第一届原子核结构与相对论重离子碰撞前沿交叉研讨会
2023 年 8 月 1-5 日，大连

原子核的形状及相关协变密度泛函理论研究

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中国科学院理论物理研究所-北京航空航天大学“彭桓武科教合作中心”

Supported by: NSFC & MOST;
HPC Cluster of KLFTP/ITP-CAS
ScGrid of CNIC-CAS

内容提要

- 原子核的形状和形变
- 多维形状约束协变密度泛函理论（MDC-CDFT）与原子核的奇特形状
- 形变连续谱相对论Hartree-Bogoliubov理论（DRHBc）与形变晕
- 总结与展望

P. A. Butler & W. Nazarewicz, Intrinsic reflection asymmetry in atomic nuclei, Rev. Mod. Phys. 68 (1996) 349

周善贵，形变原子核中的晕现象，10000个科学难题-物理学卷，科学出版社，2009，682-683

SGZ, Multidimensionally constrained covariant density functional theories---nuclear shapes and potential energy surfaces, Phys. Scr. 91 (2016) 063008

赵鹏巍，“奇形怪状”的原子核心，物理 48 (2019) 773-779

孙向向、周善贵,变形核中的晕现象与形状退耦合, 核技术（待发表）

关于原子核形变的几个基本概念

我们是否需要引入原子核的形变吗?

否: $$H = T + V = \sum_i T_i + \sum_{i,j} v_{i,j}$$

是: $$H_0 = \sum_i [T_i + V(i)] = \sum_i h(i)$$

原子核有固定形变吗?

