YTR, X.Y. Wu, B.N. Lu, J.M. Yao, PLB 840 (2023) 137896.

利用投影的多维形状约束协变密度泛函理论研究^{96}Zr的八极关联

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"原子核结构与相对论重离子碰撞"前沿交叉研讨会 大连

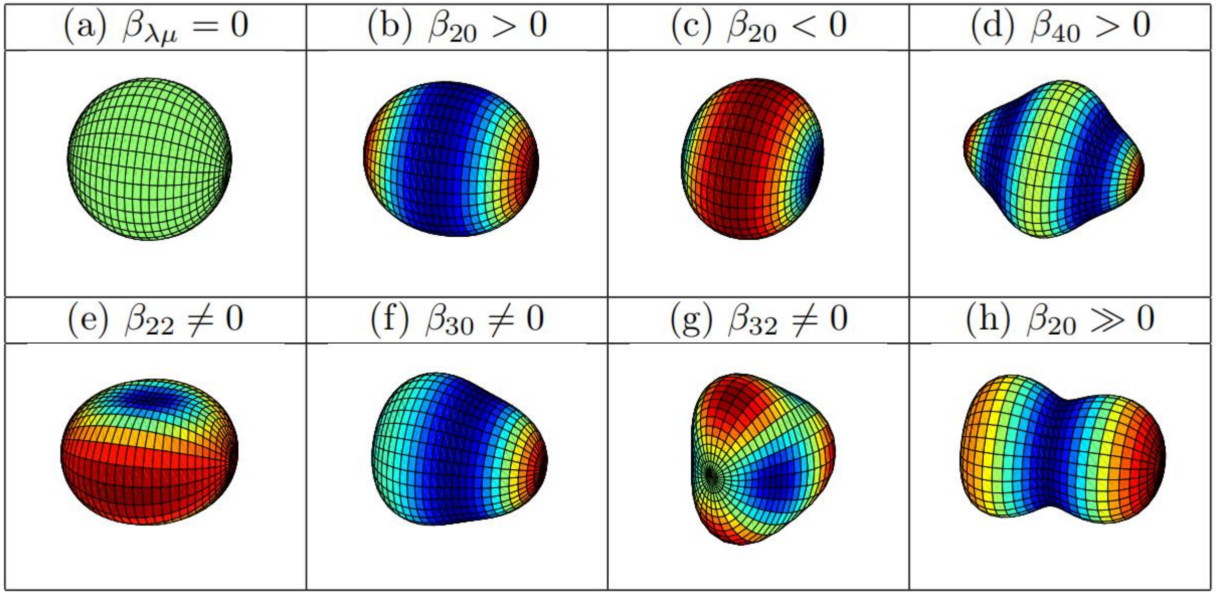
目　录

- 研究背景和意义
- p-MDCRHB模型
- 结果和讨论
- 总结和展望

原子核的形状

原子核形变参数可通过表面多极展开定义：

$$R(\theta,\varphi)=R_0\left[1+\beta_{00}+\sum_{\lambda=1}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\beta^*_{\lambda\mu}Y_{\lambda\mu}(\theta,\varphi)\right].$$



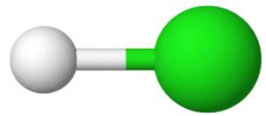
Bing-Nan Lu, 2012

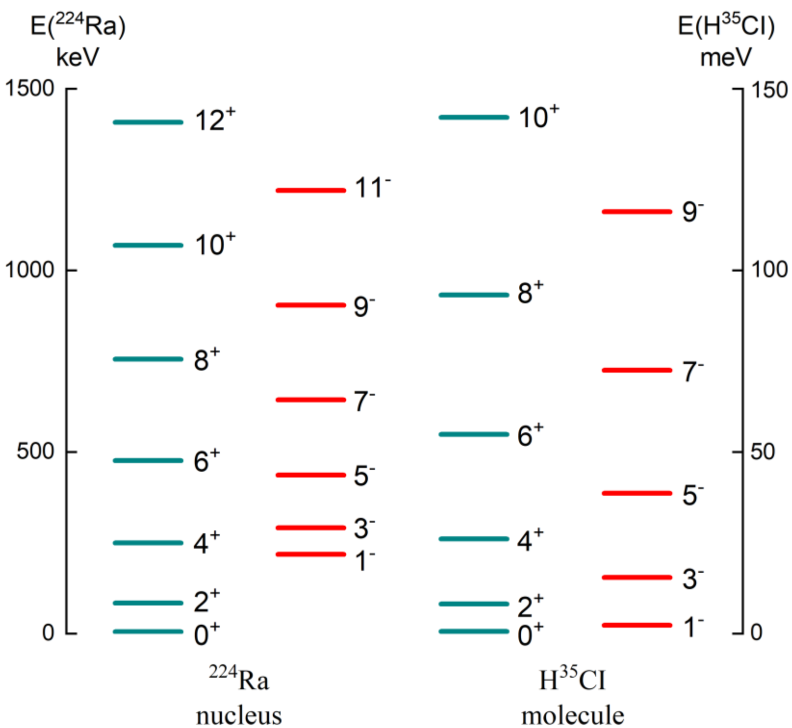
大多数原子核是轴对称形变

一些原子核（^{224}Ra, ^{144}Ba, ^{146}Ba, ^{228}Th）表现出八极形变

原子核的八极关联/形变

From Z.P. Li



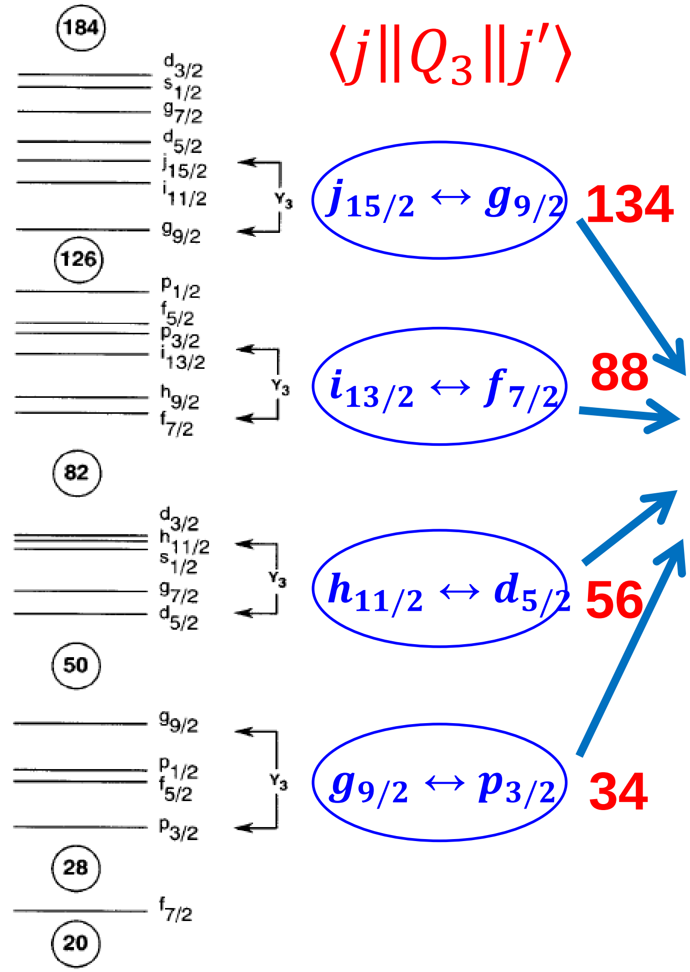


Asaro et. al., PR 92, 1495 (1953).

Stephens et. al., PR100, 1543 (1955).

Lee & Inglis, PR 108, 774 (1957).

