Two-Loop EW Corrections to Higgs Boson Pair Production: Yukawa & Self-Coupling Corrections

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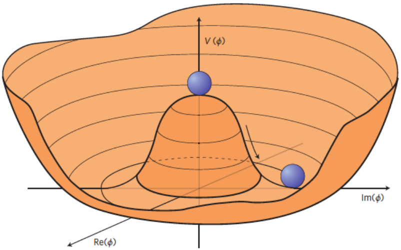
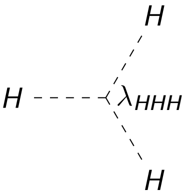
Outline

Introduction & Motivation

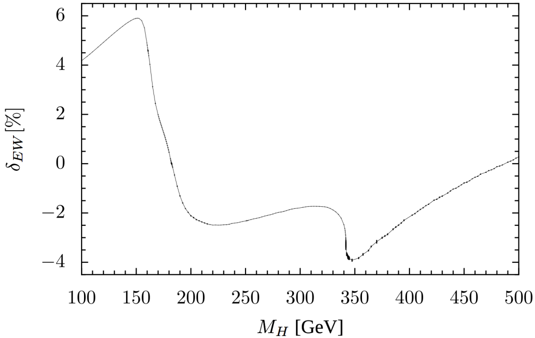
- Investigating Higgs properties in run 3 of the LHC requires precision calculations in the SM
- Gluon fusion is the dominant mechanism for producing Higgs bosons at the LHC
- Higgs pair production provides a direct way to measure the Higgs self-coupling through $\kappa_\lambda := \lambda_{HHH}/\lambda_{HHH}^{SM}$ (currently $-1.4 < \kappa_\lambda < 6.1$ [ATLAS 23] & $-1.24 < \kappa_\lambda < 6.49$ [CMS 22])





Why Electroweak?

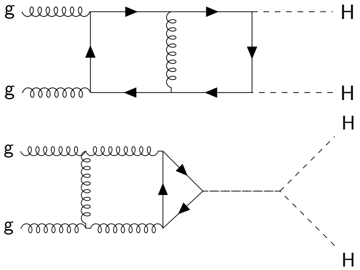
- Precision - expect magnitude of EW corrections to be $\mathcal{O}(5\%)$
- Technology - want to understand richer structure of EW corrections (e.g. many mass scales in reduction)



NLO electroweak percentage corrections to the partonic cross section $\sigma(gg \to H)$ [Actis, Passarino, Sturm, Uccirati 08]

NLO Corrections

NLO QCD: $\mathcal{O}\left(\alpha_s^2\alpha\right)$

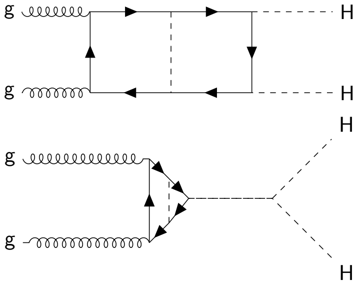


Full top mass dependence

[S. Borowka, N. Greiner, G. Heinrich, S.P. Jones, M. Kerner, J. Schlenk, U. Schubert, T. Zirke 16]

[J. Baglio, F. Campanario, S. Glaus, M. Mühlleitner, J. Ronca, M. Spira, J. Streicher 18]

NLO EW: $\mathcal{O}\left(\alpha_s\alpha^2\right)$



Heavy top mass expansion

[J. Davies, K. Schönwald, M. Steinhauser, H. Zhang 23]

Higgs tri-linear self-coupling corrections [See talk by Xiao Zhang]

Amplitude Structure

- To any number of loops, there exists a decomposition of the amplitude for gg $\rightarrow$ HH into form factors

Form Factor Decomposition

$$\mathcal{M}_{ab} = \delta_{ab}\, \epsilon_1^{\mu} \epsilon_2^{\nu}\, \mathcal{M}_{\mu\nu}$$
$$\mathcal{M}^{\mu\nu} = F_1\left(s, t, m_h^2, m_t^2, d\right)\, T_1^{\mu\nu} + F_2\left(s, t, m_h^2, m_t^2, d\right)\, T_2^{\mu\nu}$$

- The form factors F_1 and F_2 correspond to the helicity amplitudes $\mathcal{M}^{++} = \mathcal{M}^{--}$ and $\mathcal{M}^{+-} = \mathcal{M}^{-+}$ respectively

