

t̄H @ NNLO

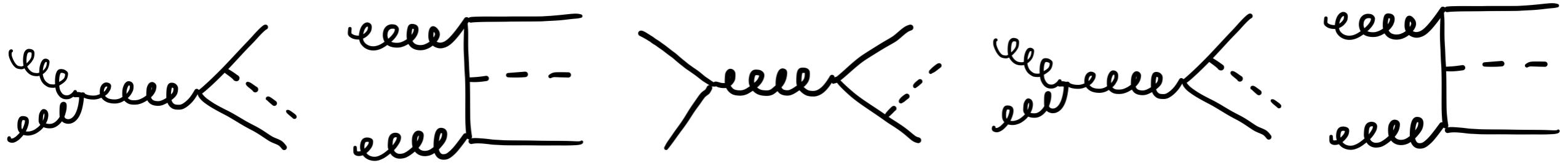


Simone Devoto



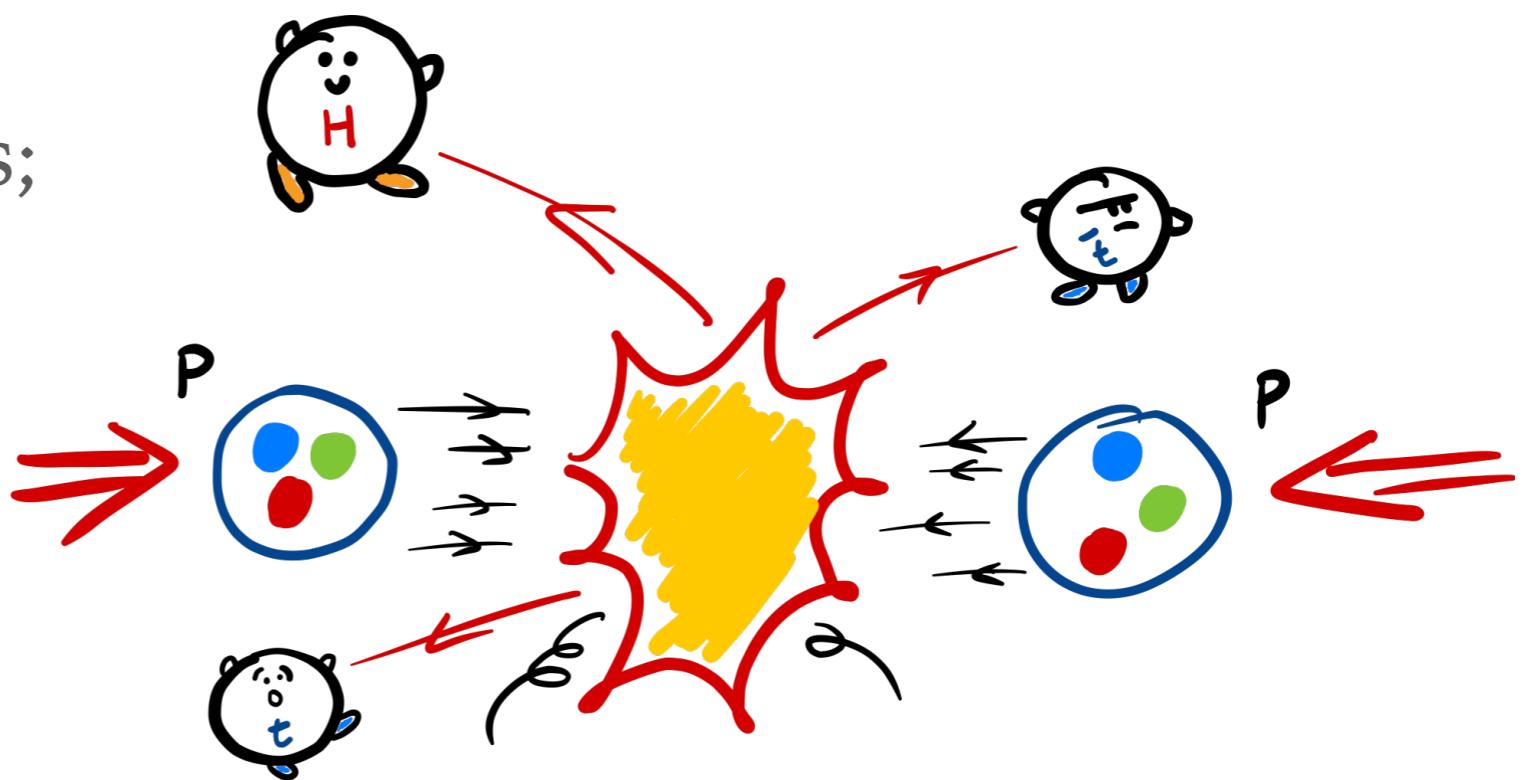
European Research Council
Established by the European Commission

In collaboration with:
S. Catani, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini



