Some Effects in Magnetized QCD Matter within NJL Model

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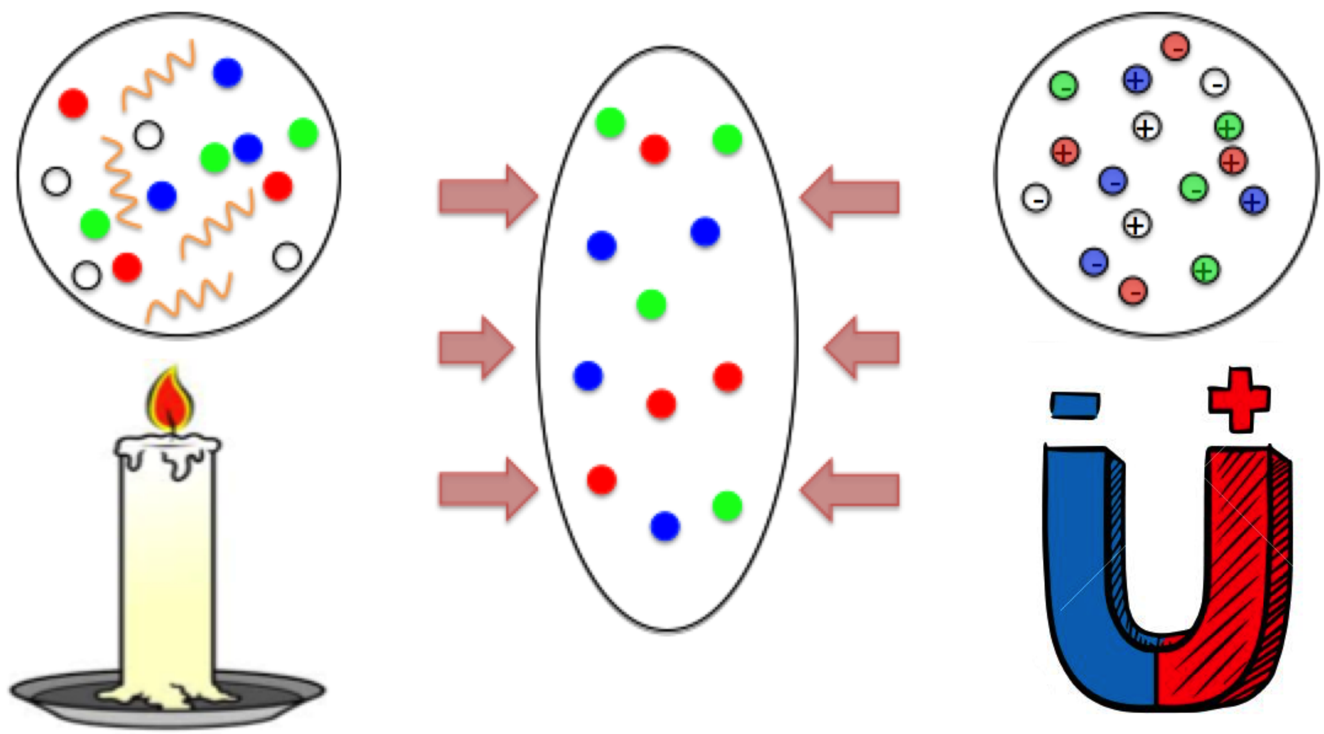
August 5, 2023 @DLUT, Dalian

OUTLINE

- Motivations
- (Inverse) Magnetic Catalysis
- Masses of pseudo-scalar mesons

WHY ELECTROMAGNETIC FIELDS

Heavy ion collisions create the strongest magnetic fields in the Laboratory.



Different excited freedoms at different environments

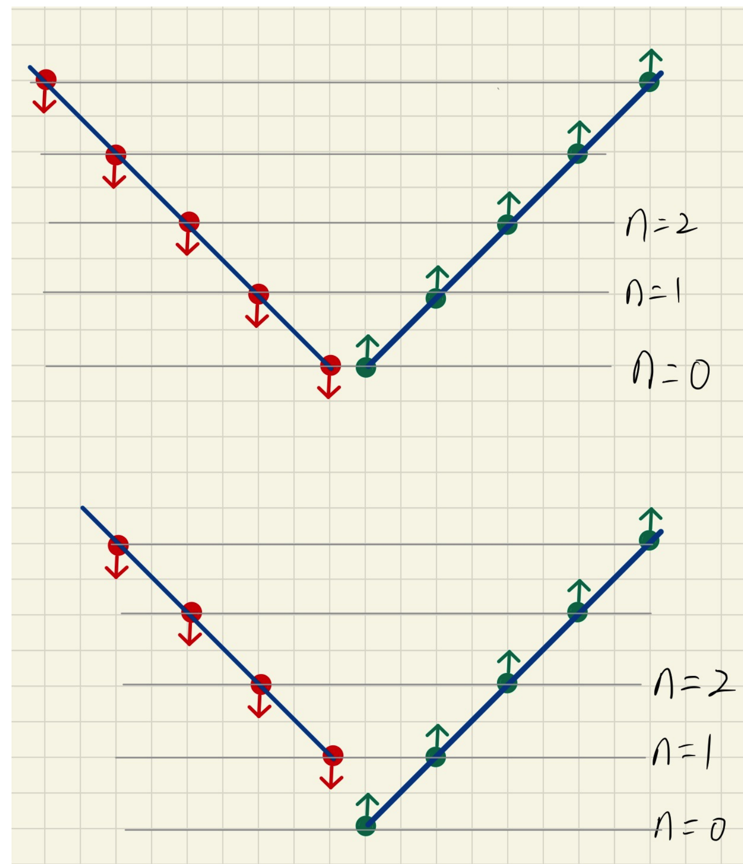
Lowest Landau Level

Free energy spectrum

$$E_n^{3+1}(k_3) = \pm\sqrt{(2n + 2s_3 + 1)|eB| + k_3^2 + m^2}$$

s_3 is the projection of the spin on the B field and $n = 0, 1, 2, \ldots$ is the orbital quantum number.

