量子动理学理论及其在相对论重离子碰撞中的应用

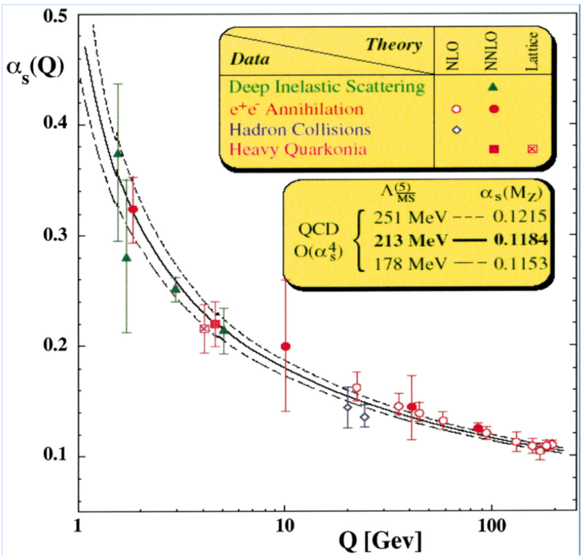
浦　实
中国科学技术大学

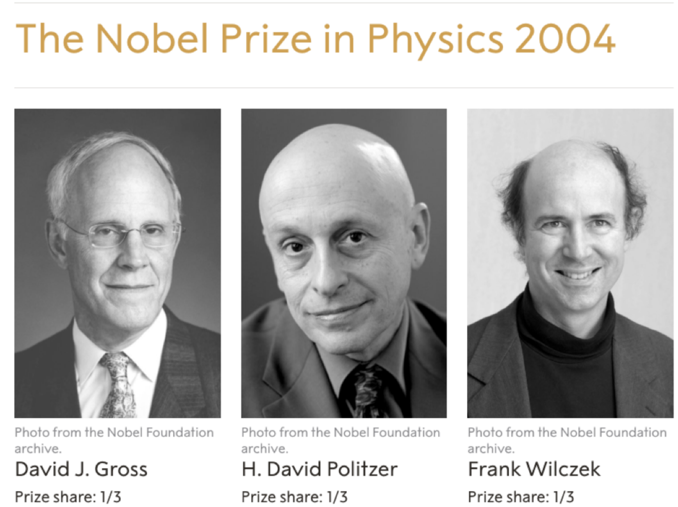
原子核结构与相对论重离子碰撞前沿交叉研讨会
大连，2023年7月31日- 8月6日

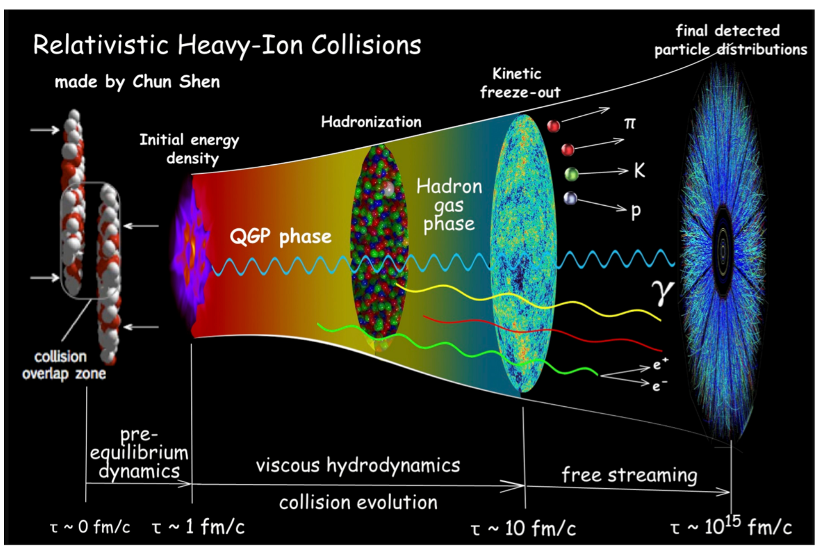
Recent invited reviews:

- Y. Hidaka, SP, Q.Wang, D.L. Yang, **Prog.Part.Nucl.Phys**. 127 (2022) 103989
- J.H. Gao, G.L. Ma, SP, Q. Wang, **Nucl.Sci.Tech.** 31 (2020) 9, 90

QCD解禁闭相变和相对论重离子碰撞

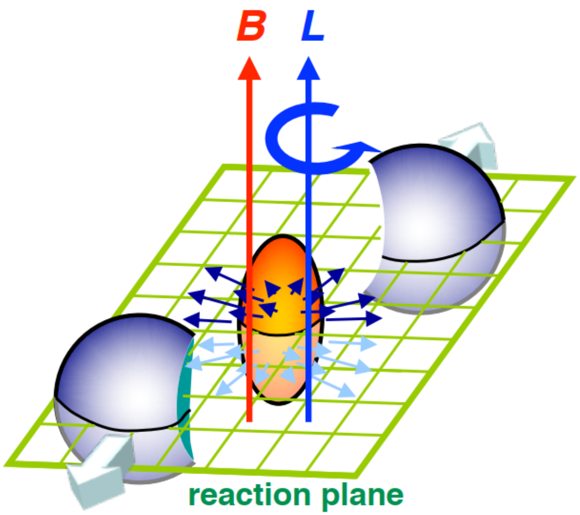






2004年量子色动力学渐进自由获得诺贝尔物理学奖。渐进自由导致的夸克解禁闭及其相变是中高能核物理领域重要基础科学问题。相对论重离子碰撞是研究该现象的重要研究手段之一。

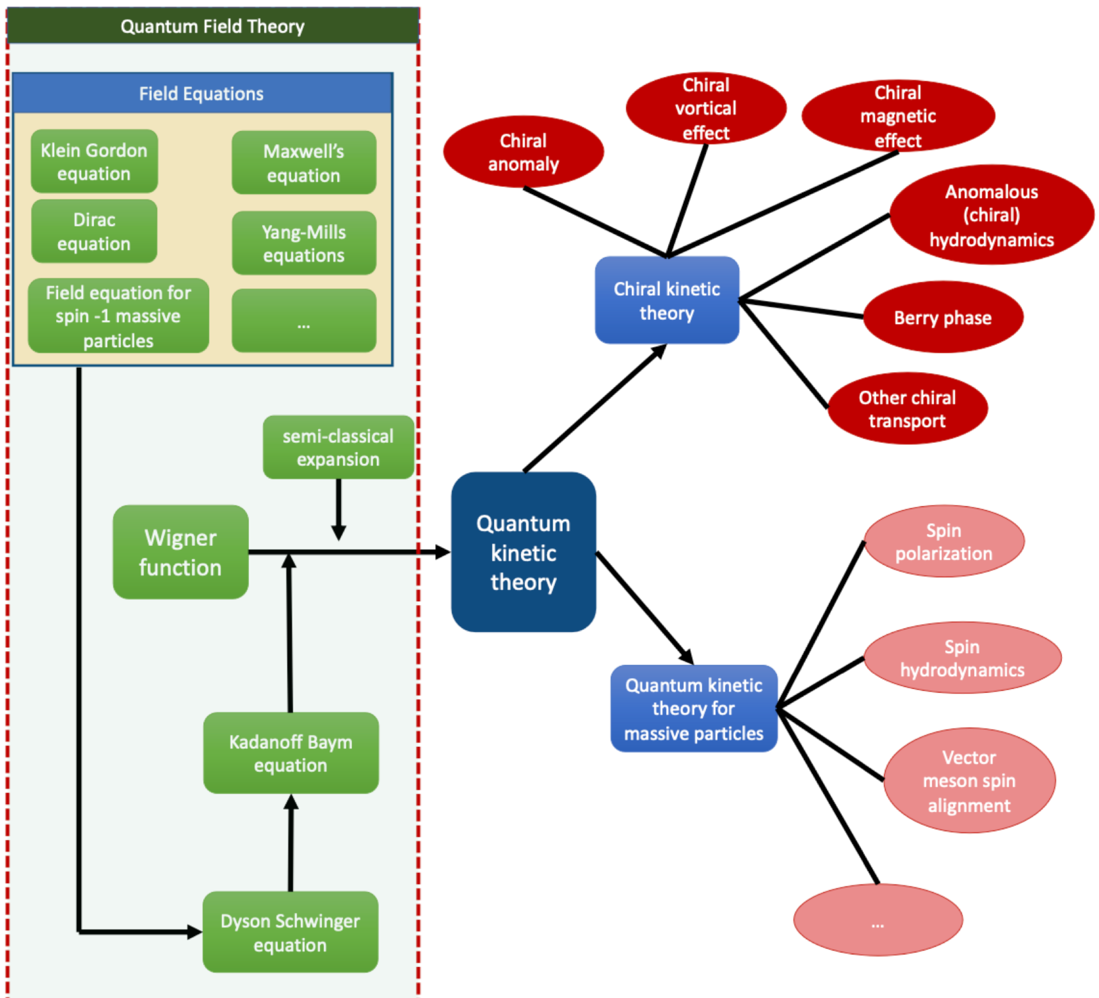
极端环境下夸克物质性质



- 相对论重离子碰撞，将产生$10^{17}-10^{18}$高斯的强磁场，这是迄今为止发现的最强磁场。
- 伴随产生高达$10^5\,\hbar$初始轨道角动量，实验测量结合理论计算估算出夸克胶子等离子体涡旋高达10^{21}次/秒，即夸克胶子等离子体是迄今为止涡旋最快的系统。
- 可以在相对论重离子碰撞中，研究极端条件下夸克物质性质。

量子动理学理论

量子动理学理论根源于量子场论，可以用于研究极端强电磁场下夸克物质量子输运及自旋极化现象。



Outline

- 相对论重离子碰撞中的手征输运
  零质量费米子的量子动力学理论

- 相对论重离子碰撞中Lambda超子极化和矢量介子自旋排列
  量子动力学理论及碰撞效应

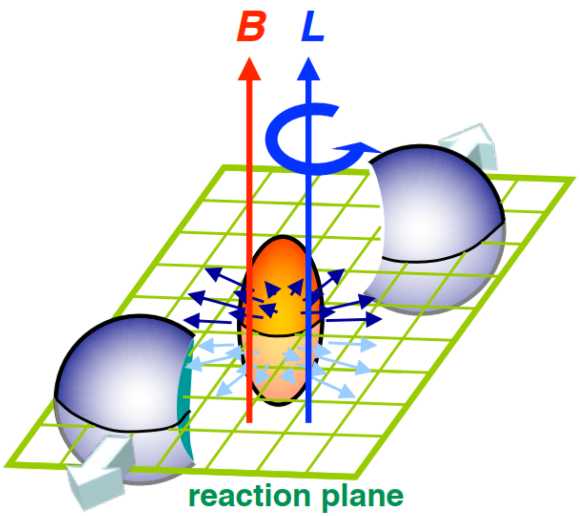
- 相对论重离子碰撞中光致双轻子产生
  散射截面中的光子Wigner函数：
  利用同量异位素碰撞研究核结构

- 总结

相对论重离子碰撞中的手征输运

零质量费米子的量子动力学理论

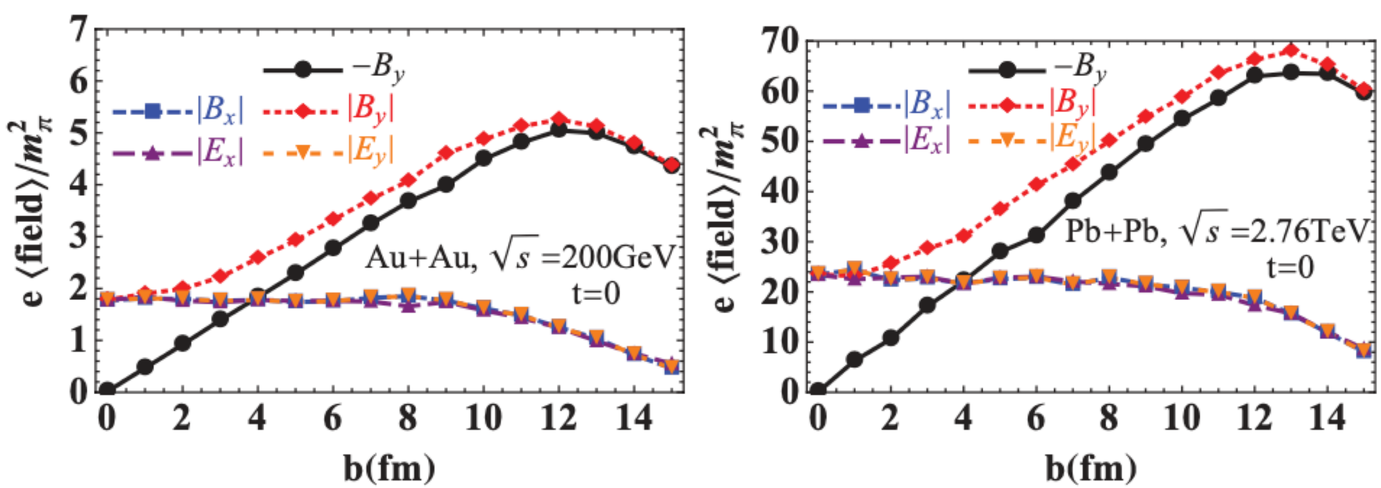
相对论重离子碰撞中产生的极强电磁场



- 相对论重离子碰撞将伴随极强电磁场的产生。
- 理论上计算方法：

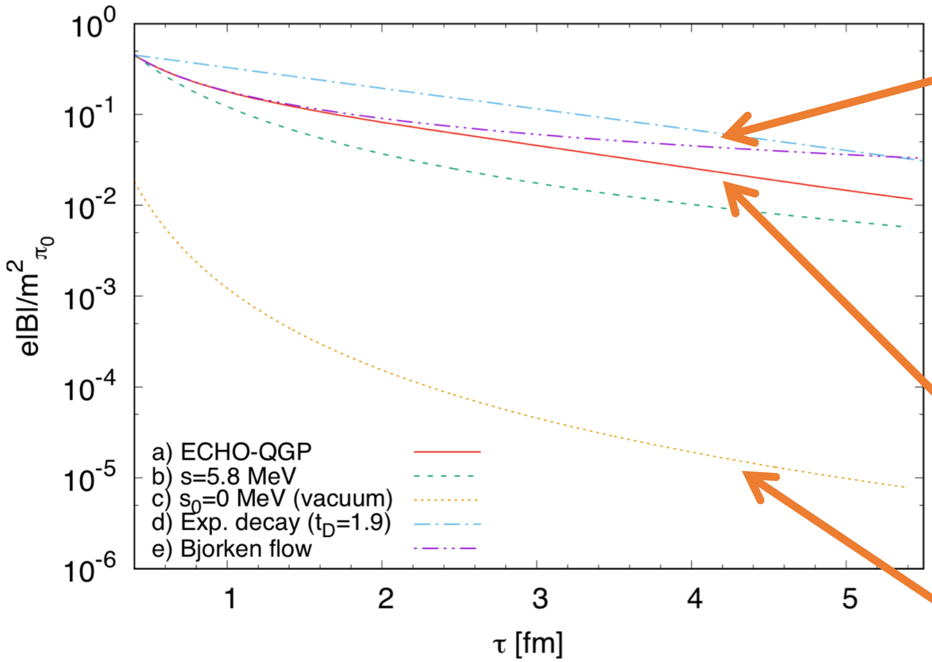
Lienard-Wiechert 势能+ 逐事例模拟

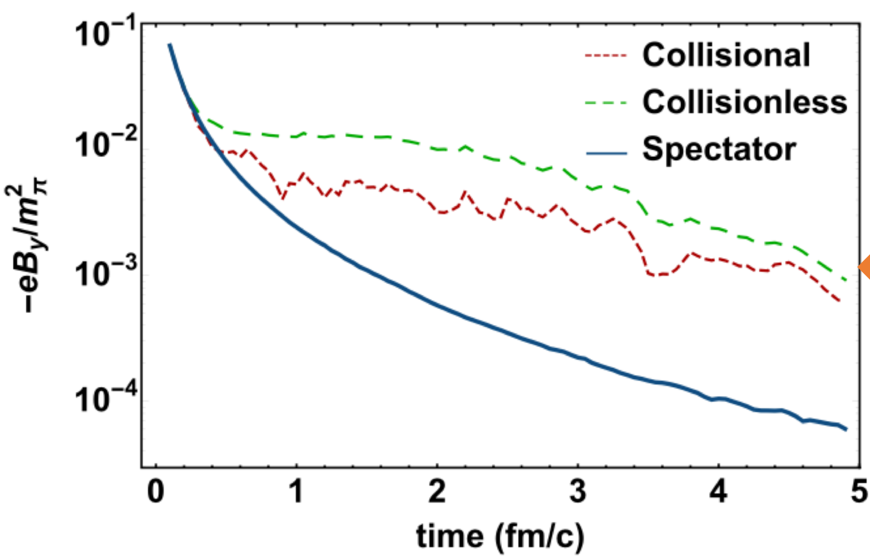
- 磁场峰值高达$10^{17}-10^{18}$高斯，这是迄今为止发现最强电磁场。



A. Bzdak, V. Skokov PRC 2012 ;W.T. Deng, X.G. Huang PRC 2012;V.Roy, SP, PRC 2015; H. Li, X.I. Sheng, Q.Wang, 2016; etc. / review: K. Tuchin 2013

电磁场在QGP中的演化





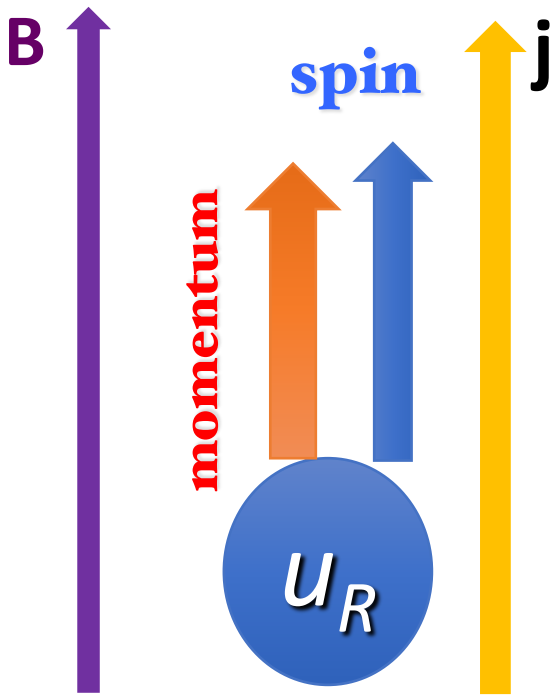
Bjorken型理想磁流体（最慢衰减模式）
Pu, Roy, Rezzolla, Rischke, PRD(2015)
磁流体+手征磁效应+手征反常
Siddique, Wang, Pu, Wang, PRD (2019)
Peng, Wu, Wang, She, Pu, PRD(2023)

ECHO-QGP磁流体模拟
Inghirami, Zanna, Moghaddam,Becattini, Bleicher, EPJC(2016)

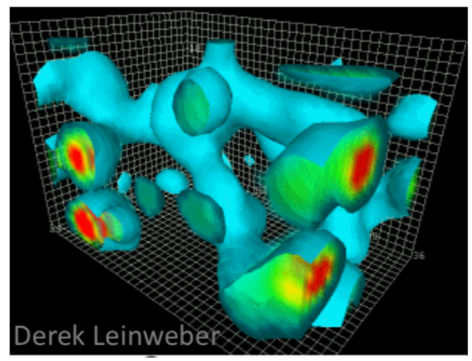
真空中衰减 (最快衰减模式)
Kharzeev, McLerran,Warringa,NPA(2008)

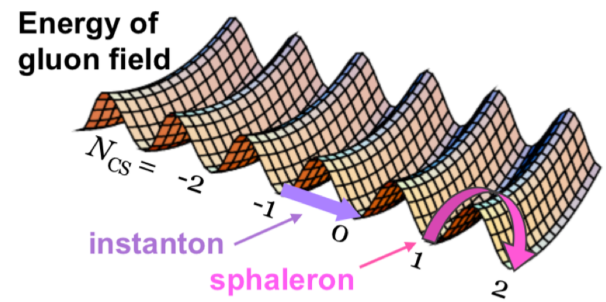
相对论玻尔兹曼方程与Maxwell方程耦合+ 2-2 领头Log阶pQCD散射矩阵元
Zhang, Sheng, Pu, Chen, Peng, Wang, Wang, PRR (2022)
Also see:
Wang, Zhao, Xu, Greiner, Zhuang, PRCL (2022); Li, Huang, PRD (2023);

手征磁效应



- 强磁场
- 手征费米子（零质量费米子）
- 由于**QCD**真空涨落导致左右手粒子数不同





- 手征磁效应：磁场可以诱导出电流

$$\mathbf{j} = \frac{e^2}{2\pi^2}\mu_5 \mathbf{B},$$

与手征反常、**QCD**真空涨落密切相关

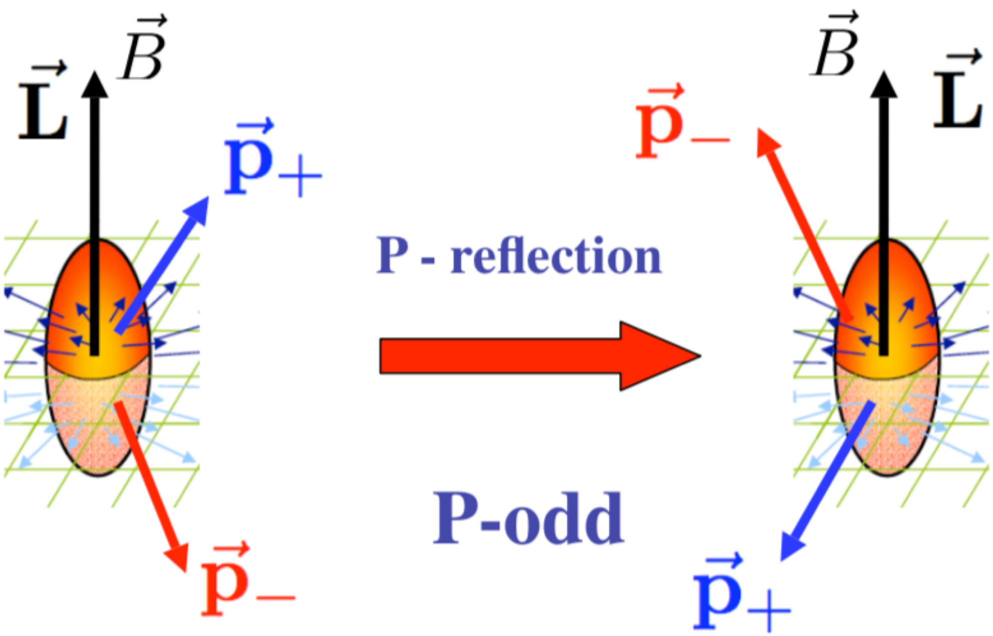
Kharzeev, Fukushima, Warrigna, (08,09), etc. ...