$\langle j \| Q_3 \| j' \rangle$



费米面附近

($\Delta l = \Delta j = 3$)

能级强八极耦合

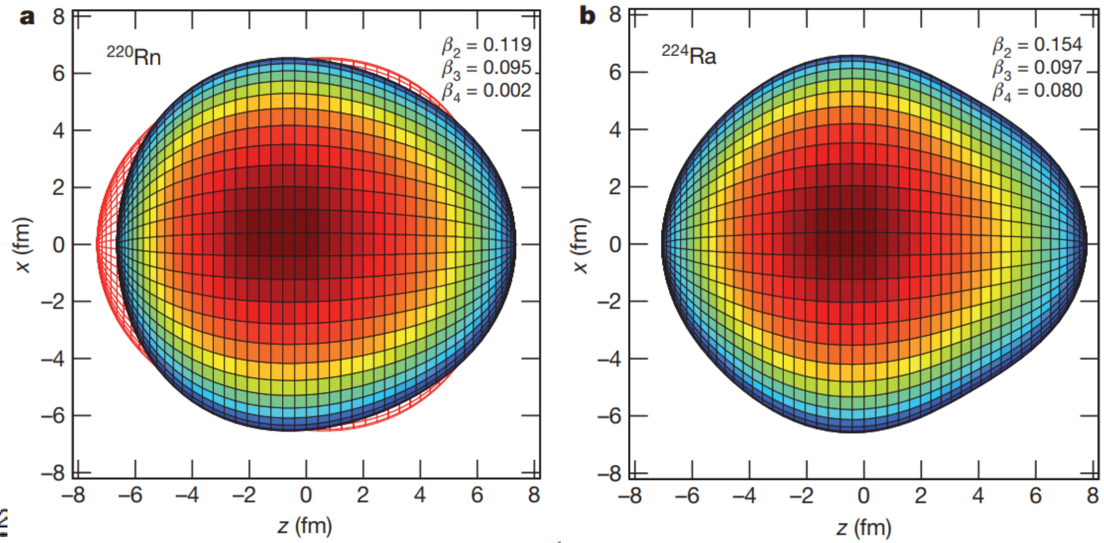
Butler & Nazarewicz, RMP68, 349 (1996).

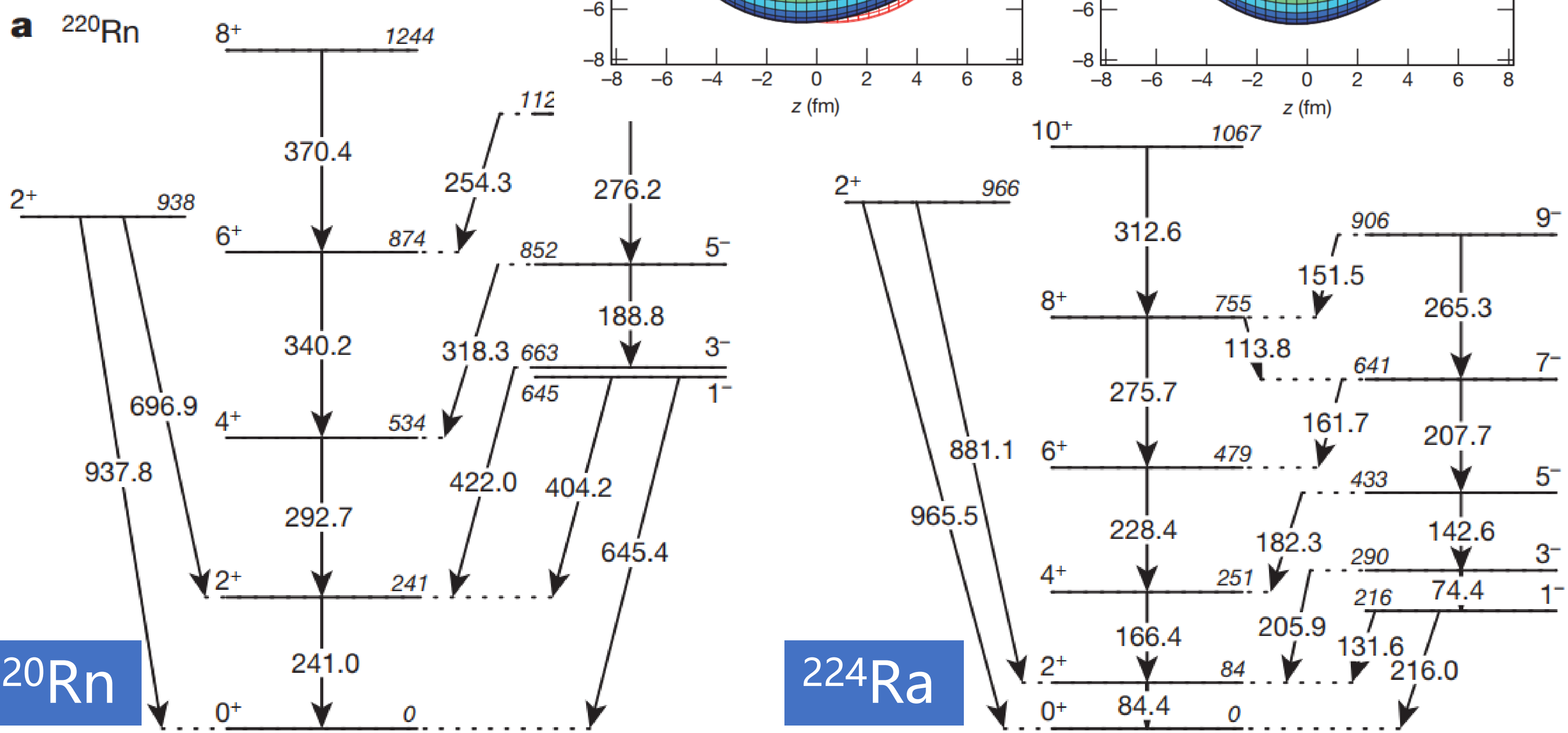
原子核的八极关联/形变

转动谱，B(E3)