LLL approximation is valid in strong eB-field with $E_0^{3+1}(k_3) = \pm|k_3|$ for light particles.



☞ *state in one dimension & spin in one direction*

LLL-DOMINANT PHENOMENA

State of the art & preliminary works in strongly interacting matter under intensive magnetic field

DIMENSION REDUCTION

- QCD phase diagram — Gusynin et al.'96,'15
- first-principle calculations — Bali et al.'12,'13
- instanton in magnetic fields — Basar et al.'11,'12

PARITY-ODD

- chiral magnetic effect (transfer spin to momentum) — Fukushima et al.'08; Kharzeev'15
- chiral instability in magnestars and black holes — Ohnishi et al.'14; Gorbar et al.'22
- axion electrodynamics — Ferrer et al.'17,'18

MODEL OF NAMBU–JONA-LASINO

- low energy effective model with four-fermion contact interactions

$$\mathcal{L} = \bar{\psi}\left(\not{D} - m\right)\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}\, i\gamma_5\vec{\tau}\psi\right)^2\right]$$

- after the Hubbard Stratonovich transformation, this is equivalent to

$$\mathcal{L}_{eff} = \frac{\sigma^2 + \pi^2}{4G_S} + \ln\det\left(i\not{D} + m_0 - \sigma - i\gamma_5\pi\right)$$

- the following composite fields were introduced

$$\sigma \sim -G_S\left\langle\bar{\psi}\psi\right\rangle, \qquad \pi \sim -G_S\left\langle\bar{\psi}\, i\gamma_5\psi\right\rangle.$$

- at $B = 0$, the free energy:

$$\mathcal{F} = \frac{M^2}{2G_S} + \frac{1}{(4\pi)^2}\left[\frac{\Lambda^4}{2} - 2\Lambda^2M^2 + \frac{M^4}{2} + M^4\ln\frac{M^2}{\Lambda^2}\right]$$

MODIFICATION OF THE GAP EQUATION WITH eB

- gap equation in NJL model

$$M\left[\frac{4\pi^2}{G_S}-\Lambda^2+M^2\ln\frac{M^2}{\Lambda^2}\right]=0$$

- nontrivial solution for

$$G>G_{cr}=\frac{4\pi^2}{\Lambda^2}$$

- gap equation for nonzero B

$$\frac{4\pi^2}{G}-\Lambda^2+M^2\ln\frac{M^2}{\Lambda^2}-|2q_fB|\left[\zeta^{(1,0)}(0,x_f)+x_f-\frac{2x_f-1}{2}\ln x_f\right]=0$$

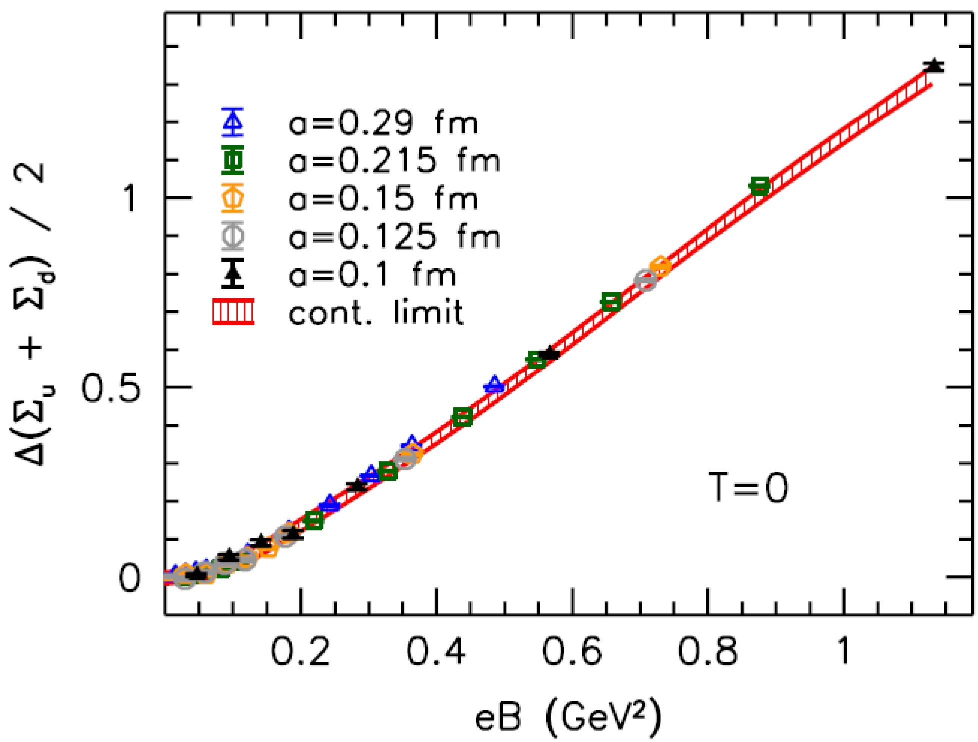
- solution exists for $G<G_{cr}$

$$M^2=\frac{|q_fB|}{\pi}\exp\left[-\frac{1}{|q_fB|}\left(\frac{4\pi^2}{G}-\Lambda^2\right)\right]$$