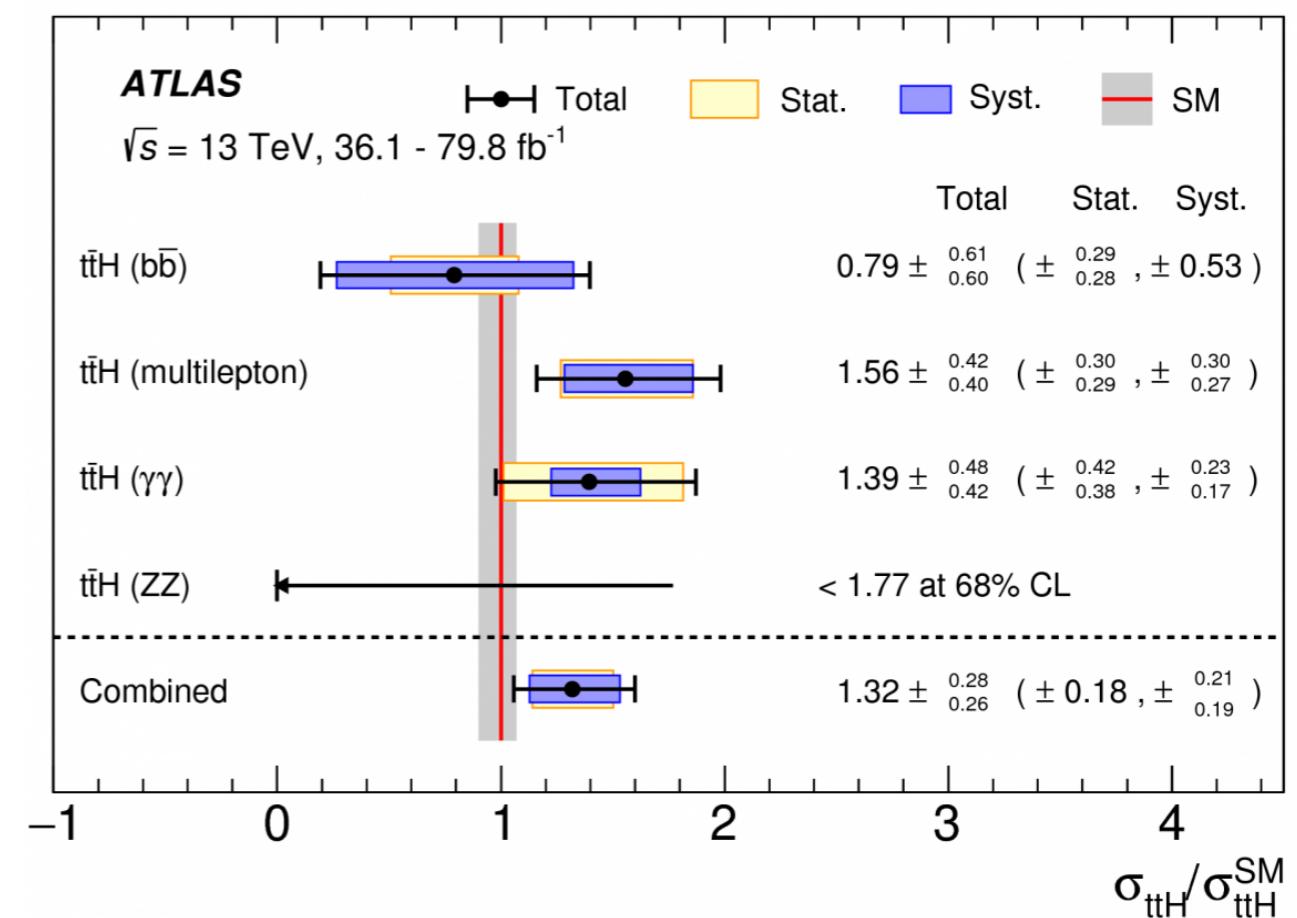
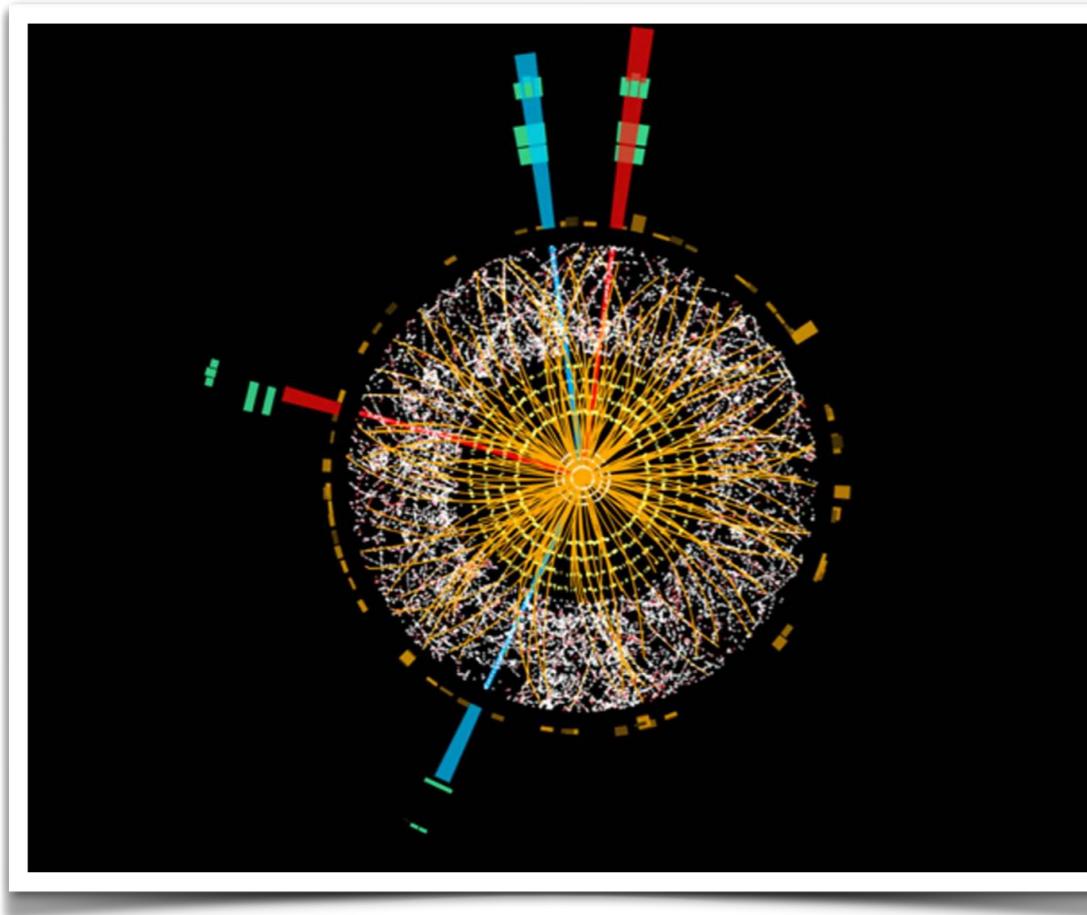
CONTENTS

- **Introduction;**
- **$t\bar{t}H$ @ NNLO, theory bottlenecks:**
 - subtraction;
 - two-loop amplitudes;
- **Results.**



INTRODUCTION

- The **discovery of the Higgs boson** in 2012 confirmed one of the most glaring predictions of the Standard Model.
- The **study of the Higgs boson** properties is one of the priorities of LHC.
- Special role played by the **top quark**: strong coupling because of the top mass!
- $t\bar{t}H$ production allows direct measurement of the **top-quark Yukawa coupling!** (possible window on new physics scenarios...)



STATUS OVERVIEW

THEORY

► NLO QCD:

[W. Beenakker, S. Dittmaier, M. Krämer, B. Plumper, M. Spira, and P. Zerwas; 0107081, 0211352], [L. Reina and S. Dawson; 0107101], [L. Reina, S. Dawson, and D. Wackerlo; 0109066], [S. Dawson, L. Orr, L. Reina, and D. Wackerlo; 0211438], [S. Dawson, C. Jackson, L. Orr, L. Reina, and D. Wackerlo; 0305087], [A. Denner and R. Feger, 1506.07448];

► NLO EW:

[S. Frixione, V. Hirschi, D. Pagani, H. Shao, and M. Zaro; 1407.0823, 1504.03446], [Y. Zhang, W.-G. Ma, R.-Y. Zhang, C. Chen, and L. Guo; 1407.1110];

► NLO QCD + EW:

[A. Denner, JN. Lang, M. Pellen, and S. Uccirati; 1612.07138];

