

# Charting the Higgs self-coupling boundaries

Gauthier Durieux  
(CP3 – UCLouvain)

*Gegenbauer Goldstones*, JHEP 01 (2022) 076, [2110.06941]

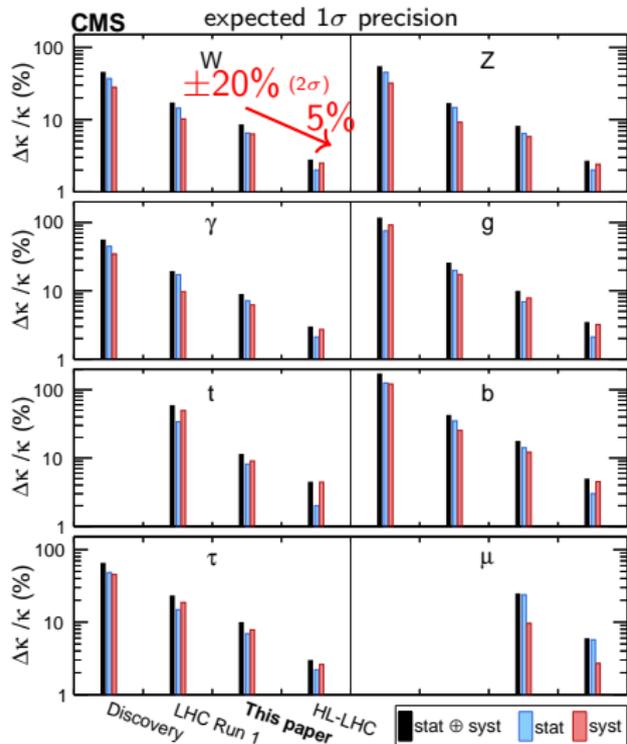
*Gegenbauer's Twin*, JHEP 05 (2022) 140, [2202.01228]

*Charting the Higgs self-coupling boundaries*, JHEP 12 (2022) 148, [2209.00666]

with Matthew McCullough and Ennio Salvioni

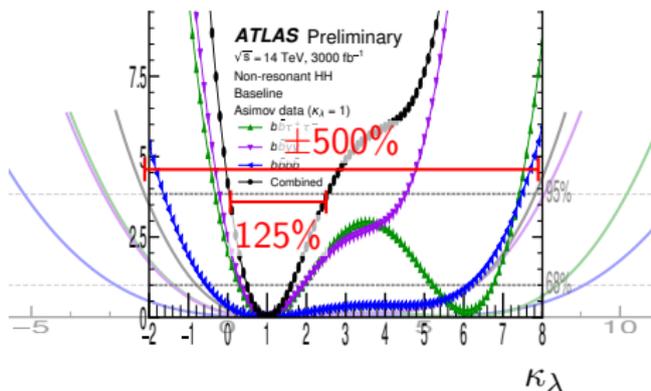


# LHC present and future



[CMS-HIG-22-001]

Any class of models where  $\delta\kappa_\lambda$  nevertheless relevant?



[HDBS-2022-03]

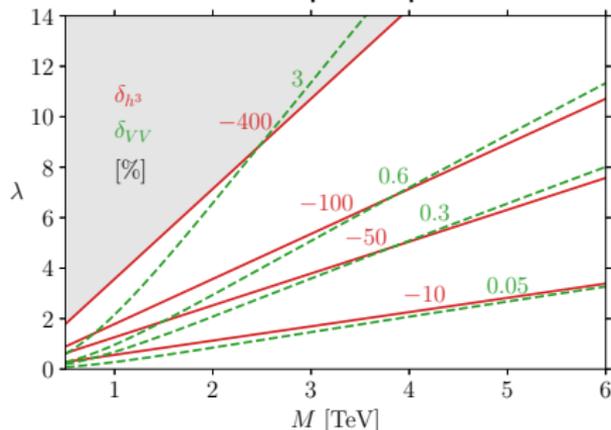
[ATL-PHYS-PUB-2022-053]

# Structurally large $\delta\kappa_\lambda/\delta\kappa_V$

see also: [Di Luzio, Gröber, Spannowsky '17]  
[Gupta, Rzehak, Wells '13] [Falkowski, Rattazzi '19]  
[Logan, Rental '15] [Chala, Krause, Nardini '18] [etc.]

