# EFT for (Higgs) beyond dim 6

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- Massless amplitude basis
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# Massless Amplitude Basis

# Effective Field Theory

- Effective field theory (EFT) has wide application in physics

High energy physics | Lower energy QCD

Gravity | Condense matter

- Effective field theory is especially useful in new physics search

Electroweak precision measurement | Dark matter search | Lower energy EFT in flavor physics

- Null signals of new physics in experiments indicate that new physics scale may be very high

**EFTs become important to probe new physics!!!**

# Challenge in EFT Basis Construction

- Effective field theory

Wilson coefficients

A complete set of EFT bases

$$\mathscr{L}_{EFT} = \mathscr{L}_{renormalizable} + \sum \frac{c_i}{\Lambda^{d-4}} \mathcal{O}_i^d$$

Scale

$\bigcup$

All lower energy effects of any UV theory

- Difficulties in constructing a complete set of operator bases

Eliminate redundancy from the constraints:

Equation of motion (EOM)

Integration by part (IBP)

Unsolved problem in traditional field theory!!!

- Recent progress: Hilbert series technique

**2, 84, 30, 993, 560, 15456, 11962, 261485, ...: Higher dimension operators in the SM EFT**
Brian Henning (Yale U.), Xiaochuan Lu (UC, Davis), Tom Melia (UC, Berkeley and LBNL, Berkeley), Hitoshi Murayama (UC, Berkeley and LBNL, Berkeley and Tokyo U., IPMU) (Dec 10, 2015)
Published in: *JHEP* 08 (2017) 016, *JHEP* 09 (2019) 019 (erratum) • e-Print: 1512.03433 [hep-ph]

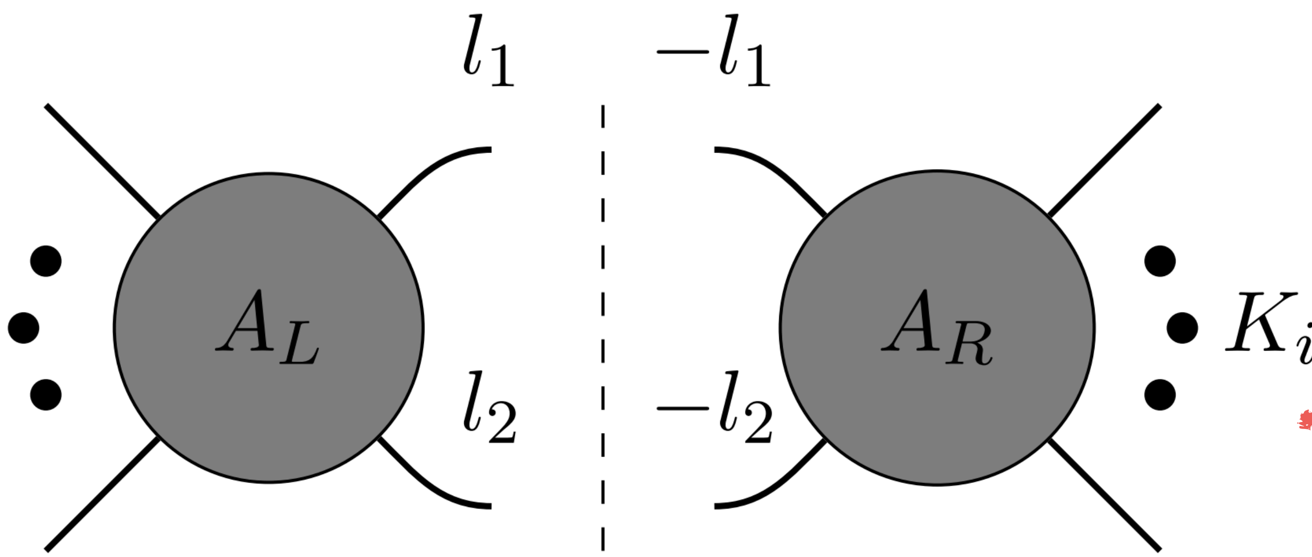
**Low-derivative operators of the Standard Model effective field theory via Hilbert series methods**
Landon Lehman (Notre Dame U.), Adam Martin (Notre Dame U.) (Oct 1, 2015)
Published in: *JHEP* 02 (2016) 081 • e-Print: 1510.00372 [hep-ph]

Only count the number of bases!!!

