QCD thermodynamics and observables via functional QCD approach

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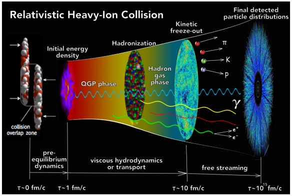
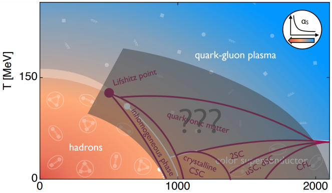
Introduction

The phenomena of QCD can be mapped into the phase diagram:

- The chiral phase transition driven by different combination of temperature and chemical potential; studies on the critical end-point (CEP);
- Fine structures to classify: moat regime, inhomogeneous phase, chiral spin symmetric phase, color superconductivity etc.
- Bridge the gap between theory and experiment using thermodynamic observables.

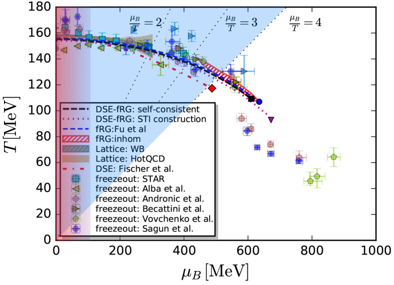




The Present and Future of QCD, QCD Town Meeting White Paper, 2023
Correlations in a moat regime, F. Rennecke, ECT*Trento, 2023

Towards finite density QCD: state-of-the-art predictions

2+1-flavour QCD phase diagram:



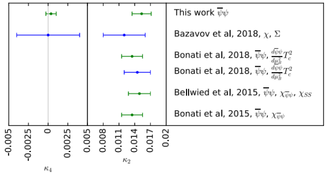
$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(0)}\right)^2 + \lambda \left(\frac{\mu_B}{T_c(0)}\right)^4 + \cdots$$

Wuppertal-Budapest, PLB 751 (2015) 559

hotQCD, PLB 795 (2019) 15

W.-j. Fu, Pawlowski and Rennecke, PRD 101 (2020) 054032

F. Gao and Pawlowski, PLB 820 (2021) 136584



$T_c(0) = 156.5 \pm 1.5$ MeV;

lattice:

$\kappa_{\mathrm{WB}} = 0.0149(21)$, $\kappa_{\mathrm{hQCD}} = 0.015(4)$;
reliablity: $\mu_B/T \lesssim 2$.

fQCD:

$\kappa_{\mathrm{fRG}} = 0.0141(2)$, $\kappa_{\mathrm{DSE}} = 0.0147(5)$;
reliablity: $\mu_B/T \lesssim 4$.

Great opportunity for a combined analysis: lattice QCD + functional QCD.

Towards finite density QCD EoS

Currently, the functional QCD approaches can only calculate the quark potential directly, while the gluon sector still awaits further investigations.

· Baryon number density ρ_B method + Lattice, suitable for the RHIC BES region:

$$P(T,\mu_B) = P_{Latt.}(T,\mathbf{0}) + \int_0^{\mu_B} \rho_B(T,\mu)\, d\mu;$$

· Towards high density region: compact star:

$$P_q(\mu_q, T) = P_q(\mu_{q,0}, 0) + \int_{\mu_{q,0}}^{\mu_{\rm UV}} {\rm d}\mu' \rho_q(\mu', 0) + \int_0^T dT' s_q^{\rm free}(\mu_{\rm UV}, T') + \int_{\mu_{\rm UV}}^{\mu_q} {\rm d}\mu' \rho_q(\mu', T).$$

Isserstedt, Fischer and Steinert, PRD 103 (2021) 054012;

F. Gao and Y.-x. Liu, PRD 94 (2016) 9, 094030;

YL, F. Gao, B.-c. Fu, H.-c. Song and Y.-x. Liu, in preparation;

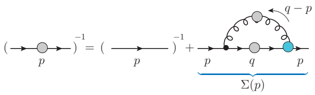
F. Gao, PRL 128, 131301 (2022);

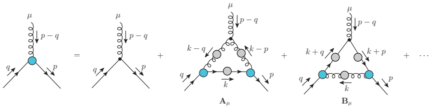
H. Chen, Baldo, Burgio, and Schulze, PRD 86 (2012) 045006.

Dyson-Schwinger equations (DSEs) and functional renormailization group (fRG) approach are the nonperturbative approach in continuum QCD which contain the features of both confinement and chiral symmetry breaking.

Truncation is required in functional QCD methods as the equations are not closed.

