2023原子核结构与相对论重离子碰撞前沿交叉研讨会

Alpha-cluster, neutron skin and symmetry energy
—— 核结团, 中子皮和对称能

Chang Xu (许昌)

***Nanjing University* (南京大学)**

1. **Alpha cluster structure and motion in *light nuclei***
2. ***Heavy nuclei*: alpha cluster formation and decay**
3. **Impact of alpha-cluster correlation: *PREX-2 and CREX***
4. ***Summary* and possible strategy for future studies**

南京大学

集团结构与运动(α集团)

原子核运动模式

1. 单粒子运动:原子核壳模型

The Nobel Prize in Physics 1963


Eugene Paul Wigner
Prize share: 1/2

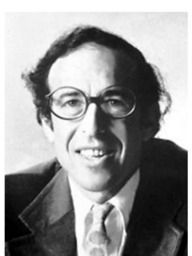

Maria Goeppert Mayer
Prize share: 1/4


J. Hans D. Jensen
Prize share: 1/4

2. 集体运动:原子核集体模型

The Nobel Prize in Physics 1975

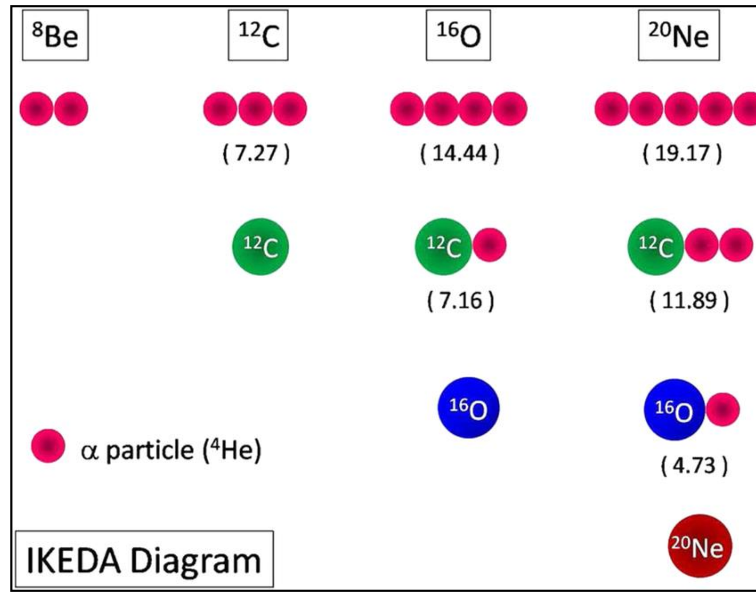

Aage Niels Bohr
Prize share: 1/3


Ben Roy Mottelson
Prize share: 1/3

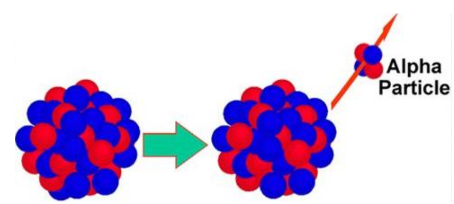

Leo James Rainwater
Prize share: 1/3

3. α集团结构和运动

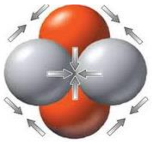
轻核α集团结构



重核和超重核α集团衰变



一个α集团：强关联量子四体系统 (张量&短程关联)



强相互作用

$$V_{ij} = \sum_{p=1,18} v_p(r_{ij}) O^p_{ij}$$

$$O^{p=1,\dots,14}_{ij} = 1,\ \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j,\ \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j,\ (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j),\ S_{ij},\ S_{ij}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j),\ \boldsymbol{L}\cdot\boldsymbol{S},\ \boldsymbol{L}\cdot\boldsymbol{S}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j),\ L^2,\ L^2(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j),\ L^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j),\ L^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j),\ (\boldsymbol{L}\cdot\boldsymbol{S})^2,\ (\boldsymbol{L}\cdot\boldsymbol{S})^2(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j).$$

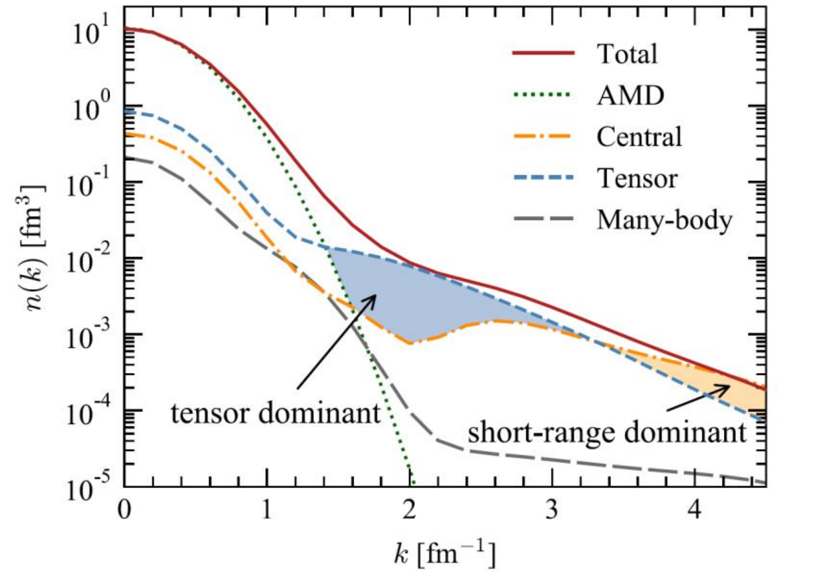
$$O^{p=15,\dots,18}_{ij} = T_{ij},\ T_{ij}(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j),\ T_{ij}S_{ij},\ (\tau_{zi} + \tau_{zj})$$

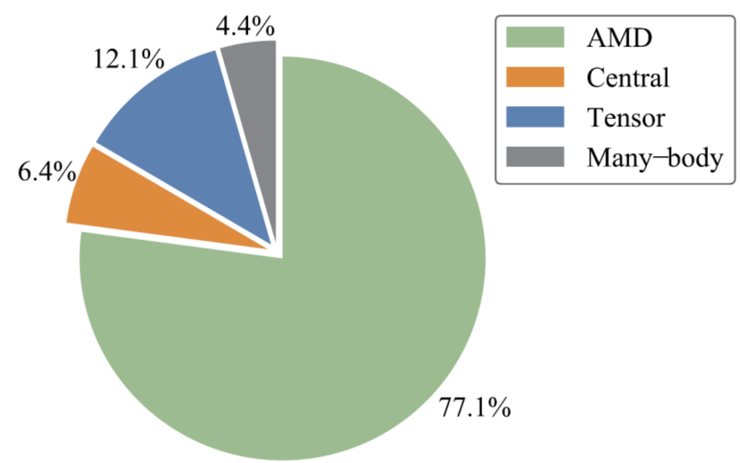
AMD: $|\Psi_0\rangle = n_0\, |\Psi_{\rm AMD}\rangle, \quad |\Psi_S\rangle = n_S F_S\, |\Psi_{\rm AMD}\rangle$

Central: $|\Psi_1\rangle = n_1 (1 - |\Psi_0\rangle\langle\Psi_0|)\, |\Psi_S\rangle,$

Tensor: $|\Psi_2\rangle = n_2\, F_D\, |\Psi_0\rangle,$

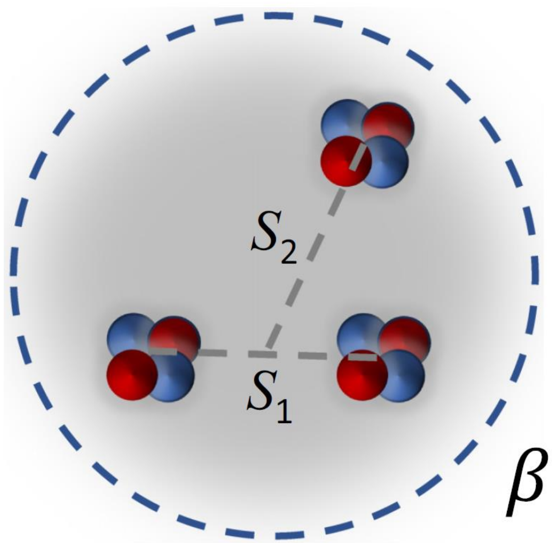
Many-body: $|\Psi_3\rangle = n_3 (1 - |\Psi_0\rangle\langle\Psi_0| - |\Psi_1\rangle\langle\Psi_1| - |\Psi_2\rangle\langle\Psi_2|)\, |\Psi\rangle.$





Lv(吕梦蛟) et.al *Physics Letters B 805 (2020) 135421*

多个α集团：轻核的非局域结构与运动

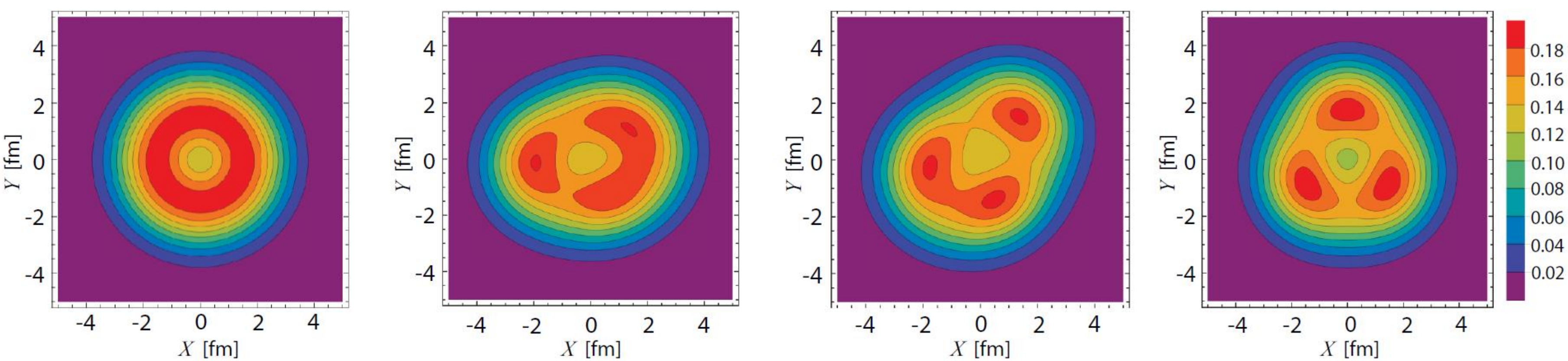


^{12}C中非局域化运动：Pauli Blocking

$$\Phi(\boldsymbol{\beta}, \boldsymbol{S}_1, \boldsymbol{S}_2) = \int d^3R_1 d^3R_2 \exp\left[-\frac{(\boldsymbol{R}_1 - \boldsymbol{S}_1)^2}{2\boldsymbol{\beta}^2} - \frac{2(\boldsymbol{R}_2 - \boldsymbol{S}_2)^2}{3\boldsymbol{\beta}^2}\right] \Phi^B(\boldsymbol{R}_1, \boldsymbol{R}_2)$$

$$\propto \phi_G \mathcal{A}\left\{\exp\left[-\frac{(\boldsymbol{\xi}_1 - \boldsymbol{S}_1)^2}{B^2} - \frac{(\boldsymbol{\xi}_2 - \boldsymbol{S}_2)^2}{3/4\, B^2}\right]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\},$$

$$\Phi^B(\boldsymbol{R}_1, \boldsymbol{R}_2) \propto \phi_G \mathcal{A}\left\{\exp\left[-\frac{(\boldsymbol{\xi}_1 - \boldsymbol{R}_1)^2}{b^2} - \frac{(\boldsymbol{\xi}_2 - \boldsymbol{R}_2)^2}{3/4\, b^2}\right]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\},$$



Zhou(周波) et.al PHYSICAL REVIEW C **99**, 051303(R) (2019)

多个α集团：轻核的非局域结构与运动

PRL **110,** 262501 (2013) PHYSICAL REVIEW LETTERS week ending 28 JUNE 2013

Nonlocalized Clustering: A New Concept in Nuclear Cluster Structure Physics

Bo Zhou,[1,2,3,*] Y. Funaki,[3,†] H. Horiuchi,[2,4] Zhongzhou Ren,[1,5,‡] G. Röpke,[6] P. Schuck,[7,8] A. Tohsaki,[2] Chang Xu,[1] and T. Yamada[9]

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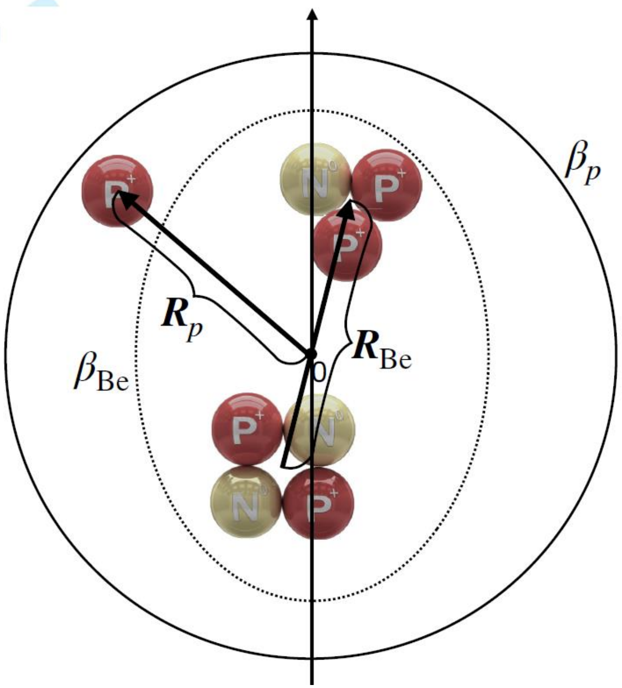
(Received 5 April 2013; revised manuscript received 17 May 2013; published 24 June 2013)

We investigate the $\alpha + {}^{16}\mathrm{O}$ cluster structure in the inversion-doublet band ($K^\pi = 0_1^\pm$) states of $^{20}\mathrm{Ne}$ with an angular-momentum-projected version of the Tohsaki-Horiuchi-Schuck-Röpke (THSR) wave function, which was successful "in its original form" for the description of, e.g., the famous Hoyle state. In contrast with the traditional view on clusters as localized objects, especially in inversion doublets, we find that these *single* THSR wave functions, which are based on the concept of nonlocalized clustering, can well describe the $K^\pi = 0_1^-$ band and the $K^\pi = 0_1^+$ band. For instance, they have 99.98% and 99.87% squared overlaps for 1^- and 3^- states (99.29%, 98.79%, and 97.75% for 0^+, 2^+, and 4^+ states), respectively, with the corresponding exact solution of the $\alpha + {}^{16}\mathrm{O}$ resonating group method. These astounding results shed a completely new light on the physics of low energy nuclear cluster states in nuclei: The clusters are nonlocalized and move around in the whole nuclear volume, only avoiding mutual overlap due to the Pauli blocking effect.

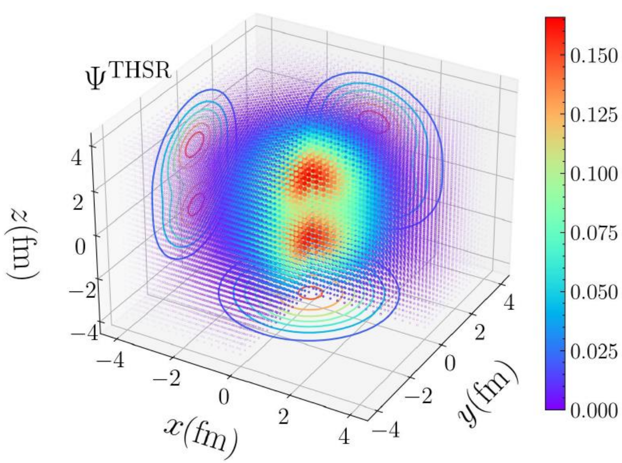
α集团+价核子：轻核的非局域结构与运动



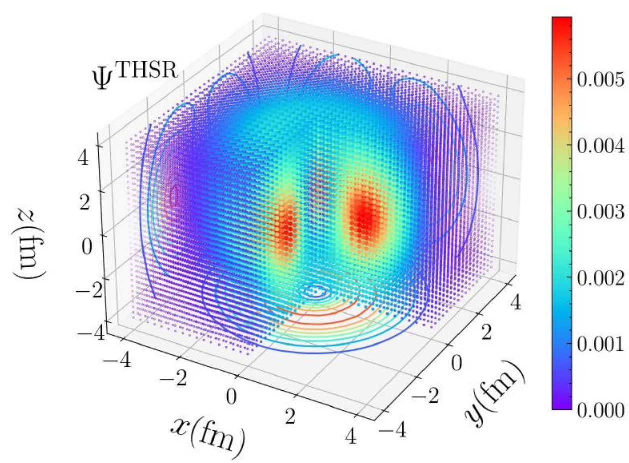
$$H=\sum_{i}^{8}T_i-T_{c.m.}+\sum_{i<j}^{8}V_{i,j}^{N}+\sum_{i<j}^{8}V_{i,j}^{C}+\sum_{i<j}^{8}V_{i,j}^{ls}$$

$$V_{ij}^{N}=\left\{V_1e^{-\alpha_1 r_{ij}^2}-V_2e^{-\alpha_2 r_{ij}^2}\right\}\times\left\{W-M\hat{P}_\sigma\hat{P}_\tau+B\hat{P}_\sigma-H\hat{P}_\tau\right\}$$

$$V_{ij}^{ls}=V_0^{ls}\left\{e^{-\alpha_1 r_{ij}^2}-e^{-\alpha_2 r_{ij}^2}\right\}\mathbf{L}\cdot\mathbf{S}\hat{P}_{31}$$



