國科大杭州高等研究院
Hangzhou Institute for Advanced Study, UCAS

Meson Structure in Nuclear Matter

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Dalian University of Technology, Dalian, August 2nd

Outline

- Motivation
- Theoretical framework
- Numerical analysis
- Perspective

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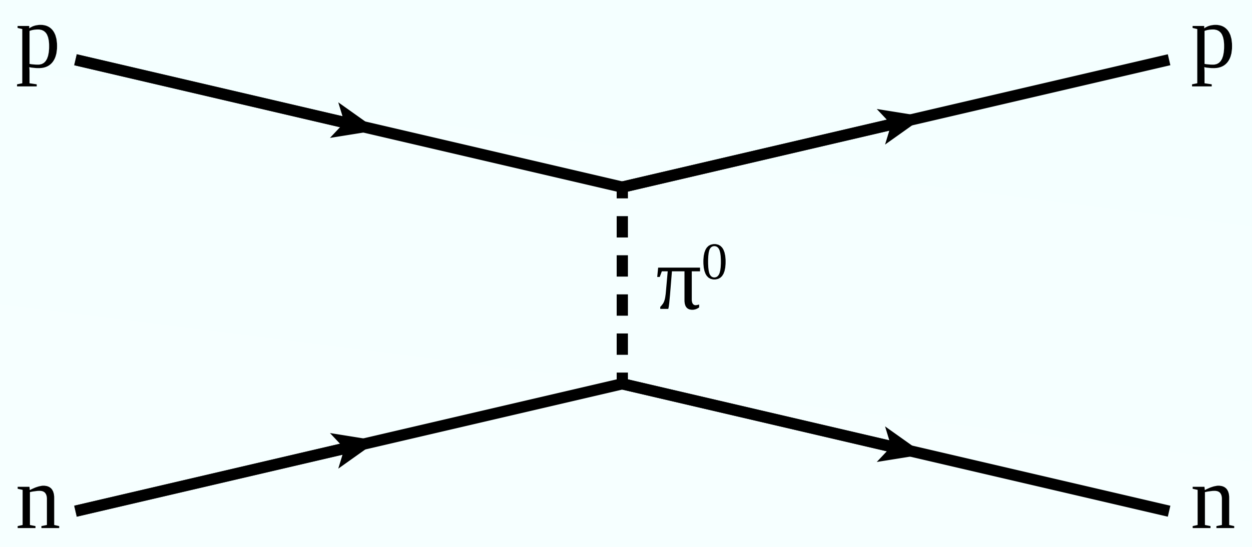
Motivation

Nucleon force parametrization



- Basic freedoms: baryons as matter fields and mesons as interaction carriers

- Baryons: N (~938MeV) include proton and neutron in two flavor case, or N with hyperons, e.g. Λ, in three flavor case;

- Mesons: π (~140MeV, 0^-) to describe r>1fm regions; σ, aka $f_0(500)$ (0^+), to describe r ~0.35fm regions; ω, ρ (~780MeV, 1^-) to describe r~0.25fm regions and many heavier mesons to describe shorter range regions, such as δ, aka $a_0(980)$ (0^+)… in two flavor cases or K mesons included within three flavor cases.

One-boson-exchange type

H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi, Nuclear Physics A 637, 435 (1998).
Y. Sugahara and H. Toki, Nuclear Physics A 579, 557 (1994).

$$\begin{aligned}\mathscr{L}_{\mathrm{O.B.E}} = \bar{\psi}\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu - g_\rho\gamma_\mu\tau_a\rho^{a\mu}\right]\psi \\ +\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\ -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 \\ -\frac{1}{4}R^a_{\mu\nu}R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho^a_\mu\rho^{a\mu}\end{aligned}$$

- It grasped the physical properties easily and quickly;
- The connection to QCD isn't obvious.

Chiral EFTs

S. Scherer and M. R. Schindler, A Primer for Chiral Perturbation Theory, Lect Notes Phys (2012).

$$\mathscr{L}^{(1)}_{\chi\mathrm{PT}} = \mathrm{Tr}\left[\bar{B}\left(i\gamma_\mu D^\mu - M_0\right)B\right] - \frac{D}{2}\mathrm{Tr}\left(\bar{B}\gamma^\mu\gamma_5\left\{u_\mu, B\right\}\right) - \frac{F}{2}\mathrm{Tr}\left(\bar{B}\gamma^\mu\gamma_5\left[u_\mu, B\right]\right)$$

$$U(x) = \exp\left(i\frac{\phi(x)}{F_0}\right)$$

Mesons

- The connection to QCD are embedded in to LECs;
- The power-counting rules;
- The difficulties of building a Lagrangian with σ.

Linear sigma model

$$\mathscr{L}_{\text{LSM}} = \frac{1}{2}\text{Tr}\left\{\bar{B}i\gamma_\mu\left[\left(D_R^\mu + D_L^\mu\right) + \gamma_5\left(D_R^\mu - D_L^\mu\right)\right]B\right\}$$
$$-\frac{g}{2}\text{Tr}\left\{\bar{B}\left[\left(\Phi + \Phi^\dagger\right) + \gamma_5\left(\Phi - \Phi^\dagger\right)\right]B\right\}$$

Linear meson matrix

- Chiral representations of hadrons;
- No obvious power-counting rules;
- Lagrangian with σ mesons.

Sigma meson problems

R. L. Workman et al., Particle Data Group, PTEP 2022, 083C01 (2022) and references therein

- A large decay width scalar particle: hard to identify in experiments

$f_0(500)$ T-MATRIX POLE $\sqrt{s}$	$(400 - 550) - i(200 - 350)$ MeV
$f_0(500)$ BREIT-WIGNER MASS	400 to 800 MeV
$f_0(500)$ BREIT-WIGNER WIDTH	100 to 800 MeV

Z. Xiao and H. Q. Zheng, Nucl. Phys. A 695, 273 (2001)
F. E. Close and N. A. Tornqvist, J. Phys. G28, R249 (2002)
C. Amsler and N. A. Tornqvist, Phys. Rept. 389, 61 (2004)
D. V. Bugg, Phys. Rept. 397, 257 (2004)
E. Klempt and A. Zaitsev, Phys. Rept. 454, 1 (2007)
J. R. Pelaez, Phys. Rept. 658, 1 (2016)

- P-wave problems in understanding light scalar mesons with naive quark model: one expects non-$\bar{q}q$ state, maybe hadronic molecules or multi-quark states

Theoretical Framework

An extended linear sigma model

- σ meson problems
- Reasonable power-counting rules
- SU(3) three flavor cases
- Include heavy mesons

Chiral Representation

- (pseudo-)scalar mesons:

$$\Phi = S + iP = \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

2-quark state 4-quark state

$$\Phi \to g_L \Phi g_R^\dagger, \quad \hat{\Phi} \to g_L \hat{\Phi} g_R^\dagger$$

$$\mathrm{U(1)_A} : \Phi \to e^{2iv}\Phi, \quad \hat{\Phi} \to e^{-4iv}\hat{\Phi}$$

- (axial-)vector mesons:

$$R^\mu = V^\mu - A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu + \rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu + a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu +} - a_1^{\mu +} & K^{*\mu +} - K_1^{\mu +} \\ \rho^{\mu -} - a_1^{\mu -} & \frac{\omega_N^\mu - \rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu - a_1^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} - K_1^{\mu 0} \\ K^{*\mu -} - K_1^{\mu -} & \bar{K}^{*\mu 0} - \bar{K}_1^{\mu 0} & \omega_S^\mu - f_{1S}^\mu \end{pmatrix}$$

$$L^\mu = V^\mu + A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu + \rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu + a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu +} + a_1^{\mu +} & K^{*\mu +} + K_1^{\mu +} \\ \rho^{\mu -} + a_1^{\mu -} & \frac{\omega_N^\mu - \rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu - a_1^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} + K_1^{\mu 0} \\ K^{*\mu -} + K_1^{\mu -} & \bar{K}^{*\mu 0} + \bar{K}_1^{\mu 0} & \omega_S^\mu + f_{1S}^\mu \end{pmatrix}$$

$$L_\mu \to g_L L_\mu g_L^\dagger, \quad R_\mu \to g_R R_\mu g_R^\dagger$$

Baryons:

$$B_N \equiv \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

$$N_R^{(RR)} \to g_R N_R^{(RR)} g_R^\dagger, \quad N_L^{(RR)} \to g_L N_L^{(RR)} g_R^\dagger$$

$$N_R^{(LL)} \to g_R N_R^{(LL)} g_L^\dagger, \quad N_L^{(LL)} \to g_L N_L^{(LL)} g_L^\dagger$$

$$N_R^{(RR)} \to e^{-3iv} N_R^{(RR)}, \quad N_L^{(RR)} \to e^{-iv} N_L^{(RR)},$$

$$N_R^{(LL)} \to e^{iv} N_R^{(LL)}, \quad N_L^{(LL)} \to e^{3iv} N_L^{(LL)}$$

Diquark approximation

$$B = \frac{1}{\sqrt{2}} \left(N^{(RR)} - N^{(LL)} \right)$$

$$= \frac{1}{\sqrt{2}} \left(N_L^{(RR)} + N_L^{(RR)} - N_R^{(LL)} - N_L^{(LL)} \right)$$

Power-counting rules

A. The order of a effective term is determined by the number of quarks and antiquarks; H. Fariborz, R. Jora, and J. Schechter, Physical Review D 77, 034006 (2008).

B. Double trace terms are neglected because of large N_c suppression

D. Parganlija, F. Giacosa, and D. H. Rischke, Physical Review D 82, 054024 (2010).

C. Vector meson contributions varying with nucleon density are at the order of ρ, while scalar meson contributions are behaving more like constants in chiral limits. So, the terms containing vector mesons should be given priority at the same quark number order to describe density effects.

Lagrangian

H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 72, 034001 (2005).
H. Fariborz, R. Jora, and J. Schechter, Physical Review D 77, 034006 (2008).
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 79, 074014 (2009).
D. Parganlija, P. Kovács, G. Wolf, F. Giacosa, and D. H. Rischke, Physical Review D 87,014011 (2013).
L. Olbrich, Phenomenology of baryons in the extended linear sigma model, Master's thesis,Goethe U., Frankfurt (main), 2015.
L. Olbrich, M. Zétényi, F. Giacosa, and D. H. Rischke, Phys. Rev. D 93, 034021 (2016).