平衡形变和形变涨落

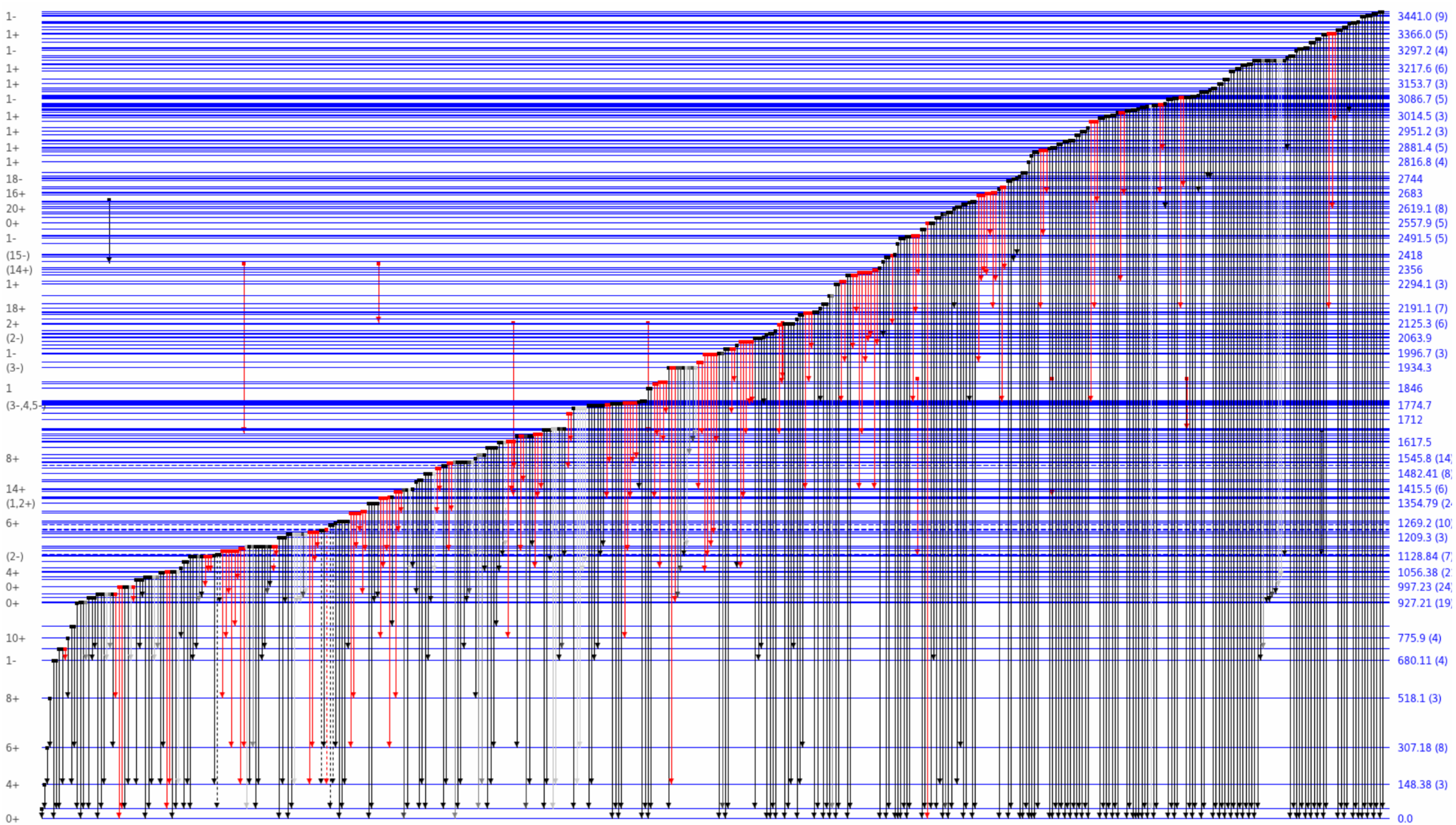
对称性自发破缺和恢复

内禀系⇔实验室系

平均势场与密度分布

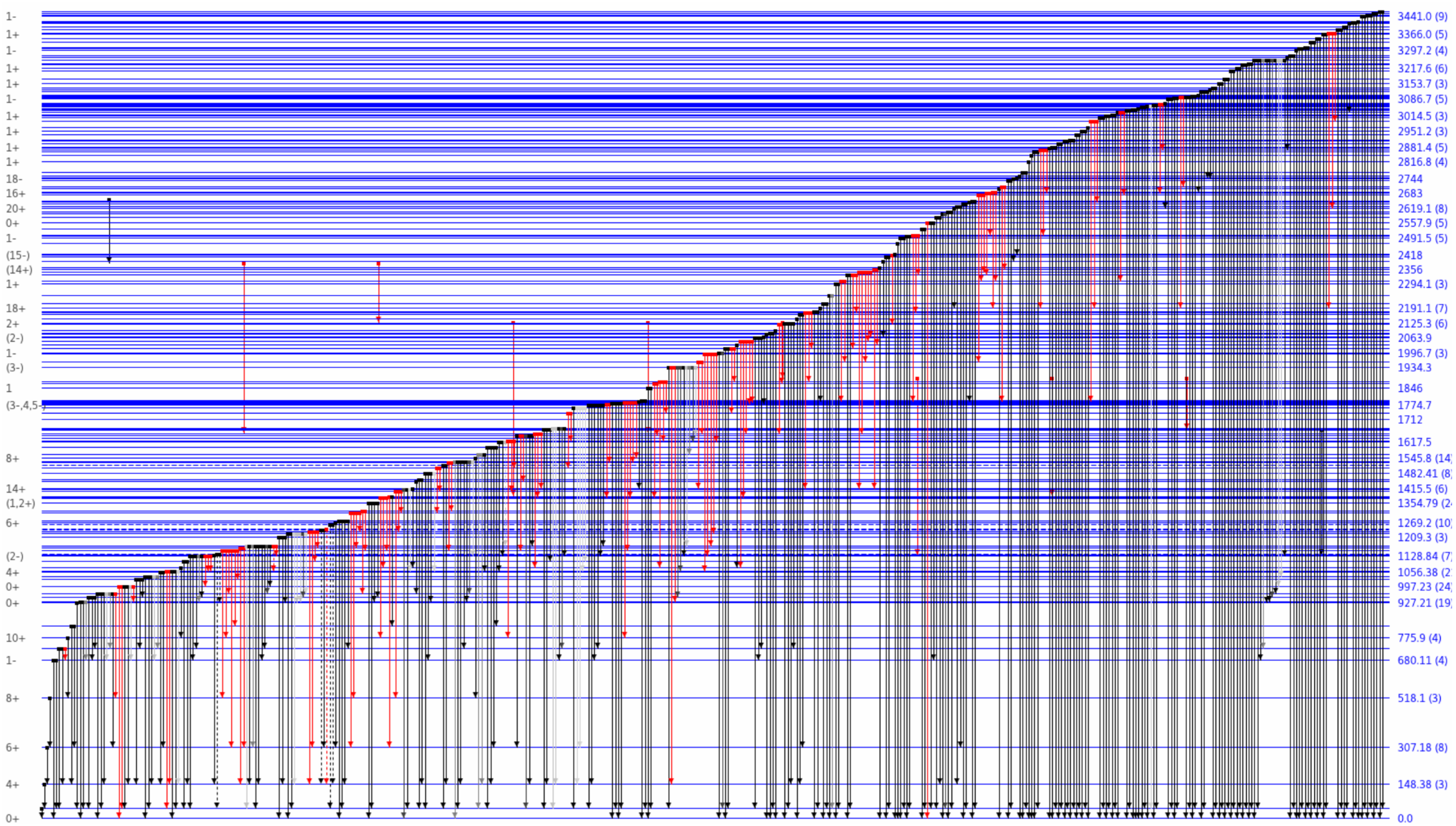
自洽 ~ 非自洽

^{238}U: 能级与转动态

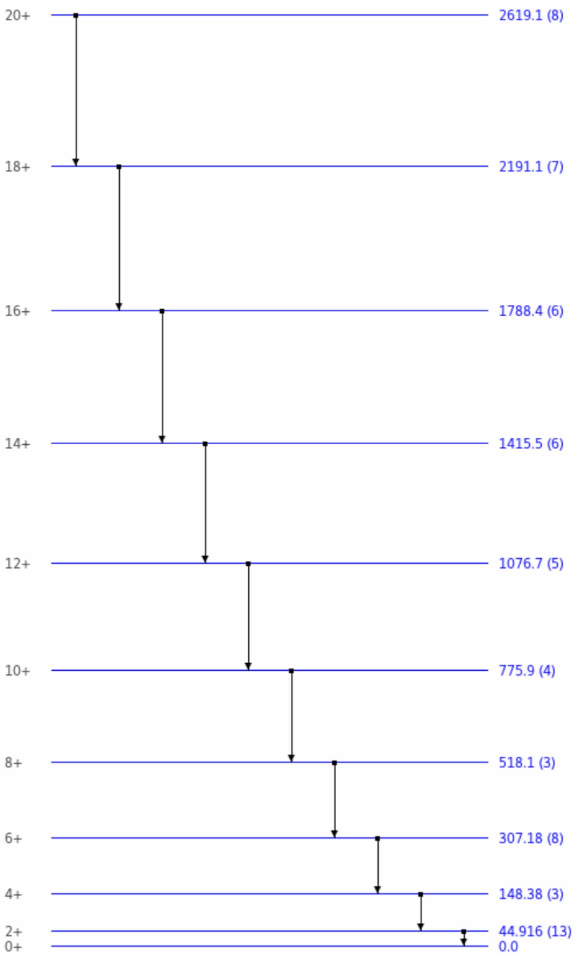


^{238}U的基态与199个激发态

^{238}U: 能级与转动态



^{238}U的基态与199个激发态



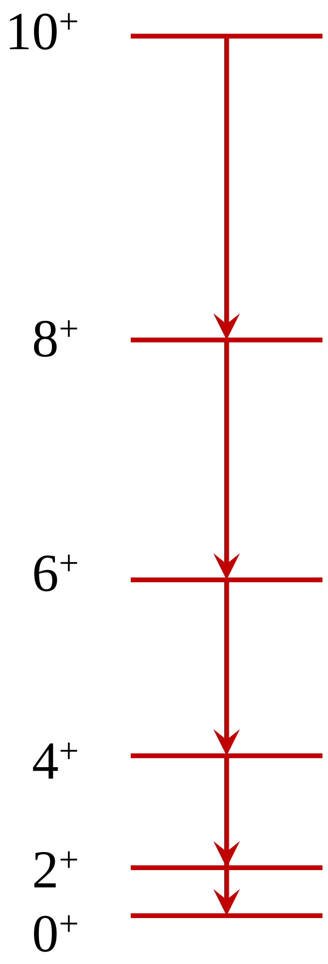
^{238}U的一个转动带

原子核的转动

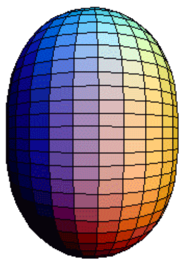
$$E(\omega) = \frac{1}{2}\mathcal{J}\omega^2$$

$$I(\omega) = \mathcal{J}\omega$$

$$E(I) = \frac{1}{2\mathcal{J}}I(I+1)$$



“Rotation”



原子核的转动

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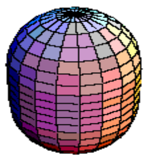
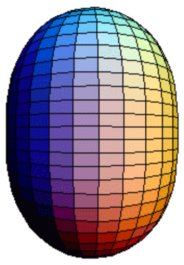
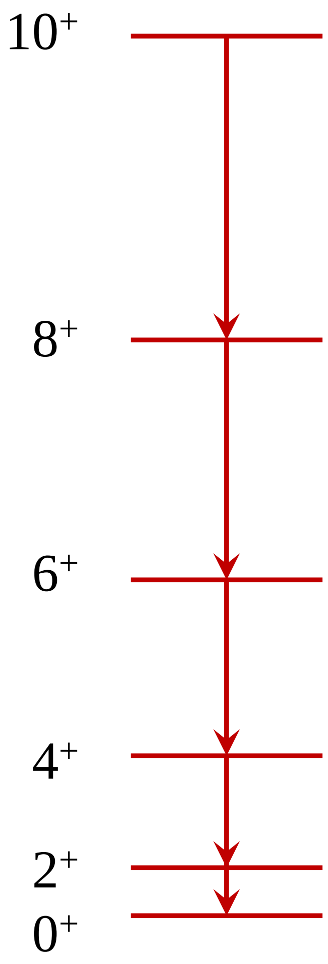


“Rotation”





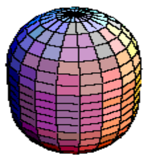
原子核形状的变化

$$E(\omega) = \frac{1}{2}\mathcal{J}\omega^2$$

$$I(\omega) = \mathcal{J}\omega$$

$$E(I) = \frac{1}{2\mathcal{J}}I(I+1)$$

“Vibration”



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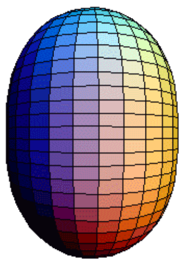
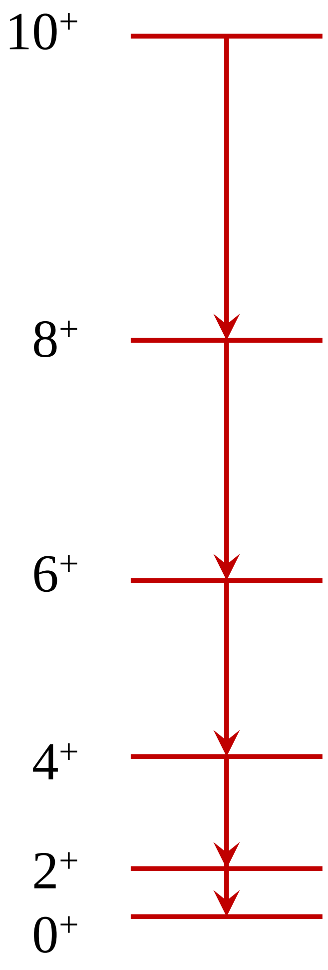




原子核转动谱公式

- Bohr-Mottelson

$$E_I = A\xi^2 + B\xi^4 + C\xi^6 + D\xi^8 + \cdots$$

$$\xi^2 = I(I+1)$$

- Harris

$$E = \alpha\omega^2 + \beta\omega^4 + \gamma\omega^6 + \cdots$$

- *ab* 公式（Holberg-Lipas; 吴-曾)

$$E_I = a\left[\sqrt{1+bI(I+1)} - 1\right]$$

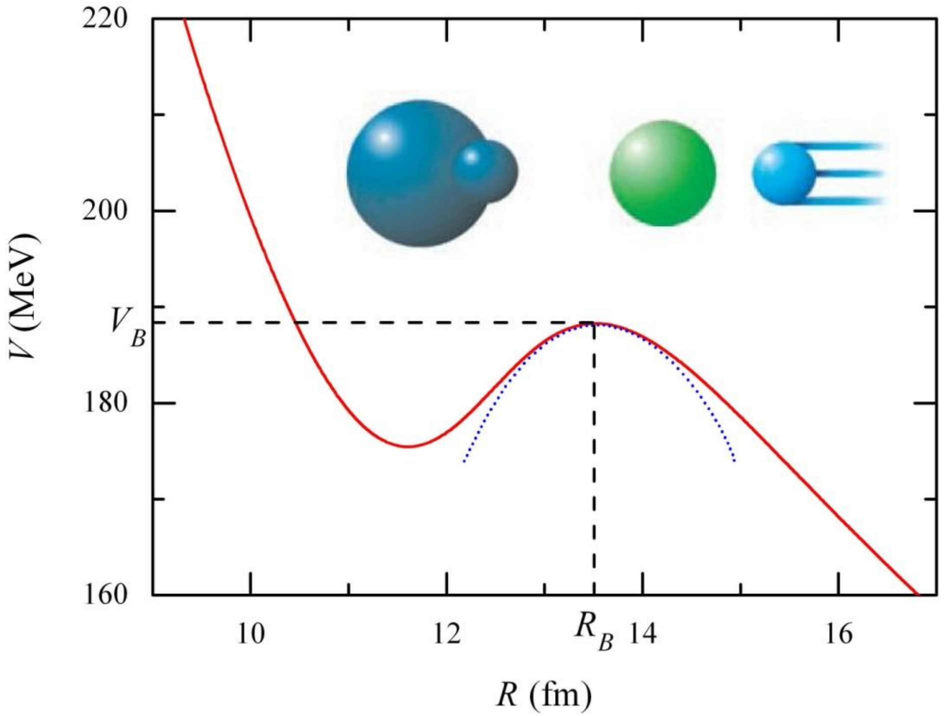
- *pq*公式（曾-刘-赵)

$$x = qI(I+1)$$

$$E(I) = p\left(\left\{\left(\frac{x}{2}\right)^2 + \left[\left(\frac{x}{2}\right)^4 + \left(\frac{x}{3}\right)^3\right]^{1/2}\right\}^{1/3} + \left\{\left(\frac{x}{2}\right)^2 - \left[\left(\frac{x}{2}\right)^4 + \left(\frac{x}{3}\right)^3\right]^{1/2}\right\}^{1/3}\right)$$