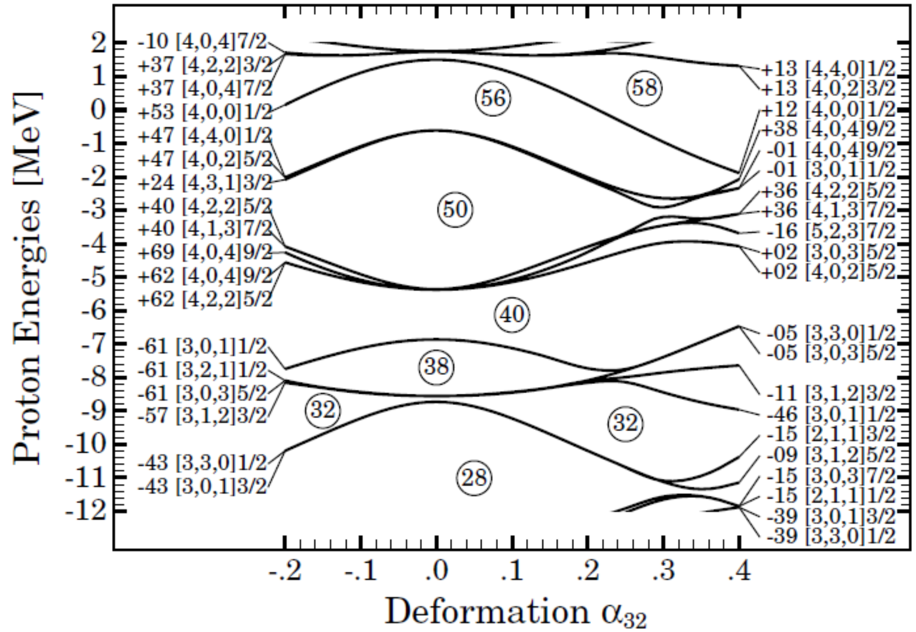
八极振动 & 稳定八极形变

L. P. Gaffney, P. A. Butler, M. Scheck et al., Nature 497 (2013) 199.





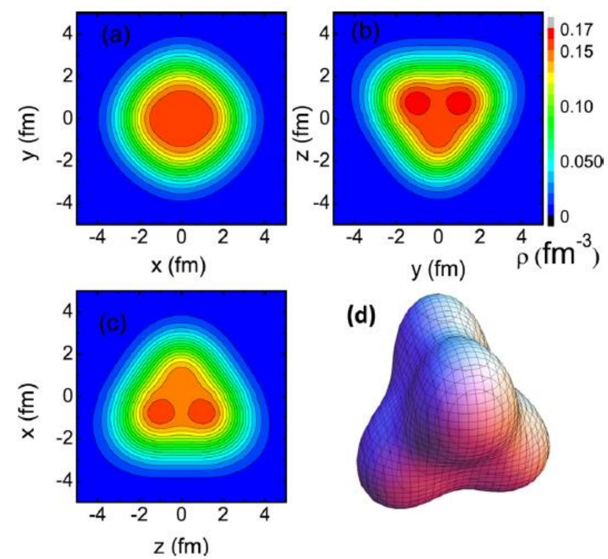
原子核的八极关联/形变



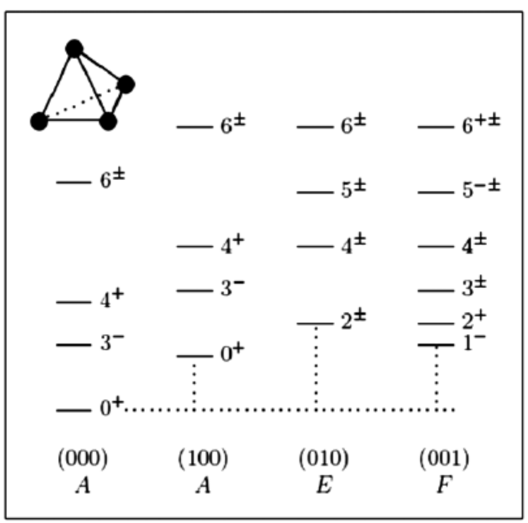
T_D^d symmetry

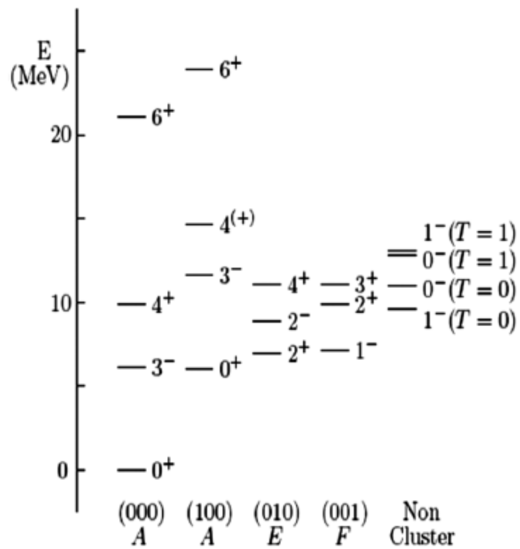
Tetrahedral magic numbers：
16, 20, 32, 40, 56, 64, 70, 90,
112 and 136

Dudek et al., PRL 88 (2002) 252502.
Dudek et al., PS 89 (2014) 054007.



Wang et al., PLB 790 (2019) 498.

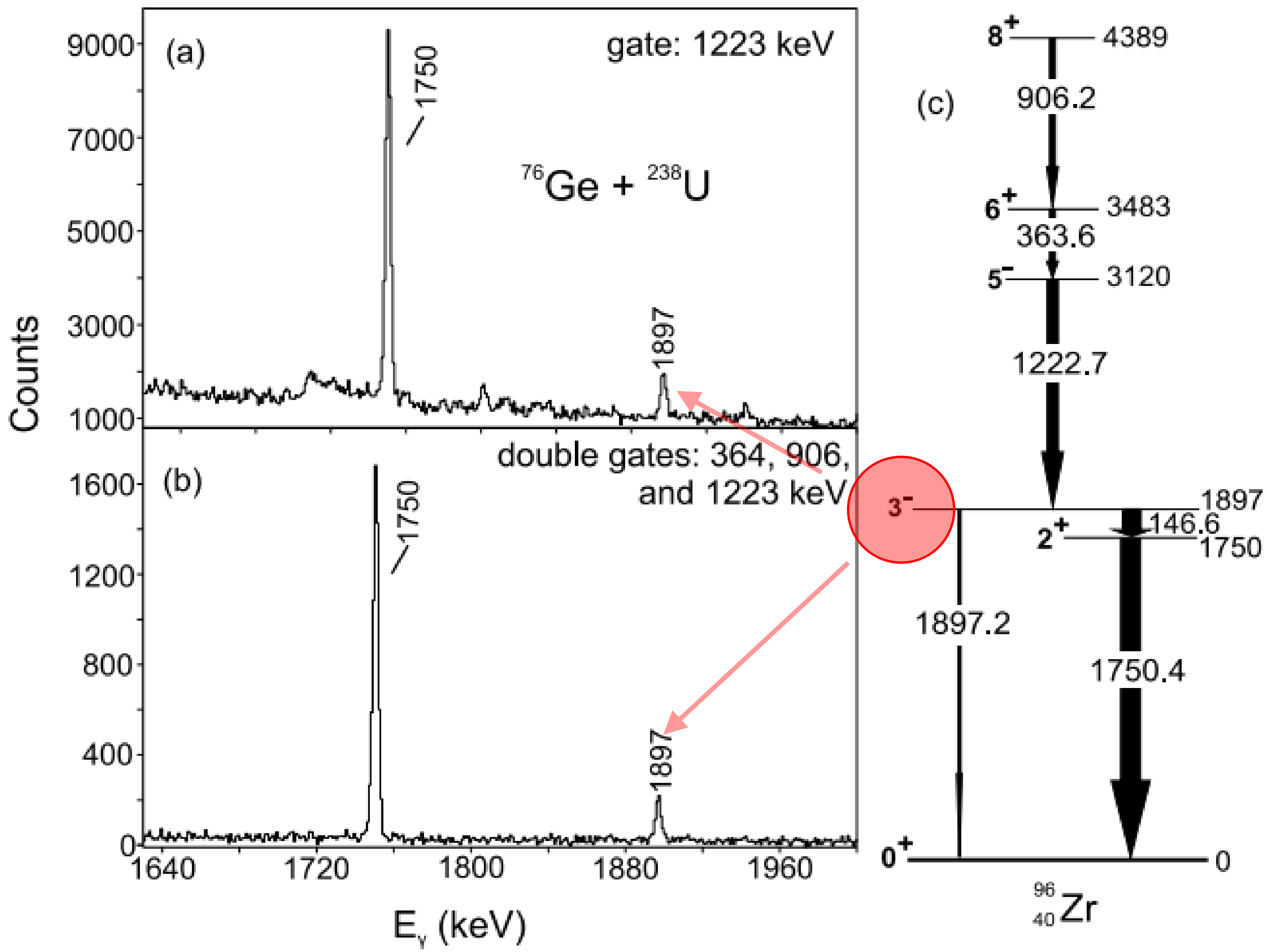




low-lying spectrum of ^{16}O

Bijker & Iachello, PRL 112 (2014) 152501.

^{96}Zr 形状的实验研究



L.W. Iskra, R. Broda, R.V.F. Janssens et al., PLB 788 (2019) 396.

