

# 3HDM with a CP symmetry of order 4: a phenomenological update

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Based on: [Zhao, Ivanov, Pasechnik, Zhang, JHEP 04 \(2023\) 116 = arXiv:2302.03094](#)  
[Liu, Ivanov, Gonçalves, work in progress, and earlier papers.](#)



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# $N$ -Higgs-doublet models

Higgses can come in **generations**  $\rightarrow$  NHDMs [T.D.Lee 1973, Weinberg 1976, ...]

SM

|                              |                            |                            |                    |
|------------------------------|----------------------------|----------------------------|--------------------|
| $u$<br>up                    | $c$<br>charm               | $t$<br>top                 | $\gamma$<br>photon |
| $d$<br>down                  | $s$<br>strange             | $b$<br>bottom              | $Z$<br>Z boson     |
| $\nu_e$<br>electron neutrino | $\nu_\mu$<br>muon neutrino | $\nu_\tau$<br>tau neutrino | $W$<br>W boson     |
| $e$<br>electron              | $\mu$<br>muon              | $\tau$<br>tau              | $g$<br>gluon       |

+



Multi-Higgs-doublet models

|                              |                            |                            |                    |
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+



# 3HDM vs 2HDM

2HDMs explored in thousands of papers [Branco et al, 1106.0034].

What new can 3HDMs bring? [Ivanov, 1702.03776]

- More options for model-building (scalar and fermion)  $\Rightarrow$  richer pheno!
  - ▶ new options for  $CP$  violation [Branco, Gerard, Grimus, 1984];
  - ▶ an exotic  $CP$  symmetry of order 4 [Ivanov, Silva, 2015];
  - ▶ combining features of 2HDM: NFC + CPV [Weinberg, 1976; Branco, 1979], scalar DM + CPV [Grzadkowski et al, 2009].
  - ▶ astroparticle consequences: various dark sectors [Cordero et al, 2017]; new options for baryon asymmetry [Davoudiasl, Lewis, Sullivan, 2019]; multi-step phase transitions.

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  - ▶ astroparticle consequences: various dark sectors [Cordero et al, 2017]; new options for baryon asymmetry [Davoudiasl, Lewis, Sullivan, 2019]; multi-step phase transitions.
- Many symmetry options tested starting from late 1970's [many classic papers].

Symmetries = the single most powerful novelty of the 3HDMs.

A dilemma in symmetry-based multi-Higgs model building:

- Large symmetry groups  $\rightarrow$  very few free parameters, nicely calculable, very predictive, but **conflicts experiment**.
- Small symmetry groups  $\rightarrow$  many free parameters, compatible with experiment but not quite predictive.

I will show a peculiar model based on three Higgs doublets (3HDM) which

- **assumes very little**: the minimal model realizing a particular symmetry;
- this symmetry is unusual: **CP-symmetry of order 4 (CP4)**;
- **remarkable connections** between the scalar and Yukawa sectors  $\rightarrow$  **predictions**.

A good balance between minimality, predictive power, and theoretical appeal.

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# A hidden gem: CP4 3HDM

CP4 = CP symmetry of order 4 = you need to apply it **four times** to get the identity.

# Higher order CP

In QFT, CP is not uniquely defined *a priori*.

- phase factors  $\phi(\vec{r}, t) \xrightarrow{CP} e^{i\alpha} \phi^*(-\vec{r}, t)$  [Feinberg, Weinberg, 1959].

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- With  $N$  scalar fields  $\phi_i$ , the general CP transformation is  $\phi_i \xrightarrow{CP} X_{ij} \phi_j^*$ ,  $X \in U(N)$ .
- If  $\mathcal{L}$  is invariant under CP with any  $X$ , it is explicitly CP-conserving [Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].

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- If  $\mathcal{L}$  is invariant under CP with any  $X$ , it is explicitly CP-conserving [Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].
- Squaring general CP  $\rightarrow$  a family transformation:

$$\phi_i \xrightarrow{CP} X_{ij} \phi_j^* \xrightarrow{CP} X_{ij} (X_{jk}^* \phi_k) = (XX^*)_{ik} \phi_k.$$

It can happen that  $(CP)^2 = XX^* \neq \mathbb{I}$  but  $(CP)^k = \mathbb{I}$  for  $k > 2$ .

CP-symmetry can be of a higher order  $k > 2$ .

