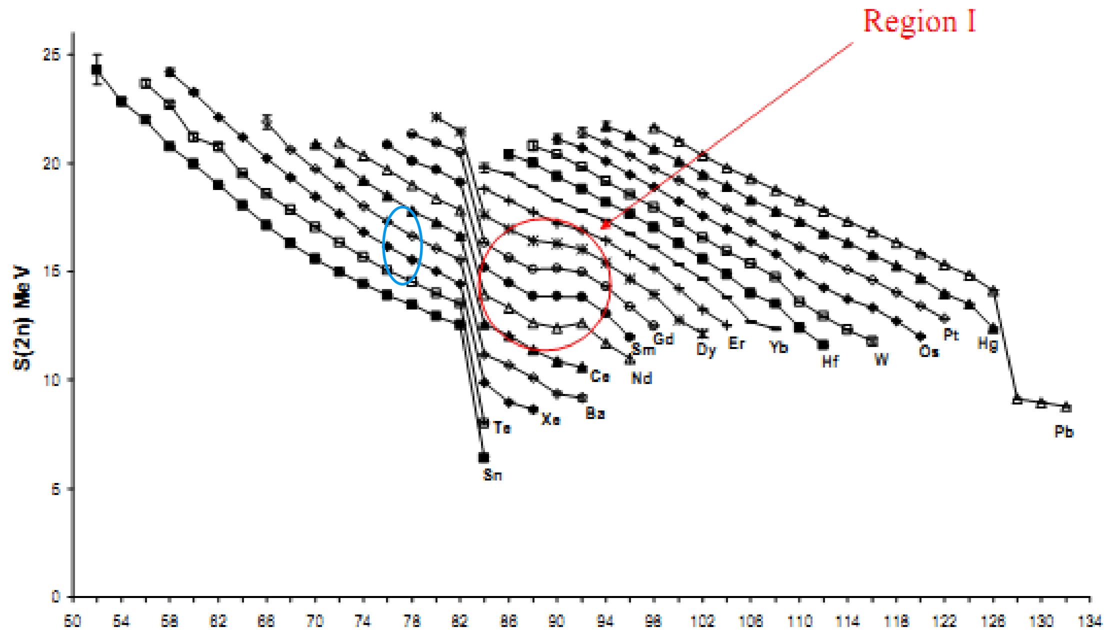
How to “see” SPT

$$S_{2n}(N) = E_B(N+1) - E_B(N)$$

1st order

$$S_{2n} \propto \frac{\partial E}{\partial g}$$



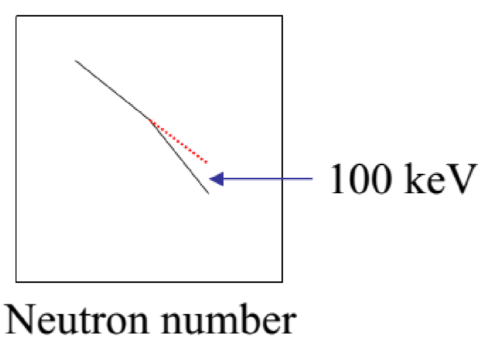


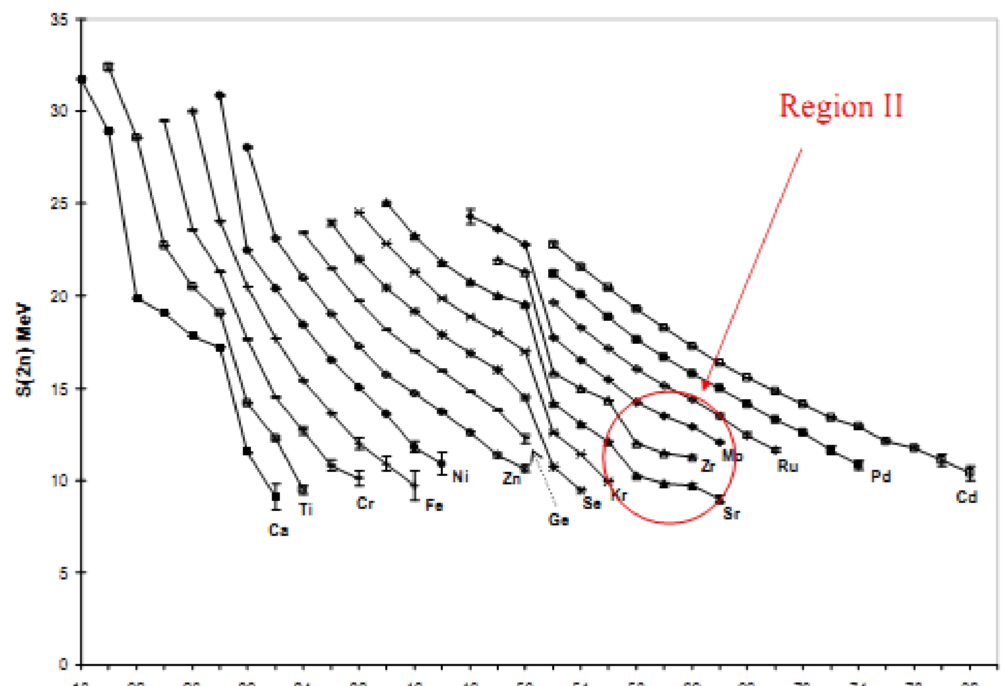
Region I: Nd-Sm-Gd-Dy

Region II: Zr-Sr

2nd order

$$S_{2n} \propto \frac{\partial E}{\partial g}$$





SPT in 2n transfer reaction

$${}^{A}_{Z}X_{N} + 2\mathrm{n} \leftrightarrow {}^{A+2}_{Z}X_{\overline{N}}$$

$$P^{(0)}_{+,\nu,0} = t_{a_\nu} s^\dagger A(\Omega_\nu, N_\nu), \quad P^{(0)}_{-,\nu,0} = t_{a_\nu} A(\Omega_\nu, N_\nu) s, \qquad (1)$$

