NLO Electroweak corrections to $gg \to HH$ and $gg \to Hg$

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Based on [JHEP 08 (2022) 259] & [JHEP 10 (2023) 033]

Karlsruher Institut für Technologie

Higgs 2023 - Beijing

Institute for Theoretical Particle Physics

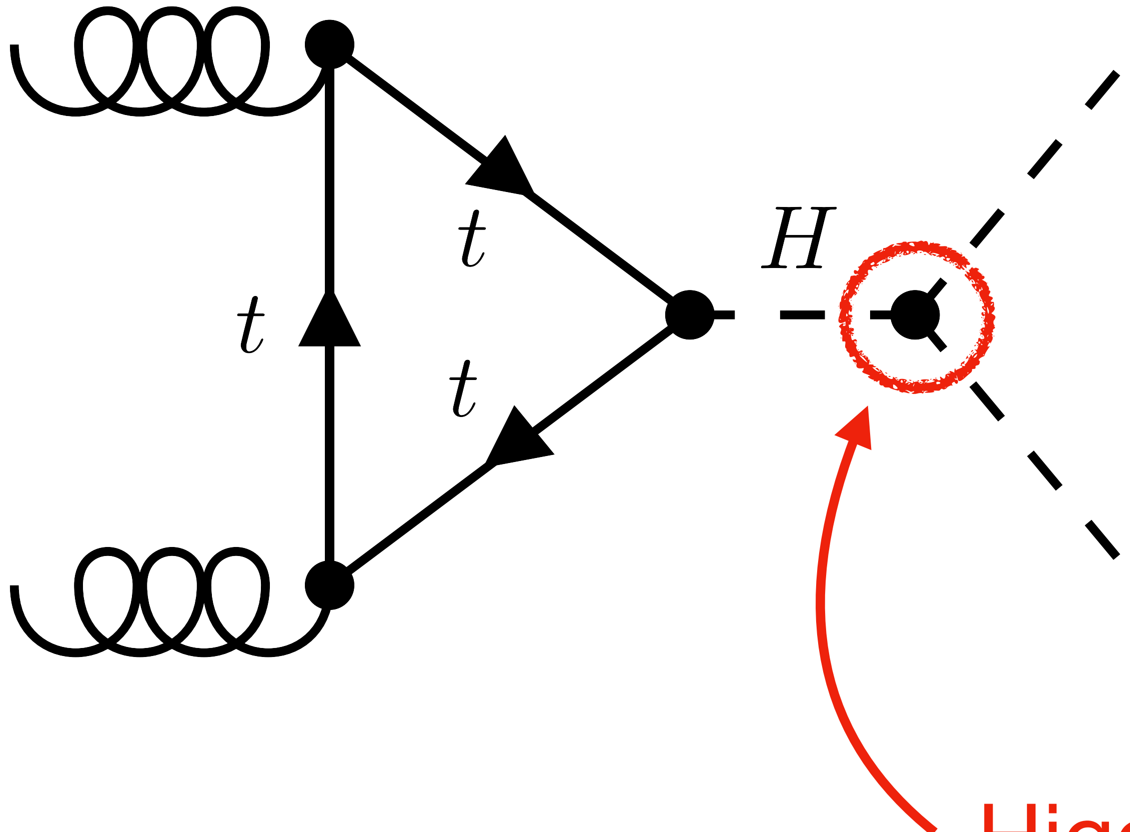
Motivation of $gg \to HH$: study Higgs self-coupling

- Probe Higgs self-coupling in pair productions, and compare with the Standard Model value

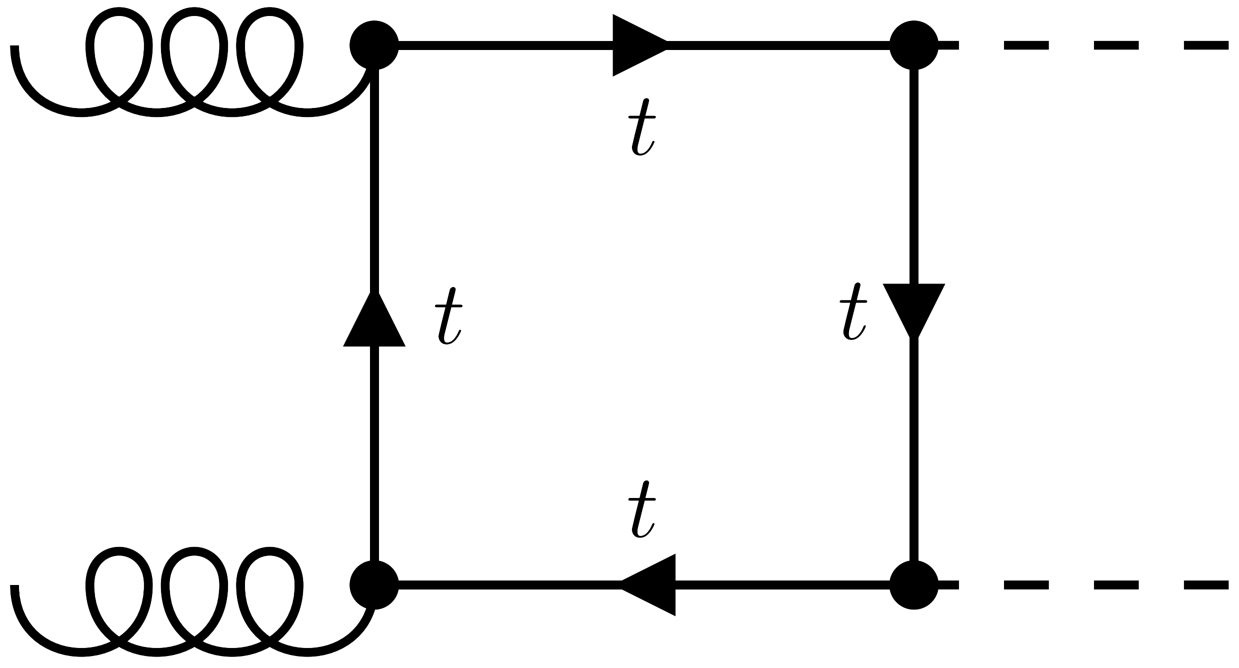
  $\lambda = m_H^2/(2v^2) \approx 0.13$ in the Higgs potential

$$V(H) \;=\; \frac{1}{2}\, m_H^2\, H^2 + \lambda\, v\, H^3 + \frac{\lambda}{4}\, H^4$$

- Gluon-fusion channel dominates Higgs boson pair production at LHC



Higgs self-coupling λ

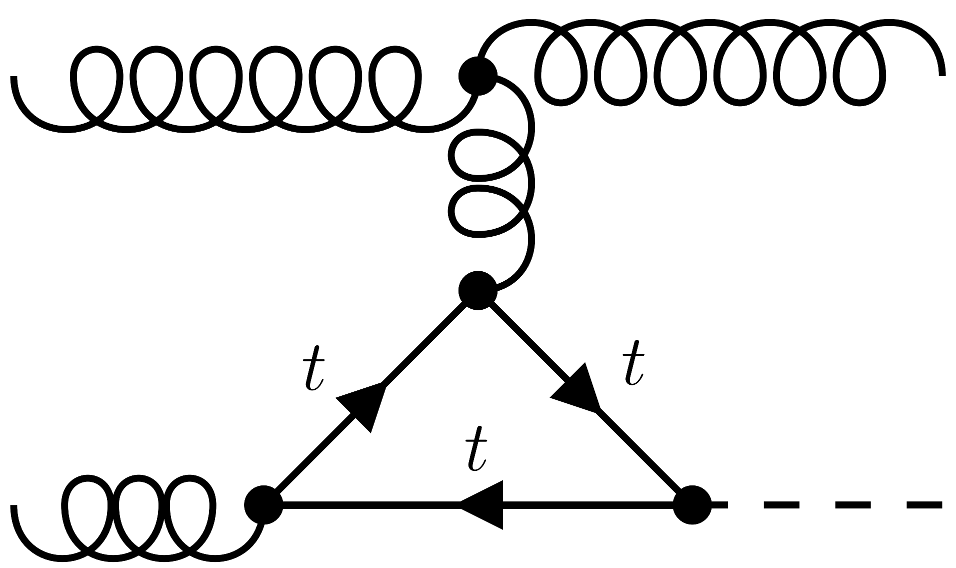


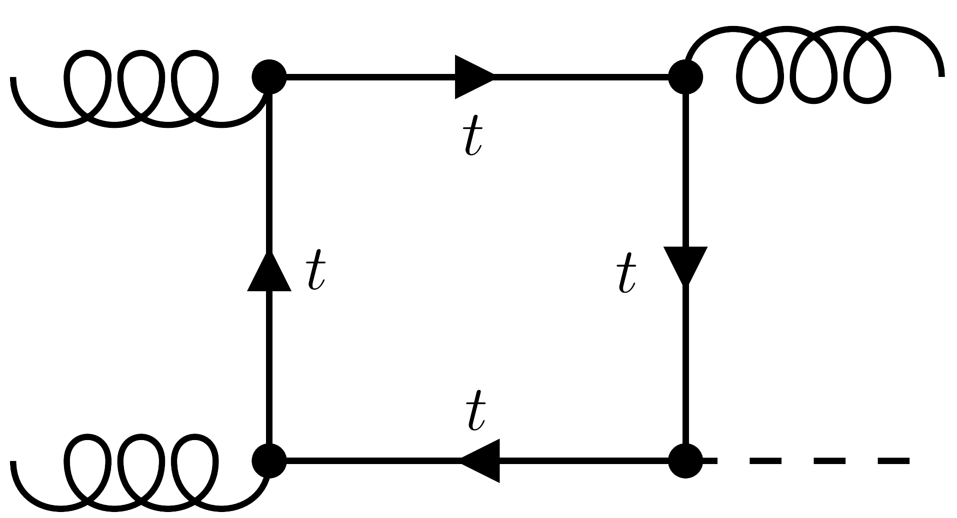
New Physics opportunity $\Rightarrow$ Talk by Teppei Kitahara

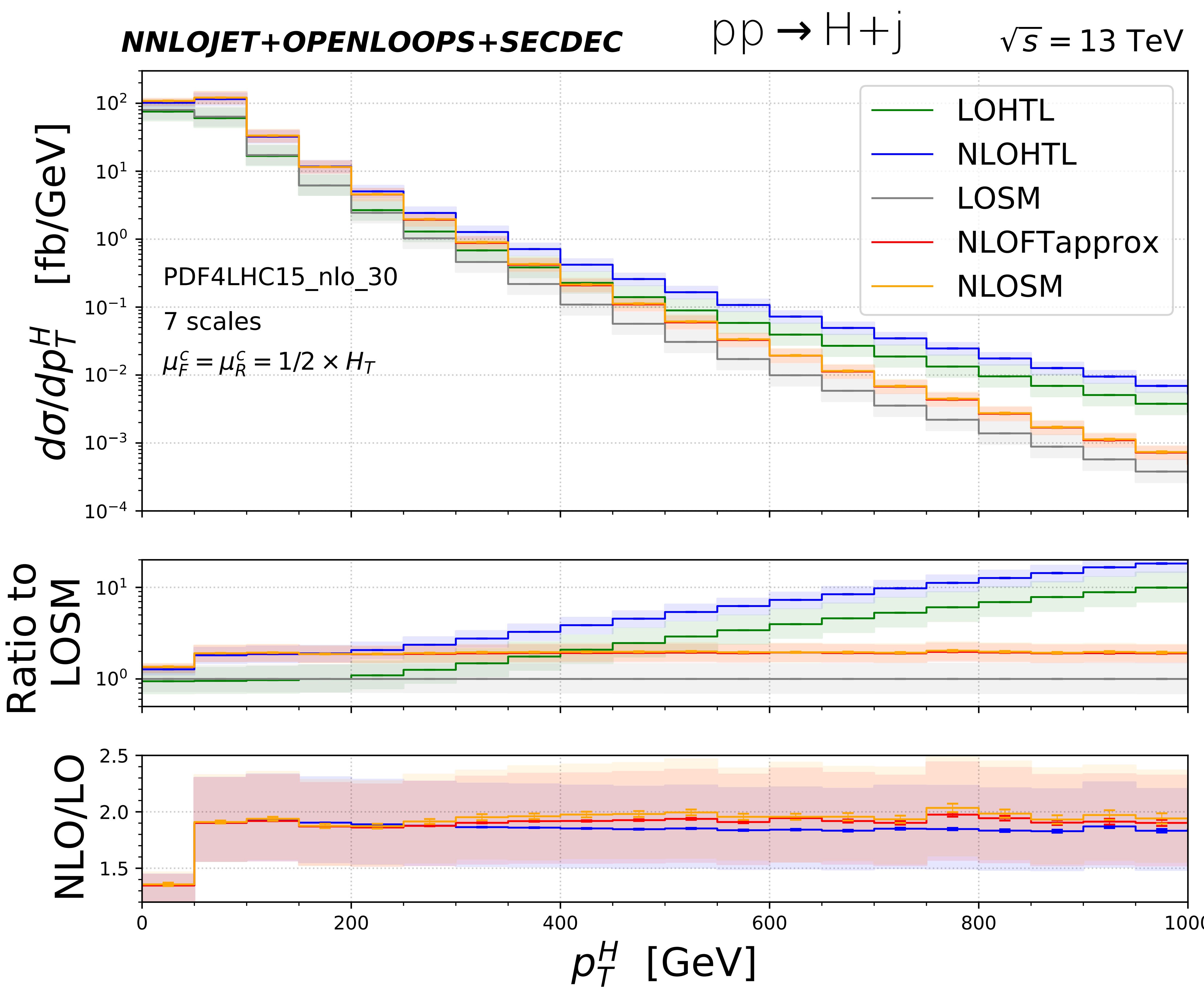
[Iguro, Kitahara, Omura, **Zhang**, ***Phys.Rev.D 107 (2023) 7, 075017***]

Motivation of $gg \to Hg$: study p_T^H spectrum

- Dominant channel for Higgs boson production with large transversal momentum p_T^H at LHC







p_T^H spectrum:
crucial for studying
Higgs boson properties

NLOSM ⇒ NLO QCD with
top-mass dependence

See also
[Jones, Kerner, Luisoni, 18']
[Bonciani, Del Duca et. al. 23']

Overview for Higgs + jet ⇒ Talk by Xuan Chen

[Chen, Huss, Jones, Kerner, Lang, Lindert, **Zhang**, ***JHEP 03 (2022) 096***]

Overview of QCD and EW calculations (HH)

- **NLO QCD with full m_t-dependence are known**
  - Expansion-by-Region & Numerical approaches: [Daoson, Dittmaier, Spira, 98'], [Grigo, Hoff, Melnikov, Steinhauser, 13'], [Borowka, Geiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke, 16'], [Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher, 18'], [Degrassi, Giardino, Gröber, Bonciani, 18'], [Xu, Yang, 18'], [Davies, Mishima, Steinhauser, Wellmann, 18', 19'], [Davies, Heinrich, Jones, Kerner, Mishima, Steinhauser, Wellmann, 19'], [Bellafronte, Degrassi, Giardino, Gröber, Vitti, 22']