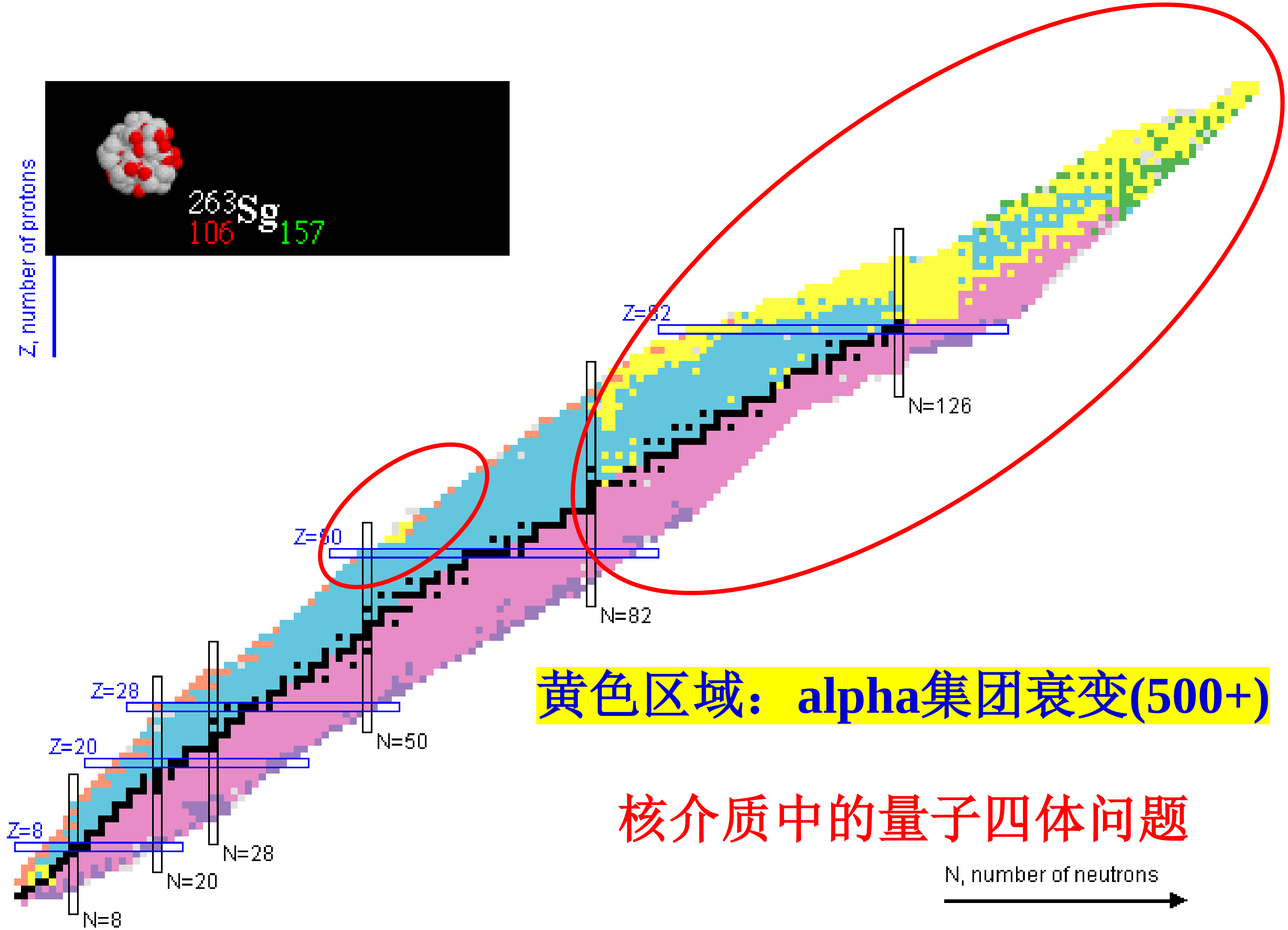
(a) Ψ^{THSR} matter density distribution



(b) Ψ^{THSR} valence density distribution

Lei (雷松矩) et.al

核素图上的α集团衰变



黄色区域：alpha集团衰变(500+)

核介质中的量子四体问题

Xu and Ren, PRC 68 (2003) 034319, newly discovered alpha decay of ^{209}Bi: long-lived alpha emitter

PHYSICAL REVIEW C **68**, 034319 (2003)

α decay of odd-A nuclei with an extra nucleon outside a closed shell

Chang Xu[1] and Zhongzhou Ren[1,2,*]
[1] *Department of Physics, Nanjing University, Nanjing 210008, China*
[2] *Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou 730000, China*
(Received 19 May 2003; published 18 September 2003)

The newly discovered α decay of ^{209}Bi [Marcillac *et al.*, Nature (London) **422**, 876 (2003)] is investigated in the cluster model of α decay. It is found that the cluster model can reproduce the data of this longest-lived α emitter in all known α-decay nuclei. This decay belongs to a special class of α decays occurring in odd-A nuclei with an extra nucleon outside a closed shell. By combining the cluster model of α decay with a microscopic model of preformation α cluster, we can successfully describe the half-lives of odd-A $N=127$ isotones. The cluster model of the favored α decays is interestingly generalized to the hindered α decays of odd-A nuclei.

Ren et al., PRC 70 (2004) 034304, Density-Dependent Cluster Model (DDCM): new model ^{4}He, ^{14}C decay

PHYSICAL REVIEW C **70**, 034304 (2004)

New perspective on complex cluster radioactivity of heavy nuclei

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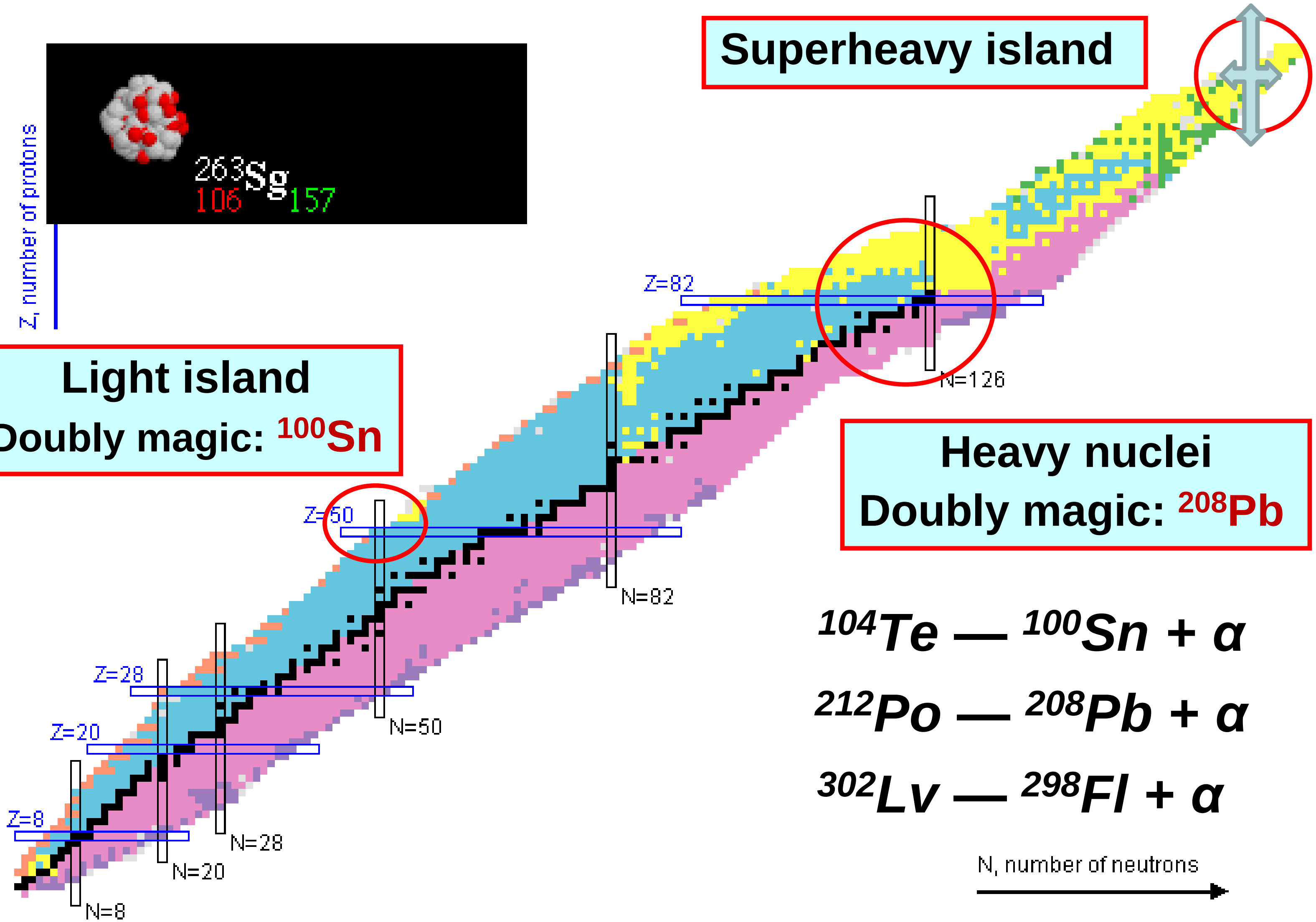
[2]*Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou 730000, China*

(Received 15 June 2004; published 14 September 2004)

Experimental data of complex cluster radioactivity (^{14}C–^{34}Si) are systematically analyzed and investigated with different models. The half-lives of cluster radioactivity are well reproduced by a new formula between half-lives and decay energies and by a microscopic density-dependent cluster model with the renormalized M3Y nucleon-nucleon interaction. The formula can be considered as a natural extension of both the Geiger-Nuttall law and the Viola-Seaborg formula from simple α decay to complex cluster radioactivity where different kinds of clusters are emitted. It is useful for experimentalists to analyze the data of cluster radioactivity. A new linear relationship between the decay energy of cluster radioactivity and the number of α particles in the cluster is found where the increase of decay energy for an extra α particle is between 15 and 17 MeV. The possible physics behind this new linear relationship is discussed.

理想体系：核介质中的α集团形成与发射



Superheavy island

Light island

Doubly magic: ^{100}Sn

Heavy nuclei

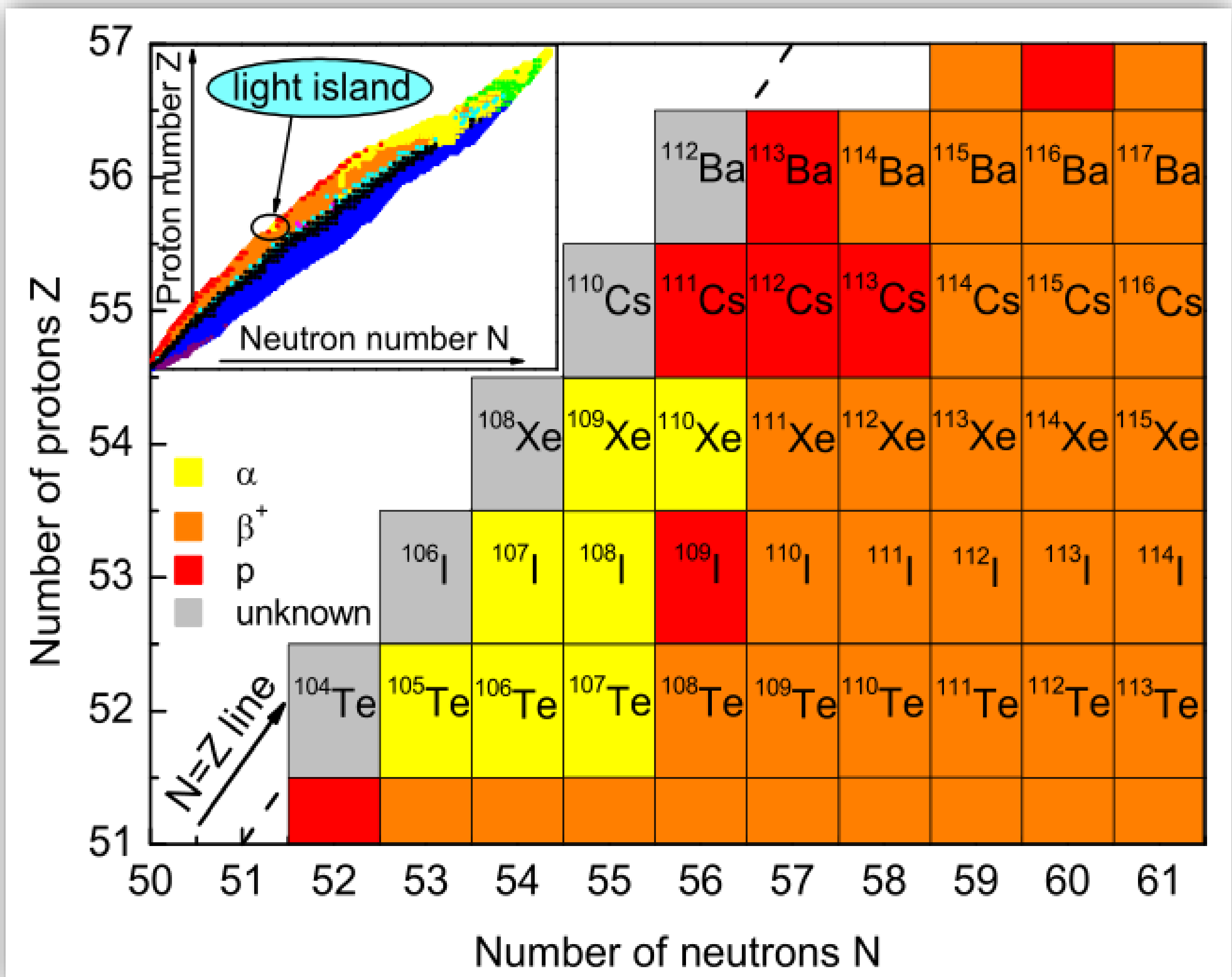
Doubly magic: ^{208}Pb

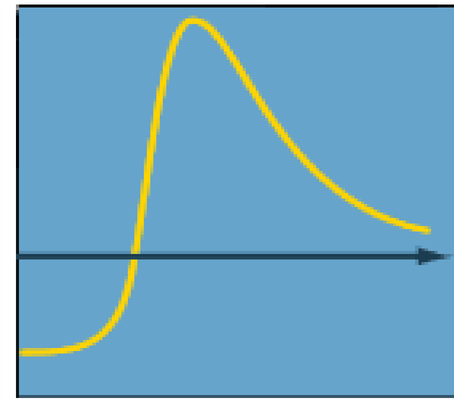
$$^{104}Te — {}^{100}Sn + \alpha$$

$$^{212}Po — {}^{208}Pb + \alpha$$

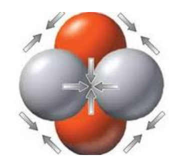
$$^{302}Lv — {}^{298}Fl + \alpha$$

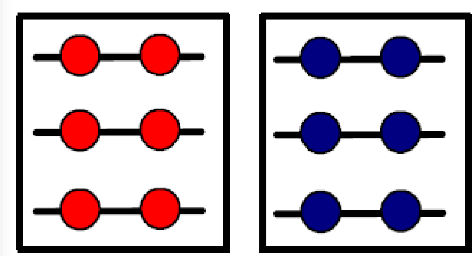
^{100}Sn附近轻岛： α集团衰变





^{104}Te

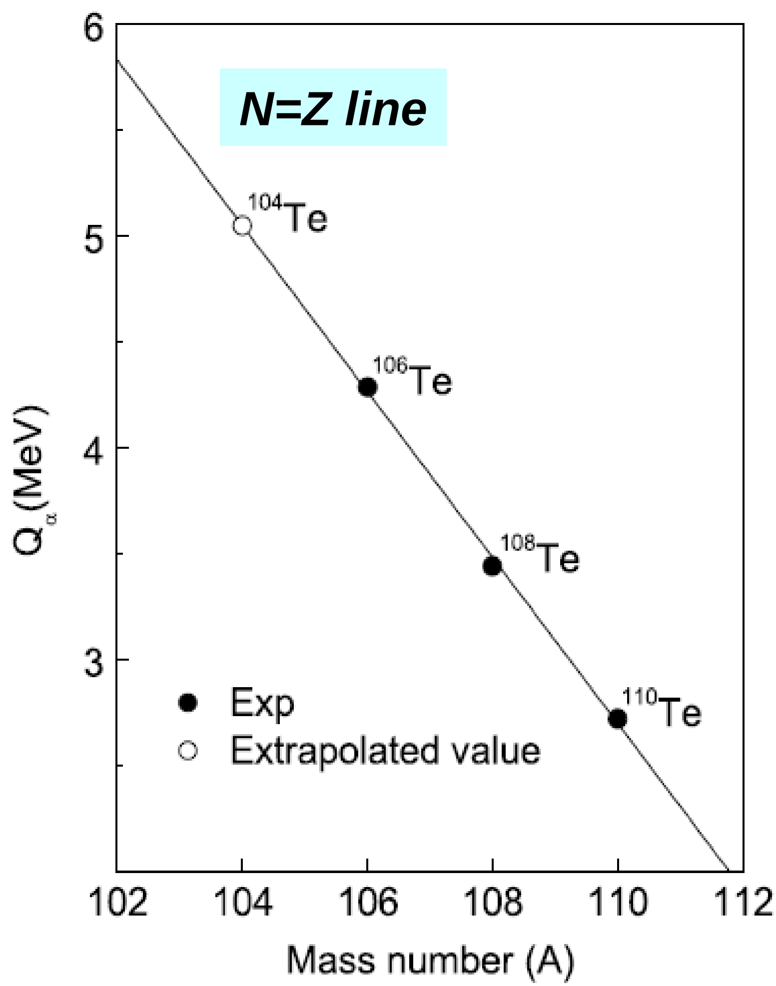


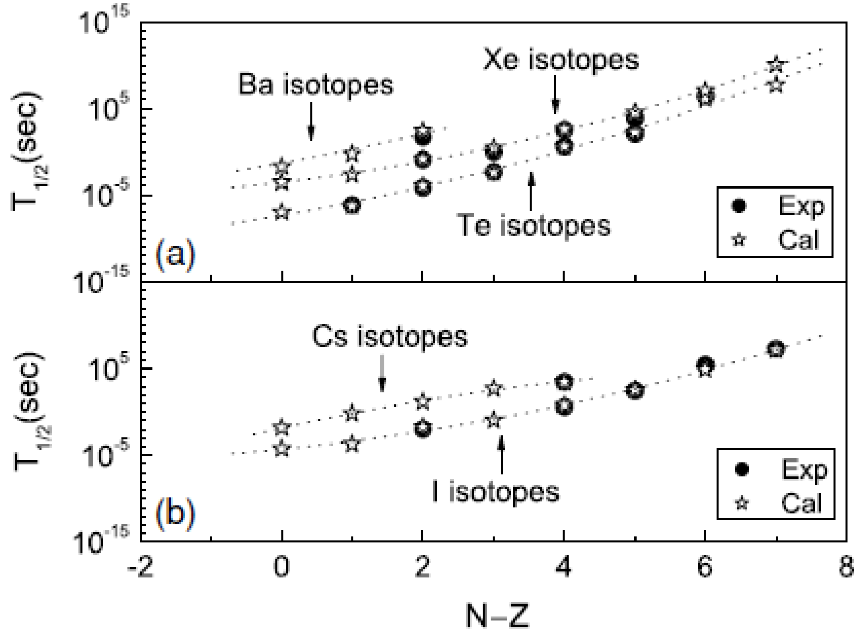


PHYSICAL REVIEW C **74**, 037302 (2006)

Half lives of α-emitters approaching the $N = Z$ line

Chang Xu[1] and Zhongzhou Ren[1,2,3]
[1]*Department of Physics, Nanjing University, Nanjing 210008, People's Republic of China*
[2]*Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou 730000, People's Republic of China*