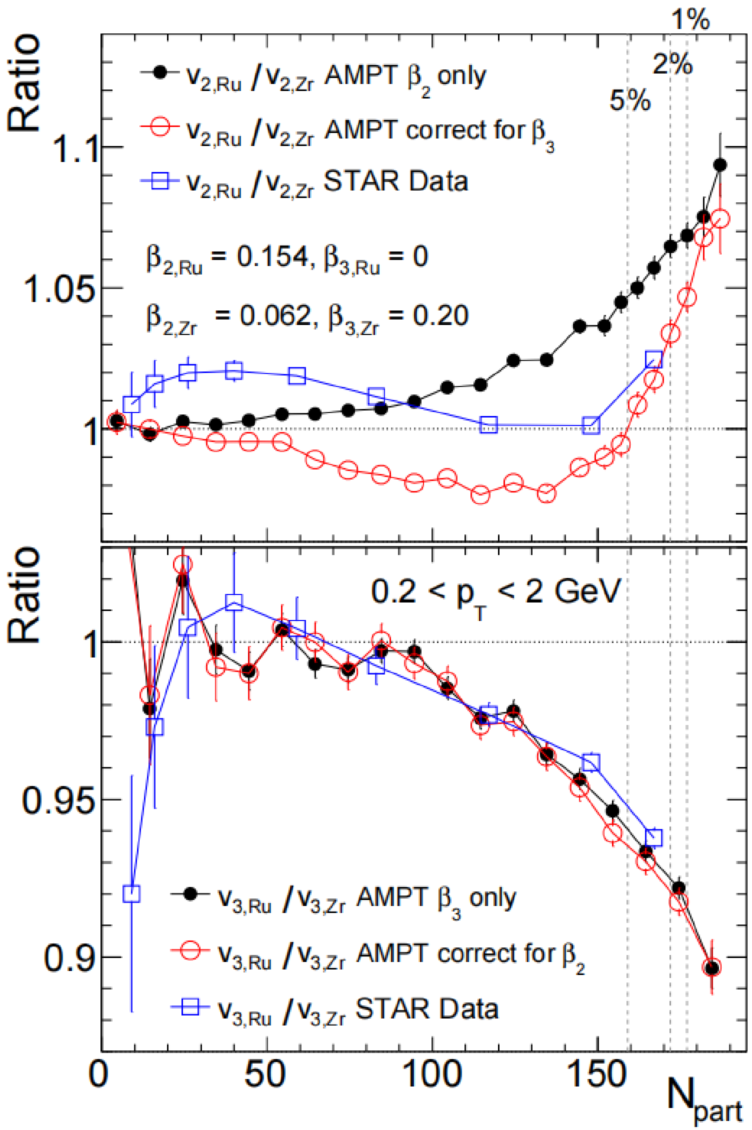
^{96}Zr 形状的实验研究

重离子碰撞实验提取原子核形变信息：

$$\varepsilon_n^2 = a'_n + \sum_{m,k=2}^{4} b'_{n;mk}\beta_m\beta_k, \quad v_n^2 = a_n + \sum_{m,k=2}^{4} b_{n;mk}\beta_m\beta_k,$$

$$\frac{\varepsilon^2_{n,Y}}{\varepsilon^2_{n,X}} = 1 + \frac{b'_n}{a'_n}(\beta^2_{n,Y} - \beta^2_{n,X})/(1 + \frac{b'_n}{a'_n}\beta^2_{n,X}) \approx 1 + \frac{b'_n}{a'_n}(\beta^2_{n,Y} - \beta^2_{n,X}),$$

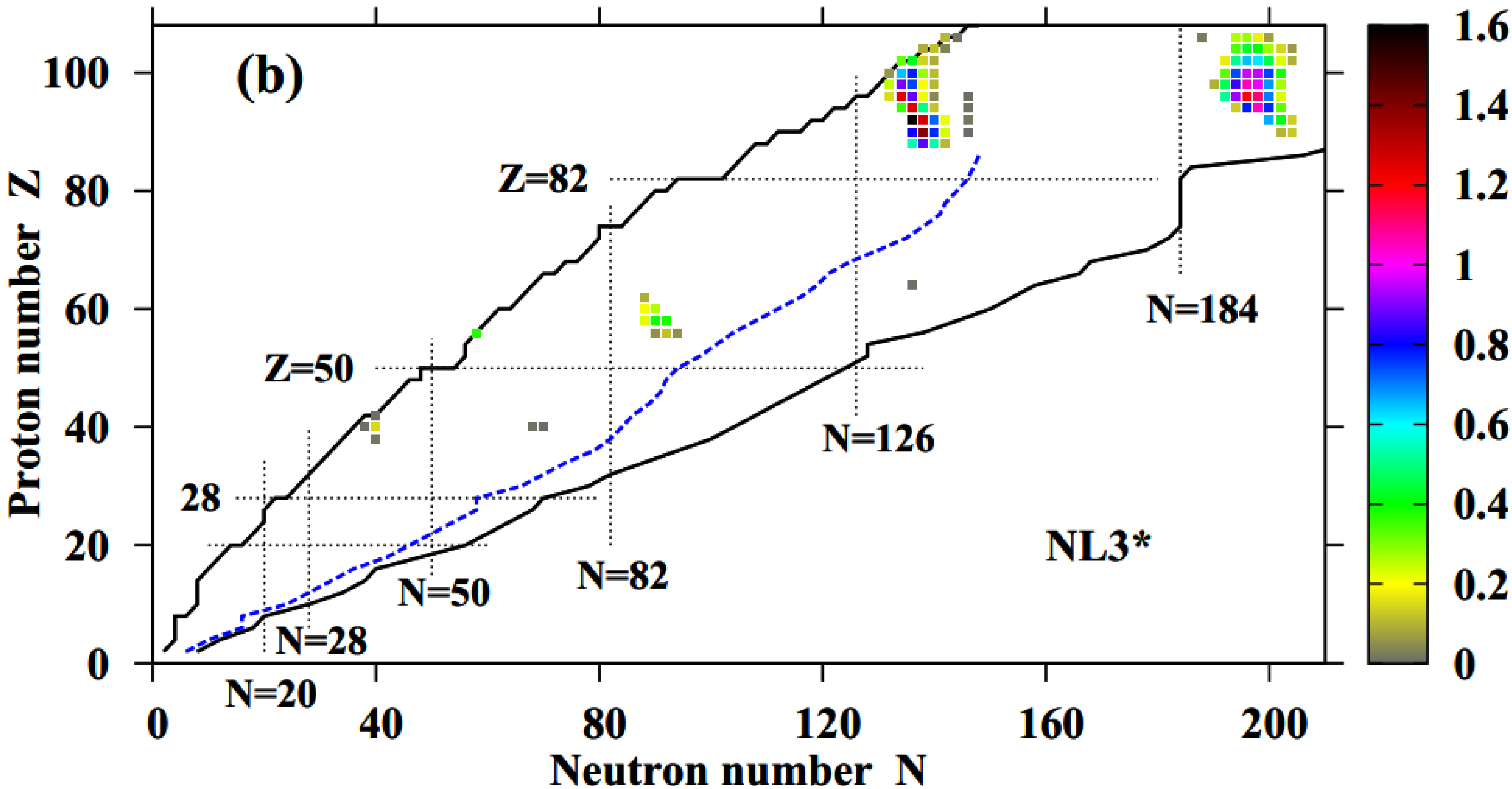
$$\frac{v^2_{n,Y}}{v^2_{n,X}} = 1 + \frac{b_n}{a_n}(\beta^2_{n,Y} - \beta^2_{n,X})/(1 + \frac{b_n}{a_n}\beta^2_{n,X}) \approx 1 + \frac{b_n}{a_n}(\beta^2_{n,Y} - \beta^2_{n,X}).$$



C. Zhang & J. Jia, PRL128（2022）022301.

^{96}Zr 形状的理论研究

平均场模型计算得到的基态没有八极形变.