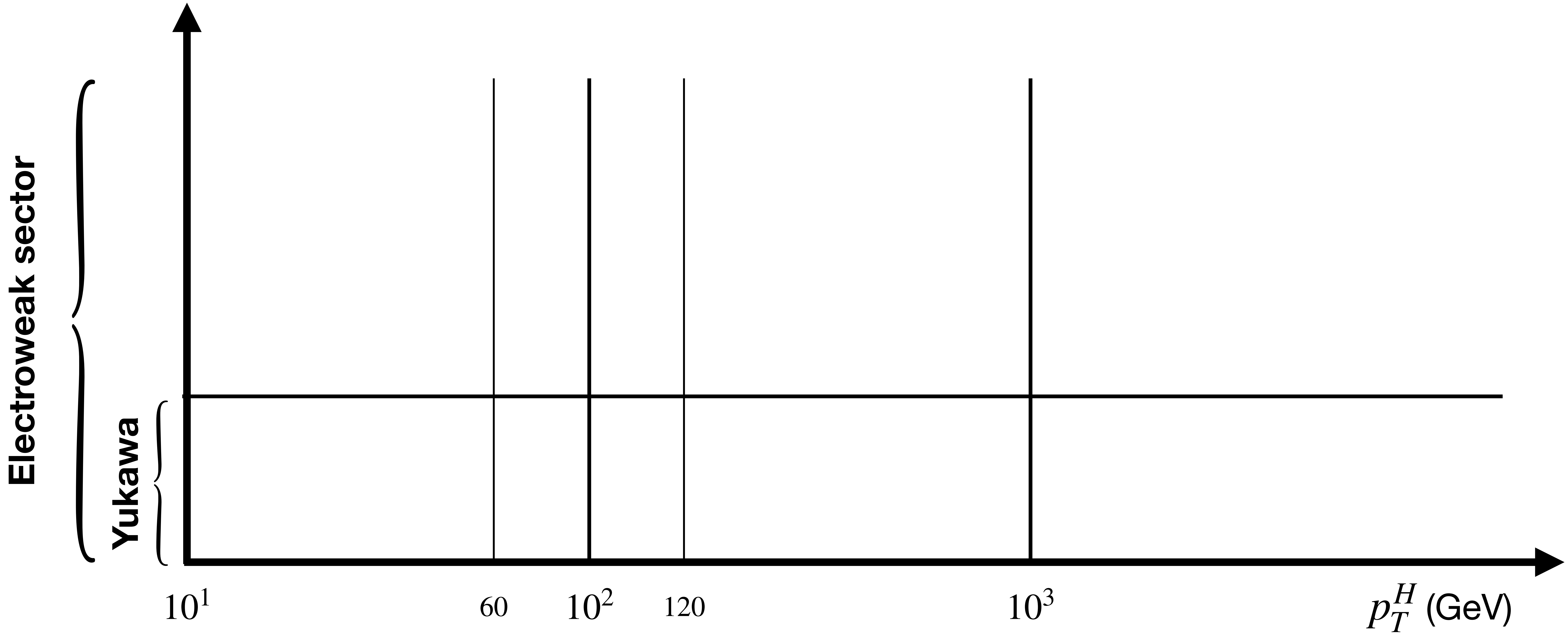
- **QCD beyond NLO are available in large-m_t and small-t limit / expansion**
  - At NNLO: [de Florian, Mazzitelli, 13'], [Grigo, Melnikov, Steinhauser, 14' and Hoff, 15'], [Davies, Herren, Mishima, Steinhauser, 19', 21'], [Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli, 18'], [Davies, Schönwald, Steinhauser, 23']
  - At N^{3}LO: [Spira, 16'], [Gerlach, Herren, Steinhauser, 18'], [Banerjee, Borowka, Dhabi, Gehrmann, Ravindran, 18'], [Chen, Li, Chao, Wang, 19']

- **NLO EW become available recently in** [Also see talks by Xiao Zhang and Thomas Stone]
  - Higgs self-coupling corrections [Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao, 19']
  - Yukawa-top corrections in high-energy expansion [Davies, Mishima, Schönwald, Steinhauser, Zhang, 22'] and large-m_t limit [Mühlleitner, Schlenk, Spira, 22']
  - Full top-induced EW corrections in large-m_t expansion [Davies, Schönwald, Steinhauser, Zhang, 23']

This talk

Recent NLO EW calculations

JHEP 08 (2022) 259 & JHEP 10 (2023) 033



Recent NLO EW calculations

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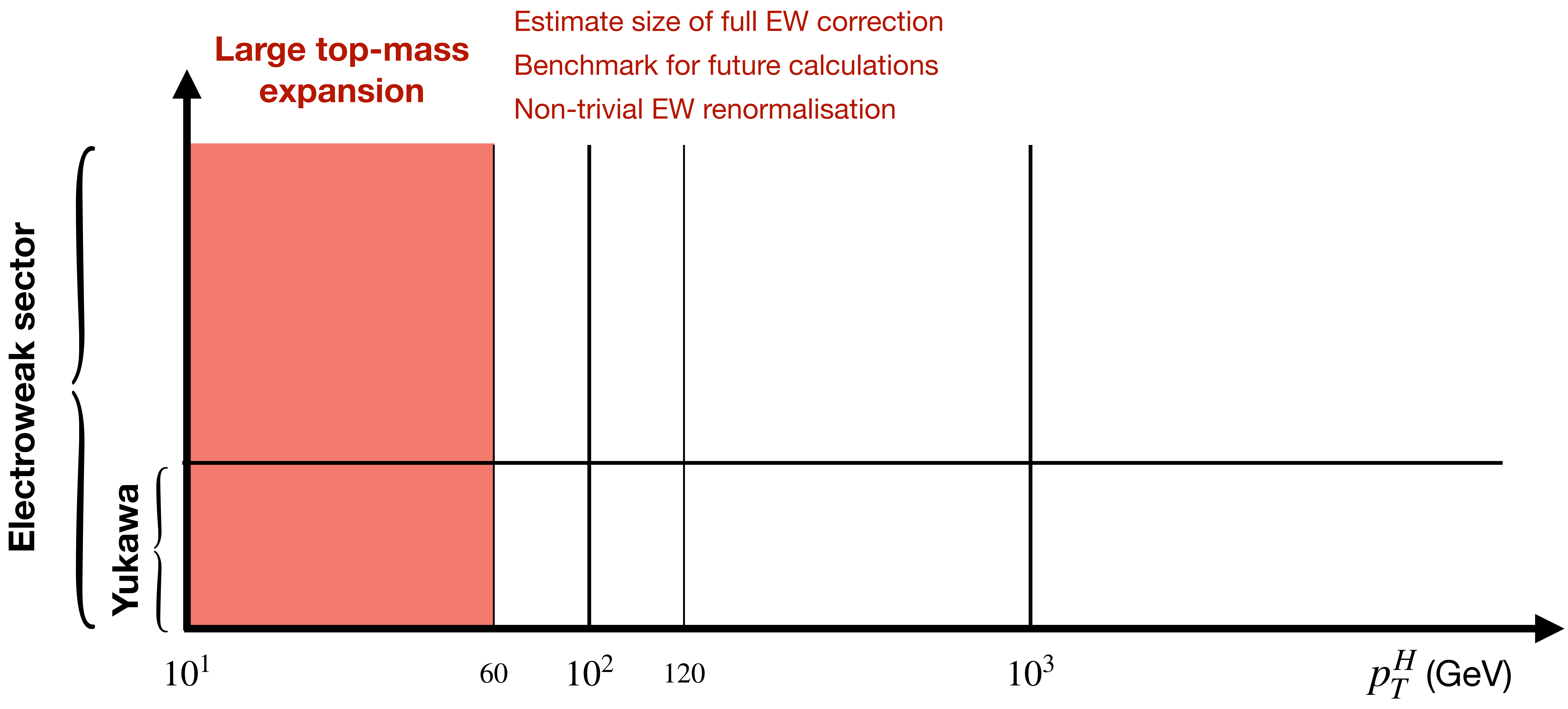

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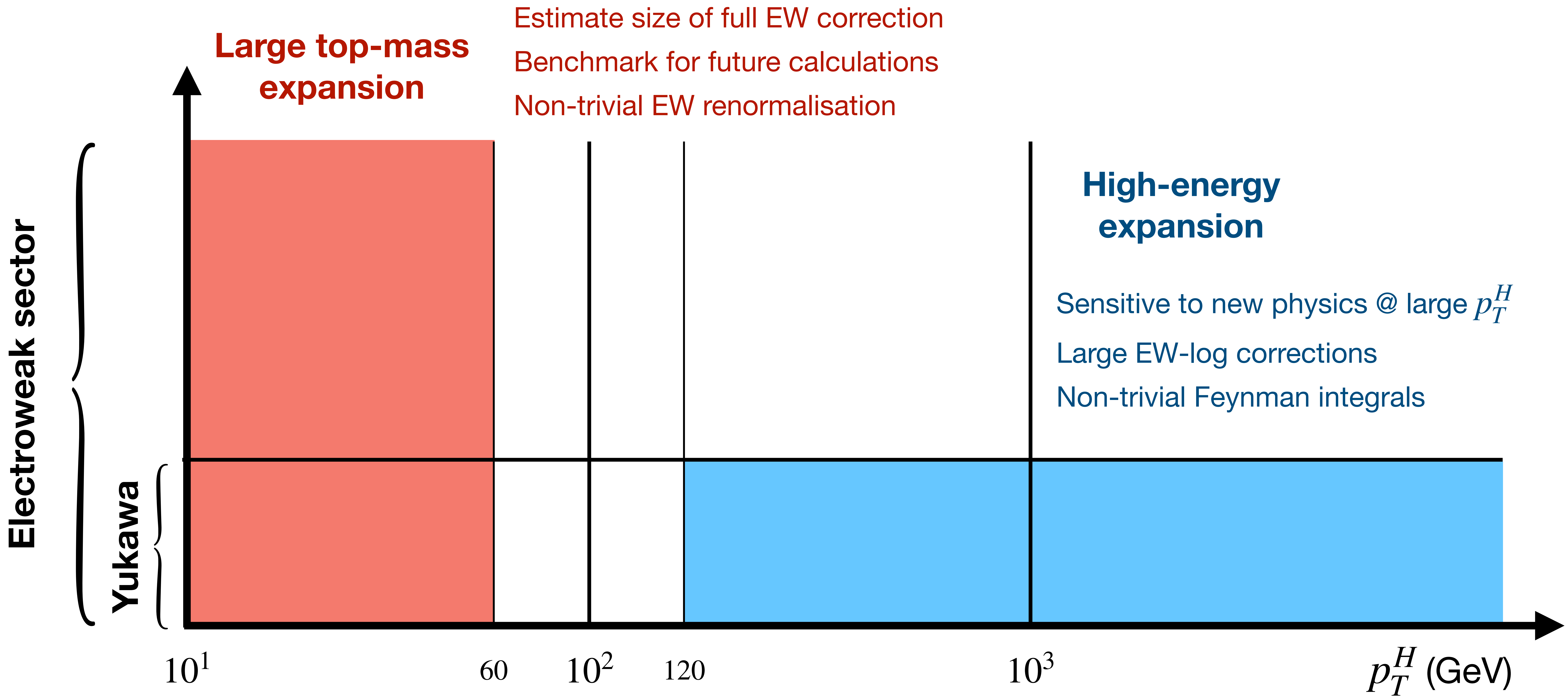
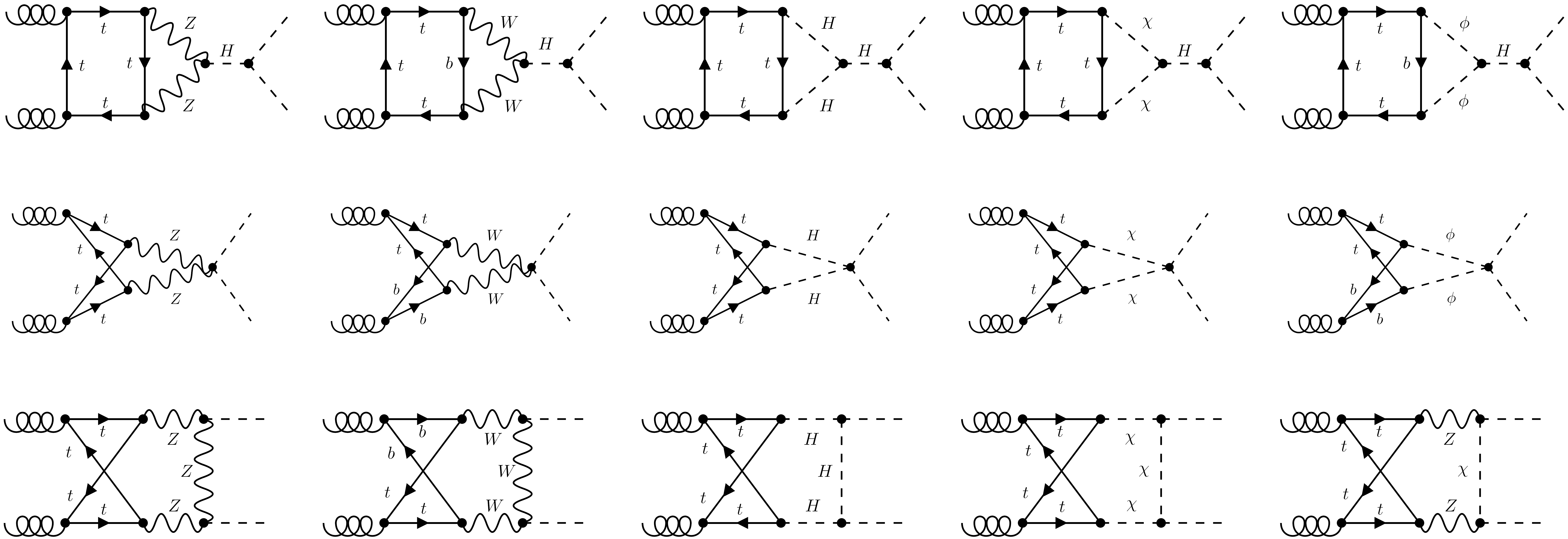


Part 1: Full NLO EW corrections in large-m_t limit (HH)

[Davies, Schönwald, Steinhauser, **Zhang**, ***JHEP 10 (2023) 033***]

- Sample two-loop diagrams involving SM fields: $\{t, b, H, \gamma, Z, W^\pm, \chi, \phi^\pm\}$ and ghosts: $\{u^\gamma, u^Z, u^\pm\}$

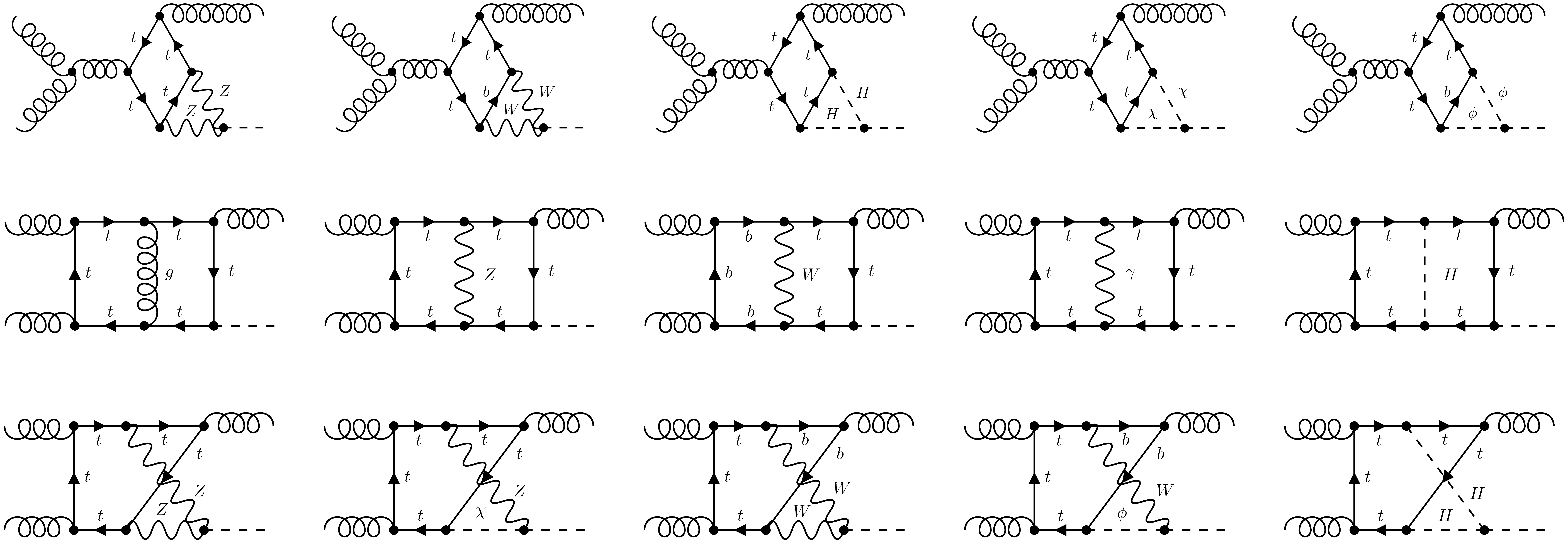


- **Aim:** analytic large-m_t expansion in $\sqrt{s} \leq 300\,\mathrm{GeV}$ region