loop factor (or  $v^2/M_X^2$ ) allowed dimensionally btw.  $H^6$  and  $D^2H^4$

custodial weak-quadruplet scalar



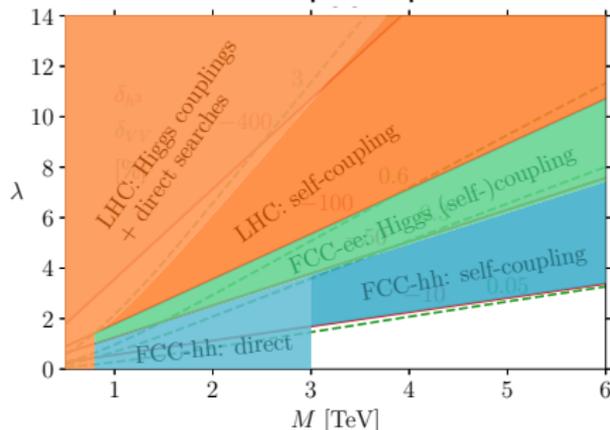
$$\lambda H^* H^* (\epsilon H) \Phi + \lambda \frac{1}{\sqrt{3}} H^* H^* H^* \tilde{\Phi}$$

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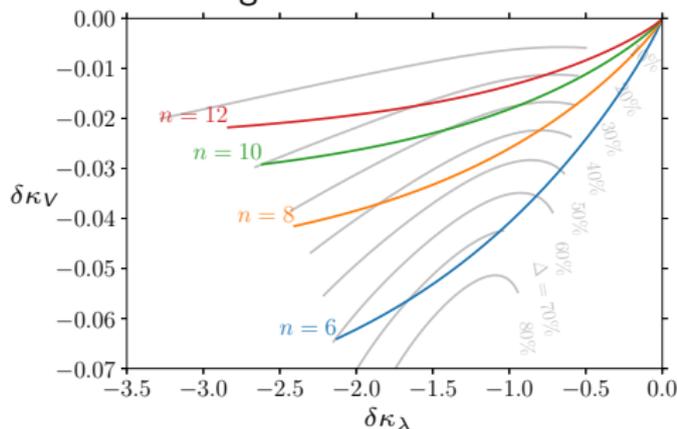
custodial weak-quadruplet scalar



$$\lambda H^* H^* (\epsilon H) \Phi + \lambda \frac{1}{\sqrt{3}} H^* H^* H^* \tilde{\Phi}$$

- $\dim \gg 6$  operators may be very relevant
- vacuum stability limiting the  $\delta\kappa_\lambda/\delta\kappa_V$  ratio

Gegenbauer's Twin



large representations!

# Naturalness exacerbated

$$\frac{m_h^2}{M_X^2} \ll 1$$

$$\delta\kappa_V \ll 1$$

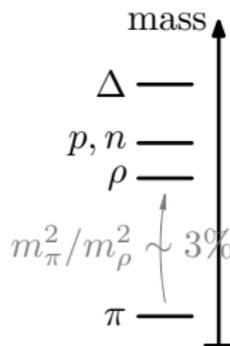
# Composite Higgs

the Higgs as  
pseudo-Nambu-Goldstone boson (pNGB)  
of a new strong sector

e.g. global  $SO(5) \rightarrow SO(4)$  spontaneous breaking  
at scale  $f$

small mass from  
explicit  $SO(5)$  breaking  
by e.g. the SM

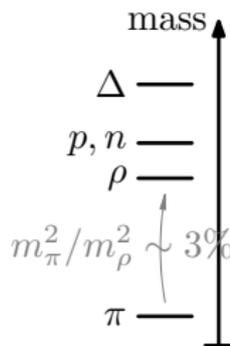
small  $\delta\kappa_V$  implies  $v^2/f^2 \ll 1$   
and requires fine-tuning in minimal models



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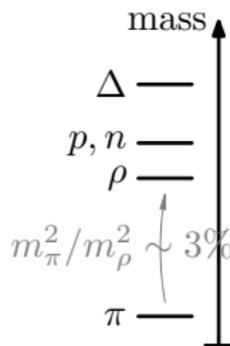
small(ish)  $m_h^2/M_X^2!$