温凯、张振华、赵恩广、周善贵，2012，《中国科学》第42卷第22-31页

重离子碰撞——俘获过程



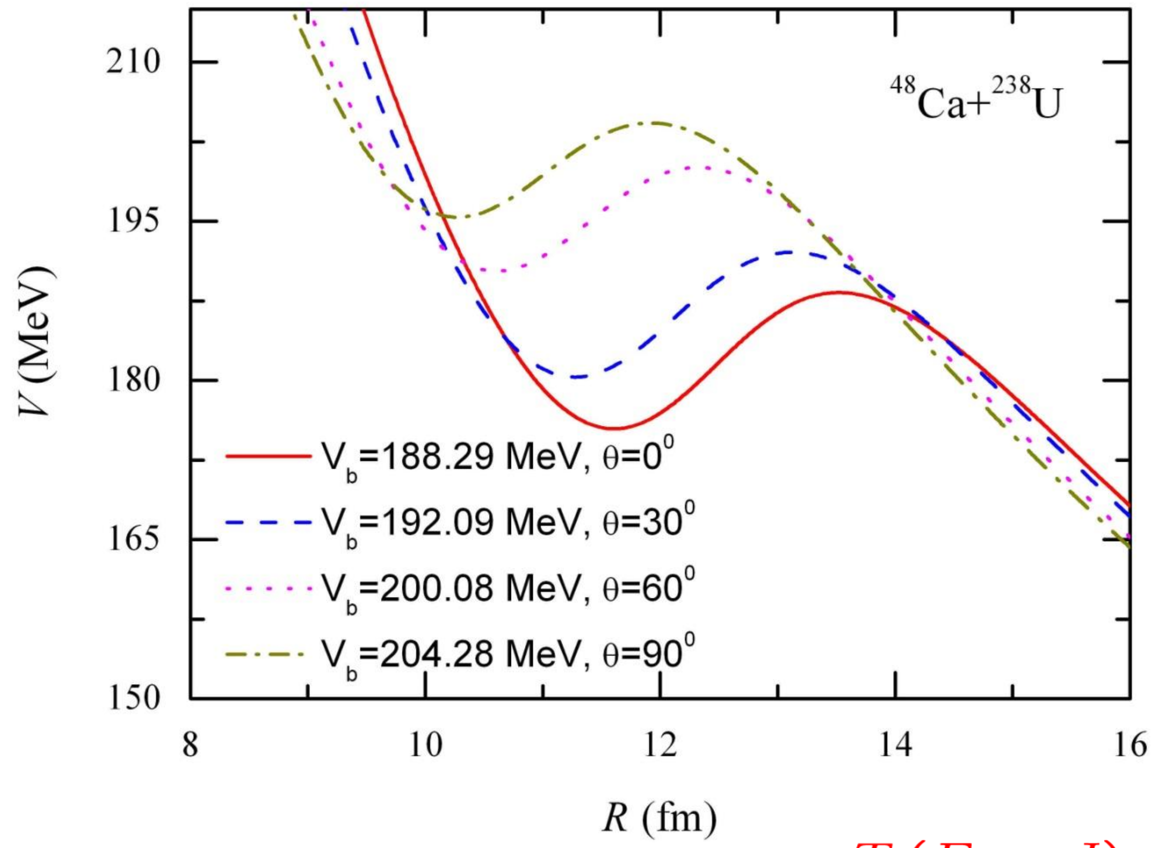
- Path integral method
- WKB approximation
- Hill-Wheeler formula
- New formula by Li et al.
- ...

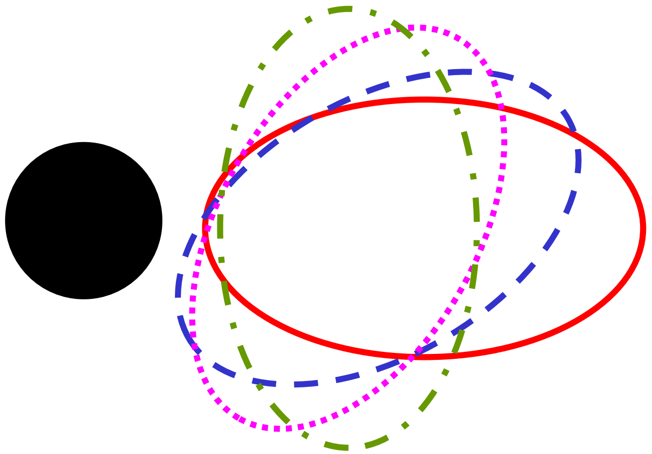
$$\sigma_{\mathrm{cap}}\left(E_{\mathrm{cm}}, J\right)=\frac{\pi \hbar^2}{2 \mu E_{\mathrm{cm}}}(2 J+1) T\left(E_{\mathrm{cm}}, J\right)$$

$$T\left(E_{\mathrm{cm}}, J\right)=\left(1+\exp \left[-\frac{2 \pi}{\hbar \omega}\left(E_{\mathrm{cm}}-E_{\mathrm{rot}}-V_B\right)\right]\right)^{-1}$$

Hill_Wheeler1953_PR089-1102

形变和取向 — 位垒分布

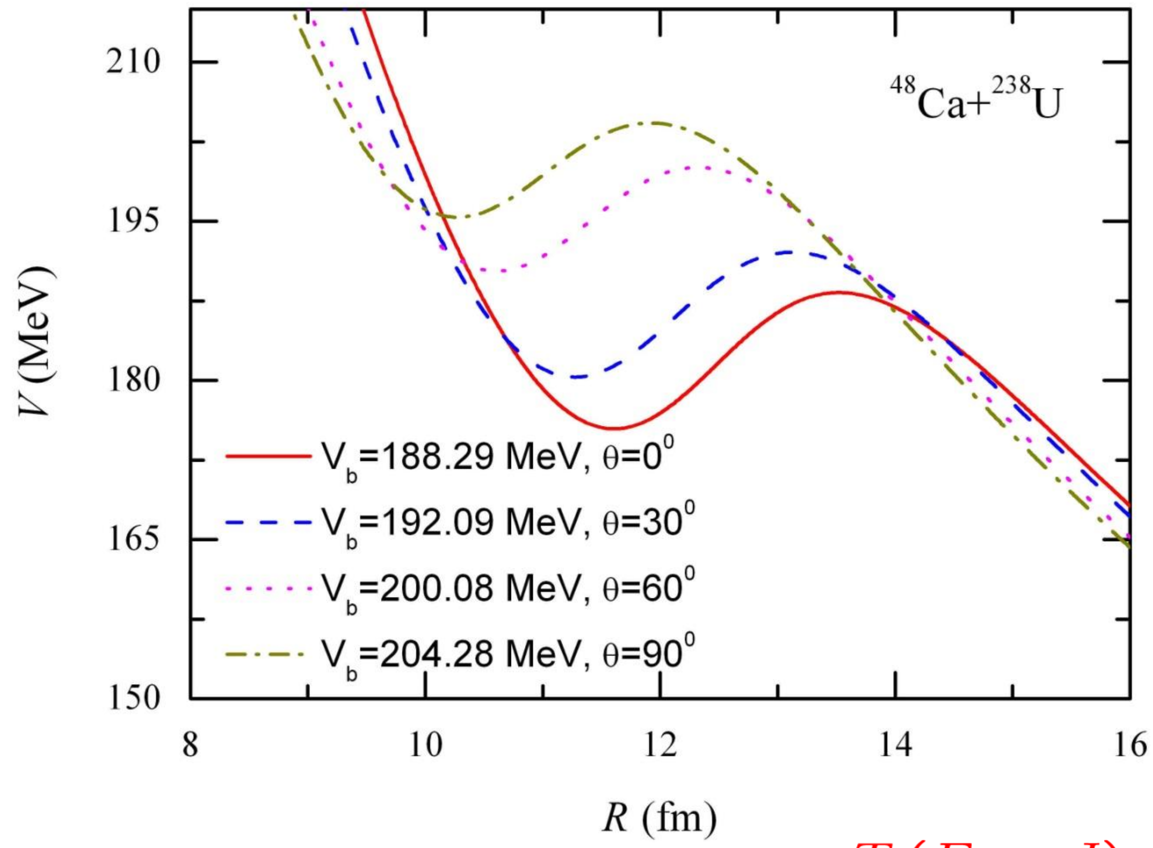


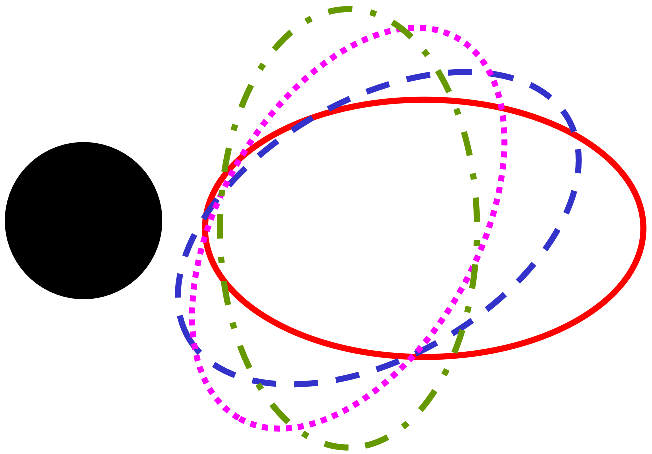


$$T\,(E_{\rm cm},J) = \int dB\ f(B)\ T\,(E_{\rm cm},B,J)$$

形变和取向 — 位垒分布

Coupling effects due to rotation, vibration, nucleon transfer, …
are taken into account empirically by introducing a barrier distribution





$$T\left(E_{\rm cm}, J\right) = \int dB\ f(B)\ T\left(E_{\rm cm}, B, J\right)$$

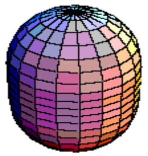
原子核的转动耦合

$$E(\omega) = \frac{1}{2}\mathcal{J}\omega^2$$

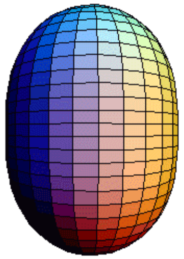
$$I(\omega) = \mathcal{J}\omega$$

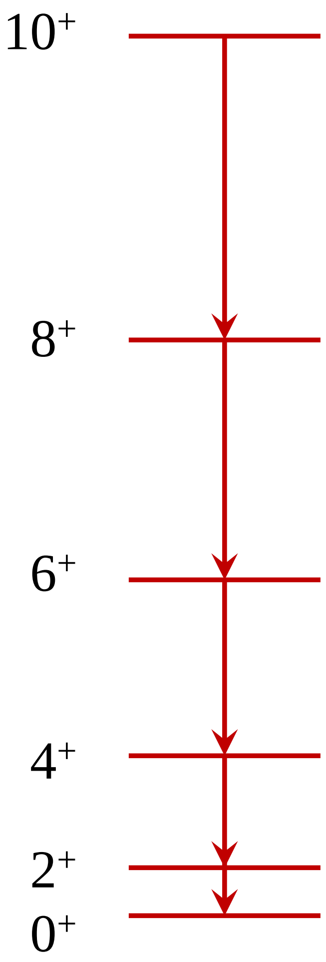
$$E(I) = \frac{1}{2\mathcal{J}}I(I+1)$$

“Vibration”