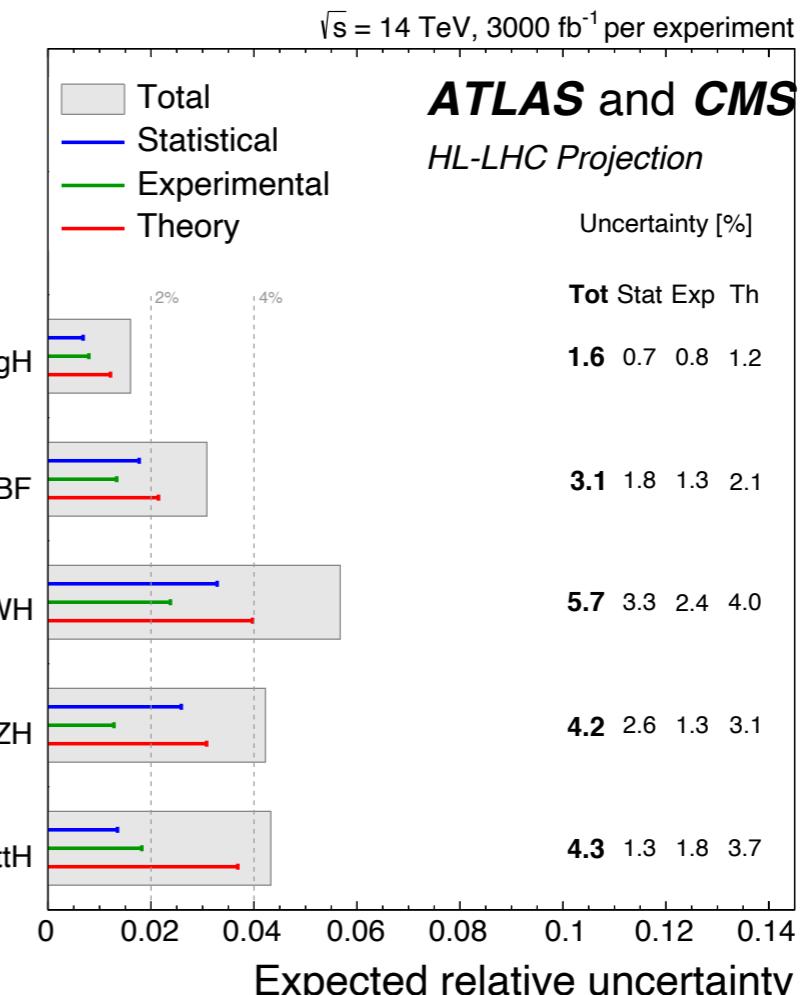
► Resummation of soft gluons:

[A. Kulesza, L. Motyka, T. Stebel, and V. Theeuwes; 1509.02780, 1704.03363], [A. Broggio, A. Ferroglio, B. D. Pecjak, A. Signer, and L. L. Yang; 1510.01914], [A. Broggio, A. Ferroglio, B. D. Pecjak, and L. L. Yang; arXiv:1611.00049], [A. Broggio, A. Ferroglio, R. Frederix, D. Pagani, B. D. Pecjak, and I. Tsinikos; 1907.04343], [W.-L. Ju and L. L. Yang; 1904.08744], [A. Kulesza, L. Motyka, D. Schwartländer, T. Stebel, and V. Theeuwes; 2001.03031]

► Theoretical uncertainties $\mathcal{O}(10\%)$

EXPERIMENTS

- ATLAS collaboration: [1806.00425];
- CMS collaboration: [1804.02610].

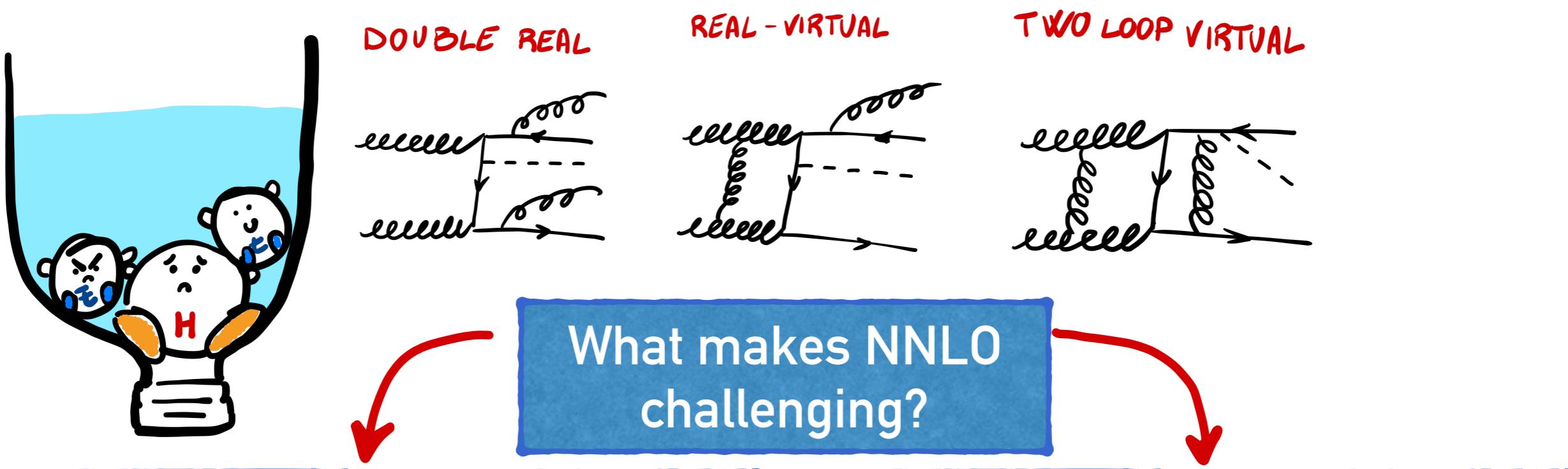


[M.Cepeda et al.: arXiv 1902.00134]

- Current experimental uncertainties $\mathcal{O}(20\%)$
- Expected at the end of HL-LHC $\mathcal{O}(2\%)$

THEORY BOTTLENECKS

- First steps: computation of NNLO QCD off-diagonal channels [S. Catani, I. Fabre, M. Grazzini, S. Kallweit : arXiv:2102.03256]
- Full NNLO corrections: [S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, C. Savoini: [2210.07846](#)] THIS TALK!



Subtraction procedure

- We use **q_T -subtraction**;
- We **generalised** the method to this class of processes.

Two loop amplitudes

- Not known: current frontier!
- We develop a **soft Higgs approximation**.

q_T SUBTRACTION FORMALISM

[S. Catani, M. Grazzini Phys.Rev.Lett. 98 (2007)]

$$d\sigma_{NNLO}^F = d\sigma_{NNLO}^F \Big|_{q_T=0} + d\sigma_{NNLO}^F \Big|_{q_T \neq 0}$$

$$d\sigma_{NLO}^{F+jets}$$

$$d\sigma_{NNLO}^F = \mathcal{H}_{NNLO}^F \otimes d\sigma_{LO}^F + \left[d\sigma_{NLO}^{F+jets} - d\sigma_{NLO}^{CT} \right]$$

HARD COLLINEAR COEFFICIENT

Contains information on virtual corrections to the process.

$$\mathcal{H}_{NNLO}^F = H^{(2)} \delta(1 - z_1) \delta(1 - z_2) + \delta \mathcal{H}^{(2)}$$

Contains the genuine **2-loop contribution**:

$$H^{(2)} = \frac{2 \operatorname{Re}(\mathcal{M}^{(2)}(\mu_{IR}, \mu_R) \mathcal{M}^{(0)})}{|\mathcal{M}^{(0)}|^2}$$

- APPROXIMATED -

Includes:

- one-loop squared contribution;
- **soft parton contribution**.