Part 1: Full NLO EW corrections in large-m_t limit (H+jet)

[Davies, Schönwald, Steinhauser, **Zhang**, ***JHEP* 10 (2023) 033**]

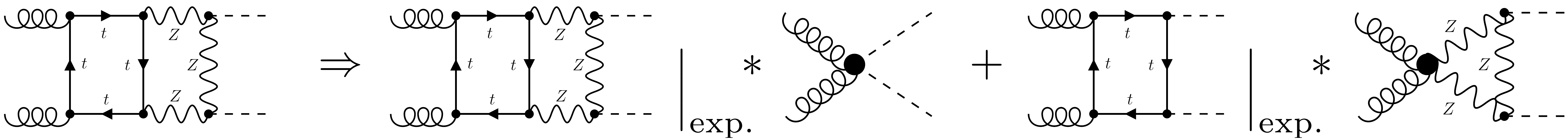
- Sample two-loop diagrams involving SM fields: $\{t, b, H, \gamma, Z, W^\pm, \chi, \phi^\pm\}$ and ghosts: $\{u^\gamma, u^Z, u^\pm\}$



- **Aim:** analytic large-m_t expansion in $\sqrt{s} \leq 300\,\mathrm{GeV}$ region

Large-m_t expansion and EW renormalisation

- Expansion hierarchy: $m_t^2 \gg \xi_W m_W^2, \xi_Z m_Z^2 \gg s, t, m_H^2, m_W^2, m_Z^2$
- Expand and calculate in **general R_ξ gauge**



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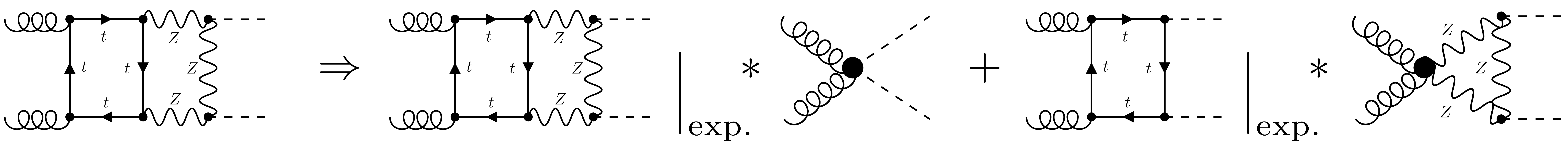


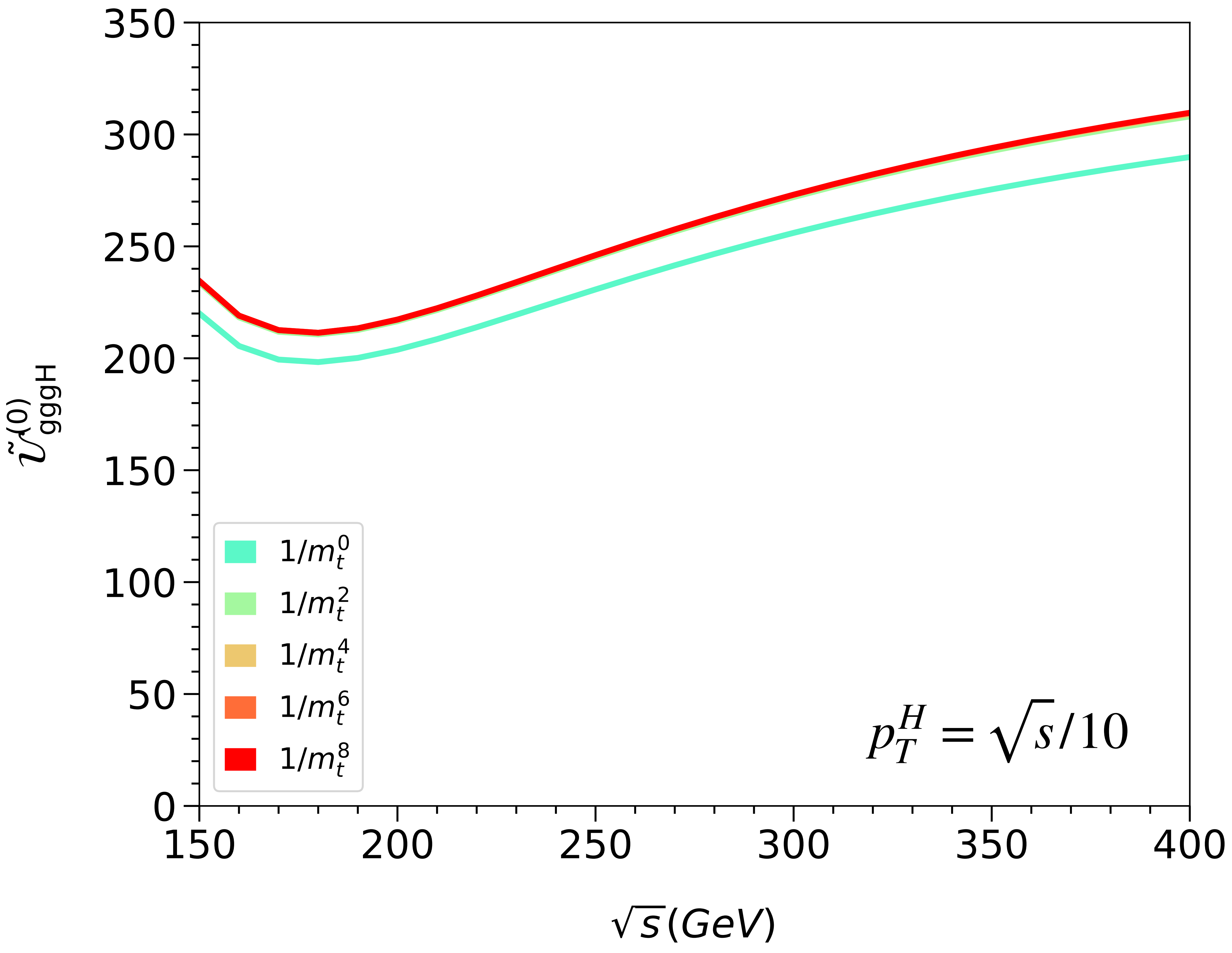
- **On-Shell renormalise** input parameters$\{e, m_W, m_Z, m_t, m_H\}$ in G_μ scheme
- ξ_W, ξ_Z **cancel** after external Higgs **fields OS renormalisation** $\Rightarrow$ gauge parameter independent

Matrix elements for H+jet $(gg \to gH)$ @ LO & NLO QCD

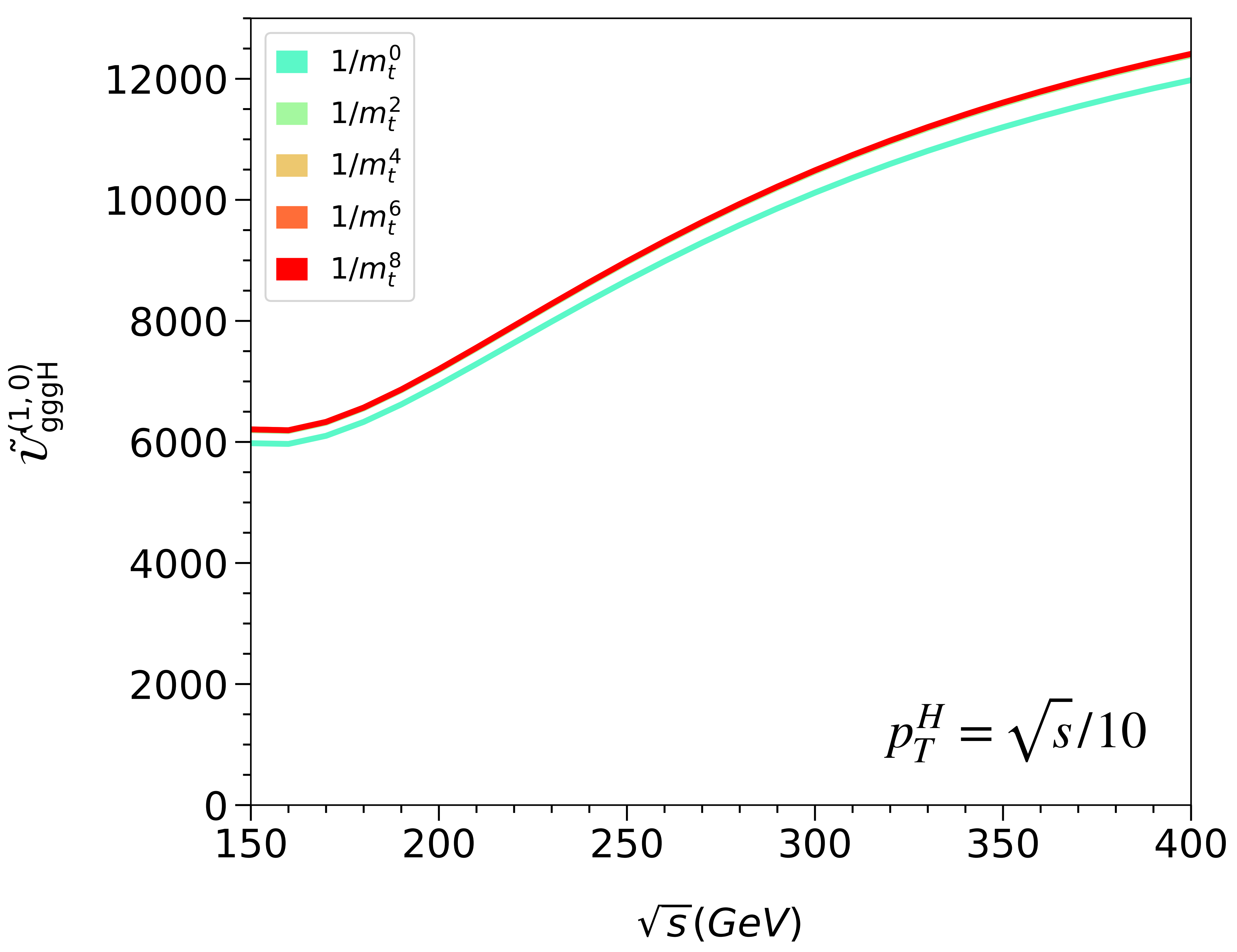
[Davies, Schönwald, Steinhauser, **Zhang**, *JHEP 10 (2023) 033*]

$$\mathcal{M} = \frac{1}{8^2\,2^2}\sum_{\rm col}\sum_{\rm pol}|\mathcal{A}|^2 = \frac{3}{32}\left(X_0^{\rm gggH}\right)^2 s\,\tilde{U}_{\rm gggH}$$

$$\tilde{\mathcal{U}}_{\rm gggH} = \tilde{\mathcal{U}}^{(0)}_{\rm gggH} + \frac{\alpha_s(\mu)}{\pi}\tilde{\mathcal{U}}^{(1,0)}_{\rm gggH} + \frac{\alpha}{\pi}\tilde{\mathcal{U}}^{(0,1)}_{\rm gggH}$$



$\tilde{\mathcal{U}}^{(0)}_{\rm gggH}$ plot @ **LO**



$\tilde{\mathcal{U}}^{(1,0)}_{\rm gggH}$ plot @ **NLO QCD**

Rapid convergence observed

Matrix elements for H+jet $(gg \to gH)$ @ NLO EW

[Davies, Schönwald, Steinhauser, **Zhang**, ***JHEP 10 (2023) 033***]

$$\mathcal{M} = \frac{1}{8^2\,2^2}\sum_{\text{col}}\sum_{\text{pol}} |\mathcal{A}|^2 = \frac{3}{32}\left(X_0^{\text{gggH}}\right)^2 s\,\tilde{U}_{\text{gggH}}$$