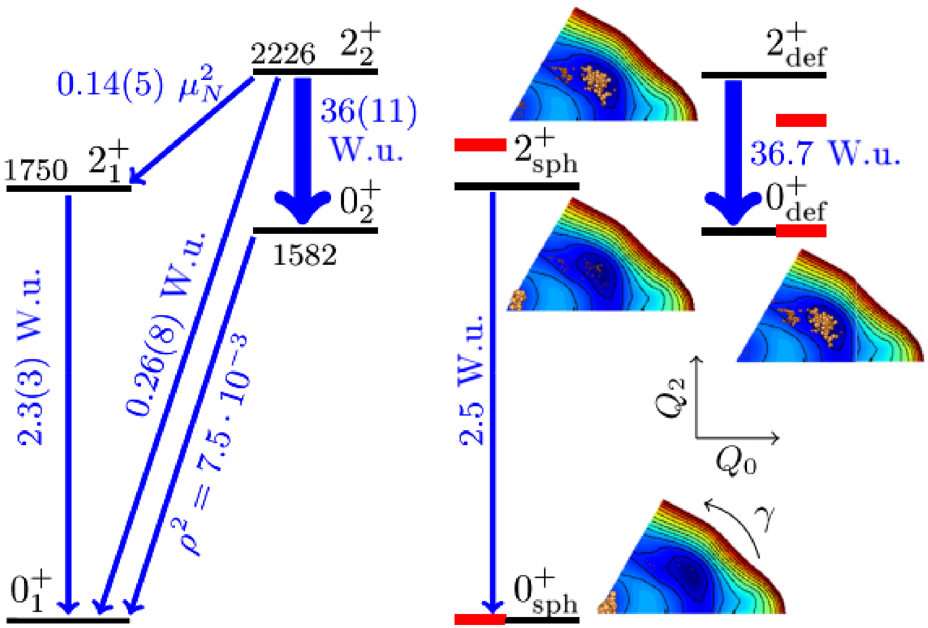
$$\Delta E_{\mathrm{oct}} = E^{\mathrm{oct}}(\beta_2, \beta_3) - E^{\mathrm{quad}}(\beta_2', \beta_3' = 0)$$

S. E. Agbemava, A. V. Afanasjev, P. Ring, PRC 93 （2016）044304.

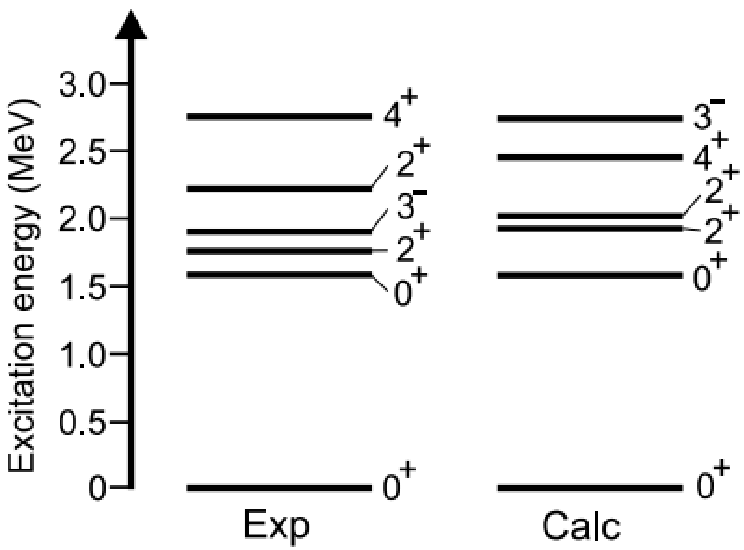
^{96}Zr 形状的理论研究

三轴形变的壳模型计算表明^{96}Zr的低激发态存在球形和三轴形变的形状共存, 基态带为球形振动带

八极形变的 MCSM 给出的 B(E3:3⁻→0+) = 46.6 W. u. 和实验值 42(3) W.u. 很接近，但 3⁻ 激发能偏高



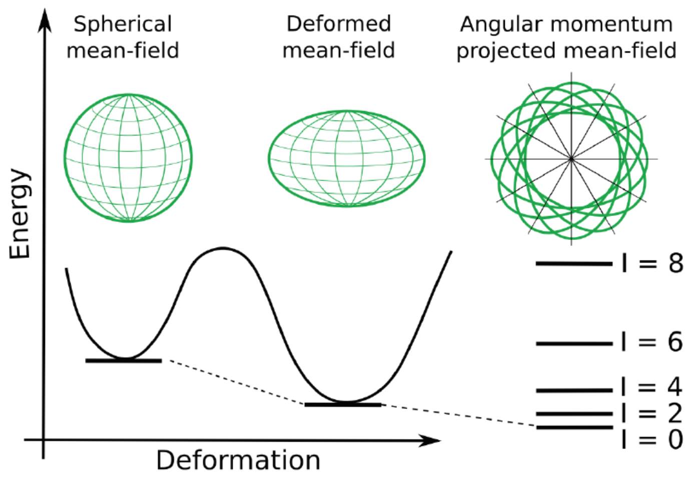
C. Kremer, S. Aslanidou, S. Bassauer et al., PRL 117 (2016) 172503.



L.W. Iskra, R. Broda and R.V.F. Janssens et al., PLB 788 (2019) 396.

基于密度泛函方法研究原子核低激发谱

- 静态平均场模型不能用于描述原子核激发态
- 投影+GCM是原子核谱学研究的常用方法 (PAV)
- 基于相对论平均场的投影+GCM方法：



Basis	(β_{20})	(β_{20}, β_{22})	(β_{20}, β_{30})	$(\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32})$
HO基展开	Niksic_Vretenar(2006) RMF+BCS+AMP+GCM	Yao_Meng_Ring_Arteaga(2009) (RMF+BCS+AMP+GCM)	Yao_Zhou_Li(2015) RMF+BCS+AMP+PP+PNP+GCM	Wang_Lv(2022) RHB+AMP+PP
WS基展开	Sun_Zhou(2021) RHB+AMP	×	×	×

目　录

p-MDCRHB模型

构造点耦合的拉氏量:

$$\mathcal{L} = \mathcal{L}^{\text{free}} + \mathcal{L}^{\text{4f}} + \mathcal{L}^{\text{hot}} + \mathcal{L}^{\text{der}} + \mathcal{L}^{\text{em}}$$

$$\mathcal{L}^{\text{free}} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi$$

$$\begin{aligned}\mathcal{L}^{\text{4f}} = & -\tfrac{1}{2}\alpha_S(\bar{\psi}\psi)(\bar{\psi}\psi) - \tfrac{1}{2}\alpha_V(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi) \\ & - \tfrac{1}{2}\alpha_{TS}(\bar{\psi}\vec{\tau}\psi)(\bar{\psi}\vec{\tau}\psi) - \tfrac{1}{2}\alpha_{TV}(\bar{\psi}\vec{\tau}\gamma_\mu\psi)(\bar{\psi}\vec{\tau}\gamma^\mu\psi)\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{\text{der}} = & -\tfrac{1}{2}\delta_S\partial_\nu(\bar{\psi}\psi)\partial^\nu(\bar{\psi}\psi) - \tfrac{1}{2}\delta_V\partial_\nu(\bar{\psi}\gamma_\mu\psi)\partial^\nu(\bar{\psi}\gamma^\mu\psi) \\ & - \tfrac{1}{2}\delta_{TS}\partial_\nu(\bar{\psi}\vec{\tau}\psi)\partial^\nu(\bar{\psi}\vec{\tau}\psi) \\ & - \tfrac{1}{2}\delta_{TV}\partial_\nu(\bar{\psi}\vec{\tau}\gamma_\mu\psi)\partial^\nu(\bar{\psi}\vec{\tau}\gamma_\mu\psi),\end{aligned}$$

$$\mathcal{L}^{\text{hot}} = -\tfrac{1}{3}\beta_S(\bar{\psi}\psi)^3 - \tfrac{1}{4}\gamma_S(\bar{\psi}\psi)^4 - \tfrac{1}{4}\gamma_V[(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)]^2$$

$$\mathcal{L}^{\text{em}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\frac{1-\tau_3}{2}\bar{\psi}\gamma^\mu\psi A_\mu$$

P. W. Zhao, Z. P. Li, J. M. Yao et al., PRC 82 (2010) 054319.

p-MDCRHB模型

Relativistic Hartree Bogoliubov 方程:

$$\int d^3 \boldsymbol{r}' \begin{pmatrix} \hat{h} - \lambda & \Delta \\ -\Delta^* & -\hat{h} + \lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

$$\hat{h}(\boldsymbol{r}) = \boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta (M + S(\boldsymbol{r})) + V(\boldsymbol{r}),$$

$$\Delta(\boldsymbol{r}_1\sigma_1, \boldsymbol{r}_2\sigma_2) = \int d^3\boldsymbol{r}_1' d^3\boldsymbol{r}_2' \sum_{\sigma_1'\sigma_2'} V(\boldsymbol{r}_1\sigma_1, \boldsymbol{r}_2\sigma_2, \boldsymbol{r}_1'\sigma_1', \boldsymbol{r}_2'\sigma_2')\kappa(\boldsymbol{r}_1'\sigma_1', \boldsymbol{r}_2'\sigma_2'),$$