Note: $x_f=M_f^2/(2eB)$

MAGNETIC CATALYSIS

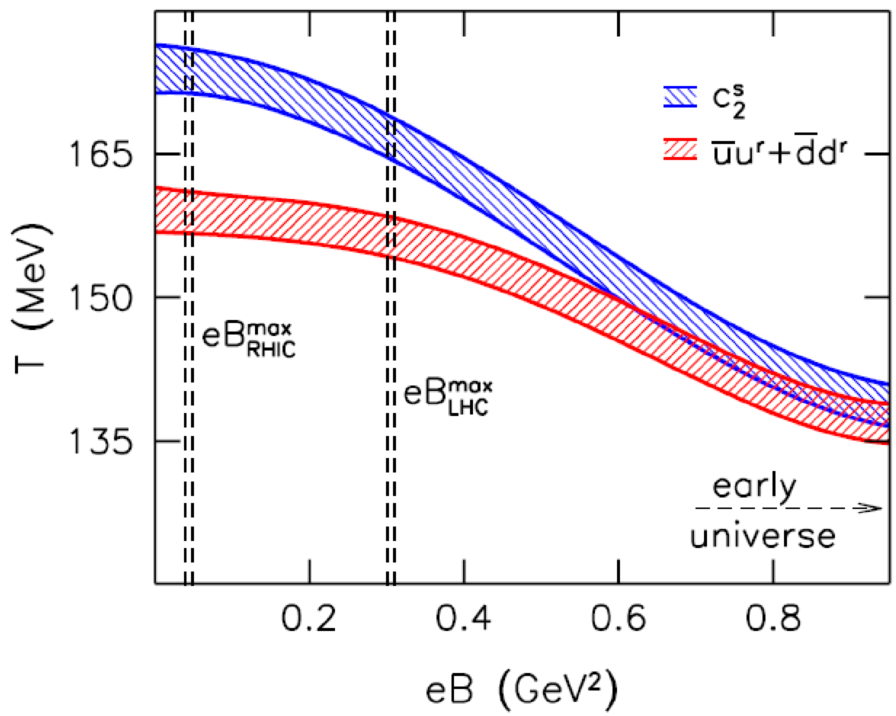
The infrared dynamics generates a linear-dependent quark mass even at the weakest attractive interactions Gusynin, Miransky, and Shovkovy, NPB 462(1996)249



Bali, Bruckmann et al., PRD 86(2012)071502(R)

PSEUDO-CRITICAL TEMPERATURE

Low energy effective models of QCD predict(ed) an increasing critical temperature $T_\chi(B)$, but



Bali, Bruckmann et al., JHEP 02(2012)044

INVERSE MAGNETIC CATALYSIS

- magnetic inhibition

  Fukushima and Hidaka, PRL 110(2013)031601

- mass gap in large N_c limit

  Kojo and Su, PLB 720(2013)192

- contribution from sea quarks

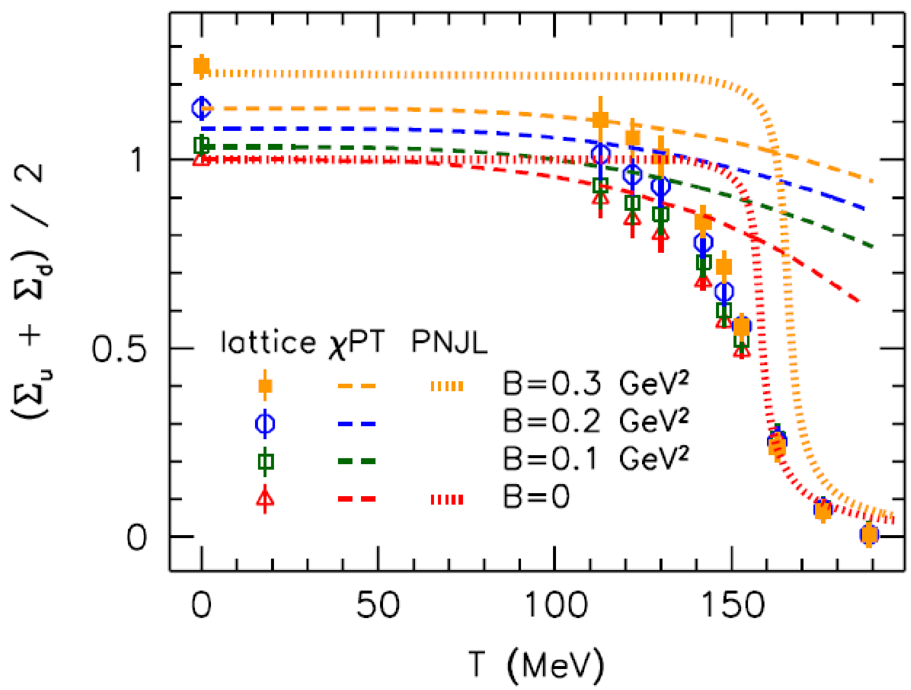
  Bruckmann, Endrodi and Kovacs, JHEP 04(2013)112