- (pseudo-)scalar meson sector:

$$\mathscr{L}_{\mathrm{M}} = -\frac{1}{2}\operatorname{Tr}\left(D_\mu \Phi D^\mu \Phi^\dagger\right) - \frac{1}{2}\operatorname{Tr}\left(D_\mu \hat{\Phi} D^\mu \hat{\Phi}^\dagger\right) - V_0(\Phi, \hat{\Phi})$$

$$V_0 = -c_2 \operatorname{Tr}\left(\Phi\Phi^\dagger\right) + c_4 \operatorname{Tr}\left(\Phi\Phi^\dagger\Phi\Phi^\dagger\right) + d_2 \operatorname{Tr}\left(\hat{\Phi}\hat{\Phi}^\dagger\right) + e_3\left(\epsilon_{abc}\epsilon^{def}\Phi^a_d\Phi^b_e\hat{\Phi}^c_f + \text{H.c.}\right)$$

$$+c_3\left[\gamma_1 \ln\left(\frac{\det\Phi}{\det\Phi^\dagger}\right) + (1-\gamma_1)\,\gamma_1 \ln\left(\frac{\operatorname{Tr}\left(\Phi\hat{\Phi}^\dagger\right)}{\operatorname{Tr}\left(\hat{\Phi}\Phi^\dagger\right)}\right)\right]^2$$

- (axial-)vector meson sector:

$$\mathscr{L}_V = -\frac{1}{4}\operatorname{Tr}\left(R^2_{\mu\nu} + L^2_{\mu\nu}\right) + i\frac{g_2}{2}\left\{\operatorname{Tr}\left(L_{\mu\nu}\left[L^\mu, L^\nu\right]\right) + \operatorname{Tr}\left(R_{\mu\nu}\left[R^\mu, R^\nu\right]\right)\right\}$$

$$+h_2 \operatorname{Tr}\left(\left|L_\mu\Phi\right|^2 + \left|\Phi R_\mu\right|^2\right) + \hat{h}_2 \operatorname{Tr}\left(\left|L_\mu\hat{\Phi}\right|^2 + \left|\hat{\Phi}R_\mu\right|^2\right)$$

$$+2h_3 \operatorname{Tr}\left(L_\mu\Phi R^\mu\Phi^\dagger\right) + 2\hat{h}_3 \operatorname{Tr}\left(L_\mu\hat{\Phi}R^\mu\hat{\Phi}^\dagger\right)$$

$$+g_3\left[\operatorname{Tr}\left(L_\mu L_\nu L^\mu L^\nu\right) + \operatorname{Tr}\left(R_\mu R_\nu R^\mu R^\nu\right)\right]$$

$$+g_4\left[\operatorname{Tr}\left(L_\mu L^\mu L_\nu L^\nu\right) + \operatorname{Tr}\left(R_\mu R^\mu R_\nu R^\nu\right)\right]$$

- Baryon sector:

$$\mathscr{L}_{\mathrm{B}} = \frac{1}{2}\operatorname{Tr}\left\{\bar{B} i\gamma_\mu\left[\left(D^\mu_R + D^\mu_L\right) + \gamma_5\left(D^\mu_R - D^\mu_L\right)\right]B\right\}$$

$$-\frac{g}{2}\operatorname{Tr}\left\{\bar{B}\left[\left(\Phi+\Phi^\dagger\right) + \gamma_5\left(\Phi-\Phi^\dagger\right)\right]B\right\}$$

Keys to obtain mass spectrum

◉ Spontaneous symmetry breaking down from $\mathrm{SU}(3)_{\mathrm{L}} \otimes \mathrm{SU}(3)_{\mathrm{R}}$ to $\mathrm{SU}(3)_{\mathrm{V}}$

$$\langle S_a^b \rangle = \alpha_a \delta_a^b, \quad \left\langle \hat{S}_a^b \right\rangle = \beta_a \delta_a^b$$

◉ Mixing between 2-quark and 4-quark configurations

$$\begin{bmatrix} \phi_{i,j} \\ \hat{\phi}_{i,j} \end{bmatrix} = R \begin{bmatrix} \phi'_{i,j} \\ \hat{\phi}'_{i,j} \end{bmatrix} = \begin{bmatrix} \cos\theta_{i,j} & -\sin\theta_{i,j} \\ \sin\theta_{i,j} & \cos\theta_{i,j} \end{bmatrix} \begin{bmatrix} \phi'_{i,j} \\ \hat{\phi}'_{i,j} \end{bmatrix}$$

◉ Two scalar meson sets with different mass

$$m_{s,0,\pm} = -c_2 + 6\alpha^2 c_4 + d_2 + 4\beta e_3 \pm \sqrt{\left(-c_2 + 6\alpha^2 c_4 + d_2 + 4\beta e_3\right)^2 + 4\left(c_2 d_2 - 6\alpha^2 c_4 d_2 + 4 e_3\left(4\alpha^2 e_3 - \beta d_2\right)\right)}$$

$$m_{s,8,\pm} = -c_2 + 6\alpha^2 c_4 + d_2 - 2\beta e_3 \pm \sqrt{\left(c_2 - 6\alpha^2 c_4 - d_2 + 2\beta e_3\right)^2 + 4\left(c_2 d_2 - 6\alpha^2 c_4 d_2 + 2 e_3\left(2\alpha^2 e_3 + \beta d_2\right)\right)}$$

◉ Mixing angles

$$\theta_{s,0} = -\frac{1}{2}\arctan\left(\frac{8\alpha e_3}{c_2 - 6\alpha^2 c_4 - 4\beta e_3 + d_2}\right)$$

$$\theta_{s,8} = -\frac{1}{2}\arctan\left(\frac{4\alpha e_3}{c_2 - 6\alpha^2 c_4 + 2\beta e_3 + d_2}\right)$$

$$g_{a_0 NN} = -\frac{1}{\sqrt{2}} g \cos\tilde{\theta}_{s,8}$$

$$g_{f_0 NN} = -\frac{g}{\sqrt{6}} \cos\tilde{\theta}_{s,8}$$

$$g_{\sigma NN} = -\frac{g}{\sqrt{3}} \cos\tilde{\theta}_{s,0}$$

$$g_{\rho NN} = g_{\omega NN} = \frac{c}{2}$$

Choice of the light set

Method to handle dense environment

- Hidden strange channel approximation: drop out K mesons and hyperons.
- The mesons heavier than 1GeV are neglected.
- Relativistic mean field approximation: static fields to drop out kinematic terms of meson freedoms; $\langle \hat{P} \rangle = 1 \rightarrow$ drop out pseudo-scalar and axial-vector meson freedoms; $\langle \hat{I}_3 \rangle = 0 \rightarrow$ drop out $\rho^{\pm}$, $a_0^{\pm}$.
- The relevant LECs are α, β, c_2, c_4, d_2, e_3, h_2, $\hat{h}_2$, g_3 and c.

Numerical Analysis

Hadron Mass spectrum

Parameter space

Nuclear matter properties at saturation density

Parameter results

Y. Sugahara and H. Toki, Nuclear Physics A 579, 557 (1994).
H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi, Nuclear Physics A 637, 435 (1998).
F. Li, B. J. Cai, Y. Zhou, W. Z. Jiang, and L.-W. Chen, Astrophys. J. 929, 183 (2022).
G. A. Lalazissis, J. Konig, and P. Ring, Phys. Rev. C 55, 540 (1997).
F. J. Fattoyev, C. J. Horowitz, J. Piekarewicz, and G. Shen, Phys. Rev. C 82, 055803(2010).

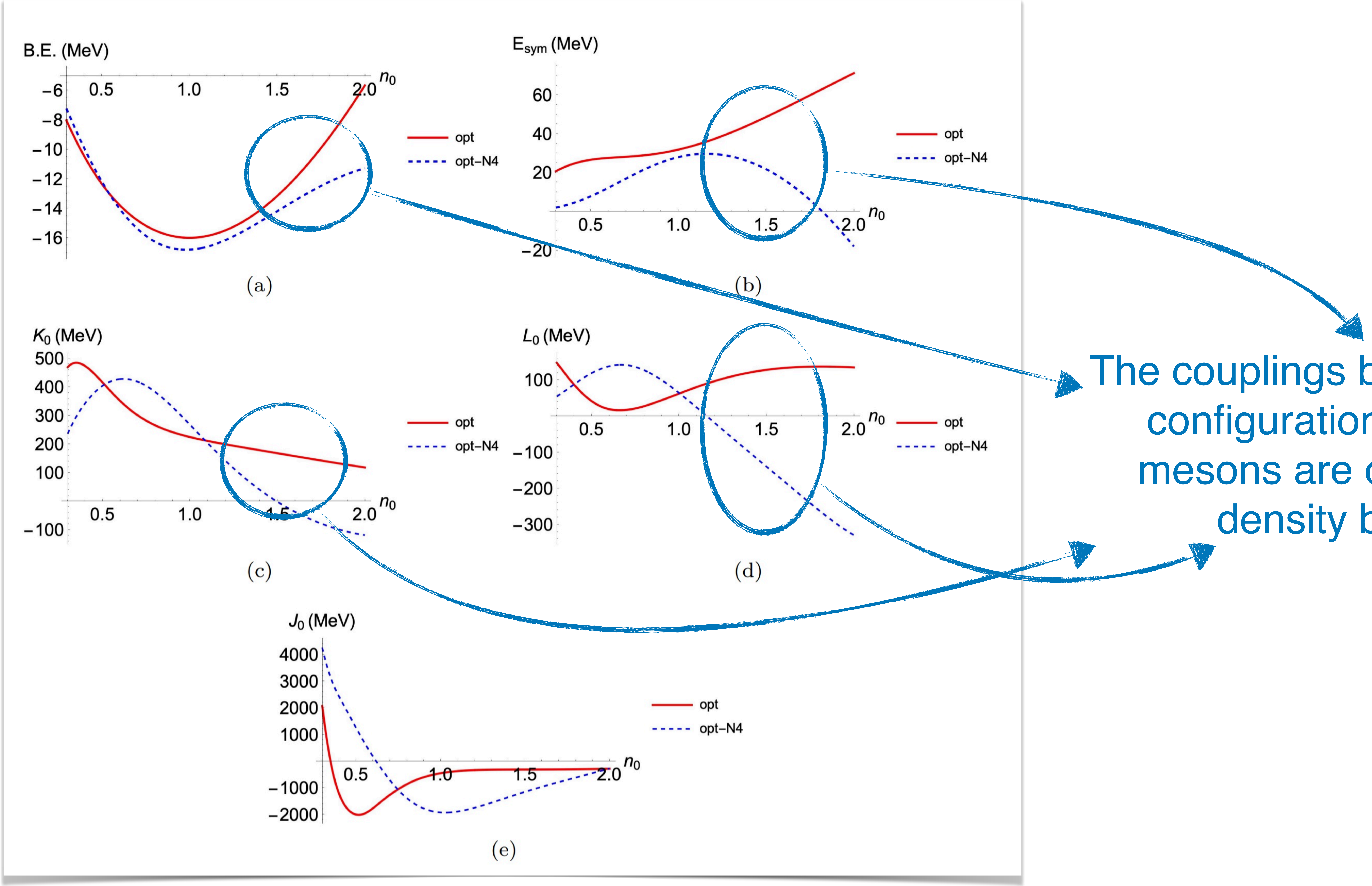
$$|g_{\sigma NN}| \approx 12$$
$$|g_{\omega NN}| \approx 13$$