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small(ish)  $m_h^2/M_X^2!$

and require **no** small  $\delta\kappa_V$  or  $v/f!$  models

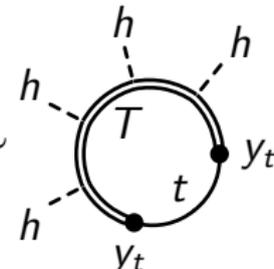
# Minimal composite Higgs

$$V(h) \sim \text{[Diagram]} + \dots \sim \kappa \frac{y_t^2 N_c}{16\pi^2} f^2 M_T^2 \left( -\sin^2 \frac{h}{f} + \delta \sin^4 \frac{h}{f} \right)$$

$$\rightarrow \frac{v^2}{f^2} = \sin^2 \frac{\langle h \rangle}{f} = \frac{1}{2\delta} \quad \text{vs.} \quad |\delta \kappa_V| \simeq \frac{v^2}{2f^2} \lesssim 5\%$$

$$\rightarrow \frac{m_h^2}{M_T^2} = \kappa \frac{4y_t^2 N_c}{16\pi^2} \left( 1 - \frac{1}{2\delta} \right) \quad \text{vs.} \quad M_T \gtrsim 1.5 \text{ TeV}$$

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Gegenbauer fix

$$\rightarrow \frac{v^2}{f^2} = \sin^2 \frac{\langle h \rangle}{f} = \frac{1}{2\delta} \quad \left| \delta \kappa v \right| \simeq \frac{v^2}{2f^2} \lesssim 5\%$$

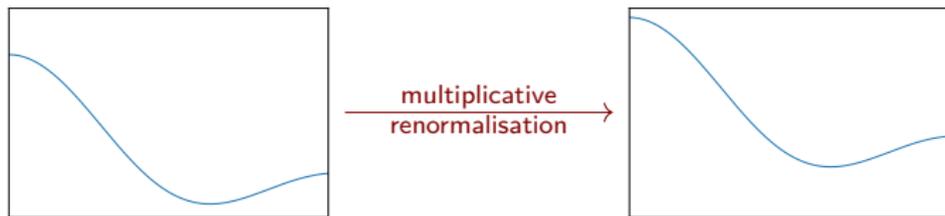
$$\rightarrow \frac{m_h^2}{M_T^2} = \kappa \frac{4y_t^2 N_c}{16\pi^2} \left( 1 - \frac{1}{2\delta} \right) \quad \kappa \lesssim 0.10 \quad M_T \gtrsim 1.5 \text{ TeV}$$

Twin fix

Few percent fine-tuning wrt.  $\delta \lesssim 1$ ,  $\kappa \simeq 1$  expectation

# Structurally small vev

radiatively stable  
low-energy pNGB potential



with deepest minimum close to the origin

## Radiatively stable $SO(N + 1) \rightarrow SO(N)$ potentials

$$\vec{\phi} \equiv \left( \frac{\vec{h}}{h} \sin \frac{h}{f}, \cos \frac{h}{f} \right), \quad h \equiv |\vec{h}|$$

Linear one-loop correction to  $V(\frac{h}{f})$ :

$$\frac{\Lambda^2}{32\pi^2 f^2} \left( V'' + (N - 1) \cot \frac{h}{f} V' \right)$$

Radiative stability at one-loop and linear order order if  $\propto V$

Differential equation of Gegenbauer polynomials

$$V\left(\frac{h}{f}\right) \propto G_n^{(N-1)/2}\left(\cos \frac{h}{f}\right)$$

# Radiatively stable $SO(N + 1) \rightarrow SO(N)$ potentials

Explicit  $SO(N + 1) \rightarrow SO(N)$  breaking by an irrep spurion  $K$ :

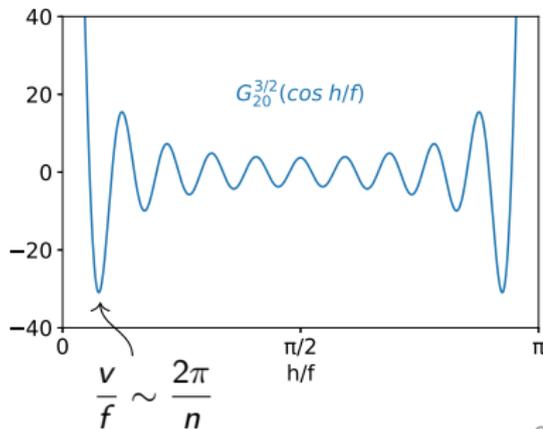
$$K^{i_1 \dots i_n} \phi_{i_1} \cdots \phi_{i_n} \quad (\text{symmetric traceless})$$

$$\vec{\phi} \equiv \left( \frac{\vec{h}}{h} \sin \frac{h}{f}, \cos \frac{h}{f} \right), \quad h \equiv |\vec{h}|$$

No other invariant, linear in  $K$ , can be constructed,  
so all-loop linear renormalisation can only be multiplicative.