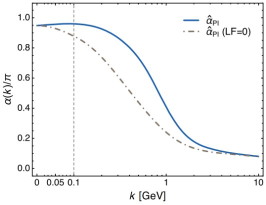
- How to generally evaluate the truncation?
- How to reduce the higher order correction and make the truncation controllable?

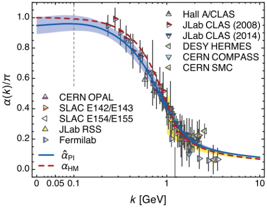




The truncation of the functional QCD methods

Hints from effective charge:





D. Binosi et al, PRD 96, 054026 (2017); A. Aguilar et al, PRD 80, 085018 (2009);

A. Deur et al, PPNP 90, 1 (2016).

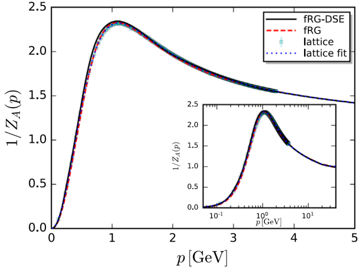
The infrared fixed point defines an expansion; connection to the (p)NJL model, Holographic QCD, etc.

· M. Huang, P.-f. Zhuang, Symmetry 15 (2023) 2, 541
· G.-y. Shao, X.-r. Yang, C.-l. Xie, W.-b. He, Eur.Phys.J.Plus 138 (2023) 1, 44
· Y.-q. Zhao, S. He, D.-f. Hou, L. Li, Z.-b. Li, JHEP 04 (2023) 115
· H.-l. Chen, X.-g. Huang, J.-f. Liao, Lect. Notes Phys. 987 (2021) 349-379

The truncation that describes both the vacuum and the phase transition region:

- Describe the running mass of quark and gluon: propagator;
- Describe the structure of the vertex.

The Yang-Mills sector is relatively separable. One can apply the data in vacuum:

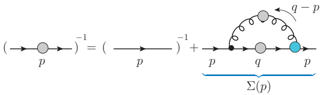


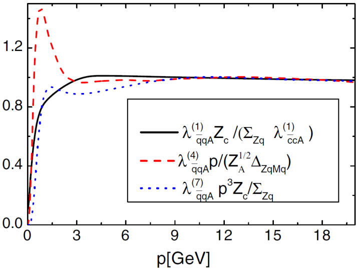
Lattice:
Duarte et al, PRD 94, 074502 (2016),
Boucaud et al, PRD 98, 114515 (2018),
Zafeiropoulos et al, PRL122, 162002 (2019)
fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005
fRG-DSE:
F. Gao, Pawlowski, PRD 102, 034027 (2020)

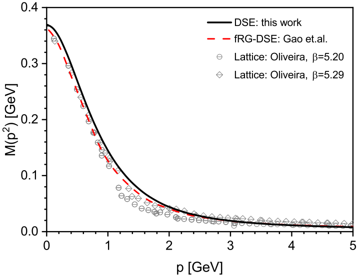
then compute finite (T, μ) effect:

$$D_{\mu\nu}^{-1}(k)|_{T,\boldsymbol{\mu}} = D_{\mu\nu}^{-1}(k)|_{0,\mathbf{0}} + \Delta\Pi_{\mu\nu}^{\mathrm{gauge}}(k) + \Delta\Pi_{\mu\nu}^{\mathrm{qrk}}(k),$$

*Ghost propagator changes little at finite temperature and density.







Quark-gluon vertex (tran. part) in Landau gauge:

$$\Gamma^\mu(q,-p) = \sum_{i=1}^{8} \lambda_i(q,-p) P^{\mu\nu}(q-p) \mathcal{T}_i^\nu(q,-p)\,,$$

optimized structures: λ_1 and λ_4 [a]:

$$\mathcal{T}_1(p,q) = -i\gamma^\mu, \quad \mathcal{T}_4^\mu(p,q) = (\not{p}+\not{q})\gamma^\mu\,,$$

$$\lambda_1(p,q) = F(k^2)\frac{A(p^2)+A(q^2)}{2},$$

$$\lambda_4(p,q) = \left[Z(k^2)\right]^{-1/2} \frac{B(p^2)-B(q^2)}{p^2-q^2}.$$

STI-RG relation well satisfied for $\gtrsim 3$ GeV [b];
Obtained quark condensate
$\Delta_{l,\chi} = -(273.9\,\mathrm{MeV})^3$, comparable with lattice: $\Delta_l = -(272(5)\,\mathrm{MeV})^3$ at $\mu_{\text{lat.}} = 2\,\mathrm{GeV}$.