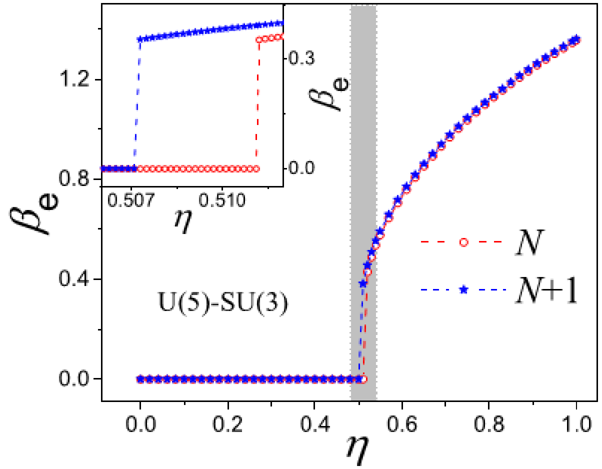
$$P^{(2)}_{+,\nu,\mu} = t_{b_\nu} d^\dagger_\mu A(\Omega_\nu, N_\nu), \quad P^{(2)}_{-,\nu,\mu} = t_{b_\nu} A(\Omega_\nu, N_\nu) \tilde{d}_\mu \qquad (2)$$

$$I^a(N+1, L' \to N, L) = \frac{1}{2L'+1} |\langle N, L \| P_- \| N+1, L' \rangle|^2$$

$$A(\Omega_\nu, N_\nu) = \left(\Omega_\nu - N_\nu - \frac{N_\nu}{N}\hat{n}_d\right)^{\frac{1}{2}} \left(\frac{N_\nu + 1}{N+1}\right)^{\frac{1}{2}}$$

$$|N; g\rangle = \frac{1}{\sqrt{N!}} (B^\dagger_g)^N |0\rangle$$

$$B^\dagger_g = \frac{1}{\sqrt{1+\beta^2}} \left[s^\dagger + \beta \cos\gamma d^\dagger_0 + \frac{1}{\sqrt{2}} \beta \sin\gamma (d^\dagger_{-2} + d^\dagger_{+2}) \right]$$



$$\langle N; g | s | N+1; g' \rangle = \langle N+1; g' | s^\dagger | N; g \rangle = \frac{\sqrt{N+1}}{\sqrt{1+\beta'^2}} \left[\frac{1 + \beta\beta' \cos(\gamma - \gamma')}{\sqrt{(1+\beta'^2)(1+\beta^2)}} \right]^N,$$

$$\langle N; g | d_\mu | N+1; g' \rangle = \langle N+1; g' | d^\dagger_\mu | N; g \rangle = \frac{\sqrt{N+1}}{\sqrt{1+\beta'^2}} \left[\frac{1 + \beta\beta' \cos(\gamma - \gamma')}{\sqrt{(1+\beta'^2)(1+\beta^2)}} \right]^N \times \left[\beta' \cos\gamma' \delta_{\mu,0} + \frac{1}{\sqrt{2}} \beta' \sin\gamma' (\delta_{\mu,2} + \delta_{\mu,-2}) \right].$$

ground to ground

Intrinsic matrix elements of boson operators

ZY, Iachello, PRC 95 (2017) 034306

$$B^\dagger_{\beta'_v} = \frac{1}{\sqrt{1+\beta'^2}}\Big[-\beta' s^\dagger + \cos\gamma' d^\dagger_0 + \frac{1}{\sqrt{2}}\sin\gamma'(d^\dagger_{-2} + d^\dagger_{+2})\Big],$$

$$|N+1; g'\rangle = \frac{1}{\sqrt{(N+1)!}}(B^\dagger_{g'})^{N+1}|0\rangle$$

$$|N+1; \beta'_v\rangle = \frac{1}{\sqrt{(N+1)}}(B^\dagger_{\beta'_v})B_{g'}|N+1; g'\rangle$$

$$|N+1; \gamma'_v\rangle = \frac{1}{\sqrt{(N+1)}}(B^\dagger_{\gamma'_v})B_{g'}|N+1; g'\rangle$$

$$|N+1; 2\beta'_v\rangle = \frac{1}{\sqrt{2(N+1)N}}(B^\dagger_{\beta'_v t})^2(B_{g'})^2|N+1; g'\rangle.$$

$$B^\dagger_{g'} = \frac{1}{\sqrt{1+\beta'^2}}\Big[s^\dagger + \beta'\cos\gamma' d^\dagger_0 + \frac{1}{\sqrt{2}}\beta'\sin\gamma'(d^\dagger_{-2} + d^\dagger_{+2})\Big].$$

$$B^\dagger_{\gamma'_v} = \frac{1}{\sqrt{2}}\cos\gamma'(d^\dagger_{+2} + d^\dagger_{-2}) - \sin\gamma' d^\dagger_0.$$

ground to excited

For the (t,p) transfer reaction between ground bands and excited (e) bands, one can find

$$\langle N+1;\beta'_v|s^\dagger|N;g\rangle = [N\beta\cos(\gamma-\gamma') - (N+1)\beta' - \beta\beta'^2\cos(\gamma-\gamma')]\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{(\sqrt{1+\beta^2})^N}\left(\frac{1}{\sqrt{1+\beta'^2}}\right)$$

$$\langle N+1;\beta'_v|d^\dagger_\mu|N;g\rangle = \frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{(\sqrt{1+\beta^2})^N}\left(\frac{1}{\sqrt{1+\beta'^2}}\right)^{N+1}\Big\{[N\beta\beta'\cos\gamma\cos\gamma' - N\beta'^2 + 1 + \beta\beta'\cos(\gamma -$$

$$\times\left[\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\sin\gamma'(\delta_{\mu,2}+\delta_{\mu,-2})\right] + N\beta\beta'\sin\gamma\sin\gamma'\cos\gamma'\Big\},$$

$$\langle N+1;\gamma'_v|s^\dagger|N;g\rangle = N\beta\sin(\gamma-\gamma')\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{[\sqrt{(1+\beta^2)(1+\beta'^2)}]^N},$$

$$\langle N+1;\gamma'_v|d^\dagger_\mu|N;g\rangle = \left[\frac{1+\beta\beta'\cos(\gamma-\gamma')}{\sqrt{(1+\beta^2)(1+\beta'^2)}}\right]^N\left[\frac{\cos\gamma'}{\sqrt{2}}(\delta_{\mu,2}+\delta_{\mu,-2}) - \sin\gamma'\delta_{\mu,0} + N\beta\sin(\gamma-\gamma')\frac{\beta'\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\beta'\sin\gamma'(\delta_{\mu,-2}+\delta_{\mu,2})}{1+\beta\beta'\cos(\gamma-\gamma')}\right],$$

$$\langle N+1;2\beta'_v|s^\dagger|N;g\rangle = \sqrt{\frac{N}{2}}[\beta\cos(\gamma-\gamma')-\beta']\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-2}}{\sqrt{(1+\beta^2)^N(1+\beta'^2)^{N+1}}} \times\{(N-1)[\beta\cos(\gamma-\gamma')-\beta'] - 2\beta'[1+\beta\beta'\cos(\gamma-\gamma')]\},$$

$$\langle N+1;2\beta'_v|d^\dagger_\mu|N;g\rangle = \sqrt{\frac{N}{2}}[\beta\cos(\gamma-\gamma')-\beta']\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-2}}{\sqrt{(1+\beta^2)^N(1+\beta'^2)^{N+1}}}\left[\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\sin\gamma'(\delta_{\mu,2}+\delta_{\mu,-2})\right] \times\{2[1+\beta\beta'\cos(\gamma-\gamma')] + (N-1)\beta'[\beta\cos(\gamma-\gamma')-\beta']\}.$$