Also see the works done by the groups in PKU, Tsinghua Uni, Fudan Uni., IMP, SINAP, USTC, CCNU, SUSY, SCNU…

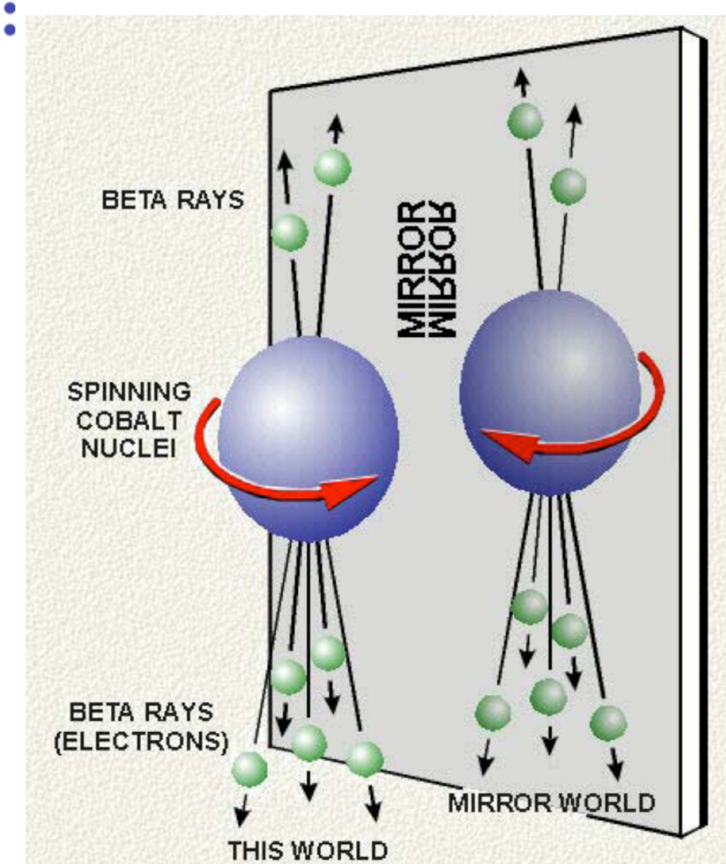
电荷分离暗示强相互作用宇称破缺

Slides from Kharzeev's talk at 26th Winter Workshop on Nuclear Dynamics (2010)

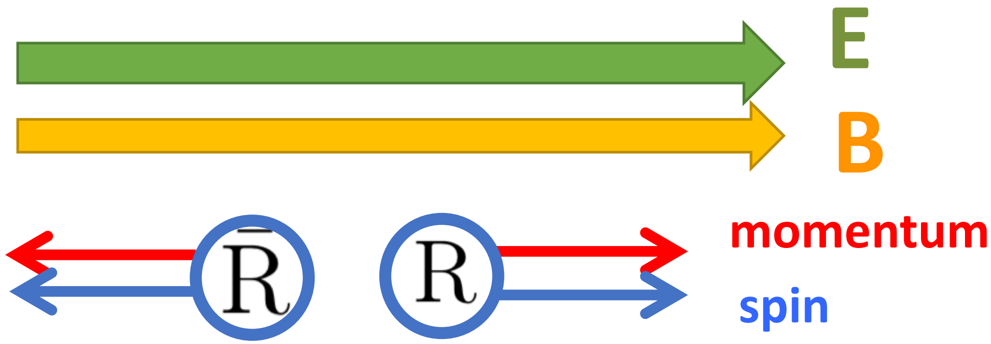
Charge separation = parity violation:



$$\mathcal{P}: \quad \vec{p} \to -\vec{p}; \quad \vec{B} \to \vec{B}; \quad \vec{L} \to \vec{L}$$

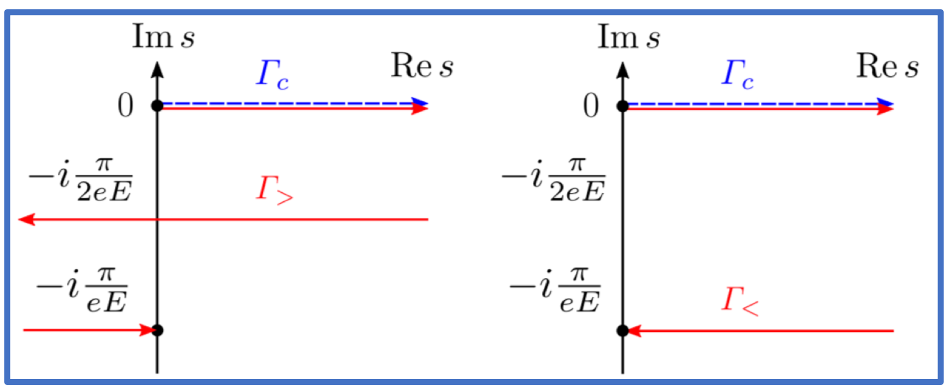


手征磁效应与Schwinger对产生机制深层关联



$$\frac{1}{2}\partial_t n_5 = \text{Schwinger Pair Production rate}$$

Fukushima, Kharzeev, Warringa, PRL(2010)



- 我们利用非微扰方法计算了稳恒平行强电磁场下粒子对实时产生
- 轴矢流 Ward 恒等式

$$\partial_\mu j_5^\mu = \frac{e^2 EB}{2\pi^2}\exp\left(-\frac{\pi m^2}{eE}\right)$$

- 手征磁效应质量修正

$$j^3 = \frac{e^2 EB}{2\pi^2}\coth\left(\frac{B}{E}\pi\right)\exp\left(-\frac{\pi m^2}{eE}\right)t$$

- 强场下手征凝聚

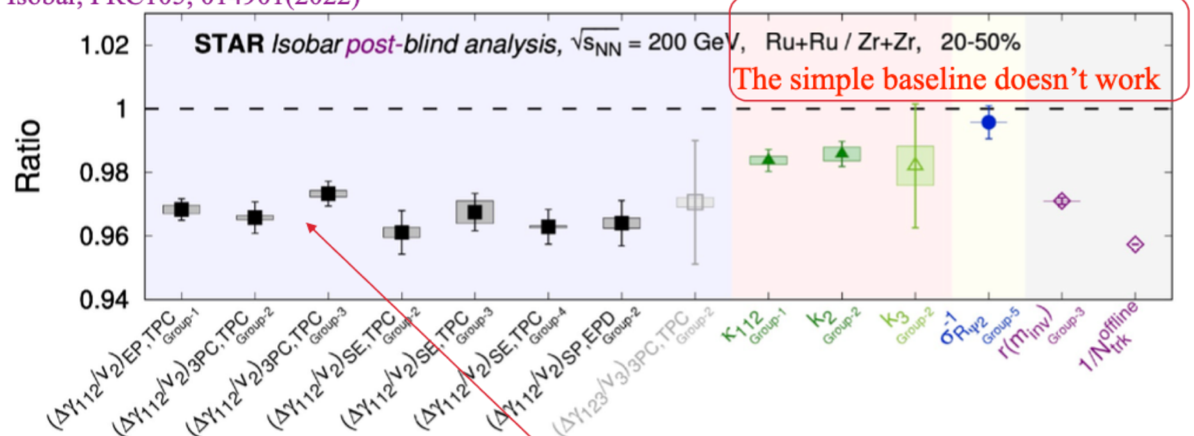
Copinger, Fukushima, SP, PRL(2018)

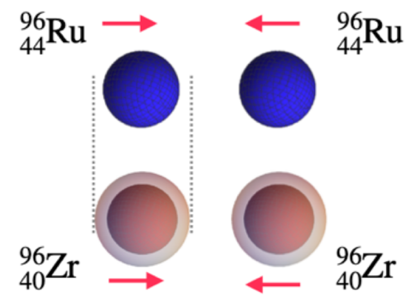
Also see recent review: Copinger, SP, IJMPA (2020)

手征磁效应与同量异位素碰撞

需要未来从实验与理论方面更系统地研究同量异位素碰撞。

STAR, Isobar, PRC105, 014901(2022)





$$\Delta\gamma_{bkg} = \langle\cos(\varphi_\alpha+\varphi_\beta-2\Psi_{RP})\rangle = \frac{N_{cluster}}{N_\alpha N_\beta} \times \langle\cos(\varphi_\alpha+\varphi_\beta-2\Psi_{cluster})\rangle \times v_{2,cluster}$$

Multiplicity differences

Flow differences

Copy from Hao-jie Xu's slides

The multiplicity and v2 differences from isobar structure are crucial for the CME search in the isobar collisions at RHIC

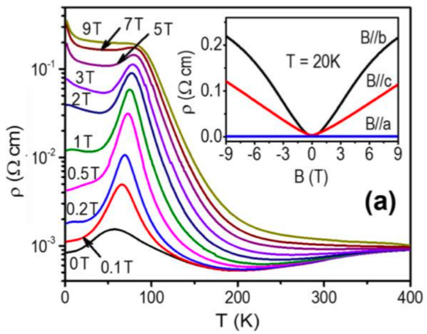
自洽地加入核结构信息是理解同量异位素实验结果关键之一：

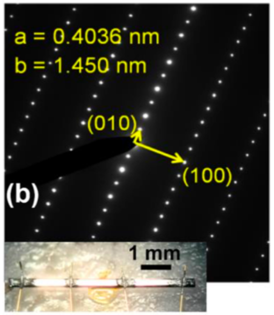
Xu, et al., PRL 2018; Li, et al, PRC 2018; Zhang, Jia, PRL 2022; Deng, Huang, Ma, Wang PRC 2016; Zhao, Xu, Liu, Song, PLB 2023 …

凝聚态物理实验中手征磁效应的观测

Dirac Semi-metal:

- $ZrTe_5$: Nature Physics, 12 , 550–554, (2016)





- Na_3Bi: Science 350, 413
- Cd_3As_2: Nature Commun. 6, 10137

Weyl Semi-metal:

- TaAs: PRX 5, 031023
- NbAs: arXiv: 1506.02283
- NbP: arXiv: 1504.07398
- TaP: arXiv: 1506.06577

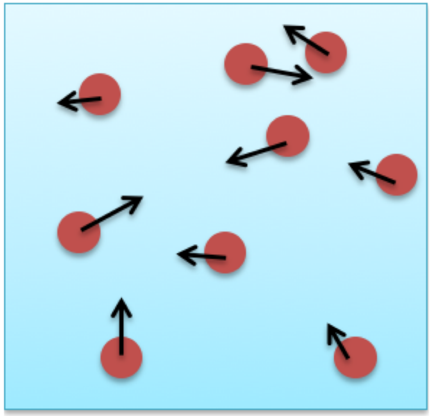
动理学理论

- 基本假设: 粒子平均自由程 >> 粒子碰撞特征长度

- 研究目标：分布函数 f(x,p,t)

在相空间(x+dx, p+dp)可以找到粒子的个数/几率

如：Fermi-Dirac 分布



- 统计物理中描述分布函数演化方程： Boltzmann 方程

该方程描述分布函数随时空的演化

Chiral kinetic theory 手征动理学方程

- **Hamiltonian formulism, effective theory**

  Son, Yamamoto, PRL, (2012); PRD (2013)

- **Path integration**

  Stephanov, Yin, PRL (2012);

  Chen, Son, Stephanov, Yee, Yin, PRL, (2014);

  J.W. Chen, J.Y. Pang, SP, Q. Wang, PRD (2014)

- **Wigner function (Quantum field theory)**
  - **hydrodynamics, equilibrium**

    Gao, Liang, Pu, Q. Wang, X.N. Wang, PRL 109, 232301 (2012) ;

    J.W. Chen, SP, Q. Wang, X.N. Wang, PRL (2013);
  - **out-of-equilibrium, quantum field theory**

    Y. Hidaka, SP, D.L. Yang, PRD(RC) (2017)
  - **Other studies**

    A.P. Huang, S.Z. Su, Y. Jiang, J.F. Liao, P.F. Zhuang, arXiv:1801.03640

- **World-line formulism**

  N. Muller, R, Venugopalan PRD 2017

Also see recent review:

Gao, Liang, Wang, Int.J.Mod.Phys A 36 (2021), 2130001

Hidaka, SP, D.L. Yang, Q. Wang, a Prog.Part.Nucl.Phys. 127 (2022) 103989

为了描述手征输运等量子效应，我们需要将量子修正自洽地加入动理学理论

Wigner 函数定义及其物理含义

- **Wigner 算符:**

$$\hat{W}_{\alpha\beta} = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y}\, \bar{\psi}_{\beta}(x_+)U(x_+, x_-)\psi_{\alpha}(x_-), \quad x = x_+ + x_-, \quad y = x_+ - x_-$$

规范链： $U(x_+, x_-) \equiv e^{-iQ\int_{x-}^{x+} dz^{\mu}A_{\mu}(z)},$

- **Wigner 函数:**

$$W(x,p) = \left\langle : \hat{W}(x,p) : \right\rangle$$

<>: 系综平均； ：：正规乘积

- **物理含义: 量子场论版本的“密度矩阵”**

Vasak, Gyulassy, Elze, Ann. Phys. (N.Y.) 173, 462 (1987);

Elze, Heinz,Phys.Rep.183,81(1989).

无相互作用情况下Wigner函数满足的主方程

- By using the Dirac equation, one can get the master equations for Wigner function without interactions,

$$\gamma_\mu\left(p^\mu + \frac{i}{2}\nabla^\mu\right)W(x,p) = 0, \qquad \nabla^\mu \equiv \partial_x^\mu - QF^\mu{}_\nu\partial_p^\nu$$

- Matrix decomposition

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^5\gamma^\mu\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right],$$

Charge current

$$\mathscr{V}^\mu = \int \frac{d^4y}{(2\pi)^4}e^{-ip\cdot y} <: \bar{\psi}_\beta\left(x+\frac{1}{2}y\right)\gamma^\mu U\left(x+\frac{1}{2}y, x-\frac{1}{2}y\right)\psi_\alpha\left(x-\frac{1}{2}y\right) :>$$

Chiral current

$$\mathscr{A}_\mu = \int \frac{d^4y}{(2\pi)^4}e^{-ip\cdot y} <: \bar{\psi}_\beta\left(x+\frac{1}{2}y\right)\gamma^\mu\gamma^5 U\left(x+\frac{1}{2}y, x-\frac{1}{2}y\right)\psi_\alpha\left(x-\frac{1}{2}y\right) :>$$

$\hbar$ 展开下逐级自洽求解Wigner函数

- **We consider the $\hbar$ (gradient) expansion,**

$$\mathscr{J}^s_\mu(x,p) = \frac{1}{2}[\mathscr{V}_\mu(x,p) + s\mathscr{A}_\mu(x,p)],$$

$s =$ +, right handed; -, left handed

$$\mathscr{J}^s_\mu(x,p) = \mathscr{J}^s_{\mu,(0)}(x,p) + \mathscr{J}^s_{\mu,(1)}(x,p) + \cdots ,$$

- **Solutions up to $\hbar^2$,**

$$\mathscr{J}^\rho_{(0)}(x,p) = p^\rho f_{(0)}(x,p)\delta(p^2),$$

$$\mathscr{J}^\mu_{(1)} = -\frac{s}{2}\widetilde{\Omega}^{\mu\lambda}p_\lambda f'_{(0)}\delta(p^2) + s\widetilde{F}^{\mu\nu}p_\nu f_{(0)}\delta'(p^2).$$

$$\begin{aligned}
\mathscr{J}^{(2)}_\mu &= X^{(2)}_\mu\delta(p^2) + \frac{s}{2p^2}\epsilon_{\mu\nu\rho\sigma}p^\nu\nabla^\rho\mathscr{J}^\sigma_{(1)} \\
&= \frac{1}{4p^2}\left(p_\mu\Omega_{\gamma\beta}p^\beta - p^2\Omega_{\gamma\mu}\right)\Omega^{\gamma\lambda}p_\lambda f''_{(0)}\delta(p^2) \\
&\quad + \frac{1}{(p^2)^2}\left(p_\mu F_{\gamma\beta}p^\beta - p^2F_{\gamma\mu}\right)\Omega^{\gamma\lambda}p_\lambda f'_{(0)}\delta(p^2) \\
&\quad + \frac{2}{(p^2)^3}\left(p_\mu F_{\gamma\beta}p^\beta - p^2F_{\gamma\mu}\right)F^{\gamma\lambda}p_\lambda f_{(0)}\delta(p^2),
\end{aligned}$$

$$\widetilde{F}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$$\widetilde{\Omega}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\rho\sigma}\Omega_{\rho\sigma}$$

$$\delta'(x) = -(1/x)\delta(x).$$

各种宏观流及手征反常

- **Integral over momentum p, we get CME and other quantum transport effects**

$$j_s^\mu = \int d^4p \mathscr{J}_s^\mu = n_s u^\mu + \xi_{B,s} B^\mu + \xi_s \omega^\mu,$$

Charge current

$$j^\mu = \sum_{s=\pm} j_s^\mu = n u^\mu + \xi_B B^\mu + \xi \omega^\mu,$$

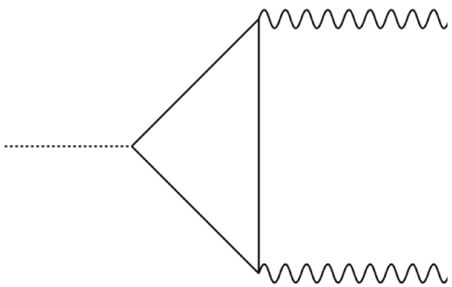
Chiral current

$$j_5^\mu = \sum_{s=\pm} s j_s^\mu = n_5 u^\mu + \xi_{B5} B^\mu + \xi_5 \omega^\mu,$$

$$\xi_B = \frac{e}{2\pi^2}\mu_5, \quad \xi = \frac{1}{\pi^2}\mu\mu_5, \quad \xi_{B5} = \frac{e}{2\pi^2}\mu, \quad \xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}\left(\mu^2 + \mu_5^2\right)$$

- **We also reproduce the chiral anomaly from the kinetic theory.**

$$\partial_\mu j^\mu = 0, \qquad \partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2} E \cdot B.$$



Gao, Liang, Pu, Q. Wang, X.N. Wang, PRL 109, 232301 (2012)

Chiral kinetic equation 手征动理学方程

- Chiral kinetic theory is a useful tool to study CME.

$$\sqrt{G}\partial_t f + \sqrt{G}\dot{\mathbf{x}} \cdot \nabla_x f + \sqrt{G}\dot{\mathbf{p}} \cdot \nabla_p f = C[f].$$

- Particle's effective velocity:

$$\sqrt{G}\dot{\mathbf{x}} = \frac{\partial \varepsilon}{\partial \mathbf{p}} + \hbar \left(\frac{\partial \varepsilon}{\partial \mathbf{p}} \cdot \mathbf{\Omega} \right) \mathbf{B} + \hbar \mathbf{E} \times \mathbf{\Omega},$$

- Effective force:

$$\sqrt{G}\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{B} + \hbar (\mathbf{E} \cdot \mathbf{B}) \mathbf{\Omega},$$

- Berry curvature

$$\mathbf{\Omega} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}, \qquad \sqrt{G} = 1 + \hbar \mathbf{B} \cdot \mathbf{\Omega},$$

Chen, SP, Q. Wang, X.N. Wang, PRL (2013);

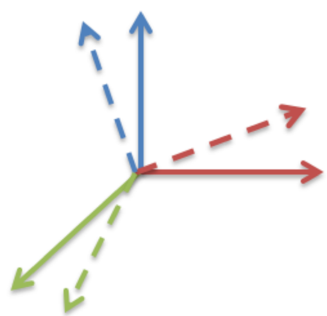
非平庸Lorentz变换及Side-jump效应

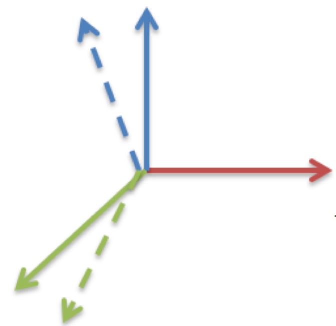
- **The subgroup for Lorentz group for massless fermions and massive fermions are different.**

Massive particles:
(Rest frame)

$$p^\mu = (m, 0, 0, 0)$$

Subgroup: SO(3)





Massless particles:
(No rest frame)

$$p^\mu = (|p_z|, 0, 0, p_z)$$

Subgroup: ISO(2)

- **From quantum field theory, the distribution function is no longer a scalar.**

$$f'(x', p', t') = f(x', p', t') + \hbar N^\mu (\partial_\mu^x + F_{\nu\mu}\partial_p^\nu) f,$$

Infinitesimal Lorentz Transform

$$\delta \mathbf{x} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|},$$

$$\delta \mathbf{p} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|} \times \mathbf{B}$$

Chen, Son, Stephanov, PRL, (2015); Y. Hidaka, SP, D.L. Yang, PRD (2016)

其它各种手征输运

- **Chiral vorticial effects**

Vilenkin PRD (1979) ; Erdmenger, et. al. JHEP (2009); Banerjee, et. al. JHEP(2011); Son, Surowka, PRL (2009); Landsteiner, et. al. PRL(2011);…

- **Chiral electric separation effects**

Huang, Liao, PRL (2013); SP, Wu, Yang, PRD (2014);…

- **Chiral Hall separation effects**

SP, Wu, Yang, PRD (2015);…

- **Other electromagnetic responses**

Chen, Ishii, SP, Yamamoto, PRD (2016); Gorbar, et. al. PRD (2016), PRD (2017), PRL (2017); Hidaka, SP, Yang, PRD (2017), PRD (2018); …

- **Effects from non-abelian Berry phase**

Chen, Pang, SP, Wang, PRD (201;3); SP, Yamamoto, NPB (2018);…

Also see reviews:

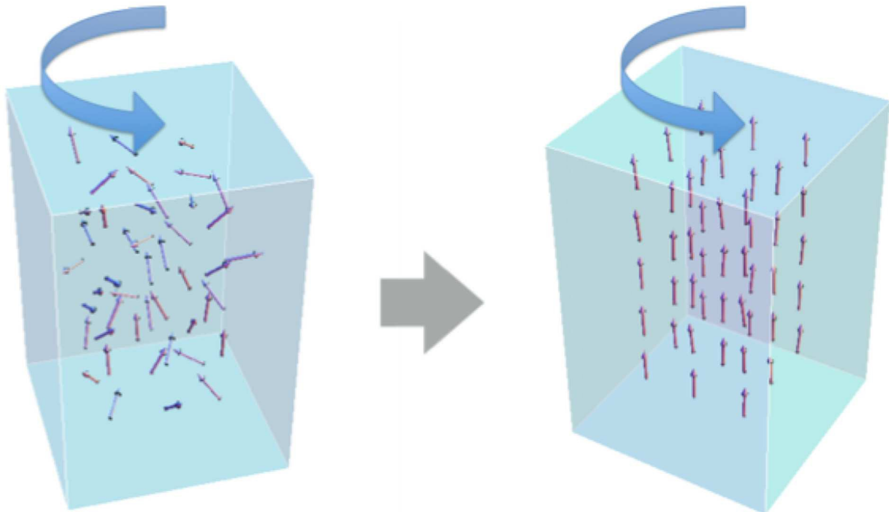
Y. Hidaka, SP, Q.Wang, D.L. Yang, Prog.Part.Nucl.Phys. 127 (2022) 103989

J.H. Gao, G.L. Ma, SP, Q. Wang, Nucl.Sci.Tech. 31 (2020) 9, 90

相对论重离子碰撞中Lambda超子极化和矢量介子自旋排列

量子动力学理论及碰撞效应

Barnet效应及Einstein-de Hass效应



Barnett effect:

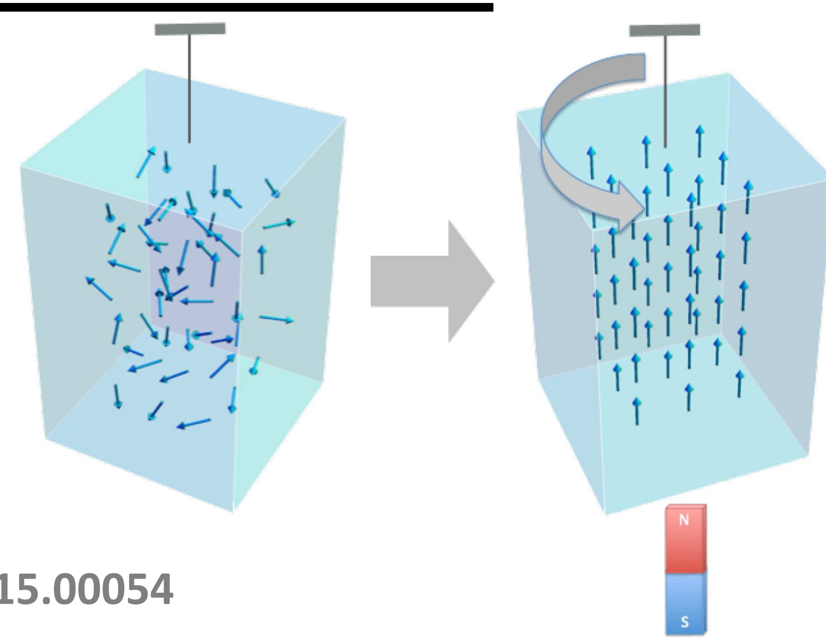
Rotation $\Longrightarrow$ Magnetization

Barnett, Magnetization by rotation, Phys Rev. (1915) 6:239–70.