Coupling Structures

$$F_i \sim y_t^2 F^{(0)}_{i,y_t^2} + y_t \lambda F^{(0)}_{i,y_t\lambda} + y_t^4 F^{(1)}_{i,y_t^4} + y_t^3 \lambda F^{(1)}_{i,y_t^3\lambda} + y_t^2 \lambda^2 F^{(1)}_{i,y_t^2\lambda^2} + y_t \lambda^3 F^{(1)}_{i,y_t\lambda^3}$$

Master Integrals

- Obtain F_1 & F_2 from $\mathcal{M}_{\mu\nu}$ using projectors [Glover, van der Bij 88]
- Form factors are linear combinations of scalar Feynman integrals ($\Sigma_{i=1}^{\mathcal{O}(1000\text{s})} c_i I_i$)
- We can express each complicated Feynman integral in terms of a finite set of master integrals

Master Integral Decomposition

$$\forall i : I_i = \Sigma_{j=1}^{494} \alpha_{ij} M_j$$

Integral Reduction

- How do we determine $\{\alpha_{ij}\}$?
- We can use integration-by-parts (IBP) reduction rules to rewrite Feynman integrals in terms of other ones

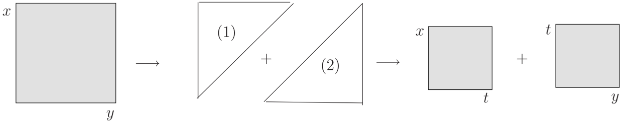
Integration-by-Parts Identity

$$\forall j, n : \int \prod_{i=1}^{L} [\mathrm{d}k_i] \frac{\partial}{\partial k_j^\mu} \frac{q^\mu}{\mathcal{D}_{1,n}^{\alpha_{1,n}} \ldots \mathcal{D}_{p,n}^{\alpha_{p,n}}} = 0$$ [Tkachov 81; Chetyrkin 81]

- We reduce these integrals using Kira [Maierhoefer, Usovitsch, Uwer 17; Maierhofer, Usovitsch 18; Klappert, Lange, Maierhofer, Usovitsch 20] and Ratracer [Magerya 22] via functional reconstruction with finite fields

Numerical Evaluation of Master Integrals

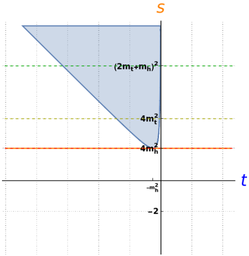
- Sector Decomposition (pySecDec [SecDec Collaboration 22])



- Series Solutions of Differential Equations (DiffExp [Hidding 20])

Differential Equation System

$$\mathrm{d}\vec{f} = \mathrm{d}\tilde{\mathbf{A}}\vec{f}$$



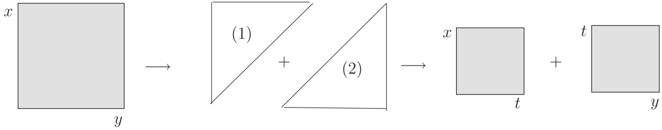
Sector Decomposition

Feynman-Parameterised Integral

$$I \sim \int_{\mathbb{R}^N_{>0}} [\mathrm{d}\mathbf{x}]\, \mathbf{x}^{\boldsymbol{\nu}} \frac{\mathcal{U}(\mathbf{x})^{N-(L+1)D/2}}{(\mathcal{F}(\mathbf{x},\mathbf{s})-i\epsilon)^{N-LD/2}} \delta\left(1-\boldsymbol{\alpha}\cdot\mathbf{x}\right)$$

Singularities from two effects:

- Subset of parameters x_i go to zero at the same time
- $\mathcal{F}$ polynomial goes to zero *inside* domain of integration



[Slide inspired by S.P. Jones]

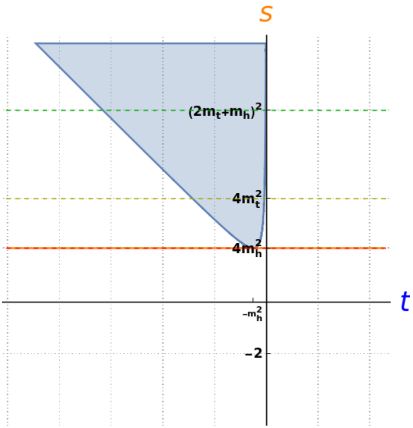
Differential Equations

- It was noted that master integrals could be solved as a system of differential equations [Kotikov 91]