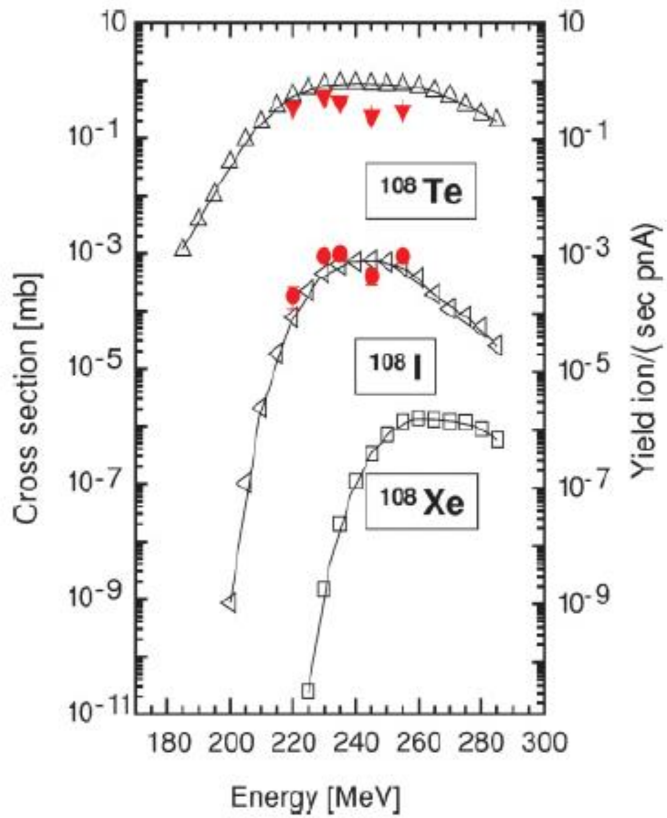
asymmetry along the isotopic chains [7]. To improve the agreement between experiment and theory, we therefore use the isospin-dependent preformation factor $P_\alpha = c_1 + c_2(N - Z)$ instead of the constant one [19] for each kind of nuclei [e.g., a linear dependence $P_\alpha^{ee} = 0.73 - 0.09 \times (N - Z)$ for the even-even nuclei]. As expected, the corresponding theoretical

PHYSICAL REVIEW C 77, 034301 (2008)

University of Tennessee

Toward ^{100}Sn: Studies of excitation functions for the reaction between ^{58}Ni and ^{54}Fe ions

A. Korgul,[1,2,3,4] K. P. Rykaczewski,[5] C. J. Gross,[5] R. K. Grzywacz,[3] S. N. Liddick,[3,6] C. Mazzocchi,[3,7] J. C. Batchelder,[8] C. R. Bingham,[3] I. G. Darby,[3] C. Goodin,[4] J. H. Hamilton,[4] J. K. Hwang,[4] S. V. Ilyushkin,[9] W. Królas,[10] and J. A. Winger[2,8,11]



beam energy around 240 MeV will maximize the production of the $A = 108$ isobar ^{108}Xe in the ^{58}Ni+^{54}Fe reaction. The cross section for the 4n evaporation channel can be expected at the (sub)nanobarn level, see Fig. 4. At $\sigma = 1$ nb, the implantation of about 20 ^{108}Xe ions can be achieved in 100 hr with 50 pnA beam intensity and a 300 μg/cm^2 ^{54}Fe target. The targets rotating with the speed corresponding to a linear velocity for the irradiated spot of about 0.3 m/s can withstand this high beam intensity, see, e.g., Ref. [32]. The predicted half-lives of ^{108}Xe and ^{104}Te are of the order of 50 μs and 10 ns, respectively [14,15]. Using digital pulse processing and recording decay signal waveforms, one should be able to identify the pileup of two α signals at the sum energy around 10 MeV [15].

[14] C. Xu and Z. Ren, Phys. Rev. C **74**, 037302 (2006).
[15] P. Mohr, Eur. Phys. J. A **31**, 23 (2007).

PHYSICAL REVIEW LETTERS **121**, 182501 (2018)

Editors' Suggestion   Featured in Physics

Superallowed α Decay to Doubly Magic ^{100}Sn

K. Auranen,[1,*] D. Seweryniak,[1] M. Albers,[1] A. D. Ayangeakaa,[1,†] S. Bottoni,[1,‡] M. P. Carpenter,[1] C. J. Chiara,[1,2,§] P. Copp,[1,3] H. M. David,[1,∥] D. T. Doherty,[4,¶] J. Harker,[1,2] C. R. Hoffman,[1] R. V. F. Janssens,[5,6] T. L. Khoo,[1] S. A. Kuvin,[1,7] T. Lauritsen,[1] G. Lotay,[8] A. M. Rogers,[1,**] J. Sethi,[1,2] C. Scholey,[9] R. Talwar,[1] W. B. Walters,[2] P. J. Woods,[4] and S. Zhu[1]

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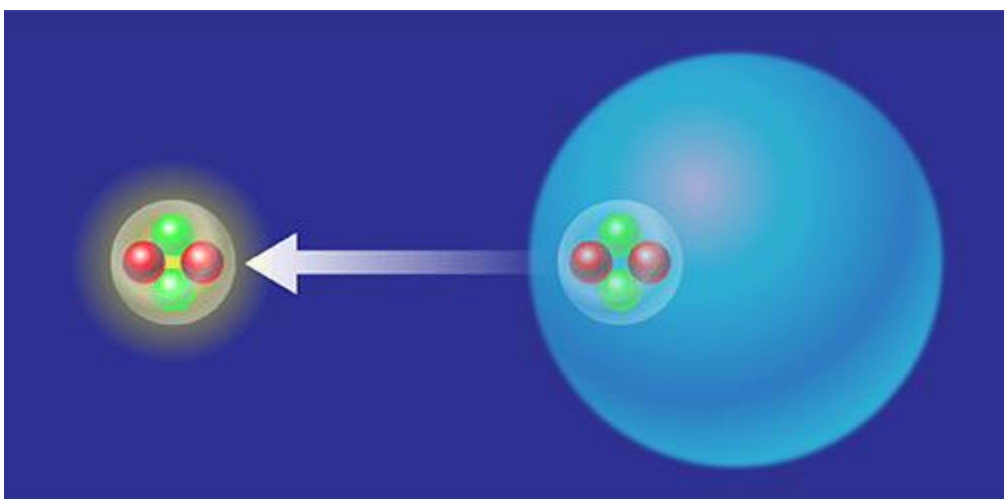


The Fastest Alpha Emitter

"Tellurium-104 is now also the fastest known alpha emitter—though this finding is more fun than fundamental."

APS physics | Physics

Synopsis: The Fastest Alpha Emitter

October 30, 2018

The detection of unusually fast alpha emission from a heavy isotope could lead to new ways of testing the nuclear shell model.

with iron. They then looked for two alpha particles: one from xenon-108 decaying to tellurium-104, the other from tellurium-104 decaying to tin-100. So far, they have detected two of these double-alpha events, and they have placed an upper limit of 18 ns on the tellurium-104 half-life. The measured lifetime limit is in line with shell-model calculations, which predict that alpha preformation in tellurium-104 is several times more likely than in the alpha emitter polonium-212, a benchmark for shell-model calculations. Tellurium-104 is now also the fastest known alpha emitter—though this finding is more fun than fundamental.

2018: ^{104}Te <18ns (experiment)

PHYSICAL REVIEW LETTERS **121,** 182501 (2018)

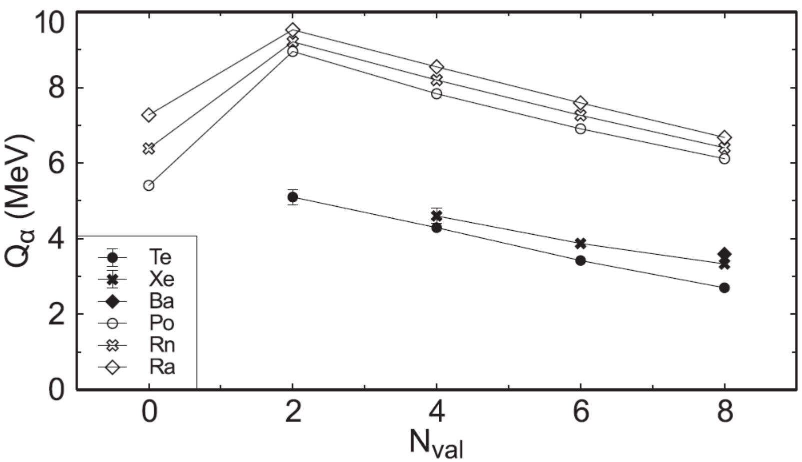
Editors' Suggestion | Featured in Physics

Superallowed α Decay to Doubly Magic ^{100}Sn

Chain	Nuclide	E_α (keV)	$T_{1/2}$	b_α (%)
$N = Z$	^{108}Xe	4400(200)	58^{+106}_{-23} μs	100[a]
$N = Z$	^{104}Te	4900(200)	<18 ns	100[a]
$N = Z + 2$	^{114}Ba	3480(20) [17]	380^{+190}_{-110} ms [17]	0.9(3) [35]
$N = Z + 2$	^{110}Xe	3720(20) [17]	95^{+25}_{-20} ms [17]	64(35) [35]
$N = Z + 2$	^{106}Te	4128(9) [36]	70^{+20}_{-15} μs [17]	100 [35]
$N = Z + 4$	^{112}Xe	3216(7) [36]	2.7(8) s [37]	$0.8^{+1.1}_{-0.5}$ [36]
$N = Z + 4$	^{108}Te	3314(4) [20]	2.1(1) s [37]	49(4) [36]

suddenly. The present data are in agreement with this linear trend, and therefore with the extrapolated values of $Q_\alpha(^{104}Te) = 5.053$ MeV and $Q_\alpha(^{108}Xe) = 4.440$ MeV [29]. Furthermore, the folding potential calculations

[29] C. Xu and Z. Ren, Phys. Rev. C **74**, 037302 (2006).



α集团衰变：双幻数核^{100}Sn附近

PHYSICAL REVIEW C **100**, 034315 (2019)

Search for α decay of ^{104}Te with a novel recoil-decay scintillation detector

Y. Xiao,[1] S. Go,[1,2] R. Grzywacz,[1,3] R. Orlandi,[4] A. N. Andreyev,[4,5] M. Asai,[4] M. A. Bentley,[5] G. de Angelis,[6] C. J. Gross,[3] P. Hausladen,[3] K. Hirose,[4] S. Hofmann,[7] H. Ikezoe,[4] D. G. Jenkins,[5] B. Kindler,[7] R. Léguillon,[4] B. Lommel,[7] H. Makii,[4] C. Mazzocchi,[8] K. Nishio,[4] P. Parkhurst,[9] S. V. Paulauskas,[1] C. M. Petrache,[10] K. P. Rykaczewski,[3] T. K. Sato,[4] J. Smallcombe,[4] A. Toyoshima,[4] K. Tsukada,[4] K. Vaigneur,[11] and R. Wadsworth[5]

[1]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
[2]*Department of Physics, Kyushu University, Fukuoka 819-0395, Japan*
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α集团衰变：双幻数核^{100}Sn附近

I. INTRODUCTION

In the α-decay island northeast of ^{100}Sn, valence protons and neutrons are expected to occupy the same single-particle orbitals outside the $N = Z = 50$ doubly magic nucleus ^{100}Sn. The additional interaction between protons and neutrons may lead to the enhanced pre-formation of an α particle and therefore to the enhancement of α-decay probability, the so-called superallowed α decay [1]. Extensive experimental efforts have been made in this region, providing evidence of such enhancement [2–6]. The ultimate evidence would be the observation of accelerated α decay of ^{104}Te ($N = Z = 52$) with two protons and two neutrons occupying the same single-particle orbitals. When α clusterization is included, the estimated half-life would be as short as 50 ns [7], which makes the measurement of ^{104}Te decay very difficult. The indirect production of this isotope through the synthesis of the longer-lived α-decay precursor ^{108}Xe, whose half-life is estimated to be 0.15 ms [7] by the same model with enhanced preformation, would enable the study of ^{104}Te using the in-flight electromagnetic separation technique. Even in this case, the short half-life of ^{104}Te is a challenge for today's detection techniques and requires the use of a fast response detection method to be able to separate the α decay of ^{108}Xe and the fast α decay of ^{104}Te. Semiconductor detectors, e.g., double-sided strip detectors (DSSDs), are widely used as implantation detectors for such measurements of ions and charged particle

The short half-life of 104Te: challenge for today's detection techniques

the 100 ns limit to resolve two consecutive pulses remains a challenge. In addition, the expensive DSSDs are susceptible to radiation damage. A recent measurement [10] resulted with the half-life estimate $T_{1/2} < 18$ ns for ^{104}Te based on

[7] C. Xu and Z. Ren, Phys. Rev. C **74**, 037302 (2006).

Dear Friends

It seems that Riken PAC appreciated the importance of this proposal and awarded it an S category. I would like to stress how important was the work of our theory friends to lay foundation for this proposal. The S grade would not be possible without their recent work on new perspective on alpha decay theory.

Let's hope that the cross section to produce 108 Xe from 124xe will be favorable for us.

With best regards

Robert

2023

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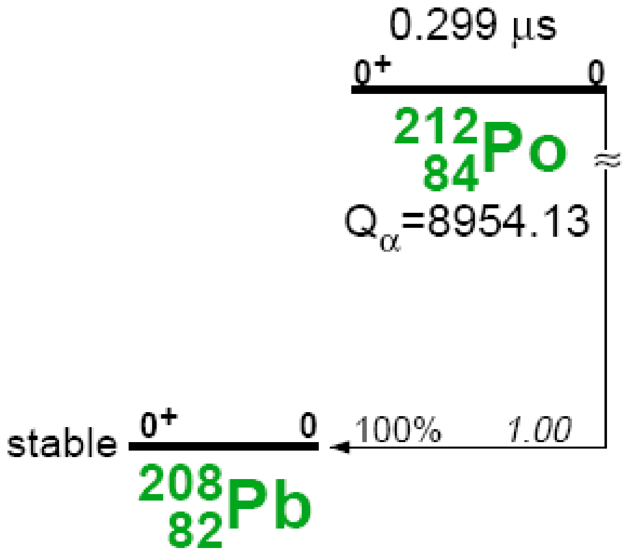
α集团衰变：双幻数核^{208}Pb附近



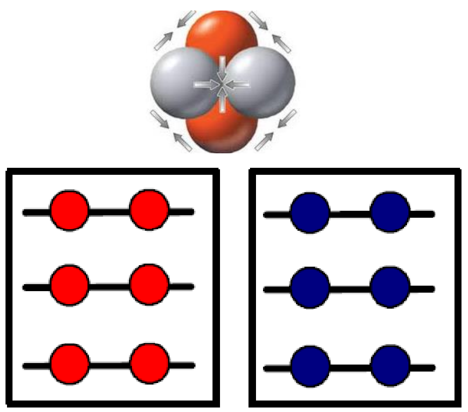
Spherical

Doubly magic

Only one decay channel

Accurate experimental data

……

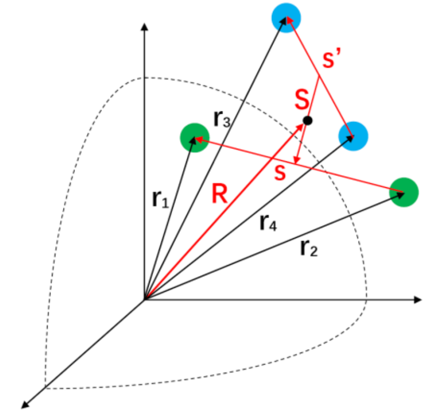


Microscopic calculation of alpha cluster formation and decay in ^{212}Po

Quartetting wave function approach

2n+2p+core: subdivide the W.F. into an intrinsic part and a c.o.m part

$$\Psi(\mathbf{R},\mathbf{s}_j) = \varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})\,\Phi(\mathbf{R})$$



equation for the c.m. motion

$$-\frac{\hbar^2}{2Am}\nabla_R^2\Phi(\mathbf{R}) - \frac{\hbar^2}{Am}\int ds_j \varphi^{\rm intr,*}(\mathbf{s}_j,\mathbf{R})[\nabla_R\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})][\nabla_R\Phi(\mathbf{R})]$$

$$-\frac{\hbar^2}{2Am}\int ds_j \varphi^{\rm intr,*}(\mathbf{s}_j,\mathbf{R})\left[\nabla_R^2\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})\right]\Phi(\mathbf{R}) + \int dR'\, W(\mathbf{R},\mathbf{R}')\Phi(\mathbf{R}') = E\,\Phi(\mathbf{R})$$

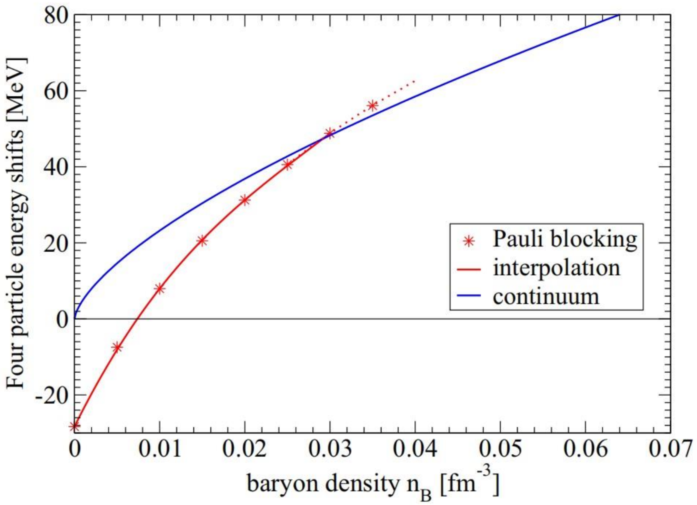
$$\mathbf{r}_{n,\uparrow} = \mathbf{R}+\mathbf{S}/2+\mathbf{s}/2$$
$$\mathbf{r}_{n,\downarrow} = \mathbf{R}+\mathbf{S}/2-\mathbf{s}/2$$
$$\mathbf{r}_{p,\uparrow} = \mathbf{R}-\mathbf{S}/2+\mathbf{s}'/2$$
$$\mathbf{r}_{p,\downarrow} = \mathbf{R}-\mathbf{S}/2-\mathbf{s}'/2$$

equation for the intrinsic motion

$$-\frac{\hbar^2}{Am}\Phi^*(\mathbf{R})[\nabla_R\Phi(\mathbf{R})][\nabla_R\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})] - \frac{\hbar^2}{2Am}|\Phi(\mathbf{R})|^2\nabla_R^2\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})$$

$$+\int dR'\, ds_j'\, \Phi^*(\mathbf{R})\left\{T\left[\nabla_{s_j}\right]\delta(\mathbf{R}-\mathbf{R}')\delta(\mathbf{s}_j-\mathbf{s}_j') + V(\mathbf{R},\mathbf{s}_j;\mathbf{R}',\mathbf{s}_j')\right\}\Phi(\mathbf{R}')\varphi^{\rm intr}(\mathbf{s}_j',\mathbf{R}') = F(\mathbf{R})\varphi^{\rm intr}(\mathbf{s}_j,\mathbf{R})$$