Einstein-de Haas effect:

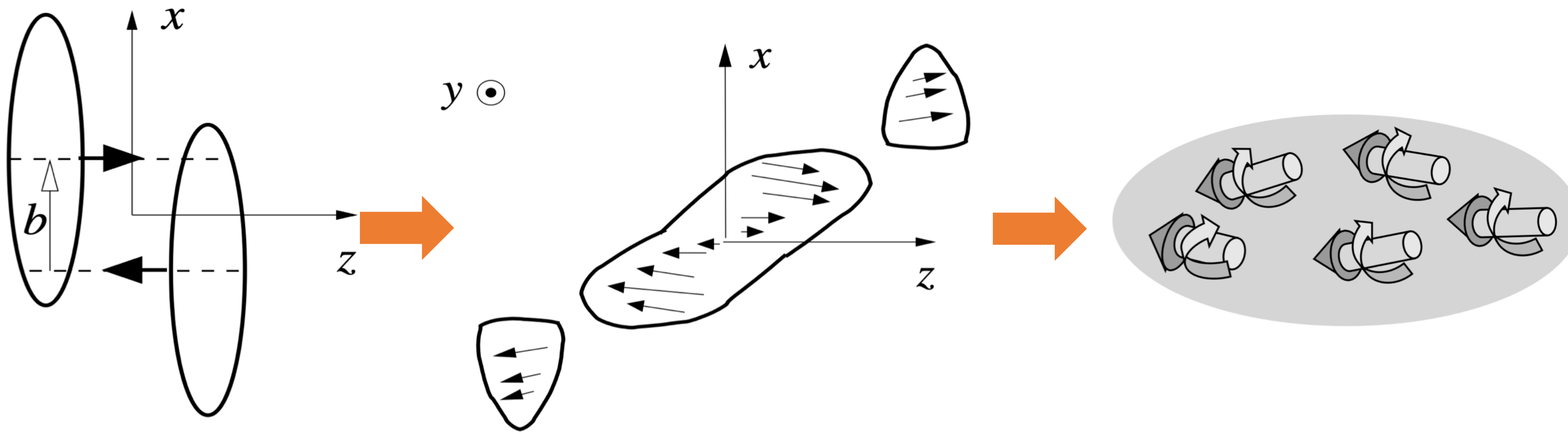
Magnetization $\Longrightarrow$ Rotation

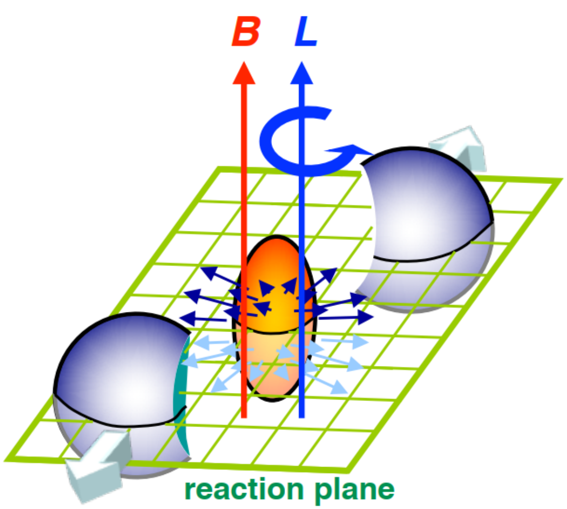
Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents. Verh Dtsch Phys Ges. (1915) 17:152.



Figures: copy from paper doi: 10.3389/fphy.2015.00054

重离子碰撞中初始轨道角动量导致末态粒子极化



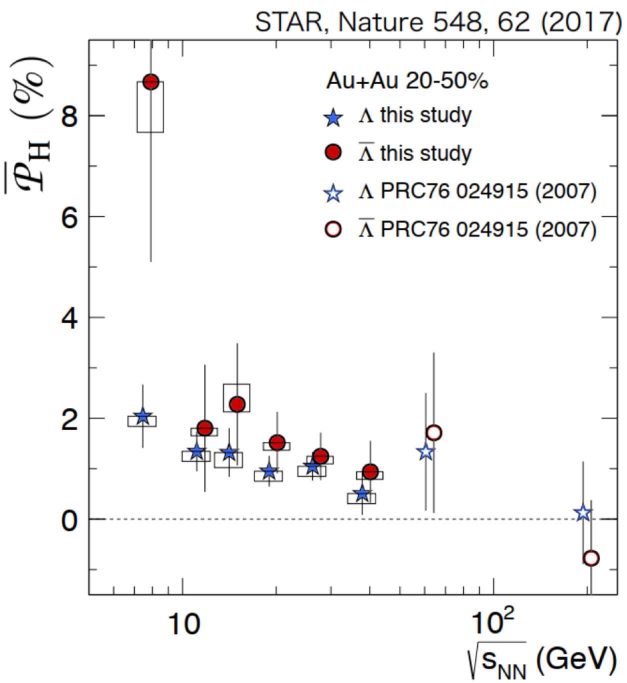


- **Huge global orbital angular momenta ($L \sim 10^5 \hbar$) are produced in HIC.**

- **Global orbital angular momentum leads to the polarizations of Λ hyperons and vector mesons through spin-orbital coupling.**

  Liang, Wang, PRL (2005); PLB (2005);
  Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Λ和$\overline{\Lambda}$ 超子的整体极化

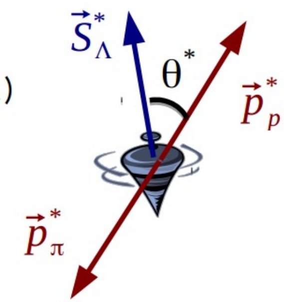


parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

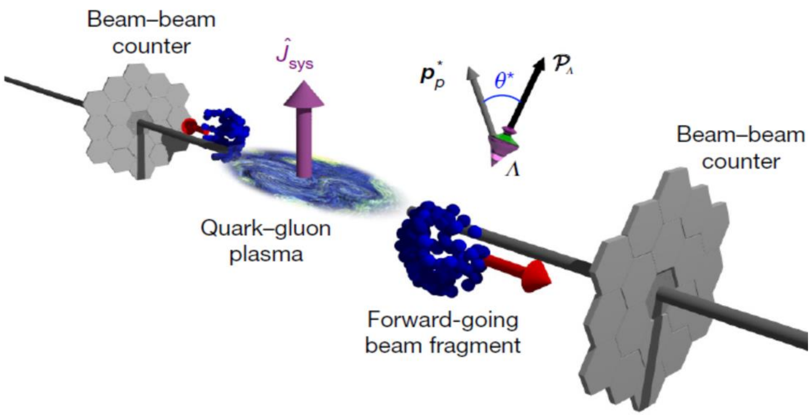
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}_{\mathbf{\Lambda}} \cdot \mathbf{p}_{\mathbf{p}}^*)$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



$$\Lambda \rightarrow p + \pi^+$$

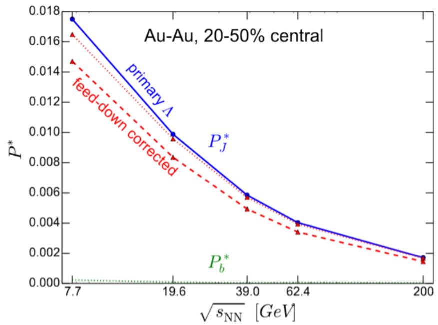
(BR: 63.9%, cτ~7.9 cm)



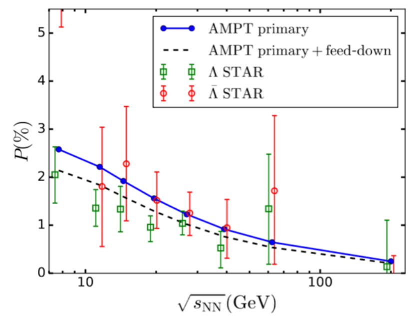
- **STAR实验结果可以反推出，QGP的涡旋可以高达ω = (9 ± 1)x10²¹/s。**
- **这是迄今为止发现的旋转最快的系统。**

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)
…

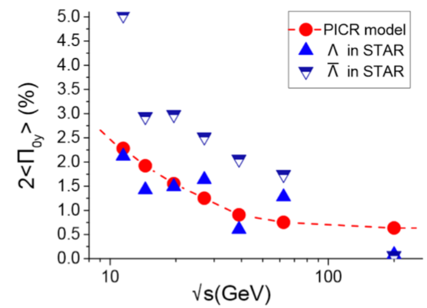
各类唯象模型对整体极化的计算



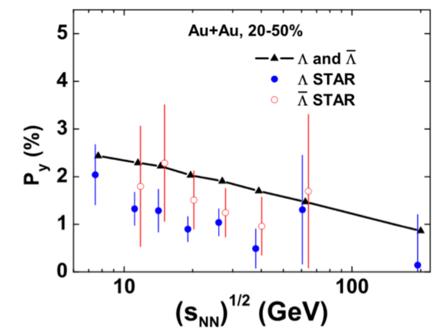
Karpenko, Becattini, EPJC(2017)



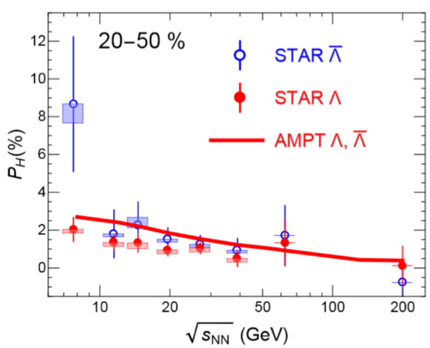
Li, Pang, Wang, Xia PRC(2017)



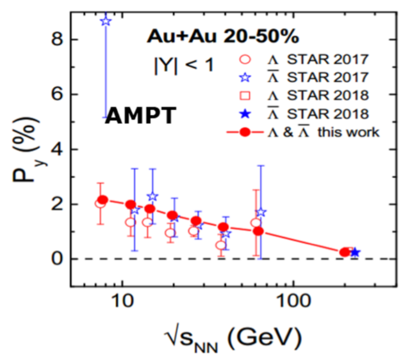
Xie, Wang, Csernai, PRC(2017)



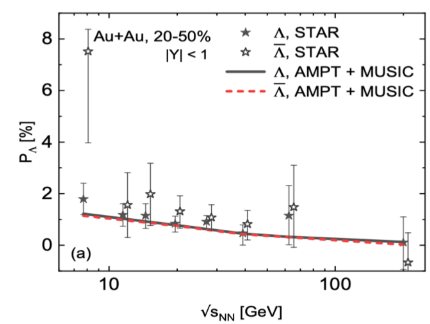
Sun, Ko, PRC(2017)



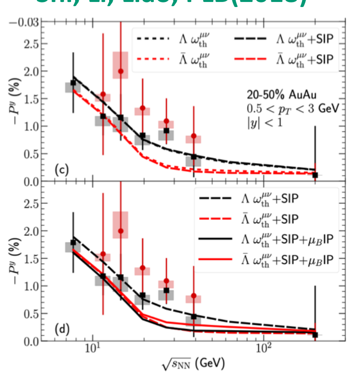
Shi, Li, Liao, PLB(2018)



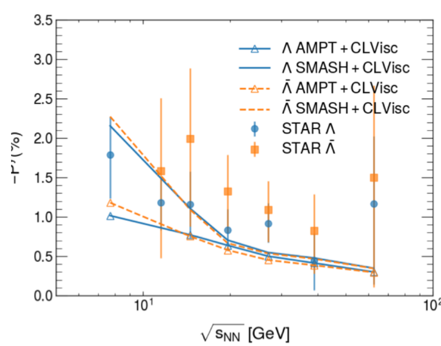
Wei, Deng, Huang, PRC(2019)



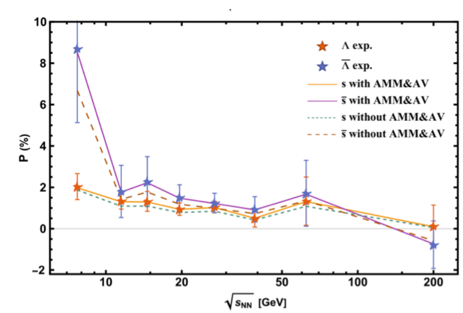
Fu, Xu, Huang, Song, PRC (2021)



S. Ryu, V. Jupic, C. Shen, PRC (2021)

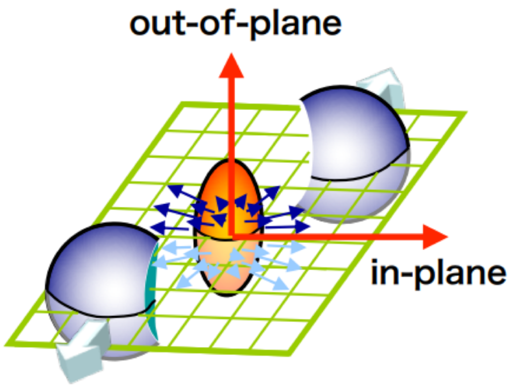


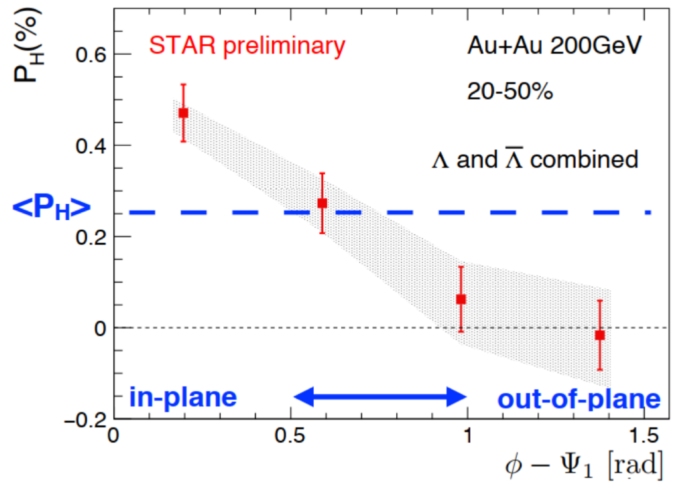
Y.X. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

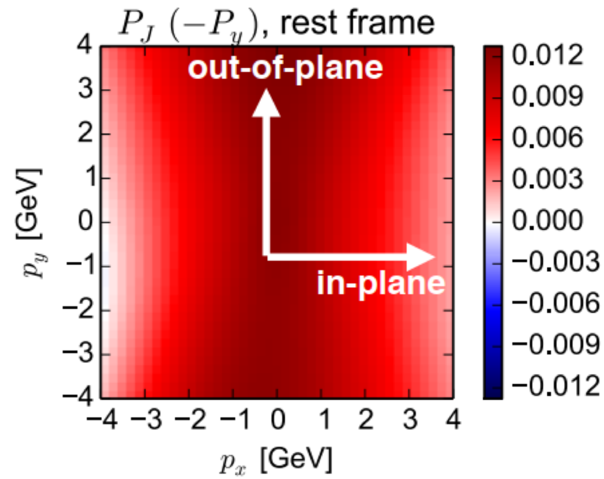


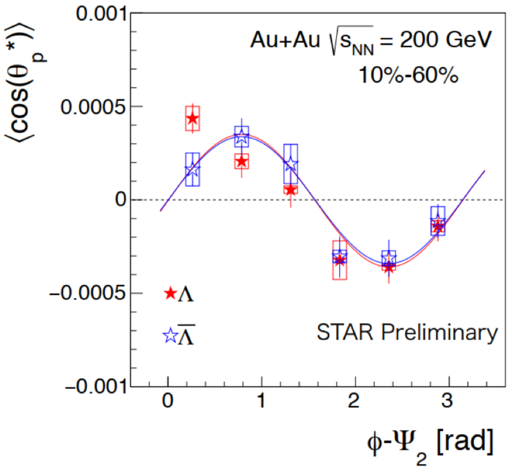
Xu, Lin, Huang, Huang, PRDL (2022)

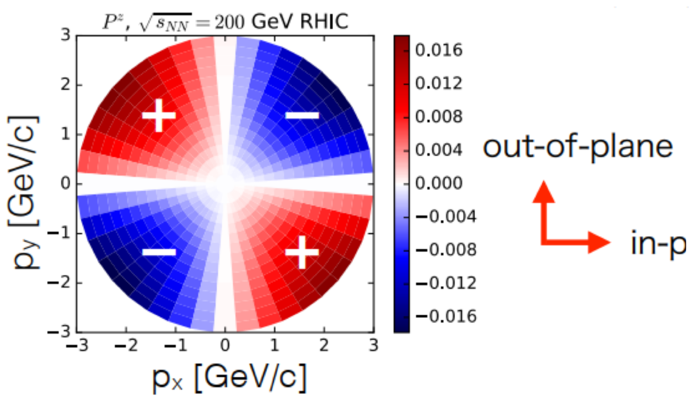
局域极化与符号疑难

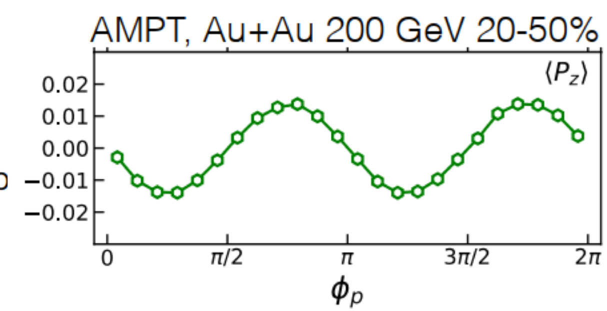












- **UrQMD :**
  Becattini, Karpenko, PRL (2018)
- **AMPT:**
  Xia, Li, Tang, Wang, PRC (2018)

理论研究进展

- **Spin hydrodynamics (macroscopic approach)**
  Florkowski, Friman, Jaiswal, Ryblewski, Speranza (2017-2018);
  Montenegro, Tinti, Torrieri (2017-2019);
  Hattori, Hongo, Huang, Matsuo, Taya PLB(2019) ; arXiv: 2201.12390; arXiv: 2205.08051
  Fukushima, SP, Lecture Note (2020); PLB(2021); Wang, Fang, SP, PRD(2021); Wang, Xie, Fang, SP, PRD (2022); …
  S.Y. Li, M.A Stephanov, H.U Yee, arXiv:2011.12318
  D. She, A. Huang, D.F. Hou, J.F Liao, arXiv: 2105.04060
  Weickgenannt, Wanger, Speranze, Rischke, PRD 2022; PRD 2022; Weickgennatt, Wanger, Speranza, PRD 2022; arXiv:2306.05936;
  Peng, Zhang, Sheng, Wang, CPL 2021
- **Quantum kinetic theory with collisions (microscopic approach)**
  Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD 100, 056018 (2019)
  Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.
  Liu, Mameda, Huang, arXiv:2002.03753.
  Gao, Liang, PRD 2019
  Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019) ；Z.Y. Wang, arXiv:2205.09334;
  Li ,Yee, PRD100, 056022 (2019)
  Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184; Lin, Wang, arXiv:2206.12573
  Fang, SP, Yang, PRD (2022)
- **Other approaches:**
  Side-jump effect Liu, Sun, Ko PRL(2020)
  Mesonic mean-field Csernai, Kapusta, Welle, PRC(2019)
  Using different vorticity Wu, Pang, Huang, Wang, PRR (2019)
- **Recent reviews:**
  Gao, Ma, SP, Wang, NST (2020)
  Gao, Liang, Wang, IJMPA (2021)
  Hidaka, SP, Yang, Wang, PPNP (2022)

极化矢量与相空间轴矢量流

- **The polarization tensor is connected to the axial current in phase space by modified Cooper-Frye formula**
  Karpenko, Becattini, EPJC. (2017); Fang, Pang, QW, Wang, PRC (2016)

$$\mathcal{S}^{\mu}(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^{\mu}(p, X)}{2m_{\Lambda} \int d\Sigma \cdot \mathcal{N}(p, X)},$$

极化矢量 ~ 相空间的正则自旋（轴矢量流）

- **For massless fermions, the left and right handed currents can be derived by quantum kinetic theory,**

$$\mathcal{J}_5^{\mu} = \mathcal{J}_{\text{thermal}}^{\mu} + \mathcal{J}_{\text{shear}}^{\mu} + \mathcal{J}_{\text{accT}}^{\mu} + \mathcal{J}_{\text{chemical}}^{\mu} + \mathcal{J}_{\text{EB}}^{\mu},$$

Y. Hidaka, SP, and D.L. Yang, Phys. Rev. D97, 016004 (2018)

导致极化的各种效应

$$\mathcal{S}^{\mu}(\mathbf{p}) = \mathcal{S}^{\mu}_{\text{thermal}} + \mathcal{S}^{\mu}_{\text{shear}} + \mathcal{S}^{\mu}_{\text{accT}} + \mathcal{S}^{\mu}_{\text{chemical}} + \mathcal{S}^{\mu}_{\text{EB}}$$

Y. Hidaka, SP, D.L. Yang, PRD97, 016004 (2018); C. Yi, SP, D.L. Yang, PRC 2021

Thermal vorticity 热涡旋

$$\mathcal{S}^{\mu}_{\text{thermal}}(\mathbf{p}) = \frac{\hbar}{8m_{\Lambda}N}\int d\Sigma^{\sigma}p_{\sigma}f_V^{(0)}(1-f_V^{(0)})\epsilon^{\mu\nu\alpha\beta}p_{\nu}\partial_{\alpha}\frac{u_{\beta}}{T},$$

Shear viscous tensor 剪切粘滞张量

$$\mathcal{S}^{\mu}_{\text{shear}}(\mathbf{p}) = -\frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_V^{(0)}(1-f_V^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}}{(u\cdot p)T}\frac{1}{2}\{p^{\sigma}(\partial_{\sigma}u_{\nu}+\partial_{\nu}u_{\sigma})-Du_{\nu}\}$$

Fluid acceleration 流体加速

$$\mathcal{S}^{\mu}_{\text{accT}}(\mathbf{p}) = -\frac{\hbar}{8m_{\Lambda}N}\int d\Sigma\cdot pf_V^{(0)}(1-f_V^{(0)})\frac{1}{T}\epsilon^{\mu\nu\alpha\beta}p_{\nu}u_{\alpha}\left(Du_{\beta}-\frac{1}{T}\partial_{\beta}T\right),$$

Gradient of chemical potential 化学势/温度梯度

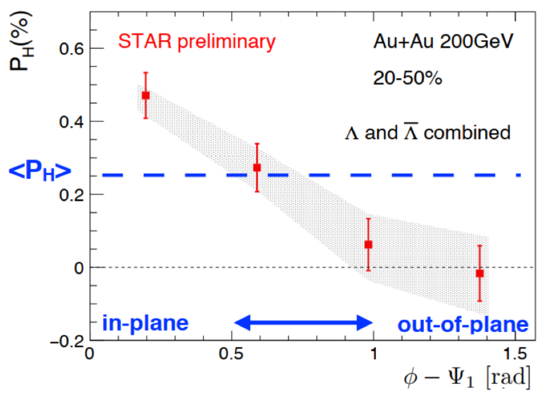
$$\mathcal{S}^{\mu}_{\text{chemical}}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_V^{(0)}(1-f_V^{(0)})\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}\partial_{\nu}\frac{\mu}{T},$$

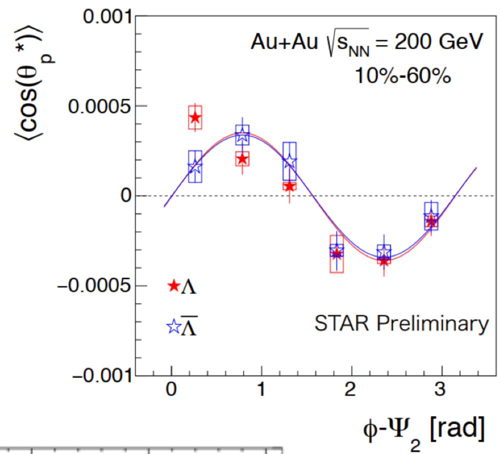
Electromagnetic fields 电磁场

$$\mathcal{S}^{\mu}_{\text{EB}}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot pf_V^{(0)}(1-f_V^{(0)})\left(\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}E_{\nu}+\frac{B^{\mu}}{T}\right)$$

由剪切粘滞张量导致的极化效应

$$\mathcal{S}^{\mu}_{\text{shear}}(\mathbf{p}) = -\frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot p f_V^{(0)}(1-f_V^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}}{(u\cdot p)T}\frac{1}{2}\{p^{\sigma}(\partial_{\sigma}u_{\nu}+\partial_{\nu}u_{\sigma})-Du_{\nu}\}$$





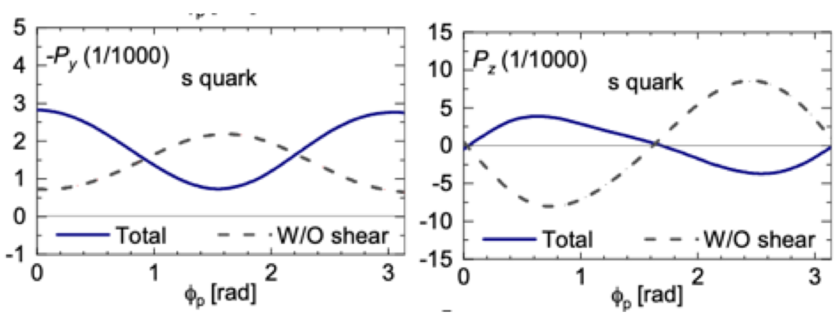
Early works:
(thermal vorticity only)

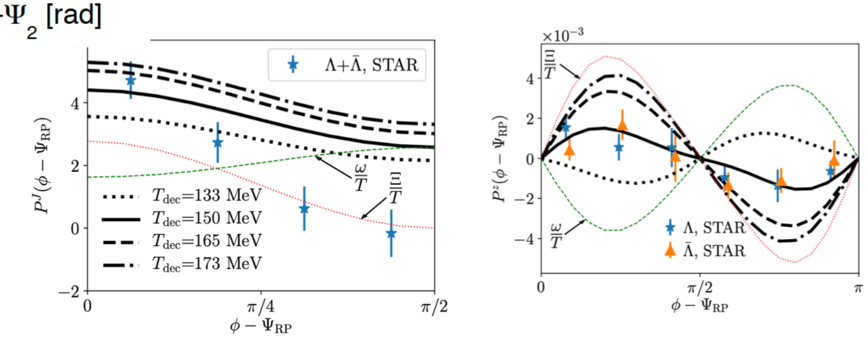
- **UrQMD :**

Becattini, Karpenko, PRL (2018)