点耦合的矢量势和标量势:

$$S = \alpha_S \rho_S + \alpha_{TS} \vec{\rho}_{TS} + \beta_S \rho_S^2 + \gamma_S \rho_S^3 + \delta_S \Delta \rho_S + \delta_{TS} \vec{\rho}_{TS} \cdot \vec{\tau},$$

$$V = \alpha_V \rho_V + \alpha_{TV} \vec{\rho}_{TV} \cdot \vec{\tau} + \gamma_V \rho_V^3 + \delta_V \Delta \rho_V + \delta_{TV} \vec{\rho}_{TV} \cdot \vec{\tau}.$$

有限力程可分离对力:

$$V(\boldsymbol{r}_1\sigma_1, \boldsymbol{r}_2\sigma_2, \boldsymbol{r}_1'\sigma_1', \boldsymbol{r}_2'\sigma_2') = -G\delta(\boldsymbol{R} - \boldsymbol{R}')P(\boldsymbol{r})P(\boldsymbol{r}')\frac{1 - P_\sigma}{2},$$

Y. Tian, Z. Y. Ma and P. Ring, PLB 676 (2009) 44.

p-MDCRHB模型

形变参数:

$$\beta^{\tau}_{\lambda\mu} = \frac{4\pi}{3N_{\tau}R^{\lambda}} Q^{\tau}_{\lambda\mu} \longleftarrow \quad Q^{\tau}_{\lambda\mu} = \int d^3 \boldsymbol{r} \rho^{\tau}_{V}(\boldsymbol{r}) r^{\lambda} Y_{\lambda\mu}(\Omega)$$

角动量和宇称投影:

$$\hat{P}^{J}_{MK} = \frac{2J+1}{8\pi^2} \int \mathrm{d}\Omega D^{J*}_{MK}(\Omega) \hat{R}(\Omega)$$

$$|\Psi^{JM\pi}_{\alpha,q}\rangle = \sum_{K} f^{JK\pi}_{\alpha} \hat{P}^{J}_{MK} \hat{P}^{\pi} |\Phi(q)\rangle$$

$$\hat{P}^{\pi} = \frac{1}{2}(1 + \pi \hat{P})$$

粒子数约束:

$$\mathcal{H}' = \mathcal{H} - \lambda_p [Z(\mathbf{r}; q, q'; \Omega) - Z_0] - \lambda_n [N(\mathbf{r}; q, q'; \Omega) - N_0]$$

Hill-Wheeler 方程:

$$\sum_{K'} \left\{ \mathcal{H}'^{J\pi}_{KK'}(q;q) - E^{J\pi}_{\alpha} N^{J\pi}_{KK'}(q;q) \right\} f^{JK'\pi}_{\alpha} = 0$$

K. Wang and B.N. Lu, CTP 74 (2022) 015303.

目　录

- 研究背景和意义
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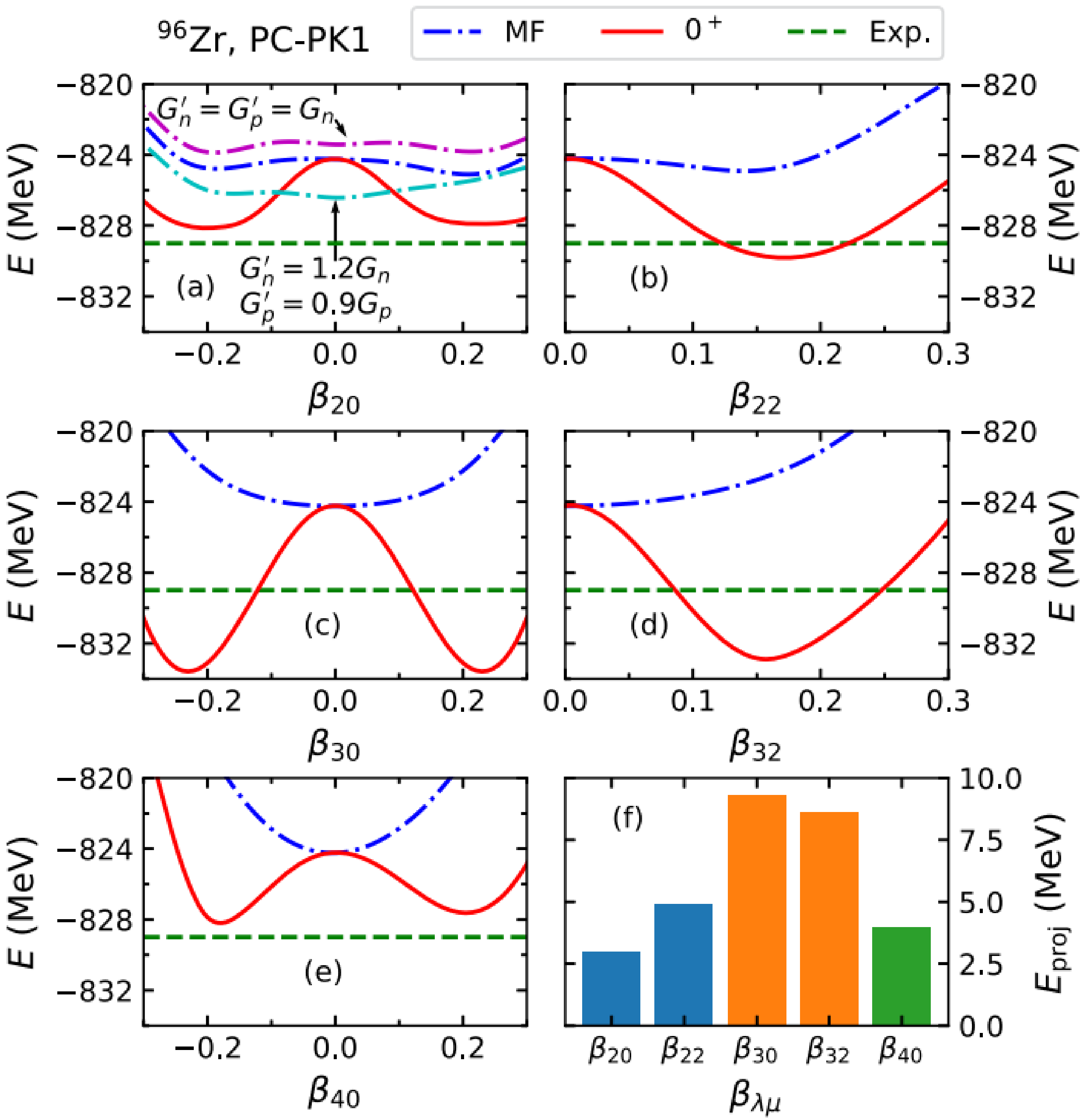
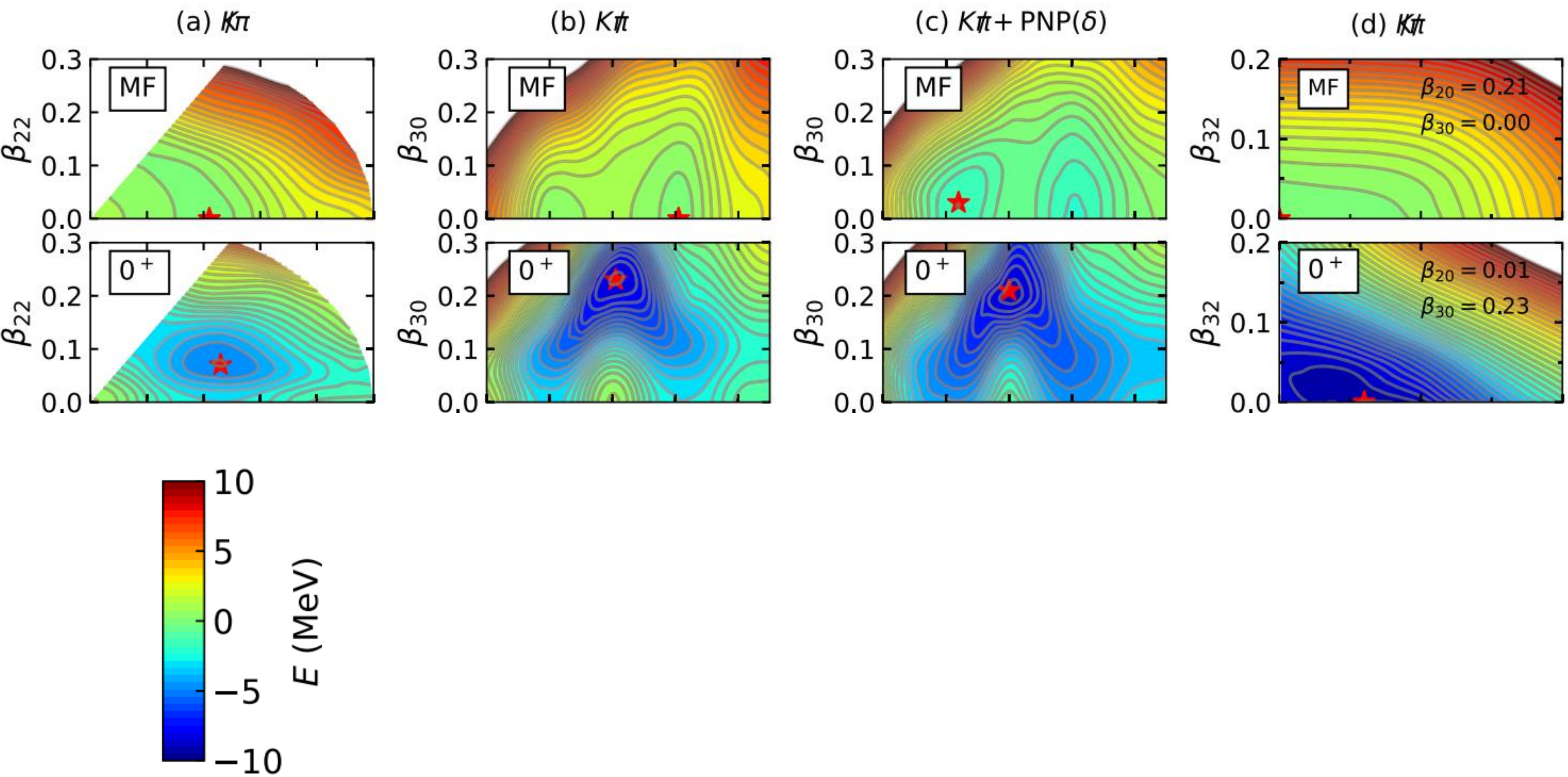
- 总结和展望