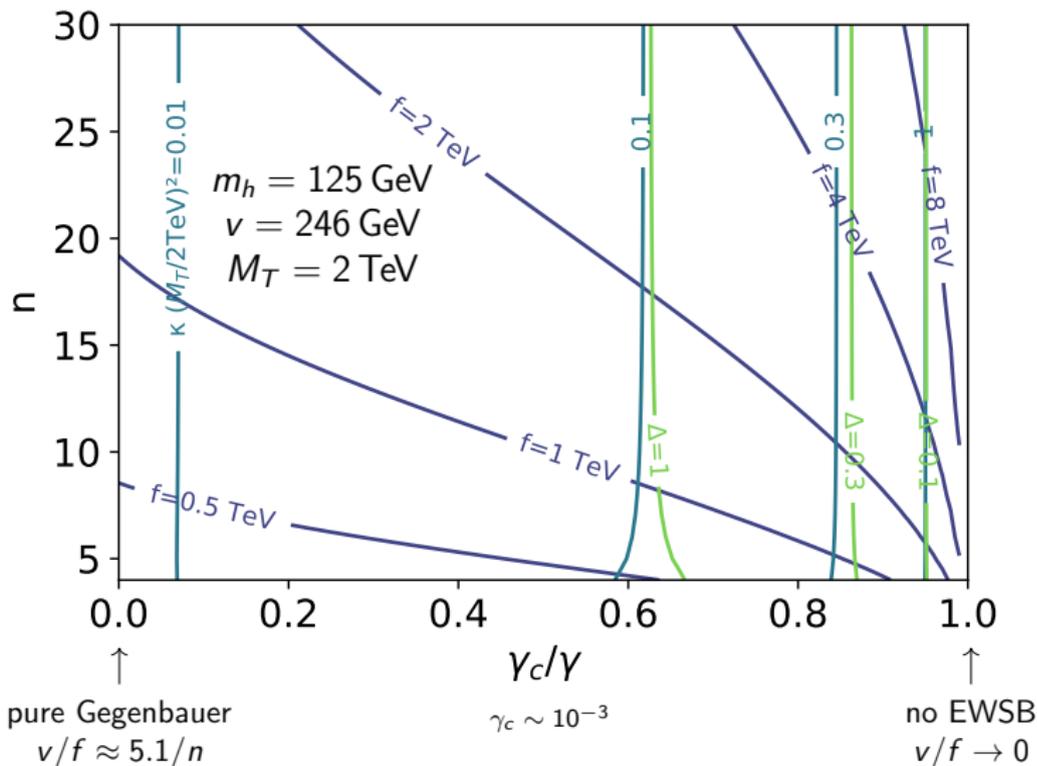
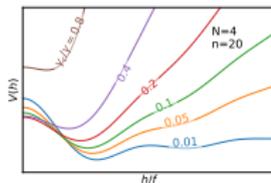
Obtain Gegenbauer polynomials:

$$K^{i_1 \dots i_n} \phi_{i_1} \cdots \phi_{i_n} \propto G_n^{(N-1)/2} \left( \cos \frac{h}{f} \right)$$