- **AMPT:**

Xia, Li, Tang, Wang, PRC (2018)





s quark scenarios (Thermal vorticity + shear)

Fu, Liu, Pang, Song, Yin, PRL 2021

Isothermal equilibrium
(Thermal vorticity + shear)

Becattini, Buzzegoli, Palermo, Inghirami, Karpenko, PRL 2021

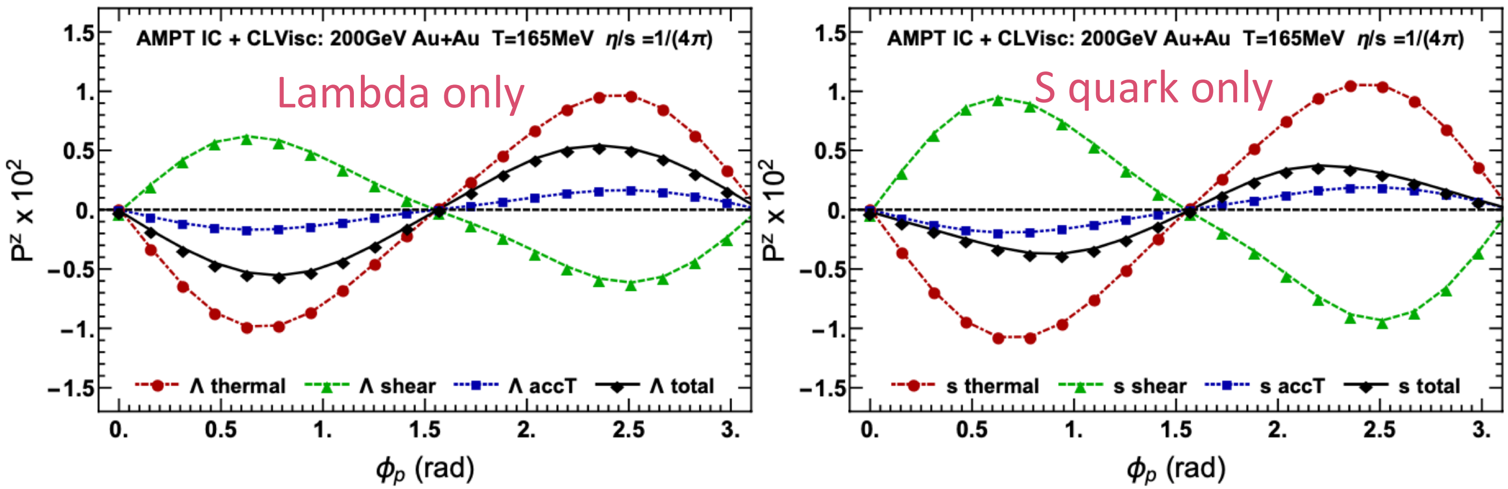
Also see:

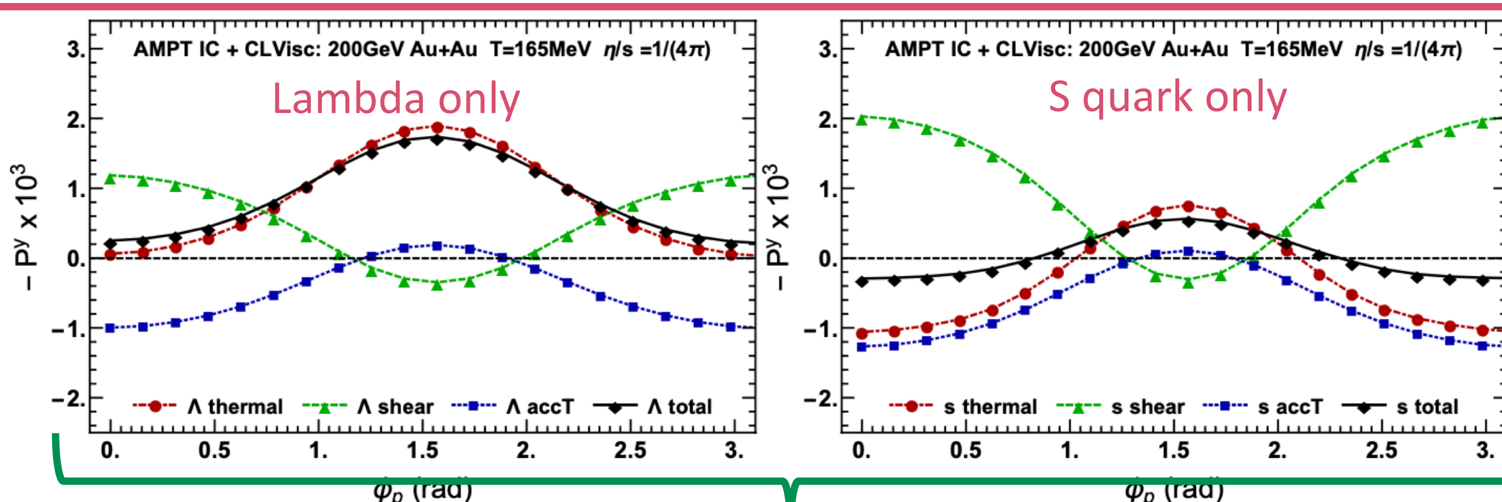
Yi, Pu, Yang, PRC (2021); Yi, Wu, Qin, Pu, PRC (2022)
Ryu, Jupic, Shen, PRC (2021)

由剪切粘滞导致的极化效应

Polarization along beam direction

C. Yi, SP, D.L. Yang, PRC 2021



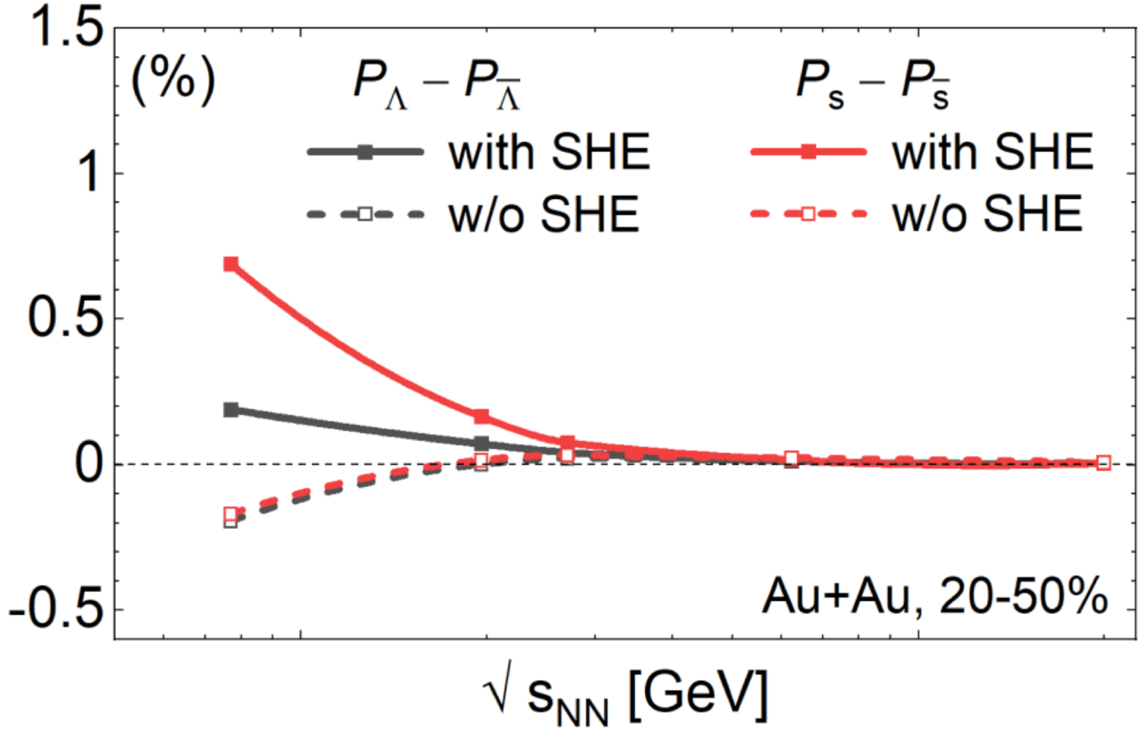


Polarization along out-of-plane direction

由化学势梯度导致的极化/自旋Hall效应

$$\mathcal{S}^{\mu}_{\text{chemical}}(\mathbf{p}) = \frac{\hbar}{4m_{\Lambda}N}\int d\Sigma\cdot p f_V^{(0)}(1-f_V^{(0)})\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}\boxed{\partial_{\nu}\frac{\mu}{T}},$$

Y. Hidaka, SP, D.L. Yang, PRD97, 016004 (2018); C. Yi, SP, D.L. Yang, PRC 2021



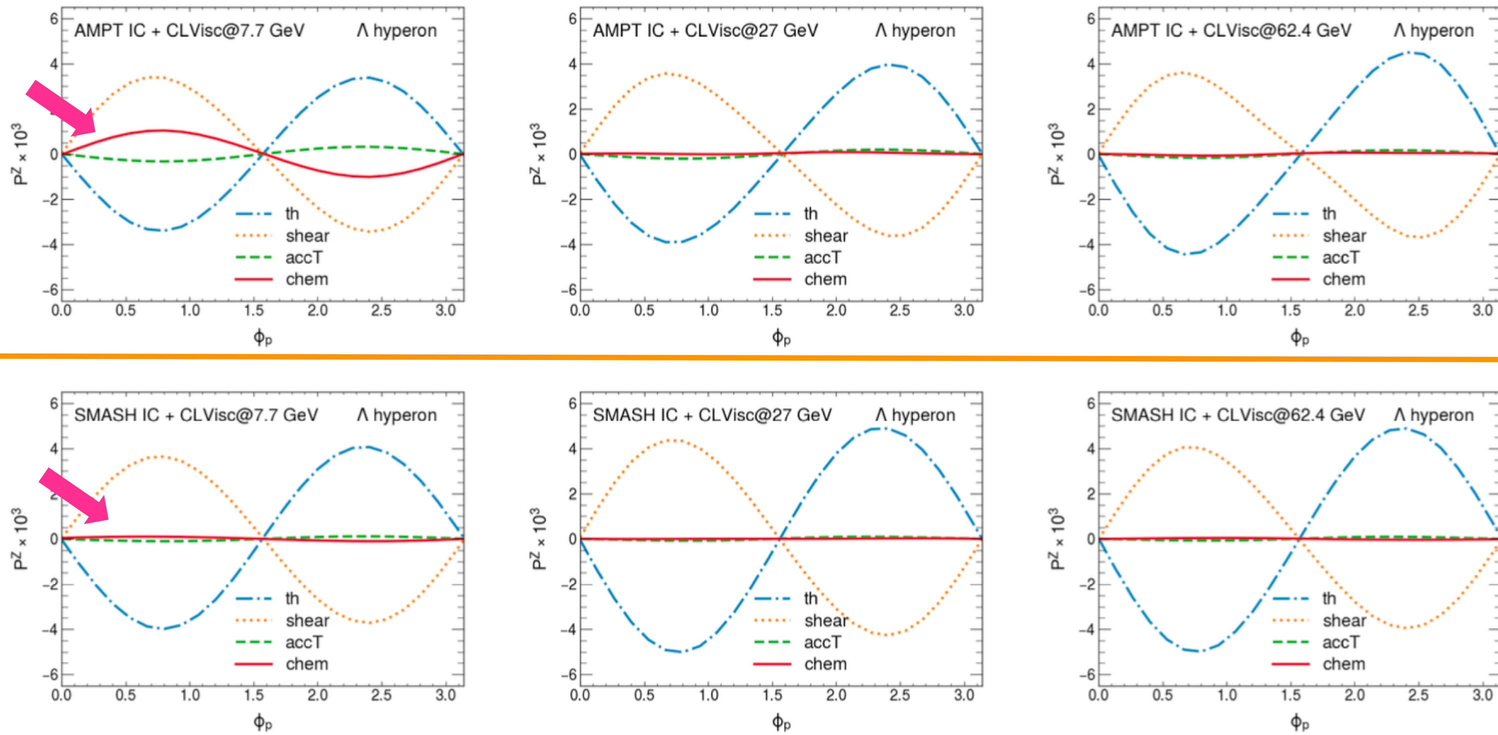
Fu, Pang, Song, Yin, 2208.00430

由化学势梯度导致的极化/自旋Hall效应: 探测初始条件

7.7 GeV | 27 GeV | 62.4 GeV

From AMPT Initial condition

From SMASH Initial condition



Red lines: contributions from spin Hall effect

Polarization induced by SHE is almost zero at 27, 62.4GeV and it depends on the initial conditions at 7.7 GeV. For SMASH, Pz is still almost vanishing at 7.7 GeV.

X.Y. Wu, C. Yi, G.Y. Qin, SP, PRC (2022)

量子动理学理论及碰撞项修正

- **Collision term with quantum corrections**

  Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD (2019); PRL (2021)

  Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.

  Liu, Mameda, Huang, arXiv:2002.03753.

  Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019); Wang, Guo, Zhuang, EPJC (2021);Wang, Zhuang, arXiv:2105.00915

  Li ,Yee, PRD100, 056022 (2019)

  Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184

  Fang, SP, Yang, PRD (2022)

  Z.Y. Wang, arXiv:2205.09334; Lin, Wang, arXiv:2206.12573

  …

Recent reviews:

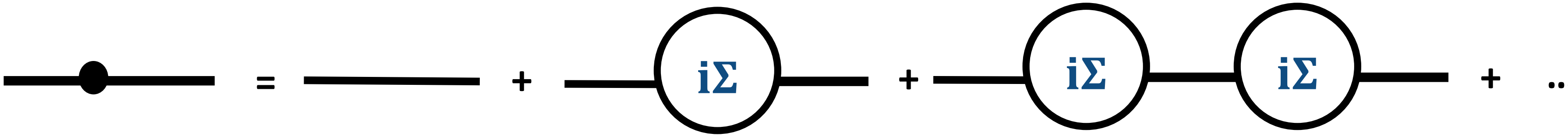
Gao, Ma, SP, Wang, NST 31 (2020) 9, 90

Gao, Liang, Wang, IJMPA 36 (2021), 2130001

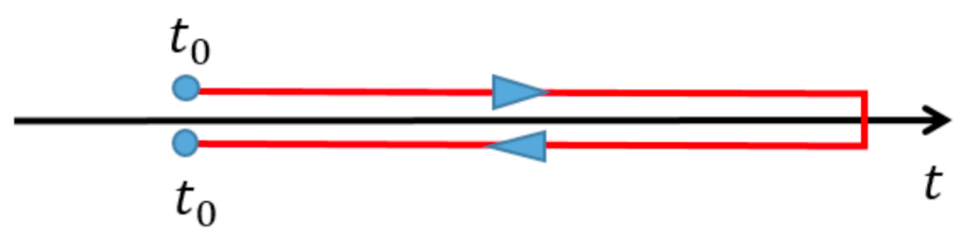
Hidaka, SP, Yang, Wang, arXiv:2201.07644

Closed-time-path(闭时) Wigner函数

$$S = S_0 - iS_0\Sigma S = S_0 - iS\Sigma S_0,$$



- S: full propagator
- S_0: free propagator
- Σ: self energy



$$G(x_1,x_2) = \begin{pmatrix} G^{++}(x_1,x_2) & G^{+-}(x_1,x_2) \\ G^{-+}(x_1,x_2) & G^{--}(x_1,x_2) \end{pmatrix} = \begin{pmatrix} G^{F}(x_1,x_2) & \pm G^{<}(x_1,x_2) \\ G^{>}(x_1,x_2) & G^{\overline{F}}(x_1,x_2) \end{pmatrix}$$

$$\begin{aligned} G^{F}_{\alpha\beta}(x_1,x_2) &= \langle T\psi_\alpha(x_1)\overline{\psi}_\beta(x_2)\rangle, \\ G^{\overline{F}}_{\alpha\beta}(x_1,x_2) &= \langle T_A\psi_\alpha(x_1)\overline{\psi}_\beta(x_2)\rangle, \\ G^{<}_{\alpha\beta}(x_1,x_2) &= \langle \overline{\psi}_\beta(x_2)\psi_\alpha(x_1)\rangle, \\ G^{>}_{\alpha\beta}(x_1,x_2) &= \langle \psi_\alpha(x_1)\overline{\psi}_\beta(x_2)\rangle, \end{aligned}$$

$$\begin{aligned} i\sigma\cdot D_{x_1} S^< &= \Sigma^R S^< - \Sigma^< S^A, \\ -S^< i\sigma\cdot \overleftarrow{D}^\dagger_{x_2} &= -S^R\Sigma^< + S^<\Sigma^A, \end{aligned}$$

NJL相互作用类型的碰撞项(例)

- **An example for collision kernel of NJL type interactions:**

Eq. for Particle distribution function

$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[f^{(1)}(x,p)\right] = -\frac{1}{\pi\hbar}\int_0^\infty dp_0\, \mathrm{Im}\mathrm{Tr}\left(I^{(2)}_{\mathrm{coll}}\right) - \frac{1}{2\pi\hbar m}\mathrm{Re}\,\mathrm{Tr}\left(\gamma\cdot\partial_x I^{(1)}_{\mathrm{coll}}\right)$$

$$\equiv \mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right) + \mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll,}\,\nabla}\right) + \mathscr{C}_{\mathrm{scalar}}\left(I^{(0)}_{\mathrm{coll,PB}}\right) + \mathscr{C}_{\mathrm{scalar}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right),$$

Eq. for Spin distribution function

$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[n_j^{(+)\mu}\tau_j^T f^{(1)}(x,p)\right] = \frac{1}{2\pi\hbar m}\int_0^\infty dp_0\left[\epsilon^{\mu\nu\alpha\beta}p_\nu \mathrm{Im}\mathrm{Tr}\left(\sigma_{\alpha\beta} I^{(2)}_{\mathrm{coll}}\right) + \mathrm{Re}\,\mathrm{Tr}\left(\gamma^5\partial_x^\mu I^{(1)}_{\mathrm{coll}}\right)\right]$$

$$\equiv \mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right) + \mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll,}\,\nabla}\right) + \mathscr{C}^\mu_{\mathrm{pol}}\left(I^{(0)}_{\mathrm{coll,PB}}\right) + \mathscr{C}^\mu_{\mathrm{pol}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right).$$

$$\mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right) \quad \mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll,}\,\nabla}\right) \quad \mathscr{C}_{\mathrm{scalar}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right) \quad \mathscr{C}_{\mathrm{scalar}}\left(I^{(0)}_{\mathrm{coll,PB}}\right)$$

$$\mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right) \quad \mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll,}\,\nabla}\right) \quad \mathscr{C}^\mu_{\mathrm{pol}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right) \quad \mathscr{C}^\mu_{\mathrm{pol}}\left(I^{(0)}_{\mathrm{coll,PB}}\right)$$

Perturbative Correction to Ordinary terms

Non-local terms related to the space derivatives may be the key to describe the spin-orbital transformation.

Sheng, Weickgenannt, Speranza, Rischke, Wang PRD (2021)

规范相互作用下量子动理学理论：自旋Boltzmann方程(I)

- **We have derived collision kernel for QED in HTL approximation.**

Eq. for Particle distribution function

$$(p\cdot\partial)f_V^<(x,p)=\mathcal{C}_V^{\rm HTL}[f_V]+\mathcal{O}(\hbar^2),$$

Eq. for Spin distribution function

$$(p\cdot\partial)f_A^<(x,p)+\hbar\partial_\mu S^{\mu\nu}_{(u)}\partial_\nu f_V^<(x,p)=\mathcal{C}_A^{\rm HTL}[f_V,f_A]+\mathcal{O}(\hbar^2),$$

- **The real QED type collision kernel for axial part:**

$$\begin{aligned}
\mathcal{C}_A[f_V,f_A]=&-\frac{e^4\delta(p^2)}{8\pi^2|\boldsymbol{p}|}\ln\frac{T}{m_D}\Bigg\{\frac{2\pi^2}{3\beta^2}|\boldsymbol{p}|F(p)f_A^<(p)+\frac{\pi^2}{3\beta^2}|\boldsymbol{p}|^2F(p)[(\hat{p}_\perp\cdot\partial_{p_\perp})-\frac{1}{\beta}(\partial_{p_\perp}\cdot\partial_{p_\perp})]f_A^<(p)\\
&-\frac{2\pi^2}{3\beta^2}|\boldsymbol{p}|^2f_A^<(p)(\hat{p}_\perp\cdot\partial_{p_\perp})f_V^<(p)+\hbar F(p)|\boldsymbol{p}|H_{3,\alpha}\partial^\alpha_{p_\perp}f_V^<(p)\\
&-\hbar\frac{\pi^2}{12\beta^2}F(p)|\boldsymbol{p}|\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\nu}u_\beta\partial_{p_\perp,\rho}\partial_\alpha f_V^<(p)+\hbar\frac{\pi^2}{6\beta^3}\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\rho}u_\beta\partial_{p_\perp,\nu}\partial_\alpha f_V^<(p)\\
&+\hbar\frac{\pi^2}{6\beta^2}\epsilon^{\mu\xi\lambda\kappa}p_\lambda u_\kappa\partial_\xi f_V^<(p)\partial_{p_\perp,\mu}f_V^<(p)\\
&-\hbar\frac{\pi^2}{12\beta^3}|\boldsymbol{p}|\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\nu}u_\beta\hat{p}_{\perp,(\gamma}g_{\lambda)\rho}\hat{p}_{\perp,\lambda}\partial^\lambda_{p_\perp}\partial^\gamma_{p_\perp}\partial_\alpha f_V^<(p)\Bigg\}+\mathcal{O}(\hbar^2).
\end{aligned}$$

S. Fang, SP, D.L. Yang, PRD (2022)

自旋Boltzmann方程 (II): 粒子数平衡态下自旋演化方程

- **We have proved that dynamical spin polarization for a probe is much slower than its thermalization.**

$$\frac{\text{Spin polarization time}}{\text{Thermalization time}} \approx \frac{\Gamma_A(p)}{\Gamma_V(p)} \approx \frac{\hbar H_{3,\alpha}}{T^2|\boldsymbol{p}|} \sim \mathcal{O}\left(\frac{\partial}{|\boldsymbol{p}|}\right),$$

Also see Wang, Guo, Zhuang, EPJC (2021);Wang, Zhuang, arXiv:2105.00915

- **We also derive the Boltzmann equation for spin evolution:**

$$(p\cdot\partial)f_A^<(x,p) + \hbar\partial_\mu S_{(u)}^{\mu\nu}\partial_\nu f_{V,leq}^<(x,p) = \mathcal{C}_A^{\text{HTL}}[f_{V,leq}, f_A] + \mathcal{O}(\hbar^2),$$

$$\begin{aligned} C_A^{\text{HTL}}[f_{V,leq}, f_A] = &-\frac{e^4}{16\pi^3}\frac{\pi^2}{3\beta^2}\ln\frac{T}{m_D}\Big\{2\left(f_{V,leq}^>(p) - f_{V,leq}^<(p)\right) + 2|\boldsymbol{p}|\beta f_{V,leq}^<(p)f_{V,leq}^>(p) \\ &+|\boldsymbol{p}|\left[\left(f_{V,leq}^>(p) - f_{V,leq}^<(p)\right)\hat{p}_\perp\cdot\partial_{p_\perp} - \frac{1}{\beta}(\partial_{p_\perp}\cdot\partial_{p_\perp})\right]\Big\} f_A^<(p) \\ &+\hbar\frac{e^4}{16\pi^3|\boldsymbol{p}|}\frac{\pi^2}{3\beta^3}\ln\frac{T}{m_D}S_{(u)}^{\alpha\nu}\Omega_{\alpha\nu}f_{V,leq}^<(p)f_{V,leq}^>(p) + \mathcal{O}(\hbar^2), \end{aligned}$$

S. Fang, SP, D.L. Yang, PRD (2022)

自旋Boltzmann方程 (III): 相互作用对自旋分布的修正

Interaction correction to spin polarization in HTL approximation:

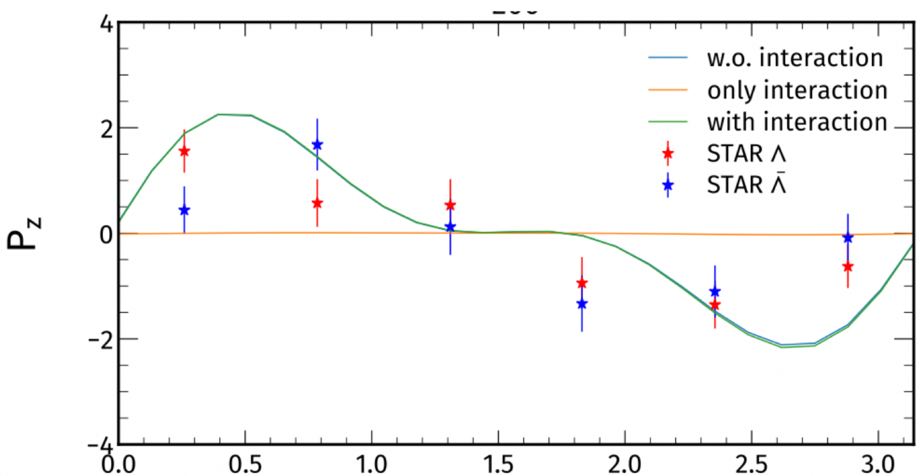
$$
\begin{aligned}
&\delta f_{\mathrm{A}}^{<}(x,p)\\
=\quad & -\frac{\hbar}{2} f_{V,\mathrm{leq}}^{<}(x,p) f_{V,\mathrm{leq}}^{>}(x,p) \frac{48\pi\beta^2}{e^4 \ln\frac{T}{m_{\mathrm{D}}}} \left\{ -(-\frac{27\zeta(3)}{\pi^2\beta} + E_{\mathbf{p}}) \frac{2\ln 2}{3} (\omega_\rho \nabla^\rho \beta - \beta \nabla_\alpha \omega^\alpha) \right.\\
& +(-\frac{27\zeta(3)}{\pi^2\beta} + E_{\mathbf{p}}) \left(\frac{5}{3} - \frac{360\zeta(3)}{7\pi^4} \ln 2 \right) \beta \omega_\rho \nabla^\rho \alpha\\
& + \left[-\frac{45\zeta(3) - 7\pi^2 \ln 2}{14\pi^2 \ln 2} \epsilon^{\mu\nu\alpha\beta} u_\beta \nabla_\nu \alpha \nabla_\mu \beta - \frac{3\pi^2}{10\ln 2} \beta \sigma^{\mu\alpha} \omega_\mu \right] p_{\langle\alpha\rangle}\\
& + \left[-\frac{\beta}{4} \left(\omega^{\langle\alpha} \nabla^{\rho\rangle} \beta + \frac{1}{2} \beta \nabla^{\langle\rho} \omega^{\alpha\rangle} + \epsilon^{\mu\nu\sigma\langle\rho} \sigma_\mu^{\alpha\rangle} u_\sigma \nabla_\nu \beta \right) + \frac{3645\zeta^2(3) + 7\pi^6}{1512\pi^4\zeta(3)} \beta^2 \epsilon^{\mu\nu\sigma\langle\rho} \sigma_\mu^{\alpha\rangle} u_\sigma \nabla_\nu \alpha \right.\\
& \left.\left. + \frac{10935\zeta^2(3) - 14\pi^6}{756\pi^4\zeta(3)} \beta^2 \omega^{\langle\alpha} \nabla^{\rho\rangle} \alpha \right] p_{\langle\alpha} p_{\rho\rangle} + \frac{7\pi^4}{10125\zeta(5)} \beta^3 \omega^{\langle\rho} \sigma^{\mu\lambda\rangle} p_{\langle\mu} p_\rho p_{\lambda\rangle} \right\},
\end{aligned}
$$

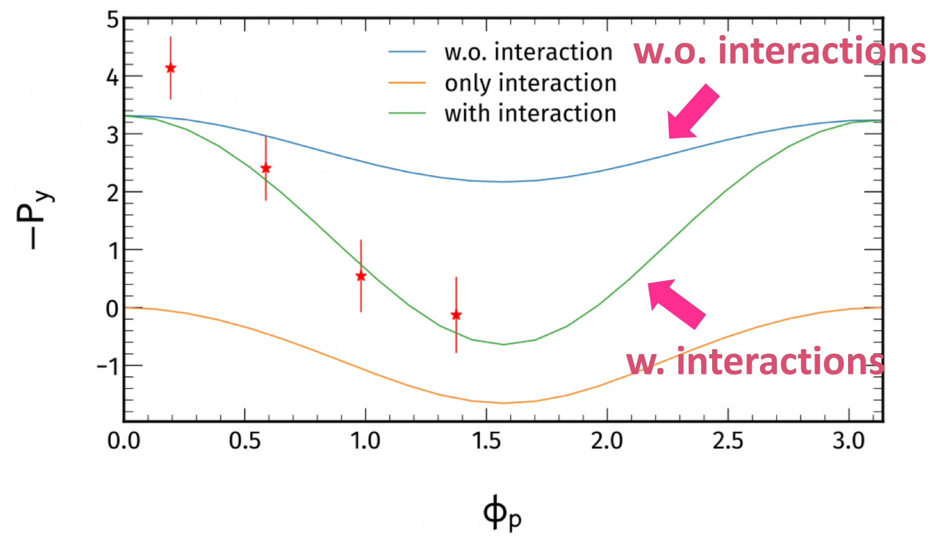
S. Fang, SP, et. al. , in preparation

Chirality2023 报告内容

相互作用对局域极化的影响(Preliminary)

200GeV, CLVisc hydrodynamics + AMPT initial + EoS: NEOS-BQS





- We consider the s quark polarization (s quark scenario):
  - Mass of s quark: 0.4 GeV
- For qq -> qq type effective interactions, we find that the interactions can modify the local polarization.

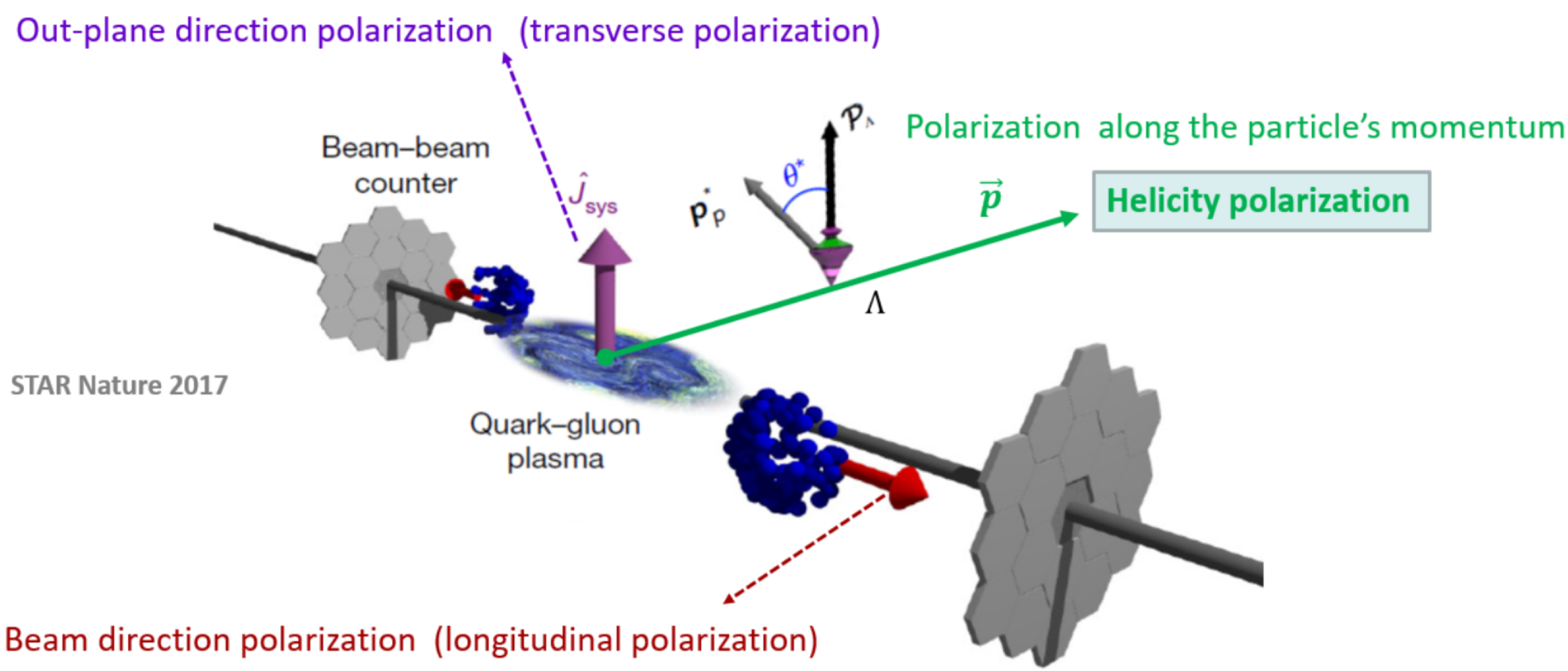
More systemic studies are needed!

in preparation

螺旋度极化 (I)

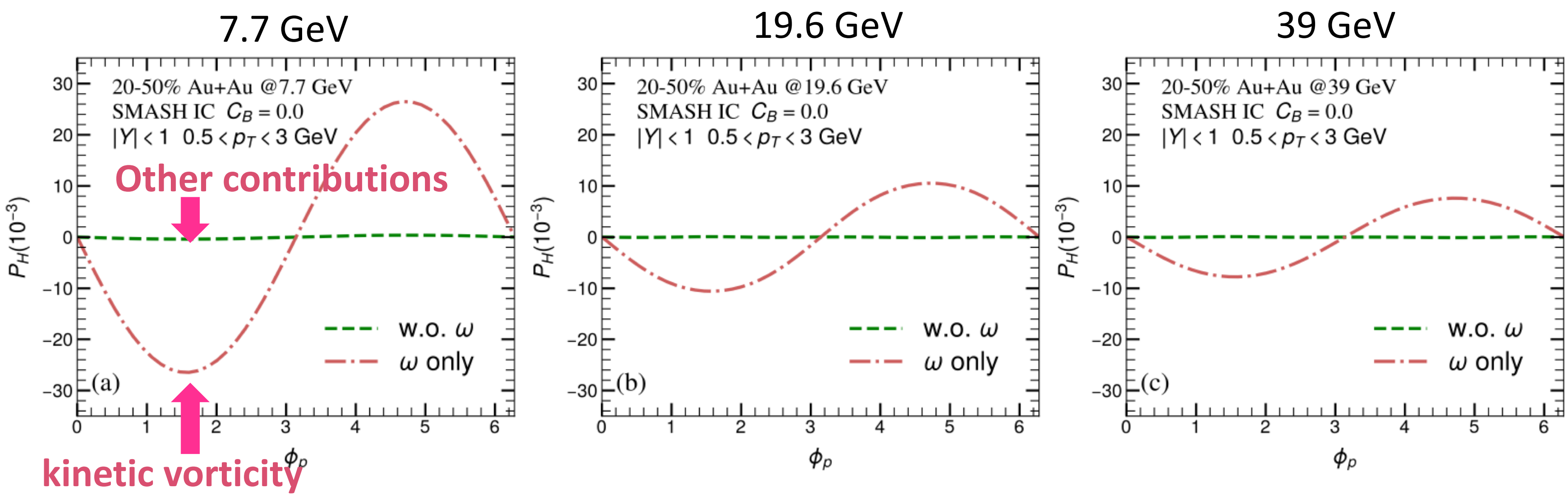
- The original idea for helicity polarization is proposed by Becattini, Buzzegoli, Palermo, Prokhorov, PLB(2021) and Gao, PRD(2021); Yi, Pu, Gao, Yang, PRC (2022)
- to probe the initial chiral chemical potential.
- Helicity instead of spin is widely-used in high energy spin physics.

$$\mathcal{S}^h = \widehat{\mathbf{p}}\cdot\boldsymbol{\mathcal{S}}(\mathbf{p}) = \widehat{p}^x\mathcal{S}^x + \widehat{p}^y\mathcal{S}^y + \widehat{p}^z\mathcal{S}^z,$$



利用螺旋度极化去探测涡旋精细结构

$$S^h(\mathbf{p}) = S^h_{\text{thermal}} + S^h_{\text{shear}} + S^h_{\text{accT}} + S^h_{\text{chemical}} + S^h_{\text{EB}}$$



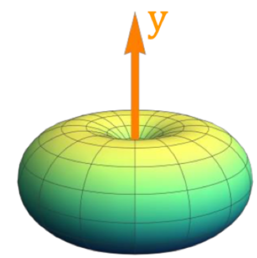
- **Helicity polarization induced by kinetic vorticity dominates at low energy collisions.**

- **A possible way to probe the fine structure of kinetic vorticity by mapping the simulations of helicity polarization to the future measurements?**
  Yi, Pu, Gao, Yang, PRC (2022); Yi, Wu, Yang, Gao, SP, Qin, arXiv:2304.08777

矢量介子的自旋排列

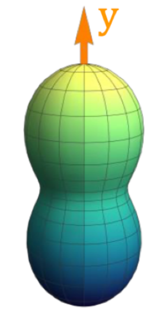
- **Spin density matrix**

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{1,0} & \rho_{1,-1} \\ \rho^*_{1,0} & \rho_{00} & \rho_{0,-1} \\ \rho^*_{1,-1} & \rho^*_{0,-1} & \rho_{-1,-1} \end{pmatrix}$$



$\rho_{00} < 1/3$

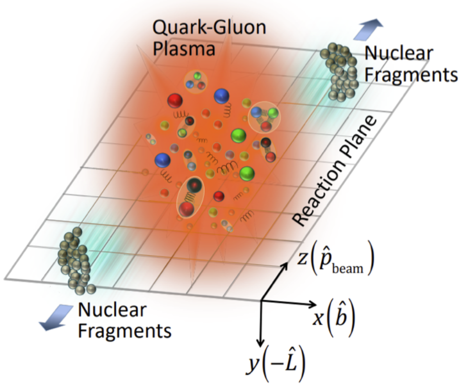
横向极化

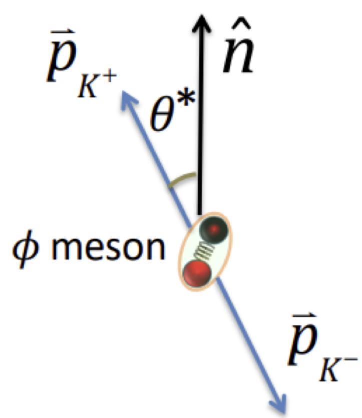


$\rho_{00} > 1/3$

纵向极化

- **Spin-1 meson decays to two spin-0 particles, the cross section is given by**

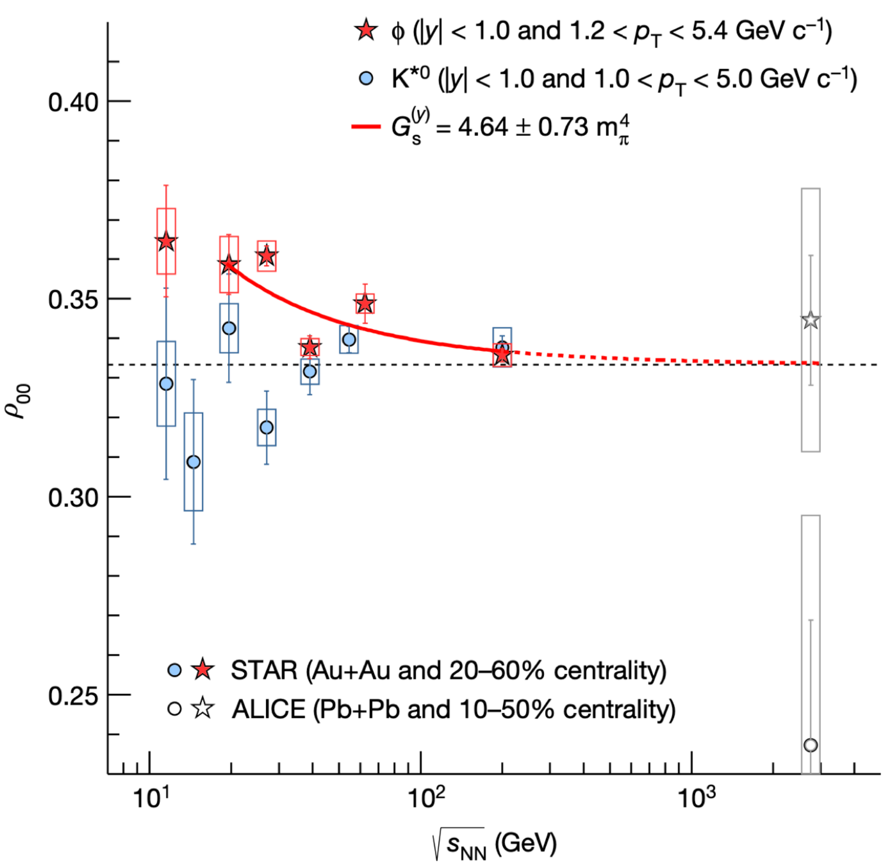




$$\frac{dN}{d(\cos\theta^*)} \propto (1-\rho_{00}) + (3\rho_{00}-1)\cos^2\theta^*,$$

Phi介子矢量自旋排列

STAR Collaboration, Nature 614, 244 (2023)



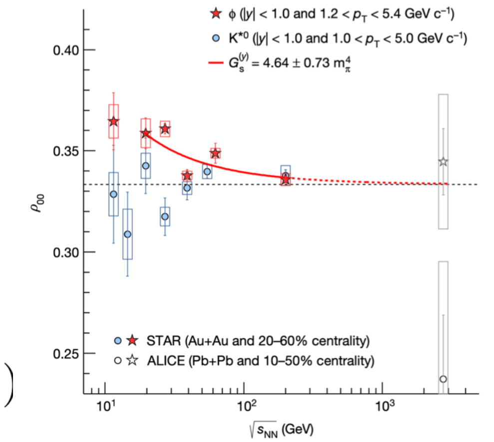
Physics Mechanisms	(ρ_{00})
c_Λ: Quark coalescence vorticity & magnetic field[1]	< 1/3 (Negative ~ 10^{-5})
c_ε: Vorticity tensor[1]	< 1/3 (Negative ~ 10^{-4})
c_E: Electric field[2]	> 1/3 (Positive ~ 10^{-5})
Fragmentation[3]	> or, < 1/3 (~ 10^{-5})
Local spin alignment and helicity[4]	< 1/3
Turbulent color field[5]	< 1/3
c_ϕ: Vector meson strong force field[6]	> 1/3

[1] Liang, Wang, PLB 629, 20 (2005); Yang et al., PRC 97, 034917 (2018); Xia et al., PLB 817, 136325 (2021); Beccattini et al., PRC 88, 034905 (2013)
[2] Sheng et al., PRC 101, 096005 (2020); Yang et al., PRC 97, 034917 (2018)
[3] Liang, Wang, PLB 629, 20 (2005);
[4] Xia et al., PLB 817, 136325 (2021); Gao, PRD 104, 076016 (2021)
[5] Muller, Yang, PRD 105, L011901 (2022)
[6] Sheng et al., PRD 101, 096005 (2020); PRD 102, 056013 (2020)

利用量子动理学理论框架来研究矢量介子自旋排列

- **Spin density matrix (normalized MVSD) for phi mesons given by spin Boltzmann equation:**

$$s + \bar{s} \rightleftharpoons \phi$$



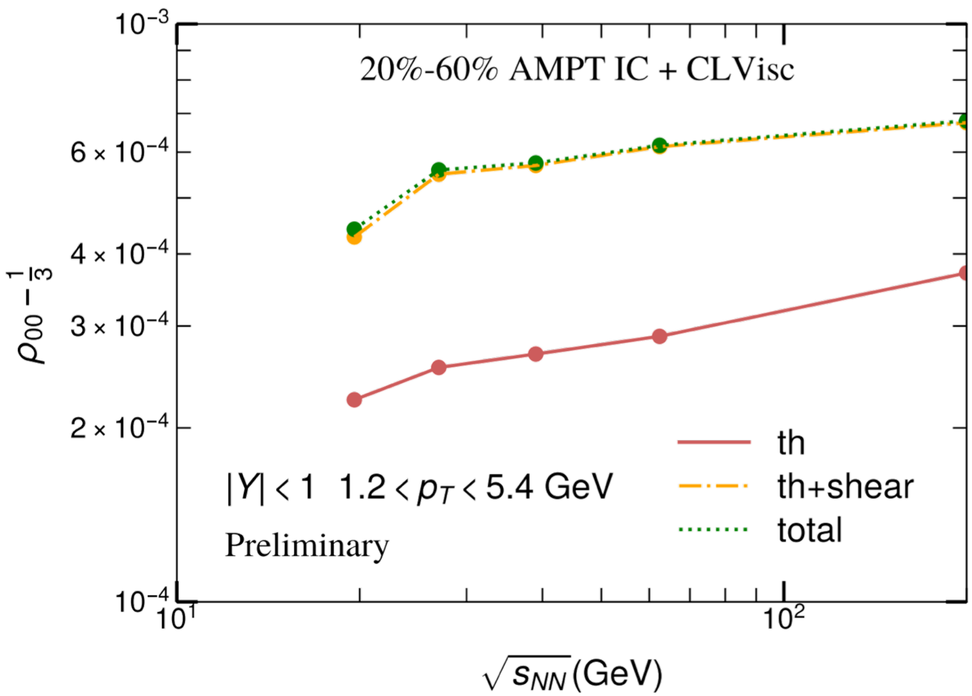
$$\begin{aligned}\rho^{\phi}_{\lambda_1\lambda_2}(x,\mathbf{p}) \propto & \frac{\Delta t}{32}\int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E^{\bar{s}}_{p'}E^{s}_{\mathbf{p}-\mathbf{p}'}E^{\phi}_{p}} f_{\bar{s}}(x,\mathbf{p}') f_s(x,\mathbf{p}-\mathbf{p}') \\ & \times 2\pi\hbar\delta\left(E^{\phi}_{p} - E^{\bar{s}}_{p'} - E^{s}_{\mathbf{p}-\mathbf{p}'}\right)\epsilon^{*}_{\alpha}(\lambda_1,\mathbf{p})\,\epsilon_{\beta}(\lambda_2,\mathbf{p}) \\ & \times \mathrm{Tr}\left\{\Gamma^{\beta}(p'\cdot\gamma - m_{\bar{s}})\left[1+\gamma_5\gamma\cdot P^{\bar{s}}(x,\mathbf{p}')\right]\Gamma^{\alpha}\right. \\ & \left.\times\left[(p-p')\cdot\gamma + m_s\right]\left[1+\gamma_5\gamma\cdot P^{s}(x,\mathbf{p}-\mathbf{p}')\right]\right\},\end{aligned}$$

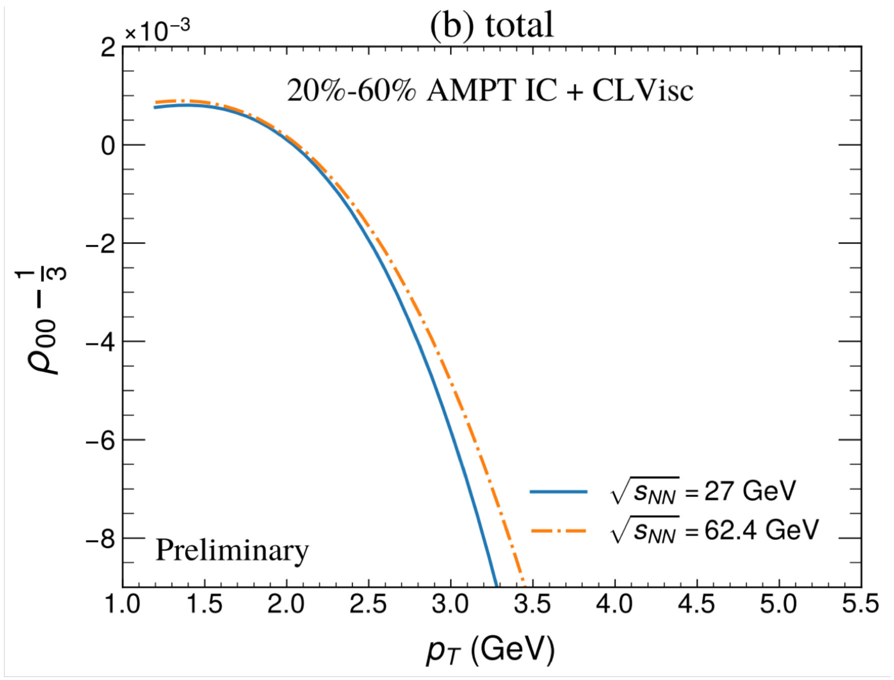
Spin polarization of s and $\bar{s}$ quarks

Sheng, Oliva, Liang, Q. Wang, and X.-N. Wang, PRD (2022); PRL (2023)

各类流体效应导致的极化效应对矢量介子自旋排列的影响

$$\mathcal{S}^{\mu}(\mathbf{p}) = \mathcal{S}^{\mu}_{\text{thermal}} + \mathcal{S}^{\mu}_{\text{shear}} + \mathcal{S}^{\mu}_{\text{accT}} + \mathcal{S}^{\mu}_{\text{chemical}} + \mathcal{S}^{\mu}_{\text{EB}}$$





- **The total hydrodynamic contributions to $\rho_{00}-1/3$ are positive and at the order of 10^{-4}.**

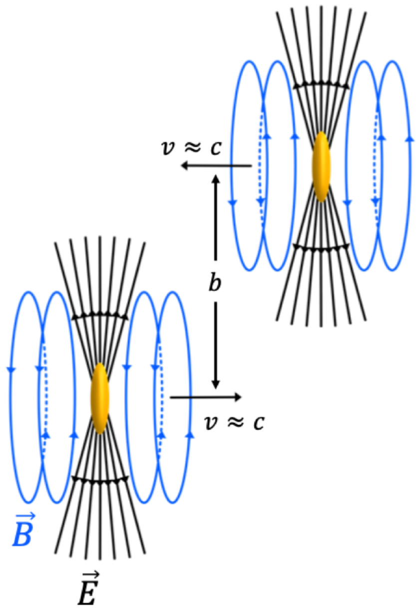
Yi，Wu, et. al, in preparation

相对论重离子碰撞中光致双轻子产生

散射截面中的光子Wigner函数：

利用同量异位素膨胀研究核结构

超偏心/边缘碰撞



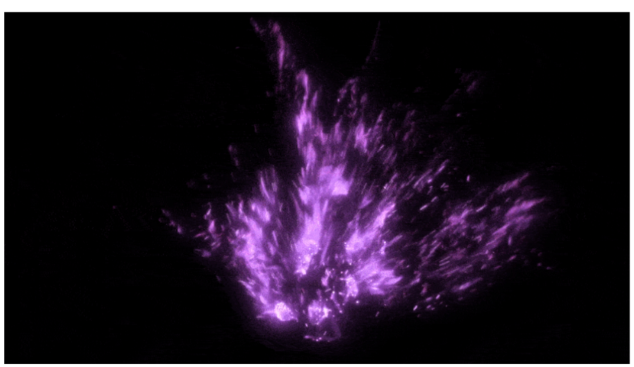
- **Ultra-Peripheral Collisions (UPC): the impact parameter is larger than 2 times the radius of a nucleus**
- **Since the QCD effects are higher orders and QED effects are enhanced by the Ze, UPC provides a nice platform to study the strong EB effects.**

  $Z\alpha \approx 1$ → High photon density
- **Because the relativity, the photon (EB fields) are almost real. Photon-photon, photon-nuclear interactions**

Scientists Generate Matter Directly From Light – Physics Phenomena Predicted More Than 80 Years Ago

TOPICS: Antimatter Atomic Physics Brookhaven National Laboratory DOE Popular

By BROOKHAVEN NATIONAL LABORATORY JULY 30, 2021

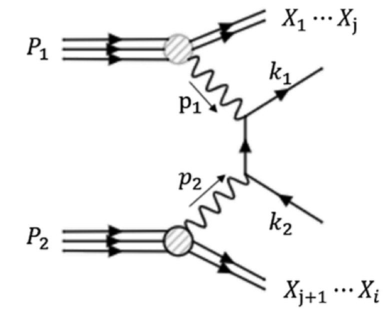


Abstract energy concept illustration.