- EXACT -

SOFT PARTON CONTRIBUTION

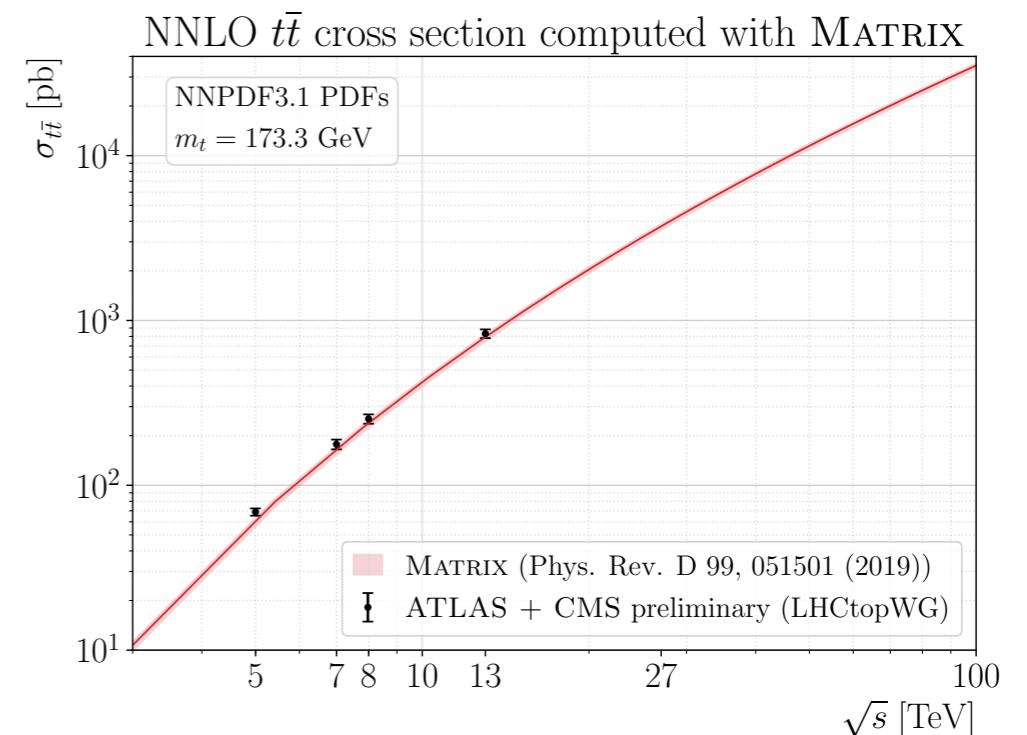
[S. Catani, SD, M. Grazzini , J. Mazzitelli: [2301.11786](#)
SD, J. Mazzitelli, In preparation]

The soft contribution from a massive final state was a key ingredient to extend q_T subtraction to a massive coloured final state.

Soft contributions to heavy-quark (Q) production

- Applied to top pair and bottom pair production:
[S. Catani, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, H. Sargsyan:
2019, 2020];
- Mostly **analytic** expressions;
- Assumption of $Q\bar{Q}$ **back-to-back** at LO.

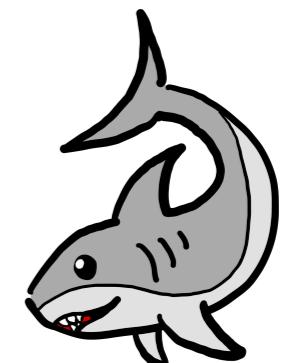
[S. Catani, SD, M. Grazzini ,
J. Mazzitelli: [2301.11786](#)]



NEW: generalisation to $Q\bar{Q}F$ kinematics

- removed the back-to-back assumption;
- Extra contribution computed **numerically**;
- On-the-fly numerical integration implemented in a **library**: **SHARK**
Soft function for **H**eavy quark production in **A**Rbitrary **K**inematics

[SD, J. Mazzitelli, IN PREPARATION]



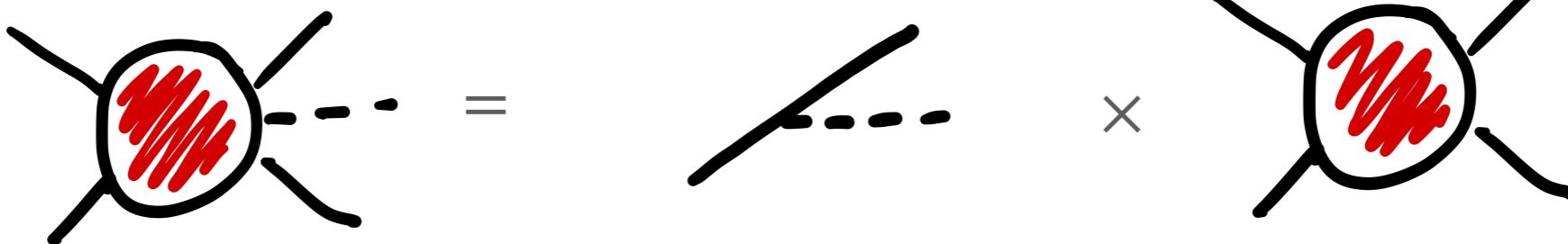
2-LOOP CONTRIBUTION

[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli, C. Savoini: [2210.07846](#)]

Process: $c(p_1) + \bar{c}(p_2) \rightarrow t(p_3) + \bar{t}(p_4) + H(k)$

Soft Higgs approximation: $k \rightarrow 0, m_H \ll m_t$

$$\lim_{k \rightarrow 0} \mathcal{M}_{t\bar{t}H}(\{p_i\}, k) = F(\alpha_S(\mu_R); m_t/\mu_R) \frac{m_t}{v} \sum_{i=3,4} \frac{m_t}{p_i \cdot k} \mathcal{M}_{t\bar{t}}(\{p_i\})$$



- The formula captures the leading behaviour in the **soft limit** $k \rightarrow 0$: the emission from highly **off-shell top propagators** is not captured.
- Amplitudes for the process $c\bar{c} \rightarrow t\bar{t}$ **available** in the literature [Czakon (2008); Barnreuther et al. (2013)]
- A similar approximation has been recently successfully applied also for the computation of **$t\bar{t}W$ production** [L. Buonocore, SD, M. Grazzini, S. Kallweit, J. Mazzitelli, L. Rottoli, C. Savoini: [2306.16311](#)]

2-LOOP CONTRIBUTION

[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli, C. Savoini: [2210.07846](#)]

subtraction scale μ_{IR}
(we use $\mu_{IR} = Q_{t\bar{t}H}$)

We apply the approximation both on the numerator
and denominator of $H_{t\bar{t}H}^{(2)}$: effectively a reweighting.

$$\mathcal{H}_{NNLO}^{t\bar{t}H} = H_{t\bar{t}H}^{(2)} \delta(1 - z_1) \delta(1 - z_2) + \delta \mathcal{H}_{t\bar{t}H}^{(2)}$$

$$H_{t\bar{t}H}^{(2)} = \frac{2 \operatorname{Re}(\mathcal{M}_{t\bar{t}H}^{(2)}(\mu_{IR}, \mu_R) \mathcal{M}_{t\bar{t}H}^{(0)})}{|\mathcal{M}_{t\bar{t}H}^{(0)}|_{soft}^2}$$

$$\lim_{k \rightarrow 0} \mathcal{M}_{t\bar{t}H}(\{p_i\}, k) = F(\alpha_S(\mu_R); m_t/\mu_R) \frac{m_t}{v} \sum_{i=3,4} \frac{m_t}{p_i \cdot k} \mathcal{M}_{t\bar{t}}(\{p_i\})$$

- To use the approximation, we need a **recoil prescription** to map the $t\bar{t}H$ kinematics into a $t\bar{t}$ kinematics ($Q_{t\bar{t}H} \rightarrow Q_{t\bar{t}}$);
- We use the **q_T recoil prescription**:
 - We reabsorb the Higgs momentum equally in the initial-state parton momenta;
 - We leave unchanged the top and anti-top momenta.