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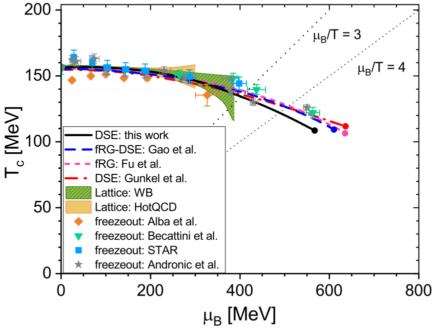
[a] **YL**, F. Gao, Y.-x. Liu, Pawlowski, in preparation

[b] F. Gao, PRD 102, 034027 (2020)

2+1-flavour phase diagram:



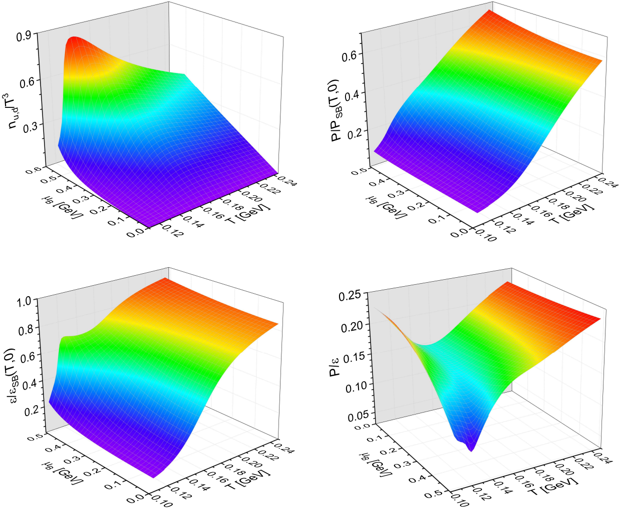
The fQCD computations of chiral phase transition are converging:

- $T_c = 157\,\text{MeV}$, $\kappa \sim 0.017$;
- Estimated range of CEP:
  $T \in (100, 110)$ MeV
  $\mu_B \in (550, 650)$ MeV;
- $\sqrt{s_{NN}} \in 2 \sim 4$ GeV.

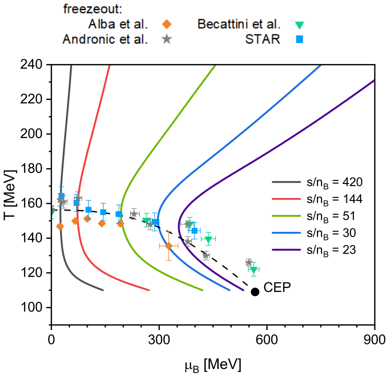
W.-j. Fu et al, PRD 101, 054032 (2020)
F. Gao and J. M. Pawlowski, PLB 820, 136584(2021)
P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

EoS from the optimized DSE truncation

$$P(T,\mu_B) = P_{Latt.}(T,\mathbf{0}) + \int_0^{\mu_B} \rho_B(T,\mu)\, d\mu; \qquad \rho_B = 2\,\rho_{u,d} = -2\,\mathrm{Tr}[\gamma_4 S_{u,d}].$$



EoS from the optimized DSE truncation



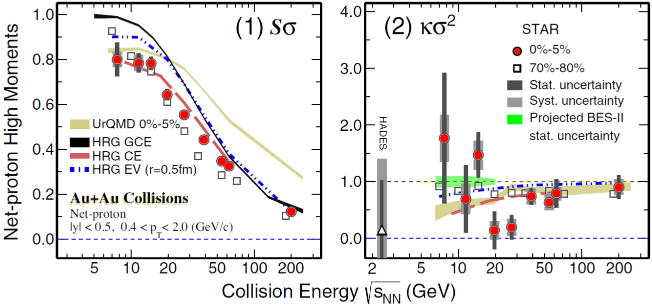
The trajectories at $s/n_B = 420$, 144, 51 and 30 meet with the freezeout points at $\sqrt{s_{\mathrm{NN}}} = 200$, 62.4, 19.6 and 11.5 GeV, respectively; in agreement with lattice results [a] [b].

Highly relevant for hydrodynamic simulations at finite density.