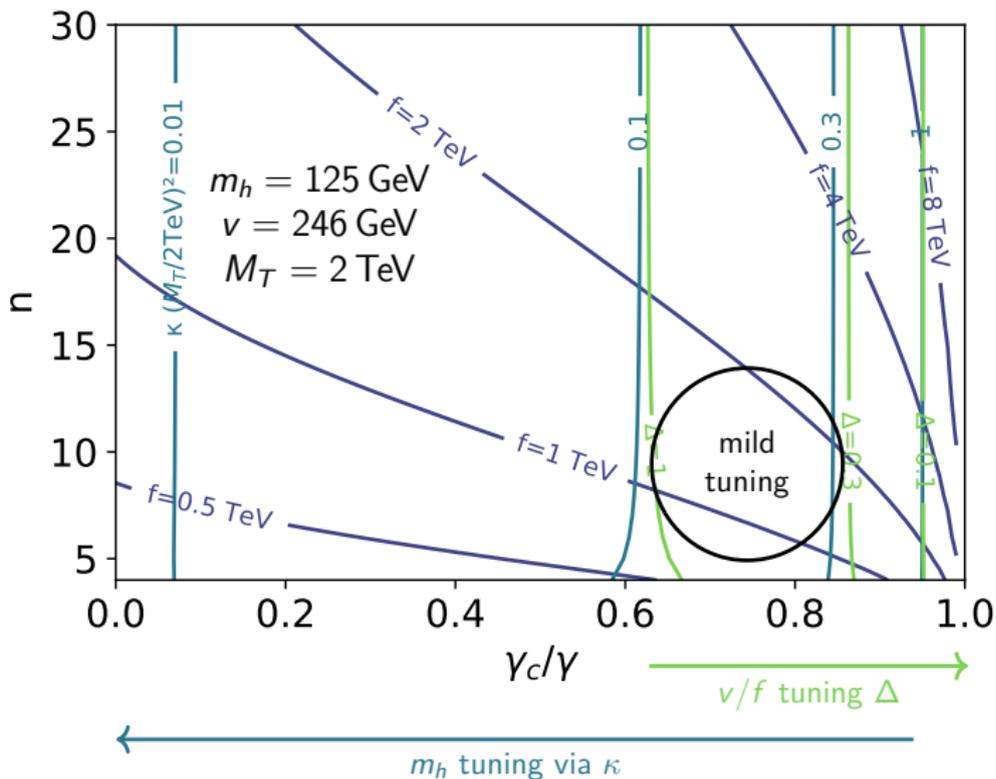
# The Gegenbauer Higgs

$$V(h) = \kappa \frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \left[ \sin^2 \frac{h}{f} + \gamma G_n^{3/2} \left( \cos \frac{h}{f} \right) \right]$$



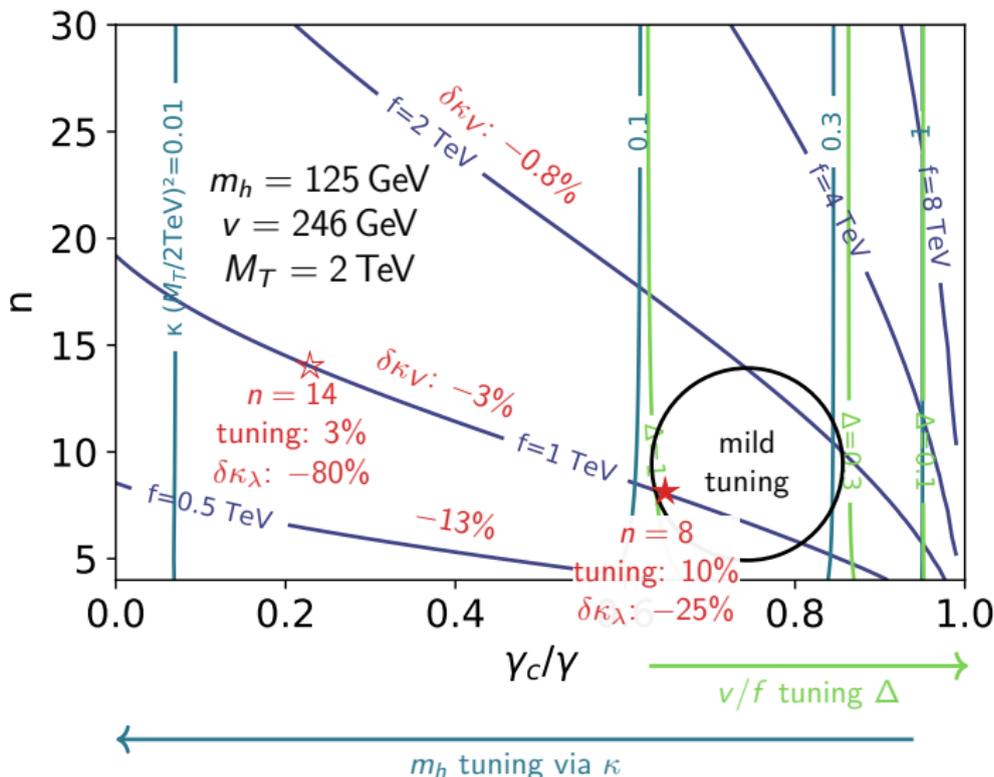
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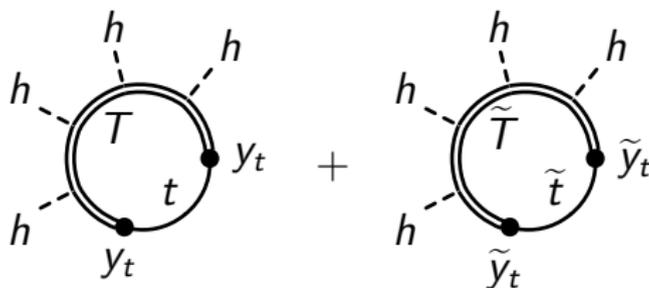