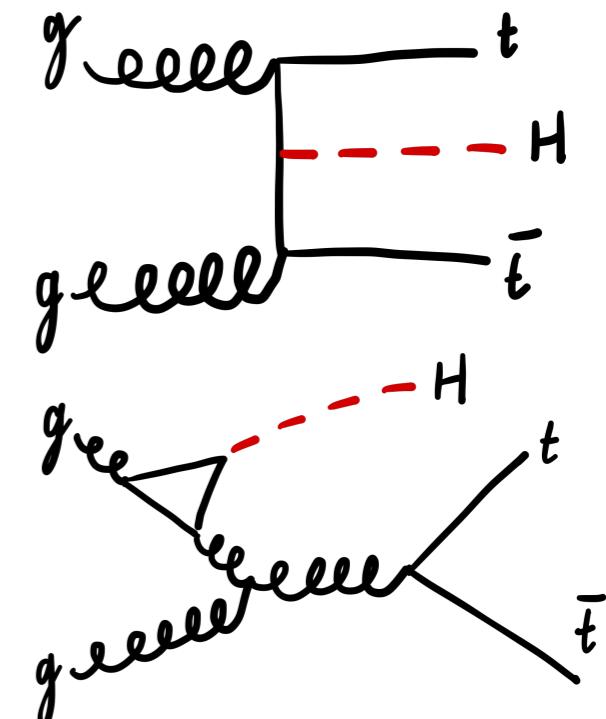
TESTING THE APPROXIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli, C. Savoini: [2210.07846](#)]

To validate our procedure: test the approximation at NLO!

$\Delta\sigma_{\text{NLO},H} [\text{fb}]$	13 TeV		100 TeV	
	gg	$q\bar{q}$	gg	$q\bar{q}$
Exact	88.62	7.826	8205	217.0
Soft Approximation	61.92	7.413	5612	206.0
Difference	30.1%	5.27%	31.6%	5.06 %

- Deviation w.r.t. exact computation is about **30%** for the **gg channel** and **5%** for the **$q\bar{q}$ channel**;
- Deviation **independent** of kinematic variables;
- Better agreement** for $q\bar{q}$ channel can be explained by the presence, both at LO and NLO, of diagrams where a **Higgs boson is radiated from a virtual top** only present in the gg channel.



UNCERTAINTIES ESTIMATION

[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli, C. Savoini: [2210.07846](#)]

How to estimate the NNLO uncertainties?

- We use the **deviation from the exact results at NLO** as a **lower bound** on the NNLO uncertainty;
- We multiply by a **tolerance factor of 3**;
- We combined **linearly** the uncertainty for the gg and $q\bar{q}$ channel;

How to test the NNLO uncertainties?

- Check the effect of using **different recoil prescription**;
- Check the effect of using a **different subtraction scales** $\mu_{IR} \rightarrow 2 \mu_{IR}$,
 $\mu_{IR} \rightarrow 1/2 \mu_{IR}$.

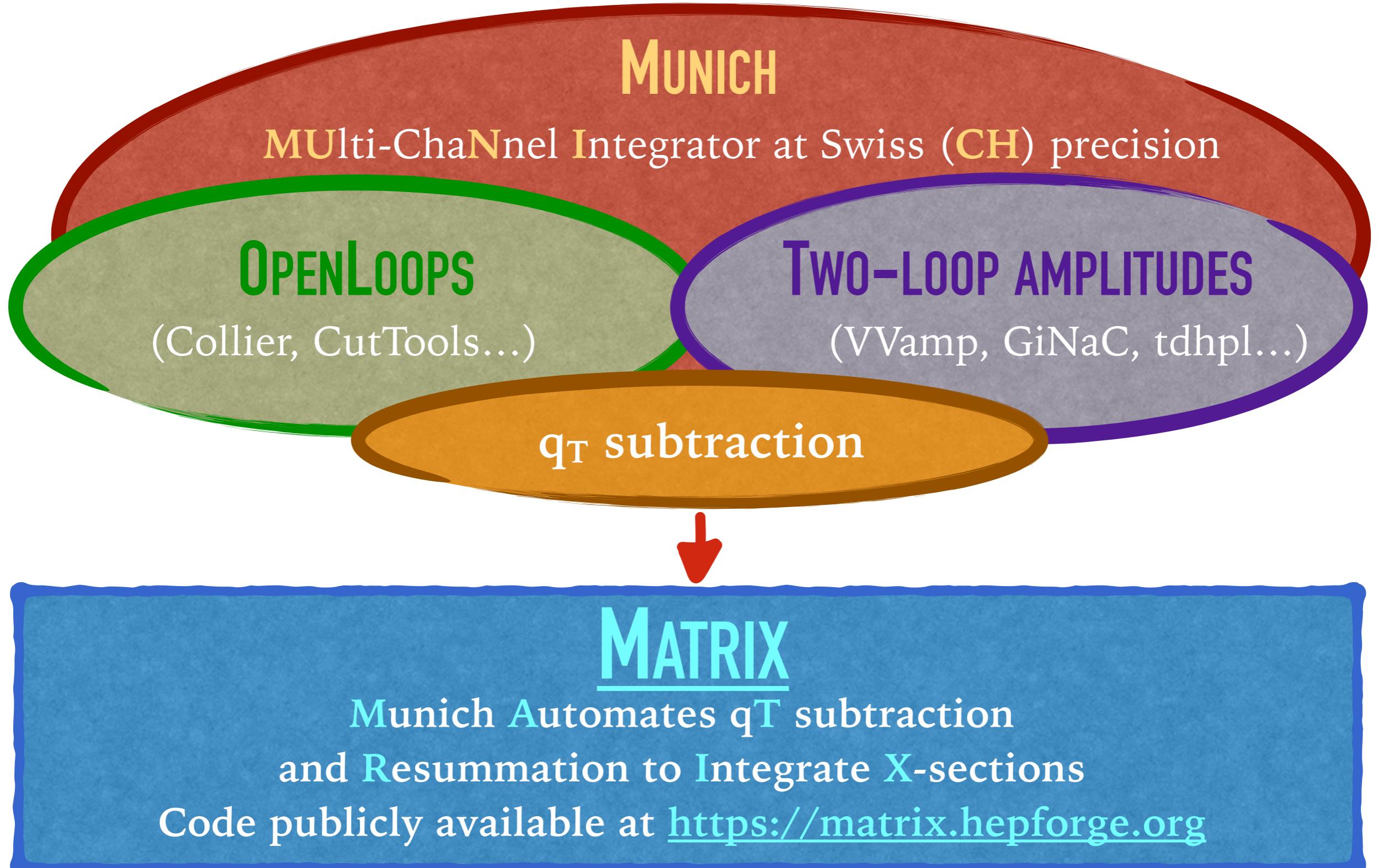
Final uncertainty:

- $\pm 15\%$ on $\Delta\sigma_{NNLO}$
- $\pm 0.6\%$ on σ_{NNLO}

Effect on the total cross section modulated by the (small) contribution of the hard factor: about 1% of the LO cross section in the gg and 2-3% in the $q\bar{q}$ channel.