α-cluster approaching the core



$$\mathbf{p}_1 = \mathbf{P}/4 + \mathbf{k}/2 + \mathbf{k}_{12}, \qquad \mathbf{p}_2 = \mathbf{P}/4 + \mathbf{k}/2 - \mathbf{k}_{12},$$

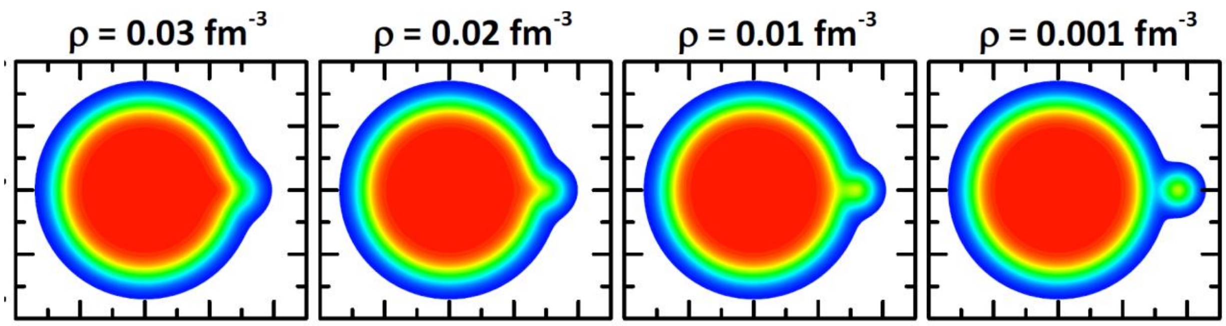
$$\mathbf{p}_3 = \mathbf{P}/4 - \mathbf{k}/2 + \mathbf{k}_{34}, \qquad \mathbf{p}_4 = \mathbf{P}/4 - \mathbf{k}/2 - \mathbf{k}_{34}.$$

$$\frac{\hbar^2}{2m}[k^2 + 2k_{12}^2 + 2k_{34}^2]\tilde{\varphi}^{\rm intr}(\mathbf{k},\mathbf{k}_{12},\mathbf{k}_{34},\mathbf{P}) + \int \frac{d^3k'}{(2\pi)^3}\frac{d^3k'_{12}}{(2\pi)^3}\frac{d^3k'_{34}}{(2\pi)^3}\tilde{V}_4(\mathbf{k},\mathbf{k}_{12},\mathbf{k}_{34};\mathbf{k}',\mathbf{k}'_{12},\mathbf{k}'_{34};\mathbf{P})\tilde{\varphi}^{\rm intr}(\mathbf{k}',\mathbf{k}'_{12},\mathbf{k}'_{34},\mathbf{P}) = \tilde{W}(\mathbf{P})\tilde{\varphi}^{\rm intr}(\mathbf{k},\mathbf{k}_{12},\mathbf{k}_{34},\mathbf{P})$$

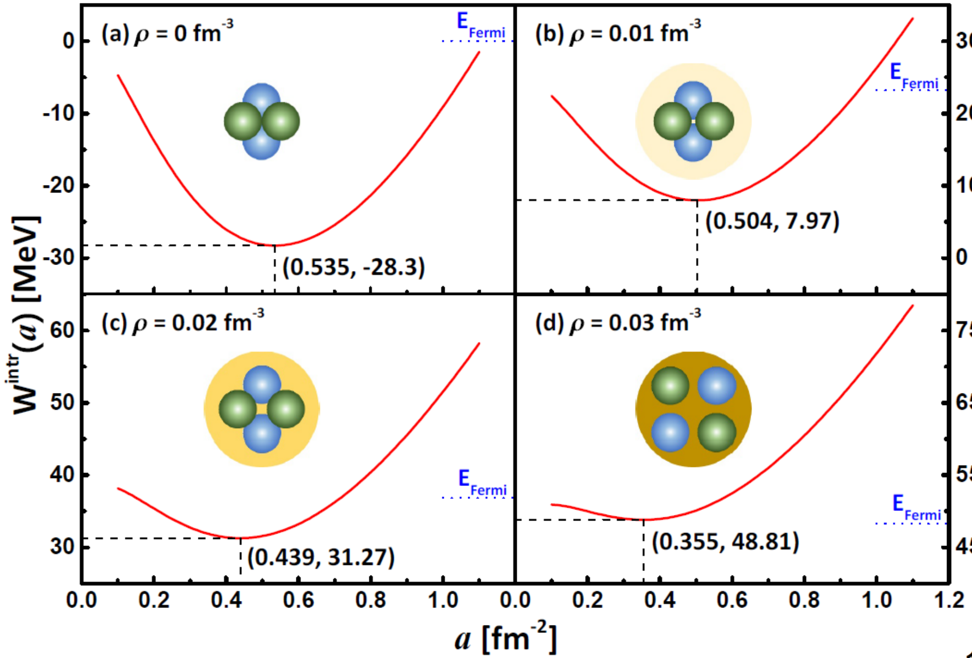
$$\tilde{W}^{\rm intr}(\mathbf{P}) = \frac{\hbar^2}{2m}\int \frac{d^3k}{(2\pi)^3}\frac{d^3k_{12}}{(2\pi)^3}\frac{d^3k_{34}}{(2\pi)^3}\left[k^2 + 2k_{12}^2 + 2k_{34}^2\right]|\tilde{\varphi}^{\rm intr}(\mathbf{k},\mathbf{k}_{12},\mathbf{k}_{34},\mathbf{P})|^2 + \int \frac{d^3k}{(2\pi)^3}\frac{d^3k_{12}}{(2\pi)^3}\frac{d^3k_{34}}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\frac{d^3k'_{12}}{(2\pi)^3}\frac{d^3k'_{34}}{(2\pi)^3}\tilde{\varphi}^{\rm intr,*}(\mathbf{k},\mathbf{k}_{12},\mathbf{k}_{34},\mathbf{P}) \times \tilde{V}_4^{\rm intr}(\mathbf{k},\mathbf{k}_{12},\mathbf{k}_{34};\mathbf{k}',\mathbf{k}'_{12},\mathbf{k}'_{34};\mathbf{P})\tilde{\varphi}^{\rm intr}(\mathbf{k}',\mathbf{k}'_{12},\mathbf{k}'_{34},\mathbf{P})$$

$$\tilde{W}^{\rm intr,bound}(n_B) = \tilde{W}^{\rm Pauli}(n_B) - 28.3\,{\rm MeV} \qquad \tilde{W}^{\rm intr,free}(n_B) = 2E_{\rm Fermi}(n_n) + 2E_{\rm Fermi}(n_p) = \frac{\hbar^2}{m}\left[2(\frac{3}{2}\pi^2 n_B)^{2/3}\right]$$

α-cluster dissolves



free α-cluster

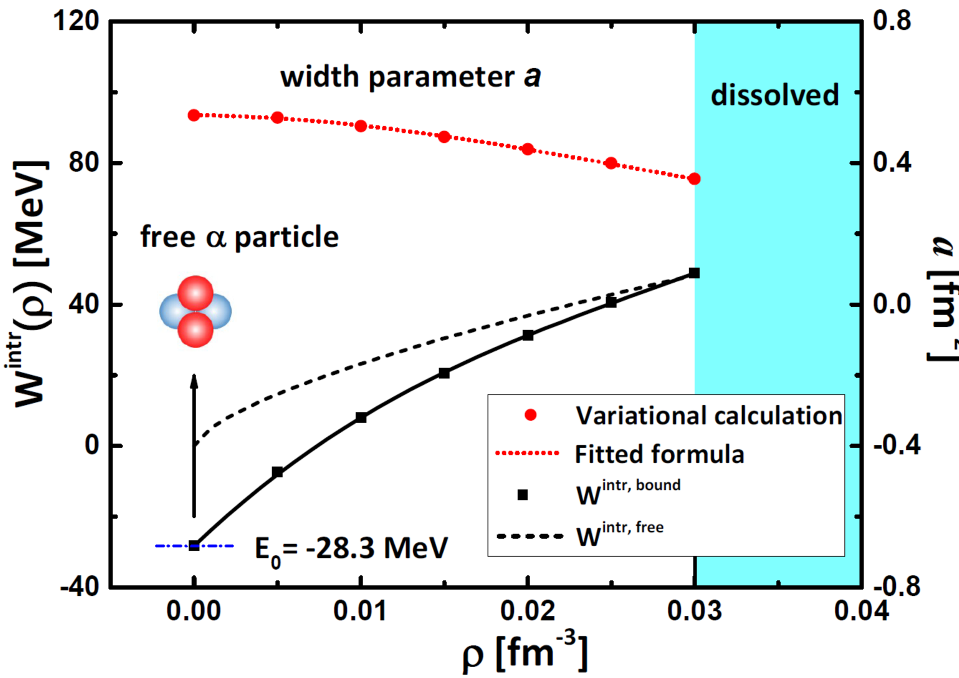


Strong binding of α-cluster is gradually reduced

Energy shift due to Pauli blocking after it feels the tail of the core density

Eventually α-cluster dissolves. Before that, remains a relatively compact entity with small extension even up to the critical density

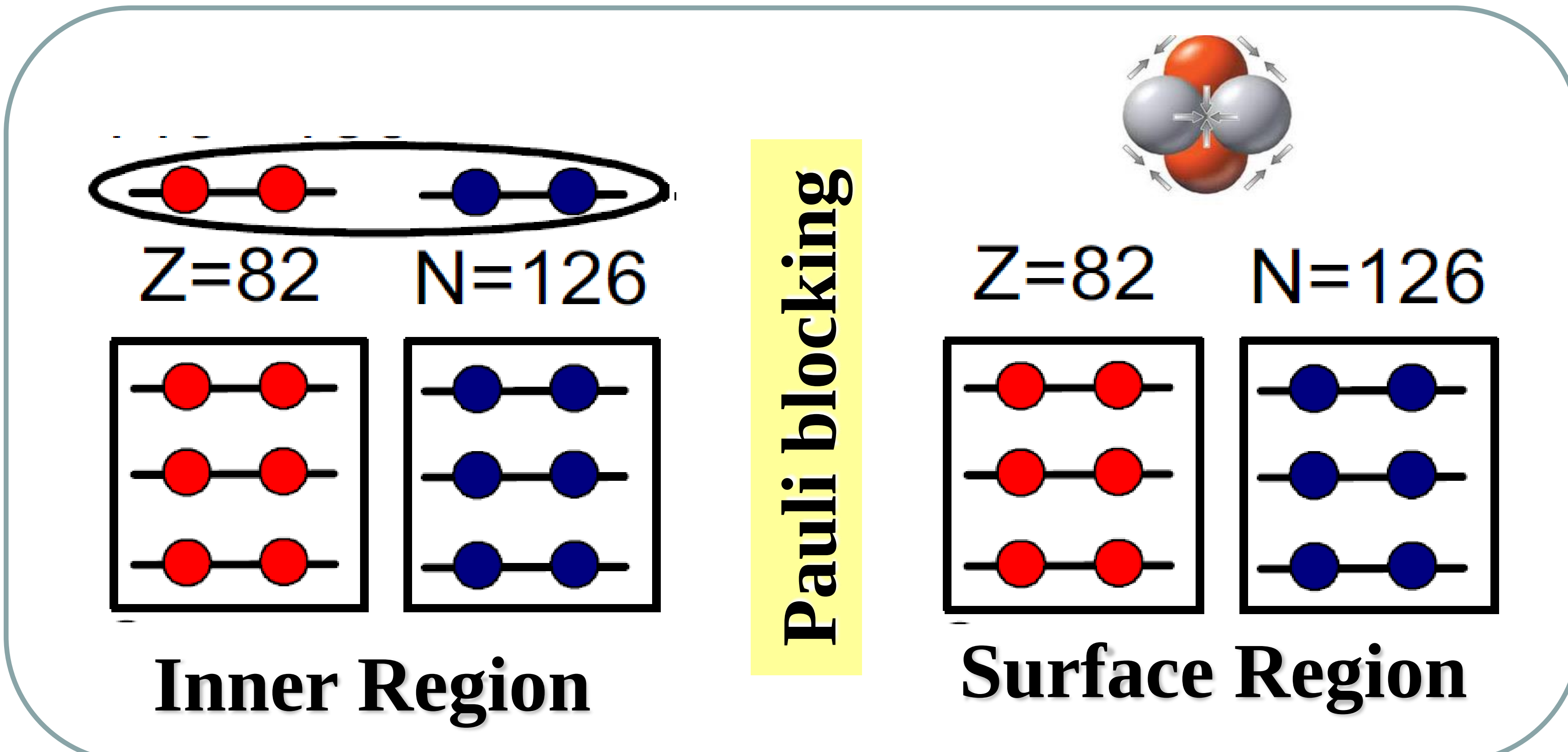
Four nucleons go over into single particle states with pair correlations in the open shells on top of the core



Alpha cluster formation and decay

——Quartetting wave function approach

Intrinsic bound-state W. F. transforms at critical density into an unbound 4 nucleon shell-model state



Alpha cluster formation and decay

——*Quartetting wave function approach*

The quartet wave function with the Jacobi-Moshinsky coordinates

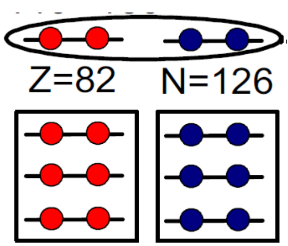
$$\begin{aligned}
\Phi_{\text{quartet}}(\mathbf{R},\mathbf{S},\mathbf{s},\mathbf{s}') = \;& \Phi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) \\
= \;& \sum_{J_{12},M_{12},J_{34},M_{34}} \langle J_{12},M_{12},J_{34},M_{34}|J,M\rangle \\
& \mathcal{A}_{12}\Big\{\sum_{m_{s1},m_{s2}}\sum_{m_1,m_2,m_{l1},m_{l2}} \langle j_1,m_1,j_2,m_2|J_{12},M_{12}\rangle \left\langle l_1,m_{l1},\frac{1}{2},m_{s1}|j_1,m_1\right\rangle \\
& \left\langle l_2,m_{l2},\frac{1}{2},m_{s2}|j_2,m_2\right\rangle R_{n_1l_1}(r)Y_{l_1m_{l1}}(\theta_{r1},\varphi_{r1})R_{n_2l_2}(r)Y_{l_2m_{l2}}(\theta_{r2},\varphi_{r2})\Big\} \\
& \mathcal{A}_{34}\Big\{\sum_{m_{s3},m_{s4}}\sum_{m_3,m_4,m_{l3},m_{l4}} \langle j_3,m_3,j_4,m_4|J_{34},M_{34}\rangle \left\langle l_3,m_{l3},\frac{1}{2},m_{s3}|j_3,m_3\right\rangle \\
& \left\langle l_4,m_{l4},\frac{1}{2},m_{s4}|j_4,m_4\right\rangle R_{n_3l_3}(r)Y_{l_3m_{l3}}(\theta_{r3},\varphi_{r3})R_{n_4l_4}(r)Y_{l_4m_{l4}}(\theta_{r4},\varphi_{r4})\Big\}
\end{aligned}$$

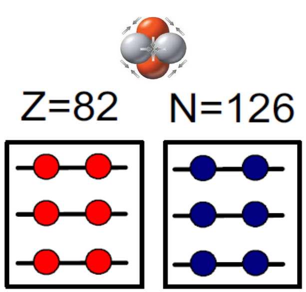
The c.o.m. wave function of quartet is obtained from the integral

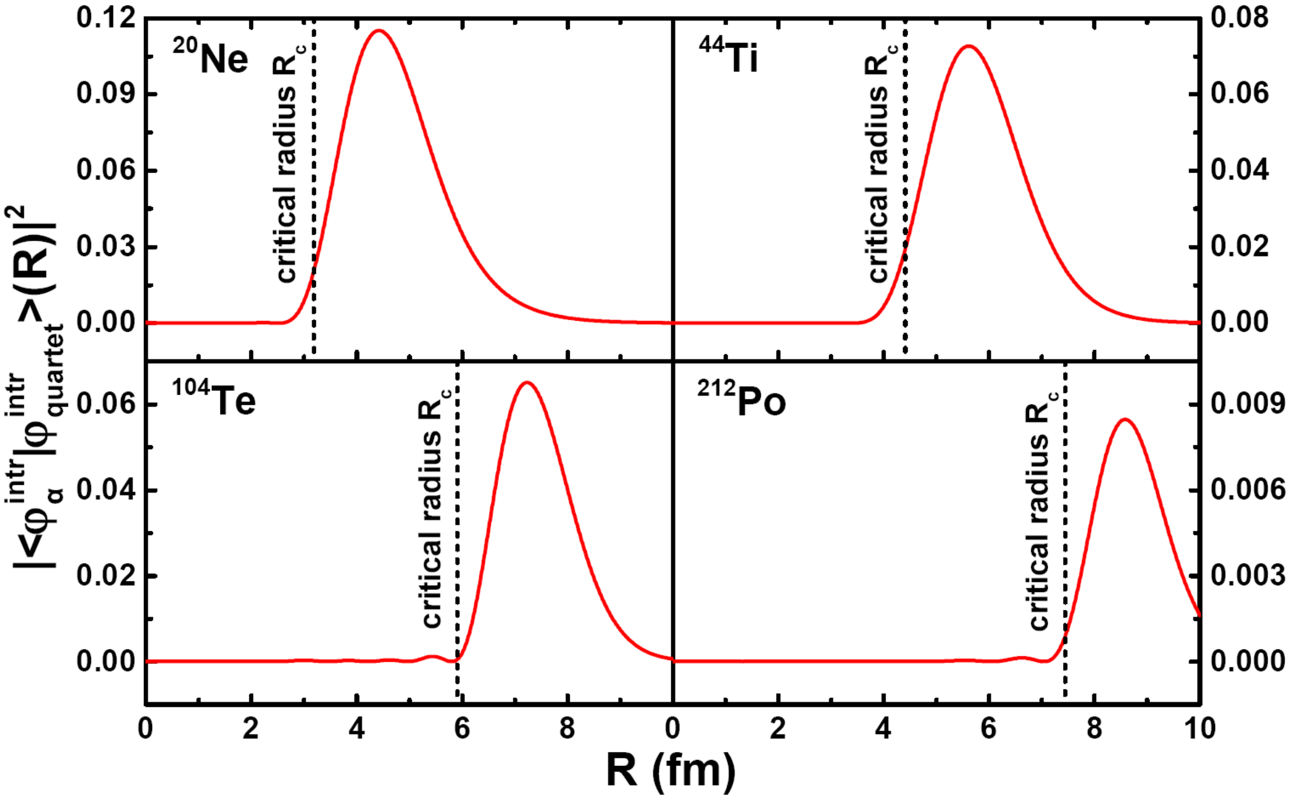
$$\Psi^{\text{com}}_{\text{quartet}}(\mathbf{R}) = [\int d^3S\, d^3s\, d^3s'\, |\Phi_{\text{quartet}}(\mathbf{R},\mathbf{S},\mathbf{s},\mathbf{s}')|^2]^{1/2}$$