STAR Collaboration, "Measurement of e+e− Momentum and Angular Distributions from Linearly Polarized Photon Collisions", Phys. Rev. Lett 127, 052302

超偏心碰撞中的光致双轻子产生

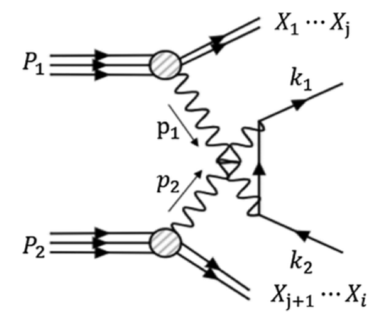
Equivalent photon approximation (EPA)



- Baltz, Gorbunov, Klein, Nystrand, PRC 80, 044902 (2009)
- Zha, Ruan, Tang, Xu, Yang, PLB 781, 182 (2018)
- Zha, Brandenburg, Tang, Xu, PLB 800 (2020) 135089

Based on QED calculations:

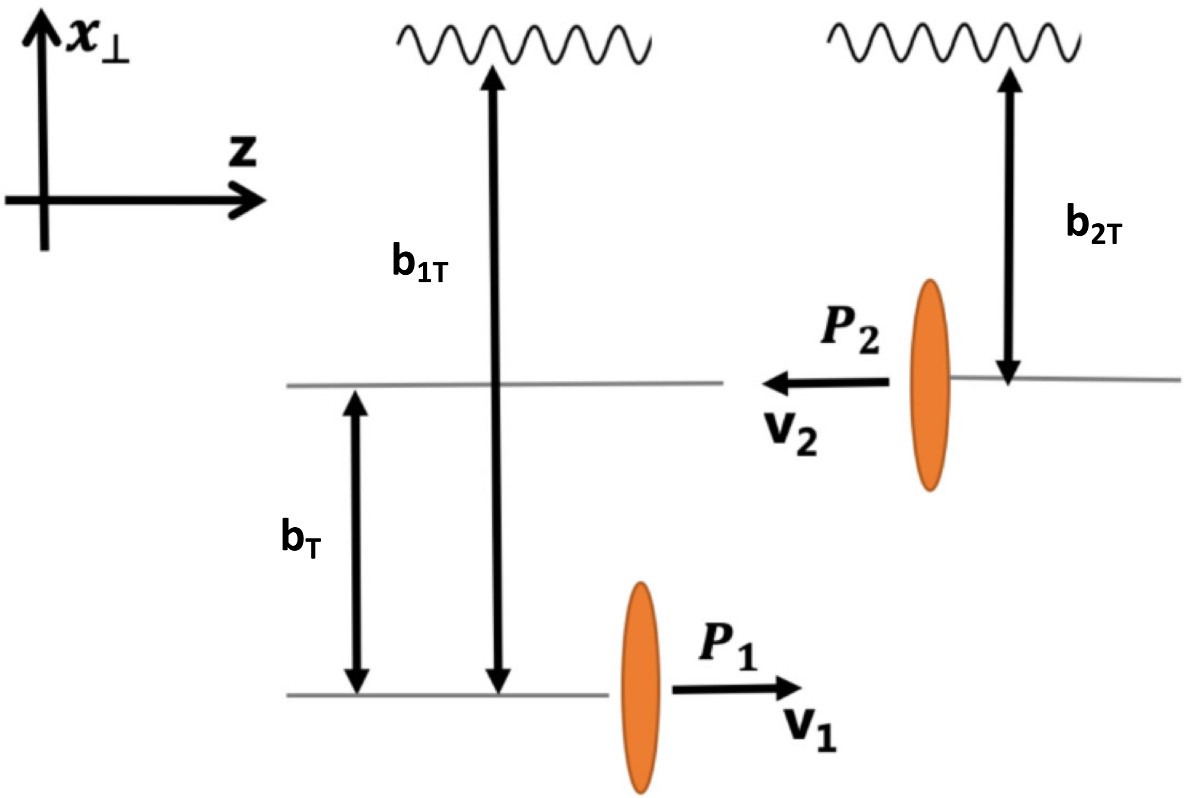
- Transverse momentum dependent (TMD) formulism
  - C. Li, J. Zhou and Y. J. Zhou, Phys. Lett. B 795, 576 (2019)
  - Klein, Muller, Xiao, Yuan, PRL 122 (2019) 13, 132301; PRD 102 (2020) 9, 094013
  - Xiao, Yuan, Zhou, PRL 125 (2020) 23, 232301



- QED in classical field approximation
  - Vidovic, Greiner, Best, Soff, PRC (1993)
  - W. Zha, J. D. Brandenburg, Z. Tang and Z. Xu, PLB 800 (2020) 135089
- QED with wave packet description of nuclei
  - Wang, SP, Wang, PRD (2021); Wang, SP, Zhang, Wang, PRD (2022);
  - Lin, Wang, Wang, Xu, SP, Wang, PRD (2022)

基于原子核波包展开的模型

- How to consider the space and momentum dependence for photons?



- Three related impact parameters: b_T, b_{1T}, b_{2T}

类似QCD因子化的微分散射截面

- **Our general expression for differential cross section is as follows.**

R.J. Wang, SP, Q. Wang, PRD 2021

$$
\begin{aligned}
\frac{d\sigma}{d^3k_1 d^3k_2} \approx & \frac{1}{32(2\pi)^6}\frac{1}{E_{k1}E_{k2}}\int d^2\mathbf{b}_T d^2\mathbf{b}_{1T} d^2\mathbf{b}_{2T}\int d^4p_1 d^4p_2 \\
& \times\delta^{(2)}\left(\mathbf{b}_T-\mathbf{b}_{1T}+\mathbf{b}_{2T}\right)(2\pi)^4\delta^{(4)}\left(p_1+p_2-k_1-k_2\right) \\
& \times\int\frac{d^2\mathbf{P}_{(1+1')T}}{(2\pi)^2}\frac{d^2\mathbf{P}_{(2+2')T}}{(2\pi)^2}\frac{1}{v\sqrt{E_{P1}E_{P2}E_{P1'}E_{P2'}}} \\
& \times G^2\left[(P_1'^z-P_{A1}^z)^2\right]\phi_T(\mathbf{P}_{1T})\phi_T(\mathbf{P}_{2T})\phi_T^*(\mathbf{P}'_{1T})\phi_T^*(\mathbf{P}'_{2T}) \\
& \times\mathcal{S}_{\sigma\mu}(p_1,\mathbf{b}_{1T})\mathcal{S}_{\rho\nu}(p_2,\mathbf{b}_{2T}) \\
& \times L^{\mu\nu;\sigma\rho}\left(p_1,p_2;p_1-P_1+P'_1,p_2-P_2+P'_2;k_1,k_2\right),
\end{aligned}
$$

Fluctuations from wave package: we neglect this part contributions so far.

Photon Wigner function: provides information of space and momentum for photons

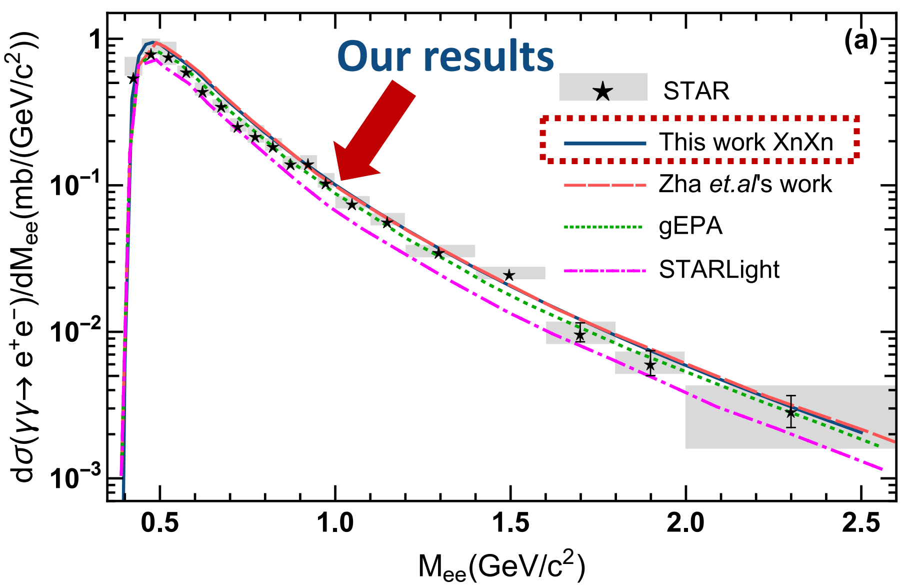
Lepton tensor: Can be computed by perturbative theories

$$\mathcal{S}_{\sigma\mu}(p_1,\mathbf{b}_{1T}) \equiv \int\frac{d^2\mathbf{\Delta}_{1T}}{(2\pi)^2}\int\frac{d^4y_1}{(2\pi)^4}e^{ip_1\cdot y_1}\langle P'_1|A_\sigma^\dagger(0)A_\mu(y_1)|P_1\rangle e^{-i\mathbf{b}_{1T}\cdot\mathbf{\Delta}_{1T}}$$

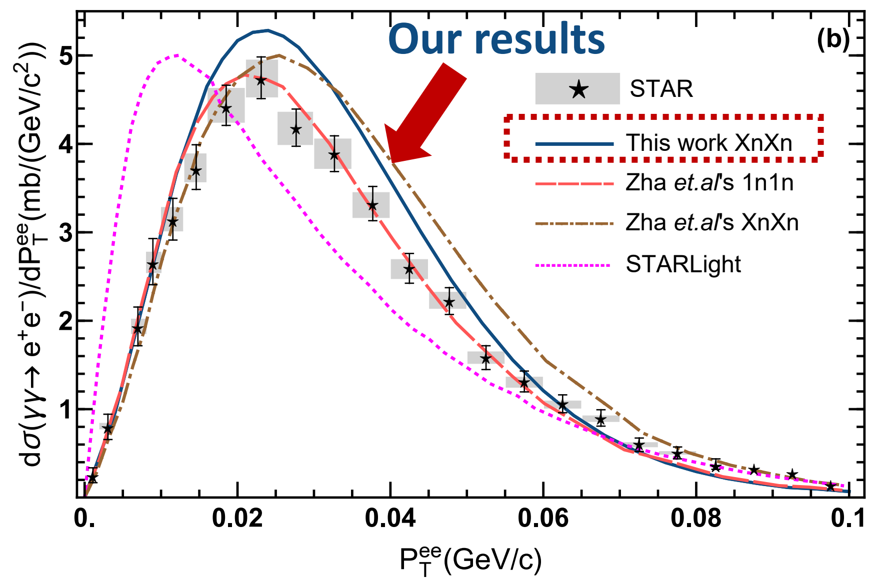
Transverse Momentum Dependent (TMD) Photon Distribution

Klein, Muller, Xiao, Yuan, PRL (2019)

超偏心碰撞下理论模型与实验数据对比



Invariant mass of dilepton

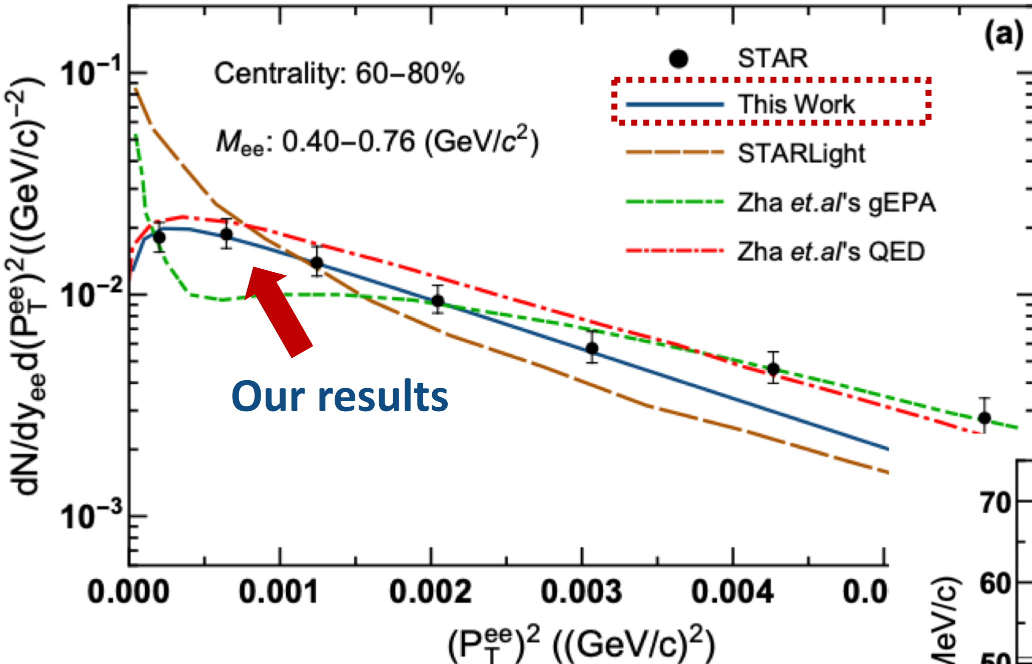


Total transverse momentum of dilepton

- **Our results for UPC agree with experimental data.**
- **Differences between our results and data may come from the higher order corrections.**

R.J. Wang, SP, Q. Wang, PRD 104 (2021) 5, 056011

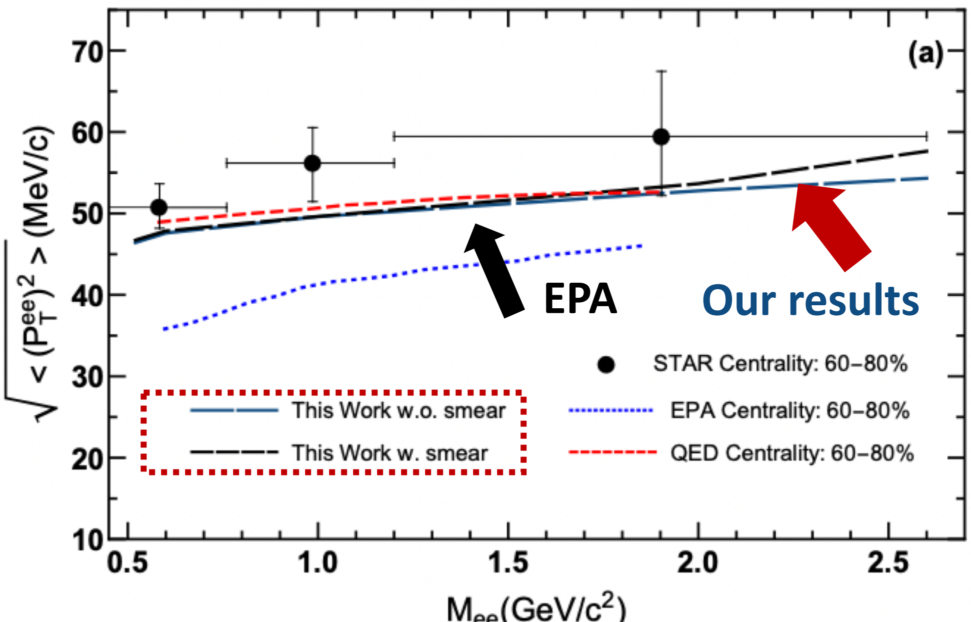
偏心碰撞下理论模型与实验数据对比



Square of Total transverse momentum of dilepton

- The calculation based on QED can explain the curve at low P^2_T .

- The study of $<P^2_T>$ as a function of Mee is an nice example to show that the information of transverse momentum of photons is essential.

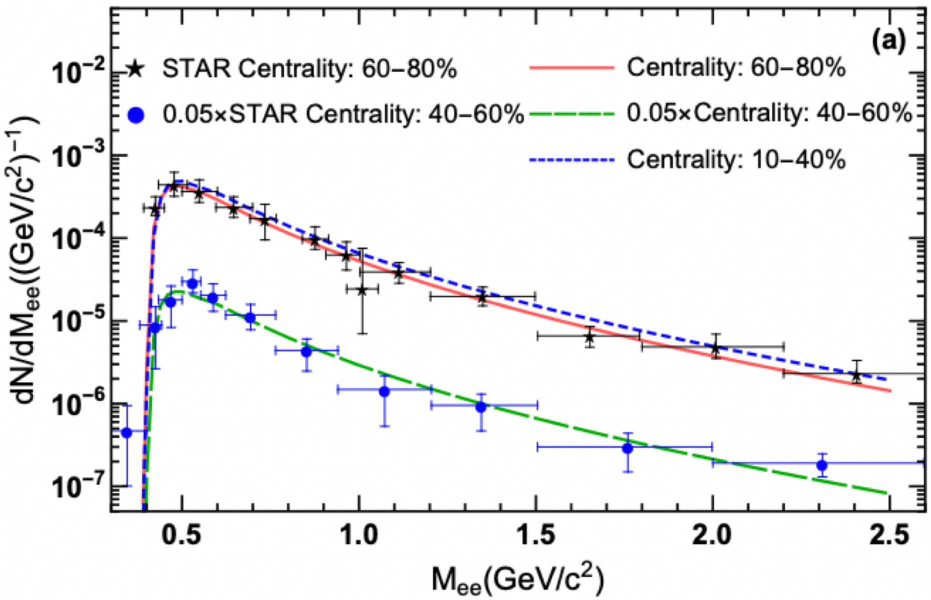


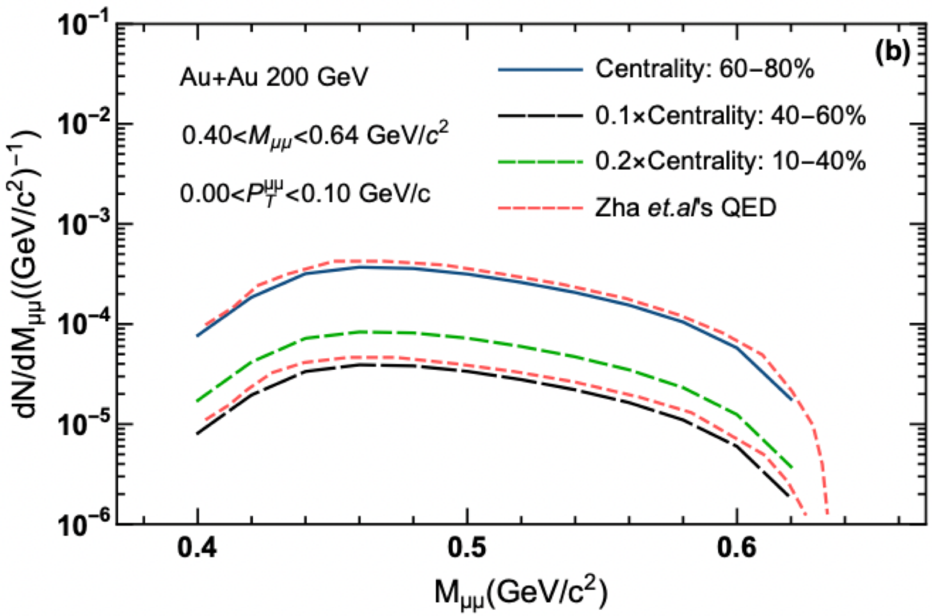
Invariant mass of dilepton

R.J. Wang, SP, Y.F. Zhang, Q. Wang, PRD 106 (2022) 3, 034025

偏心碰撞下理论模型与实验数据对比

e+ e- **μ+ μ-**

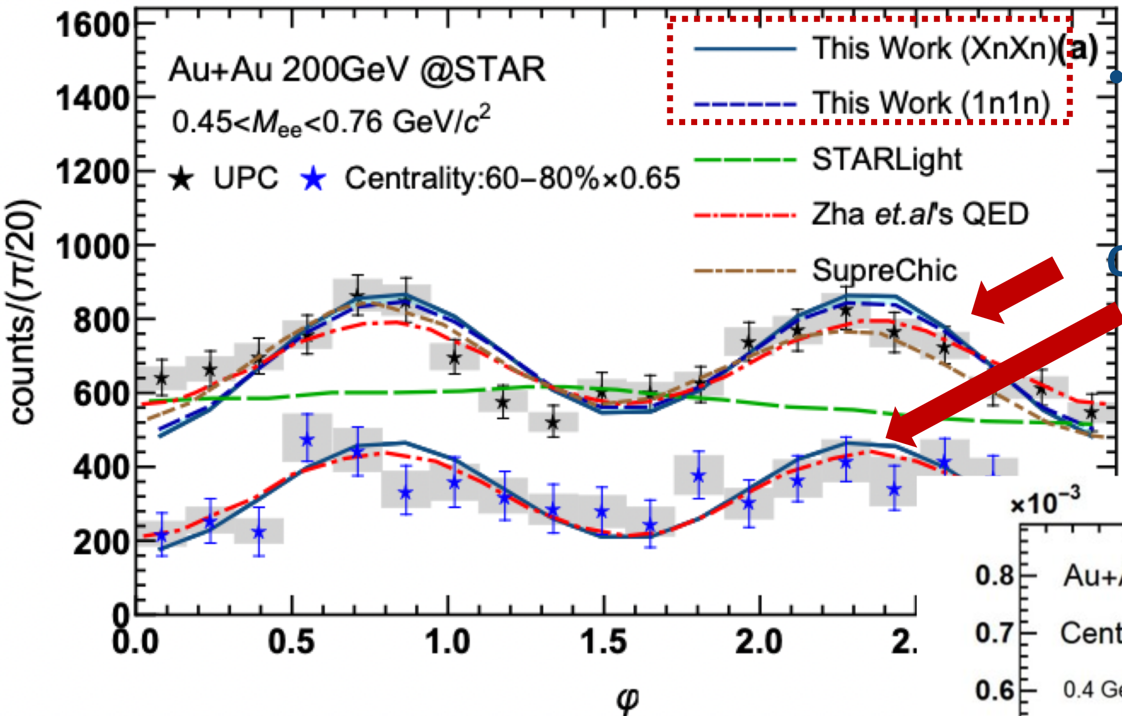




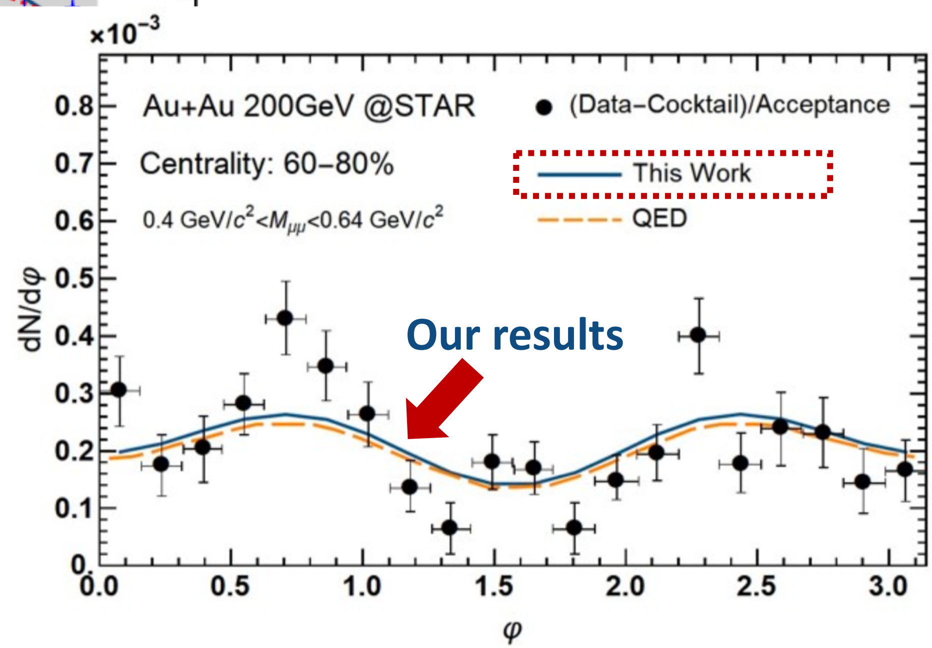
- **Invariant mass distributions for both e+e- and μ+μ- pairs agree with the data.**

R.J. Wang, SP, Y.F. Zhang, Q. Wang, PRD 106 (2022) 3, 034025

超偏心和偏心碰撞中末态双轻子角度分布



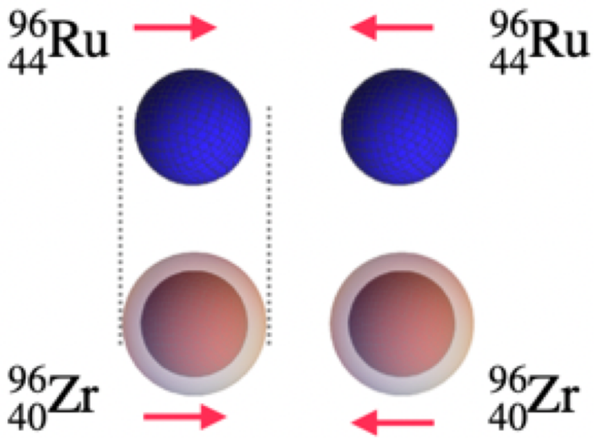
- Our results agree with the experimental data for $e+e-$.