THE MATRIX PROJECT

[M.Grazzini, S. Kallweit,
M. Wiesemann: 1711.06631]



RESULTS

[*S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli, C. Savoini: 2210.07846*]

Our Setup

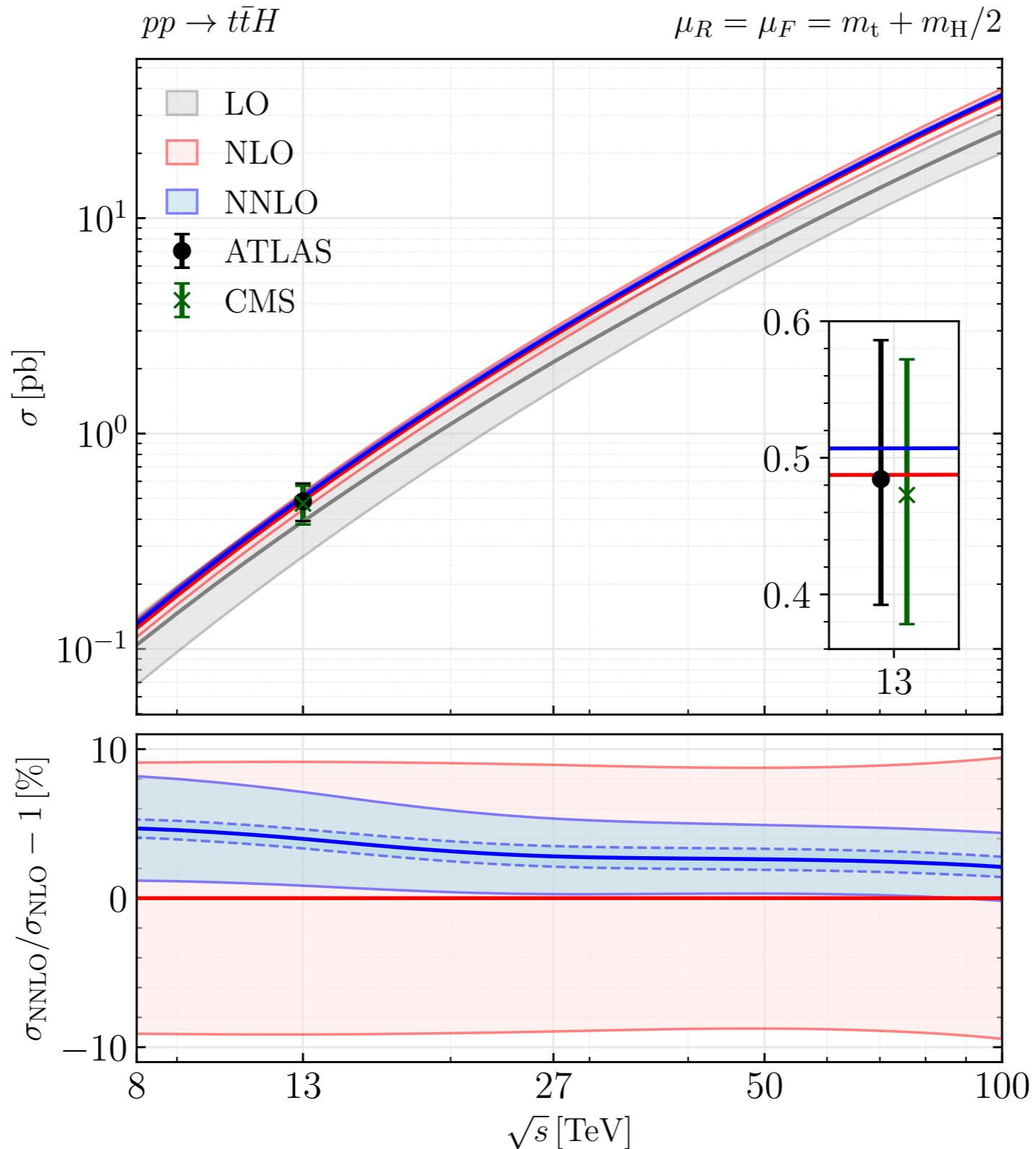
PdF set	NNPDF31 (NNLO)with 3 loops α_s	G_F	$1.16639 \cdot 10^{-5} \text{ GeV}^{-2}$
μ_R	$(m_t + m_H)/2$	m_H	125 GeV
μ_F	$(m_t + m_H)/2$	m_t	173.3 GeV

- **NNLO corrections:** **+4%** (13 TeV), **+2%** (100 TeV);
- Reduction of **scale uncertainties**;
- Soft approximation uncertainty significantly **smaller** than remaining perturbative uncertainties.
- Scale uncertainties are highly **asymmetric**: we consider their **symmetrised** version our estimate of **perturbative uncertainty**.

$\sigma [\text{pb}]$	13 TeV	100 TeV	
σ_{LO}	$0.3910^{+31.3\%}_{-22.2\%}$	$25.38^{+21.1\%}_{-16.0\%}$	Scale uncertainties
σ_{NLO}	$0.4875^{+5.6\%}_{-9.1\%}$	$36.43^{+9.4\%}_{-8.7\%}$	Statistical + soft Higgs uncertainties
σ_{NNLO}	$0.5070(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)^{+0.1\%}_{-2.2\%}$	