(C) $\phi'_g(N+1) \to \phi_e(N)$

For the (p,t) transfer reaction between ground bands and excited bands, one can find

$$\langle N;\beta_v|s|N+1;g'\rangle = \frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{(\sqrt{1+\beta^2})^N}\left(\frac{1}{\sqrt{1+\beta'^2}}\right)^{N+1}\sqrt{N(N+1)}[\beta'\cos(\gamma-\gamma')-\beta],$$

$$\langle N;\beta_v|d_\mu|N+1;g'\rangle = \sqrt{N(N+1)}\frac{(1+\beta\beta'\cos(\gamma-\gamma'))^{N-1}}{(\sqrt{1+\beta^2})^N}\left(\frac{1}{\sqrt{1+\beta'^2}}\right)^{N+1}[\beta'\cos(\gamma-\gamma')-\beta] \times\left[\beta'\cos\gamma'\delta_{\mu,0} + \frac{\beta'\sin\gamma'}{\sqrt{2}}(\delta_{\mu,2}+\delta_{\mu,-2})\right],$$

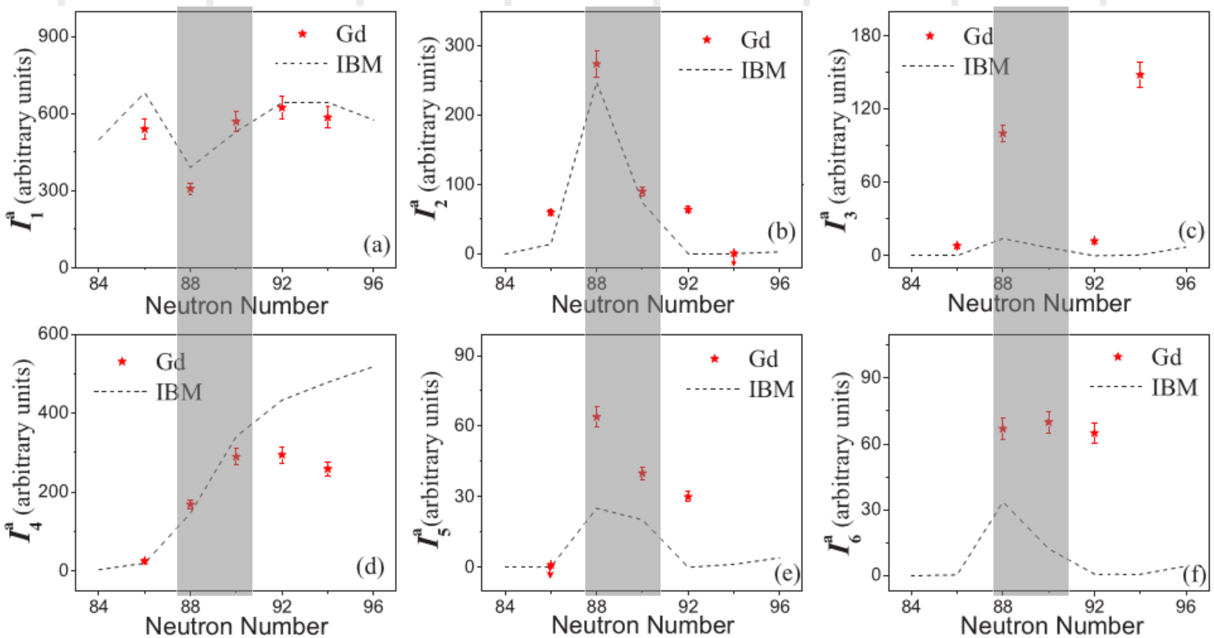
$$\langle N;\gamma_v|s|N+1;g'\rangle = \sqrt{N(N+1)}(\beta'+\beta^2\beta')\sin(\gamma'-\gamma)\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{[\sqrt{(1+\beta^2)(1+\beta'^2)}]^{N+1}},$$

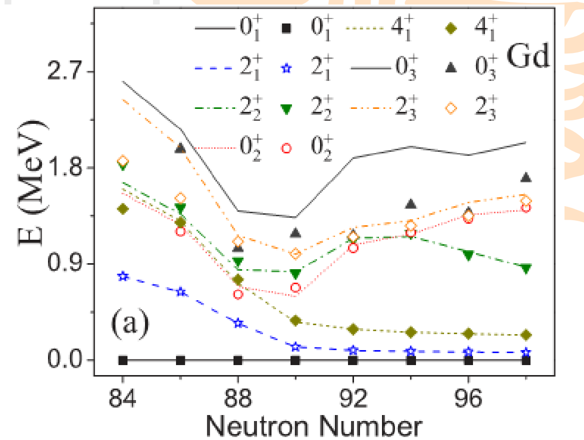
$$\langle N;\gamma_v|d_\mu|N+1;g'\rangle = \sqrt{(N+1)N(1+\beta^2)}\beta'\sin(\gamma'-\gamma)\left[\beta'\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\beta'\sin\gamma'(\delta_{\mu,2}+\delta_{\mu,-2})\right] \times\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-1}}{(\sqrt{1+\beta^2})^N}\frac{1}{(\sqrt{1+\beta'^2})^{N+1}},$$

$$\langle N;2\beta_v|s|N+1;g'\rangle = \sqrt{\frac{(N+1)N(N-1)}{2}}[\beta'\cos(\gamma-\gamma')-\beta]^2\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-2}}{\sqrt{(1+\beta^2)^N(1+\beta'^2)^{N+1}}},$$

$$\langle N;2\beta_v|d_\mu|N+1;g'\rangle = \sqrt{\frac{(N+1)N(N-1)}{2}}[\beta'\cos(\gamma-\gamma')-\beta]^2\Big[\beta'\cos\gamma'\delta_{\mu,0} + \frac{1}{\sqrt{2}}\beta'\sin\gamma'(\delta_{\mu,2}+\delta_{\mu,- \times\frac{[1+\beta\beta'\cos(\gamma-\gamma')]^{N-2}}{\sqrt{(1+\beta^2)^N(1+\beta'^2)^{N+1}}}.$$