m_N	938
m_σ	701
$m_{f_0(a_0)}$	965
$m_{f_0'(a_0')}$	1359
$m_{\sigma'}$	1526
$m_{\rho(\omega)}$	779

e_3(GeV)	-3.58
c_4	154
α(MeV)	32.3
β(MeV)	9.55
h_2	370
$\hat{h}_2$	-915
g_3	630
c	25.8

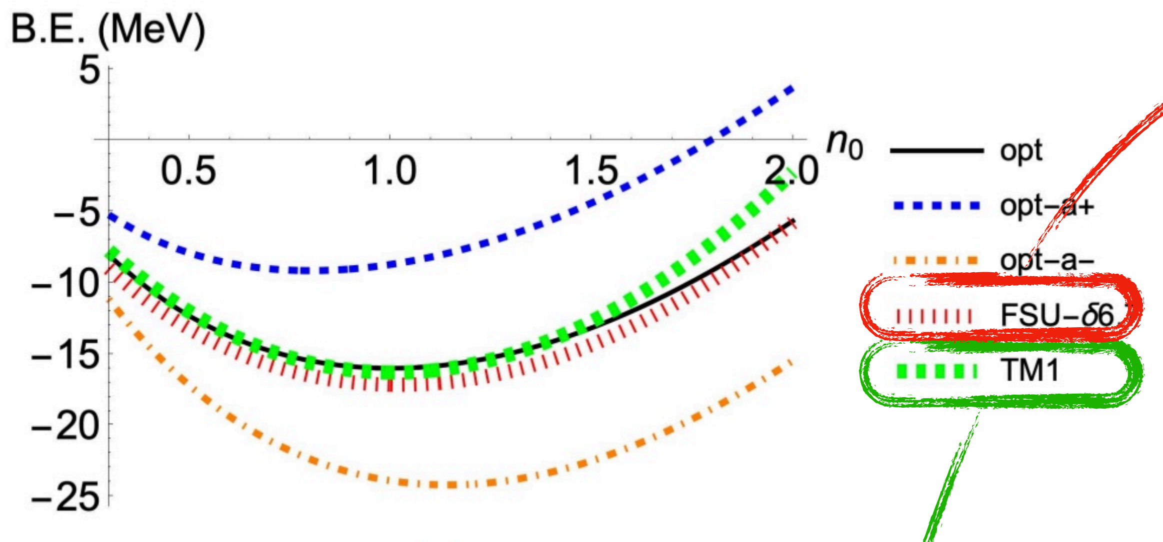
Higher order NM properties
High density region behaviors

n_0	0.155 fm^{-3}
e_0	-16.0 MeV
$E_{\rm sym}$	31.9 MeV
J_0	-449 MeV
L_0	62.7 MeV
K_0	225 MeV

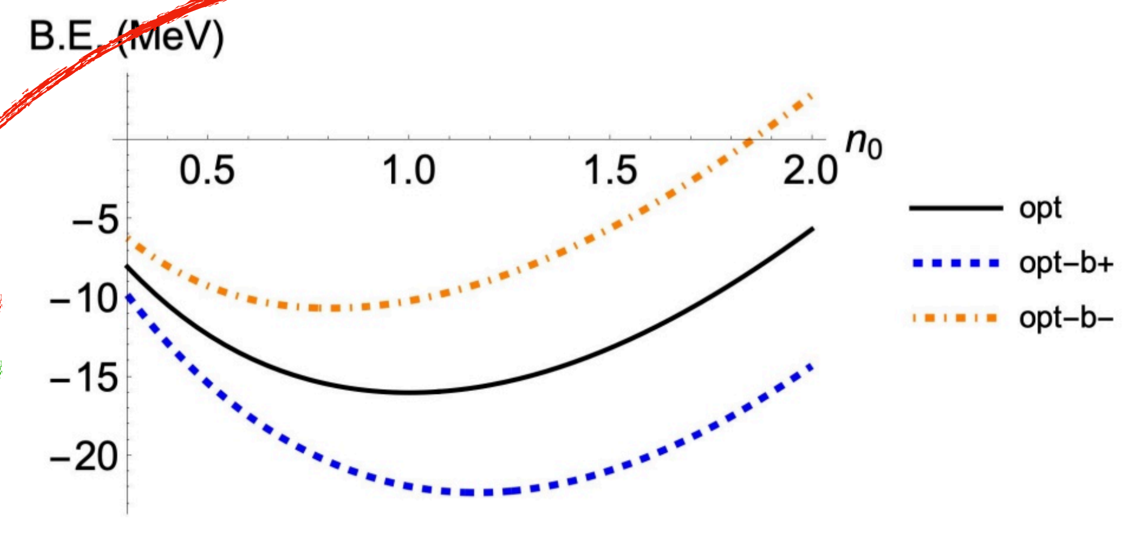


The couplings between 4-quark configurations and vector mesons are critical to high density behaviors

1% variation of parameter effect on NM properties



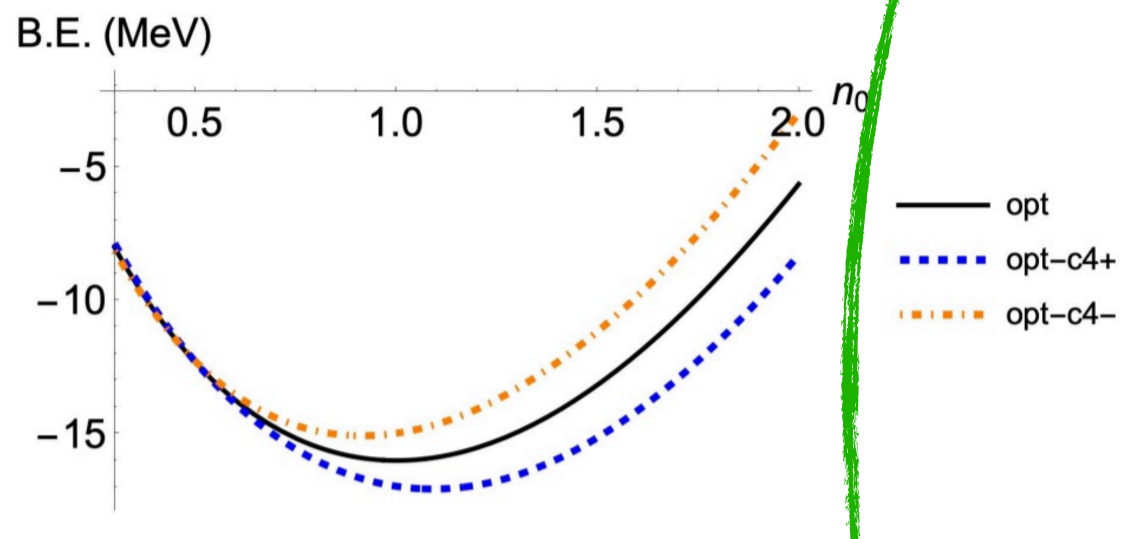
(a) α variarion.



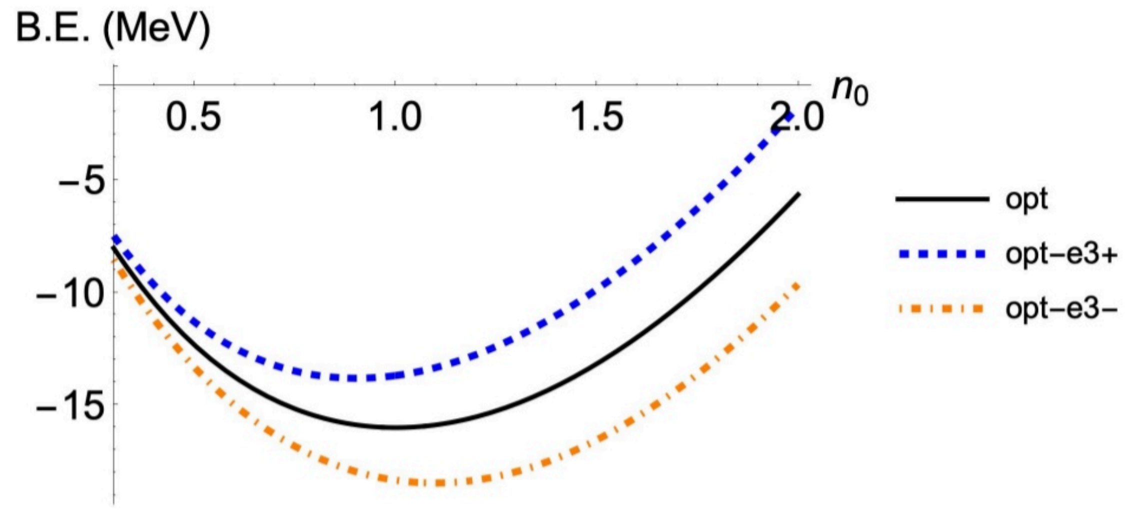
(b) β variarion.