- chirality imbalance

  Yu, Liu and Huang, PRD 90(2014)074009

THERMAL EFFECT

χPT fit up to $T = 100$ MeV:



Chiral condensates increase as eB grows at $T \leq 120$ MeV

T_{pc} shifts to lower T as eB grows

Bali, Bruckmann et al., PRD 86(2012)071502(R)

CHIRAL CHEMICAL POTENTIAL UNDER eB

time evolution of n_5: Khlebnikov and Shaposhnikov, NPB 308(1988)885

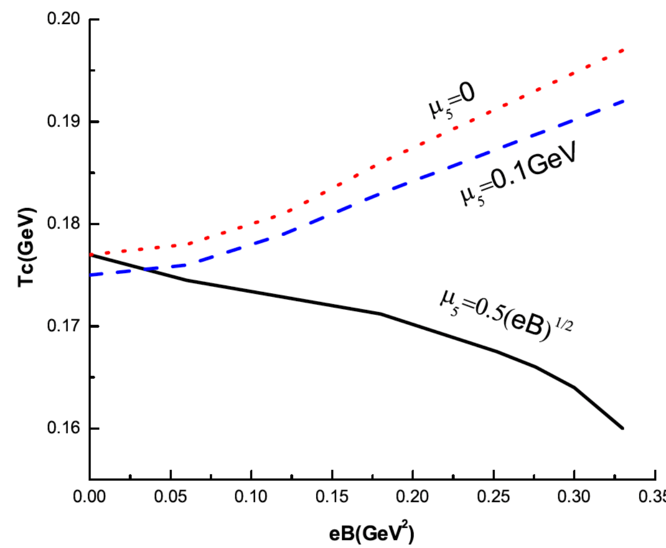
$$\frac{\partial n_5}{\partial t} = (4N_f)^2 \frac{\Gamma_{ss}}{T} \mu_5$$

sphalerons diffusion rate: Basar and Kharzeev, PRD 85(2012)086012

$$\Gamma_{ss}(eB, T) = c_1 eBT^2 + c_2 T^4$$

chiral chemical potential:

$$\mu_5 \propto \sqrt{eB}$$



JC, Chu and Huang, PRD 88(2013)054009

NAMBU-GOLDSTONE BOSON UNDER eB

- There should be massless NG bosons created due to the number of broken-symmetry generators.
- Magnetic field modifies the $SU(2)$ isospin symmetry; global chiral symmetry $U_V(1) \times U_A(1)$ breaks down to $U_{L+R}(1)$.
- Only the neutral pion is a Goldstone mode under B-field.
- Another light pseudo-NG boson, the axion may appear in a very strong magnetic field due to the $U_A(1)$.

GOLDSTONE'S THEOREM IN 2-D

There are no Goldstone Bosons in Two Dimensions*

Sidney Coleman**

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey, USA

Received February 1, 1973

This is because if such a spontaneous symmetry breaking occurred, then the corresponding Goldstone bosons, being massless, would have an infrared divergent correlation function.

In turn, the fluctuations are strong enough to destroy the spontaneous symmetry breaking.