The usual CP is of order 2. The exotic CP with  $k = 4$  is denoted CP4.

# CP4 3HDM

What is the **minimal NHDM** realizing **CP4** without accidental symmetries?

The answer was given in **Ivanov, Silva, 1512.09276**.

Consider the 3HDM with  $V = V_0 + V_1$  (notation:  $i \equiv \phi_i$ ), where

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2 \left[ (2^\dagger 2)^2 + (3^\dagger 3)^2 \right] \\ + \lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda'_3(2^\dagger 2)(3^\dagger 3) + \lambda_4 \left[ (1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1) \right] + \lambda'_4(2^\dagger 3)(3^\dagger 2),$$

with all parameters real, and

$$V_1 = \lambda_5(3^\dagger 1)(2^\dagger 1) + \frac{\lambda_6}{2} \left[ (2^\dagger 1)^2 - (3^\dagger 1)^2 \right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3) \left[ (2^\dagger 2) - (3^\dagger 3) \right] + h.c.$$

with real  $\lambda_{5,6}$  and **complex**  $\lambda_{8,9}$ . It is invariant under **CP4**  $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$  with

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix} \Rightarrow XX^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

# Different versions of CP4 3HDM

- A  $CP$ -conserving model based on CP4 **can be distinguished** from a  $CP$ -conserving model based on the usual CP [[Haber et al, 1808.08629](#)].
- **DM CP4 3HDM**: unbroken CP4 with scalar DM candidates, similar to the inert doublet model in 2HDM. We assume that  $\phi_2, \phi_3$  don't get vevs  $\rightarrow$  **scalar DM candidates** with very peculiar properties [[Ivanov, Silva, 2015](#); [Ivanov, Laletin, 2018](#)]:  
neutral scalars are neither CP even nor CP odd but are “CP-half-odd”.

Scalar DM **stabilized by a CP-symmetry!**

- **flavored CP4 3HDM**: CP4 is extended to the Yukawa sector and must be spontaneously broken  $\rightarrow$  patterns in the flavor sector [[Aranda et al, 1608.08922](#); [Ferreira et al, 1711.02042](#)]

# The flavored CP4 3HDM

## CP4-symmetric quark sector

Extending CP4 to the Yukawa sector:  $\psi_i \rightarrow Y_{ij} \psi_j^{CP}$ , where  $\psi^{CP} = \gamma^0 C \bar{\psi}^T$ .

$$-\mathcal{L}_Y = \bar{q}_L \Gamma_a d_R \phi_a + \bar{q}_L \Delta_a u_R \tilde{\phi}_a + h.c.$$

is invariant under CP4 if

$$(Y^L)^\dagger \Gamma_a Y^{dR} X_{ab} = \Gamma_b^*, \quad (Y^L)^\dagger \Delta_a Y^{uR} X_{ab}^* = \Delta_b^*.$$

Only  $X_{ab}$  is known.  $\Gamma_a, \Delta_a, Y^L, Y^{dR}, Y^{uR}$  are to be found.

Solved in [Ferreira et al, 1711.02042](#)  $\rightarrow$  only four options for  $\Gamma_a$  and for  $\Delta_a$  exist.

# CP4-symmetric quark sector

case A:  $\Gamma_1 \simeq$  arbitrary real matrix,  $\Gamma_{2,3} = 0$ .

case  $B_1$

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} -g_{22}^* & -g_{21}^* & -g_{23}^* \\ g_{12}^* & g_{11}^* & g_{13}^* \\ 0 & 0 & 0 \end{pmatrix}.$$

case  $B_2$

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{13}^* \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} g_{22}^* & -g_{21}^* & 0 \\ g_{12}^* & -g_{11}^* & 0 \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}.$$

case  $B_3$

$$\Gamma_1 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ -g_{12}^* & g_{11}^* & 0 \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{23} \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & -g_{23}^* \\ 0 & 0 & g_{13}^* \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}.$$

# CP4-symmetric quark sector

When combining up and down quarks, need to match  $\alpha_L$ : **8 combinations**.

$$(A^{down}, A^{up}), (A^{down}, B_2^{up}), (B_2^{down}, A^{up}), (B_2^{down}, B_2^{up}), \\ (B_1^{down}, B_1^{up}), (B_1^{down}, B_3^{up}), (B_3^{down}, B_1^{up}), (B_3^{down}, B_3^{up}).$$

- case (A, A) implies a **real CKM**  $\rightarrow$  disregarded.
- cases  $B_1, B_2, B_3$ : quark mass matrices

$$M_d = \frac{1}{\sqrt{2}} \sum \Gamma_a v_a, \quad M_u = \frac{1}{\sqrt{2}} \sum \Delta_a v_a^*.$$

all  $v_i$  must be nonzero to avoid mass-degenerate quarks.