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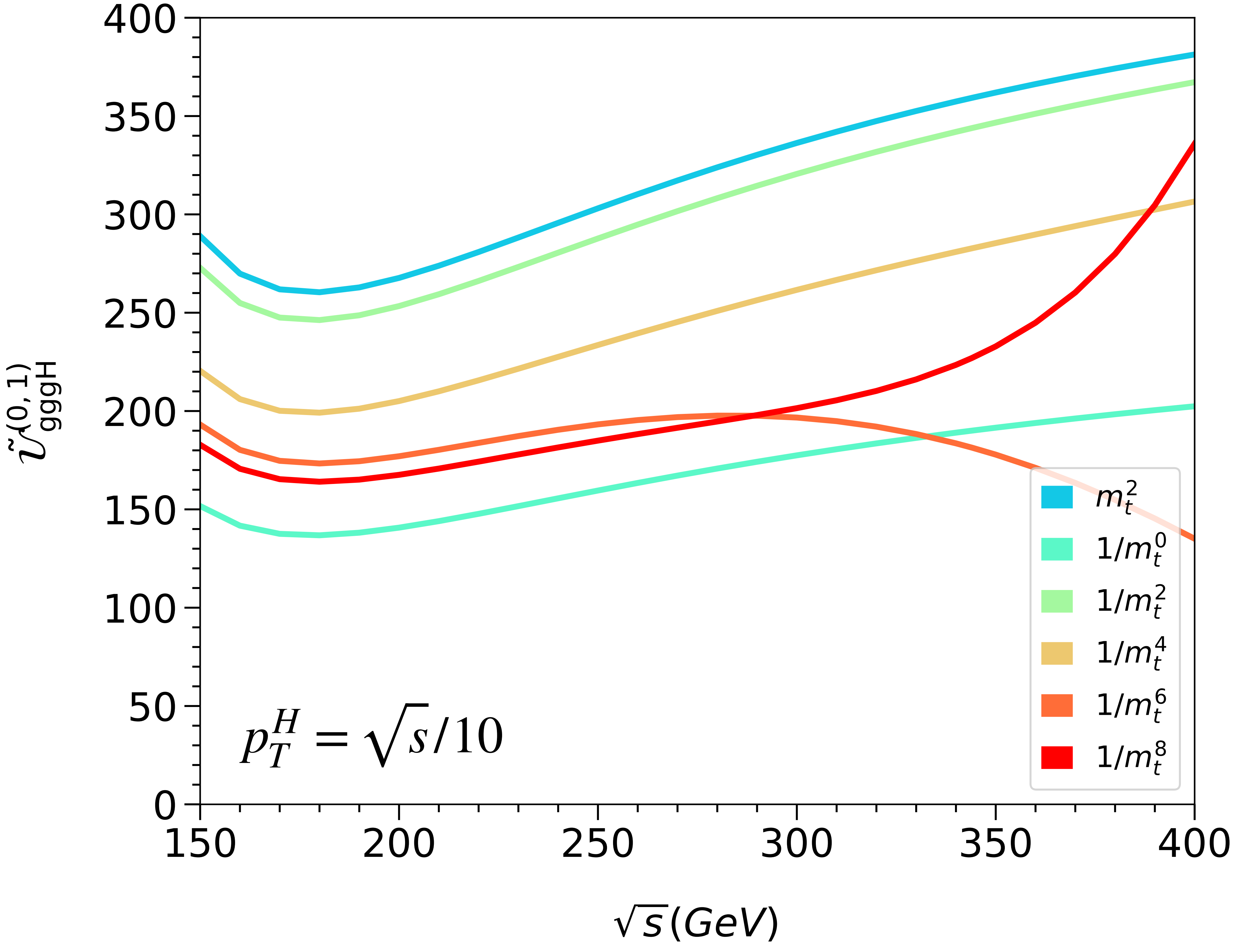
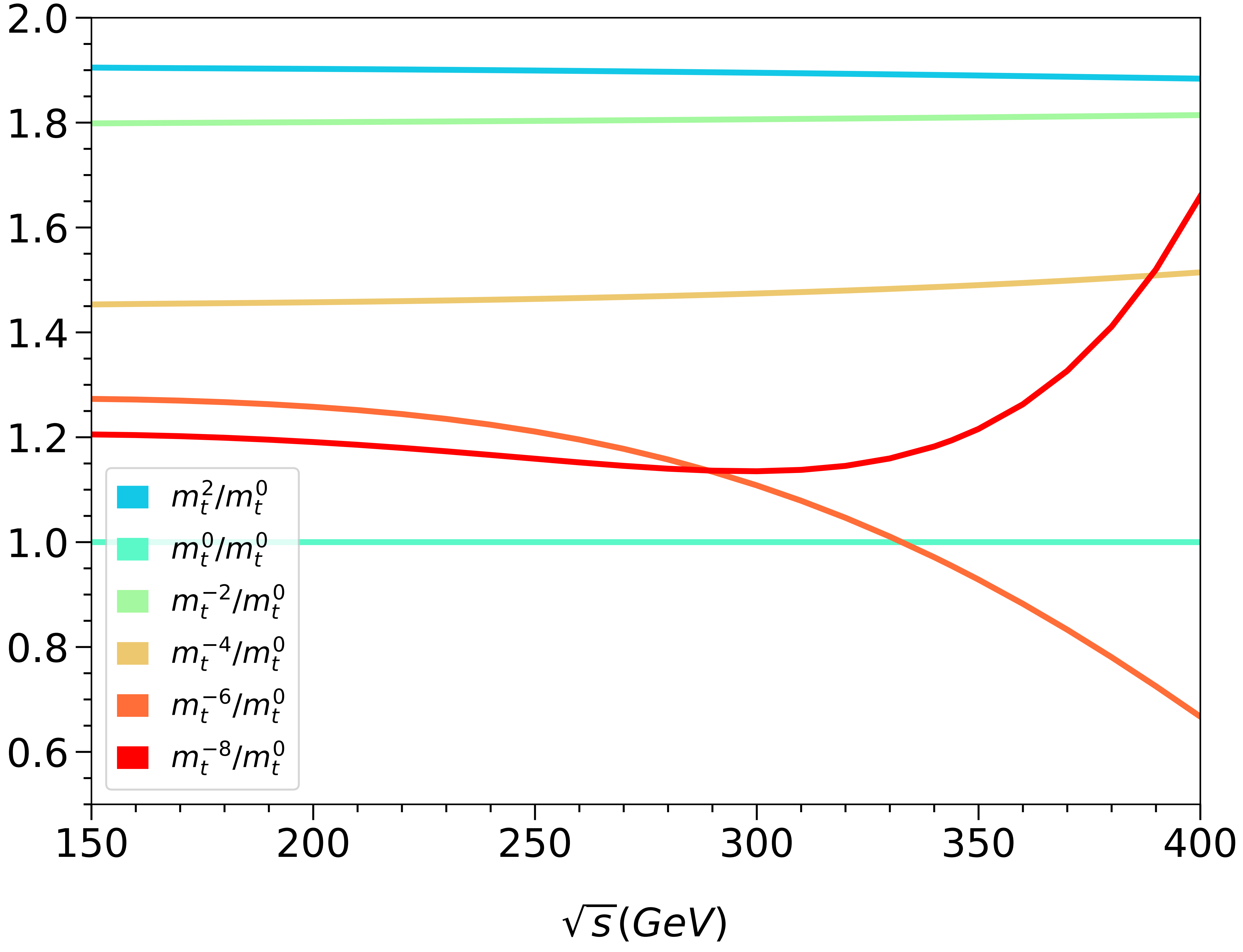


$\tilde{\mathcal{U}}^{(0,1)}_{\text{gggH}}$ plot @ **NLO EW**



$\tilde{\mathcal{U}}^{(0,1)}_{\text{gggH}}$ **convergence** plot normalised to m_t^0

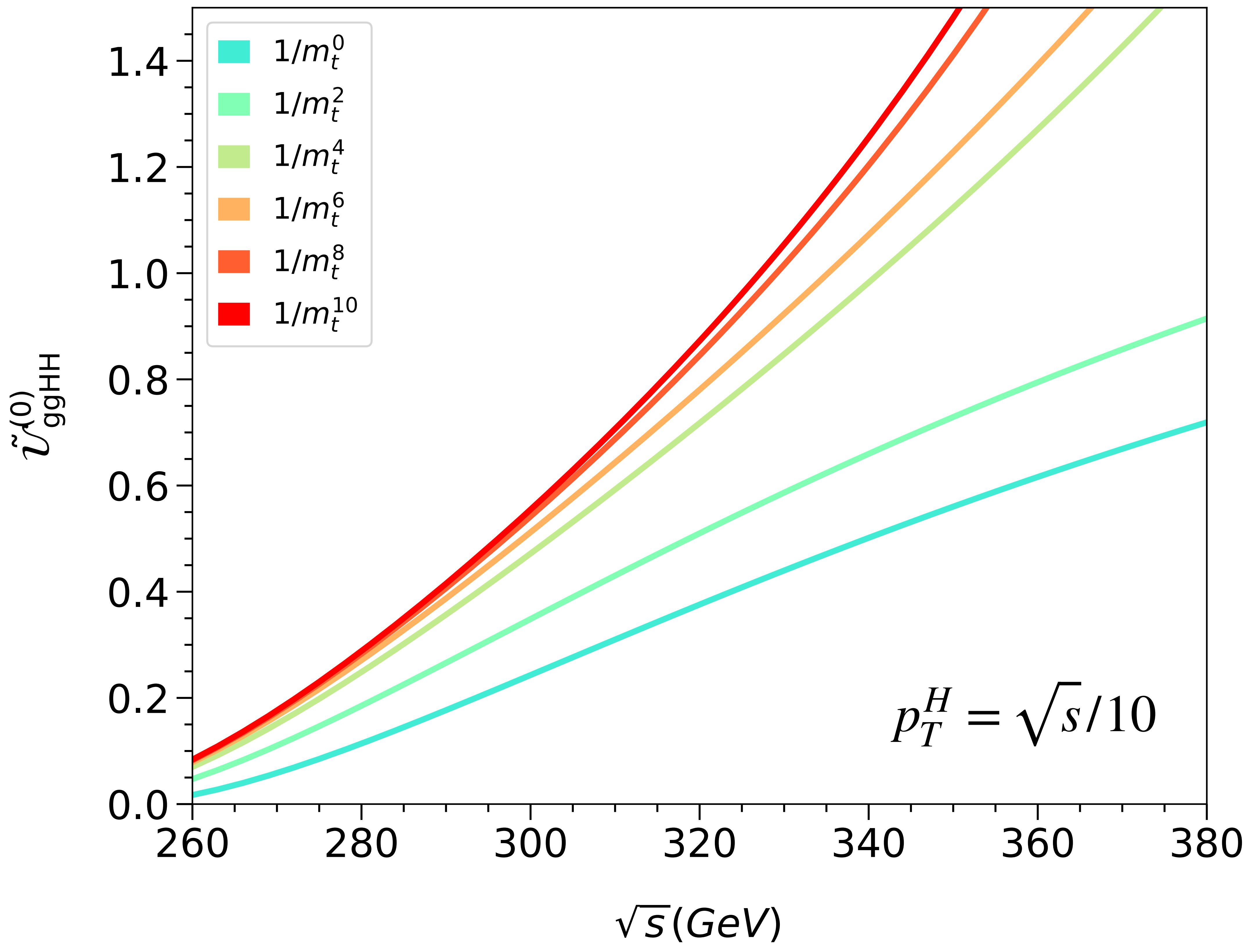
Good convergence observed, but corrections are small $\sim \mathcal{O}(1\%)$

Matrix elements for $gg \to HH$ @ LO

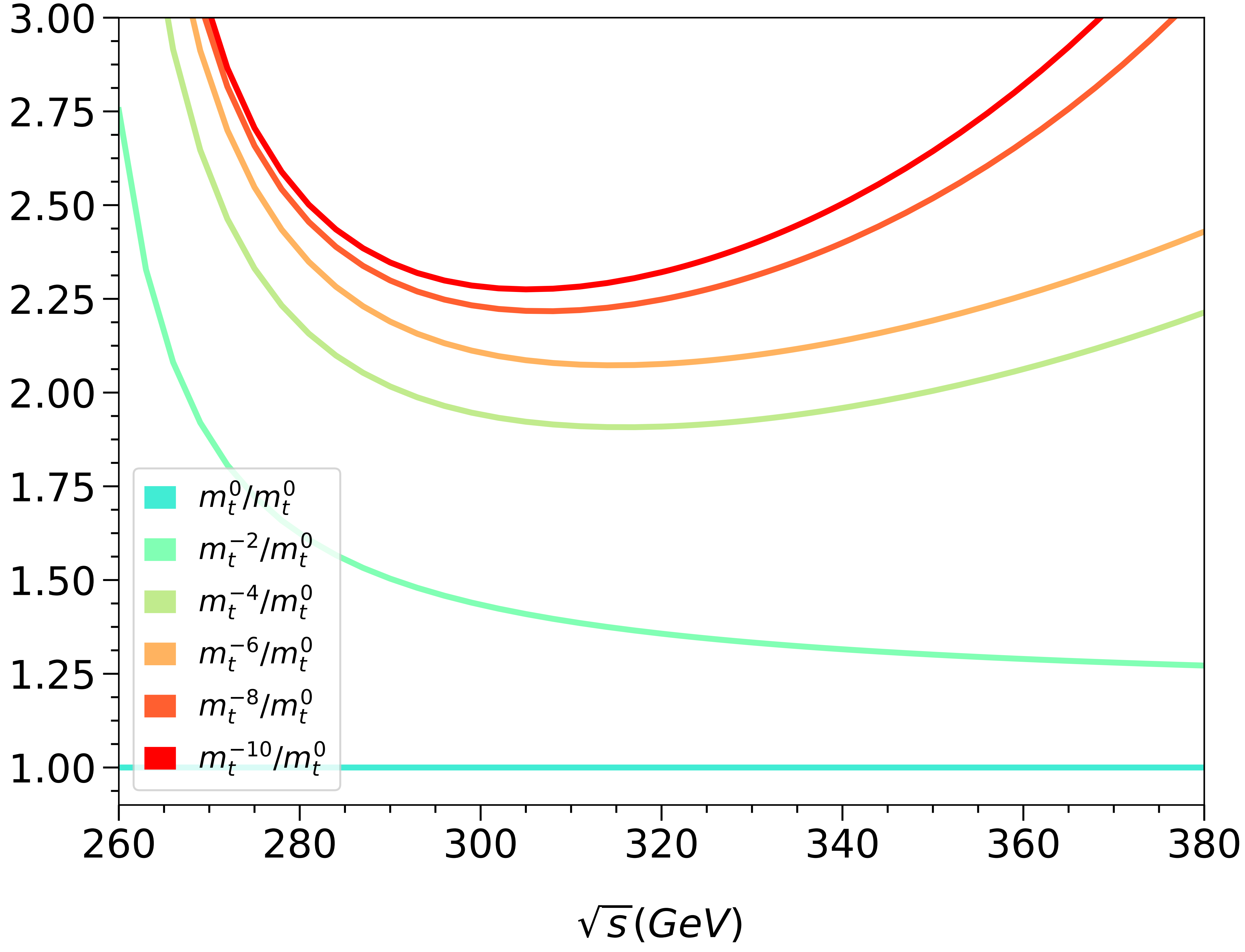
[Davies, Schönwald, Steinhauser, **Zhang**, *JHEP 10 (2023) 033*]

$$\mathcal{M} = \frac{1}{8^2\,2^2}\sum_{\rm col}\sum_{\rm pol}|\mathcal{A}|^2 = \frac{1}{16}\left(X_0^{\rm ggHH}\,s\right)^2 \tilde{U}_{\rm ggHH}$$

$$\tilde{\mathcal{U}}_{\rm ggHH} = \tilde{\mathcal{U}}^{(0)}_{\rm ggHH} + \frac{\alpha}{\pi}\tilde{\mathcal{U}}^{(0,1)}_{\rm ggHH}$$



$\tilde{\mathcal{U}}^{(0)}_{\rm ggHH}$ @ **LO**



$\tilde{\mathcal{U}}^{(0)}_{\rm ggHH}$ **convergence** plot normalised to m_t^0

Good convergence observed

Matrix elements for $gg \to HH$ @ NLO EW

[Davies, Schönwald, Steinhauser, **Zhang**, ***JHEP 10 (2023) 033***]