**How to solve it?**

# On-shell scattering amplitude

- Efficient in massless EFT calculations





Z. Bern, J. Parra-Martinez and E. Sawyer, JHEP **10**, 211 (2020) doi:10.1007/JHEP10(2020)211 [arXiv:2005.12917 [hep-ph]].
M. Jiang, T. Ma and J. Shu, [arXiv:2005.10261 [hep-ph]].
J. Elias Miró, J. Ingoldby and M. Riembau, JHEP **09**, 163 (2020) doi:10.1007/JHEP09(2020)163 [arXiv:2005.06983 [hep-ph]].
P. Baratella, C. Fernandez and A. Pomarol, Nucl. Phys. B **959**, 115155 (2020) doi:10.1016/j.nuclphysb.2020.115155 [arXiv:2005.07129 [hep-ph]].


RG-running

$$\frac{\mathrm{d}c_i(\mu)}{\mathrm{d}\log\mu} = \sum_j \frac{1}{16\pi^2}\gamma_{ij}c_j$$

Selection rules

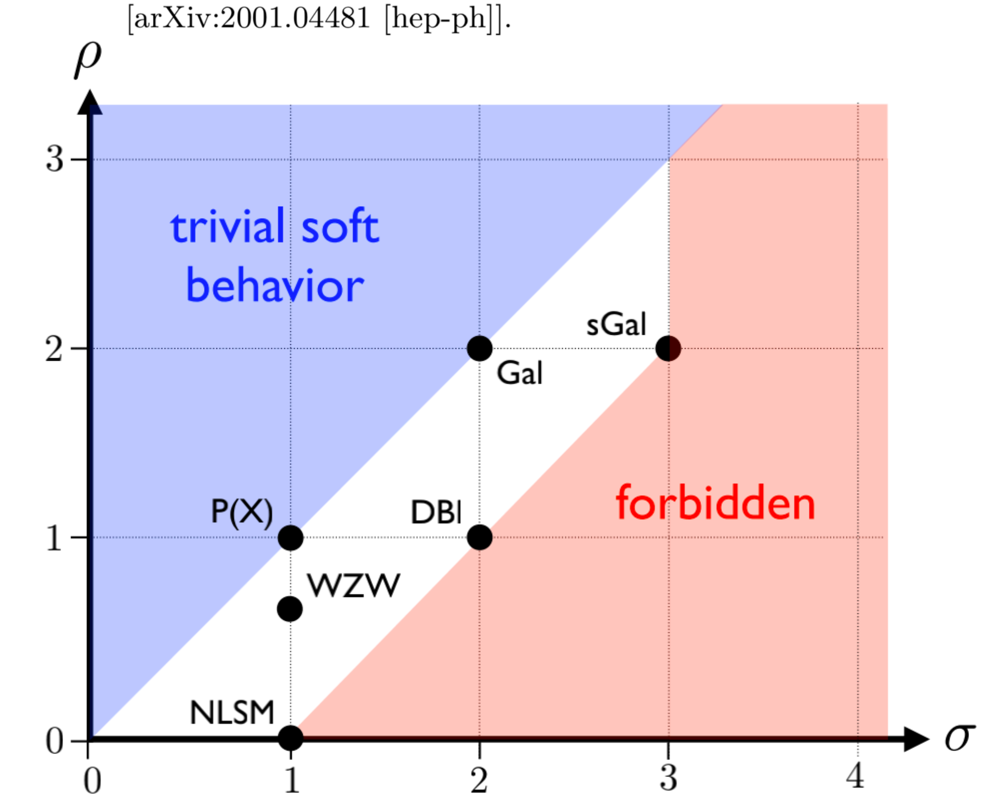
$$\gamma_{ij} = 0$$


C. Cheung and C. H. Shen, Phys. Rev. Lett. **115**, no. 7, 071601 (2015) doi:10.1103/PhysRevLett.115.071601 [arXiv:1505.01844 [hep-ph]].
M. Jiang, J. Shu, M. L. Xiao and Y. H. Zheng, [arXiv:2001.04481 [hep-ph]].