# FCNC in CP4 3HDM

- Tree-level **flavor-changing neutral couplings** (FCNC) are a generic feature of multi-Higgs models.
- Unsuppressed FCNCs conflict meson oscillation parameters  $\rightarrow$  need to be **eliminated** or **suppressed** (recent review: [Sher, 2207.06771](#)).

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- CP4 leads to **remarkably tight connections** between the Yukawa and scalar sectors  $\rightarrow$  **no built-in suppression of FCNC**.
- Avoiding FCNC from  $h_{125}$  via the **scalar alignment** condition:  $m_{11}^2 = m_{22}^2$ .
- But then the additional neutral Higgses may exhibit significant FCNCs.

Must be explored in a **full scan of the parameter space**.

# FCNC in CP4 3HDM

- In [Ferreira et al, 1711.02042](#), we reported the first pheno scan of the parameter space (theory constraints, EWPT, fermion masses and mixing, meson oscillation parameters) → many benchmark points found.
- Ruled out in [Ivanov, Obodenko, 2104.11440](#) based on the LHC Run 2 charged Higgs searches.
- Need to repeat the scan — but do it [in a smart way](#).

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- Need to repeat the scan — but do it [in a smart way](#).
- The usual scanning procedure

random seed point in  $\Gamma_i, \Delta_i \Rightarrow$  fit  $m_q, \text{CKM}$

is [very time consuming](#): many trial points are thrown away.

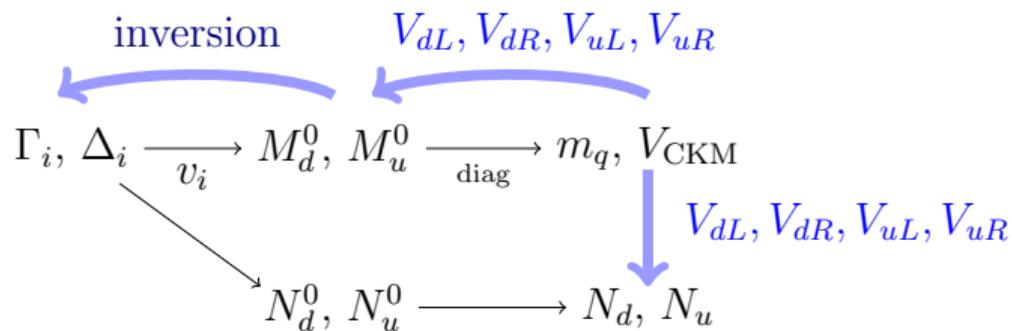
- A more efficient scanning procedure is needed:

start with  $m_q, \text{CKM} \Rightarrow$  reconstruct  $\Gamma_i, \Delta_i$

If this [inversion](#) is feasible, [every trial point will give a viable model](#).

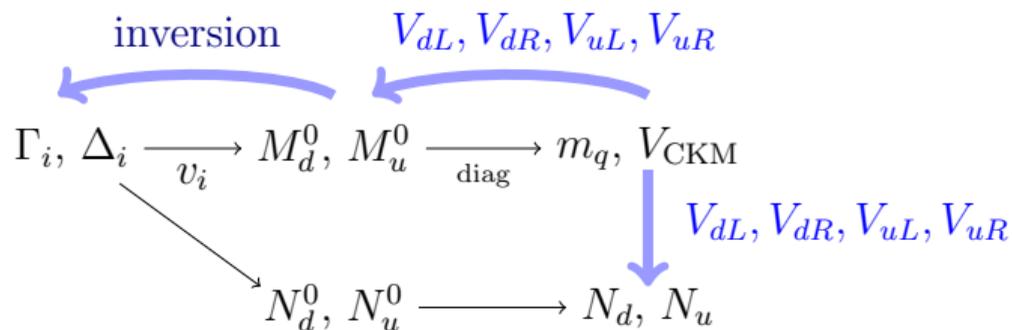
# Scanning CP4 3HDM Yukawa sector: inversion

In Zhao et al, 2302.03094 = JHEP 04 (2023) 116, we constructed an inversion procedure.