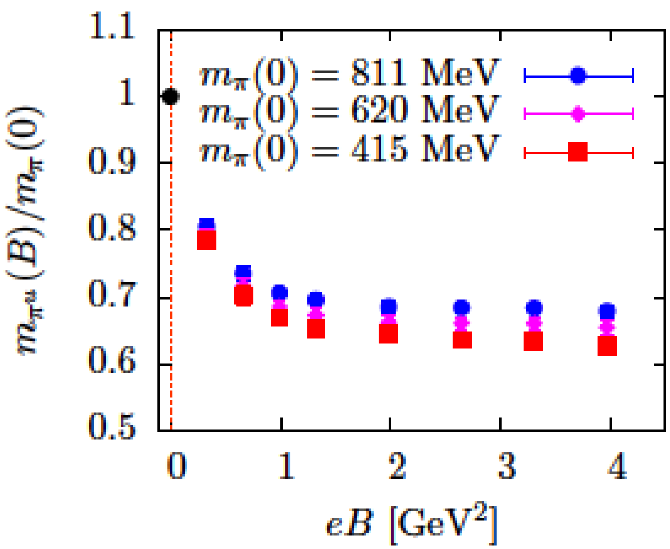
Mermin-Wagner-Coleman theorem

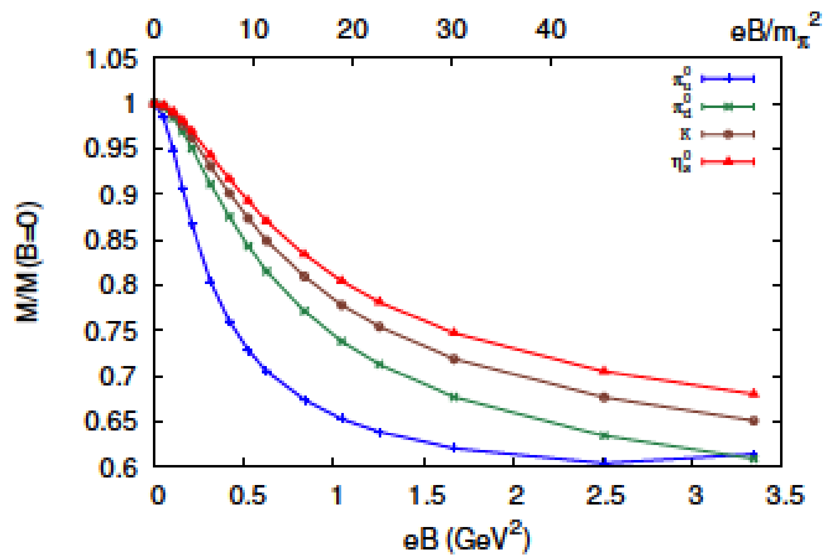
However, the MWC theorem is not applicable to the present problem. The central point is that the condensate $\langle 0|\bar{\psi}\psi|0\rangle$ and the NG modes are **neutral** in this problem and the dimensional reduction in a magnetic field does not affect the dynamics of the center of mass of **neutral** excitations. Indeed, the dimensional reduction $D \to D-2$ in the fermion propagator, in the infrared region, reflects the fact that the motion of **charged** particles is restricted in the directions perpendicular to the magnetic field. Since there is no such restriction for the motion of the center of mass of neutral excitations, their propagators have D–dimensional form in the infrared region (since the structure of excitations is irrelevant at

Gusynin, Miransky and Shovkovy, NPB 462(1996)249

LATTICE RESULTS OF NEUTRAL PION

Neutral pion monotonouslly decrease as the magnetic field grows and then saturate at a nonzero value



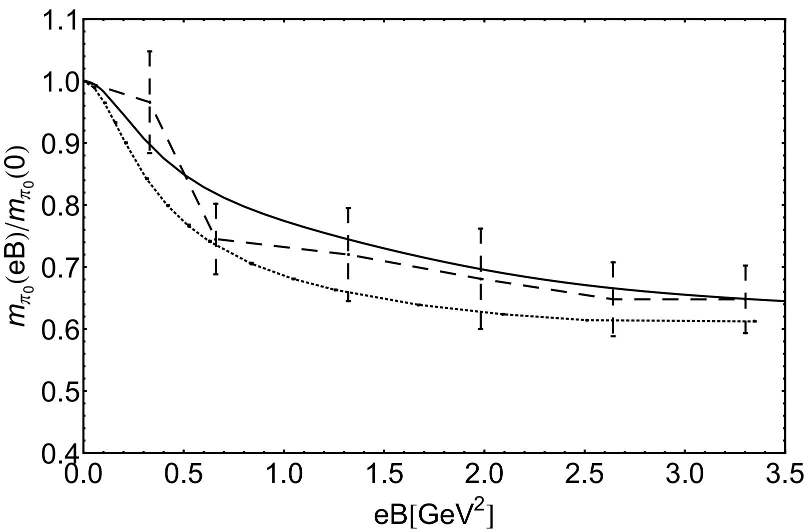


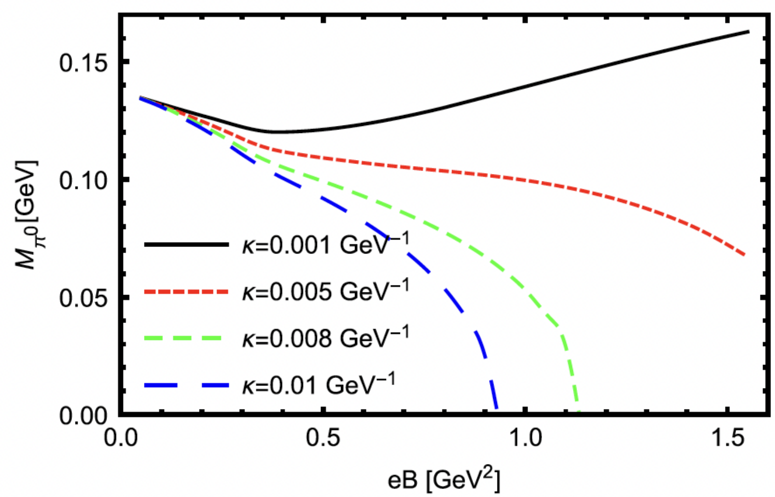
left: Bali, Brandt et al., PRD 97(2018)034505
right: Ding, Li et al., PRD 104(2021)014505