O.B.E model with ρ, ω, σ, δ and their self interactions

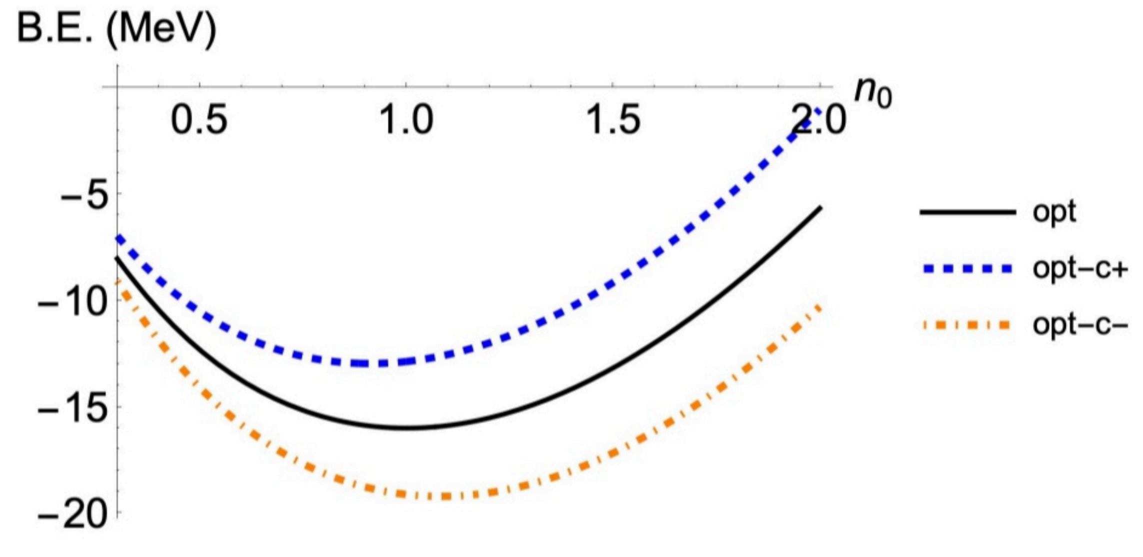
F. Li, B. J. Cai, Y. Zhou, W. Z. Jiang, and L. W. Chen, Astrophys. J. 929, 183 (2022).



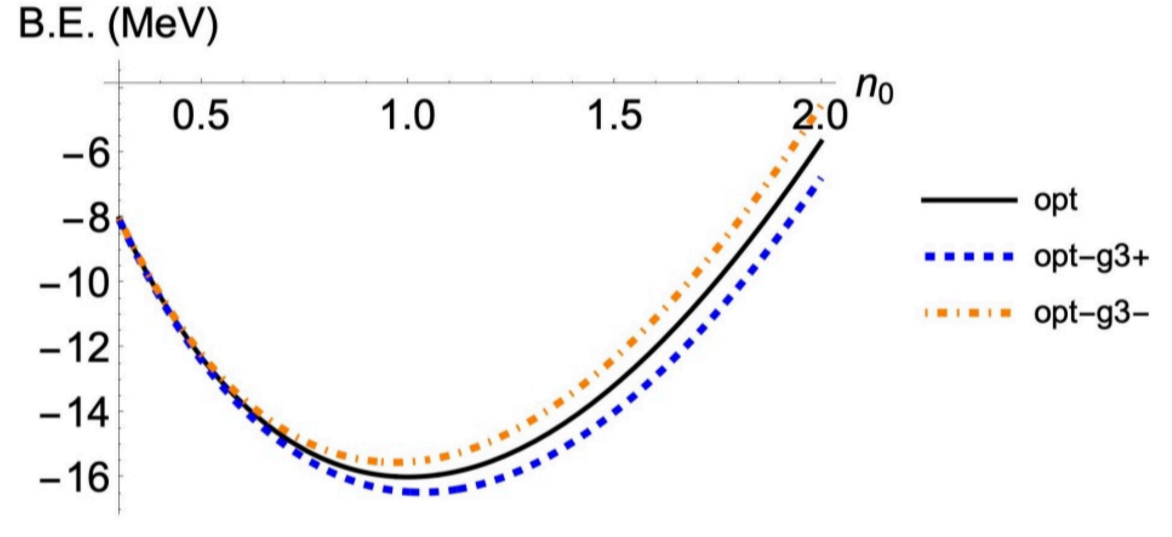
(c) c_4 variarion.



(d) e_3 variarion.



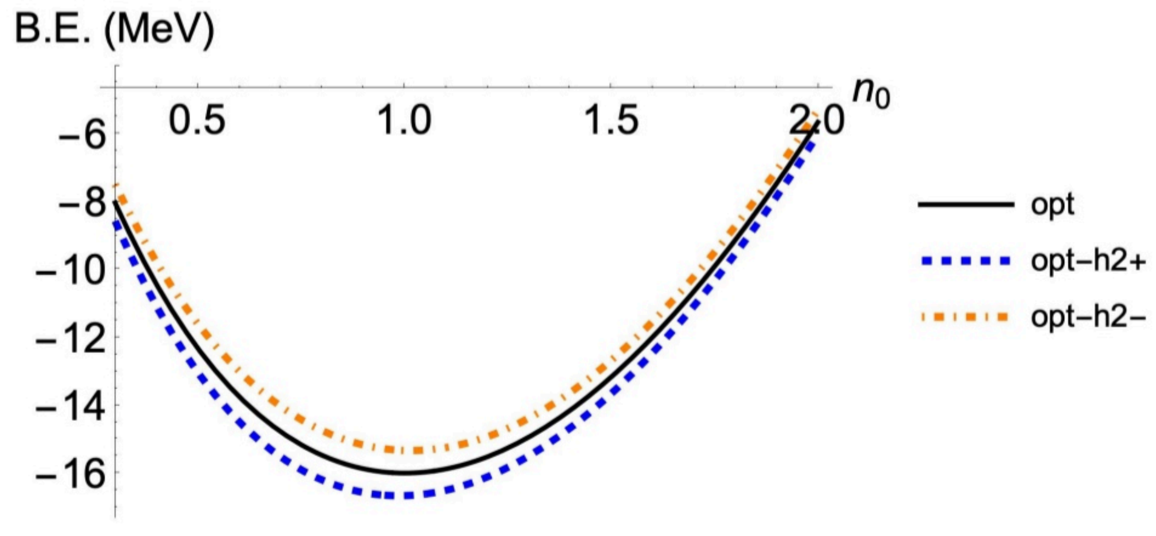
(e) c variarion.



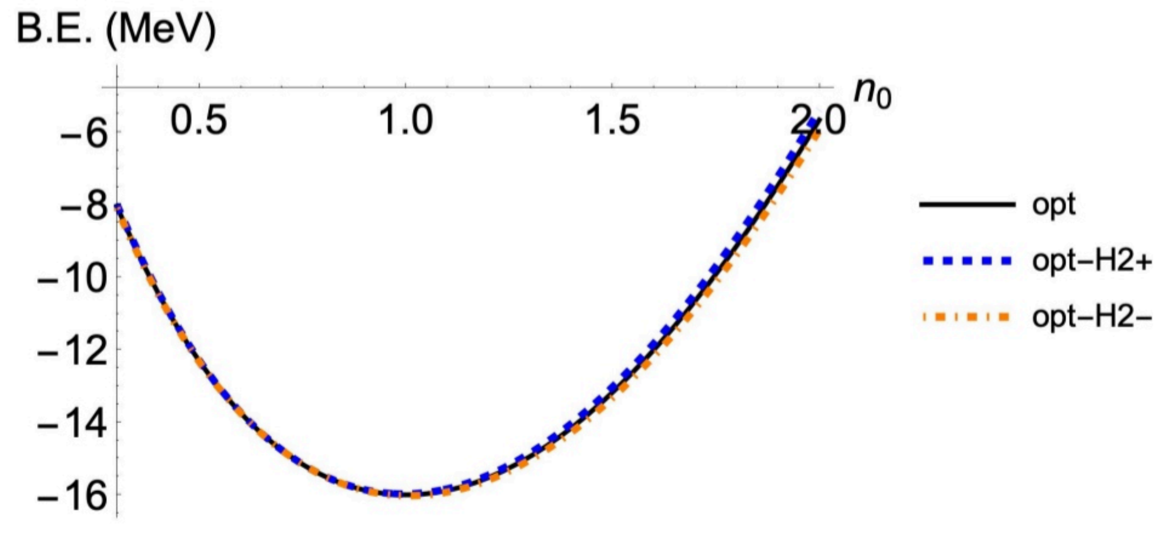
(f) g_3 variarion.

O.B.E model with ρ, ω, σ and their self interactions

Y. Sugahara and H. Toki, Nuclear Physics A 579, 557 (1994).
H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi, Nuclear Physics A 637, 435 (1998).



(g) h_2 variarion.