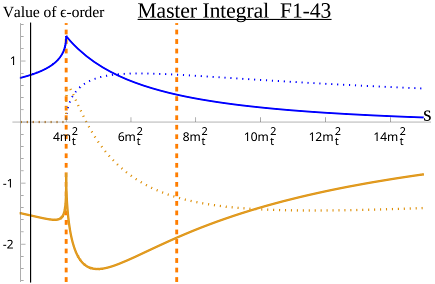
Differential Equation System

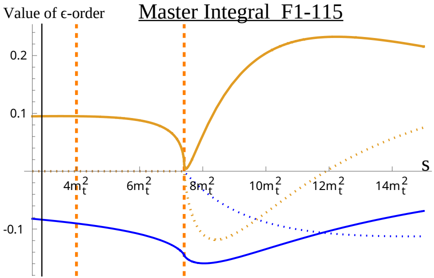
$$\mathrm{d}\vec{f} = \left(\sum_{x \in \{s,t,m_h^2\}} \mathbf{A_x} \mathrm{d}x \right) \vec{f}$$

- DiffExp [Hidding 20] is a Mathematica package which solves the differential equation system using a generalised series expansion solution



DiffExp Master Integral Example Results



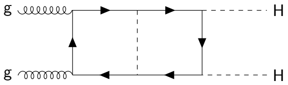


Two master integrals in the same integral family evaluated along a contour in the positive s-direction: leading order coefficient in ϵ-expansion & next order coefficient

Agreement between DiffExp & pySecDec for benchmark contours

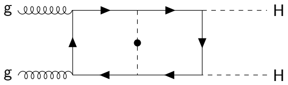
Improving the Basis: Dots & Dim-Shifts

$$F_i \sim \frac{f(\mathbf{s})}{\epsilon} M_{\text{difficult}} + \cdots$$



↙ ↘

Dots: doubling, tripling etc. propagators



Dimensional Recurrence Relation

$$M^{(d-2)} \propto D(\partial)\, M^{(d)}$$

[Tarasov 96]

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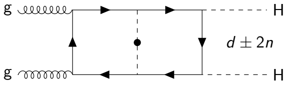
$$F_i \sim f'(\mathbf{s})\, M'_{\text{difficult}} + \cdots$$



Current Status

- We have the reduced differential equations and amplitude for an improved basis of master integrals

Basis Comparison for Virtual Correction ("Good" Point)

- Old Basis (2022): $T(F_1) = 45$ hours $T(F_2) = 347$ hours
- New Basis (2023): $T(F_1) \sim 5$ mins $T(F_2) \sim 5$ mins

- Timings are for pySecDec approach on GPU (Nvidia A100)
- Old basis did not even converge on a "bad" phase space point!
- Suggests amplitude ϵ-order is crucial consideration for basis when evaluating numerically beyond other desirable properties (e.g. d-factorising, finite etc.)

Preliminary Results

- Form factors separated by coupling structure
- Can compute for arbitrary s and t in the physical region
- Not every coupling structure appears in both form factors
- No $\frac{1}{\epsilon^2}$ poles; poles in $\frac{1}{\epsilon}$ from UV divergences only