Overlap of intrinsic W.F. and W.F. of a free alpha cluster

$$\begin{aligned}
\left\langle \varphi_\alpha^{\rm intr} \middle| \varphi_{\rm quartet}^{\rm intr} \right\rangle (R) &= \int d^3S\, d^3s\, d^3s'\, \varphi_\alpha^{\rm intr,*}(\mathbf{S},\mathbf{s},\mathbf{s}')\varphi_{\rm quartet}^{\rm intr}(\mathbf{R},\mathbf{S},\mathbf{s},\mathbf{s}') \\
&= \int d^3S\, d^3s\, d^3s'\, \frac{\mathscr{Y}_\alpha^*(\mathbf{r_1})\mathscr{Y}_\alpha^*(\mathbf{r_2})\mathscr{Y}_\alpha^*(\mathbf{r_3})\mathscr{Y}_\alpha^*(\mathbf{r_4})\Phi_{\rm quartet}(\mathbf{R},\mathbf{S},\mathbf{s},\mathbf{s}')}{\mathscr{Z}_\alpha^*(\mathbf{R})\Psi_{\rm quartet}^{\rm com}(\mathbf{R})} \\
&= \frac{64(2\pi)^9}{\mathscr{Z}_\alpha(\mathbf{R})\Psi_{\rm quartet}^{\rm com}(\mathbf{R})} \int d^3p\, \phi_{12}(\mathbf{p})\phi_{34}(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{R}}.
\end{aligned}$$







Alpha cluster formation and decay
——*Quartetting wave function approach*

First-principle approach to nuclear many-body system: several approximations performed to make the approach practicable

$$\left[-\frac{\hbar^2}{8m}\frac{\partial^2}{\partial \mathbf{R}^2}+W(\mathbf{R})\right]\Phi(\mathbf{R})=E_4\Phi(\mathbf{R}),$$

with

$$W(\mathbf{R})=E_4^{\text{intr}}(\mathbf{R})=W^{\text{ext}}(\mathbf{R})+W^{\text{intr}}(\mathbf{R})$$
$$=W^{\text{ext}}(\mathbf{R})+E_\alpha^{(0)}+W^{\text{Pauli}}(\mathbf{R})$$

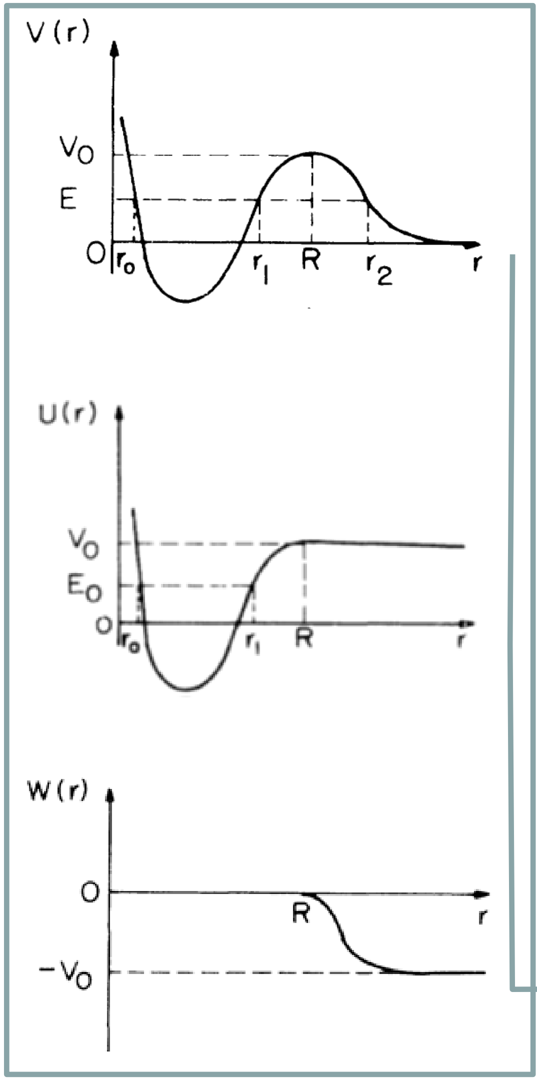
$$-\frac{\hbar^2}{4m}\int d^9 s_j \varphi^{\text{intr},*}(\mathbf{s}_j,\mathbf{R})[\nabla_R\varphi^{\text{intr}}(\mathbf{s}_j,\mathbf{R})]\times[\nabla_R\Psi^{\text{com}}(\mathbf{R})]$$
$$-\frac{\hbar^2}{8m}\int d^9 s_j \varphi^{\text{intr},*}(\mathbf{s}_j,\mathbf{R})\left[\nabla_R^2\varphi^{\text{intr}}(\mathbf{s}_j,\mathbf{R})\right]\Psi^{\text{com}}(\mathbf{R})$$

The interaction of the quartet with core contains not only the direct NN interaction but also exchange term/Pauli-blocking (genuine non-local interactions)

$$W^{\text{Pauli}}(n_B)\approx 4515.9\,\text{MeV fm}^3 n_B-100935\,\text{MeV fm}^6 n_B^2+1202538\,\text{MeV fm}^9 n_B^3$$

Alpha cluster formation and decay

——*Quartetting wave function approach*



Inside the core the intrinsic W.F. describes 4 independent nucleons in quasiparticle states, whereas it changes character if a bound state is formed on the surface region, and becomes alpha-like

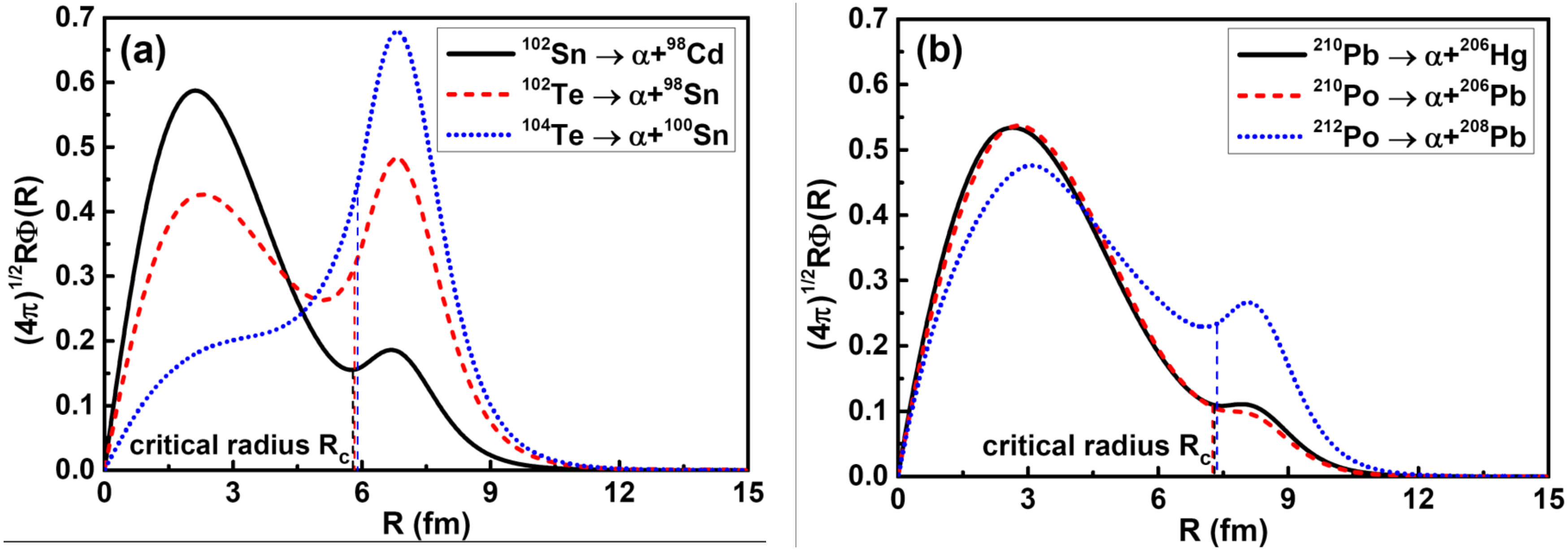
$$P_\alpha = \int_0^\infty d^3r |\Phi(r)|^2 \Theta[n_B^{\text{Mott}} - n_B(r)]$$

Decay width self-consistently obtained by solving both the c.o.m. motion equation of the quartet and the scattering state of the formed alpha-cluster

$$\Gamma = \nu \times T = \frac{4\hbar^2\alpha^2}{\mu k} |\Phi(r_{\text{sep}})\chi_k(r_{\text{sep}})|^2$$

Alpha decay in ^{104}Te, ^{210}Pb, ^{210}Po, and ^{212}Po

Parent	Z	N	Q_α [MeV]	P_α	$T_{1/2}^{\text{calc.}}$ [s]	$T_{1/2}^{\text{expt.}}$ [s]
^{102}Sn	50	52		0.0551		
^{102}Te	52	50		0.3718		
^{104}Te	52	52	4.900	0.7235	1.479×10^{-8}	$<1.8 \times 10^{-8}$
^{210}Pb	82	128	3.792	0.0176	1.777×10^{16}	3.701×10^{16}
^{210}Po	84	126	5.408	0.0137	1.060×10^{7}	1.196×10^{7}
^{212}Po	84	128	8.954	0.1045	3.395×10^{-7}	2.997×10^{-7}



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PHYSICAL REVIEW C

covering nuclear physics

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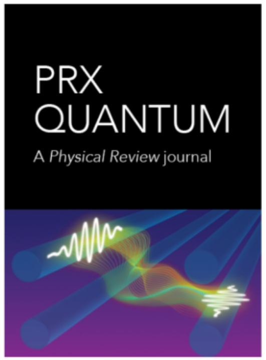


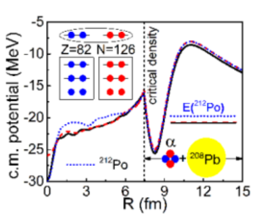
Introducing *PRX Quantum*, a new *Physical Review* journal

Opening for submissions mid-2020, *PRX Quantum* will be a highly selective, open access journal featuring quantum information science and technology research with an emphasis on lasting and profound impact. The journal expands on the excellence and innovation of *Physical Review X* (PRX).

Physical Review C 50th Anniversary Milestones

The year 2020 marks *Physical Review C*'s 50th anniversary. As part of the celebration, the editors are assembling a collection of milestone papers from PRC that remain central to current developments in nuclear physics.

Collection



高被引论文

EDITORS' SUGGESTION

α decay to a doubly magic core in the quartetting wave function approach

This microscopic calculation for the α decay of heavy nuclei provides a solution to what has long been an outstanding problem. In the authors' model, the α particle exists only below about one-fifth of saturation density, corresponding to a large radius, inside of which the α particle transitions into an unbound four-nucleon shell-model state. The model reproduces the half-life of ^{212}Po (a classic test case) as well as some neighboring nuclei, and calculations are also made for ^{104}Te.

Shuo Yang *et al.*

Phys. Rev. C **101**, 024316 (2020)

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Vol. 101, Iss. 2 — February 2020

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Vol. 100, Iss. 5 — November 2019
Vol. 100, Iss. 4 — October 2019

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50 YEARS PHYSICAL REVIEW C

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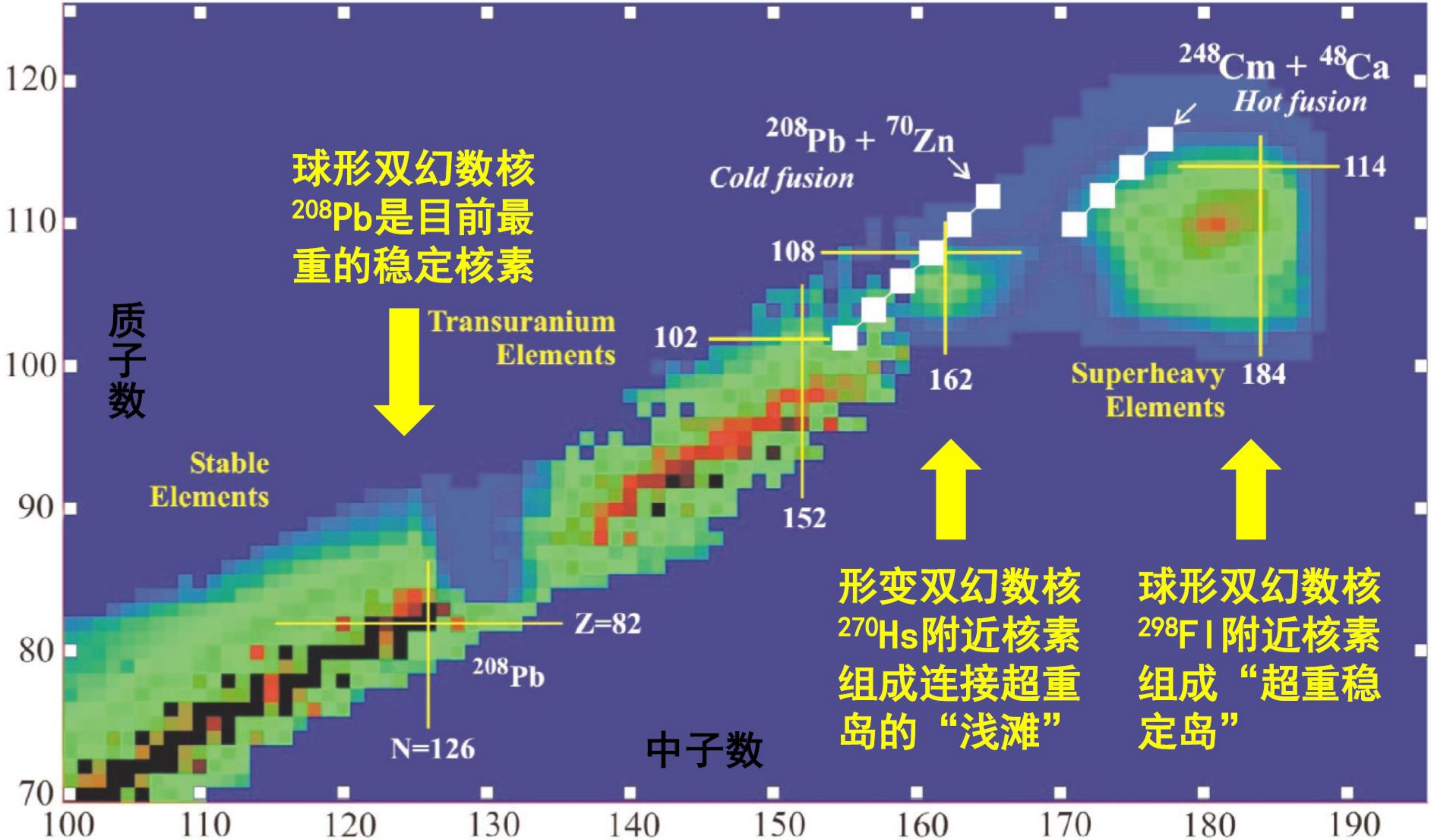
编辑评价：“该微观计算为这一长期难点问题提供了解决办法”。

PHYSICAL REVIEW RESEARCH

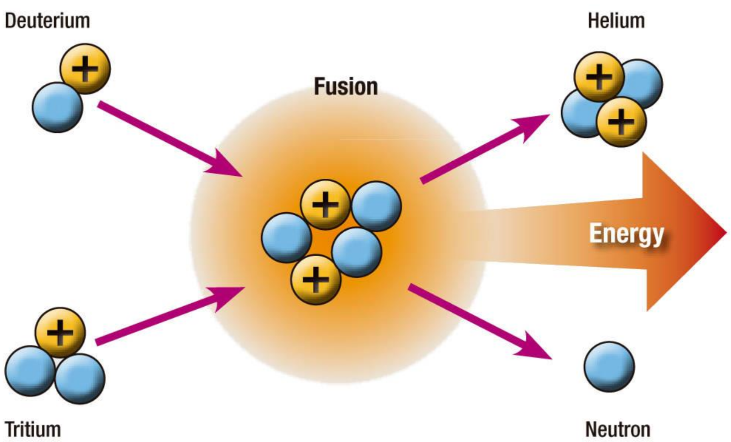
Ideal alpha cluster emitter (doubly magic core + cluster)

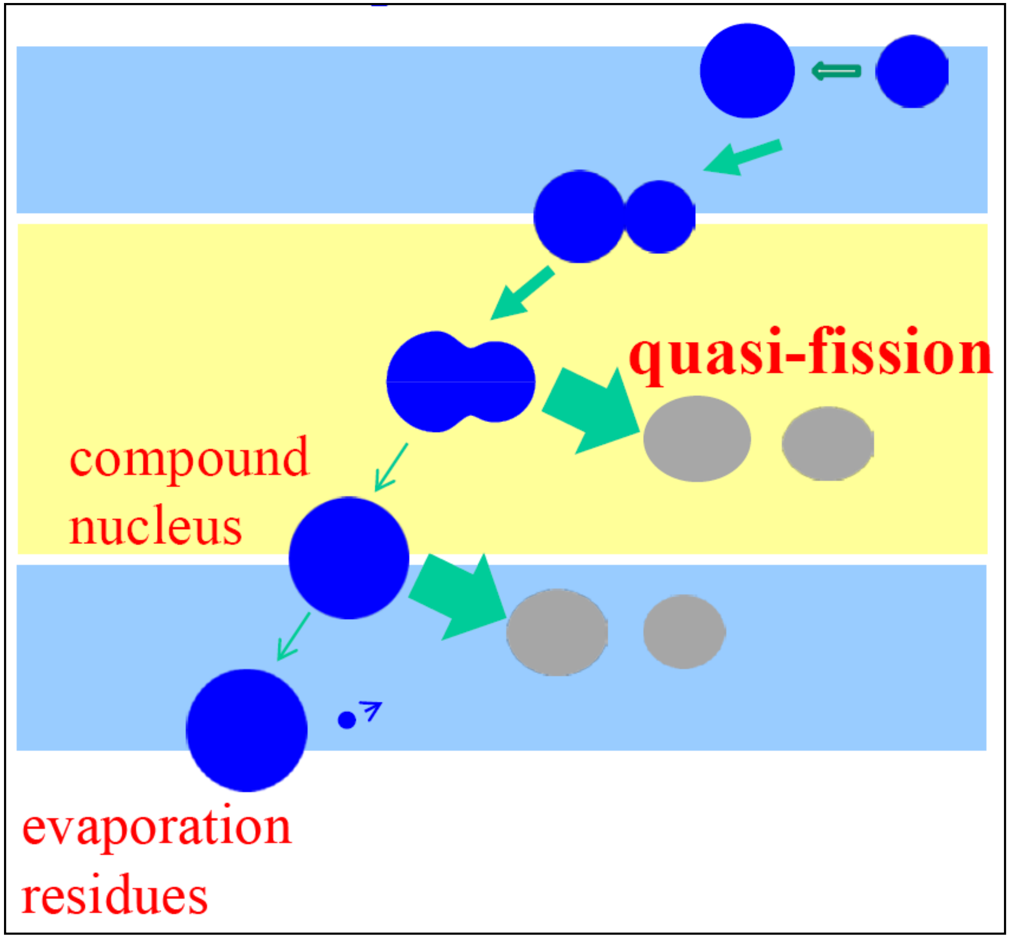
Alpha decay to a deformed magic core?