SPT in (p, t) reaction for Gd





$$I^a_1 = I(N+1, 0^+_1 \rightarrow N, 0^+_1),$$

$$I^a_2 = I(N+1, 0^+_1 \rightarrow N, 0^+_2),$$

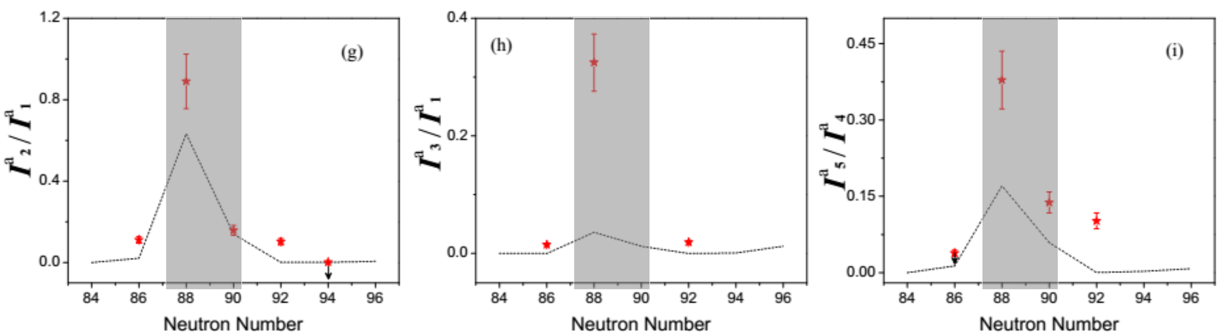
$$I^a_3 = I(N+1, 0^+_1 \rightarrow N, 0^+_3),$$

$$I^a_4 = I(N+1, 0^+_1 \rightarrow N, 2^+_1),$$

$$I^a_5 = I(N+1, 0^+_1 \rightarrow N, 2^+_2),$$

$$I^a_6 = I(N+1, 0^+_1 \rightarrow N, 2^+_3),$$

for (p, t) reactions and



$$p + {}^A_Z X_N \rightarrow {}^3_1 H_2 + {}^{A-2}_{Z} X_{N-2}$$

YZ, Iachello, PRC 95 (2017) 034306
Nomura YZ, PRC 99 (2019) 024324

Story II

Critical Point Symmetry

--A concept for "soft" nuclei (no stable shape)

VOLUME 85, NUMBER 17 PHYSICAL REVIEW LETTERS 23 OCTOBER 2000

Dynamic Symmetries at the Critical Point

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(Received 8 May 2000)

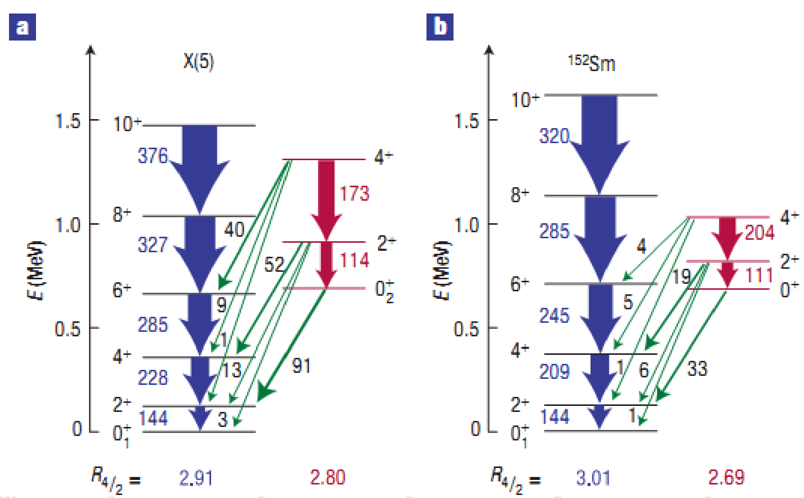
$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_\kappa \frac{Q_\kappa^2}{\sin^2(\gamma - \frac{2}{3}\pi\kappa)}\right] + V(\beta,\gamma).$$

VOLUME 87, NUMBER 5 PHYSICAL REVIEW LETTERS 30 JULY 2001

Analytic Description of Critical Point Nuclei in a Spherical-Axially Deformed Shape Phase Transition

F. Iachello

REVIEW ARTICLE **R. F. Casten**



Bohr hamiltonian. As noted above, these CPS are therefore not 'symmetries' in the strict sense of a group theoretical description as in U(5), SU(3) or O(6). They are rather solutions to a nuclear problem expressed geometrically.

A few years ago, Iachello introduced[4,5] the concept of critical-point symmetries (CPSs) to describe nuclei at the phase-transitional point. These are not dynamical symmetries with group theoretical roots, as in the IBA, but geometrical descriptions based on potentials with very simple shapes. The essence of the idea is seen in Fig. 3b. Curve 3, which represents the potential for a nucleus poised at the transition point between shapes with clear

nature physics | VOL 2 | DECEMBER 2006 |

Is CPS a group symmetry?

IBM

Arima and Iachello, 1976

$$U(6) \supset U(5) \supset SO(5) \supset SO(3),$$
$$U(6) \supset O(6) \supset SO(5) \supset SO(3),$$
$$U(6) \supset SU(3) \supset SO(3).$$

$$(d^\dagger \times \tilde{d})^{(k)}_m = \sum_{u,u'} < 2u2u'|km > d^\dagger_u \tilde{d}_{u'}, \quad k = 0,\ 1,\ 2,\ 3,\ 4$$

$$d^\dagger_u s, \quad s^\dagger \tilde{d}_u, \quad s^\dagger s$$

Eu(5)

YZ, *et al*, PRC, 90(2014); PLB,732 (2014); Symmetry (2022), 14,2219

$$\text{Eu}(5) \supset \text{SO}(5) \supset \text{SO}(3).$$

$$\text{Eu}(5) \supset \text{T}_5 \oplus_s \text{SO}(3) \supset \text{SO}(3)$$

$$\hat{Q}^{(2)}_u = \frac{1}{\sqrt{2}}[\tilde{d}_u - d^\dagger_u],$$
$$\hat{T}^{(\lambda)}_u = \sqrt{2}(d^\dagger \tilde{d})^{(\lambda)}_u, \quad \lambda = 1,3,$$