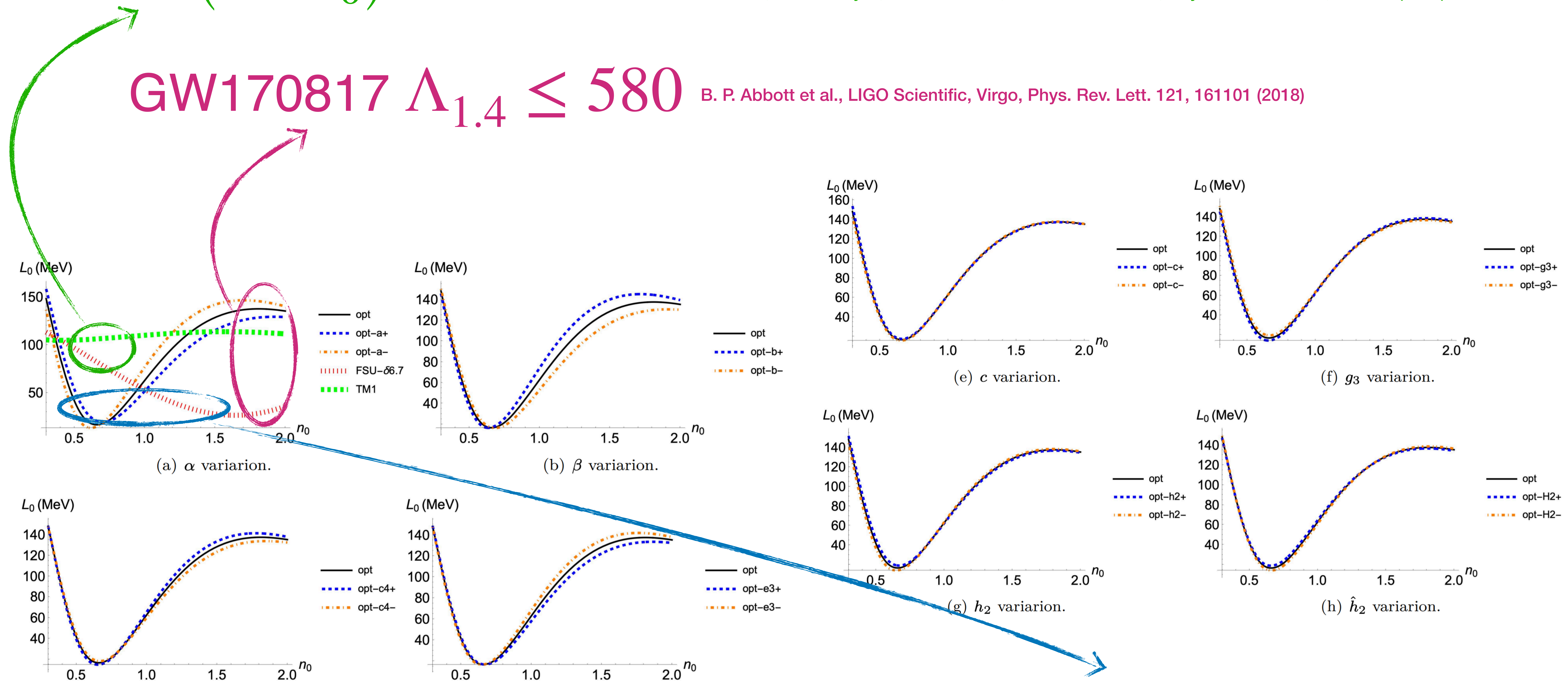


(h) $\hat{h}_2$ variarion.

The key to determine B.E. is attractive and repulsive interactions

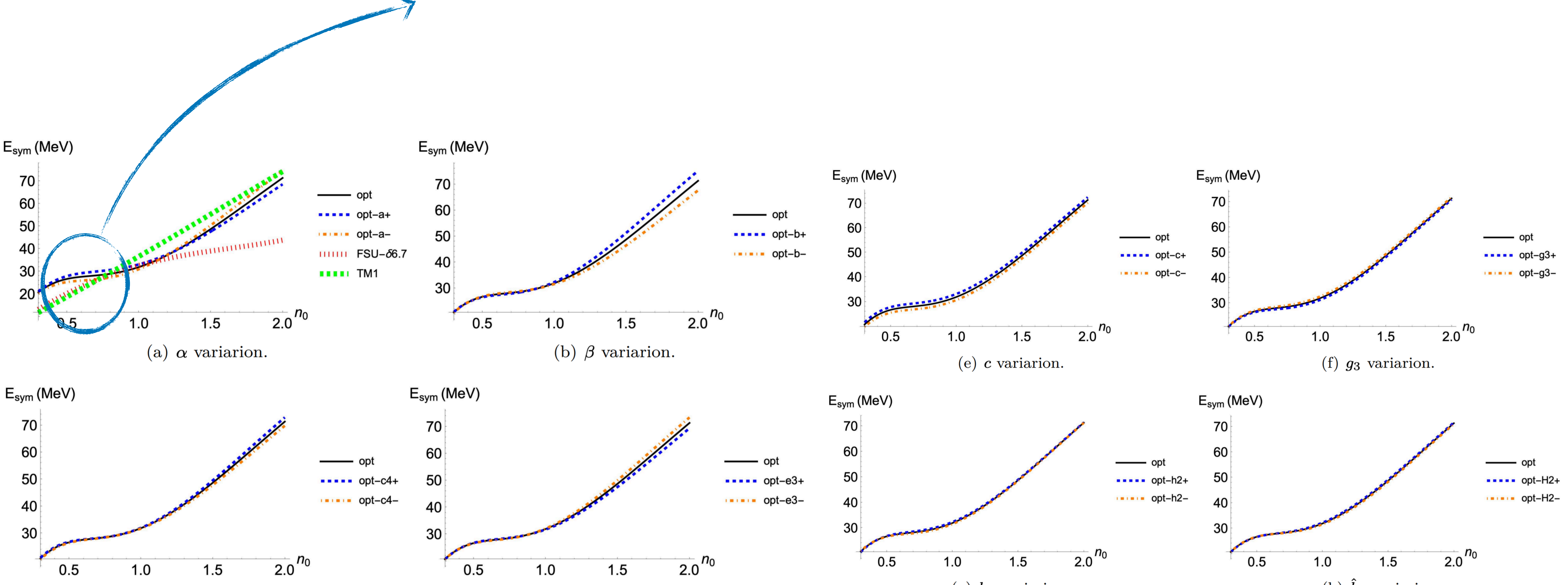
$L\left(2/3n_0\right) \geq 49\text{MeV}$ D. Adhikari et al., PREX, Phys. Rev. Lett. 126, 172502 (2021)
B. T. Reed, F. J. Fattoyev, C. J. Horowitz, and J. Piekarewicz, Phys. Rev. Lett. 126, 172503 (2021)

GW170817 $\Lambda_{1.4} \leq 580$ B. P. Abbott et al., LIGO Scientific, Virgo, Phys. Rev. Lett. 121, 161101 (2018)



U(3) symmetry → SU(3) symmetry

Grow to quickly due to vector meson U(3) constraints



(a) α variarion.

(b) β variarion.

(e) c variarion.

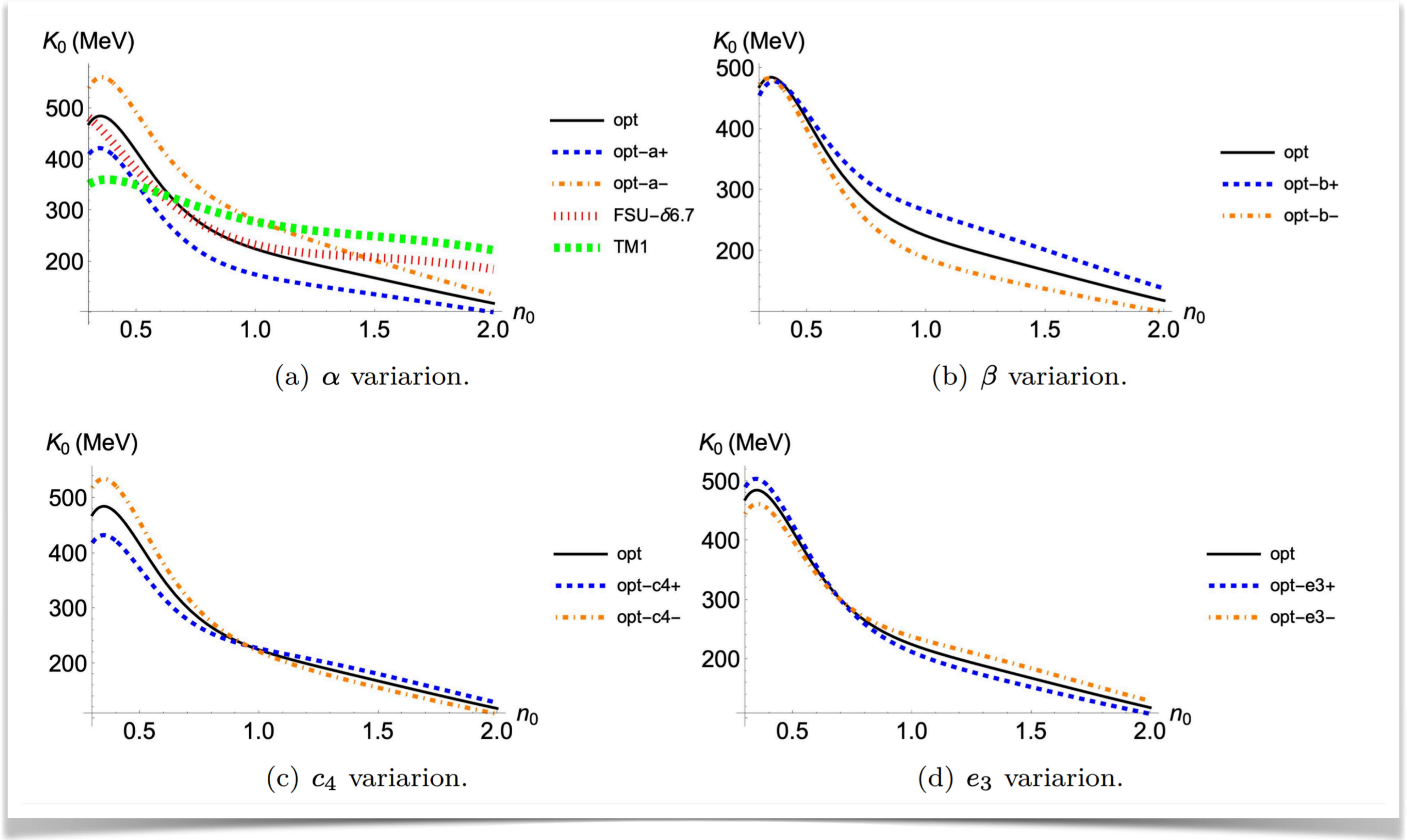
(f) g_3 variarion.

(c) c_4 variarion.