各形变对结合能的影响（1DPES）

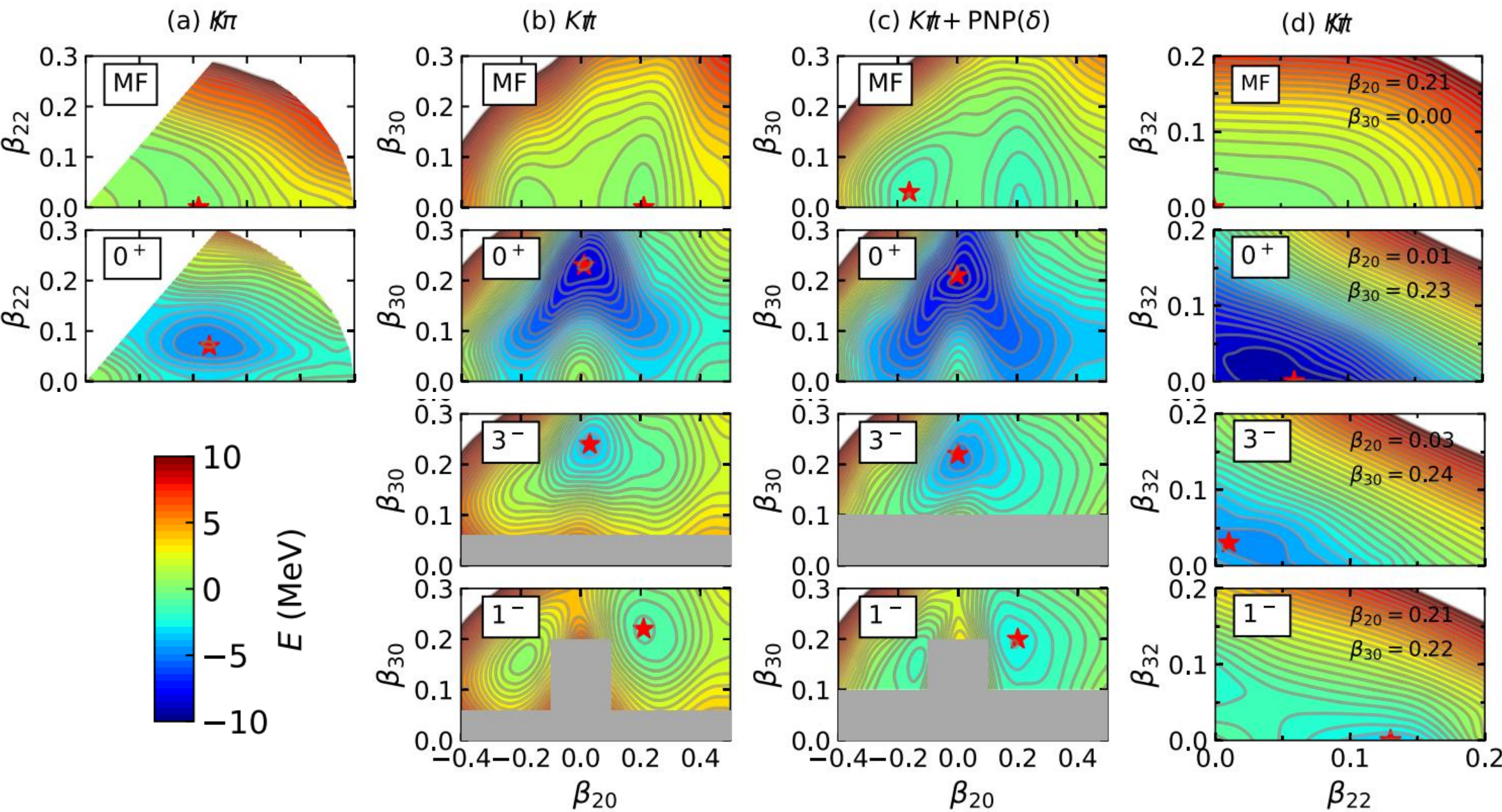


^{96}Zr的基态和激发态（2DPES）



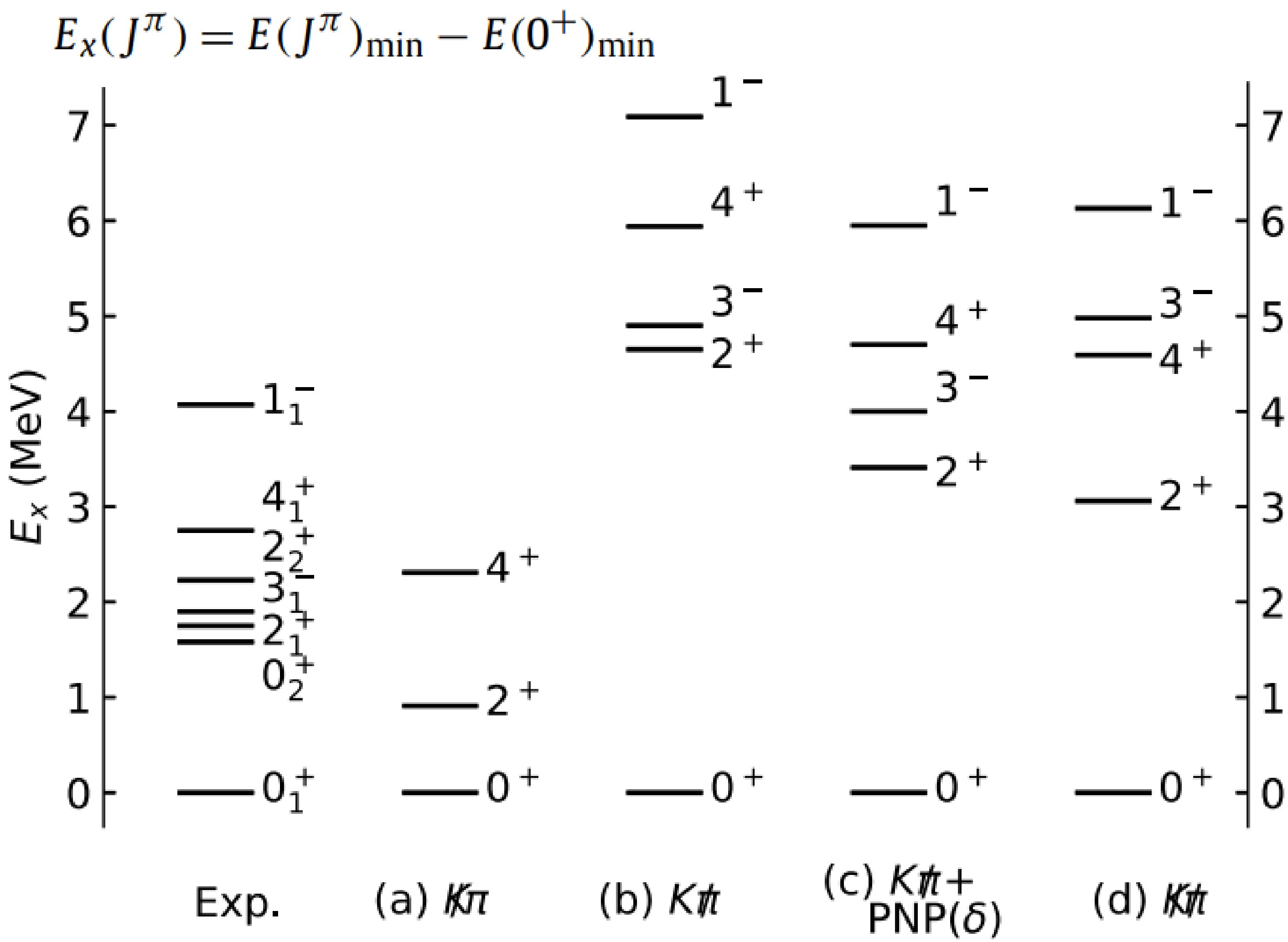
平均场层次下没有八极形变，投影给出的基态存在八极关联，需要打破轴对称和反射对称才能确定.

^{96}Zr的基态和激发态（2DPES）



平均场层次下没有八极形变，投影给出的基态存在八极关联，需要打破轴对称和反射对称才能确定． 从形变看，基态倾向于八极振动, 而非稳定八极形变.

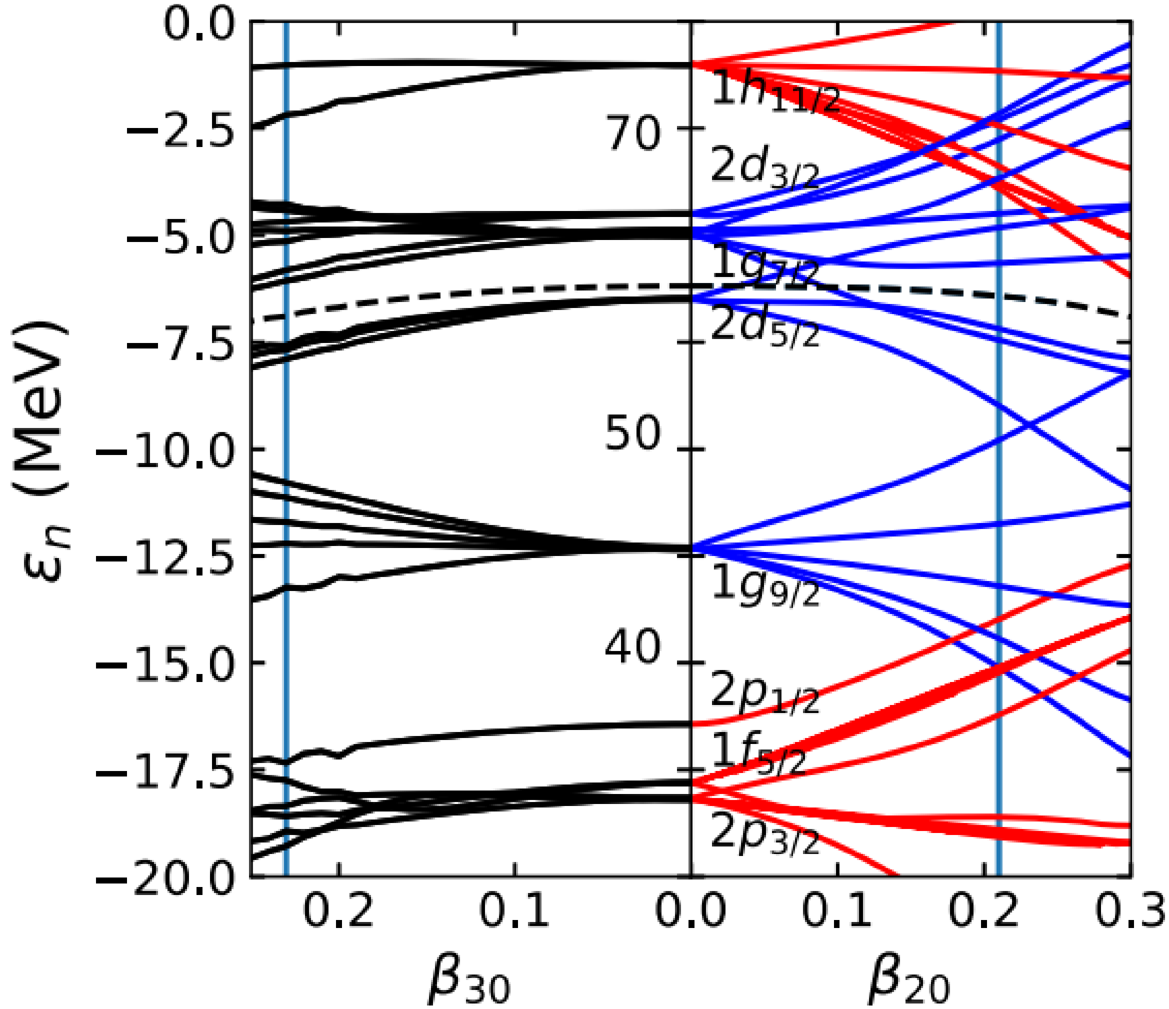
^{96}Zr的低能激发谱



八极能级顺序和实验基本一致，但激发能系统性高出 2-3 MeV.

非轴对称形变降低除 3^- 外的激发能.

^{96}Zr的单粒子能级



八极关联：质子2p3/2 → 1g9/2, 中子2d5/2→1h11/2.

总结

- 我们利用变分后投影的相对论协变密度泛函理论研究了 ^{96}Zr 的形变
- 投影后的 ^{96}Zr 基态表现出八极关联
- 寻找 ^{96}Zr 的基态需要同时打破轴对称和反射对称的限制

展望

- 考虑粒子数投影和组态混合给出低激发谱
- 高维全空间计算给出真正的基态

Thank you！