“Rotation”





Computer Physics Communications

Computer Physics Communications 123 (1999) 143–152

www.elsevier.nl/locate/cpc

A program for coupled-channel calculations with all order couplings for heavy-ion fusion reactions

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[b] Institute de Recherches Subatomiques (IReS), 23 rue du Loess, F-67037 Strasbourg Cedex 2, France
[c] Institute of Nuclear Research of the Hungarian Academy of Science, Pf. 51, H-4001 Debrecen, Hungary

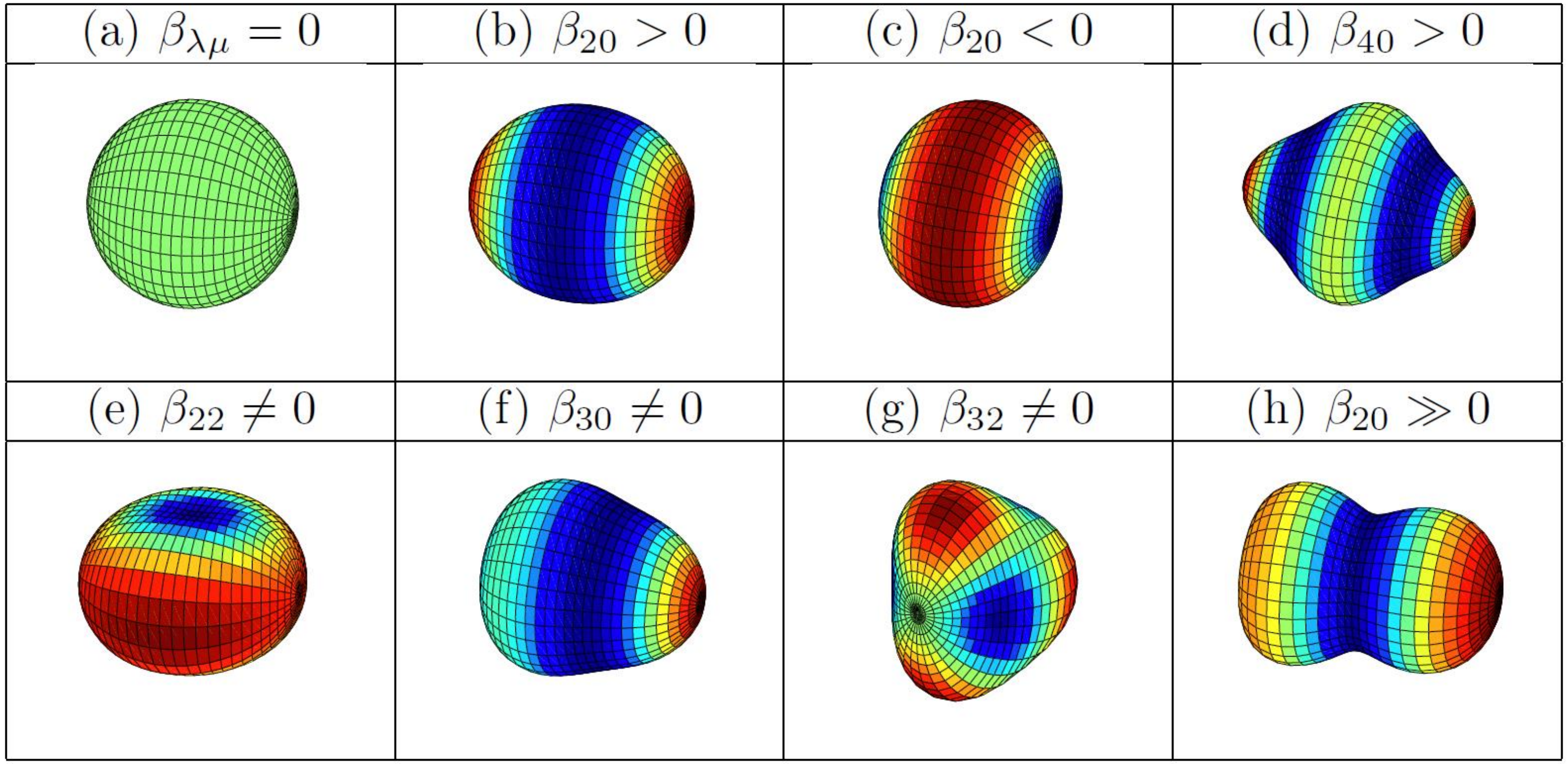
Received 6 April 1999

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{J(J+1)\hbar^2}{2\mu r^2} + V_N^{(0)}(r) + \frac{Z_P Z_T e^2}{r} + \epsilon_n - E\right]\psi_n(r) + \sum_m V_{nm}(r)\psi_m(r) = 0$$

Nuclear shapes

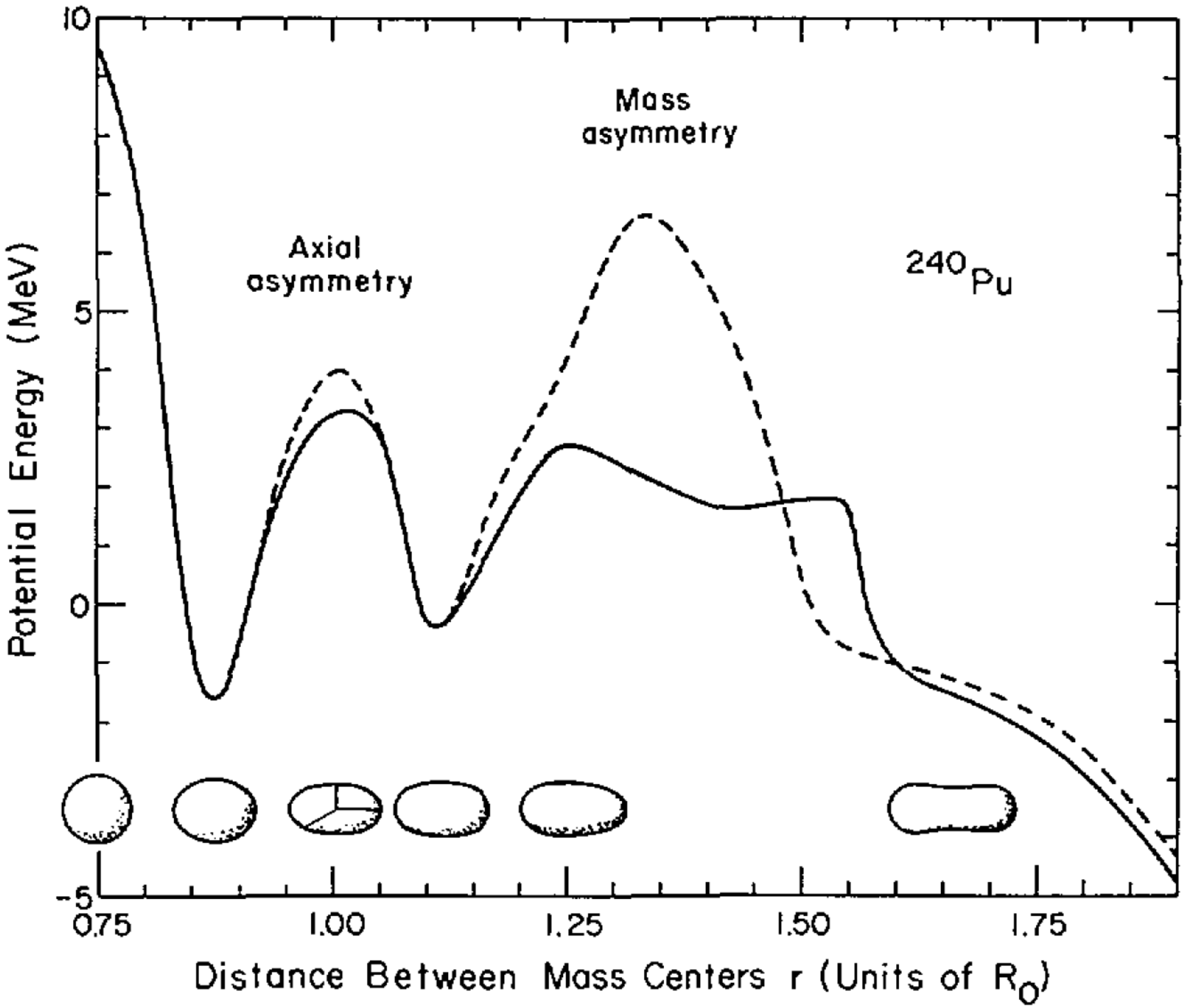
$$R(\theta,\varphi) = R_0 \left[1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta^*_{\lambda\mu} Y_{\lambda\mu}(\theta,\varphi) \right]$$

2^{λ}-pole deformation (2^{λ}-极形变)



Courtesy of Bing-Nan Lu (吕炳楠)