LET'S SPIN

Key observation: taking into account the spin effect

p-wave   anomalous magnetic momentum

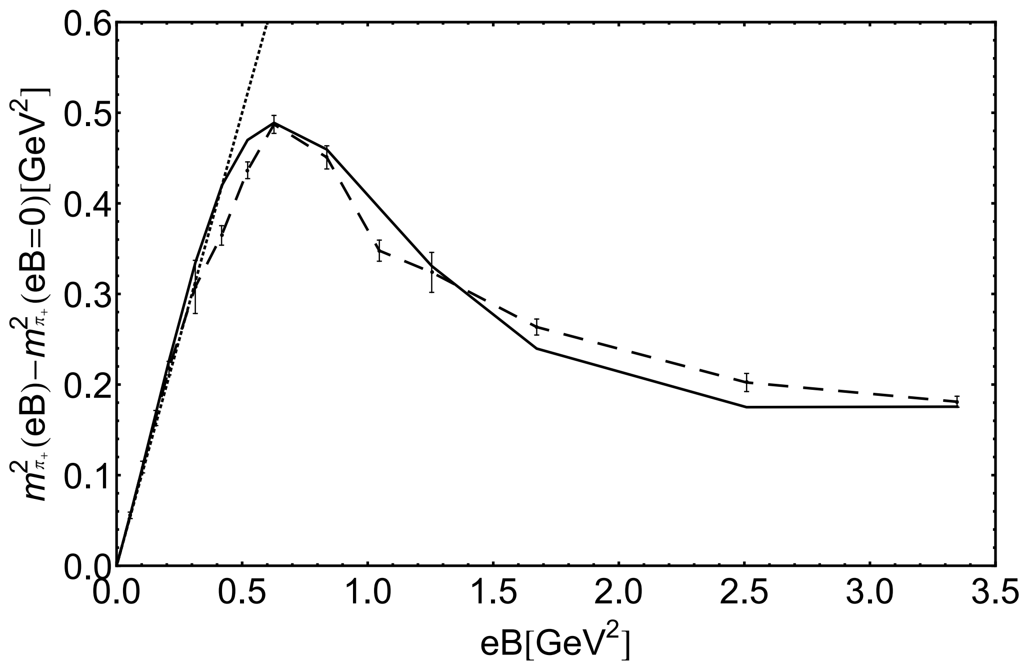




left: Xing, JC, Chang and Liu, PRD 105(2022)114003
right: Xu, JC and Huang, PRD 103(2021)076015

p-WAVE COMPONENT IN CHARGED PIONS

tune the strength of p-wave form in Bethe-Salpeter amplitude



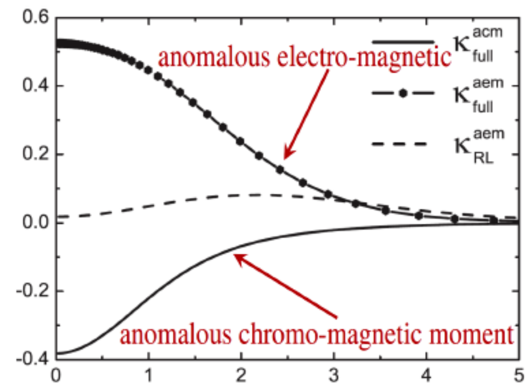
solid line: Xing, JC, Chang and Liu, PRD 105(2022)114003

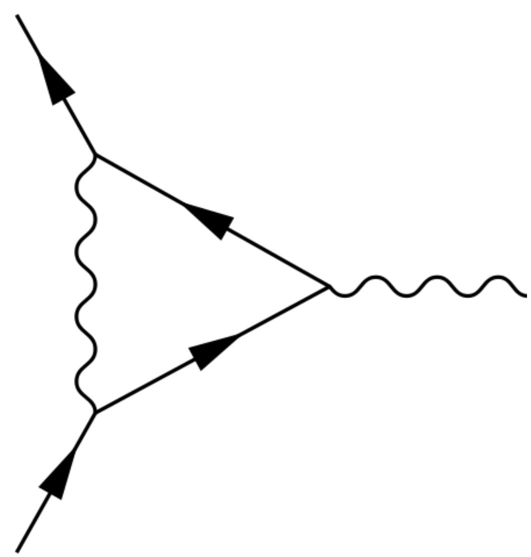
dashed line: Ding, Li et al., PRD 105(2022)034514

dotted line: point-like particle

Anomalous Magnetic Moment: source and sign

- Flavor blind $\sim \tau_0$, coming form the compensation of color interaction Chang, Liu and Roberts, PRL 106(2011)072001
- Charge dependent $\sim q_f$, due to the fluctuations of QED
- Or simply flavor dependent $\sim \text{sign}\,(q_f) = \tau_3$





ANISOTROPIC FOUR-FERMION INTERACTIONS

one-gluon exchange based four-fermion interactions

$$\mathcal{L}_{int} = g_{\shortparallel}^2 \left(\bar{\psi}\gamma_\mu^{\shortparallel}\psi\right)^2 + g_{\perp}^2 \left(\bar{\psi}\gamma_\mu^{\perp}\psi\right)^2 .$$

apply the anisotropic Fierz identities

$$\begin{aligned}
\left(\gamma_\mu^{\shortparallel}\right)_{il}\left(\gamma_{\shortparallel}^{\mu}\right)_{jk} &= \frac{1}{2}(1)_{il}(1)_{jk} + \frac{1}{2}(i\gamma_5)_{il}(i\gamma_5)_{jk} \\
&\quad + \frac{1}{4}\left(\sigma_{\perp}^{\mu\nu}\right)_{il}\left(\sigma_{\mu\nu}^{\perp}\right)_{jk} - \frac{1}{2}\left(\sigma_{\shortparallel}^{03}\right)_{il}\left(\sigma_{03}^{\shortparallel}\right)_{jk} + \dots, \\
\left(\gamma_\mu^{\perp}\right)_{il}\left(\gamma_{\perp}^{\mu}\right)_{jk} &= \frac{1}{2}(1)_{il}(1)_{jk} + \frac{1}{2}(i\gamma_5)_{il}(i\gamma_5)_{jk} \\
&\quad - \frac{1}{4}\left(\sigma_{\perp}^{\mu\nu}\right)_{il}\left(\sigma_{\mu\nu}^{\perp}\right)_{jk} + \frac{1}{2}\left(\sigma_{\shortparallel}^{03}\right)_{il}\left(\sigma_{03}^{\shortparallel}\right)_{jk} + \dots
\end{aligned}$$

differing $g_{\shortparallel,\perp}$ will manifest themselves by considering four-point coupling in tensor channel