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We expressed the FCNC matrices  $N_d, N_u$  via physical quark observables and quark rotation parameters.

# FCNC in CP4 3HDM

Let's gain some intuition with a [toy model](#):

$$V_{dL}, V_{dR}, V_{uL}, V_{uR} \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix} = \begin{pmatrix} c_\theta e^{i\alpha} & s_\theta e^{i\zeta} & 0 \\ -s_\theta e^{-i\zeta} & c_\theta e^{-i\alpha} & 0 \\ 0 & 0 & e^{i\gamma} \end{pmatrix}.$$

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Then, in the case  $B_1$ , we get

$$N_{d2} = \begin{pmatrix} m_d \cot \beta & 0 & 0 \\ 0 & m_s \cot \beta & 0 \\ 0 & 0 & -m_b \tan \beta \end{pmatrix},$$
$$N_{d3} = \frac{1}{s_\beta} \begin{pmatrix} -m_s c_{2\theta} & -m_s s_{2\theta} e^{-i(\alpha-\zeta)} & 0 \\ -m_d s_{2\theta} e^{i(\alpha-\zeta)} & m_d c_{2\theta} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

CP4 feature:  $\Phi_3 d \bar{d} \propto m_s$ , not  $m_d$ ;  $\Phi_3 d \bar{s} \propto m_s$ , not  $\sqrt{m_d m_s}$ .

Before doing a full pheno scan, the dimensionless off-diagonal elements of  $N_d$ 's and  $N_u$ 's can be constrained by  $K$ ,  $B$ ,  $B_s$  and  $D$ -meson oscillation parameters.

- What is the typical FCNC magnitude in each Yukawa sector?
- How small the FCNCs can in principle become? What controls their smallness?
- Can some CP4 3HDM Yukawa sectors be already excluded?

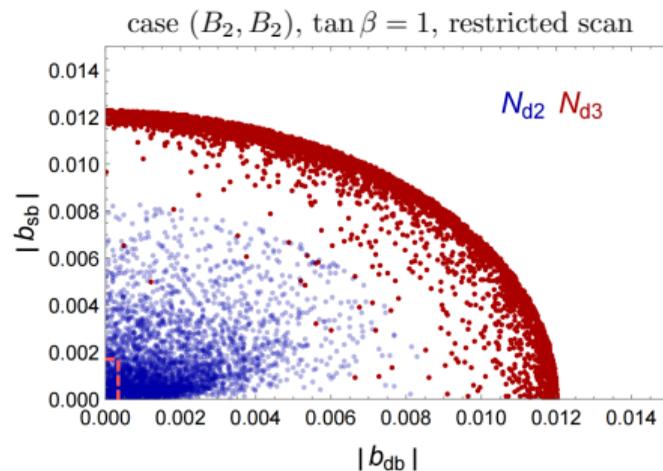
In [Zhao et al, 2302.03094](#), we explored these questions with our inversion procedure.

# Results of the numerical study 2302.03094

Out of 8 possible CP4 Yukawa sectors, **only two** are viable:  $(A, B_2)$  and  $(B_1, B_1)$ .

**The other scenarios fail!** For example,  $(B_2, B_2)$  is excluded by  $B_s$  vs.  $B$ .

Here, FCNCs from  $N_{d3}$  are far outside the box even for a 1 TeV scalar!



The full pheno scan should only be based on  $(A, B_2)$  and  $(B_1, B_1)$ .

# Conclusions

- **CP4 3HDM** is the minimal model implementing a CP symmetry of order 4 (**CP4**) without accidental symmetries.
- CP4 can be extended to the **Yukawa sector** → very characteristic flavor sector.
- Out of 8 possible CP4 invariant Yukawa sectors, only two scenarios —  $(A, B_2)$  and  $(B_1, B_1)$  — lead to viable models!
- Detailed pheno study (scalar + mesons obs. + leptons) **is underway**.

Tired of the 2HDM and singlets? Try CP4 3HDM

- based on a **single symmetry assumption**,
- quite predictive with rich phenomenology,
- **analytical insights** guide numerical exploration.