Nonaxial (β_{22} or γ) & octupole (β_{30}) shapes in PES

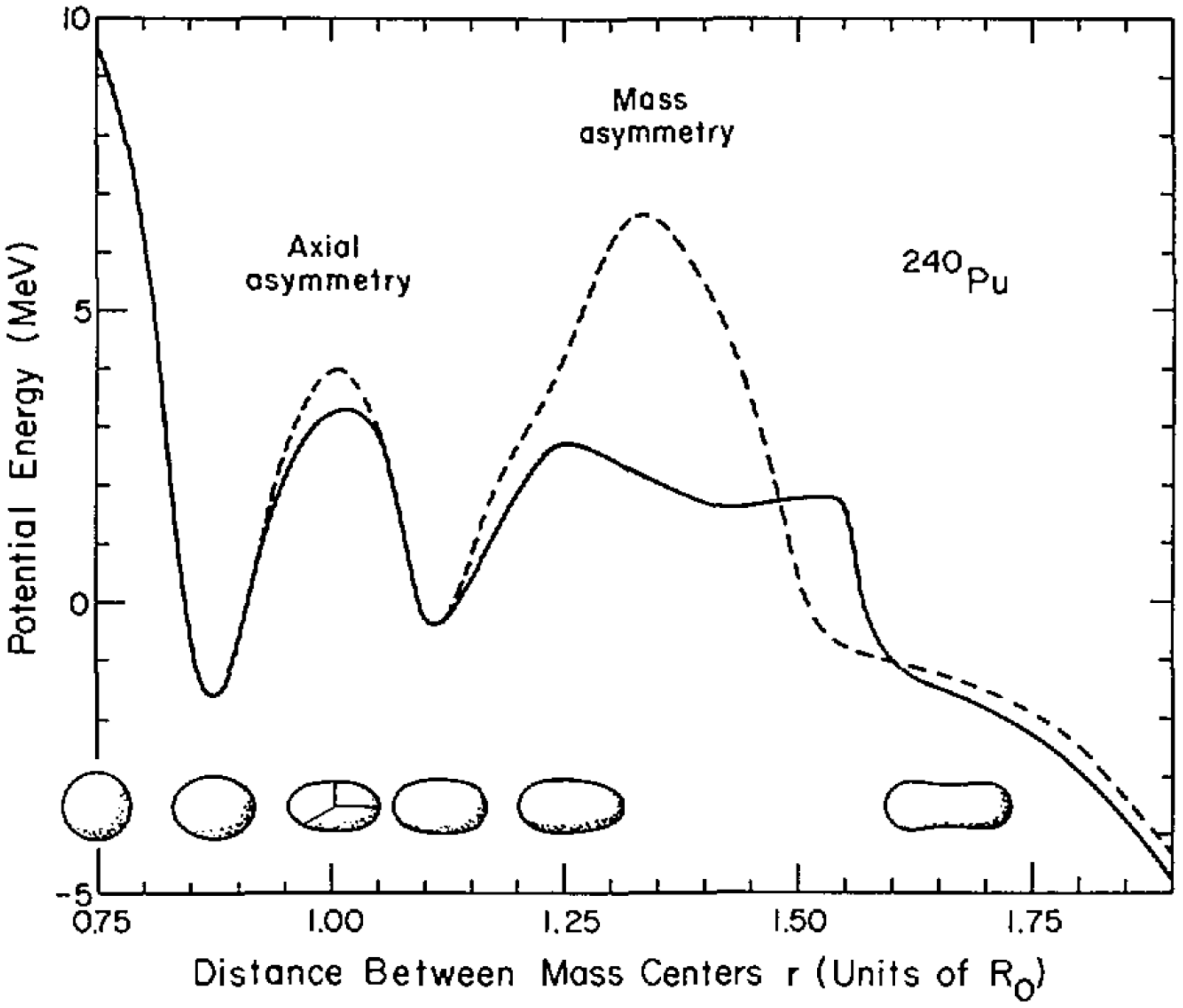


Möller_Nix 1973
IAEA-SM-174/202

Axial asymmetry plays important roles around the first barrier

Reflection asymmetry plays important roles around the second barrier

Nonaxial (β_{22} or γ) & octupole (β_{30}) shapes in PES



Möller_Nix 1973
IAEA-SM-174/202

Pashkevich1969_NPA133-400

Rutz_Maruhn_Reihard_Greiner1995_NPA590-680

Robledo_Warda2008_IJMPE17-204

Kowal_Jachikowicz_Sobiczewski2010_PRC82-014303

Li_Niksic_Vretenar_Ring_Meng2010_PRC81-064321

Abusara_Afanasjev_Ring2010_PRC82-044303

Staszczak_Baran_Nazarewicz2011_IJMPE20-552

Royer_Jaffre_Moreau2012_PRC86-044326

…

Axial asymmetry plays important roles around the first barrier

Reflection asymmetry plays important roles around the second barrier

Covariant Density Functional Theory (CDFT)

$$\mathcal{L} = \bar{\psi}_i (i\partial\!\!\!/ - M)\psi_i + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - g_\sigma\bar{\psi}_i\sigma\psi_i$$
$$- \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - g_\omega\bar{\psi}_i\omega\!\!\!/\psi_i$$
$$- \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - g_\rho\bar{\psi}_i\vec{\rho}\!\!\!/\vec{\tau}\psi_i$$
$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}_i\frac{1-\tau_3}{2}A\!\!\!/\psi_i,$$

$$(\boldsymbol{\alpha}\cdot\mathbf{p} + \beta(M + S(\mathbf{r})) + V(\mathbf{r}))\psi_i = \epsilon_i\psi_i$$
$$\left(-\nabla^2 + m_\sigma^2\right)\sigma = -g_\sigma\rho_S - g_2\sigma^2 - g_3\sigma^3$$
$$\left(-\nabla^2 + m_\omega^2\right)\omega = g_\omega\rho_V - c_3\omega^3$$
$$\left(-\nabla^2 + m_\rho^2\right)\rho = g_\rho\rho_3$$
$$-\nabla^2 A = e\rho_C$$

Serot_Walecka1986_ANP16-1

Reinhard1989_RPP52-439

Ring1996_PPNP37-193

Vretenar_Afanasjev_Lalazissis_Ring2005_PR409-101

Meng_Toki_SGZ_Zhang_Long_Geng2006_PPNP57-470

Liang_Meng_SGZ2015_PR570-1

Meng_SGZ2015_JPG42-093101

Meng (ed.), Relativistic Density Functional for Nuclear structure (World Scientific, 2016)

Λ hypernuclei

- The Lagrangian density: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Lambda$

$$\mathcal{L}_\Lambda = \bar{\psi}_\Lambda \left(i\gamma^\mu \partial_\mu - m_\Lambda - g_{\sigma\Lambda}\sigma - g_{\omega\Lambda}\gamma^\mu \omega_\mu \right) \psi_\Lambda + \frac{f_{\omega\Lambda\Lambda}}{4m_\Lambda} \bar{\psi}_\Lambda \sigma^{\mu\nu} \Omega_{\mu\nu} \psi_\Lambda,$$

- The Dirac equation for Λ

$$\left[\vec{\alpha} \cdot \vec{p} + \beta \left(m_\Lambda + S_\Lambda \right) + V_\Lambda + T_\Lambda \right] \psi_{\Lambda i} = \epsilon_i \psi_{\Lambda i}$$

$$T_\Lambda = -\frac{f_{\omega\Lambda\Lambda}}{2m_\Lambda} \beta \left(\vec{\alpha} \cdot \vec{p} \right) \omega$$

- Effective interactions

Parameter	m_Λ	$g_{\sigma\Lambda}$	$g_{\omega\Lambda}$	$f_{\omega\Lambda\Lambda}$	N-N interaction
PK1-Y1	1115.6 MeV	$0.580 g_\sigma$	$0.620 g_\omega$	$-g_{\omega\Lambda}$	PK1
NLSH-A	1115.6 MeV	$0.621 g_\sigma$	$0.667 g_\omega$	$-g_{\omega\Lambda}$	NLSH

PK1-Y1: Song_Yao_Lü_Meng2010_IJMPE19-2538
Wang_Sang_Wang_Lü2014_Commu.Theor.Phys.60-479
NLSH-A: Win_Hagino2008_PRC78-054311

MDC-CDFT (β_{20}, β_{22}, β_{30}, β_{32}, β_{40}, …)

ph channel	Non-linear	Density-dependent
Meson exchange	NL3, NL3*, PK1, …	DD-ME1, DD-ME2, …
Point Coupling	PC-F1, PC-PK1, …	DD-PC1, …

	MDC-RMF	MDC-RHB

pp channel	BCS	Bogoliubov
Constant gap	√	-
Constant strength	√	-
Delta force	√	√
Separable force	√	√

Lu_Zhao_Zhao_SGZ
2014_PRC89-014323

Zhao_Lu_Zhao_SGZ
2017_PRC95-014320

Applications of MDC-CDFTs

MultiDimensionally-Constrained Covariant Density Functional Theories

- Potential energy surface, ground state & fission properties
  - (β_{20}, β_{22}, β_{30}): 1-, 2- & 3-dim PES of ^{240}Pu & B_f's of actinides
  - (β_{20}, β_{22}): Shape polarization effect of Λ
  - (β_{20}): Superdeformed shapes in Λ hypernuclei
  - (β_{20}): Third barriers in light actinides
  - (β_{20}, β_{30}): Octupole correlations & shape transitions
  - (β_{20}, β_{22}, β_{30}): Octupole correlations in MχD
  - (β_{20}, β_{32}): Nuclear Tetrahedral shapes
  - (β_{20}, β_{22}, β_{30}): 1-, 2-, & 3-dim PES of ^{270}Hs
  - ($\beta_{\lambda\mu}$, R): Clustering, bubble & toroidal structure; GMR
- Fission dynamics based on PES from MDC-CDFTs
  - Spontaneous fission
  - Induced fission
- Angular momentum & parity projected MDC-CDFTs
  - Clustering & exotic shapes