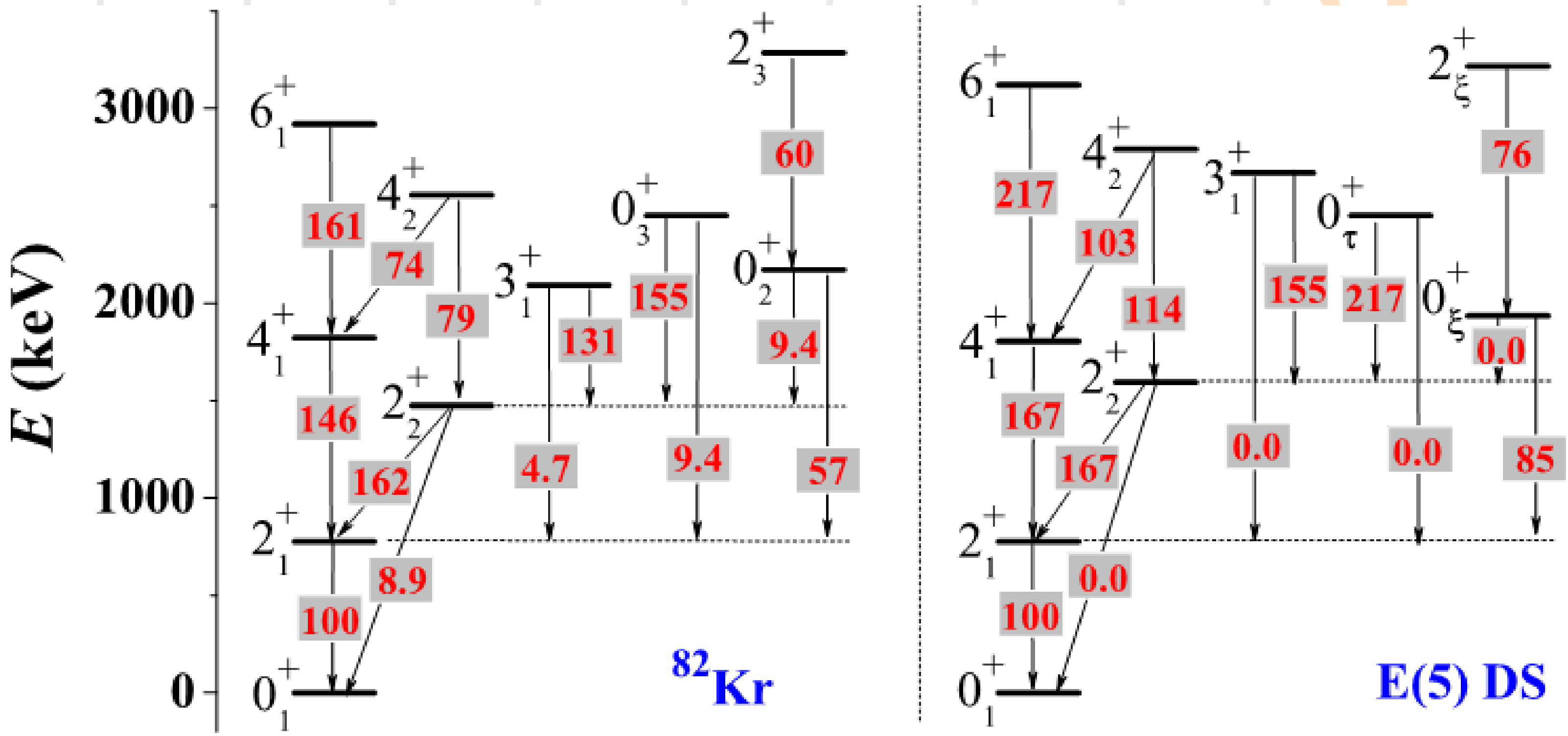
$$[\hat{Q}^{(2)}_u, \hat{Q}^{(2)}_v] = 0,$$
$$[\hat{T}^{(\lambda)}_u, \hat{Q}^{(2)}_v] = -\sqrt{\frac{4\lambda+2}{5}}\langle \lambda u 2v | 2u+v\rangle \hat{Q}^{(2)}_{u+v},$$
$$[\hat{T}^{(\lambda)}_u, \hat{T}^{(\lambda')}_{u'}] = -\sqrt{8(2\lambda+1)(2\lambda'+1)} \sum_{k=\text{odd}} \begin{Bmatrix} \lambda, \lambda', k \\ 2,2,2 \end{Bmatrix} \times \langle \lambda u \lambda' u' | k u+u' \rangle \hat{T}^{(k)}_{u+u'},$$

and

$$[\hat{Q}^{(2)}_u, \hat{C}_2[\text{Eu}(5)]] = [\hat{T}^{(\lambda)}_u, \hat{C}_2[\text{Eu}(5)]] = 0.$$

An example for Euclidean Dynamical Symmetry

$$\hat{H}_{E(5)} = a\,\hat{C}_2[E(5)] + b\,\hat{C}_2[SO(5)] + c\,\hat{C}_2[SO(3)]$$



Symmetry **2022**, *14*, 2219

Story III

Anomalous in Neutron-deficient nuclei

---New feature emerge from triaxial shape

PHYSICAL REVIEW LETTERS **121,** 022502 (2018)

Lifetime Measurements of Excited States in ^{172}Pt and the Variation of Quadrupole Transition Strength with Angular Momentum

B. Cederwall,[1,*] M. Doncel,[2] Ö. Aktas,[1] A. Ertoprak,[1,3] R. Liotta,[1] C. Qi,[1] T. Grahn,[4] D. M Cullen,[5] D. Hodge,[5] M. Giles,[5] S. Stolze,[4] H. Badran,[4] T. Braunroth,[6] T. Calverley,[4] D. M. Cox,[4,†] Y. D. Fang,[7] P. T. Greenlees,[4] J. Hilton,[4] E. Ideguchi,[7] R. Julin,[4] S. Juutinen,[4] M. Kumar Raju,[7] H. Li,[8] H. Liu,[1] S. Matta,[1] V. Modamio,[9] J. Pakarinen,[4] P. Papadakis,[4,‡] J. Partanen,[4] C. M. Petrache,[10] P. Rahkila,[4] P. Ruotsalainen,[4] M. Sandzelius,[4] J. Sarén,[4] C. Scholey,[4] J. Sorri,[4,12,§] P. Subramaniam,[1] M. J. Taylor,[11] J. Uusitalo,[4] and J. J. Valiente-Dobón[12]

TABLE I. Energies of the $2_1^+ \rightarrow 0_{gs}^+$ and $4_1^+ \rightarrow 2_1^+$ transitions (E_γ), deduced lifetime values (τ) for the 2_1^+ and 4_1^+ states, and corresponding reduced transition probabilities [$B(E2\downarrow)_{exp}$] in Weisskopf units (W.u.).

Transition	E_γ (keV)	τ (ps)	$B(E2\downarrow)_{exp}$ (W.u.)
$2_1^+ \rightarrow 0_{gs}^+$	458	15(3)	49(11)
$4_1^+ \rightarrow 2_1^+$	612	6.2(17)	27(7)

$R_{4/2}$=2.34 Normal

$B_{4/2}$=0.55 Abnormal

Normal: collective rotation or vibration!

Can "abnormal" be explained from collective R-V view?