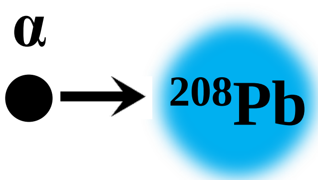
逆过程: α与原子核的熔合反应

Fusion reaction: nucleosynthesis in the early universe , energy production in stars, superheavy elements…





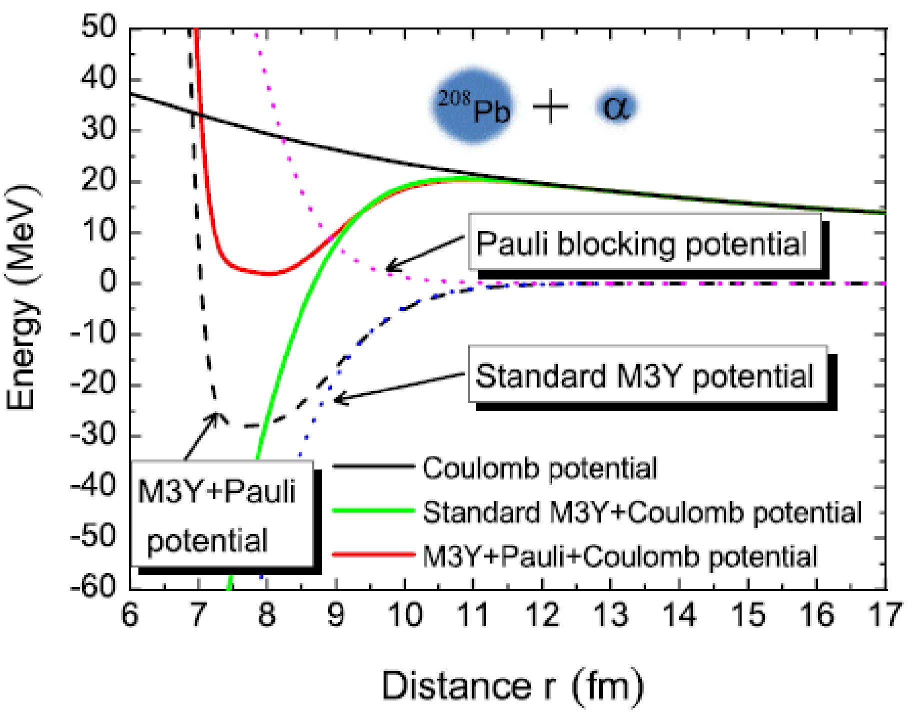
Fusion cross section of alpha-particle-induced reactions

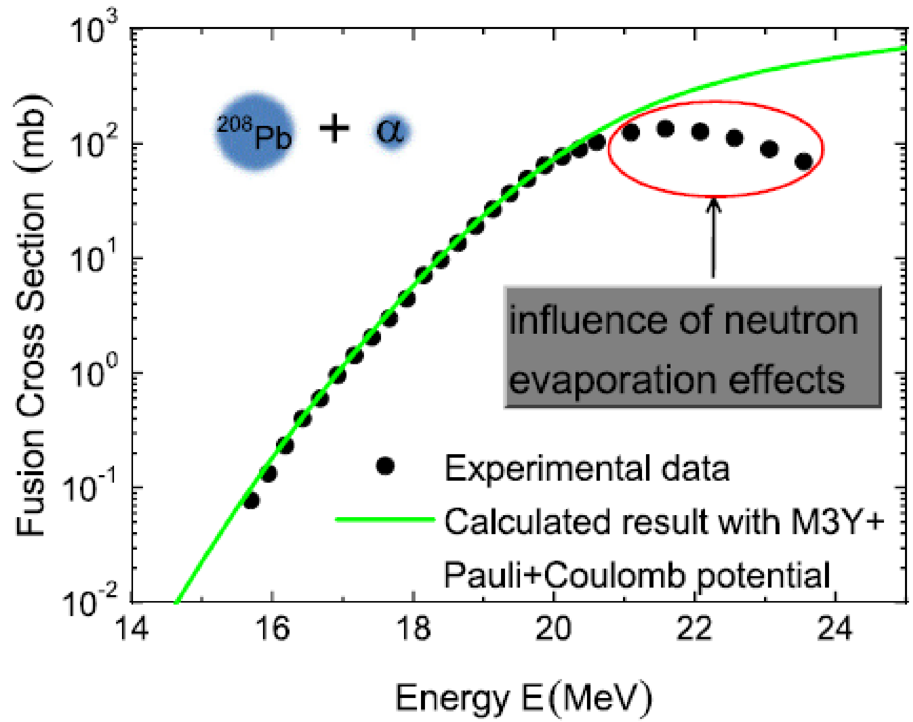


PHYSICAL REVIEW C **99**, 014607 (2019)

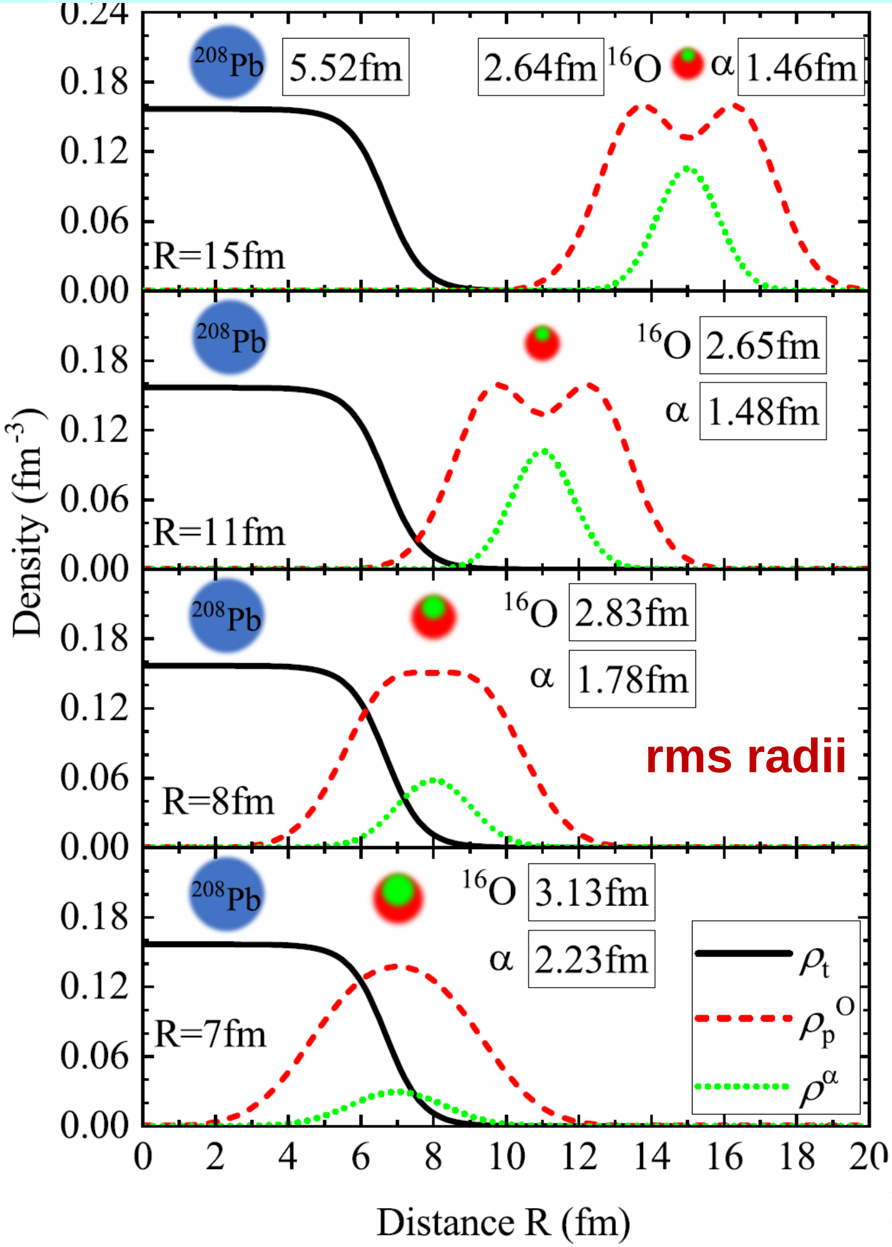
Pauli blocking effects in α-induced fusion reactions

Kaixuan Cheng and Chang Xu*
School of Physics, Nanjing University, Nanjing 210093, China

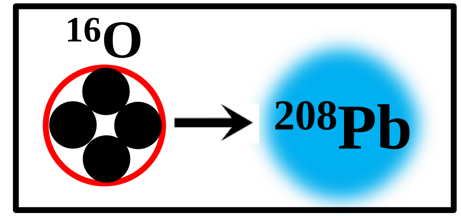


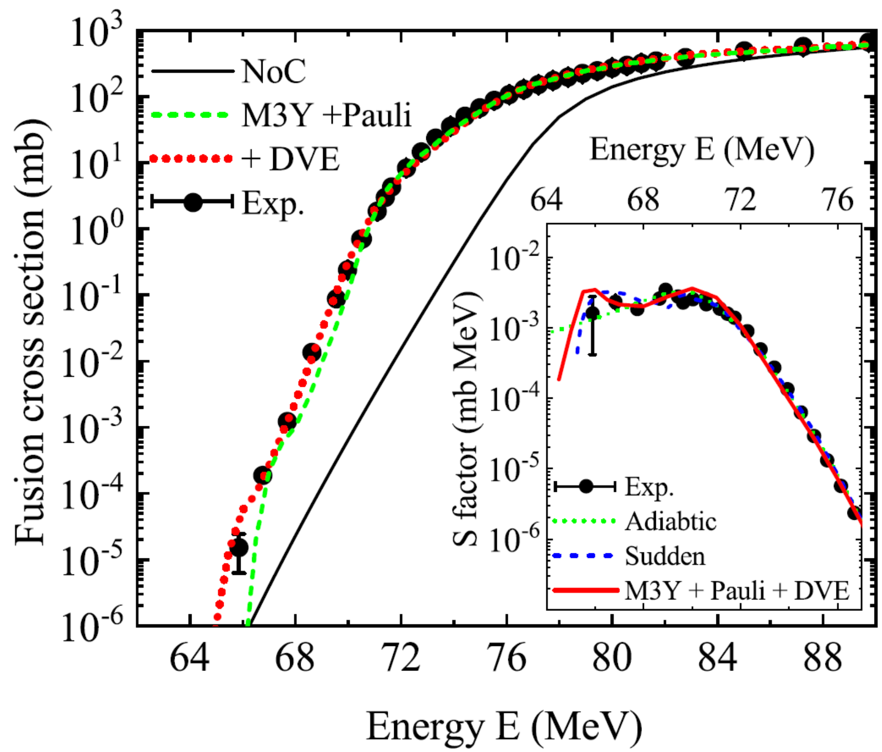


Fusion hindrance (极深垒下熔合阻碍现象)



rms radii





PHYSICAL REVIEW C **103**, 014613 (2021)

极深垒下熔合阻碍现象：2002—2021

实验家第一次发现极深垒下熔合阻碍现象

VOLUME 89, NUMBER 5 PHYSICAL REVIEW LETTERS 29 JULY 2002

Unexpected Behavior of Heavy-Ion Fusion Cross Sections at Extreme Sub-Barrier Energies

C. L. Jiang, H. Esbensen, K. E. Rehm, B. B. Back, R. V. F. Janssens, J. A. Caggiano, P. Collon, J. Greene, A. M. Heinz, D. J. Henderson, I. Nishinaka, T. O. Pennington, and D. Seweryniak
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

实验家进一步发现极深垒下熔合阻碍现象

VOLUME 93, NUMBER 1 PHYSICAL REVIEW LETTERS week ending 2 JULY 2004

Influence of Nuclear Structure on Sub-Barrier Hindrance in Ni + Ni Fusion

C. L. Jiang,[1] K. E. Rehm,[1] R. V. F. Janssens,[1] H. Esbensen,[1] I. Ahmad,[1] B. B. Back,[1] P. Collon,[2] C. N. Davids,[1] J. P. Greene,[1] D. J. Henderson,[1] G. Mukherjee,[1,*] R. C. Pardo,[1] M. Paul,[3] T. O. Pennington,[1,†] D. Seweryniak,[1] S. Sinha,[1] and Z. Zhou[1]
[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

Eur. Phys. J. A (2021) 57:235
https://doi.org/10.1140/epja/s10050-021-00536-2
THE EUROPEAN PHYSICAL JOURNAL A
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Review

2021实验家综述论文

Heavy-ion fusion reactions at extreme sub-barrier energies

C. L. Jiang[1,a], B. B. Back[1], K. E. Rehm[1], K. Hagino[2], G. Montagnoli[3], A. M. Stefanini[4]
[1] Physics Division, Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, IL 60439, USA
[2] Department of Physics, Kyoto University, Kyoto 606-8502, Japan
[3] Dipartimento di Fisica e Astronomia, Università di Padova, and INFN, Sez. di Padova, 35131 Padua, Italy
[4] INFN, Laboratori Nazionali di Legnaro, 35020 Legnaro, Padova, Italy

2021年实验家总结过去20年的相关研究。

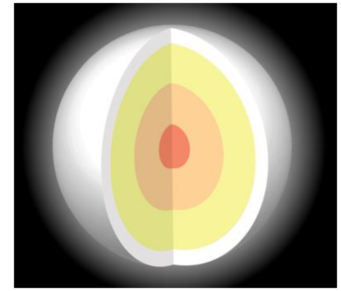
Notice that the microscopic origin of the repulsion in the overlapping region is due to the Pauli principle, as pointed out in Ref. [125] (see also Refs. [131,132]). In this model, the authors introduced a new microscopic approach to heavy-ion

131. K. Cheng, C. Xu, Phys. Rev. C **102**, 014619 (2020)
132. K. Cheng, C. Xu, Phys. Rev. C **99**, 014607 (2019)

This trend at far sub-barrier energies (no hindrance observed for ^{58}Ni $+^{64}$ Ni) suggests that, as was observed for ^{40}Ca $+^{96}$ Zr, the availability of several states following transfer with $Q > 0$ effectively counterbalances the Pauli repulsion that, in general, is predicted to reduce the tunneling probability through the Coulomb barrier [125,153].

153. Kaixuan Cheng, Xu Chang, Phys. Rev. C **102**, 014619 (2020)

热点问题之一: Alpha集团，中子皮和对称能



PHYSICAL REVIEW LETTERS **126,** 172502 (2021)

Editors' Suggestion   Featured in Physics

^{208}Pb

Accurate Determination of the Neutron Skin Thickness of ^{208}Pb through Parity-Violation in Electron Scattering

We report a precision measurement of the parity-violating asymmetry A_{PV} in the elastic scattering of longitudinally polarized electrons from ^{208}Pb. We measure $A_{PV} = 550 \pm 16(\mathrm{stat}) \pm 8(\mathrm{syst})$ parts per billion, leading to an extraction of the neutral weak form factor $F_W(Q^2 = 0.00616\ \mathrm{GeV}^2) = 0.368 \pm 0.013$. Combined with our previous measurement, the extracted neutron skin thickness is $R_n - R_p = 0.283 \pm 0.071$ fm. The result also yields the first significant direct measurement of the interior weak density of ^{208}Pb: $\rho_W^0 = -0.0796 \pm 0.0036(\mathrm{exp}) \pm 0.0013(\mathrm{theo})\ \mathrm{fm}^{-3}$ leading to the interior baryon density $\rho_b^0 = 0.1480 \pm 0.0036(\mathrm{exp}) \pm 0.0013(\mathrm{theo})\ \mathrm{fm}^{-3}$. The measurement accurately constrains the density dependence of the symmetry energy of nuclear matter near saturation density, with implications for the size and composition of neutron stars.

PREX: Thick skin in ^{208}Pb: stiff symmetry energy and large L

PHYSICAL REVIEW LETTERS **129,** 042501 (2022)

Editors' Suggestion

^{48}Ca

Precision Determination of the Neutral Weak Form Factor of ^{48}Ca

We report a precise measurement of the parity-violating (PV) asymmetry A_{PV} in the elastic scattering of longitudinally polarized electrons from ^{48}Ca. We measure $A_{PV} = 2668 \pm 106(\mathrm{stat}) \pm 40(\mathrm{syst})$ parts per billion, leading to an extraction of the neutral weak form factor $F_W(q = 0.8733\ \mathrm{fm}^{-1}) = 0.1304 \pm 0.0052(\mathrm{stat}) \pm 0.0020(\mathrm{syst})$ and the charge minus the weak form factor $F_{ch} - F_W = 0.0277 \pm 0.0055$. The resulting neutron skin thickness $R_n - R_p = 0.121 \pm 0.026(\mathrm{exp}) \pm 0.024(\mathrm{model})$ fm is relatively thin yet consistent with many model calculations. The combined CREX and PREX results will have implications for future energy density functional calculations and on the density dependence of the symmetry energy of nuclear matter.