Applications of MDC-CDFTs

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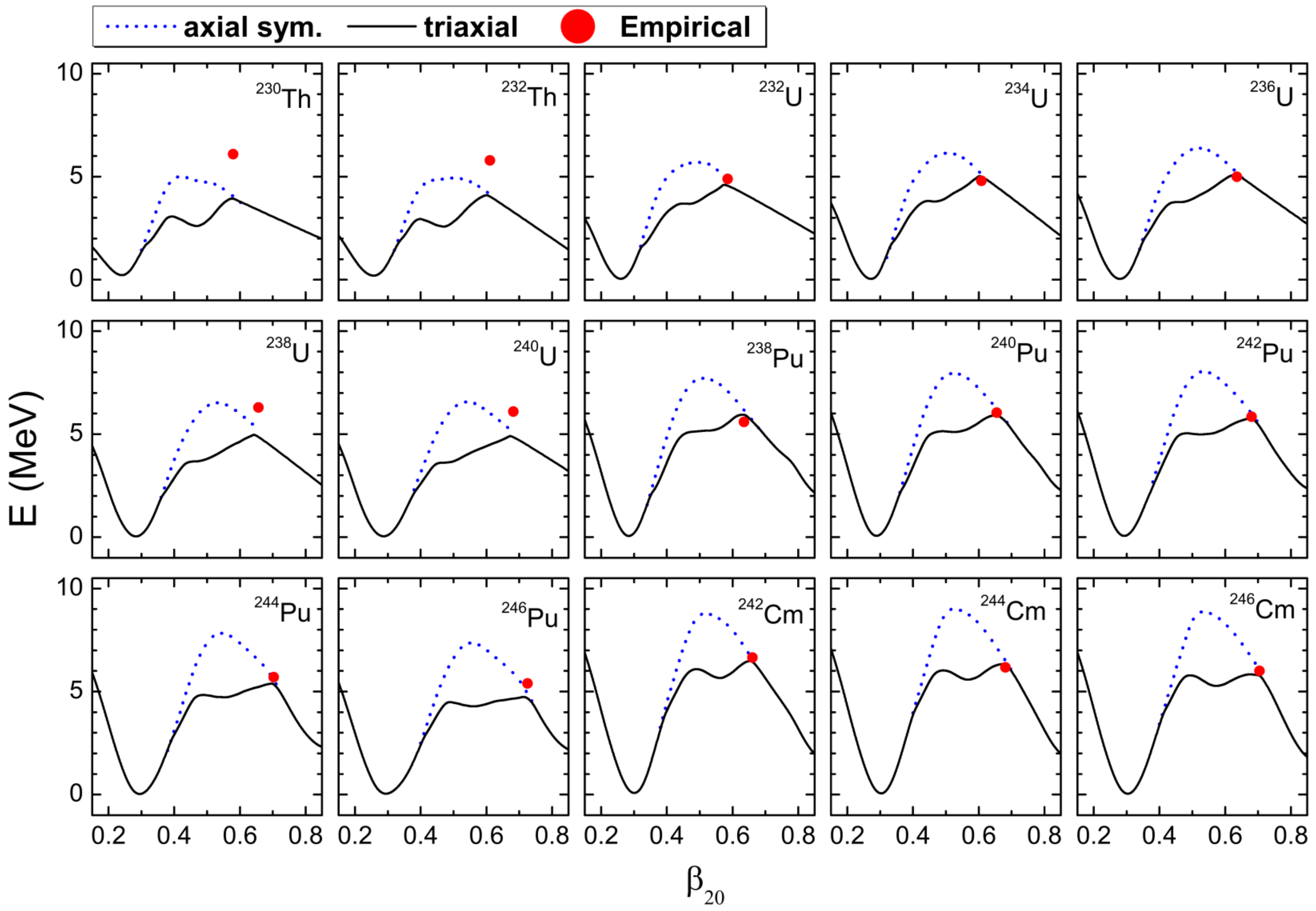
  - Clustering & exotic shapes

慧超给我布置的作业

1. ^{208}Pb 的中子皮
   - ✓ 陈列文的报告
2. ^{238}U 的形变
   - ✓ 见后
3. ^{96}Zr 和 ^{96}Ru 的形变
   - ✓ 荣宇婷的报告；见后
4. ^{16}O 的集团结构
   - ✓ 吕炳楠的报告

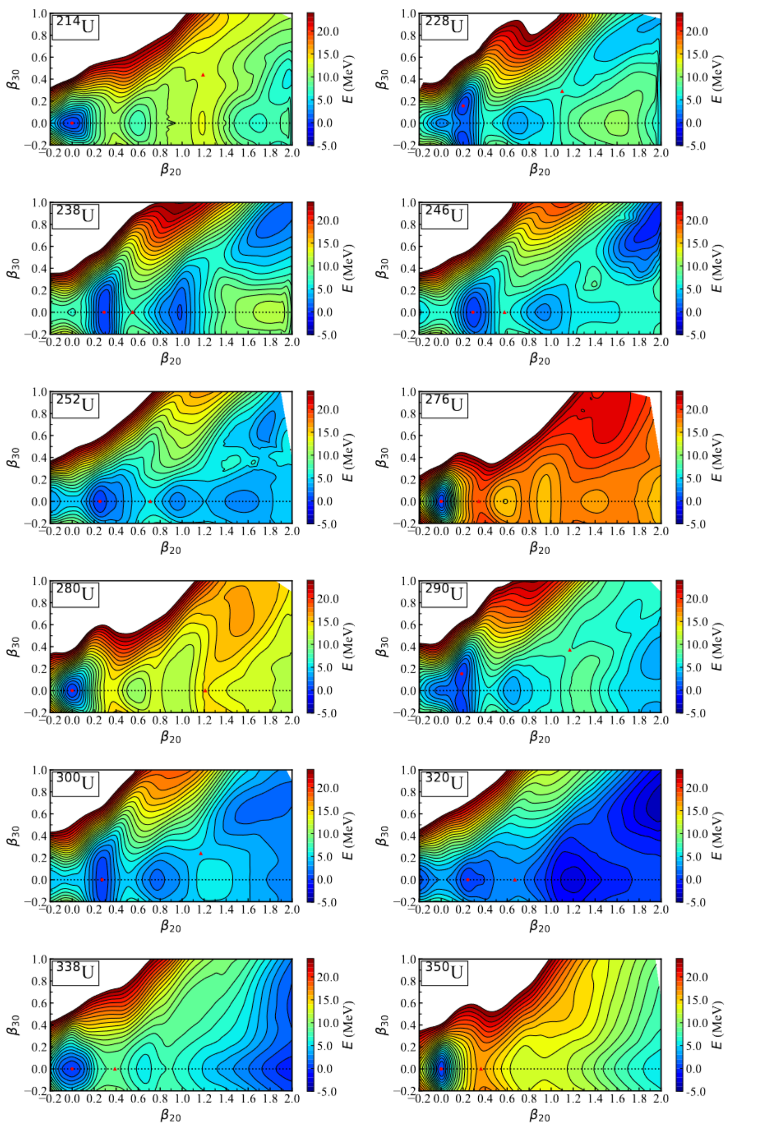
Potential energy curves of actinide nuclei



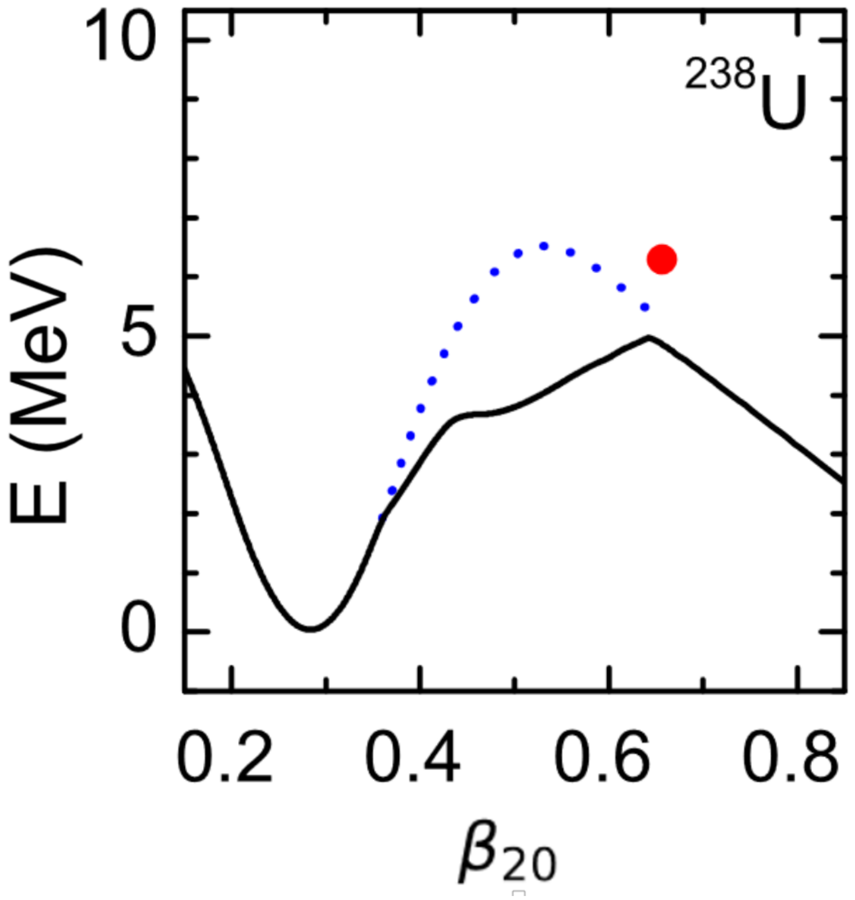
Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

Empirical values: Capote…2009 NDC110-3107 (RIPL-3)