[a] Gunther, EPJ Web of Conferences, 137 07008 (2017)

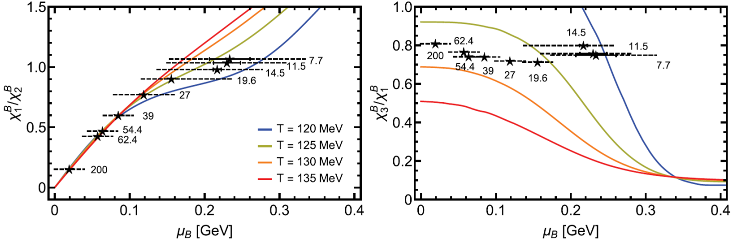
[b] Bollweg, PRD 108, 014510 (2023)

Baryon number fluctuation - (net-)proton cumulants and ratios are obsevables:



X.-f. Luo and X. Nu, Nucl. Sci. Tech. 28, 112 (2017); STAR, PRL 126, 092301 (2021); W.-j. Fu, X.-f Luo, J. M. Pawlowski, F. Rennecke, R. Wen, and S. Yin, PRD 104, 094047 (2021); Z.-b. Li, K. Xu, X.-y. Wang, M. Huang, EPJC 79, 245 (2019).

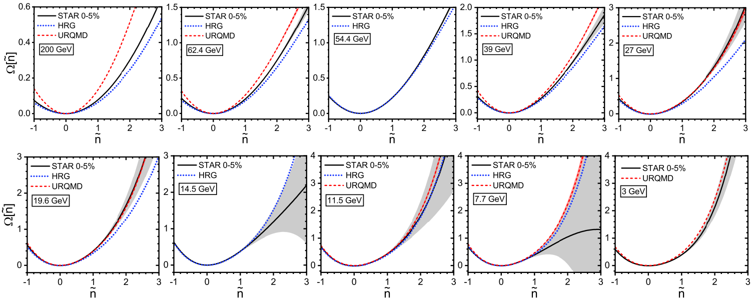
Theoretical calculations are in the (T, μ_B) space; what are the corresponding T and μ_B for the freezeout states ($\sqrt{s_{NN}}$) measured in experiment?



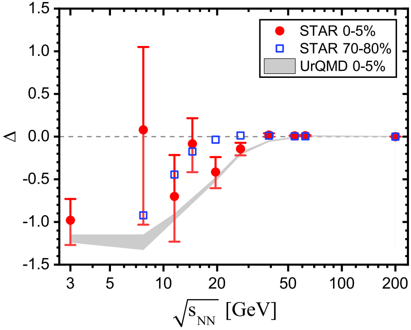
Borsanyi, PRL 113, 052301 (2014)
YL, M.-y. Chen, Z. Bai, F. Gao and Y.-x. Liu, PRD 105, 034012 (2022)

Dynamical fluctuation is also crucial to understand the exp. data; hard to calculate in first principles.
Vice versa, starting from the exp. data of full fluctuations, it is possible to extract an effective potential in near equilibrium (Lagrange transform):

YL, F. Gao, X.-f. Luo, et al, arXiv:2211.03401



Criterion for the first-order phase transition:



$$\Delta = \frac{8\kappa\sigma^2 - 21(S\sigma)^2}{12(\sigma^2/M)^4} \geq 0.$$

- The criterion can test the phase transition at a single collision energy.
- With current data, 7.7 GeV for 0-5% centrality shows signal slightly but with large error bar.

Summary

- fQCD computations of chiral phase transition are converging at small and moderate chemical potentials.
- An optimised truncation which captures the main character of QCD from vacuum to medium; accessible for the thermodynamic quantities.
- thermodynamic observables: freezeout parameters, phase transition criterion.

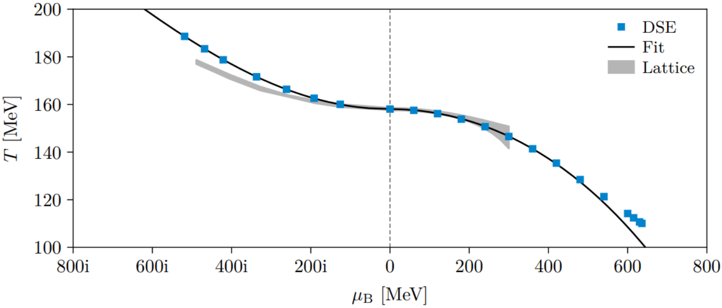
In the future:

- Combined analysis of QCD thermodynamics: lattice QCD + functional QCD + hydrodynamic simulation + experimental data;
- Studying the fine structures of QCD matter and other phase transitions besides the chiral one.