RESULTS

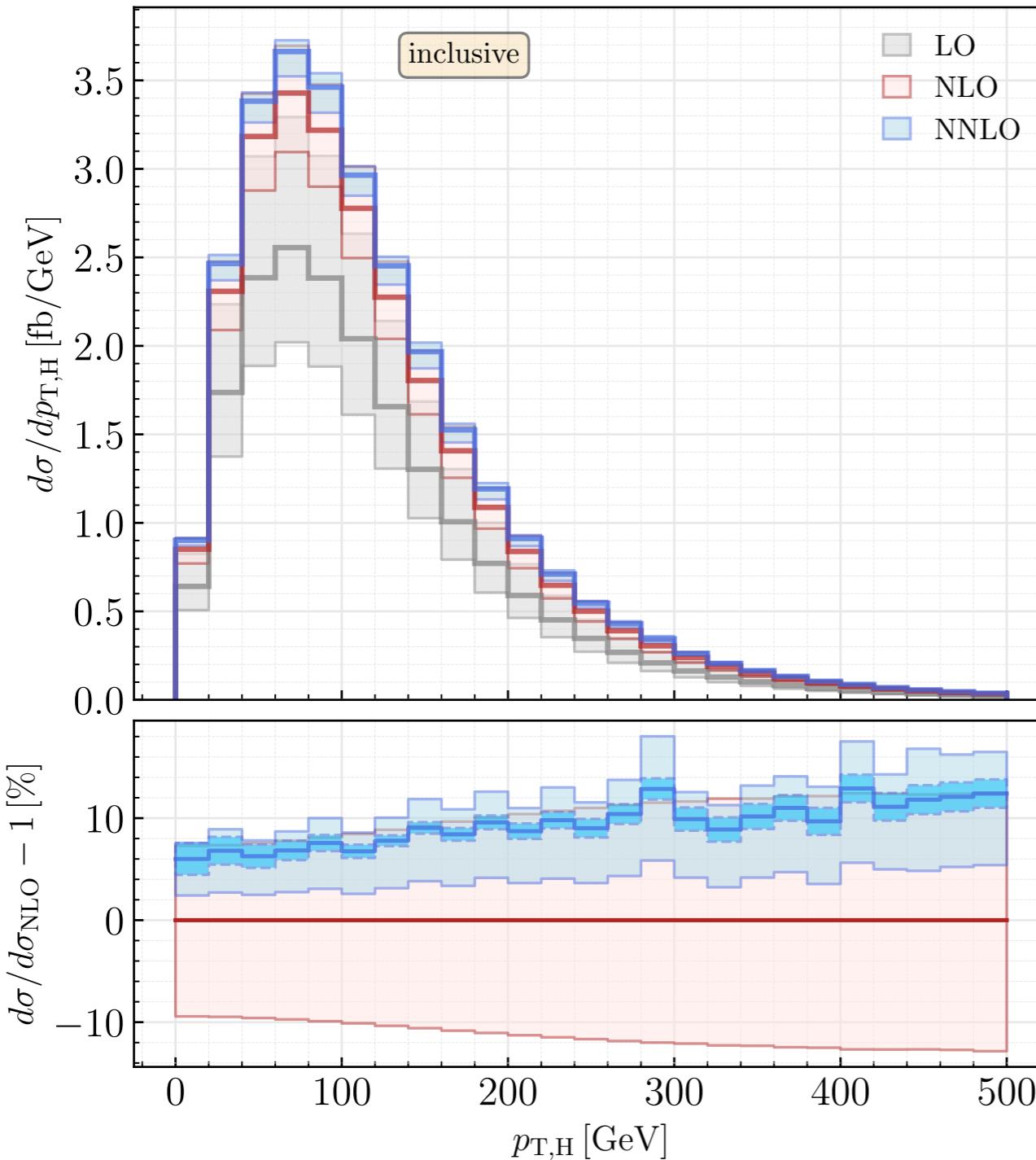
[*S. Catani, SD, M. Grazzini, S. Kallweit,
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RESULTS

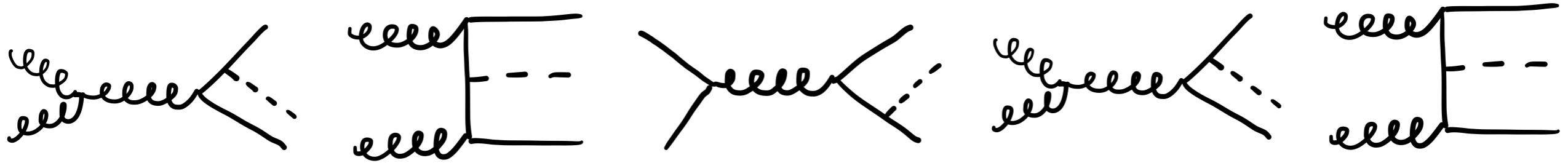
[S. Catani, SD, M. Grazzini, S. Kallweit,
J. Mazzitelli, C. Savoini: IN PREPARATION]

PDF set: NNLO NNPDF31 $m_H = 125 \text{ GeV}$, $m_t = 173.3 \text{ GeV}$
 $pp \rightarrow t\bar{t}H @ 13.6 \text{ TeV}$, $\mu_F = \mu_R = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$



-PRELIMINARY RESULTS-

- First results for **differential distributions**;
- Soft approximation uncertainty computed on a **bin-by-bin basis**;
- NLO and NNLO uncertainty bands **overlap**;
- Soft approximation uncertainty **of the same order** over all the spectrum.



SUMMARY & OUTLOOK

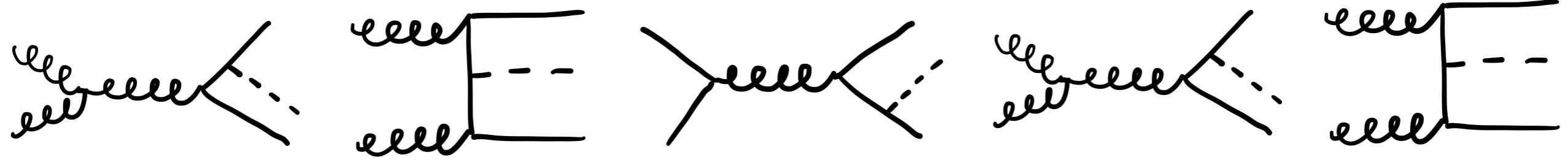
- We computed within q_T subtraction formalism the **NNLO QCD corrections** to $t\bar{t}H$ production;
- The **missing ingredients** we needed for the computation are:
 - **NNLO soft contribution** in arbitrary kinematics;
 - **two-loop amplitudes (soft Higgs boson approximation)**;
- **First** (almost) exact computation at NNLO QCD for a **$2 \rightarrow 3$ process** with massive coloured particles.



SUMMARY & OUTLOOK

- Differential distributions;
- Further phenomenological studies.

THANKS!



BACKUP SLIDES

TOTAL CROSS SECTION

	$\sqrt{s} = 13 \text{ TeV}$		$\sqrt{s} = 100 \text{ TeV}$	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\text{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\text{NLO,H}} _{\text{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\text{NNLO,H}} _{\text{soft}}$	-2.980(3)	2.622(0)	-239.4(4)	65.45(1)

► Soft Higgs approximation at LO:

- gg channel: factor 2.3 ($\sqrt{s} = 13 \text{ TeV}$)/factor 2.0 ($\sqrt{s} = 100 \text{ TeV}$)
- $q\bar{q}$ channel: factor 1.11 ($\sqrt{s} = 13 \text{ TeV}$)/factor 1.06 ($\sqrt{s} = 100 \text{ TeV}$)

► At LO there is no reweighting!