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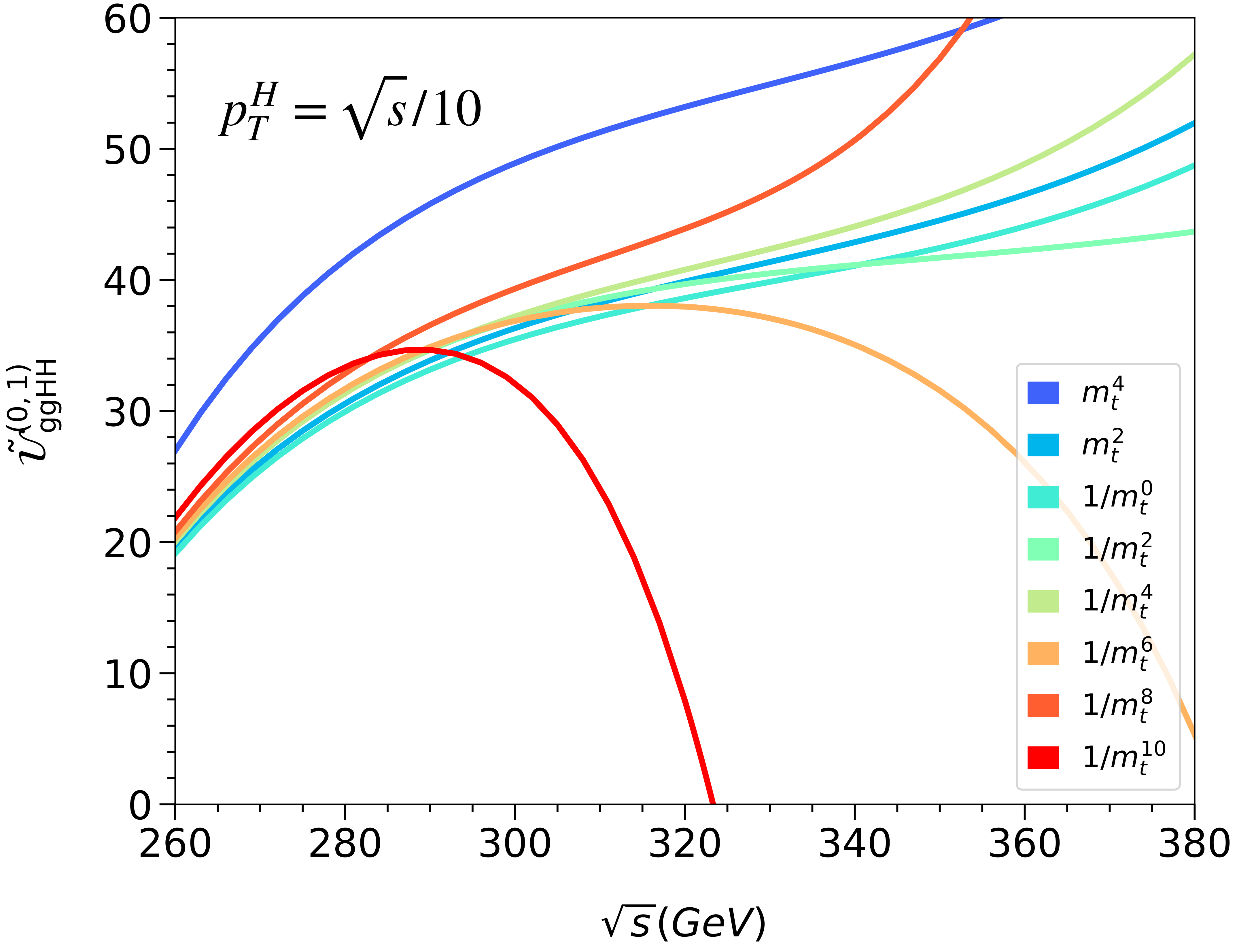
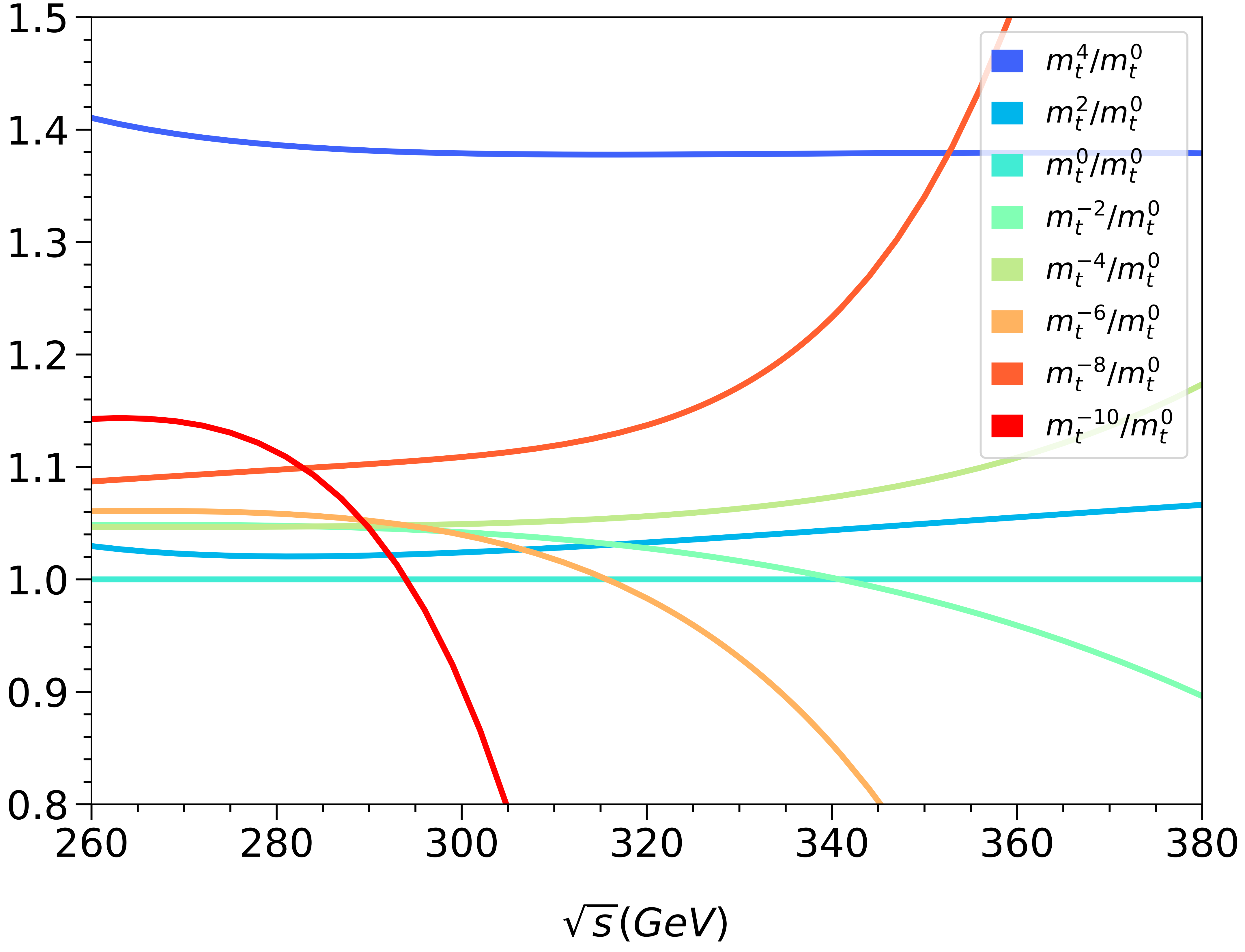


$\tilde{\mathcal{U}}^{(0,1)}_{\rm ggHH}$ plot up @ **NLO EW**



$\tilde{\mathcal{U}}^{(0,1)}_{\rm ggHH}$ **convergence** plot normalised to m_t^0

Convergence not good

Matrix elements for $gg \to HH$ @ NLO EW

[Davies, Schönwald, Steinhauser, **Zhang**, ***JHEP 10 (2023) 033***]

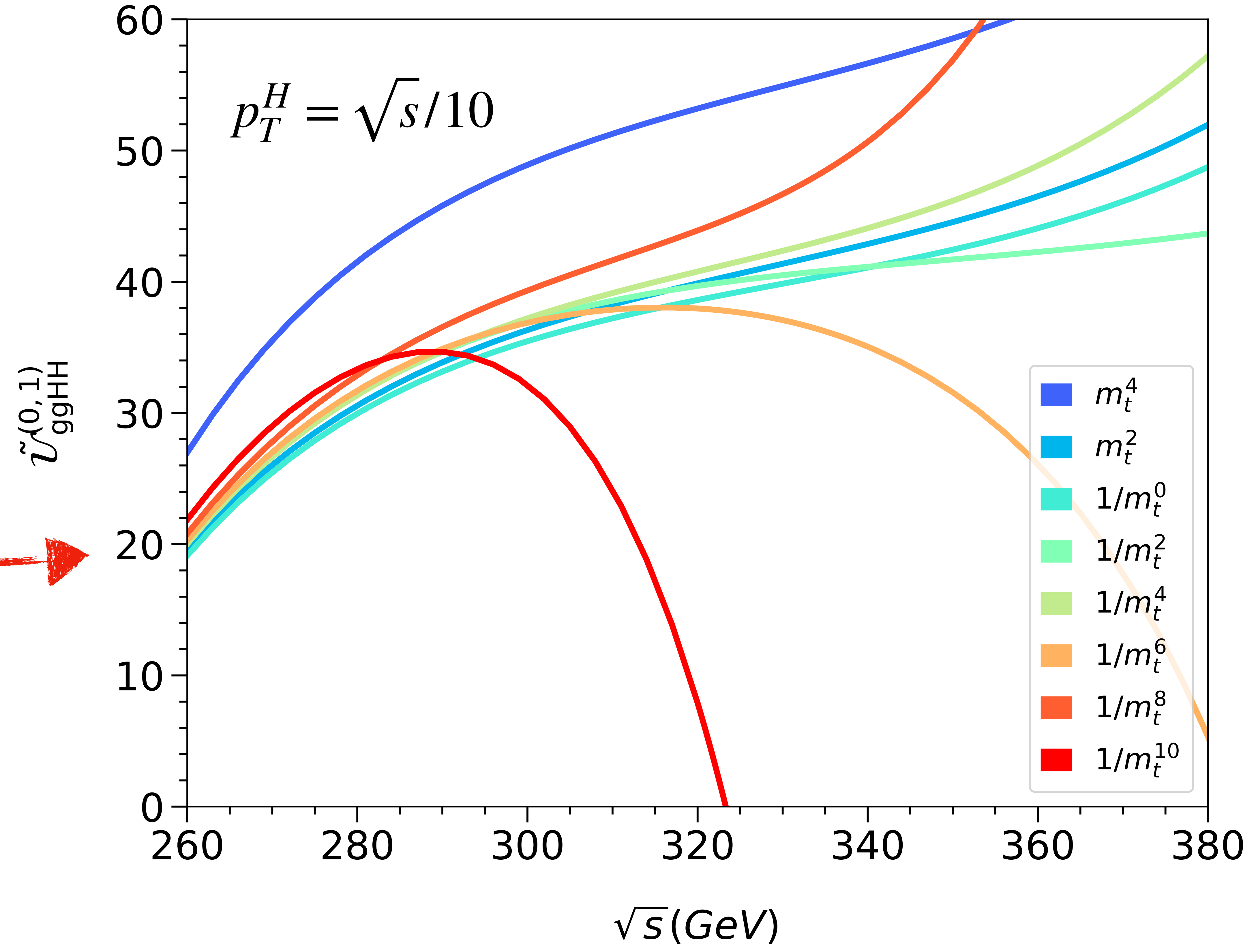
$$\mathscr{M} = \frac{1}{8^2\,2^2} \sum_{\text{col}} \sum_{\text{pol}} |\mathscr{A}|^2 = \frac{1}{16}\left(X_0^{\text{ggHH}}\, s\right)^2 \tilde{U}_{\text{ggHH}}$$

$$\tilde{\mathscr{U}}_{\text{ggHH}} = \tilde{\mathscr{U}}^{(0)}_{\text{ggHH}} + \frac{\alpha}{\pi}\tilde{\mathscr{U}}^{(0,1)}_{\text{ggHH}}$$

Famous di-Higgs destructive interference (vanishing ME at production threshold) is **lifted !!**

3-loop QCD corrections also **lifts** this destructive interference

[Grigo, Melnikov, Steinhauser, 14']

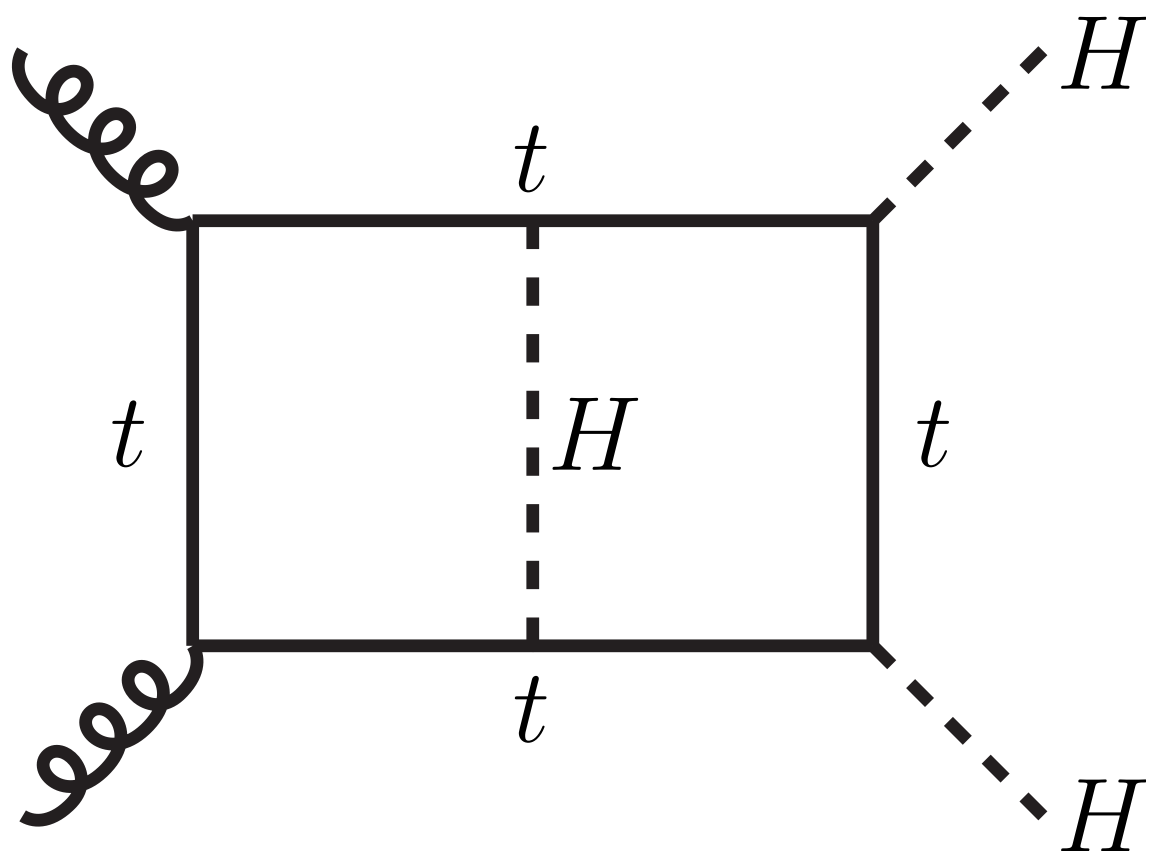


$\tilde{\mathscr{U}}^{(0,1)}_{\text{ggHH}}$ plot up to different expansion order $1/m_t^n$

Part 2: Leading Yukawa corrections at high energies

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP* 08 (2022) 259**]

- Sample two-loop diagrams contributing to leading Yukawa-top corrections $\sim \alpha_s y_t^4$

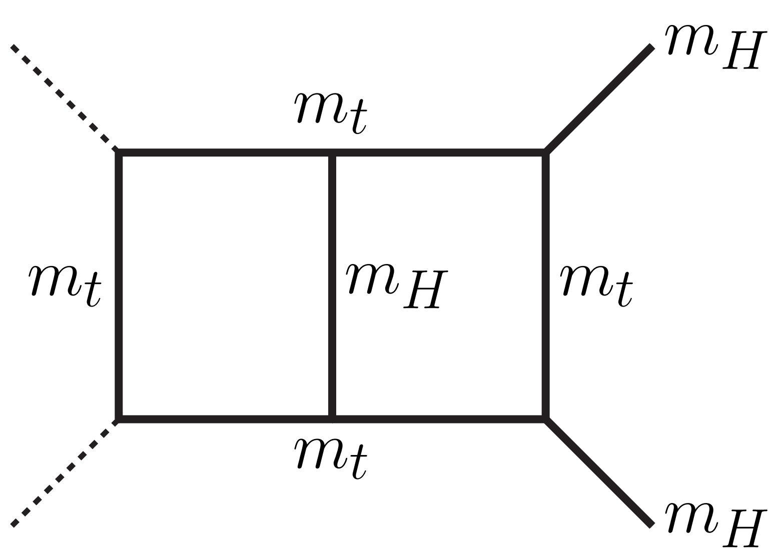


- Yukawa-top corrections are **not small**, $y_t^2 = \dfrac{\alpha\, m_t^2}{2\sin^2\theta_w\, m_W{}^2}$
- **Aim:** analytic high-energy expansion in $p_T^H \geq 120\,\mathrm{GeV}$ region