- So far, we do not know the source for the difference between our results and data for $\mu+\mu-$.

R.J. Wang, SP, Y.F. Zhang, Q. Wang, PRD 106 (2022) 3, 034025

同量异位素碰撞



- **One lesson that we have learnt from isobar collisions is the mass and charge distributions for Zr and Ru are different.**

电荷分布 ⟷ **Nuclear charge form factor**

质量分布 ⟷ **Impact parameters (centrality)**

e.g. see Xu, et al., PRL 2018; Li, et al, PRC 2018; Zhang, Jia, PRL 2022; Deng, Huang, Ma, Wang PRC 2016; Zhao, Xu, Liu, Song, PLB 2023 ...

核电荷与质量的Woods-Saxon 分布

- One can parameterize the charge and mass distribution as

$$\rho_i(\mathbf{r}) \equiv \frac{C_i}{1+\exp[(|\mathbf{r}|-R_i)/d_i]},$$

by matching the <r> and <r²>.

- The centrality and impact parameters are computed by optical Glauber model .

(a) Parameters given by energy density functional theory (DFT) ✓

(a)	R_c	d_c	R_n	d_n
Ru	5.083 fm	0.477 fm	5.093 fm	0.488 fm
Zr	4.977 fm	0.492 fm	5.022 fm	0.538 fm

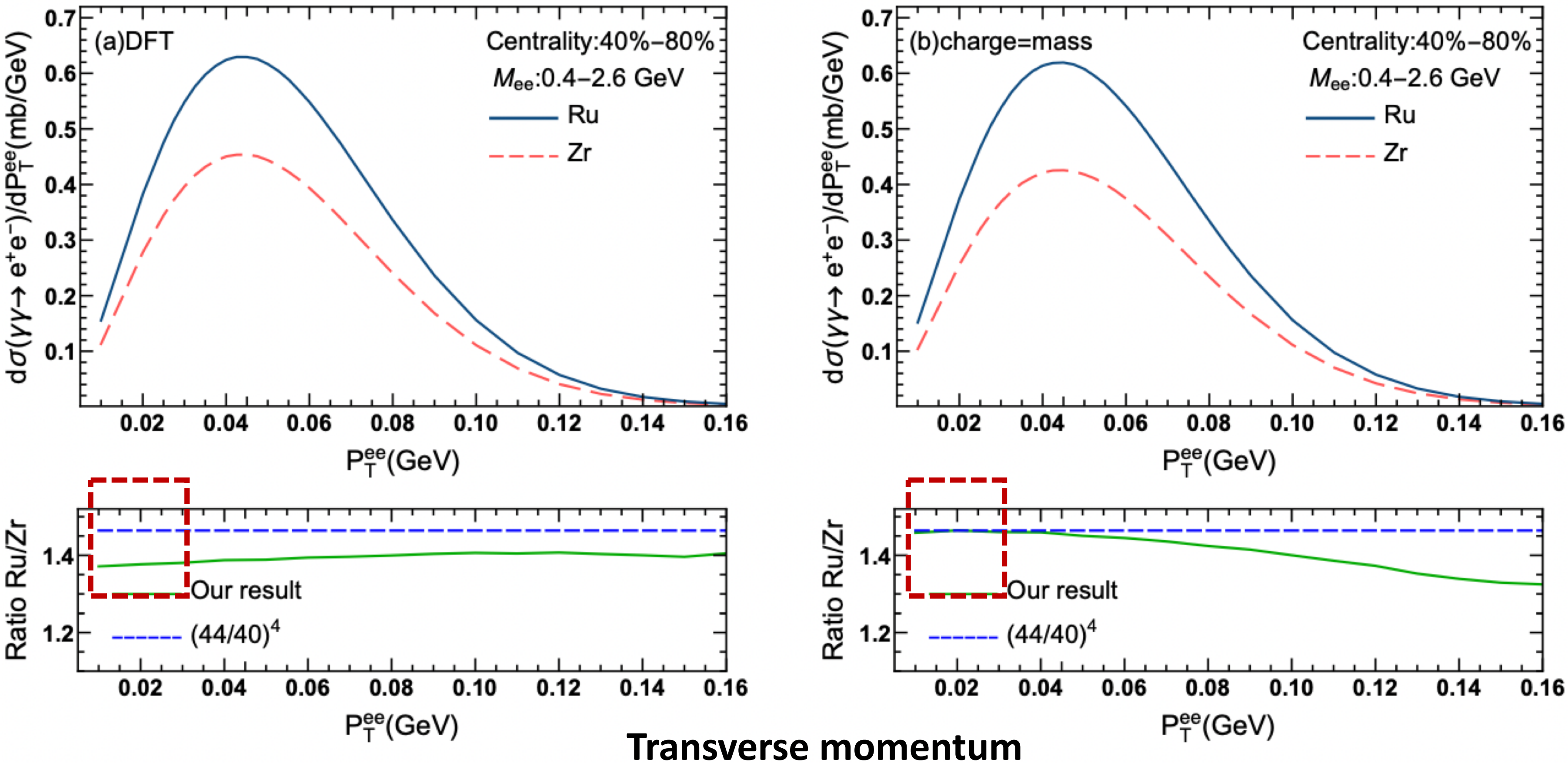
Centrality	40%	60%	70%	80%
Impact parameter	7.464 fm	9.143 fm	9.874 fm	10.563 fm
Impact parameter	7.615 fm	9.326 fm	10.073fm	10.780 fm

(b) Set the mass distribution be the same as charge distribution

(b)	R_c	d_c	R_n	d_n
Ru	5.083 fm	0.477 fm	R_c^{Ru}	d_c^{Ru}
Zr	4.977 fm	0.492 fm	R_c^{Zr}	d_c^{Zr}

Centrality	40%	60%	70%	80%
Impact parameter	7.406 fm	9.070 fm	9.797 fm	10.479 fm
Impact parameter	7.373 fm	9.030 fm	9.754 fm	10.434 fm

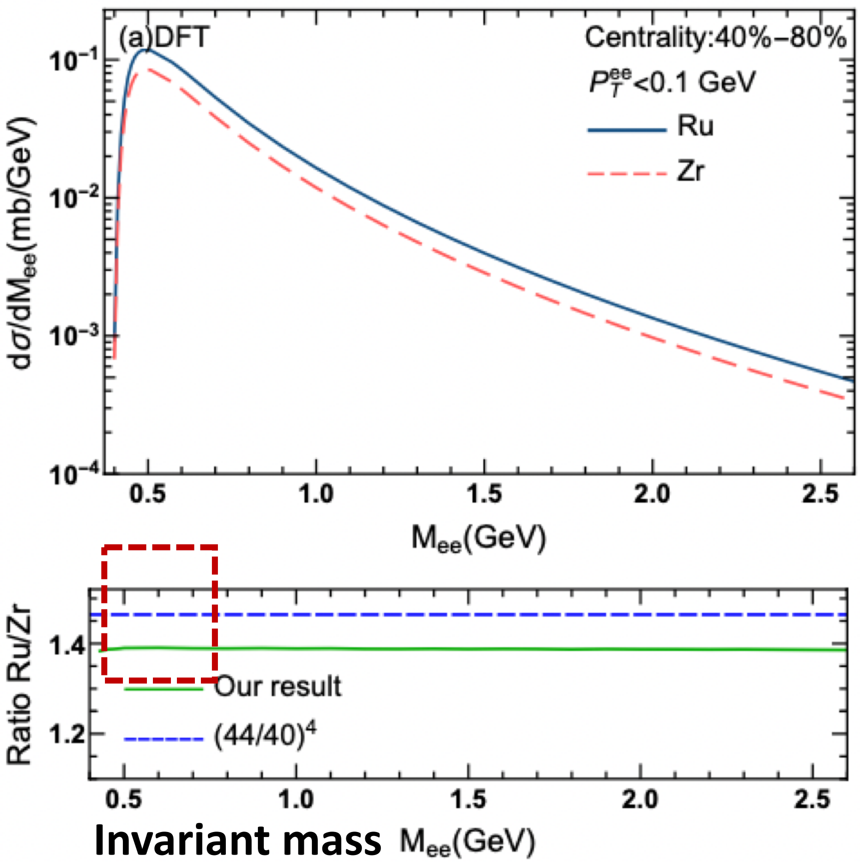
Ru+Ru 和 Zr+Zr 碰撞下各类微分截面之比 (I)

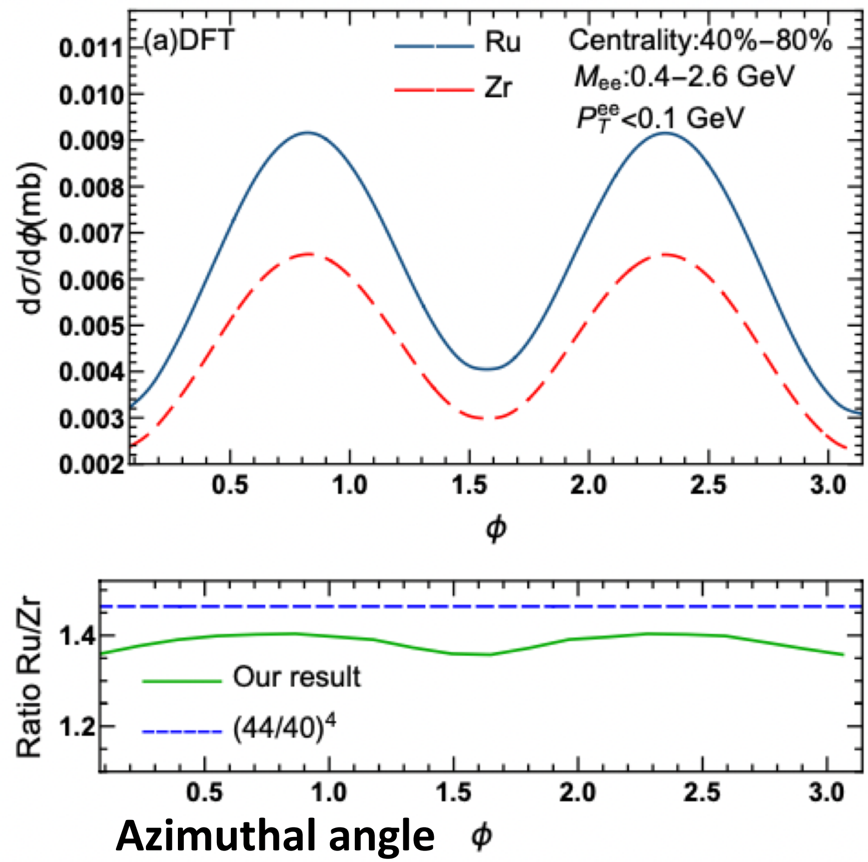


- If there is no difference between Ru and Zr, the ratio should be (44/40)^4.
- By (a) DFT, the ratio is smaller than (44/40)^4, while by (b) it is larger than (44/40)^4.

L. Shuo, R.J. Wang, J.F. Wang, H.J. Xu, SP, Q. Wang, PRD 107 (2023) 5, 054004

Ru+Ru 和 Zr+Zr 碰撞下各类微分截面之比 (II)



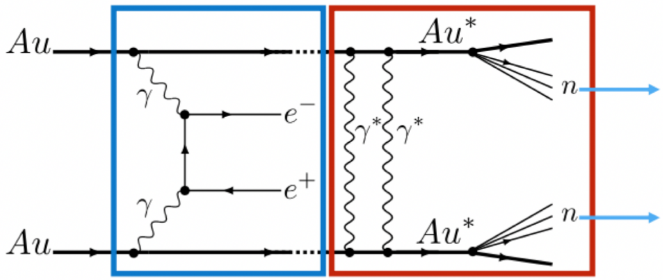


- **If there is no difference between Ru and Zr, the ratio should be (44/40)^4.**
- **By both (a) DFT, the ratios are smaller than (44/40)^4.**

L. Shuo, R.J. Wang, J.F. Wang, H.J. Xu, SP, Q. Wang, PRD 107 (2023) 5, 054004

光核反应中中子发射率

$\mathcal{P}(b_T)$:



Probability of emitting a single neutron from an excited nucleus

C. A. Bertulani and G. Baur, Phys. Rept. 163, 299 (1988).

The choice of $P(b_\perp)$:

- **Based on the Giant dipole resonance (GDR) model**

$$\mathcal{P}(b_T) = \sum_{N_\gamma=1}^{\infty} \frac{1}{N_\gamma!} w^{N_\gamma} \exp(-w) = 1 - \exp(-w),$$

$$w = 5.45 \times 10^{-5} \frac{Z^3(A-Z)}{A^{2/3} b_T^2}$$

$$\mathcal{P}(b_T) \simeq w$$

$$\mathcal{P}(b_T) = w \exp(-w)$$

Z: number of charge
A: number of nucleons
GDR does not include the information of nuclear structure.

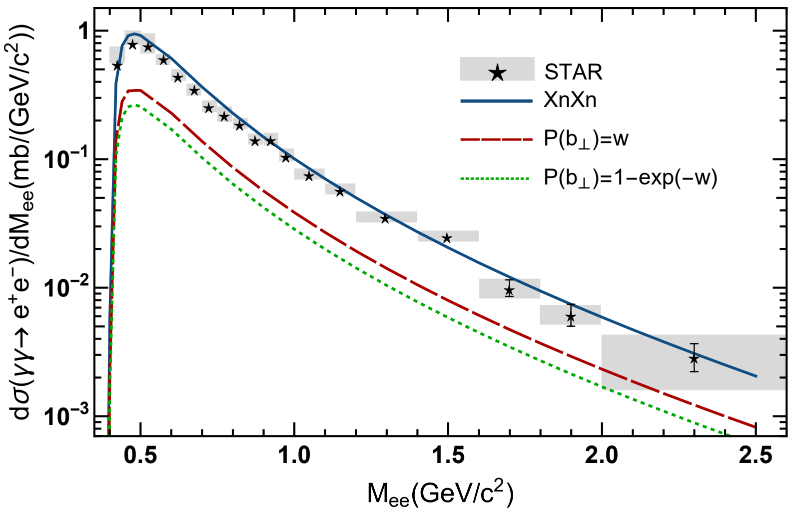
- **Based on the fixed-target experimental measurements**

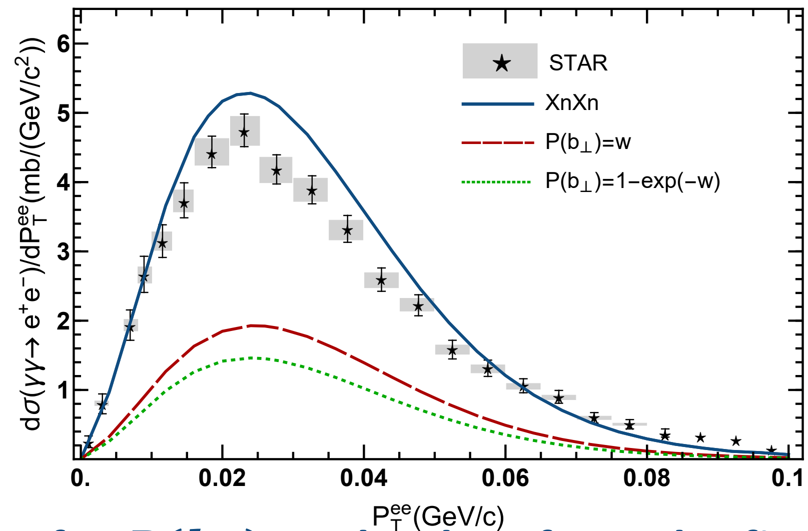
$$w_{Xn}(b_T) = \int d\omega n(\omega, b_T) \sigma_{\gamma+A\to A'+Xn}(\omega)$$

Similar to the EPA, $n(\omega, b_T)$ is photon flux, X is the number of neutrons emitted by a nuclei, $\sigma_{\gamma+A\to A'+Xn}$ is photon-nucleus cross section given by fixed-target experiments.

光核反应中中子反射率

- **The differential cross section strongly depends on the choice of $P(b_\perp)$.**





- **We need the first principle calculations for $P(b_\perp)$ or the data from the fixed target nuclear experiments.**

Nuclear	Data from exp.
Au	✓
Pb	✓
Zr	Limited
Ru	✕

2nd workshop on UPC, 2024 April (暂定)

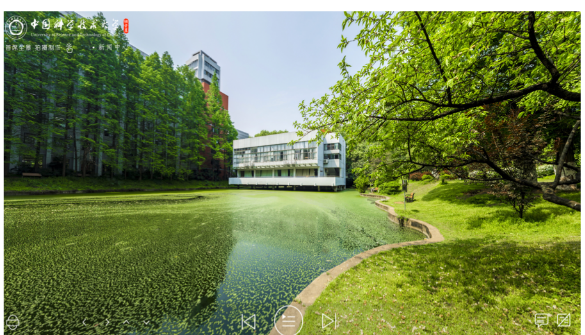
第二届超偏心/边缘碰撞研讨会

中国科学技术大学 承办

会议会务组：浦实、唐泽波、张一飞、查王妹 （暂定）

Welcome to USTC!

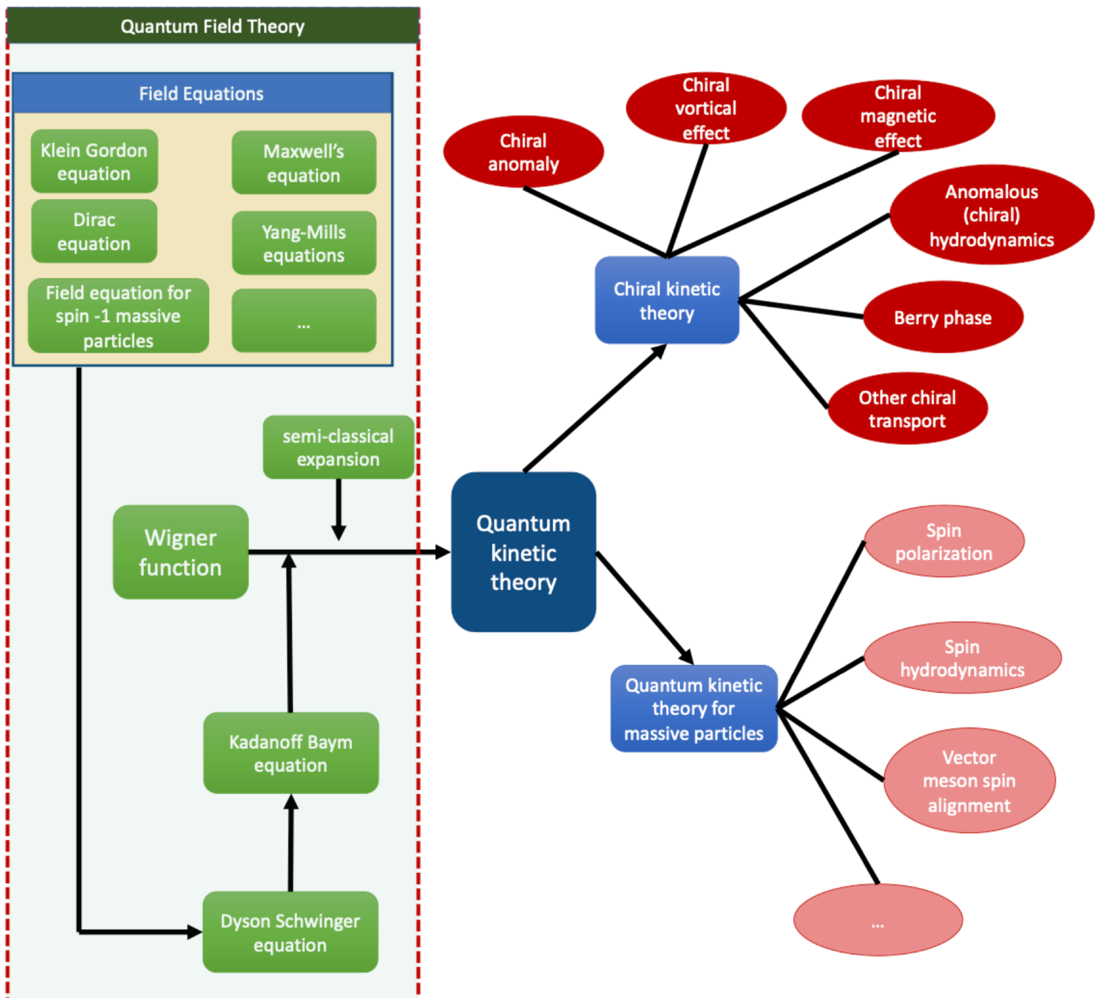


总结

量子动理学理论

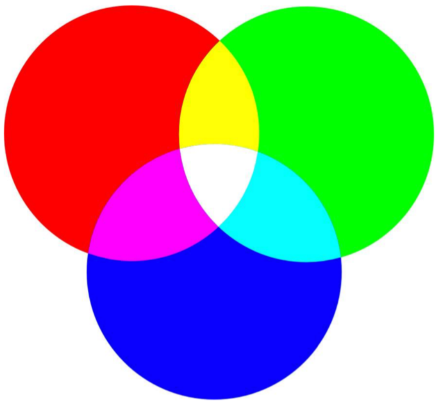
量子动理学理论根源于量子场论，可以用于研究极端强电磁场下夸克物质量子输运及自旋极化现象。



Thank you!