CHANGING THE SUBTRACTION SCALE

$$H_{t\bar{t}H}^{(2)} = \frac{2 \operatorname{Re}(\mathcal{M}_{t\bar{t}H}^{(2)}(\mu_{IR}, \mu_R) \mathcal{M}_{t\bar{t}H}^{(0)})_{soft}}{|\mathcal{M}_{t\bar{t}H}^{(0)}|_{soft}^2}$$

- The subtraction scale μ_{IR} is the scale at which the IR poles are subtracted (equivalently, at which the soft approximation is applied);
- Effect of using a different subtraction scales $\mu_{IR} \rightarrow 2\mu_{IR}$, $\mu_{IR} \rightarrow 1/2\mu_{IR}$.
 - gg channel +164%/-25% ($\sqrt{s} = 13$ TeV)
+142%/-20% ($\sqrt{s} = 100$ TeV)
 - $q\bar{q}$ channel +4%/-0% ($\sqrt{s} = 13$ TeV)
+3%/-0% ($\sqrt{s} = 100$ TeV)

SOFT HIGGS APPROXIMATION

Eikonal approximation

$$\lim_{k \rightarrow 0} \mathcal{M}_{t\bar{t}H}(\{p_i\}, k) = F(\alpha_S(\mu_R); m_t/\mu_R) \frac{m_t}{v} \sum_{i=3,4} \frac{m_t}{p_i \cdot k} \mathcal{M}_{t\bar{t}}(\{p_i\})$$

Low Energy Theorem

$$\lim_{q \rightarrow 0} \mathcal{M}^{\text{bare}}(p \rightarrow p + q) = \frac{1}{v} m_0 \frac{\partial}{\partial m_0} \mathcal{M}^{\text{bare}}(p \rightarrow p) \Big|_{p^2 = m^2}$$

$$\begin{aligned} F(\alpha_S(\mu_R); m_t/\mu_R) = & 1 + \frac{\alpha_S(\mu_R)}{2\pi} (-3 C_F) \\ & + \left(\frac{\alpha_S(\mu_R)}{2\pi} \right)^2 \left(\frac{33}{4} C_F^2 - \frac{185}{12} C_F C_A + \frac{13}{6} C_F (n_L + 1) - 6 C_F \beta_0 \ln \frac{\mu_R^2}{m_t^2} \right) + \mathcal{O}(\alpha_S^3) \end{aligned}$$

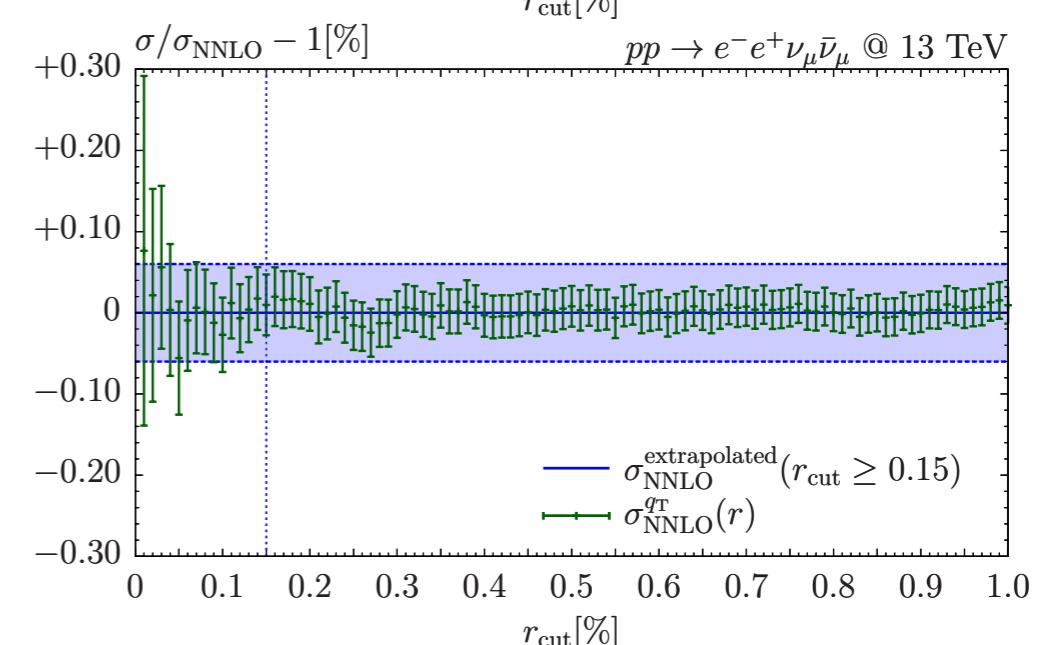
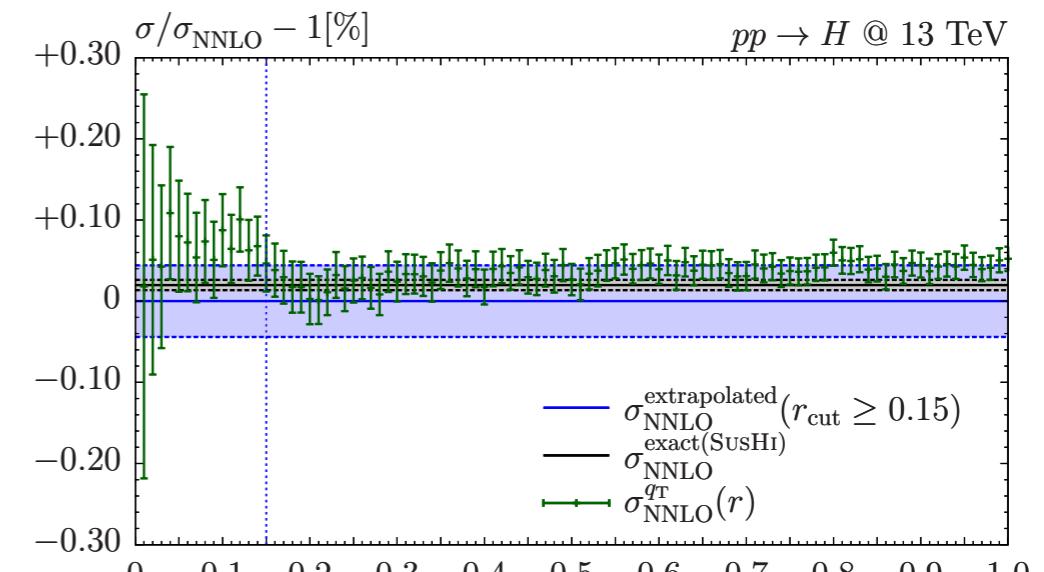
THE SLICING

$$d\sigma_{(N)NLO}^F = \mathcal{H}_{(N)NLO}^F \otimes d\sigma_{LO}^F + \boxed{d\sigma_{(N)LO}^{F+jets} - d\sigma_{(N)LO}^{CT}}$$

$d\sigma_{(N)LO}^{F+jets}$ and $d\sigma_{(N)LO}^{CT}$ are separately divergent.
In practice, q_T subtraction is implemented as a slicing method:

- introducing a cutoff $r_{cut} = Q/M$;
- performing the limit $r_{cut} \rightarrow 0$.

Quality of the $q_T \rightarrow 0$ extrapolation can be understood looking at the r_{cut} dependence



r_{cut} DEPENDENCE