Deng_SGZ arXiv:2303.13118



作业2：^{238}U



Lu_Zhao_Zhao_SGZ 2014_PRC89-014323

β_{20}: 0.29

结合能

计算：1799.08 MeV

实验：1801.69 MeV

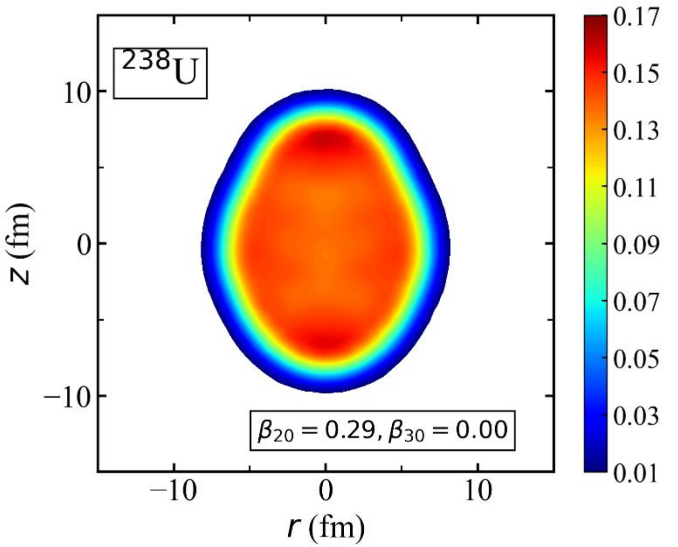
电荷半径

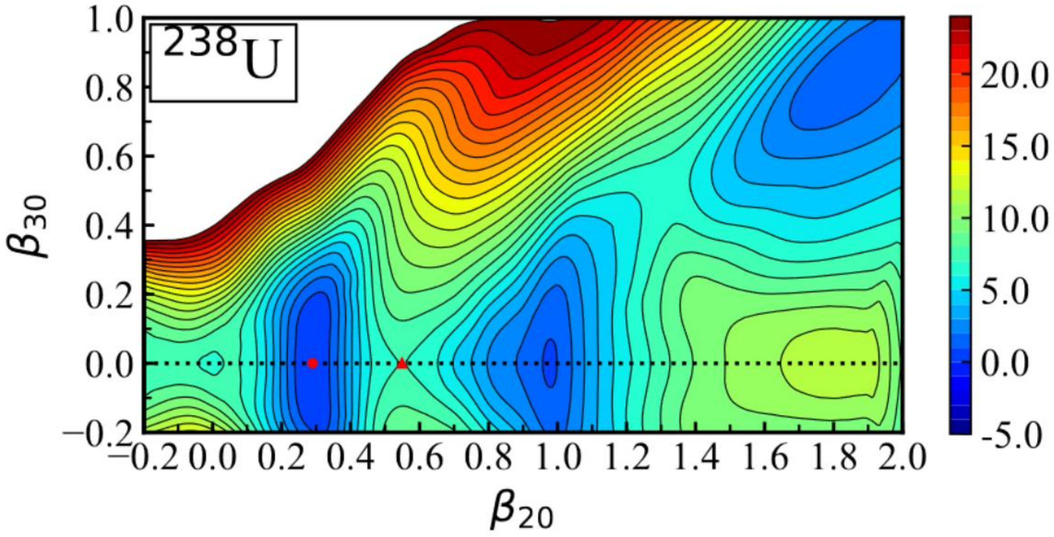
计算：5.90 fm

实验：5.86 fm

邓祥泉

Deng_SGZ arXiv:2303.13118





中子皮厚度的各向异性 & 作业2-3

- 刘鹿蒙：中子皮的角分布会依赖于自旋轨道的耦合强度；在相对论重离子碰撞中可利用旁观者中质子比探测中子皮厚度；这些不同的中子皮角分布可能会对旁观者中质子比产生一定的影响

Physics Letters B 838 (2023) 137701

Contents lists available at ScienceDirect

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journal homepage: www.elsevier.com/locate/physletb

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Measuring deformed neutron skin with free spectator nucleons in relativistic heavy-ion collisions

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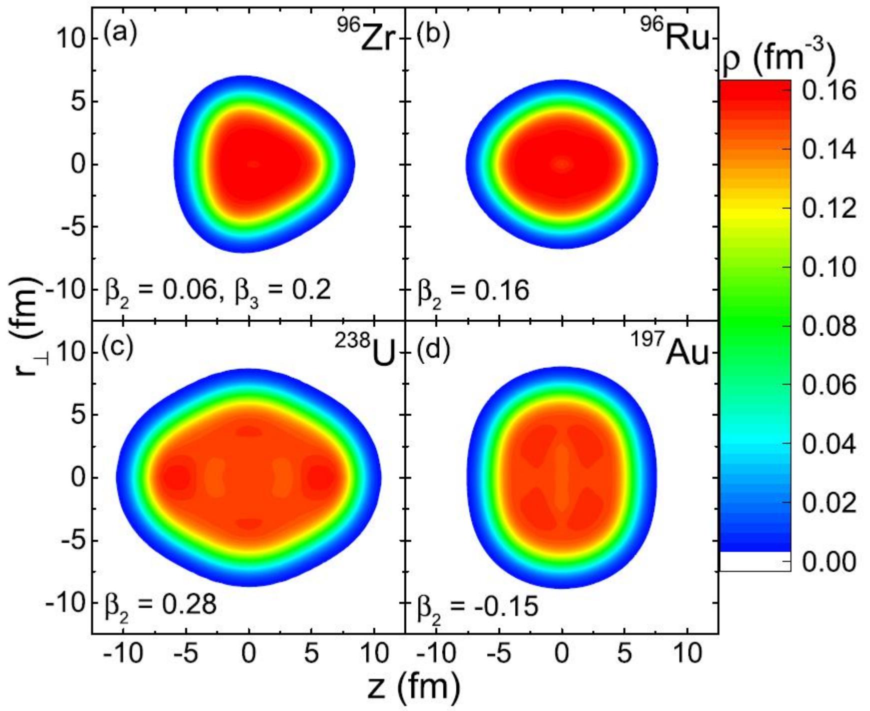
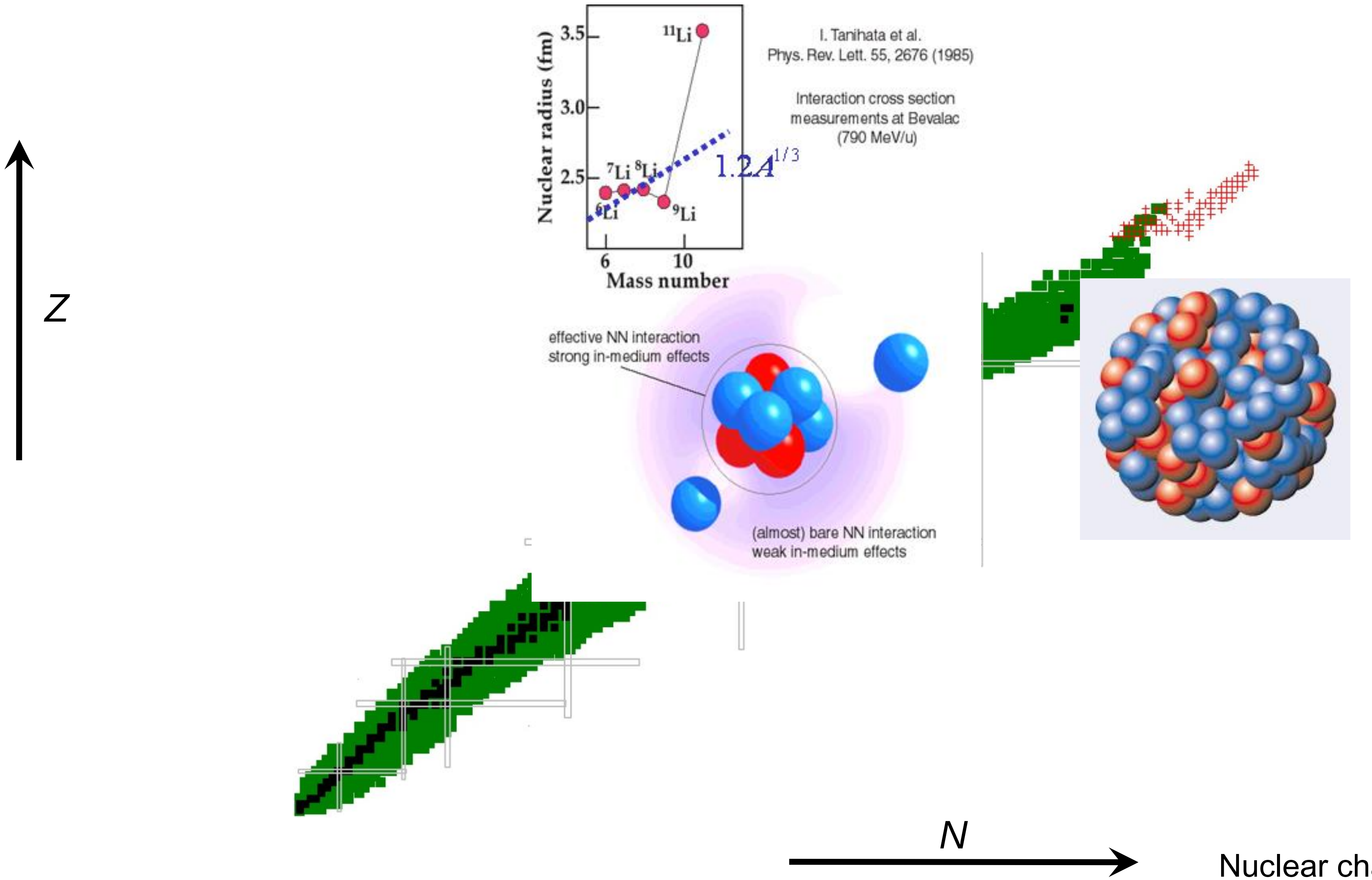
[b] School of Physics Science and Engineering, Tongji University, Shanghai 200092, China
[c] Shanghai Advanced Research Institute, Chinese Academy of Sciences, Shanghai 201210, China
[d] Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China
[e] School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 100049, China
[f] Theoretical Physics Center for Science Facilities, Institute of High Energy Physics, Beijing 100049, China
[g] Synergetic Innovation Center for Quantum Effects & Applications, Hunan Normal University, Changsha 410081, China

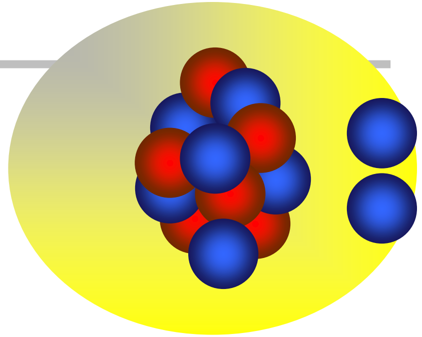


Halo: One of exotic nuclear phenomena



Nuclear chart: courtesy of Ning Wang (王宁)