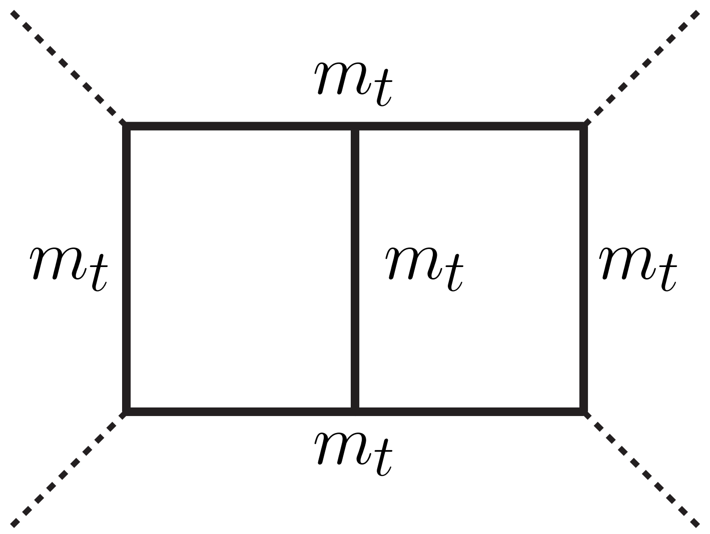
Analytic high-energy expansion to $\mathcal{O}(m_t^{120})$

1. **Expansion hierarchy:** $\left\{ s, t \gg m_t^2 \approx \left(m_H^{\text{int}}\right)^2 \gg \left(m_H^{\text{ext}}\right)^2 \right\}$ or $\left\{ s, t \gg m_t^2 \gg \left(m_H^{\text{int}}\right)^2, \left(m_H^{\text{ext}}\right)^2 \right\}$

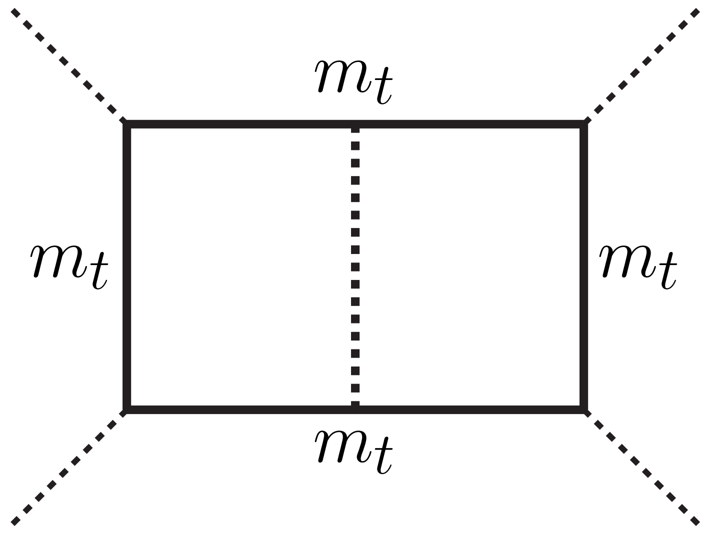
scalar master integral



kinematic invariants s, t



$s, t \gg m_t^2 \approx \left(m_H^{\text{int}}\right)^2 \gg \left(m_H^{\text{ext}}\right)^2$



$s, t \gg m_t^2 \gg \left(m_H^{\text{int}}\right)^2, \left(m_H^{\text{ext}}\right)^2$

Analytic techniques: Asymptotic expansions -> Differential equations -> Mellin-Barnes integrals

Analytic high-energy expansion to $\mathcal{O}(m_t^{120})$

1. **Expansion hierarchy:** $\left\{ s, t \gg m_t^2 \approx \left(m_H^{\text{int}}\right)^2 \gg \left(m_H^{\text{ext}}\right)^2 \right\}$

2. **System of differential equations for 140 Master Integrals** from IBP reduction

$$\frac{\partial}{\partial\left(m_t^2\right)} \mathbf{I} = M(s, t, m_t^2, \epsilon)\, \mathbf{I} \quad \text{with} \quad \mathbf{I} = \left(\mathcal{I}_1, \ldots, \mathcal{I}_{140}\right)^T$$

3. Plug in **power-log ansatz** for each master integral

$$\mathcal{I}_n = \sum C_{(n)}^{ijk}(s, t)\, \epsilon^i \left[m_t^2\right]^j \left[\log(m_t^2)\right]^k$$

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Novel Math. method & Automated algorithm

4. Solve **boundary master integrals in asymptotic limit $m_t \to 0$** with Mellin-Barnes method by **`AsyInt`** [Schönwald, **Zhang**]

   with help of **`asy.m`** [Smirnov], **`MB.m`** [Czakon], **`HarmonicSums.m`** [Ablinger], **`Sigma.m`** and **`EvaluateMultiSums.m`** [Schneider]

Analytic high-energy expansion to $\mathcal{O}(m_t^{120})$

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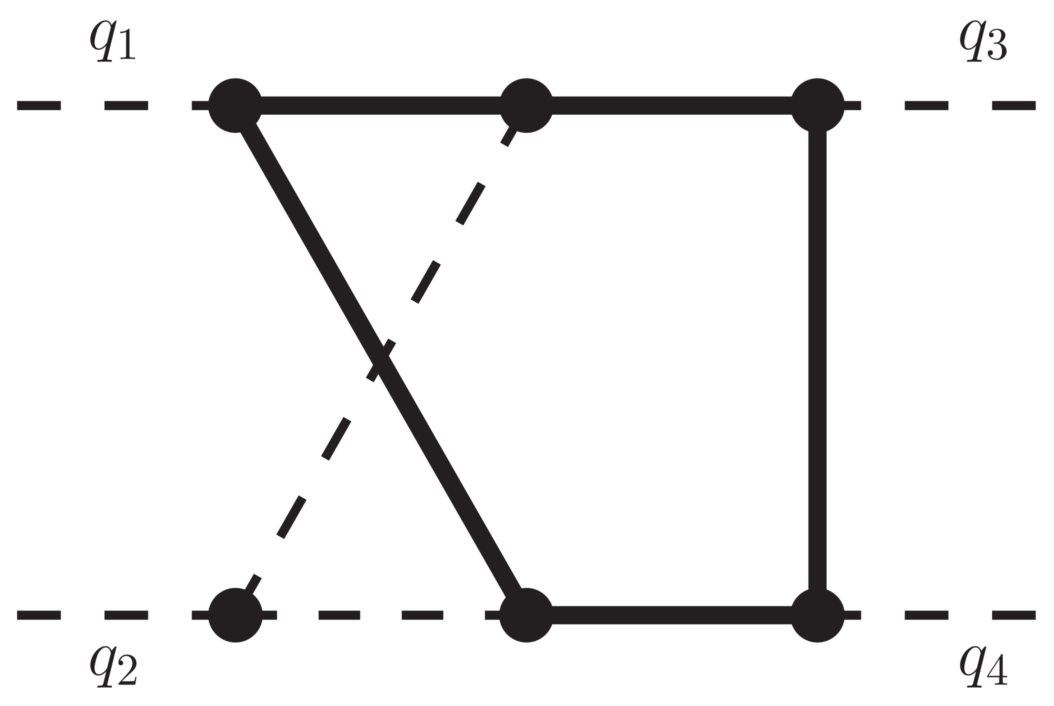
5. Apply **Padé approximations** at the level of form factors as a **precision tool**

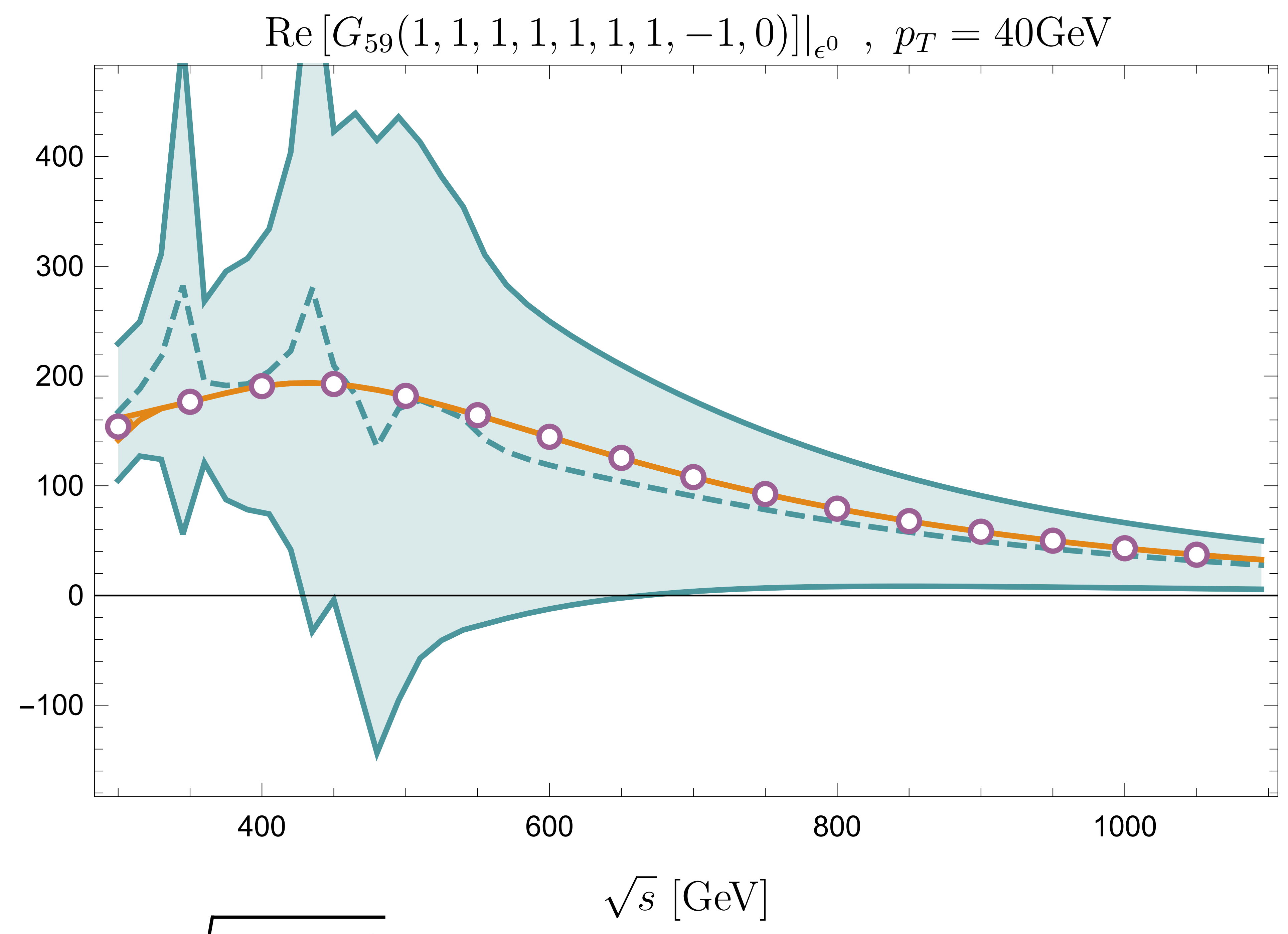
Padé as precision tool: showcase of QCD integrals

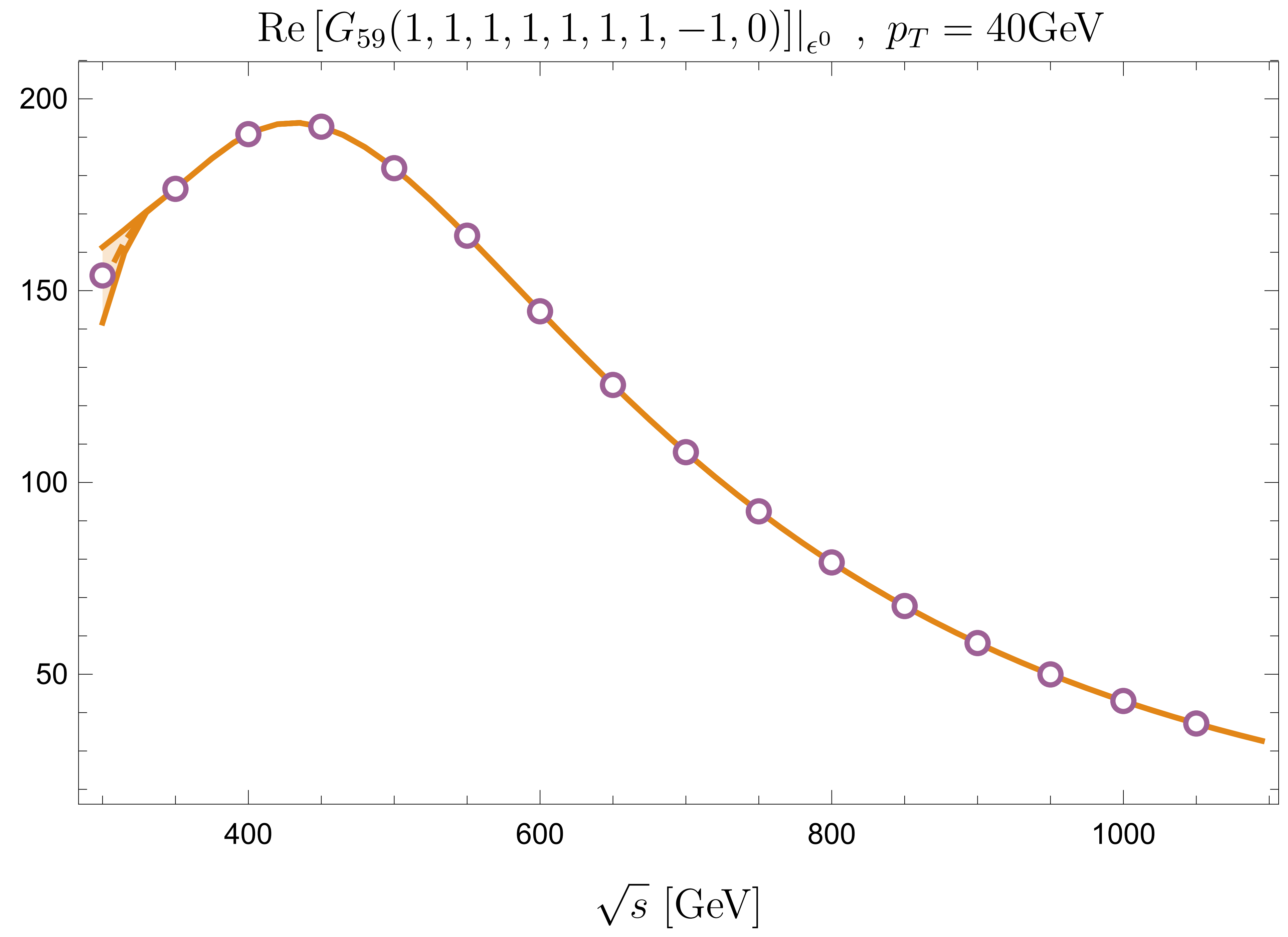
[Davies, Mishima, Schönwald, Steinhauser, ***JHEP 06 (2023) 063***]



Padé approximation:

$$\mathscr{F}^N = \lim_{x\to 1} \sum_{i=0}^{N} f_i\,(m_t^2)^i\, x^i \quad \Rightarrow \quad \mathscr{F}^N = \lim_{x\to 1} \frac{a_0 + a_1 x + \ldots + a_n x^n}{1 + b_1 x + \ldots + b_m x^m} = \lim_{x\to 1} [n/m](x)$$