Anomalous $B_{4/2} < 1.0$

PHYSICAL REVIEW C **94**, 044327 (2016)

Excited states and reduced transition probabilities in ^{168}Os

T. Grahn,[1,2,*] S. Stolze,[2] D. T. Joss,[1] R. D. Page,[1] B. Saygı,[1,†] D. O'Donnell,[1] M. Akmali,[1] K. Andgren,[3] L. Bianco,[1] D. M. Cullen,[4] A. Dewald,[5] P. T. Greenlees,[2] K. Heyde,[6] H. Iwasaki,[5] U. Jakobsson,[2] P. Jones,[2] D. S. Judson,[1] R. Julin,[2] S. Juutinen,[2] S. Ketelhut,[2] M. Leino,[2] N. Lumley,[4] P. J. R. Mason,[4,7] O. Möller,[8] K. Nomura,[5,9] M. Nyman,[2] A. Petts,[1] P. Peura,[2] N. Pietralla,[8] Th. Pissulla,[5] P. Rahkila,[2] P. J. Sapple,[1] J. Sarén,[2] C. Scholey,[2] J. Simpson,[7] J. Sorri,[2] P. D. Stevenson,[10] J. Uusitalo,[2] H. V. Watkins,[1] and J. L. Wood[11]

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(Received 19 July 2016; revised manuscript received 2 September 2016; published 28 October 2016)

The level scheme of the neutron-deficient nuclide ^{168}Os has been extended and mean lifetimes of excited states have been measured by the recoil distance Doppler-shift method using the JUROGAM γ-ray spectrometer in conjunction with the IKP Köln plunger device. The ^{168}Os γ rays were measured in delayed coincidence with recoiling fusion-evaporation residues detected at the focal plane of the RITU gas-filled separator. The ratio of reduced transition probabilities $B(E2;4_1^+ \rightarrow 2_1^+)/B(E2;2_1^+ \rightarrow 0_1^+)$ is measured to be 0.34(18), which is very unusual for collective band structures and cannot be reproduced by interacting boson model (IBM-2) calculations based on the SkM* energy-density functional.

PHYSICAL REVIEW C **100**, 034302 (2019)

$B(E2)$ anomalies in the yrast band of ^{170}Os

A. Goasduff,[1,2,*] J. Ljungvall,[1] T. R. Rodríguez,[3] F. L. Bello Garrote,[4] A. Etile,[1] G. Georgiev,[1] F. Giacoppo,[5,6] L. Grente,[7] M. Klintefjord,[4] A. Kuşoğlu,[1,8,9] I. Matea,[10] S. Roccia,[1] M.-D. Salsac,[7] and C. Sotty[1,11]

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Background: The neutron-deficient osmium isotopic chain provides a great laboratory for the study of shape evolution, with the transition from the soft triaxial rotor in ^{168}Os to the well-deformed prolate rotor in ^{180}Os, while shape coexistence appears around $N = 96$ in ^{172}Os. Therefore, the study of the Os isotopic chain should provide a better understanding of shape changes in nuclei and a detailed scrutiny of nuclear structure calculations. In this paper, the lifetimes of the low-lying yrast states of ^{170}Os have been measured for the first time to investigate the shape evolution with neutron number.
Purpose: Lifetimes of excited states in the ground-state band of ^{170}Os are measured to investigate the shape evolution with neutron number in osmium isotopes and compare with state-of-the-art calculations.
Methods: The states of interest were populated via the fusion-evaporation reaction ^{142}Nd(^{32}S, 4n) at a bombarding energy of 170 MeV at the ALTO facility from IPN (Orsay, France). Lifetimes of the 2_1^+ and 4_1^+ states in ^{170}Os were measured with the recoil-distance Doppler-shift method using the Orsay universal plunger system.
Results: Lifetimes of the two first excited states in ^{170}Os were measured for the first time. A very small $B(E2;4_1^+ \rightarrow 2_1^+)/B(E2;2_1^+ \rightarrow 0_1^+) = 0.38(11)$ was found, which is very uncharacteristic for collective nuclei. These results were compared to state-of-the-art beyond-mean-field calculations.
Conclusions: Although theoretical results give satisfactory results for the energy of the first few excited states in ^{170}Os and the $B(E2;2_1^+ \rightarrow 0_1^+)$ they fail to reproduce the very small $B(E2;4_1^+ \rightarrow 2_1^+)$, which remains a puzzle.

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **96**, 021301(R) (2017)

Reduced transition probabilities along the yrast line in ^{166}W

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(Received 7 July 2017; published 2 August 2017)

Lifetimes of excited states in the yrast band of the neutron-deficient nuclide ^{166}W have been measured utilizing the DPUNS plunger device at the target position of the JUROGAM II γ-ray spectrometer in conjunction with the RITU gas-filled separator and the GREAT focal-plane spectrometer. Excited states in ^{166}W were populated in the ^{92}Mo(^{78}Kr,4p) reaction at a bombarding energy of 380 MeV. The measurements reveal a low value for the ratio of reduced transitions probabilities for the lowest-lying transitions $B(E2;\,4^+ \rightarrow 2^+)/B(E2;\,2^+ \rightarrow 0^+) = 0.33(5)$, compared with the expected ratio for an axially deformed rotor ($B_{4/2} = 1.43$).

PHYSICAL REVIEW C **71**, 064324 (2005)