What we have aimed at



A self-consistent description of halo in deformed nuclei

- ✓ Deformation
- ✓ Continuum contribution
- ✓ Large spatial distribution
- ✓ Interplays among them

by developing a
relativistic Hartree-Bogoliubov model

RMF theories in a Woods-Saxon basis

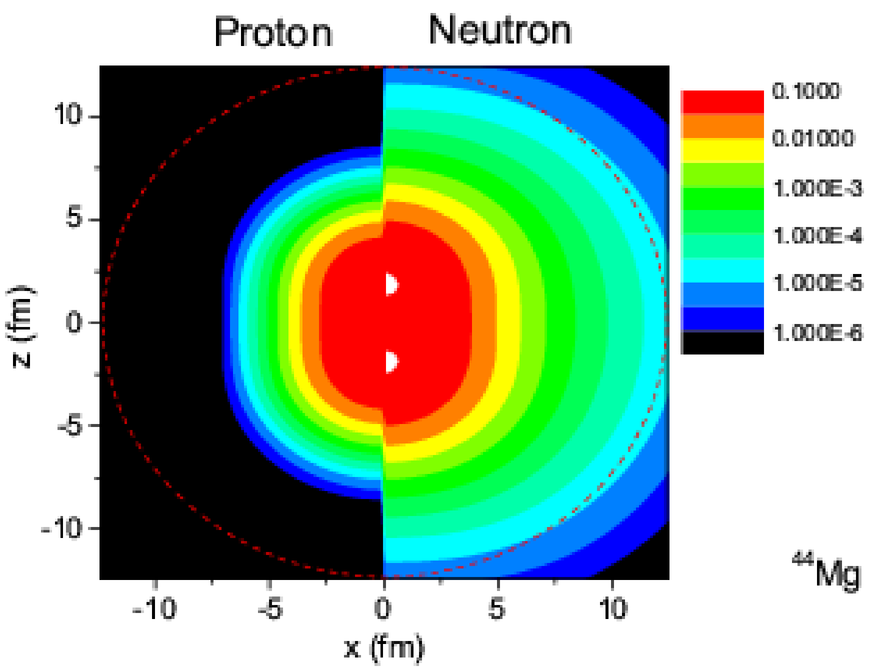
Shapes	Model	Schrödinger W-S basis	Dirac W-S basis	
Spherical	Rela. Hartree	SRH SWS	SRH DWS	√
	SGZ_Meng_Ring2003_PRC91-262501			
Axially deformed	Rela. Hartree + BCS		DRH DWS	√
	SGZ_Meng_Ring2006_AIP Conf. Proc. 865-90			
Axially deformed	Rela. Hartree-Bogoliubov		DRHB DWS	√
	SGZ_Meng_Ring 2007_ISPUN Proc. SGZ_Meng_Ring_Zhao 2010_PRC82-011301R SGZ_Meng_Ring_Zhao 2011_JPConfProc312-092067 Li_Meng_Ring_Zhao_SGZ 2012_PRC85-024312 Li_Meng_Ring_Zhao_SGZ 2012_ChinPhysLett29-042101			

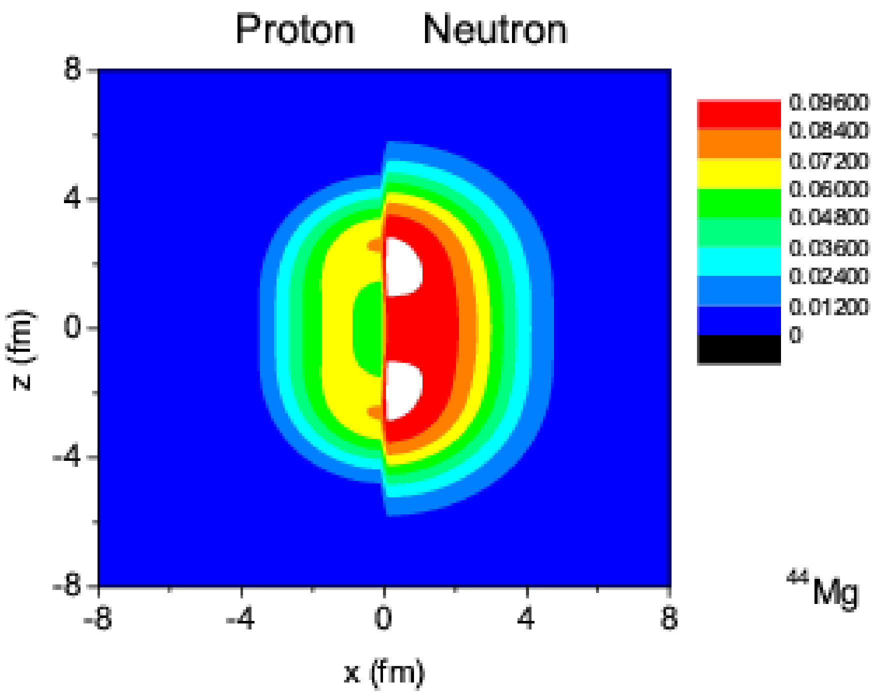
Woods-Saxon basis is a reconciler between the HO basis & *r* space

DDDRHBc: Chen_Li_Liang_Meng2012_PRC85-067301
HFB: Schunck_Egido2008_PRC77-011301R; 78-064305
RHFB: Long_Ring_Giai_Meng2010_PRC81-024308
DRHFB: Geng_Long2022_PRC105-034329
RBHF: Shen_Hu_Liang_Meng_Ring_Zhang2016_ChinPhysLett33-102103

^{44}Mg: Density distributions

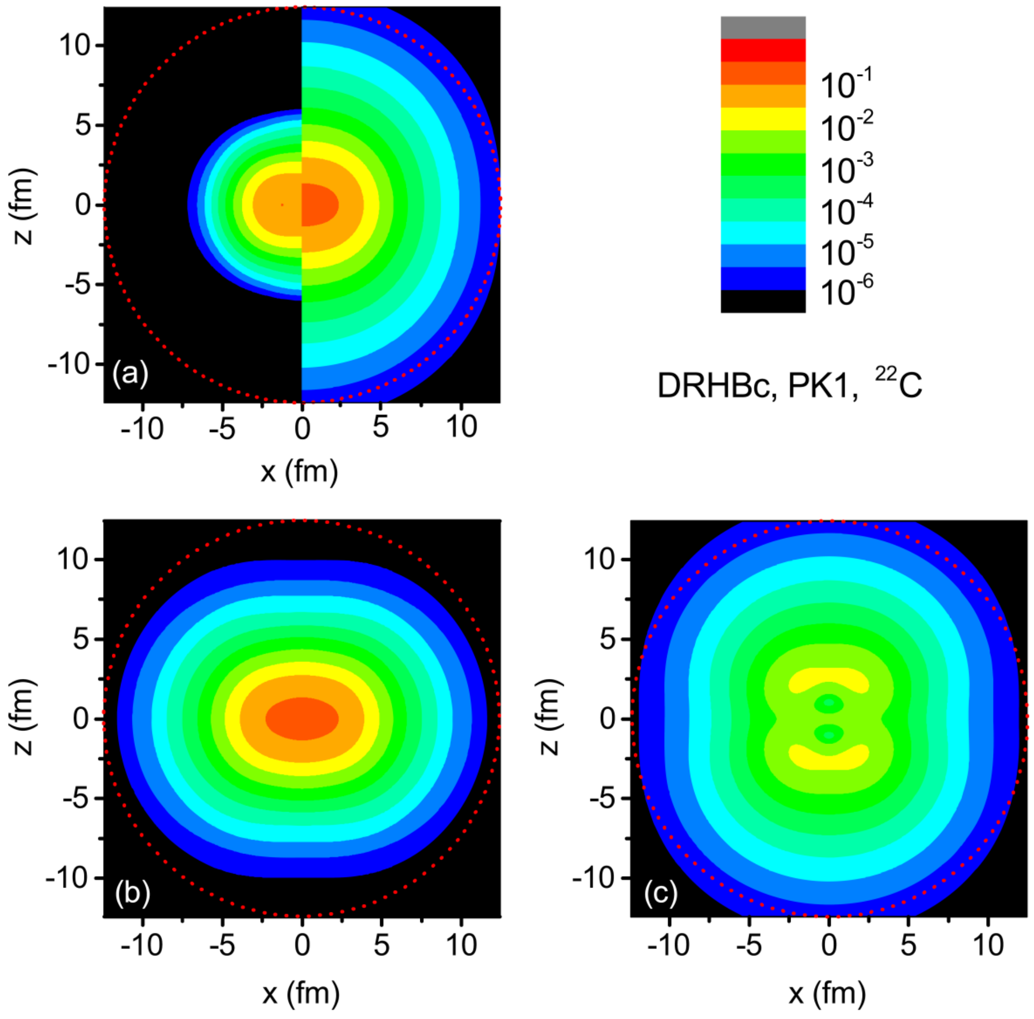
SGZ_Meng_Ring_Zhao 2010 PRC82-011301R
Li_Meng_Ring_Zhao_SGZ 2012 PRC85-024312





- Prolate deformation
- Large spatial extension in neutron density distribution

^{22}C: Halo (?) & shape decoupling



- $2s_{1/2}$: ~25% ⇒ Halo
- Mixture of $(2s_{1/2}, 1d_{5/2})$ ⇒ Prolate halo

PK1

S_{2n} = 0.43 MeV
r_m = 3.25 fm
β_2 = –0.27

Sun_Zhao_SGZ 2018_PLB785-530

内容提要

- 原子核的形状和形变
- 多维形状约束协变密度泛函理论（MDC-CDFT）与原子核的奇特形状
- 形变连续谱相对论Hartree-Bogoliubov理论（DRHBc）与形变晕
- 展望：更好地完成作业

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内容提要

Thanks

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