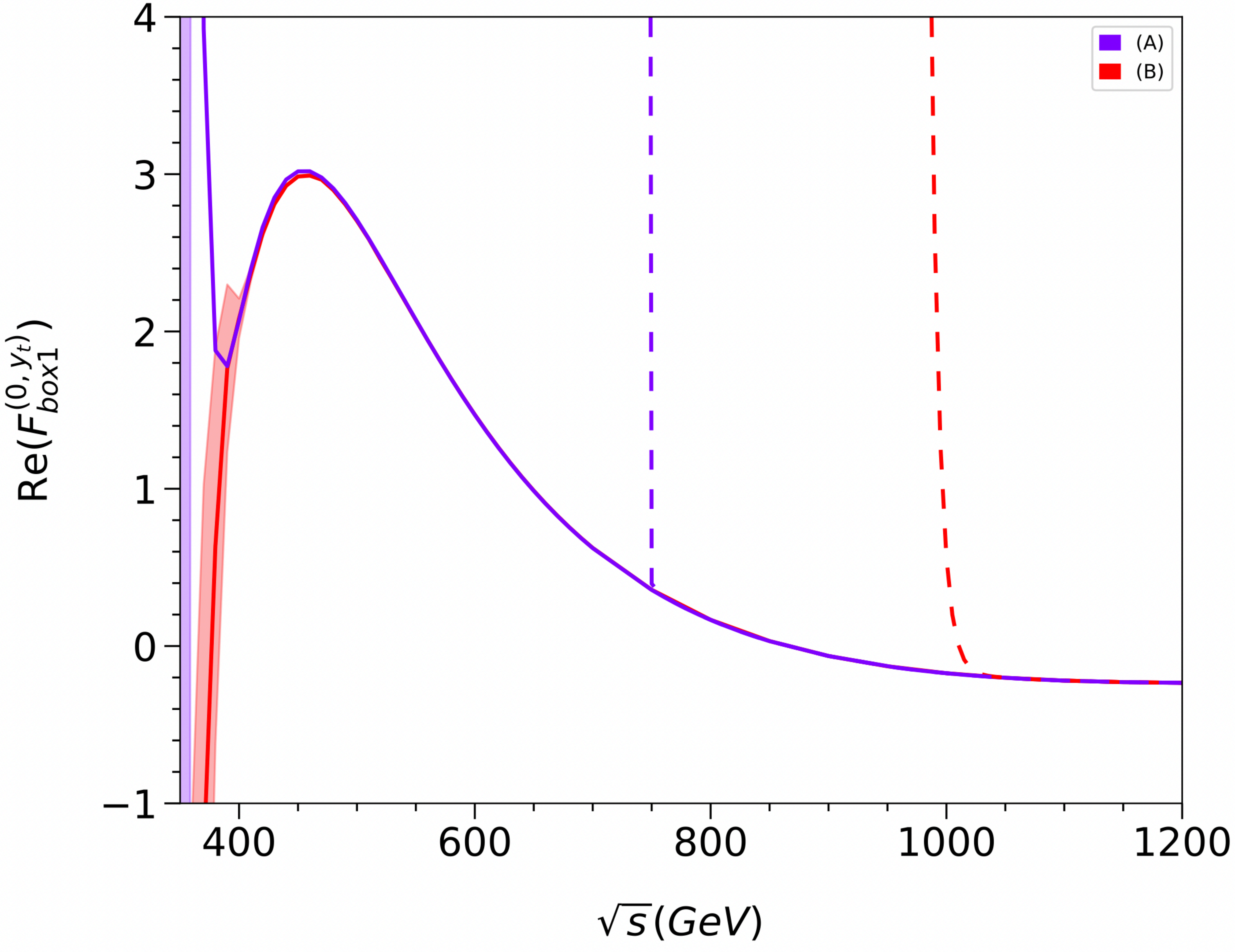


$$p_T^H = \sqrt{\frac{u\,t - m_H^4}{s}}$$

Padé(14, 16) Padé(49, 56) FIESTA

Form factors for $gg \to HH$ @ NLO Yukawa

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]



$$\mathscr{A}^{\mu\nu} = T_1^{\mu\nu}\,\mathscr{F}_1 + T_2^{\mu\nu}\,\mathscr{F}_2^{\mu\nu}$$

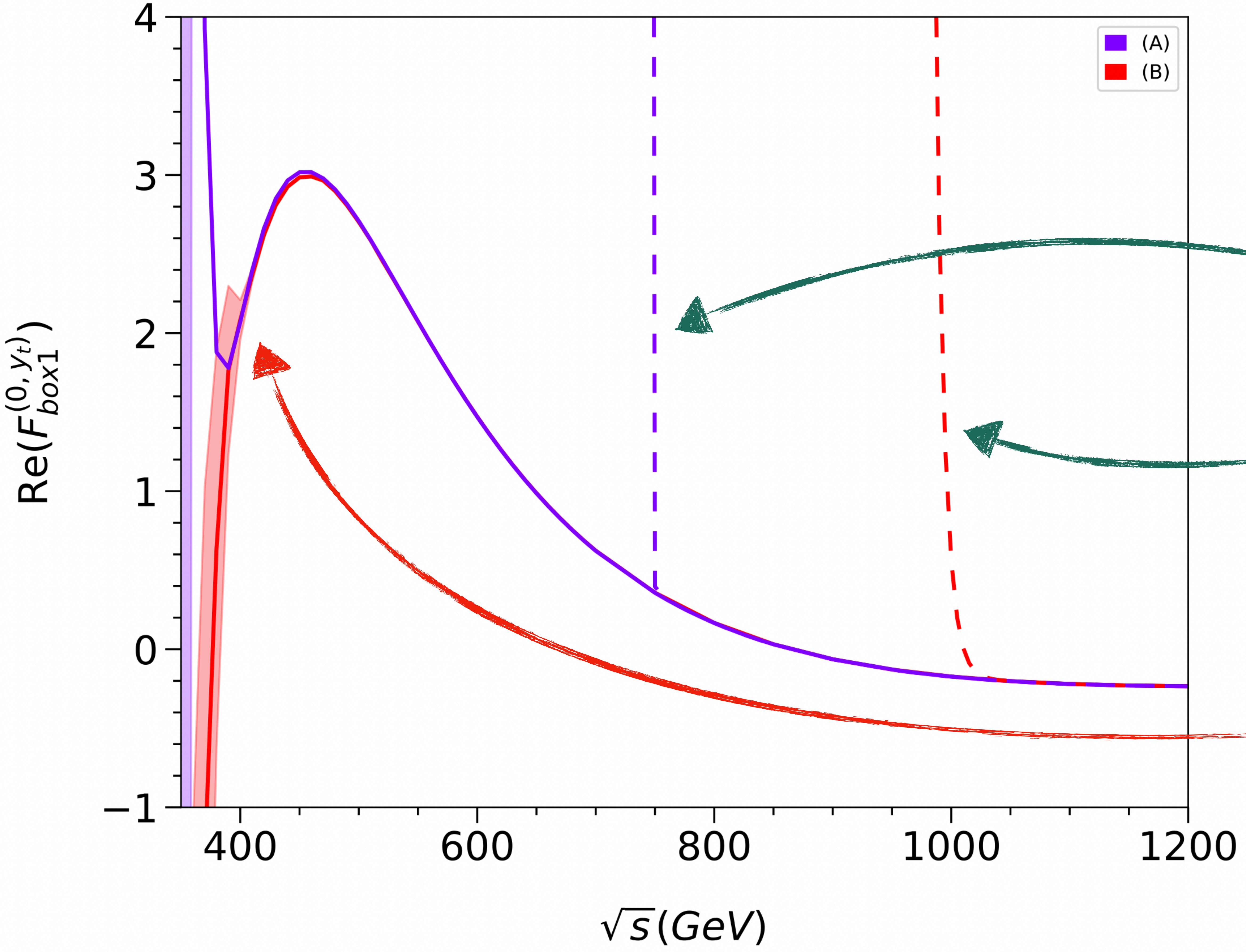
Fixed scattering angle: $\theta = \dfrac{\pi}{2}$

Solid color lines: Padé improved results using MIs from $\mathcal{O}(m_t^{116})$ in two expansion approaches

Dashed color lines: Naive expansions at high energies to $\mathcal{O}(m_t^{116})$

Form factors for $gg \to HH$ @ NLO Yukawa

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, *JHEP 08 (2022) 259*]



$$\mathscr{A}^{\mu\nu} = T_1^{\mu\nu}\mathscr{F}_1 + T_2^{\mu\nu}\mathscr{F}_2^{\mu\nu}$$

Fixed scattering angle: $\theta = \dfrac{\pi}{2}$

Naive high-energy expansions starts to diverge at $\sqrt{s} \approx 700$ & 1000 GeV

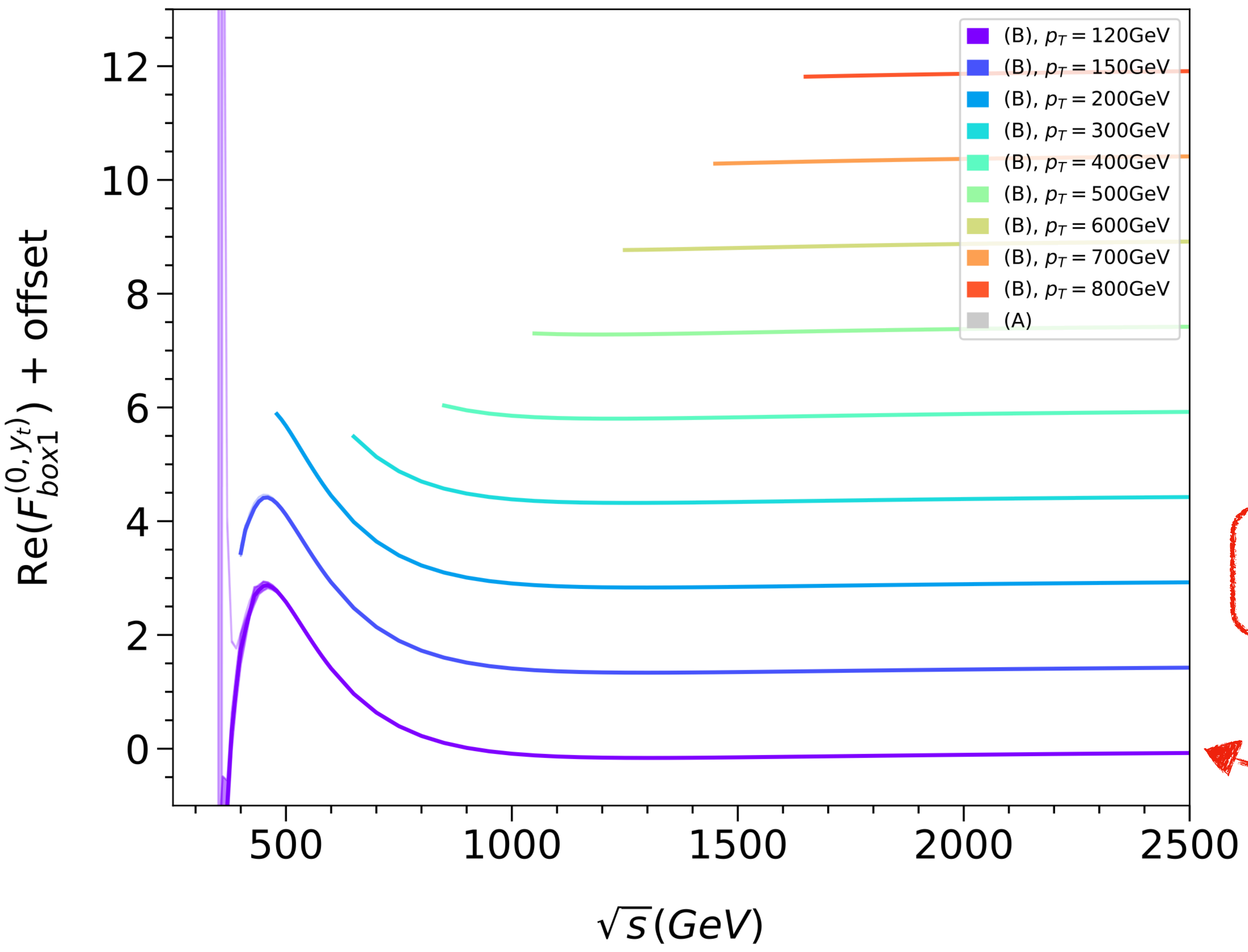
Padé improved high-energy expansions converge even at $\sqrt{s} \approx 400$ GeV

Solid color lines: Padé improved results using MIs from $\mathcal{O}(m_t^{116})$ in two expansion approaches

Dashed color lines: Naive expansions at high energies to $\mathcal{O}(m_t^{116})$

Form factors for $gg \to HH$ @ NLO Yukawa

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]



$$\mathscr{A}^{\mu\nu} = T_1^{\mu\nu}\,\mathscr{F}_1 + T_2^{\mu\nu}\,\mathscr{F}_2^{\mu\nu}$$

$$p_T^H = \sqrt{\frac{u\,t - m_H^4}{s}}$$

Padé improved high energy expansions converge even at $p_T^H = 120$ GeV

Color lines: Padé improved equal-mass δ expansions in $m_t^2 \approx \left(m_H^{\text{int}}\right)^2$ using MIs from $\mathcal{O}(m_t^{116})$

Grey lines: Coincide with colourful lines (two approaches agree)

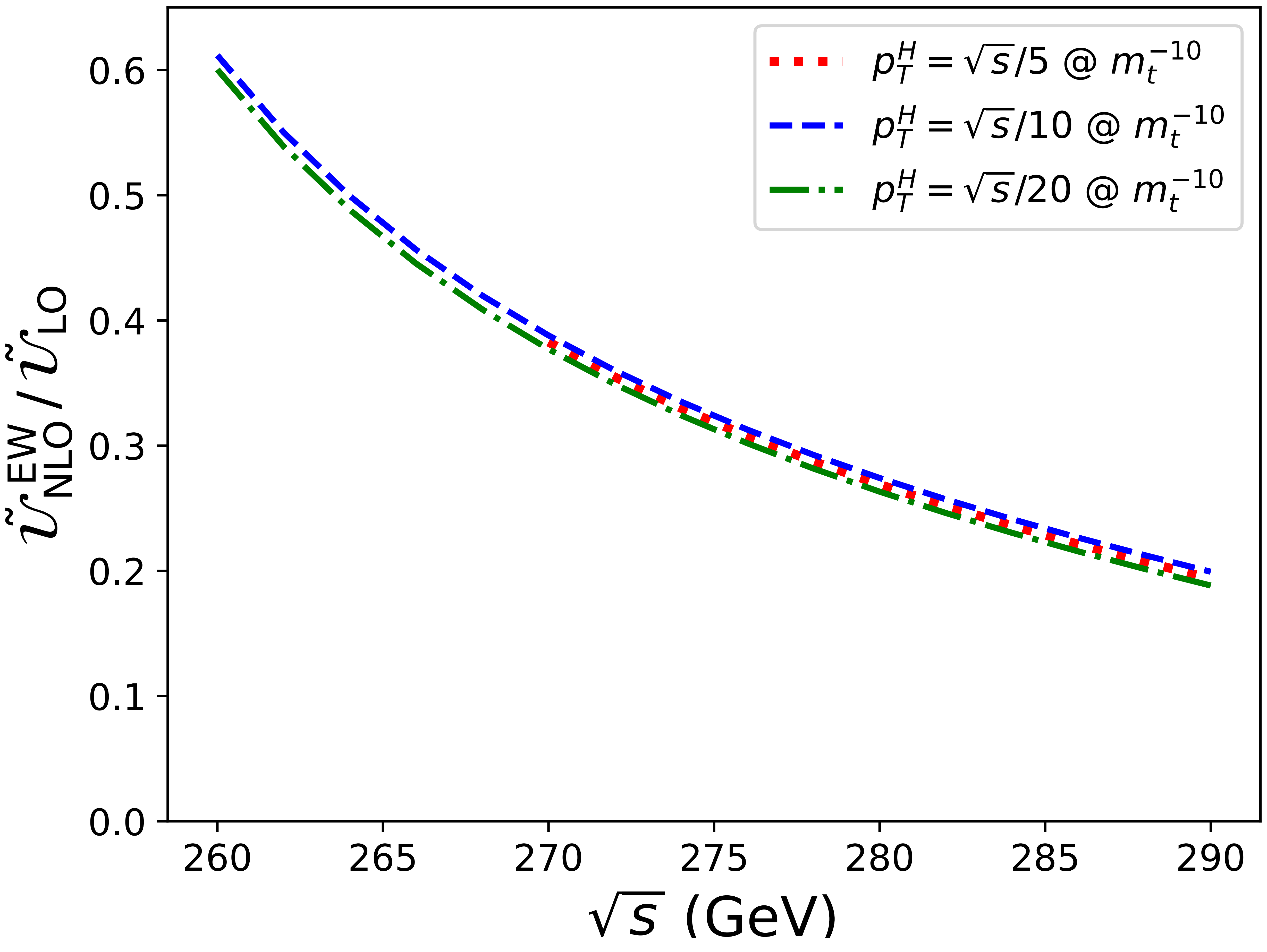
Conclusions and outlook

JHEP 08 (2022) 259 & JHEP 10 (2023) 033

- We analytically compute **NLO leading Yukawa corrections** to $gg \to HH$ in high-energy expansion
  - **Precise results for $p_T^H > 120$ GeV**
- We analytically compute **full NLO EW corrections** to $gg \to HH$ in large-m_t expansion
  - Destructive di-Higgs interference at production threshold is **lifted**
  - EW corrections can reach **a few tens of percent (positive)** w.r.t LO in this region
- We also consider **full NLO EW corrections** to $gg \to gH$ in large-m_t expansion
  - Good convergence observed, corrections are small
- Future work: complete NLO EW corrections to $gg \to HH$ in whole phase space region by including t-expansion.