Ferrer, Incera et al., PRD 89(2014)085034

DYNAMICAL AMM THROUGH THE TENSOR CHANNELS

AMM dynamically generated like dynamical mass

$$\mathcal{L}_{\rm int} = G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}\, i\gamma_5\vec{\tau}\psi\right)^2\right] + G_T\left[\left(\bar{\psi}\sigma^{12}\tau_a\psi\right)^2 + \left(\bar{\psi}\, i\gamma_5\sigma^{12}\psi\right)^2\right]$$

where $G_T \leq G_S$ since $G_S \sim g_{\shortparallel}^2 + g_{\perp}^2$ and $G_S \sim g_{\shortparallel}^2 - g_{\perp}^2$. The transverse σ^{12} index is chosen w.r.t. the magnetic field pointing in the z-direction. Note: $a = 0, 3$ for τ_a.

$$M \sim -G_S\left\langle\bar{\psi}\psi\right\rangle, \qquad \kappa \sim -G_T\left\langle\bar{\psi}\sigma^{12}\tau_a\psi\right\rangle.$$

The Dirac propagator is in the form of

$$\tilde{G}(q_f,k) = \exp\left[-\frac{\mathbf{k}_\perp^2}{|q_f|B}\right]\sum_{\pm}\sum_{n=0}^{\infty}(-1)^n\frac{\not{D}_n(q_fB,k)\,\Lambda_\pm}{k_{\shortparallel}^2 - 2n|q_f|B - M^2 + \kappa^2 \pm 2|\kappa k_{\shortparallel}|},$$

where $\Lambda_\pm = \frac{1}{2} \pm \frac{\gamma^3\gamma^5 k_0 - \gamma^0\gamma^5 k_3}{2|k_{\shortparallel}|}\mathrm{sign}(\kappa)$ and

$$\not{D}_n(q_f,k) = \left(\not{k}_{\shortparallel} + M + \kappa\left(q_f\right)\sigma\hat{F}\right)\left[P_- L_n - P_+ L_{n-1}\right] + 4\not{k}_\perp L_{n-1}^1.$$

IR-ENHANCED VEVs

The applied mean-field approximation is recognizing by the gap equations:

$$\frac{M-m}{2\,iG_S} = \text{Tr}\,G;$$
$$\frac{\kappa}{2\,iG_T} = \text{Tr}\left[\sigma^{12}G\right];$$

$$\text{Tr}\,G(k) = I_1 + I_2, \qquad \text{sign}(\kappa)\,\text{Tr}\left[\sigma^{12}G(k)\right] = I_1 + I_3.$$

Solution exists while $\text{sign}(\kappa) = -\text{sign}(q_f)$ and $\tau_a = -\tau_3$

$$I_1 = N_c \sum_{q_f} \frac{|q_f|B}{8\pi^3} \int \frac{d^2k_{\shortparallel}}{|k_{\shortparallel}| - M + \kappa} \sim (M-\kappa) \int \frac{d^2k_{\shortparallel}}{k_{\shortparallel}^2 - (M-\kappa)^2};$$
$$I_2 = N_c \sum_{q_f} \frac{|q_f|B}{8\pi^3} \sum_{\pm} \sum_{n=1}^{\infty} \int d^2k_{\shortparallel} \frac{2M}{(|k_{\shortparallel}| \pm \kappa)^2 - M_n^2};$$
$$I_3 = N_c \sum_{q_f} \frac{|q_f|B}{8\pi^3} \sum_{\pm} \sum_{n=1}^{\infty} \int d^2k_{\shortparallel} \frac{2(\kappa \pm |k_{\shortparallel}|)}{(|k_{\shortparallel}| \pm \kappa)^2 - M_n^2}.$$

JC and Yu-Xin Liu, PRD 107(2023)074038

REVISITED DIMENSION REDUCTION IN 2-POINT CORRELATION

Generalized polarization of the neutral pion

$$\frac{1}{i}\Pi^{AB} = -N_c \sum_{q_f} \mathrm{tr}\left[iG(p)\, i\gamma_5 \Gamma^A\, iG(q)\, i\gamma_5 \Gamma^B\right].$$

where $\Gamma^{(A,B)} = \left(I_4, \sigma^{12}\right)$, respectively.

$$\begin{aligned}
\frac{1}{i}\Pi^{SS} &\sim \int d^2k_{\shortparallel} \frac{1}{2\left|k_{\shortparallel} + \frac{m_\pi}{2}\right|\left(\left|k_{\shortparallel} + \frac{m_\pi}{2}\right| - M + \kappa\right)} + (m_\pi \to -m_\pi) + \ldots \\
&\sim \int \frac{d^2k_{\shortparallel}}{k_{\shortparallel}^2 - (M-\kappa)^2} + \ldots \\
&= \frac{I_1}{M-\kappa} + \frac{I_2}{M} - m_\pi^2 \langle J \rangle_0 - m_\pi^2 \langle K \rangle_n
\end{aligned}$$