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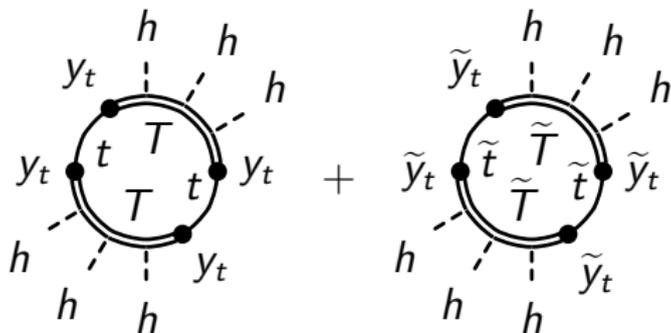
# Structurally smaller mass



$$\frac{N_c y_t^2}{16\pi^2} f^2 M_T^2 \sin^2 \frac{h}{f} + \frac{N_{\tilde{c}} \tilde{y}_t^2}{16\pi^2} f^2 M_{\tilde{T}}^2 \cos^2 \frac{h}{f}$$

if twin parity enforces  $y_t = \tilde{y}_t$  and  $M_T = M_{\tilde{T}}$   
no  $M_T^2$  sensitivity

# Structurally smaller mass



$$\frac{N_c y_t^4}{16\pi^2} f^4 \sin^4 \frac{h}{f} \log M_T \quad + \quad \frac{N_c \tilde{y}_t^4}{16\pi^2} f^4 \cos^4 \frac{h}{f} \log M_{\tilde{T}}$$

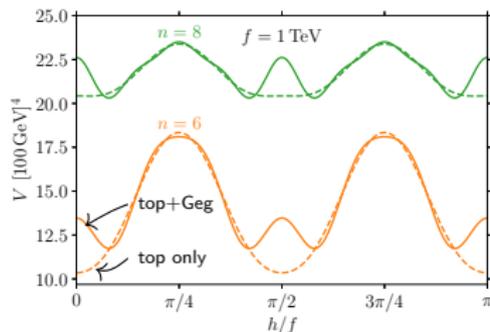
retaining  $\log M_T$  sensitivity only

# Gegenbauer's Twin

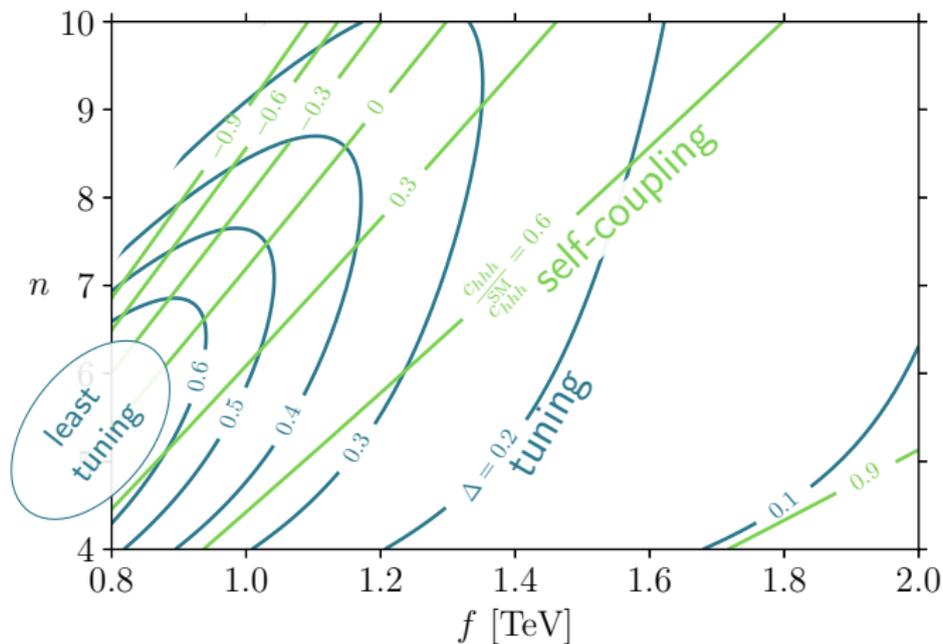
- global  $SO(8) \supset SO(4) \times \widetilde{SO(4)}$
- spontaneous  $SO(8) \rightarrow SO(7)$ 
  - 7 NGBs
  - 6 eaten by  $W^\pm, Z$  and  $\widetilde{W}^\pm, \widetilde{Z}$
  - 1 Higgs:  $\vec{\phi} = (\vec{0}_3, \sin \frac{h}{f}; \vec{0}_3, \cos \frac{h}{f})^T$  in unitary gauge
- explicit breaking from the top sector is insufficient
- explicit  $SO(8) \rightarrow SO(4) \times \widetilde{SO(4)}$ 
  - radiative stability from irrep spurion
  - $G_n^{3/2}(\cos \frac{2h}{f})$  potential