CREX: Thin skin in ^{48}Ca: soft symmetry energy and small L

NuSym22, X International Symposium on Nuclear Symmetry Energy

2022年9月26日至30日
Catania (Italy)
Europe/Rome 时区

12:00	**New models, clustering and correlations (I)**	*Wolfgang Trautmann*	*Aula Magna, Department of Physics and Astronomy "E. Majorana"*	11:30 - 13:00
13:00	**Lunch break**		*Terrace of DFA*	13:00 - 14:20
14:00 / 15:00	**New models, clustering and correlations (II)**	*Stefan Typel*	*Aula Magna, Department of Physics and Astronomy "E. Majorana"*	14:20 - 15:50

Hugenholtz, N. M.
Van Hove, L.
1958

Physica XXIV
363–376

The HVH theorem

A THEOREM ON THE SINGLE PARTICLE ENERGY IN A FERMI GAS WITH INTERACTION

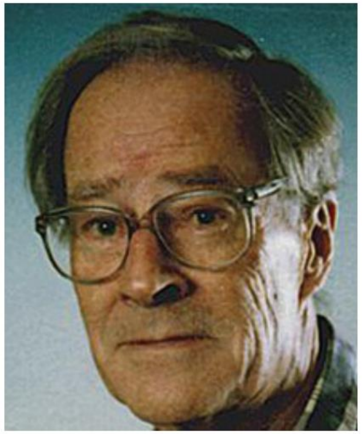
by N. M. HUGENHOLTZ and L. VAN HOVE

Instituut voor theoretische fysica der Rijksuniversiteit, Utrecht, Nederland

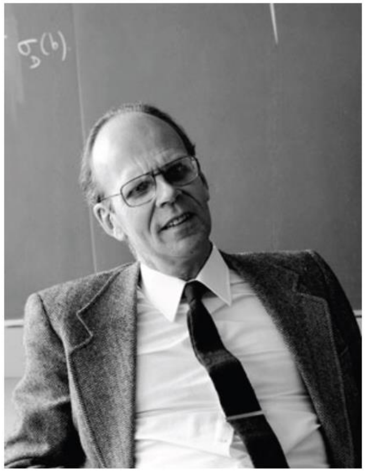
Synopsis

This paper investigates single particle properties in a Fermi gas with interaction at the absolute zero of temperature. In such a system a single particle energy has only a meaning for particles of momentum $|k|$ close to the Fermi momentum k_F. These single particle states are metastable with a life-time approaching infinity in the limit $|k| \to k_F$. The limiting value of the energy is called the Fermi energy E_F. As a special case of a more general theorem, it is shown that for a system with zero pressure (i.e. a Fermi liquid at absolute zero) the Fermi energy E_F is equal to the average energy per particle E_0/N of the system. This result should apply both to liquid He_3 and to nuclear matter.

The theorem is used as a test on the internal consistency of the theory of Brueckner [1]) for the structure of nuclear matter. It is seen that the large discrepancy between the values of E_F and E_0/N, as calculated by Brueckner and Gammel [2]), arises from the fact that Brueckner neglects important cluster terms contributing to the single particle energy. This neglection strongly affects the calculation of the optical potential.



N. M. Hugenholtz



L. Van Hove

Using the Hugenholtz–Van Hove (HVH) theorem

$$t(k_F^n) + U_n(\rho, \delta, k_F^n) = \frac{\partial \xi}{\partial \rho_n},$$
$$t(k_F^p) + U_p(\rho, \delta, k_F^p) = \frac{\partial \xi}{\partial \rho_p},$$

$$U_\tau(\rho, \delta, k) = U_0(\rho, k) + \sum_{i=1,2,3,\ldots} U_{\mathrm{sym},i}(\rho, k)(\tau\delta)^i$$
$$= U_0(\rho, k) + U_{\mathrm{sym},1}(\rho, k)(\tau\delta) + U_{\mathrm{sym},2}(k)(\tau\delta)^2 + \cdots$$

Left side

$$[t(k_F^n) - t(k_F^p)] + [U_n(\rho, \delta, k_F^n) - U_p(\rho, \delta, k_F^p)]$$
$$= \sum_{i=1,2,3,\ldots} \frac{1}{i!} \frac{\partial^i [t(k) + U_0(\rho, k)]}{\partial k^i}\Big|_{k_F} k_F^i$$
$$\times \Big[\Big(\sum_{j=1,2,3,\ldots} F(j)\delta^j\Big)^i - \Big(\sum_{j=1,2,3,\ldots} F(j)(-\delta)^j\Big)^i\Big]$$
$$+ \sum_{l=1,2,3,\ldots} U_{\mathrm{sym},l}(\rho, k_F)[\delta^l - (-\delta)^l] + \sum_{l=1,2,3,\ldots} \sum_{i=1,2,3,\ldots} \frac{1}{i!} \frac{\partial^i U_{\mathrm{sym},l}(\rho, k)}{\partial k^i}\Big|_{k_F} k_F^i$$
$$\times \Big[\Big(\sum_{j=1,2,3,\ldots} F(j)\delta^j\Big)^i \delta^l - \Big(\sum_{j=1,2,3,\ldots} F(j)(-\delta)^j\Big)^i (-\delta)^l\Big]$$
$$= \Big[\frac{2}{3} \frac{\partial [t(k) + U_0(\rho, k)]}{\partial k}\Big|_{k_F} k_F + 2U_{\mathrm{sym},1}(\rho, k_F)\Big]\delta + \cdots,$$

Isoscalar and isovector potentials:
U_0 and U_{sym}

Right side

$$\frac{\partial \xi}{\partial \rho_n} - \frac{\partial \xi}{\partial \rho_p} = \frac{2}{\rho}\frac{\partial \xi}{\partial \delta} = \sum_{i=2,4,6,\ldots} 2i\, E_{\mathrm{sym},i}(\rho)\delta^{i-1}$$
$$= 4E_{\mathrm{sym},2}(\rho)\delta + 8E_{\mathrm{sym},4}(\rho)\delta^3 + 12E_{\mathrm{sym},6}(\rho)\delta^5 + \cdots$$

Symmetry energy of any order:
E_{sym}

Analytical formula: symmetry energy and density slope

$$E_{\rm sym}(\rho_0) = \frac{1}{6}\left.\frac{\partial(t+U_0)}{\partial k}\right|_{k_F} k_F + \frac{1}{2}U_{\rm sym}(\rho_0,k_F)$$

$$= \frac{1}{3}t(k_F) + \frac{1}{6}\left.\frac{\partial U_0}{\partial k}\right|_{k_F} k_F + \frac{1}{2}U_{\rm sym}(\rho_0,k_F)$$

isoscalar

$$L(\rho_0) = \frac{1}{6}\left.\frac{\partial(t+U_0)}{\partial k}\right|_{k_F} k_F + \frac{1}{6}\left.\frac{\partial^2(t+U_0)}{\partial k^2}\right|_{k_F} k_F^2 + \frac{3}{2}U_{\rm sym}(\rho_0,k_F) + \left.\frac{\partial U_{\rm sym}(\rho_0,k)}{\partial k}\right|_{k_F} k_F$$

isovector

Lane potential

Derivative of momentum at normal density

许/李/陈, ***PRC*** **81, 044603 (2010);** ***NPA*** **865, 1 (2011);** ***PRC*** **90, 064310 (2014)**

Without α-clustering, the Rskin and RMS radii of neutrons and protons can be calculated from shell model density distributions

$$R_{\text{skin}} = r_n^{\text{rms}} - r_p^{\text{rms}}$$

$$r_n^{\text{rms}} = \left[\int r^2 \sum_{i=1}^{N} \rho_n^i(r) d^3r\right]^{1/2}, \quad r_p^{\text{rms}} = \left[\int r^2 \sum_{i=1}^{Z} \rho_p^i(r) d^3r\right]^{1/2}$$

Considering α-clustering at nuclear surface, the RMS radii become

$$r_n^{\text{rms}} = \left[\int r^2 \left(\rho_n^{\text{cluster}}(r) + \rho_n^{\text{core}}(r)\right) d^3r\right]^{1/2} = \left[\int r^2 \left(\rho_n^{\text{cluster}}(r) + \sum_{i=1}^{N-2} \rho_n^i(r)\right) d^3r\right]^{1/2}$$

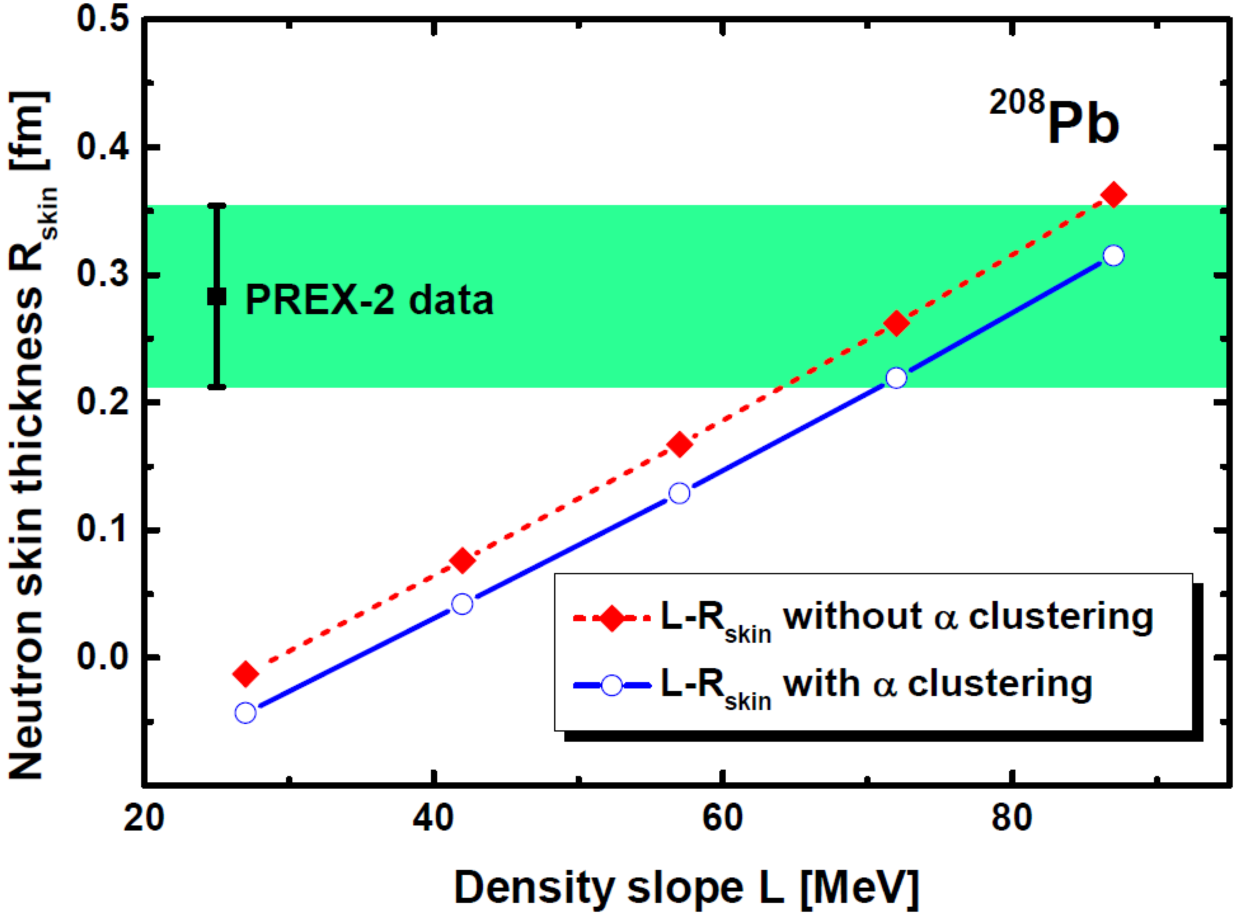
$$r_p^{\text{rms}} = \left[\int r^2 \left(\rho_p^{\text{cluster}}(r) + \rho_p^{\text{core}}(r)\right) d^3r\right]^{1/2} = \left[\int r^2 \left(\rho_p^{\text{cluster}}(r) + \sum_{i=1}^{Z-2} \rho_p^i(r)\right) d^3r\right]^{1/2}$$

Density distribution of two nucleons forming the α-cluster

$$\rho_n^{\text{cluster}}(r) = 2 \int_{R<R_c} d^3R |\Psi^{\text{com}}(\mathbf{R})|^2 \rho_n^{i=N}(\mathbf{r}) + \frac{1}{2} \int_{R>R_c} d^3R \left[|\Psi^{\text{com}}(\mathbf{R})|^2 \rho_\alpha(\mathbf{r}-\mathbf{R};\mathbf{R})\right]$$

Cluster formation probability & its spatial extension

Impact of alpha clustering on nuclear symmetry energy



Linear correlation between L and Rskin in present analysis

Impact of alpha clustering on nuclear symmetry energy

Nuclei	$R_{\rm skin}$ [fm]	L [MeV] no α-cluster	P_α	L [MeV] with α-cluster
^{208}Pb	0.283±0.071	$75.2^{+24.3}_{-24.5}$	9.3×10^{-3}	$75.3^{+24.3}_{-24.6}$
^{48}Ca	0.121±0.050	$13.2^{+25.4}_{-24.9}$	7.3×10^{-2}	$15.0^{+25.6}_{-25.0}$
*	0.071 (lower)	1.7		3.4
*	0.171 (upper)	24.8		26.8

*The correction of L due to α-clustering for the lower and upper limits of $R_{\rm skin}$ is 100% and 8%, respectively.

PREX-2: unaffected **CREX: significant**

L values deduced from PREX-2 and CREX experiments are NOT consistent with each other, even with alpha clustering

75 MeV vs 16.5 MeV

Possible strategy for future studies:

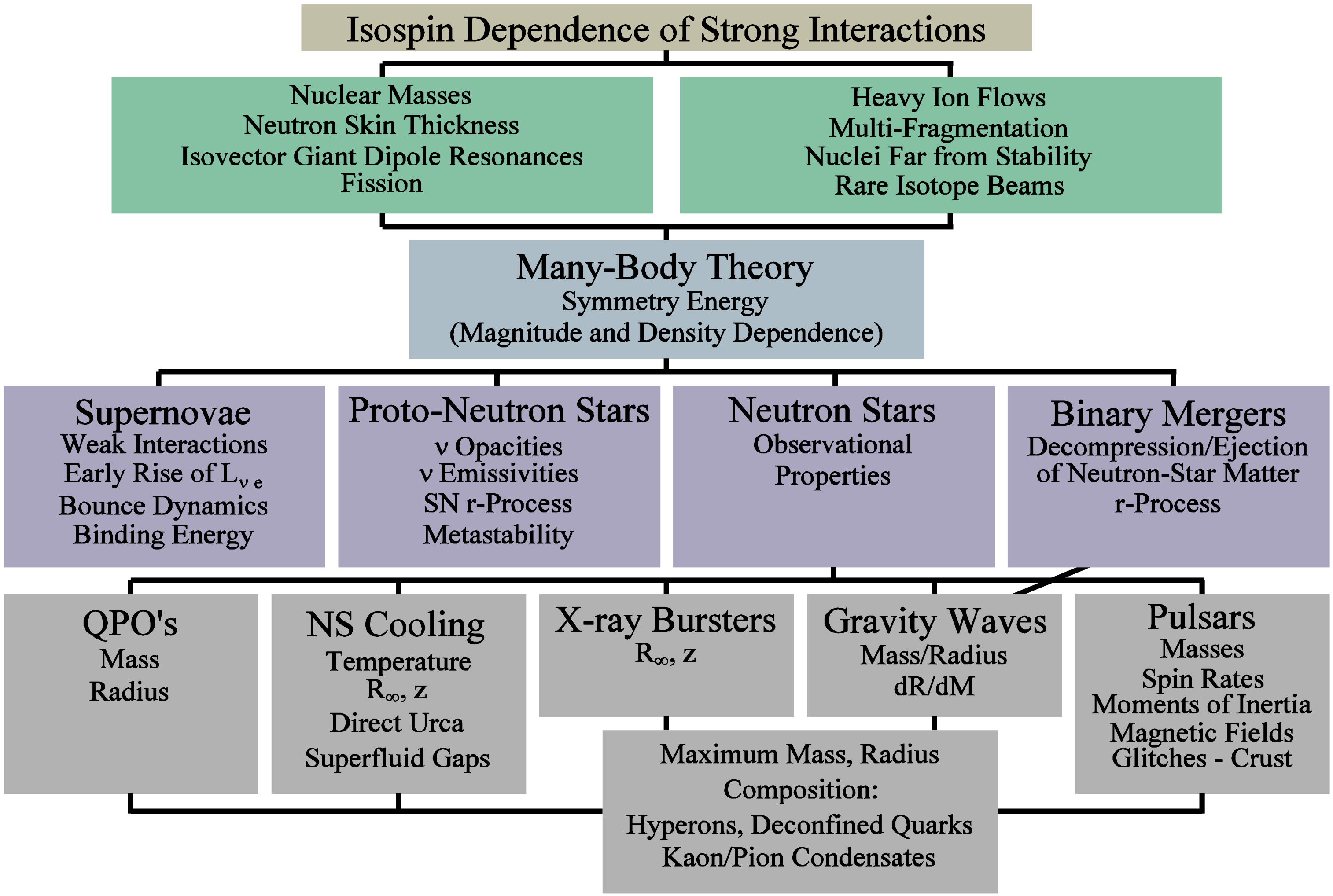
1. A better account of model-dependence in extracting R_{skin} from parity-violating asymmetry

2. State-of-art approaches for densities and improve the Gaussian ansatz

3. Exact solution of the four nucleon correlation near the critical density

Symmetry energy and its density slope

The density dependence of nuclear symmetry energy —an important issue in both nuclear physics and astrophysics



W. Steiner et.al *Phys. Rep. 411, 325 (2005)*

Symmetry energy and its density slope

symmetry energy　Isospin asymmetry

$$E(\rho_n,\rho_p) = E_0(\rho_n=\rho_p) + E_{sym}(\rho)\left(\frac{\rho_n-\rho_p}{\rho}\right)^2 + o(\delta^4)$$

Energy per nucleon in symmetric nuclear matter

Energy per nucleon in asymmetric nuclear matter

⇩

The symmetry energy can be characterized by the $E_{\mathrm{sym}}(\rho_0)$ and its slope parameter $L(\rho_0)$

$$E_{\mathrm{sym}}(\rho) = E_{\mathrm{sym}}(\rho_0) + \frac{L(\rho_0)}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + O\left[\left(\frac{\rho-\rho_0}{\rho_0}\right)^2\right]$$