JC and Yu-Xin Liu, PRD 107(2023)074038

POSSIBLE PION SUPERFLUID

In sum, the loop amplitude changes to

$$\begin{pmatrix} 1-2g\Pi^{SS} & -2g\Pi^{ST} \\ -2g\Pi^{TS} & 1-2g\Pi^{TT} \end{pmatrix} = \mathbf{A} + m_\pi^2 \mathbf{B}$$

one gets the corresponding pole of meson as

$$\begin{aligned} m_{\tilde{\pi}}^2 &= \frac{m}{-2\,igM\tilde{J}} + \frac{m+\kappa+igI_1}{M}\frac{I_1}{M\tilde{J}} + \mathcal{O}(\alpha^1) \\ m_{\tilde{\pi}}^2 &= \frac{1}{-ig\,(2\alpha-\beta)\,\tilde{J}} + \mathcal{O}(\alpha^0) \end{aligned}$$

☞ $\kappa_{\rm cr} \simeq \frac{mM}{2\,igI_1} - igI_1 - m$

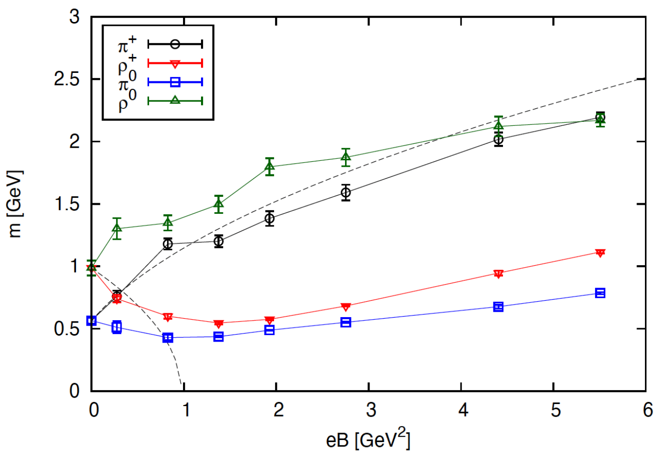
JC and Yu-Xin Liu, PRD 107(2023)074038

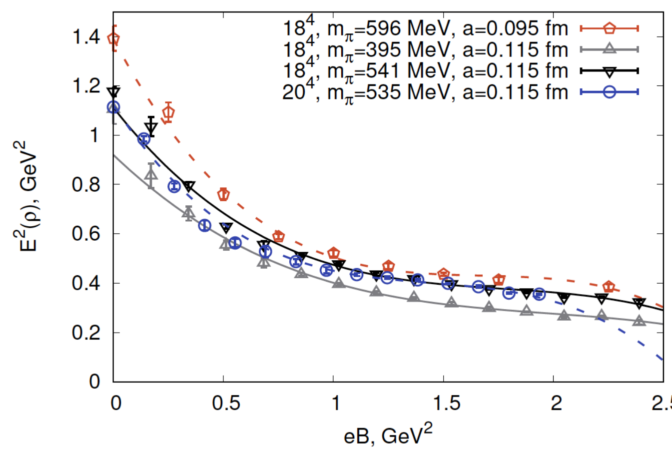
OUTLOOK: MAGNETIZED ρ

The modified energy of charged spin-1 particle:

$$E_\rho^2(eB; s_z = 0, \pm 1) = p_z^2 + (2n+1-gs_z)eB + m^2 \quad g = 2 \text{ for point particles}$$

✈ Lattice simulation shown that the polarized ρ-meson mass is decreased by the magnetic fields, and then saturated

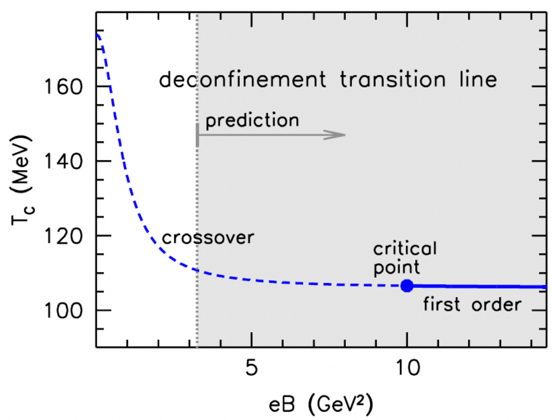


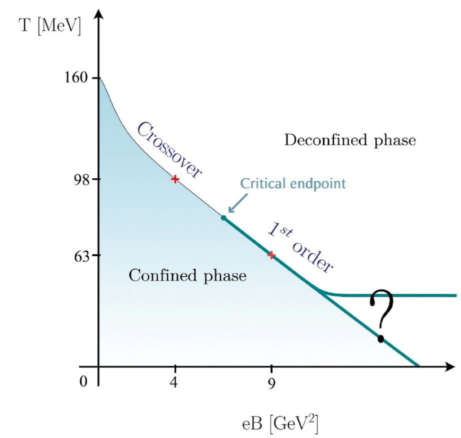


left: Hidaka and Yamamoto, PRD 87(2013)094502
right: D'Elia, Maio et al., PoS ICHEP(2022)414

OUTLOOK: FIRST-ORDER DECONFINEMENT

- first ever lattice evidence for first-order phase transition for QCD at physical masses and physical parameters
- Model has to be implemented for very large B_c





left: Endrodi, JHEP 07(2015)173
right: D'Elia, Maio et al., PRD 105(2022)034511

Thank You for Your Attention!