*E*2 transition probabilities in ^{114}Te: A conundrum

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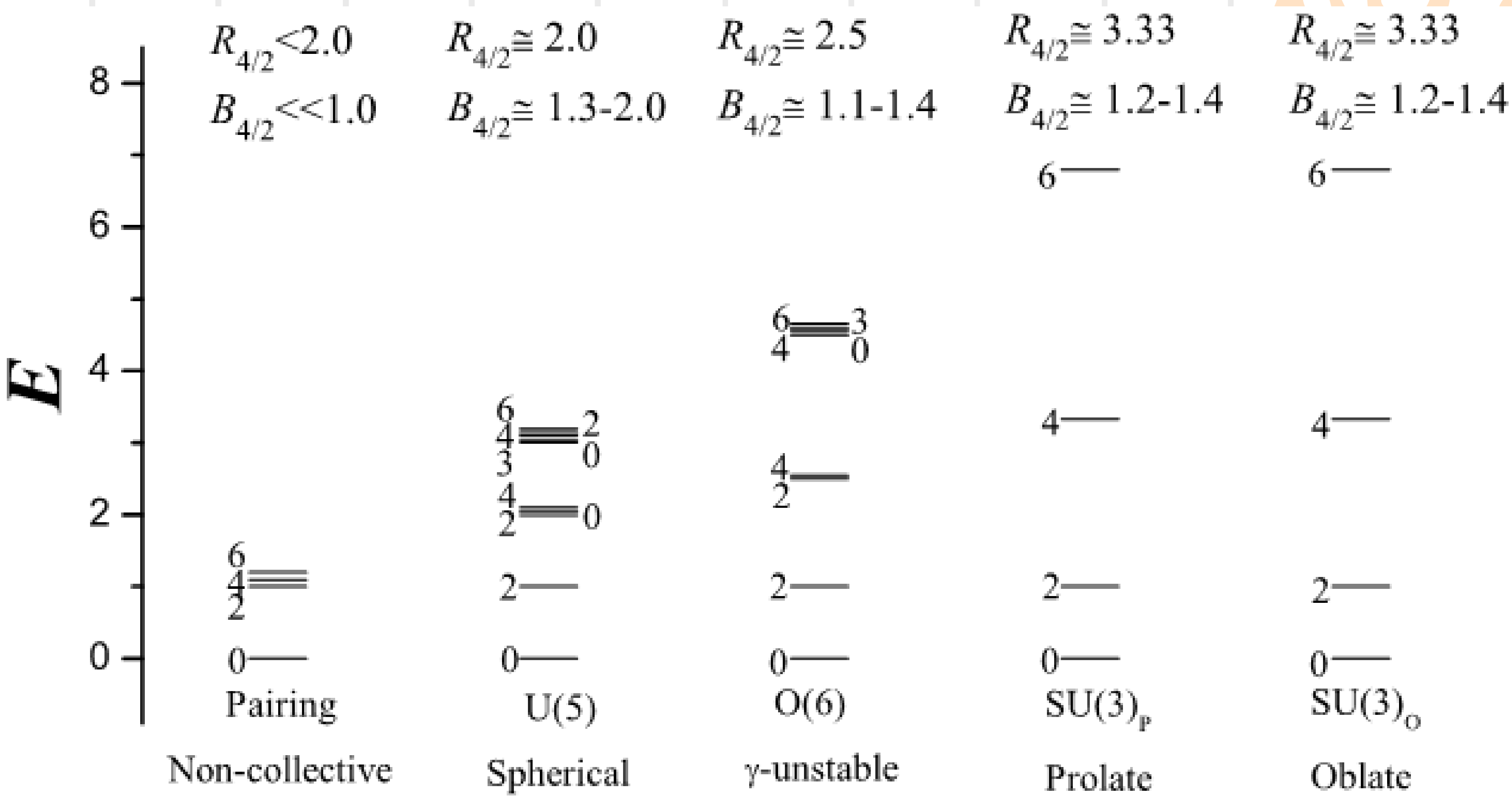
(Received 11 February 2005; published 30 June 2005)

Lifetimes in ^{114}Te were determined using the recoil distance Doppler-shift technique with a plunger device coupled to five HP Ge detectors enhanced by one Euroball cluster detector. The experiment was carried out at the Cologne FN Tandem facility using the ^{93}Nb(^{24}Mg,$p2n$) reaction at 90 MeV. The differential decay curve method in coincidence mode was employed to derive lifetimes for seven excited states, whereas the lifetime of an isomeric state was obtained in singles mode. The resulting $E2$ transition probabilities are shown to be very anomalous in comparison with the vibrational energy spacings of the ground-state band.

No answer to “abnormal” in traditional modes even with finiteness



The finite triaxial mode remains to be tested

Mean-field information

PHYSICAL REVIEW C **81**, 024310 (2010)

Mean field study of structural changes in Pt isotopes with the Gogny interaction

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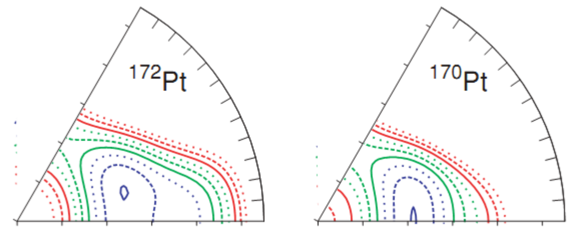
(Received 2 December 2009; published 11 February 2010)

The evolution of the nuclear shapes along the triaxial landscape is studied in the Pt isotopic chain using the self-consistent Hartree-Fock-Bogoliubov approximation based on the Gogny interaction. In addition to the parametrization D1S, the new incarnations D1N and D1M of this force are also included in our analysis to assess to which extent the predictions are independent of details of the effective interaction. The considered range of neutron numbers $88 \leqslant N \leqslant 126$ includes prolate, triaxial, oblate, and spherical ground-state shapes and serves as a detailed comparison of the predictions obtained with the new sets D1N and D1M against the ones provided by the standard parametrization Gogny-D1S in a region of the nuclear landscape for which experimental and theoretical fingerprints of shape transitions have been found. Structural evolution along the Pt chain is discussed in terms of the deformation dependence of single-particle energies.



PHYSICAL REVIEW C **100**, 034302 (2019)

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DOI: 10.1103/PhysRevC.100.034302

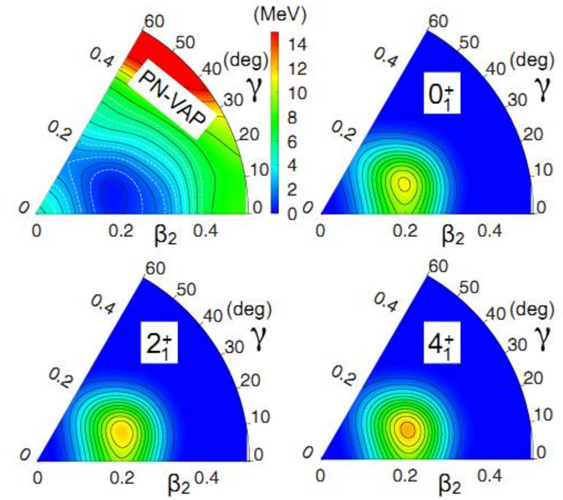
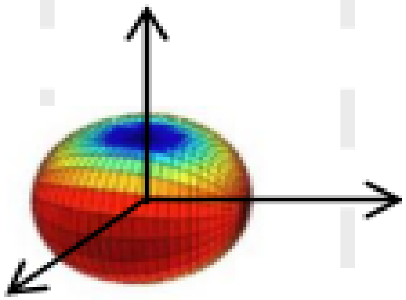


FIG. 7. The PN-VAP potential energy surface for ^{170}Os (top left) and collective wave functions for the ground state (top right) as well as the yrast 2^+ (bottom left) and 4^+ (bottom right) states. All calculated wave functions display a very similar structure.