PRELIMINARY

```
{“s": 6392./1000., 't': -1038./1000.}
AMPgs2ghh3ght@1=
  +eps^-1*(-8.0501761760992874e-01-3.8847967242213449e-01j)
  ±eps^-1*(+6.4683122144165918e-18+6.0907618364907079e-18j)
  +eps^0*(-2.8972433508074156e+00+1.0897854394801396e+01j)
  ±eps^0*(+3.6435347347217201e-07+3.4598017999757325e-07j)
AMPgs2ghhghhhght@1=
  +eps^-1*(-1.4596999451695023e+01-7.0441161054046244e+00j)
  ±eps^-1*(+1.1728681184339178e-16+1.1044087140803378e-16j)
  +eps^0*(+8.5908827457094041e+00-2.3032424593432740e+01j)
  ±eps^0*(+2.1379793495768672e-06+2.0307809283694008e-06j)
AMPgs2ghhhght2@1=
  +eps^0*(-2.9328689251200672e+01+6.2777695560284613e+01j)
  ±eps^0*(+4.2325534773895372e-05+4.7059980270574143e-05j)
AMPgs2ght4@2=
  +eps^-1*(+1.0795435677340720e+01-1.5627850302093538e+01j)
  ±eps^-1*(+6.8356820307489631e-07+8.2071745646177096e-07j)
  +eps^0*(+1.2494307071197568e+01-1.3334138655224328e+01j)
  ±eps^0*(+5.5369904028054630e-05+5.7556899298774435e-05j)
AMPgs2ght4@1=
  +eps^-1*(+8.9965909545280738e+02+2.0129450721915367e+02j)
  ±eps^-1*(+3.1141662895359273e-07+3.9765520889843626e-07j)
  +eps^0*(-3.2569582999240259e+02+7.4212291343799063e+02j)
  ±eps^0*(+1.2062257017861179e-05+1.3260886643234515e-05j)
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  ±eps^0*(+6.4890537787570324e-06+7.3624383819487271e-06j)
AMPgs2ghh2ght2@1=
  +eps^0*(+5.9931726494394134e+01-2.7172796094458263e+01j)
  ±eps^0*(+7.2171240278777598e-06+8.0229678514775212e-06j)
AMPgs2ghhght3@2=
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  ±eps^0*(+5.2242846406267860e-05+5.2540153789084503e-05j)
AMPgs2ghhght3@1=
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  ±eps^-1*(+1.2444914353984587e-11+1.1939919132080516e-11j)
  +eps^0*(-5.6920823777757072e+01-1.4139018713494380e+02j)
  ±eps^0*(+2.3643191708284244e-05+1.9217507136723785e-05j)
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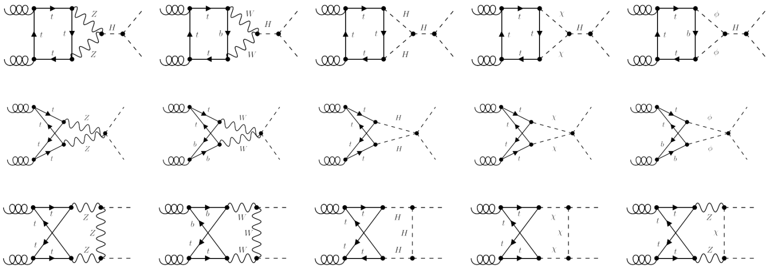
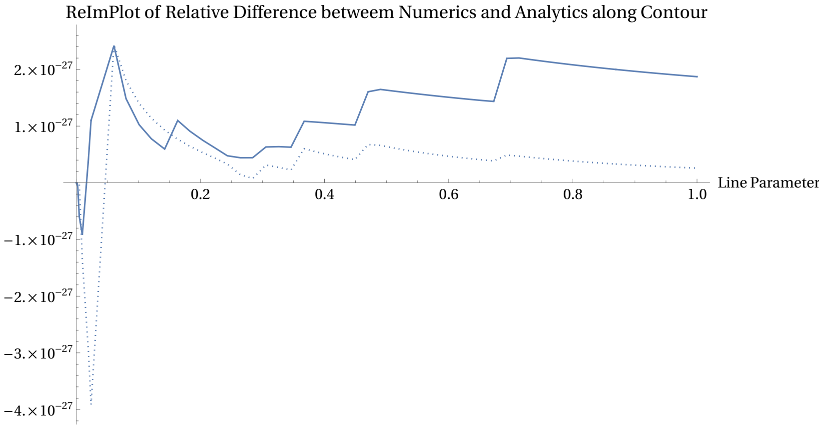
Outlook

- We are currently performing the UV renormalisation using the one-loop result we have already calculated
- We will also begin looking at the full electroweak corrections ($\sim$1000s diagrams!) retaining full top-mass dependence



[Figure from H. Zhang]

DiffExp Error Plot



Amplitude Structure

Form Factor Decomposition

$$\mathcal{M}_{ab} = \delta_{ab}\, \epsilon_1^{\mu} \epsilon_2^{\nu}\, \mathcal{M}_{\mu\nu}$$
$$\mathcal{M}^{\mu\nu} = F_1\left(s, t, m_h^2, m_t^2, d\right)\, T_1^{\mu\nu} + F_2\left(s, t, m_h^2, m_t^2, d\right)\, T_2^{\mu\nu}$$

Tensor Structures

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^{\nu} p_2^{\mu}}{p_1 \cdot p_2}$$
$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2 (p_1 \cdot p_2)} (m_h^2 p_1^{\nu} p_2^{\mu} - 2(p_1 \cdot p_3) p_3^{\nu} p_2^{\mu} - 2(p_2 \cdot p_3) p_3^{\mu} p_1^{\nu} + 2(p_1 \cdot p_2) p_3^{\nu} p_3^{\mu})$$

Projectors

$$P_1^{\mu\nu} = \frac{1}{4} \frac{d-2}{d-3} T_1^{\mu\nu} - \frac{1}{4} \frac{d-4}{d-3} T_2^{\mu\nu} \qquad P_2^{\mu\nu} = -\frac{1}{4} \frac{d-4}{d-3} T_1^{\mu\nu} + \frac{1}{4} \frac{d-2}{d-3} T_2^{\mu\nu}$$