Backup

Relativistic Boltzmann equations on GPU



Relativistic Boltzmann equations on GPU

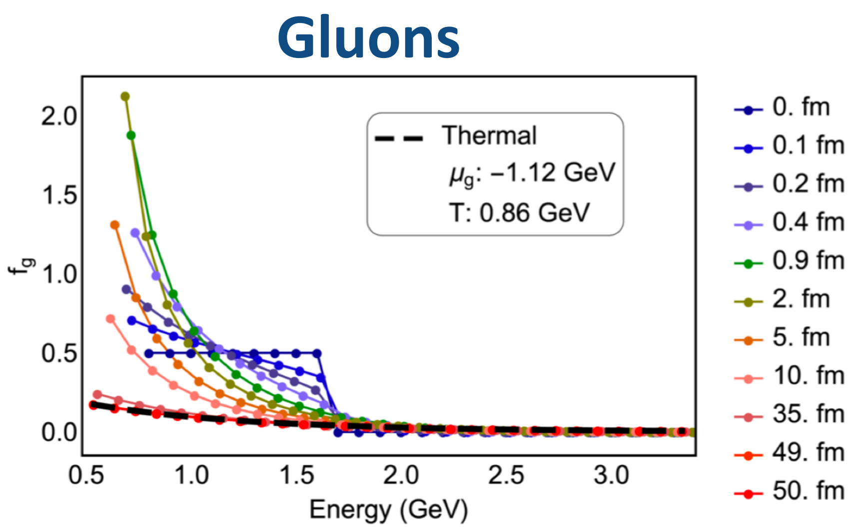
RBG

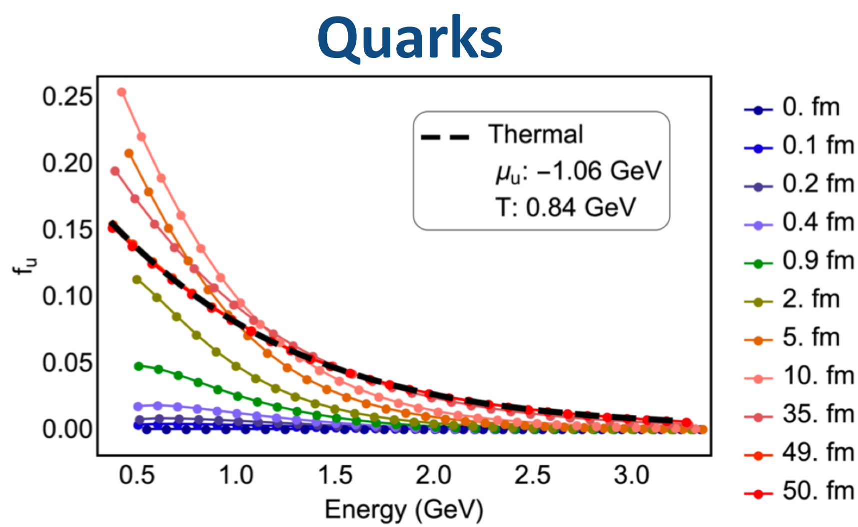
Basic, but nontrivial.

- We introduce a new numerical framework to derive full solutions of a relativistic BE on GPUs.
  - Full 2->2 collisional term:
    high dimensional integrals.
  - High performance:
    space 10x10x10,
    momentum 30x30x30,
    Time steps: 10^4-10^6,
    on one Nvidia Tesla V100 card costs a few days!
  - Particle number is strictly conserved.
  - Dynamical Debye mass

J.J Zhang, H.Z. Wu, SP, G.Y. Qin, Q. Wang, PRD (2019)

Tests for time evolution of quarks and gluons





Grids: 1 grid (space) ; momentum: 30x30x30=27,000

Phase space size: [-3fm,3fm]^3 x [-2GeV,2GeV]^3

Time step: dt=0.0005fm ; 100,000 steps

Time cost: around 50 hours on a Nvidia Tesla V100 card

J.J Zhang, H.Z. Wu, SP, G.Y. Qin, Q. Wang, PRD (2019)

RBG-Maxwell equations and v1, dv1 problem

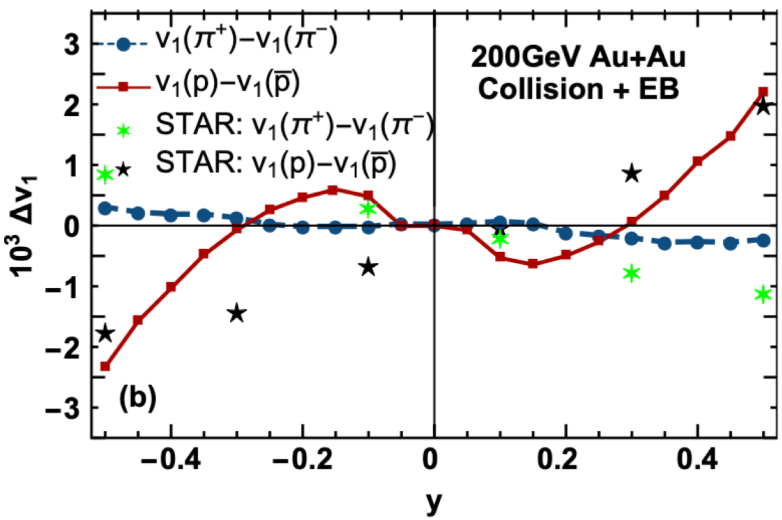
- **We solve the Boltzmann equation coupled to Maxwell equations.**

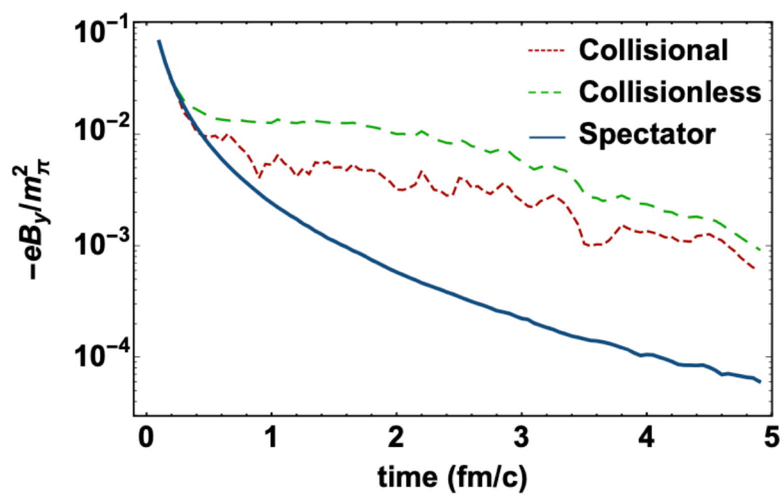
$$[p^\mu \partial_\mu + Q_a p_\mu F^{\mu\nu} \partial_{p^\nu}] f_a(t, \mathbf{x}, \mathbf{p}) = \mathcal{C}[f_a],$$
$$\partial_\mu F^{\mu\nu} = j^\nu_{\text{ext}} + j^\nu_{\text{med}},$$

(Spectators + Participant)

QCD 2->2 scattering full collision term

- **A good example to show the effects from EB fields. Qualitative consistent with exp. Could help us to understand the dv1 difference for pions and protons.**

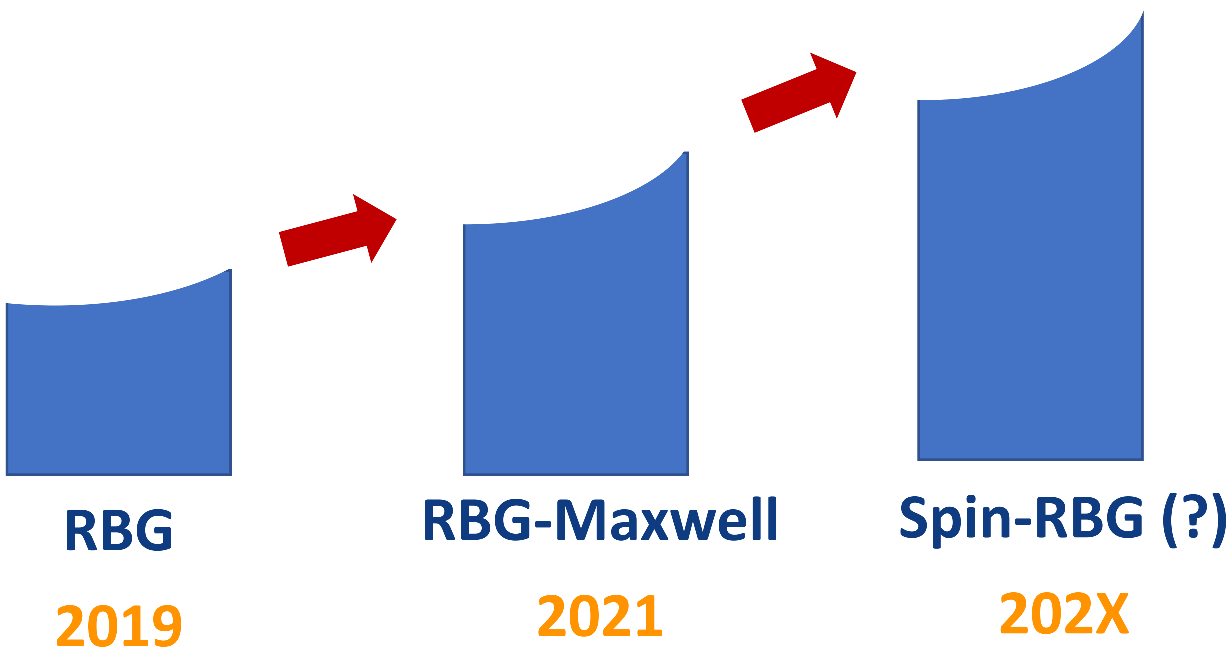




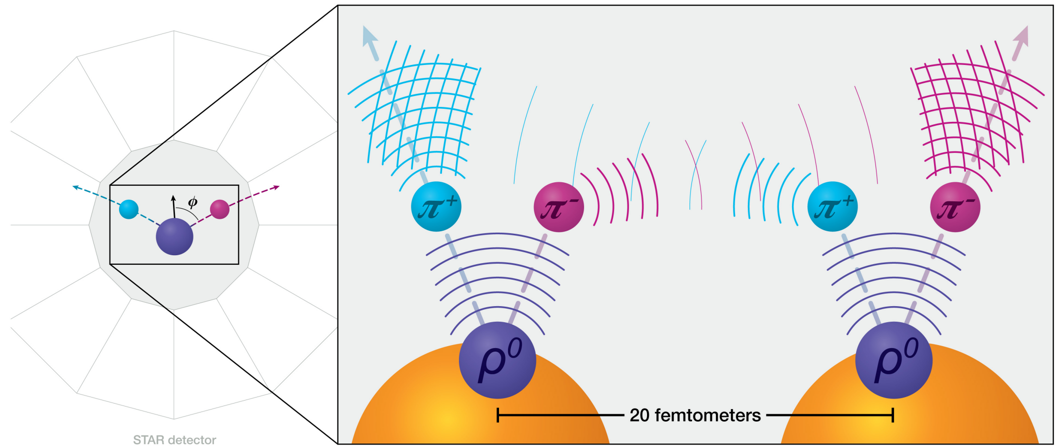
J.J. Zhang, X.L. Sheng, SP, Q. Wang, etc. , PRR 2022

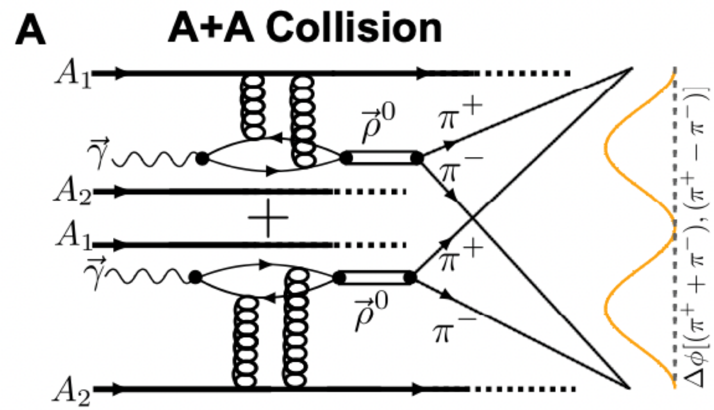
Outlook for simulation of QKT

- Remarkably, we may need to integrate (high dimensional) collision directly kernel to keep the non-local effects instead of other widely used methods.
- Future Plan:
  - Simplify collision kernel
  - Simulations for the non-local collision kernel



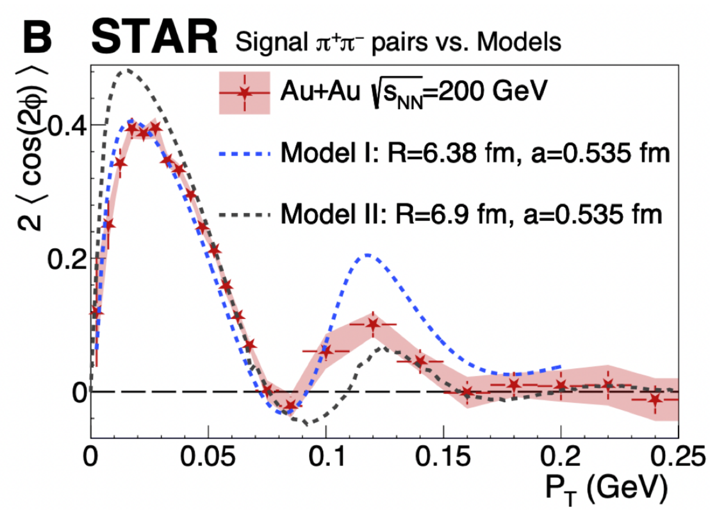
Diffractive vector meson production





$\rho^0 \to \pi^+\pi^-$

- **Azimuthal asymmetries $\cos(2\phi)$ in diffractive vector meson production in UPC**
- **STAR: Sci. Adv. 9 (2023) 1, eabq3903**
- **Theory:**
  - **Zha, Brandenburg, Ruan, Tang, Xu, PRD 2021**
  - **Xing, Zhang, Zhou, Zhou, JHEP 2020**



For $\cos(\phi)$ and $\cos(3\phi)$ related to ρ^0, see Hagiwara, Zhang, Zhou, Zhou, PRD 2021

Also see studies for J/ψ：Brandenburg, Xu, Zha, Zhang, Zhou, Zhou, PRD 2022

Equation of motion for S (I)

- We rewrite the S in CTP and get

$$
\begin{aligned}
S^< &= S_0^< - S_0^R \Sigma^R S^< + S_0^R \Sigma^< S^A - S_0^< \Sigma^A S^A \\
&= S_0^< - S^R \Sigma^R S_0^< + S^R \Sigma^< S_0^A - S^< \Sigma^A S_0^A .
\end{aligned}
$$

- Similar to the QFT at zero temperature, the free parts of retarded and advanced propagators satisfy

$$
\begin{aligned}
-i\sigma \cdot D_{x_1} S_0^R(x_1, x_2) &= \delta^{(4)}(x_1 - x_2), \\
S_0^A(x_1, x_2) i\sigma \cdot \overleftarrow{D}^\dagger_{x_2} &= \delta^{(4)}(x_1 - x_2).
\end{aligned}
$$

which leads to

$$
\begin{aligned}
i\sigma \cdot D_{x_1} S^< &= \Sigma^R S^< - \Sigma^< S^A, \\
-S^< i\sigma \cdot \overleftarrow{D}^\dagger_{x_2} &= -S^R \Sigma^< + S^< \Sigma^A,
\end{aligned}
$$

Connection to other theories

- If we consider A as background fields and take the virtuality expansion, we can reproduce the equivalent photon approximation (EPA) at LO.

$$\sigma_0(A_1A_2 \to l\bar{l}) = \int d\omega_1 d\omega_2 n_{A1}(\omega_1) n_{A2}(\omega_2) \sigma_{\gamma\gamma\to l\bar{l}}(\omega_1,\omega_2),$$

- If we integrate over the space dependence of photon Wigner function, we will reduce to the formulism introduced by Greiner etc. [Vidovic, Greiner, Best, Soff, PRC (1993)] and used by W. Zha, D. Brandenburg, Z.B. Tang, X.B. Xu's group.

- If we use the light-cone coordinates and take a twist expansion, then we can reproduce the formulism from transverse momentum dependent (TMD) parton distribution function (PDF) community (J. Zhou's and B.W. Xiao's group).

$$\frac{d\sigma_{\text{twist}\,2}}{d^3k_1 d^3k_2} = \frac{1}{2(2\pi)^{10}} Z^4\alpha^2 v \int d\omega_1 d^2\mathbf{p}_{1T} d\omega_2 d^2\mathbf{p}_{2T} \frac{1}{E_{k1}E_{k2}} \frac{p^\sigma_{1T} p^\mu_{1T} p^\rho_{2T} p^\nu_{2T}}{\omega_1^2\omega_2^2} \left|\frac{F(-p_1^2)}{-p_1^2}\right|^2 \left|\frac{F(-p_2^2)}{-p_2^2}\right|^2$$

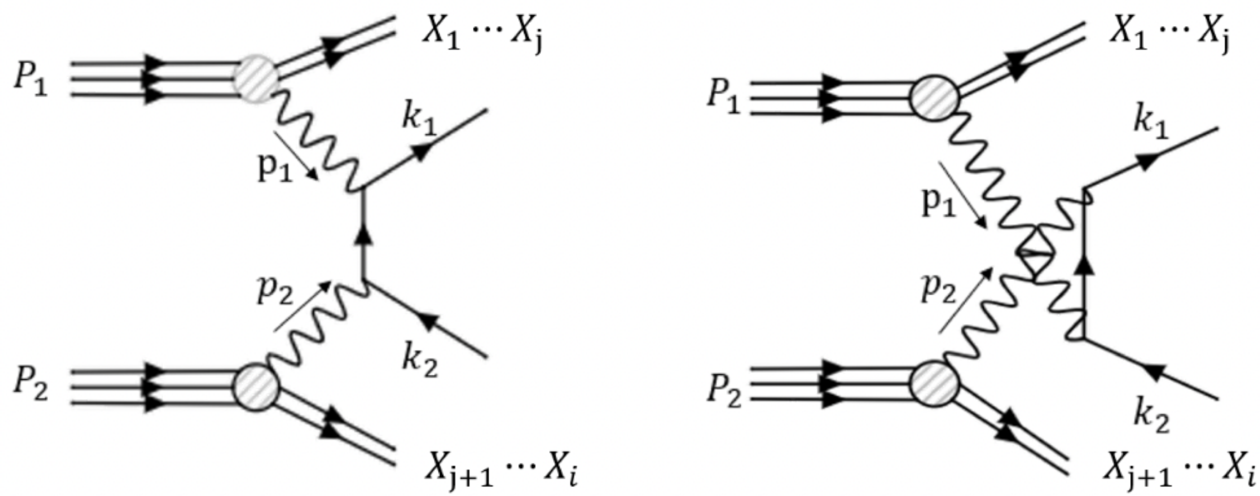
$$\times L_{\mu\nu;\sigma\rho}(p_1,p_2;p_1,p_2;k_1,k_2)(2\pi)^4\delta^4(p_1+p_2-k_1-k_2).$$

$$\sigma_{\text{twist}\,n} = \frac{Z^4\alpha^2 v}{8\pi^4} \int \frac{d^3k_1}{(2\pi)^3 2E_{k1}} \frac{d^3k_2}{(2\pi)^3 2E_{k2}} \frac{d\omega_1}{\omega_1^2} \frac{d\omega_2}{\omega_2^2} d^2p_{1T} d^2p_{2T} \left|\frac{F(-p_1^2)}{-p_1^2}\right|^2 \left|\frac{F(-p_2^2)}{-p_2^2}\right|^2 (2\pi)^4\delta^4(p_1+p_2-k_1-k_2)\mathcal{I},$$

Wave packet description of nuclei

- We start from the wave- packet description of nuclei.

$$A_1(P_{A1}) + A_2(P_{A2}) \to l(k_1) + \bar{l}(k_2) + \sum_f X_f(K_f),$$



$$|A_1\rangle = \int \frac{d^3 P_1}{(2\pi)^3\sqrt{2E_{P1}}} \phi(P_1) e^{ib\cdot P_1} |P_1\rangle ,$$

$$|A_2\rangle = \int \frac{d^3 P_2}{(2\pi)^3\sqrt{2E_{P2}}} \phi(P_2) \, |P_2\rangle ,$$

Initial state: wave packet

$$\left| l, \bar{l}, \sum_f X_f \right\rangle \equiv \left| k_1, k_2, \sum_f K_f \right\rangle_{\text{out}} ,$$

Final state

See Appendix A in R.J. Wang, SP, Q. Wang, PRD 2021

Introducing another two impact parameters

- By definition, the cross section is given by

$$\sigma = \int d^2\mathbf{b}_T \sum_{\{f\}} \int \frac{d^3k_1}{(2\pi)^3 2E_{k_1}} \frac{d^3k_2}{(2\pi)^3 2E_{k_2}} \prod_f \frac{d^3K_f}{(2\pi)^3 2E_f} \times \left| {}_{\text{out}}\left\langle k_1, k_2, \sum_f K_f \right| A_1 A_2 \rangle_{\text{in}} \right|^2$$

- Inserting the expressions of wave packet, we must have the energy momentum conservation in a delta function,

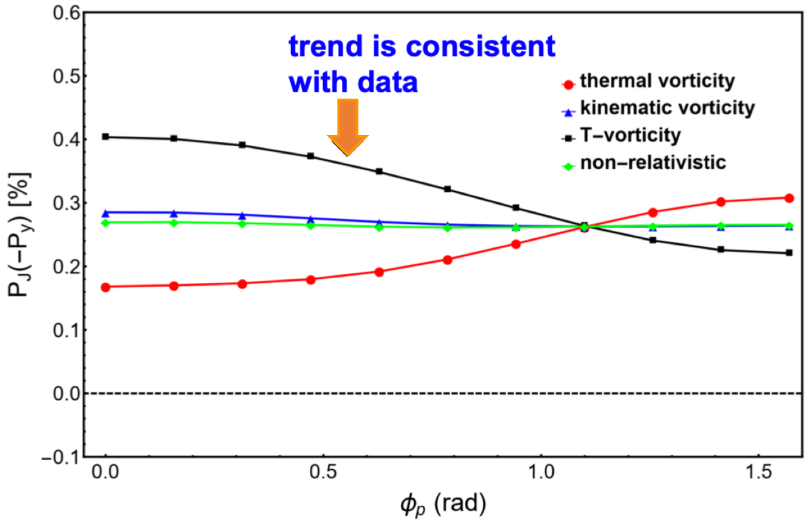
$$\delta^{(2)}\left(\mathbf{P}_{1T} + \mathbf{P}_{2T} - \mathbf{P}'_{1T} - \mathbf{P}'_{2T}\right) = \int \frac{d^2\mathbf{b}_{2T}}{(2\pi)^2} \exp\left[i\mathbf{b}_{2T} \cdot \left(\mathbf{P}_{1T} + \mathbf{P}_{2T} - \mathbf{P}'_{1T} - \mathbf{P}'_{2T}\right)\right],$$

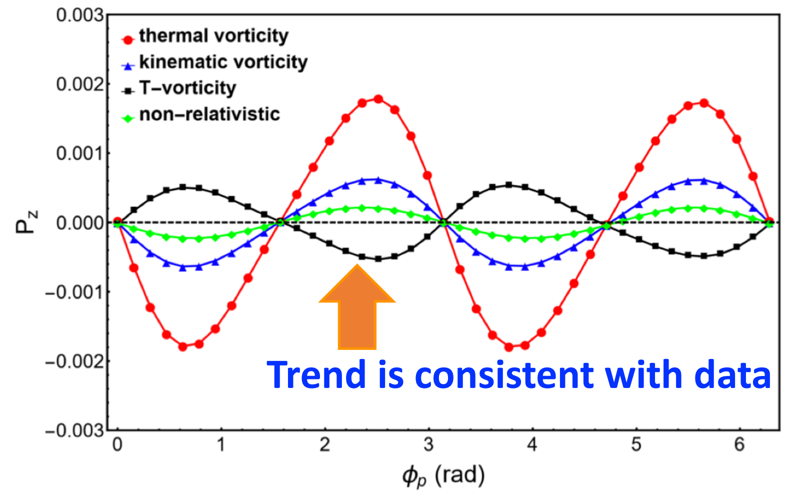
- Considering the geometry, we can use the following identity to introduce another impact parameter,

$$\int d^2\mathbf{b}_{1T} \delta^{(2)}\left(\mathbf{b}_T - \mathbf{b}_{1T} + \mathbf{b}_{2T}\right) = 1,$$

See Appendix A in R.J. Wang, SP, Q. Wang, PRD 2021

T-vorticity Vs thermal vorticity ?





Kinematic vorticity:

$$\omega_{\mu\nu}^{(K)} = -\frac{1}{2}(\partial_\mu u_\nu - \partial_\nu u_\mu)$$

T-vorticity: (black lines)

$$\omega_{\mu\nu}^{(T)} = -\frac{1}{2}[\partial_\mu(Tu_\nu) - \partial_\nu(Tu_\mu)]$$

Non-Relativistic vorticity

$$\omega_{\mu\nu}^{(NR)} = \epsilon_{\nu\mu\rho\eta}u^\rho\omega^\eta$$

Thermal vorticity:

$$\omega_{\rho\sigma}^{\rm th} = \frac{1}{2}\left[\partial_\rho\left(\frac{u_\sigma}{T}\right) - \partial_\sigma\left(\frac{u_\rho}{T}\right)\right]$$

- **Why did T-vorticity give the correct sign?**
- **How to get the contributions of T-vorticity from statistical method, quantum kinetic theory, spin hydrodynamics?**

Wu, Pang, Huang, Wang, PRR 1, 033058 (2019)