The SU(3) theory for the rotor mode

In body-fixed principle axis system



Exact

$$H_{\text{rot}} = A_1 L_1^2 + A_2 L_2^2 + A_3 L_3^2$$ **(1) Geometry**

$$L_u = \int \rho(\vec{r})(\vec{r} \times \vec{v})_u d\tau,$$

$$Q_u^c = \sqrt{16\pi/5} \int \rho(\vec{r}) r^2 Y_{2u}(\Omega) d\tau,$$

$$H_{\text{rot}} = aL^2 + bX_3^c + cX_4^c,$$ **(2) Algebraic**

$T_5 \otimes_s \text{SO}(3)$

Mapping

$$[L_u^b, L_v^b] = -\sqrt{2}\langle 1u, 1v | 1u+v \rangle L_{u+v}^b,$$

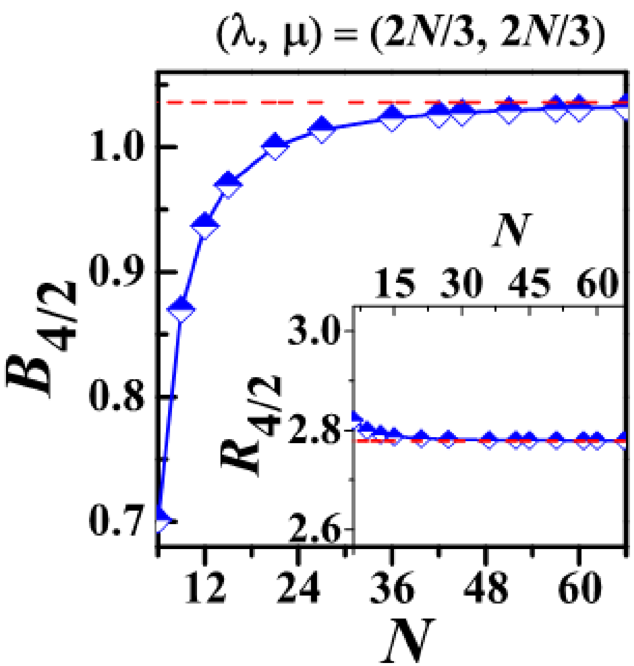
$$[L_u^b, Q_v^b] = -\sqrt{6}\langle 1u, 2v | 2u+v \rangle Q_{u+v}^b,$$

$$[Q_u^b, Q_v^b] = 3\sqrt{10}\langle 2u, 2v | 1u+v \rangle L_{u+v}^b.$$

$$H_{\text{SU}(3)} = aL_b^2 + bX_3^b + cX_4^b$$ **(3)Algebraic**

SU(3)

Deformation vs SU(3) Irreps



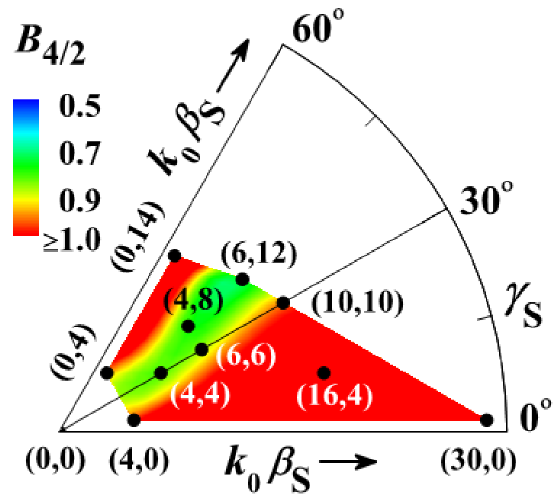


Fig. 3. Landscape of $B_{4/2}$ with $R_{4/2} \in [2.0, 3.33]$. The results are obtained from (5) with the parameters mapping from the triaxial rotor for all given (λ, μ) with $N \leq 15$.

$$\gamma_s = \tan^{-1}\left(\frac{\sqrt{3}(\mu+1)}{2\lambda+\mu+3}\right)$$

$$k_0\beta_s = \sqrt{\lambda^2+\mu^2+\lambda\mu+3\lambda+2\mu+3}$$

Key questions:

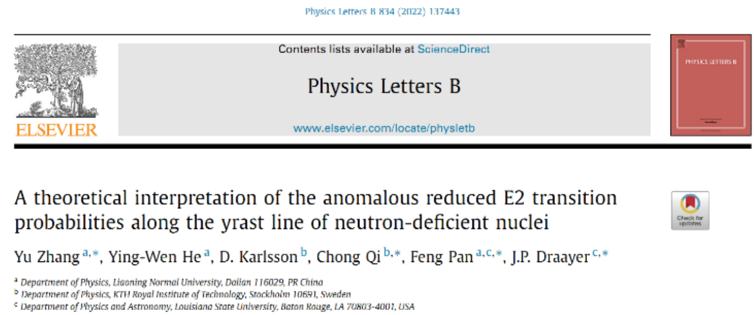
1, How to generate the required intrinsic configuration (λ,μ)?

2, What is the microscopic mechanism?

A test for the *N*=8 system

$$\hat{H} = \hat{H}_{\mathrm{CQ}} + \hat{H}_{\mathrm{Tri}}$$

$$\hat{H}_{\mathrm{CQ}} = \varepsilon \hat{n}_d + \kappa \frac{1}{N} \hat{Q}^{\chi} \cdot \hat{Q}^{\chi}$$



Physics Letters B 834 (2022) 137443

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Physics Letters B

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A theoretical interpretation of the anomalous reduced E2 transition probabilities along the yrast line of neutron-deficient nuclei

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$$(\lambda,\mu) = (16,0),(12,2),(8,4),(4,6),(0,8)$$
$$(10,0),(6,2),(2,4),(4,0),(0,2)$$

$\gamma_s = 32°$

$$\gamma_s = \tan^{-1}\left(\frac{\sqrt{3}(\mu+1)}{2\lambda+\mu+3}\right)$$

TABLE II: Rotor represents the results directly solved from the triaxial rotor Hamiltonian with the inertial parameters $A_1 : A_2 : A_3 = 20 : 1 : 39$; $(4,6)$ represents the results solved from the exact SU(3) mapping from the above triaxial rotor with the SU(3) IRREP fixed as $(\lambda_0,\mu_0) = (4,6)$; IBM_a represents the results solved from IBM Hamiltonian as discussed in the manuscript; IBM_b represents the results solved from the IBM Hamiltonian by further releasing constraints on the parameters on the basis of IBM_a.

	$E(2_1)$	$E(4_1)$	$E(6_1)$	$E(8_1)$	$E(2_2)$	$E(0_2)$	$2_1 \to 0_1$	$4_1 \to 2_1$	$6_1 \to 4_1$	$2_2 \to 0_1$	$0_2 \to 2_1$
Rotor	1	2.33	3.73	5.28	3.42	-	1.0	1.23	1.46	0.08	-
(4,6)	1	2.33	12.55	23.65	3.42	-	1.0	0.595	0.584	1.508	-
IBM_a	1	2.41	4.15	5.57	1.64	0.92	1.0	0.677	0.174	0.005	0.331
IBM_b	1	2.34	3.47	6.13	1.76	1.33	1.0	0.571	0.122	0.216	0.074
$^{172}\mathrm{Pt}$	1	2.34	3.83	6.54	-	-	1.0	0.55(19)	-	-	-

Views on shape at low-energy

1 Shape (deformation) is a central important in low-energy nuclear structural research.

2 For well-deformed nuclei, quadrupole shape can be measured by deformation (β,γ) in a quantitative way?

3 For nuclei near spherical or critical point, "shape" is more a concept guidance to better understand low-energy spectrum.

Thank You!