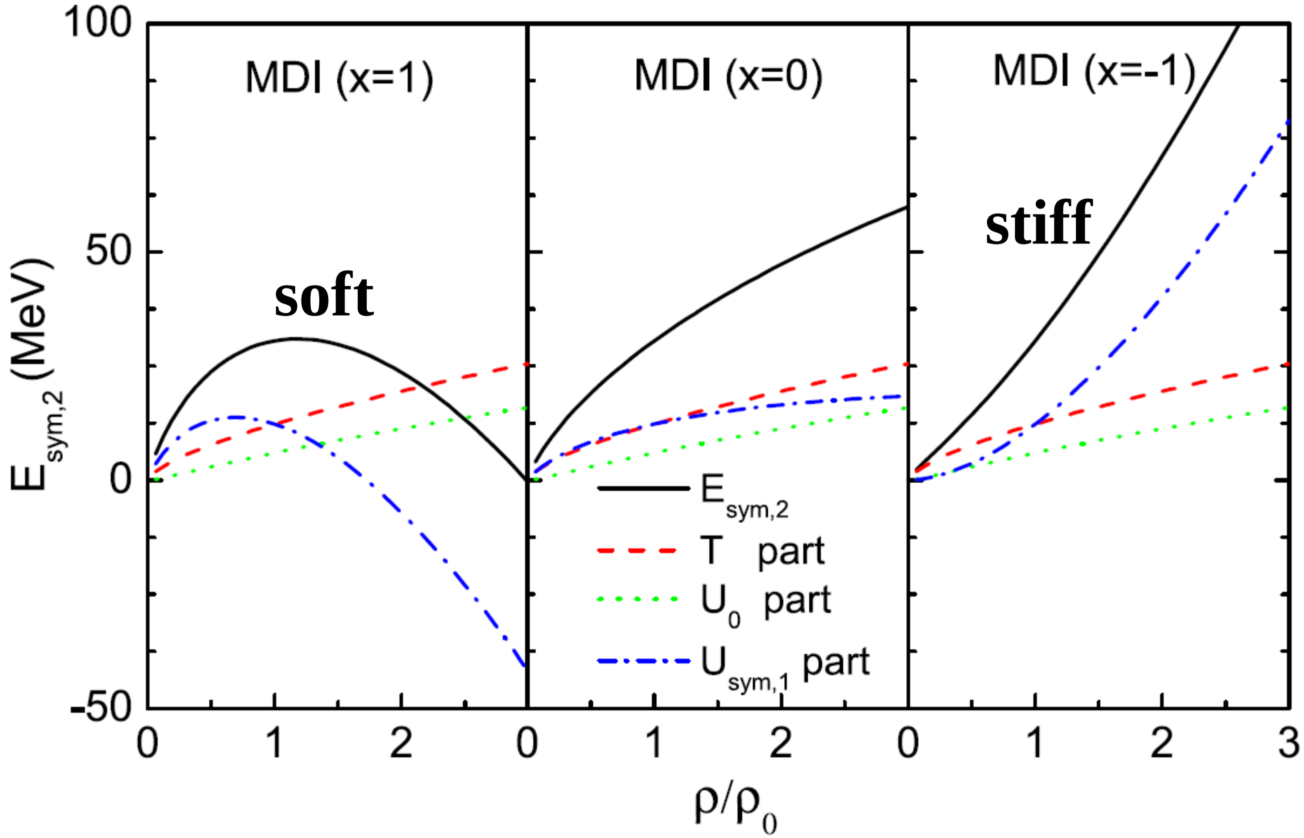
$$L(\rho_0) = 3\rho\frac{\partial E_{\mathrm{sym}}(\rho)}{\partial\rho}\bigg|_{\rho=\rho_0}$$

Derivative of density

⇩

Derivative of momentum at normal density

Decomposition of symmetry energy into kinetic, isoscalar and isovector terms



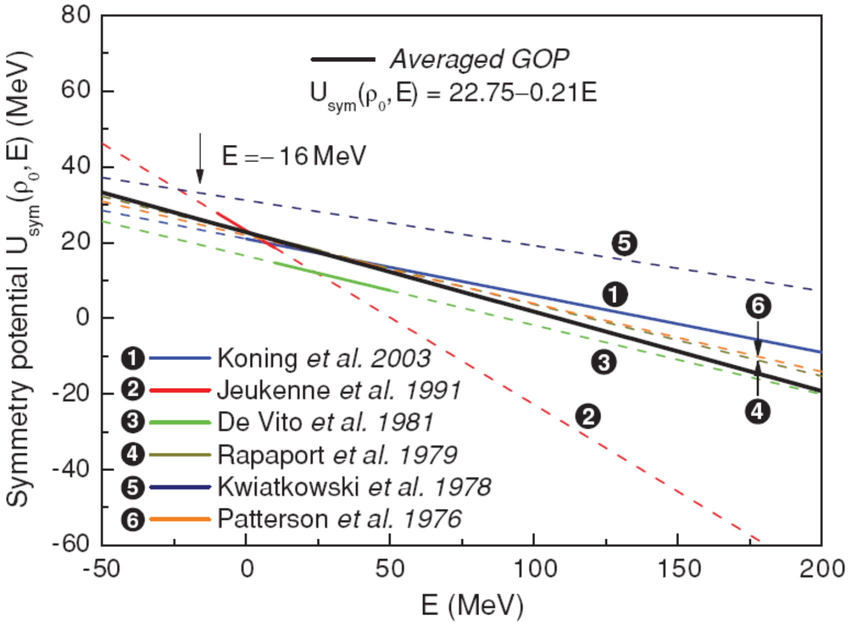
Kinetic energy and isoscalar potential contributions (relatively well constrained)

Isovector potential (uncertain)

BUU: The Momentum dependent Interaction (MDI)

Isovector potential from global optical model analysis of nuclear reactions

(1) Single particle energy levels from pick-up and stripping reaction
(2) Neutron and proton scattering on the same target at about the same energy
(3) Proton scattering on isotopes of the same element
(4) (p,n) charge exchange reactions

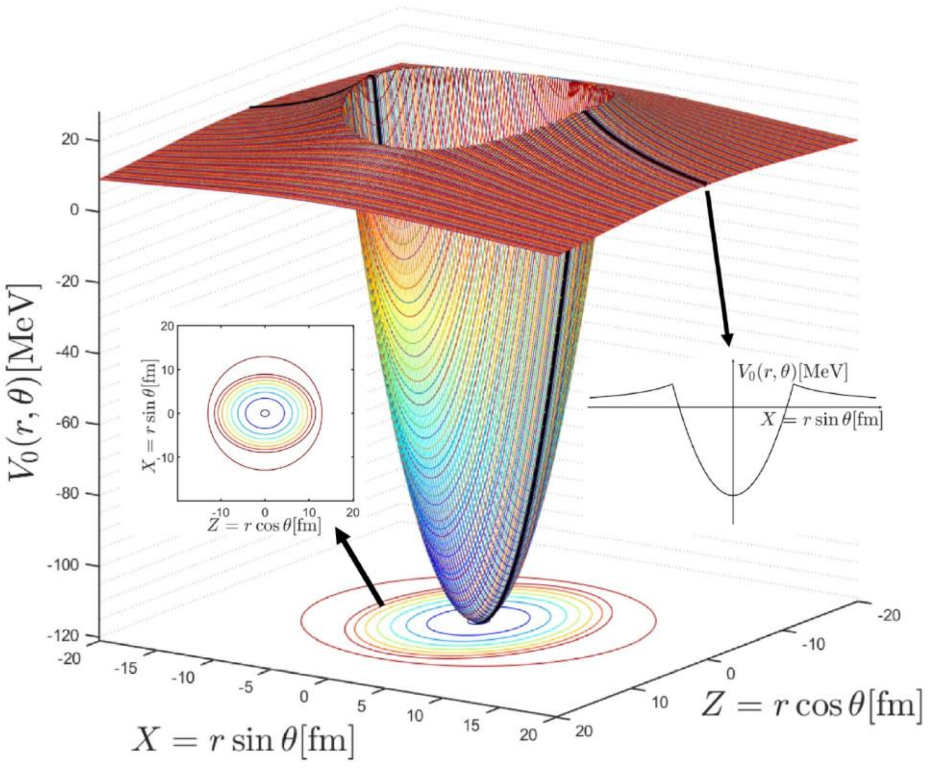


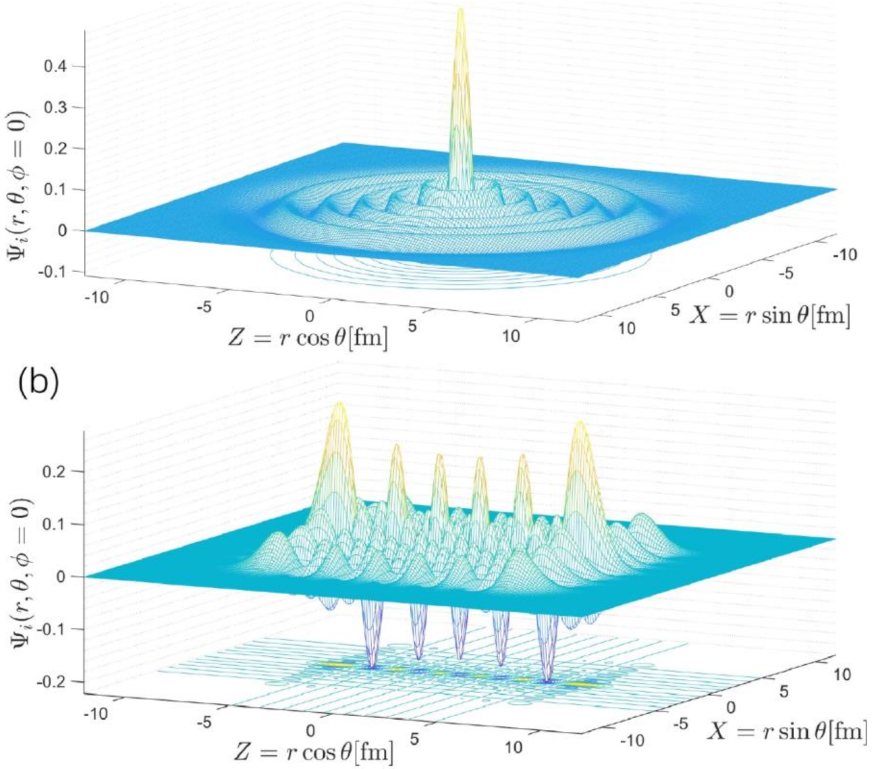
This isovector potential pushes more neutrons in finite nucleus from the inner region outwards to the surface region, and contribute to neutron skin thickness

In this sense, Usym, L, Rskin are related intrinsically

New experiment results: ^{208}Pb (PREX-2) and ^{48}Ca(CREX)

3D Approach: Quasi-stationary States of Deformed α-Emitters (PRC 107 064301 (2023))

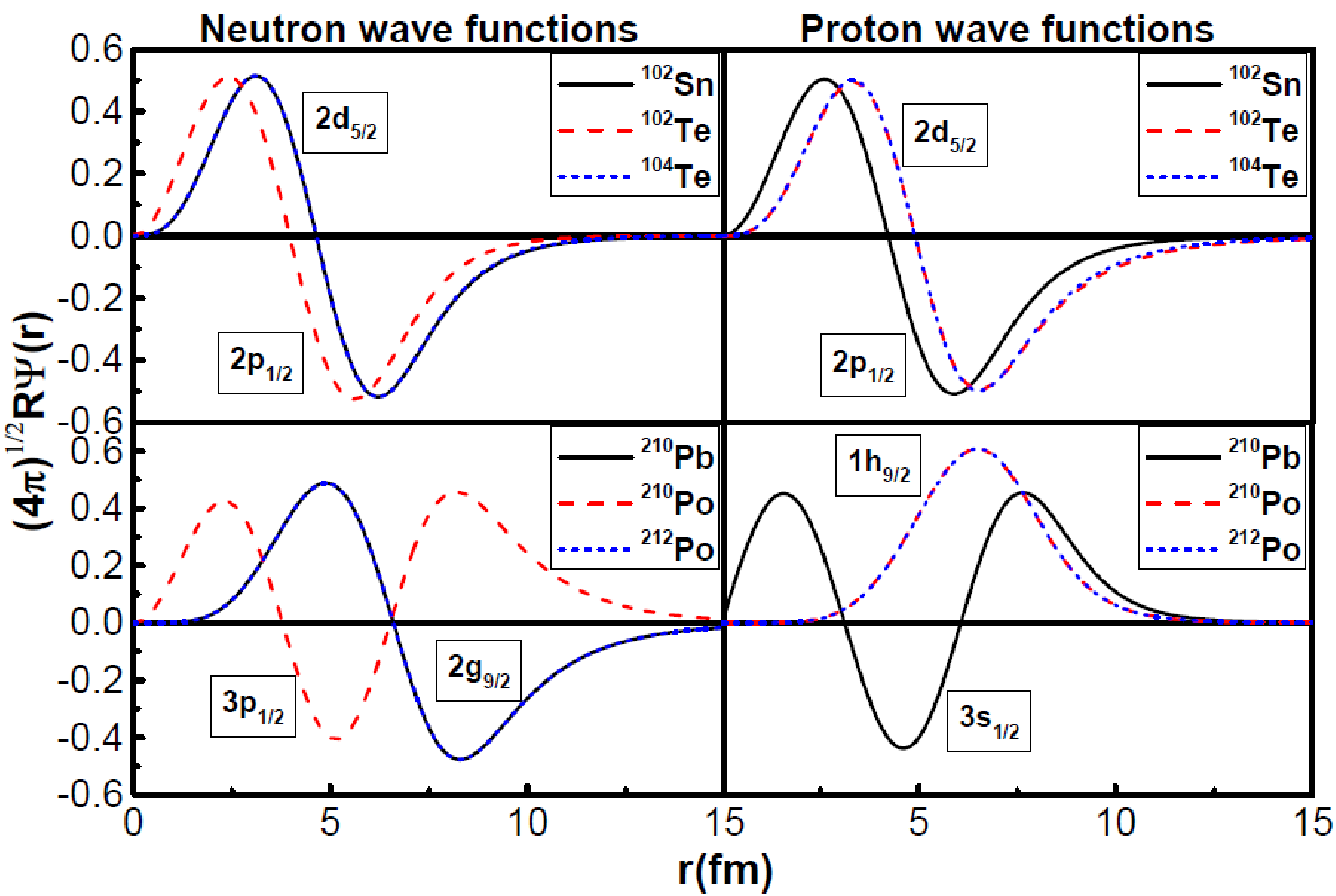




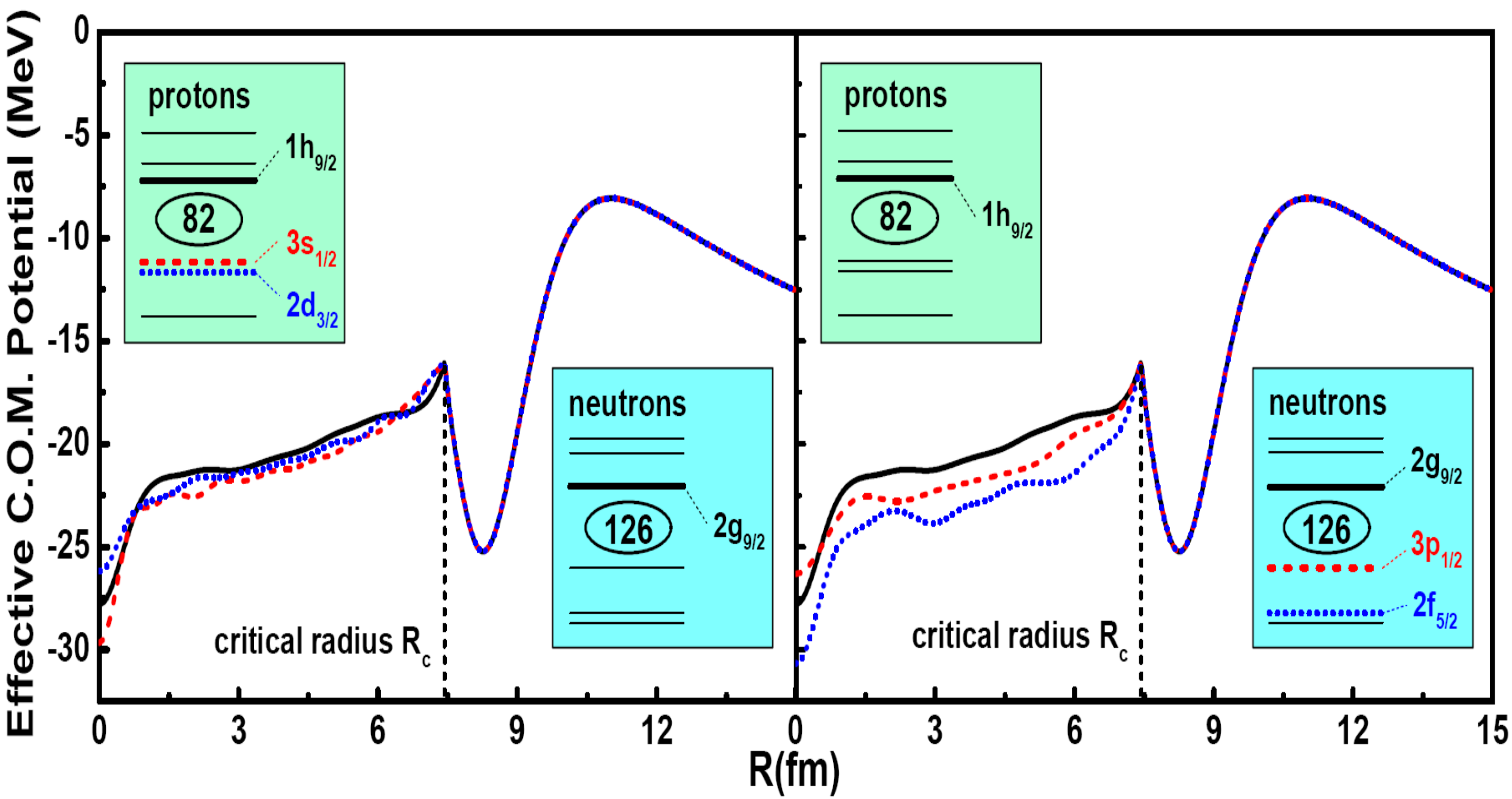
Numerical evaluation: extremely difficult

Alpha cluster formation and decay

——*Quartetting wave function approach*



Effective c.o.m potential and wave functions for ideal heavy alpha emitter ^{102}Te, ^{212}Po and their neighbors



任中洲，徐躬耦，PRC 36 (1987) 456

PHYSICAL REVIEW C VOLUME 36, NUMBER 1 JULY 1987

Reduced alpha transfer rates in a schematic model

Ren Zhong-zhou and Xu Gong-ou
Department of Physics, Nanjing University, Nanjing, China
(Received 27 January 1987)

The reduced alpha transfer rates are studied microscopically with a schematic model. Results for ground state to ground state alpha transfer reactions are given.

The model Hamiltonian is as follows:

$$H = H_0(+) + H_0(-) + H_1(+,-) , \tag{1}$$

where

$$H_0(\pm) = \pm\epsilon A(\pm) - 2\lambda_0 \left[\sum_\alpha B_\alpha^\dagger(\sigma,\pm) B_\alpha(\sigma,\pm) + \sum_\mu B_\mu^\dagger(\tau,\pm) B_\mu(\tau,\pm) \right] , \tag{2a}$$

任中洲，徐躬耦， PRC 38 (1988) 1078

PHYSICAL REVIEW C VOLUME 38, NUMBER 2 AUGUST 1988

Evidence of α correlation from binding energies in medium and heavy nuclei

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(Received 23 March 1988)

If the effect of α clustering due to the interaction of the excited correlated proton pair with correlated neutron pairs in medium and heavy nuclei were taken into consideration, quasiparticle energies would not be simply additive. The empirical values of the extra term $\delta(\alpha)$ indicate that α correlations exist to a certain extent in these nuclei.

$$\delta B = \begin{cases} \Delta & \text{even-even nuclei} \\ 0 & \text{even-odd or odd-even nuclei} \\ -\Delta & \text{odd-odd nuclei} \end{cases} \qquad (3)$$

$$\delta B = \begin{cases} \Delta + \delta(\alpha) & \text{even-even nuclei} \\ 0 & \text{even-odd or odd-even nuclei} \\ -\Delta & \text{odd-odd nuclei} \end{cases} \qquad (4)$$