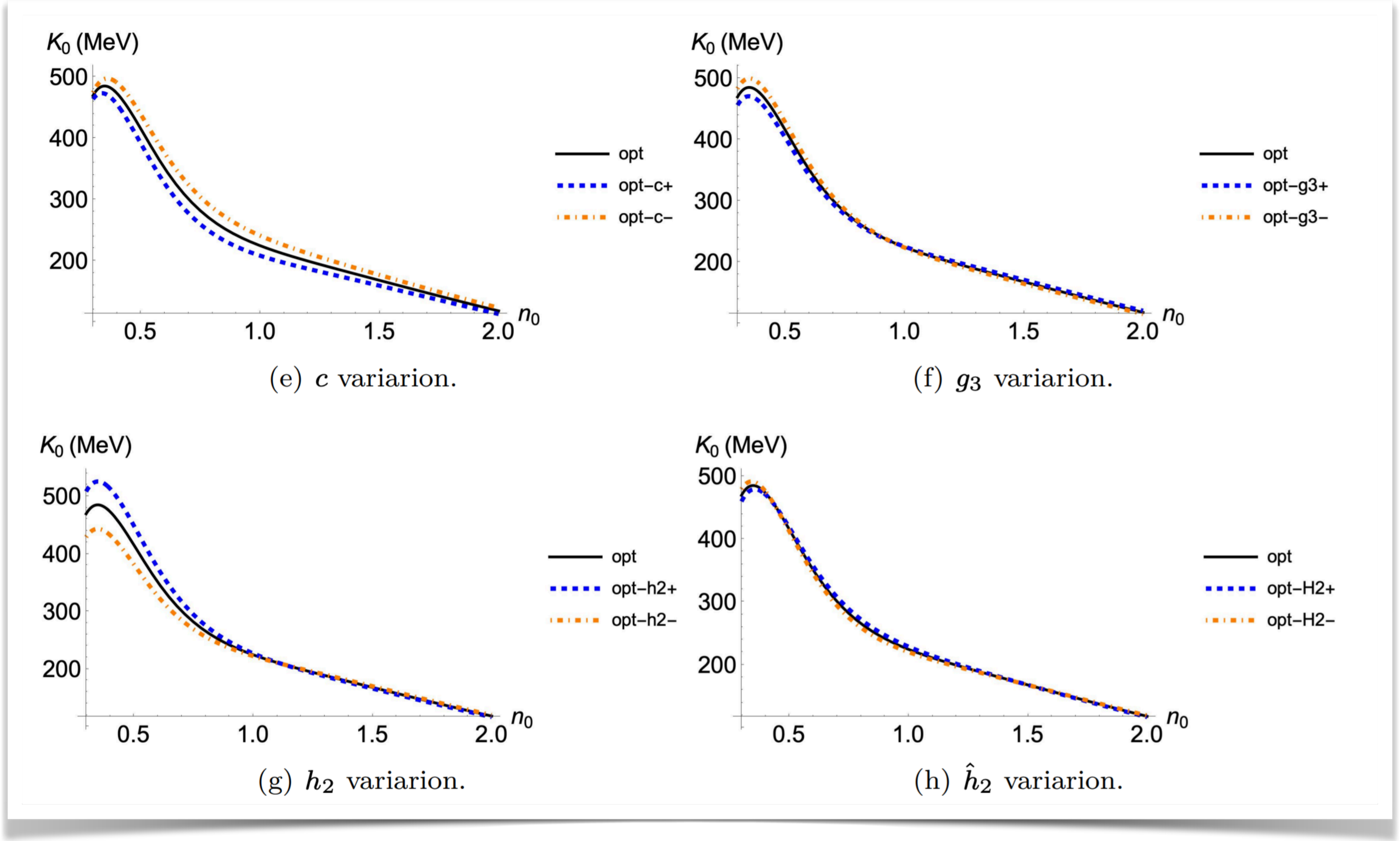
(d) e_3 variarion.

(g) h_2 variarion.

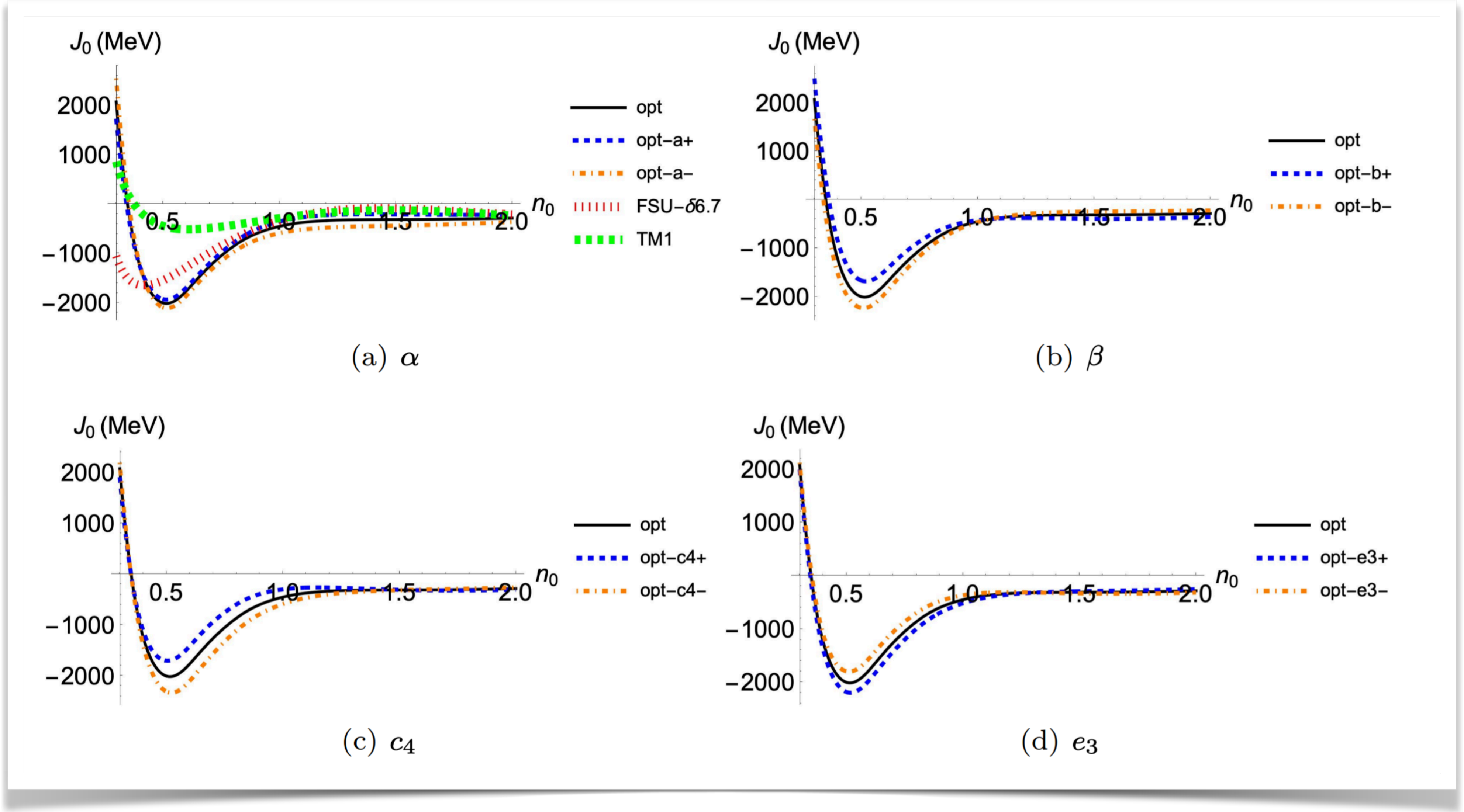
(h) $\hat{h}_2$ variarion.



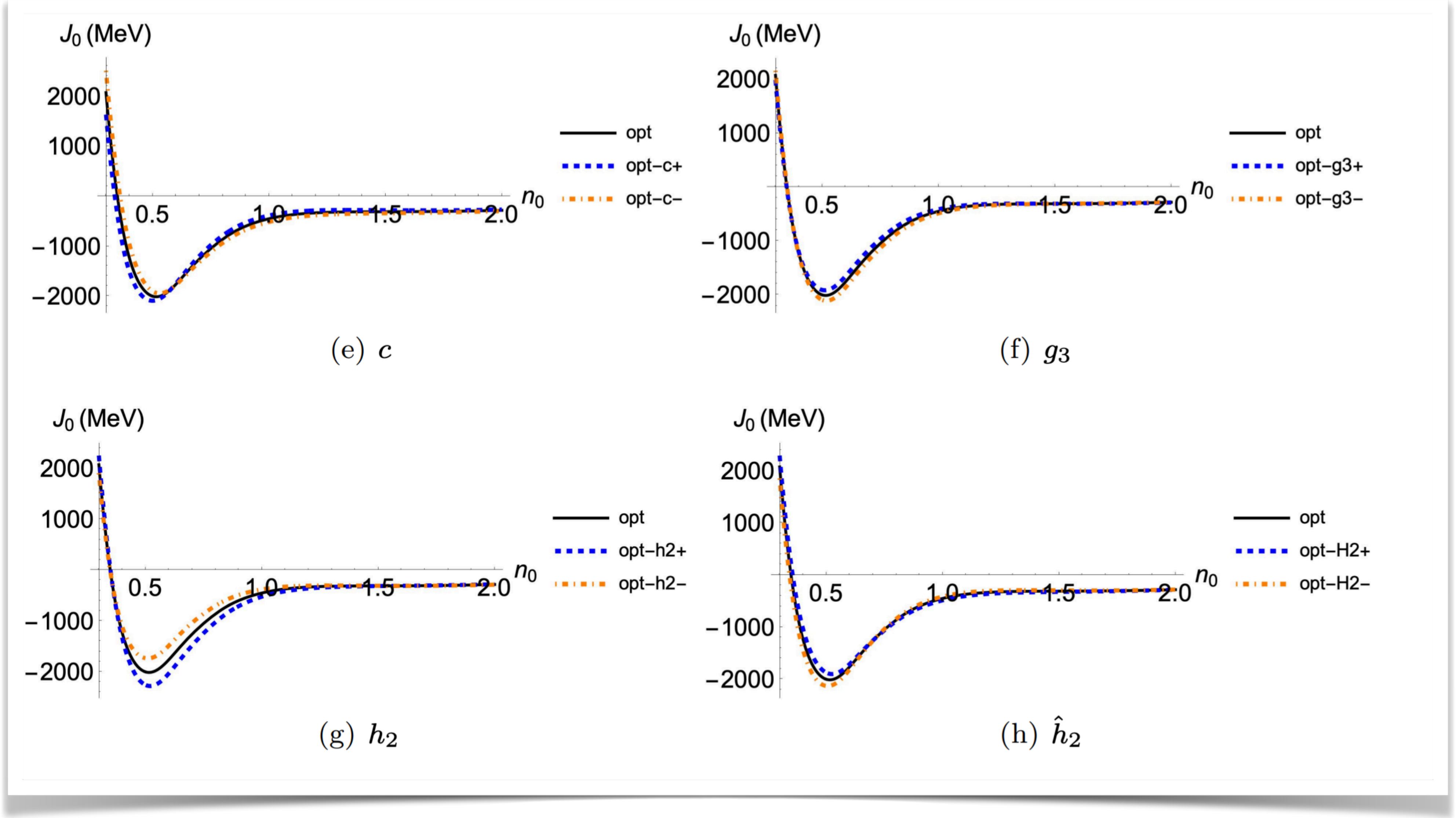
(a) α variarion. (b) β variarion. (c) c_4 variarion. (d) e_3 variarion.