Backup Slides

Ratio plot of matrix elements for NLO EW / LO



$$\mathcal{M} = \frac{1}{8^2\, 2^2} \sum_{\text{col}} \sum_{\text{pol}} |\mathcal{A}|^2 = \frac{1}{16} \left(X_0^{\text{ggHH}}\, s\right)^2 \tilde{U}_{\text{ggHH}}$$

$$\tilde{\mathcal{U}}_{\text{ggHH}} = \tilde{\mathcal{U}}^{(0)}_{\text{ggHH}} + \frac{\alpha}{\pi} \tilde{\mathcal{U}}^{(0,1)}_{\text{ggHH}}$$

$$\frac{\tilde{\mathcal{U}}^{\text{EW}}_{\text{NLO}}}{\tilde{\mathcal{U}}_{\text{LO}}} = \frac{\alpha}{\pi} \frac{\tilde{\mathcal{U}}^{(0,1)}_{\text{ggHH}}}{\tilde{\mathcal{U}}^{(0)}_{\text{ggHH}}}$$

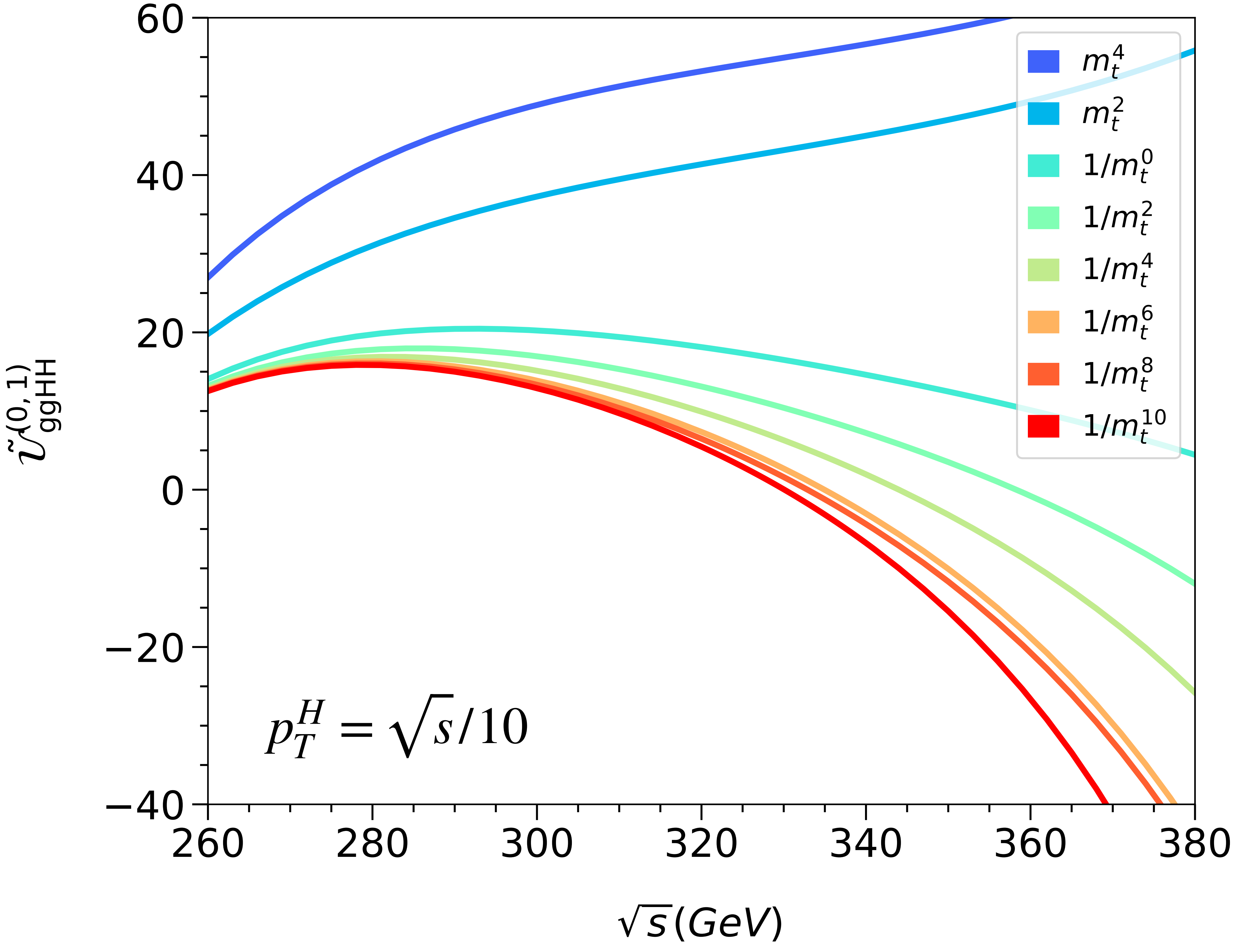
- Size of NLO EW corrections (positive) can easily reach $\mathcal{O}(10\%)$ w.r.t. LO at low-energy region
- EW effects are expected to be significant in high-energy region (stay tuned to our future papers)

Matrix elements for $gg \to HH$ @ NLO EW

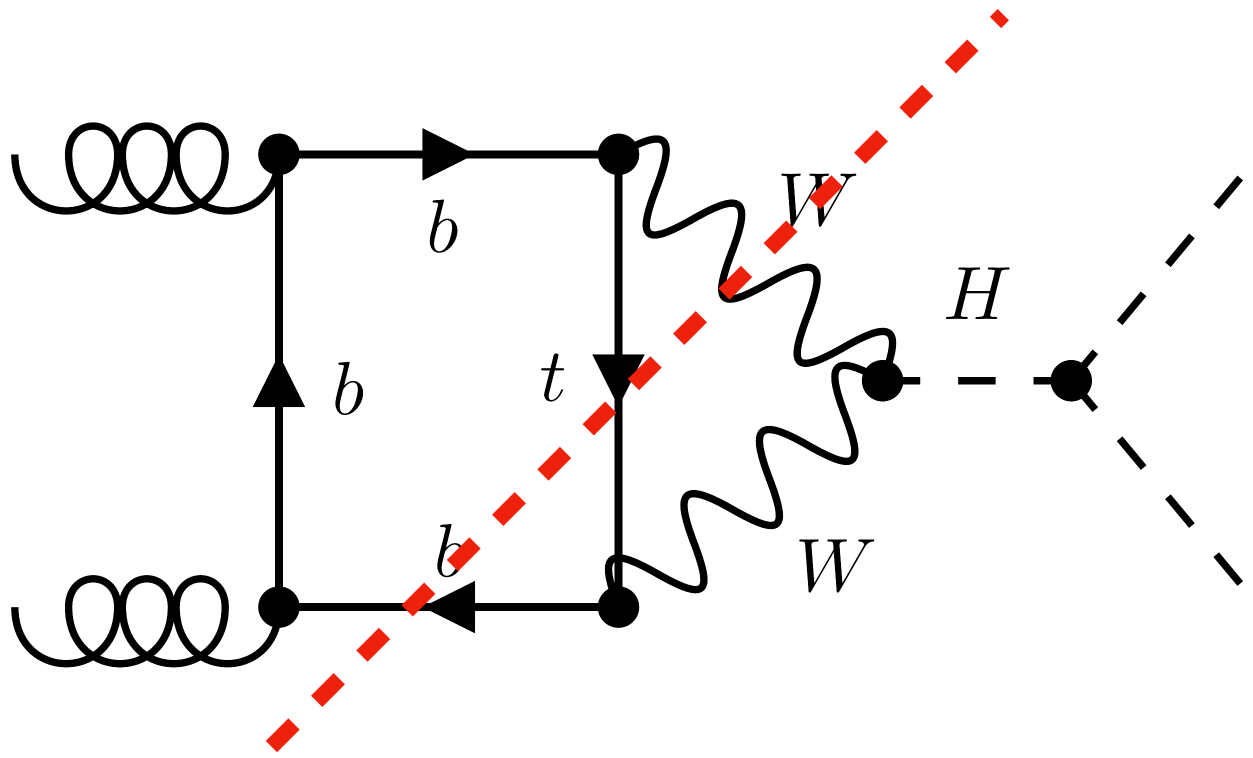
[Davies, Schönwald, Steinhauser, **Zhang**, ***JHEP 10 (2023) 033***]

$$\mathscr{M} = \frac{1}{8^2\,2^2}\sum_{\text{col}}\sum_{\text{pol}}|\mathscr{A}|^2 = \frac{1}{16}\left(X_0^{\text{ggHH}}\,s\right)^2\tilde{U}_{\text{ggHH}}$$

$$\tilde{\mathcal{U}}_{\text{ggHH}} = \tilde{\mathcal{U}}^{(0)}_{\text{ggHH}} + \frac{\alpha}{\pi}\tilde{\mathcal{U}}^{(0,1)}_{\text{ggHH}}$$



$\tilde{U}_{\text{ggHH}}$ plot up to different expansion order $1/m_t^n$

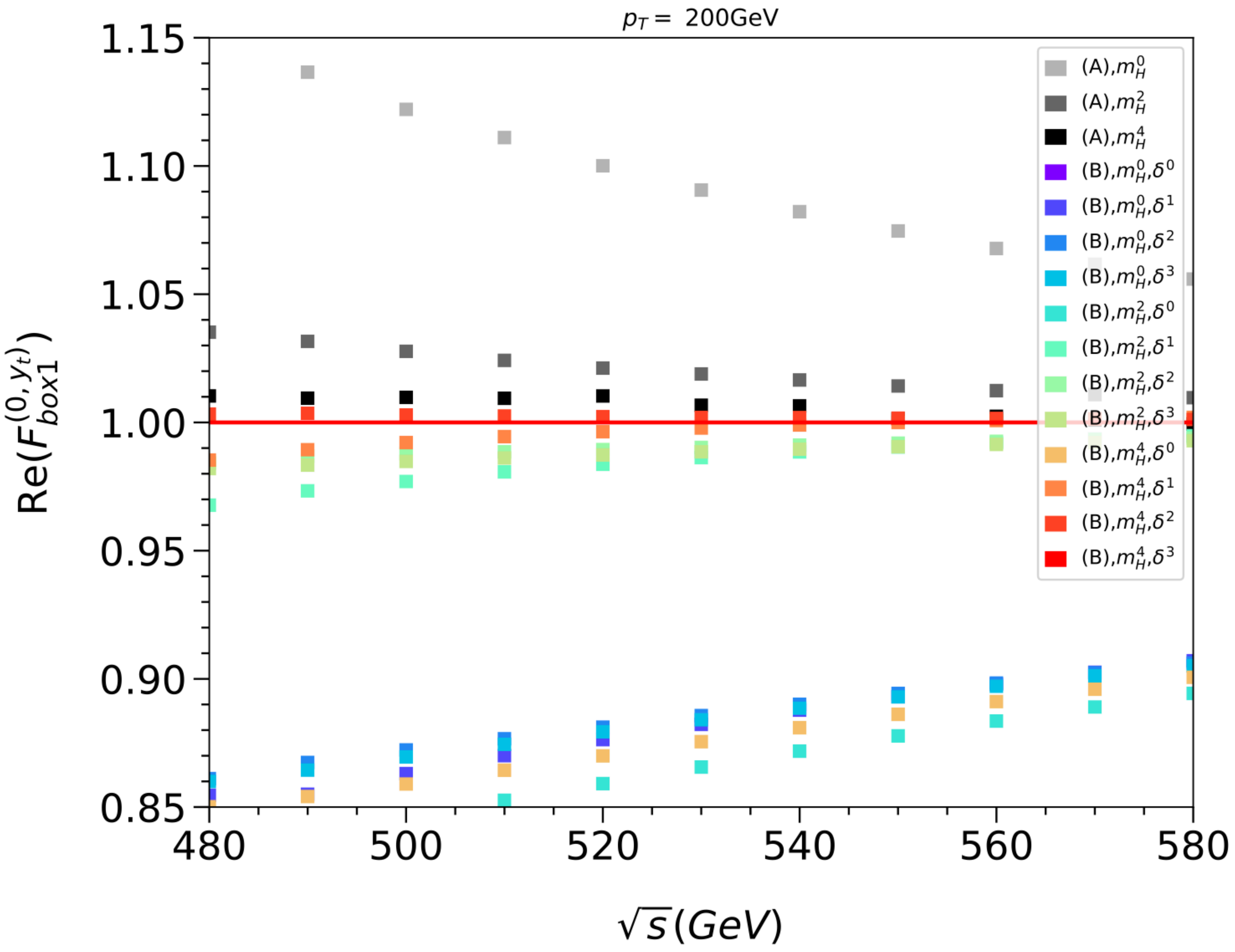


Cut through W-t-b worsen convergence at $m_t + m_b + m_W \approx 250\,\text{GeV}$

Good convergence restored by excluding W-t-b contributions

Convergence of expansions for $gg \to HH$ form factors

[Davies, Mishima, Schönwald, Steinhauser, **Zhang**, ***JHEP 08 (2022) 259***]



$$\mathscr{A}^{\mu\nu} = T_1^{\mu\nu}\,\mathscr{F}_{\mathrm{box1}} + T_2^{\mu\nu}\,\mathscr{F}^{\mu\nu}_{\mathrm{box2}}$$

The benchmark is expansion at $\mathcal{O}\left(m^4_{H(\mathrm{ext})}, \delta^3, m_t^{116}\right)$.

$$\delta = 1 - \frac{m_H^{(\mathrm{int})}}{m_t}$$

Color points: Convergence plot of different expansion orders by ratios to the benchmark at fixed $p_T^H = 200$ GeV.