Thank you for your attention!!

Back-up

Quality of the lattice extrapolation



Bernhardt and Fischer, arXiv:2305.01434

Why baryon/quark number density

Consider first-order derivative of the QCD pressure $P = T \log \mathcal{Z}$: in the partition function $\mathcal{Z}$, μ_q only couples with the quark sector:

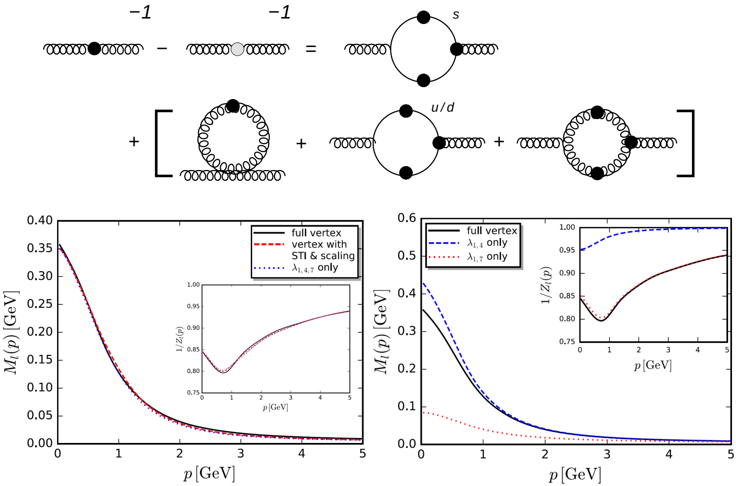
$$\mathcal{Z} = \int \mathcal{D}\bar{q}\mathcal{D}q \, e^{-\int d^4x \, \bar{q}(\gamma_\mu D_\mu - \gamma_4 \boldsymbol{\mu_q} + m)q - S_{\text{gauge}}},$$

$$\rho_q = \frac{\partial P}{\partial \mu_q} = \langle \bar{q}\gamma_4 q \rangle = -\mathrm{Tr}[\gamma_4 S_q],$$

with S_q is the quark propagator.

Similar relation for the current quark mass: $\frac{\partial P}{\partial m} = -\mathrm{Tr}[S_q] = \langle \bar{q}q \rangle$.

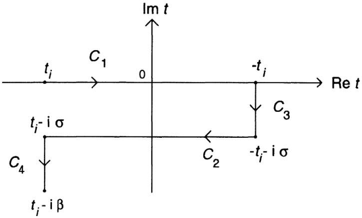
The optimised DSE truncation



F. Gao, Papavassiliou, and Pawlowski, PRD 103, 094013 (2021)
Williams, Fischer, Heupel, PRD 93, 034026 (2016)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94, 054005 (2016)

Effective potential with fluctuations

The thermodynamic quantities from functional QCD methods are computed in equilibrium via imaginary time formula.



- The effective potential is the non-equilibrium potential P(t).
- Thermodynamic relations in near equilibrium:

$$C_k(t) = \frac{1}{T}\frac{\partial^k P(t)}{\partial(\mu_B/T)^k} \,.$$

Effective potential with fluctuations

Expanding the effective potential by the rescaled dimensionless number density as the order parameter [1]:

$$\Omega[\tilde{n}] = \frac{\Phi[n_B]}{T\bar{n}} = \sum_{k=2}^{\infty} \omega_k \frac{\tilde{n}^k}{k!} ,$$

with dimensionless Taylor coefficients:

$$\omega_2(t) = \frac{1}{R_{21}(t)}, \qquad \omega_3(t) = -\frac{R_{32}(t)}{R_{21}^2(t)}, \qquad \omega_4(t) = \frac{3\,R_{32}^2(t) - R_{42}(t)}{R_{21}^3(t)},$$

$$\omega_5(t) = -\frac{15\,R_{32}^3(t) - 10R_{42}(t)R_{32}(t) + R_{52}(t)}{R_{21}^4(t)},$$

$$\omega_6(t) = \frac{105\,R_{32}^4(t) - 105R_{42}(t)R_{32}^2(t) + 10R_{42}^2(t) + 15R_{52}(t)R_{32}(t) - R_{62}(t)}{R_{21}^5(t)}, \quad \cdots ,$$

First order means the coexistence of two phases with two different number densities as two minima in the potential. A first order criterion up to forth order:

$$\Delta = \frac{1}{4}\omega_3^2 - \frac{2}{3}\omega_2\omega_4 \geq 0.$$