(e) c variarion. (f) g_3 variarion. (g) h_2 variarion. (h) $\hat{h}_2$ variarion.



(a) α (b) β (c) c_4 (d) e_3



(e) c (f) g_3 (g) h_2 (h) $\hat{h}_2$

Higher order NM properties are sensitive to multi-meson couplings

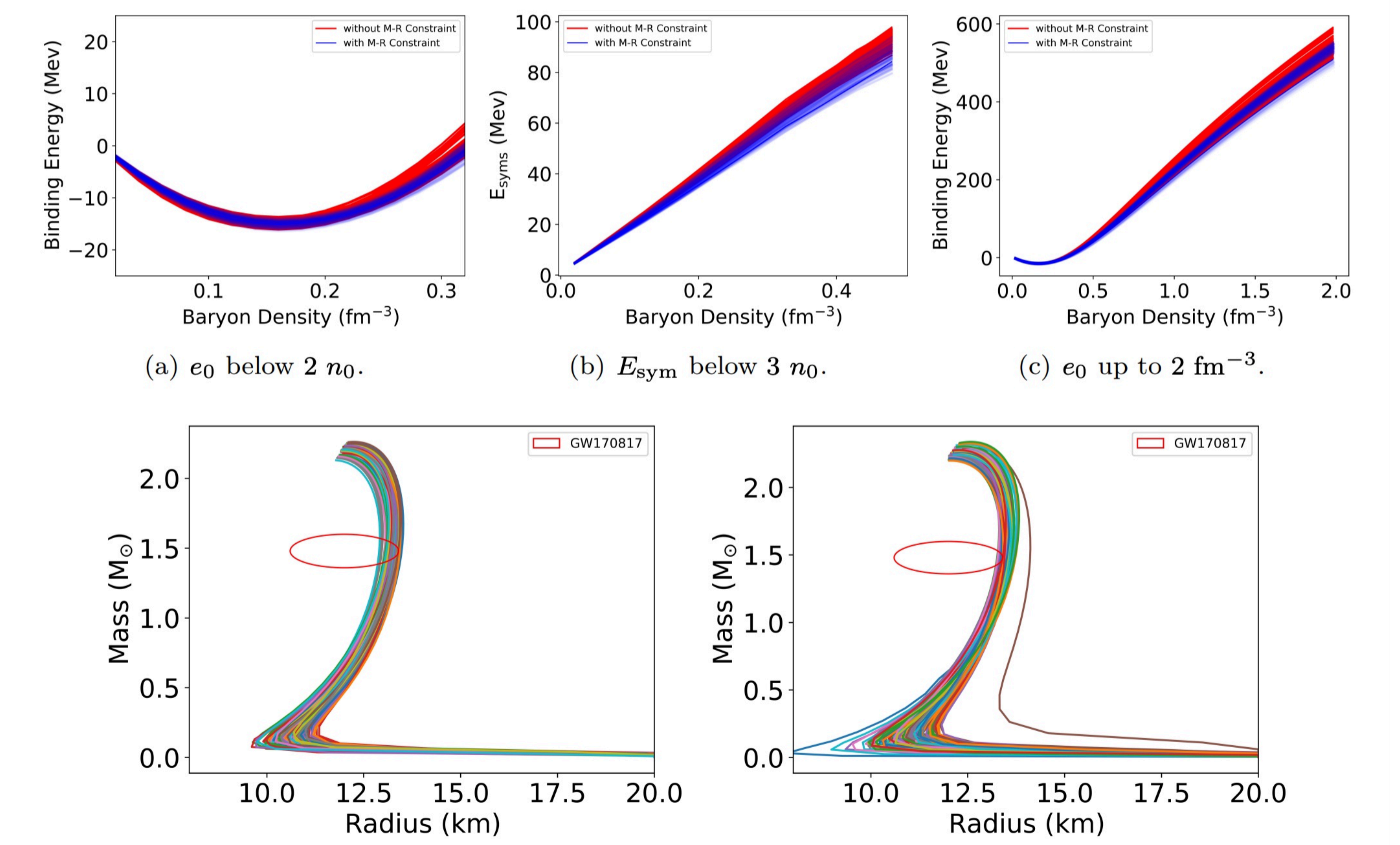
NS properties

ML experience with TM1 model

- The max mass reaches $1.98M_{\odot}$
- The radius of $1.4M_{\odot}$ is $14.5\mathrm{km}$
- The tidal deformation, $\Lambda_{1.4} = 1440$

U(3) symmetry

Handling multi-source data is important!



(a) e_0 below 2 n_0. (b) E_{sym} below 3 n_0. (c) e_0 up to 2 fm^{-3}.

(d) MR relations with MR constraints. (e) MR relations without MR constraints.

A. Mass spectrum;
B. NM properties at n_0;
C. NS MR and tidal deformation.

Perspective

I. The extended linear sigma model can be used as a tool to obtain meson spectrum and study nuclear matter properties;

II. Vector meson related couplings and $V_{\mathrm{s.b.}}$ are crucial to high density region physics;

III. This model is a promising tool to future NS studies, since it include heavy mesons and baryons in a consistent way with QCD chiral representations;

IV. The future research based on this model will be handled with our multi-source data analysis ML platform in order to understand QCD better.

Thank you!

Backup

Triple-meson couplings under RMF

$$\begin{aligned}\mathscr{L}_3 = -\frac{1}{6}\Big(&12\sqrt{3}c_4 f_0^2\alpha\sigma\cos\theta_0 - 4\sqrt{3}h_2\alpha\rho^2\sigma\cos\theta_0 + 6\sqrt{3}c_4\alpha\sigma^3\cos\theta_0 - 4\sqrt{3}h_2\alpha\sigma\omega^2\cos\theta_0 + \\ &2\sqrt{3}c_4\alpha\sigma^3\cos\left(3\theta_0\right) - 6\sqrt{3}c_4 f_0^2\alpha\sigma\cos\left(\theta_0 - 2\theta_8\right) - \sqrt{6}e_3 f_0^3\cos\theta_8 + \sqrt{6}e_3 f_0^3\cos\left(3\theta_8\right) - \\ &6\sqrt{3}c_4 f_0^2\alpha\sigma\cos\left(\theta_0 + 2\theta_8\right) - 2\sqrt{3}e_3 f_0^2\sigma\sin\theta_0 - 4\sqrt{3}\hat{h}_2\beta\rho^2\sigma\sin\theta_0 + 2\sqrt{3}e_3\sigma^3\sin\theta_0 - \\ &4\sqrt{3}\hat{h}_2\beta\sigma\omega^2\sin\theta_0 + 2\sqrt{3}e_3\sigma^3\sin\left(3\theta_0\right) - \sqrt{3}e_3 f_0^2\sigma\sin\left(\theta_0 - 2\theta_8\right) + 3\sqrt{6}c_4 f_0^3\alpha\sin\left(\theta_8\right) + \\ &2\sqrt{6}h_2 f_0\alpha\rho^2\sin\theta_8 + 2\sqrt{6}\hat{h}_2\beta f_0\rho^2\sin\theta_8 + 12\sqrt{2}h_2\alpha a_0\rho\omega\sin\theta_8 + 12\sqrt{2}\hat{h}_2\beta a_0\rho\omega\sin\theta_8 + \\ &2\sqrt{6}\hat{h}_2\alpha f_0\omega^2\sin\theta_8 + 2\sqrt{6}\hat{h}_2\beta f_0\omega^2\sin\theta_8 + 8\sqrt{3}e_3 a_0^2\sigma\cos\theta_0\cos\theta_8\sin\theta_8 + \\ &24\sqrt{3}c_4\alpha\sigma a_0^2\cos\theta_0\sin^2\theta_8 + 12\sqrt{6}e_3 f_0 a_0^2\cos\theta_8\sin\theta_8^2 - 4\sqrt{3}e_3\sigma a_0^2\sin\theta_0\sin^2\theta_8 - \\ &12\sqrt{6}c_4\alpha f_0 a_0^2\sin^3\theta_8 - \sqrt{6}c_4 f_0^3\alpha\sin\left(3\theta_8\right) + 3\sqrt{3}e_3 f_0^2\sigma\sin\left(\theta_0 + 2\theta_8\right)\Big)\end{aligned}$$

Quadruple-meson couplings under RMF

$$\mathscr{L}_4 = -\frac{1}{48}\Big(9c_4 f_0^4 - 4h_2 f_0^2\rho^2 - 4\hat{h}_2 f_0^2\rho^2 - 12g_3\rho^4 + 24c_4 f_0^2\sigma^2 - 8h_2\rho^2\sigma^2 - 8\hat{h}_2\rho^2\sigma^2 + 6c_4\sigma^4 -$$

$$4h_2 f_0^2\omega^2 - 4\hat{h}_2 f_0^2\omega^2 - 72g_3\rho^2\omega^2 - 8h_2\sigma^2\omega^2 - 8\hat{h}_2\sigma^2\omega^2 - 12g_3\omega^4 + 24c_4 f_0^2\sigma^2\cos\left(2\theta_0\right) -$$

$$8h_2\rho^2\sigma^2\cos\left(2\theta_0\right) + 8\hat{h}_2\rho^2\sigma^2\cos\left(2\theta_0\right) + 8c_4\sigma^4\cos\left(2\theta_0\right) - 8h_2\sigma^2\omega^2\cos\left(2\theta_0\right) +$$

$$8\hat{h}_2\sigma^2\omega^2\cos\left(2\theta_0\right) + 2c_4\sigma^4\cos\left(4\theta_0\right) + 8\sqrt{2}\hat{h}_2 f_0\rho^2\sigma\cos\left(\theta_0 - \theta_8\right) +$$

$$8\sqrt{2}\hat{h}_2 f_0\sigma\omega^2\cos\left(\theta_0 - \theta_8\right) - 12c_4 f_0^2\sigma^2\cos\left[2\left(\theta_0 - \theta_8\right)\right] - 12c_4 f_0^4\cos\left[2\theta_8\right] + 4h_2 f_0^2\rho^2\cos\left(2\theta_8\right) +$$