Leopold B. Gegenbauer  
1849–1903

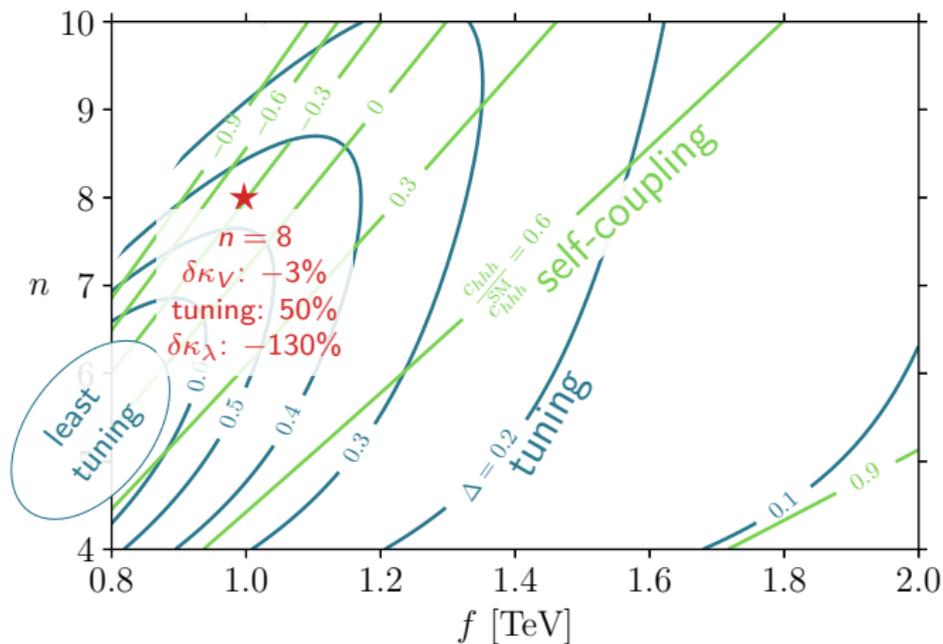


# Gegenbauer's Twin



(and possibly large  $M_T$ , with unitarity violating  $H$  scattering towards 6 TeV)

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# Charting the Higgs self-coupling boundaries

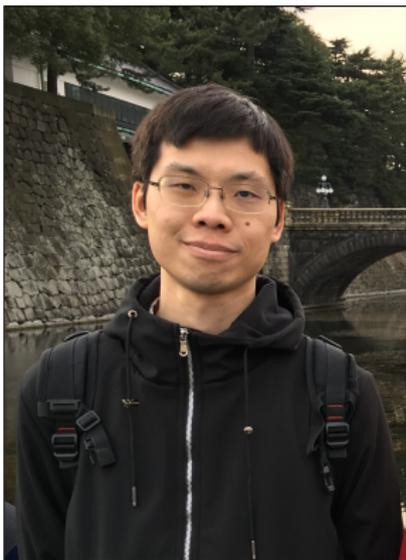
Classes of models exist with structurally large  $\delta\kappa_\lambda/\delta\kappa_V$ .

The custodial quadruplet scalar is an example.

Gegenbauer models, motivated by naturalness, have it too.

Key is an explicit breaking of the global pNGB-Higgs symmetry by a large irrep.

$\delta\kappa_\lambda$  could be the first signal of new physics!



Cen Zhang 张岑  
30 May 1984 – 9 June 2021

remembering a bright and humble friend from IHEP