$$4\hat{h}_2 f_0^2\rho^2\cos\theta_8 - 24c_4 f_0^2\sigma^2\cos\theta_8 + 4h_2 f_0^2\omega^2\cos\theta_8 + 4\hat{h}_2 f_0^2\omega^2\cos\theta_8 + 3c_4 f_0^4\cos\theta_8 -$$

$$8\sqrt{2}\hat{h}_2 f_0\rho^2\sigma\cos\left(\theta_0 + \theta_8\right) - 8\sqrt{2}\hat{h}_2 f_0\sigma\omega^2\cos\left(\theta_0 + \theta_8\right) - 12c_4 f_0^2\sigma^2\cos\left[2\left(\theta_0 + \theta_8\right)\right] +$$

$$4\sqrt{2}c_4 f_0^3\sigma\sin\left(\theta_0 - 3\theta_8\right) - 12\sqrt{2}c_4 f_0^3\sigma\sin\left(\theta_0 - \theta_8\right) - 8\sqrt{2}h_2 f_0\rho^2\sigma\sin[\theta 0 - \theta 8] -$$

$$8\sqrt{2}h_2 f_0\sigma\omega^2\sin\left(\theta_0 - \theta_8\right) + 32\sqrt{6}h_2 a_0\rho\sigma\omega\cos\theta_0\sin\theta_8 + 32\sqrt{6}\hat{h}_2 a_0\rho\sigma\omega\sin\theta_0\sin\theta_8 -$$

$$24h_2 a_0^2\rho^2\sin^2\theta_8 - 24\hat{h}_2 a_0^2\rho^2\sin^2\theta_8 - 32\sqrt{3}h_2 a_0 f_0\rho\omega\sin^2\theta_8 - 32\sqrt{3}\hat{h}_2 a_0 f_0\rho\omega\sin^2\theta_8 -$$

$$24h_2 a_0^2\omega^2\sin^2\theta_8 - 24\hat{h}_2 a_0^2\omega^2\sin^2\theta_8 + 96c_4 a_0^2\sigma^2\cos^2\theta_0\sin^2\theta_8 - 96\sqrt{2}c_4 a_0^2 f_0\sigma\cos\theta_0\sin\theta_8^3 +$$

$$24c_4 a_0^4\sin\theta_8^4 + 48c_4 a_0^2 f^2\sin\theta_8^4 + 12\sqrt{2}c_4 f_0^3\sigma\sin\left(\theta_0 + \theta_8\right) + 8\sqrt{2}h_2 f_0\rho^2\sigma\sin\left(\theta_0 + \theta_8\right) +$$

$$8\sqrt{2}h_2 f_0\sigma\omega^2\sin\left(\theta_0 + \theta_8\right) - 4\sqrt{2}c_4 f_0^3\sigma\sin\left(\theta_0 + 3\theta_8\right)\Big)$$

Numerical results

$$\mathscr{L}_3 = -m_N \left(-13.2a_0^2 f_0 + 4.41 f_0^3 + 1.84 f_0 \rho^2 + 9.72 a_0^2 \sigma + 9.72 f_0^2 \sigma - 4.06 \rho^2 \sigma + 1.96 \sigma^3 + 6.36 a_0 \rho \omega + 1.84 f_0 \omega^2 - 4.06 \omega^2 \sigma\right)$$

$$\mathscr{L}_4 = -13.9 a_0^4 - 27.8 a_0^2 f_0^2 - 13.9 f_0^4 + 116 a_0^2 \rho^2 - 38.6 f_0^2 \rho^2 + 158 \rho^4 + 90.9 a_0^2 f_0 \sigma - 30.3 f_0^3 \sigma + 98.7 f_0 \rho^2 \sigma - 74.4 a_0^2 \sigma^2 - 74.4 f_0^2 \sigma^2 - 61.2 \rho^2 \sigma^2 - 16.6 \sigma^4 - 267 a_0 f_0 \rho \omega + 342 a_0 \rho \sigma \omega - 116 a_0^2 \omega^2 - 38.6 f_0^2 \omega^2 + 946 \rho^2 \omega^2 + 98.7 f_0 \sigma \omega^2 - 61.2 \sigma^2 \omega^2 + 158 \omega^4$$

Parameter variation effects

	opt-a+	opt-a-	opt-b+	opt-b-	opt-c4+	opt-c4-	opt-e3+	opt-e3-
m_σ (MeV)	715	687	696	706	706	696	699	703
m_{a_0} (MeV)	974	956	969	960	973	956	961	968
m_ω (MeV)	789	770	777	782	779	779	779	779
σ 2-quark (%)	56.5	57.2	57.9	55.8	57.6	56.2	56.2	57.6
a_0 2-quark (%)	42.4	42.6	42.9	42.1	42.8	42.2	42.2	42.8

	opt-c+	opt-c-	opt-h2+	opt-h2-	opt-g3+	opt-g3-	opt-H2+	opt-H2-
m_ω (MeV)	779	779	784	774	780	780	778	781

Physical quantities

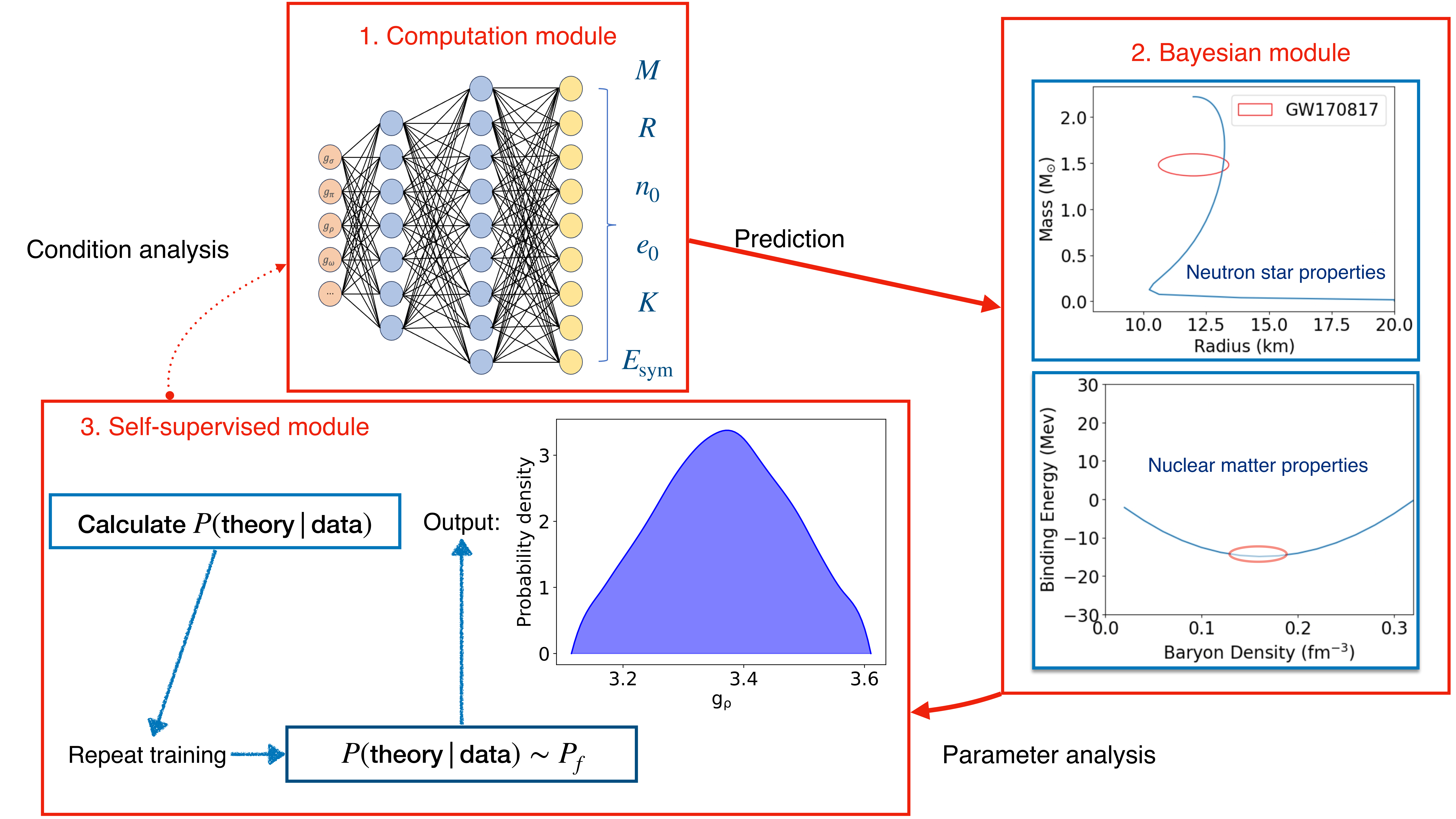
$$m_\rho = m_\sigma = \sqrt{2h_2\alpha^2 + 2\hat{h}_2\beta^2 + 2h_3\alpha^2 + 2\hat{h}_3\beta^2}$$

$$E_{\mathrm{sym}}(n) = \frac{1}{2}\frac{\partial^2 E(n,\alpha)}{\partial\alpha^2}\bigg|_{\alpha=0}$$

$$K_0 = 9n_0^2\frac{\partial^2 E(n,0)}{\partial n^2}\bigg|_{n=n_0}$$

$$J_0 = 27n_0^3\frac{\partial^3 E(n,0)}{\partial n^3}\bigg|_{n=n_0}$$

$$L_0 = 3n_0\frac{\partial E_{\mathrm{sym}}(n)}{\partial n}\bigg|_{n=n_0}$$



1. Computation module
g_σ
g_π
g_ρ
g_ω
...
M
R
n_0
e_0
K
E_{sym}
Prediction
2. Bayesian module
GW170817
Mass ($M_\odot$)
Radius (km)
Neutron star properties
Binding Energy (Mev)
Baryon Density (fm^{-3})
Nuclear matter properties
Condition analysis
3. Self-supervised module
Calculate P(theory | data)
Output:
Probability density
g_ρ
Repeat training
P(theory | data) $\sim P_f$
Parameter analysis