- Construct scalar EFT through soft limit

$$A_n \sim p^\sigma \quad \text{for} \quad p \to 0$$


C. Cheung, K. Kampf, J. Novotny and J. Trnka, Phys. Rev. Lett. **114**, no.22, 221602 (2015) doi:10.1103/PhysRevLett.114.221602 [arXiv:1412.4095 [hep-th]].
C. Cheung, K. Kampf, J. Novotny, C. H. Shen and J. Trnka, JHEP **02**, 020 (2017) doi:10.1007/JHEP02(2017)020 [arXiv:1611.03137 [hep-th]].
I. Low, Phys. Rev. D **91**, no.10, 105017 (2015) doi:10.1103/PhysRevD.91.105017 [arXiv:1412.2145 [hep-th]].
I. Low, Phys. Rev. D **91**, no.11, 116005 (2015) doi:10.1103/PhysRevD.91.116005 [arXiv:1412.2146 [hep-ph]].

# Introduction of on-shell amplitude

- Massive spinor and its little group (LG) $SU(2)_i$

Massive momentum $\quad (p_i)_{\dot\alpha\alpha} \equiv (p_i)_\mu(\sigma^\mu)_{\dot\alpha\alpha} = |i^I]_{\dot\alpha}\langle i_I|_\alpha$

Quantum number $SU(2)_l \otimes SU(2)_r \otimes SU(2)_i$

Notrivial EOM $\quad p|p^I] = m|p^I\rangle$

$|i^I]_{\dot\alpha} = (1,2,2) \quad |i^I\rangle_\alpha = (2,1,2)$

$SO(3,1)$ Lorentz

- For massless spinor, its little group is $U(1)_j$

$$|j] \to e^{-i\theta_j}|j] \qquad |j\rangle \to e^{i\theta_j}|j\rangle$$

- Minimal Lorentz scalar: spinor product

$$[ij]^{IJ} \equiv \epsilon^{\dot\alpha\dot\beta}|i^I]_{\dot\beta}|j^J]_{\dot\alpha}\,, \quad \langle ij\rangle^{IJ} \equiv \epsilon^{\alpha\beta}|i^I\rangle_\beta|j^J]_\alpha$$

- On-shell scattering amplitudes are the functions of spinor products

$$\mathcal{M}_n = \mathcal{M}_n([ij], \langle ij\rangle)$$

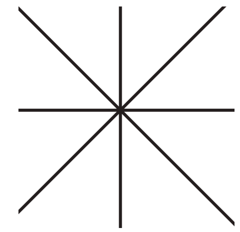

E. Witten, Commun. Math. Phys. **252**, 189 (2004) doi:10.1007/s00220-004-1187-3 [hep-th/0312171].

N. Arkani-Hamed, T. C. Huang and Y. t. Huang, arXiv:1709.04891 [hep-th].

# On-shell amplitude basis

- Efficient in constructing EFT operator bases of massless fields

T. Ma, J. Shu and M. L. Xiao, [arXiv:1902.06752 [hep-ph]].
H. Elvang, D. Z. Freedman and M. Kiermaier, JHEP **1011**, 016 (2010) doi:10.1007/JHEP11(2010)016 [arXiv:1003.5018 [hep-th]].
Y. Shadmi and Y. Weiss, arXiv:1809.09644 [hep-ph].

$\mathcal{O}_{d\geq 4}$ $\longleftrightarrow$ 

Operator base

Unfactorizable amplitude base $\longleftrightarrow$ Polynomial of spinor products

- Massless amplitude base is free of EOMs automatically

Null EOM $\quad p\,|p] = 0, \quad p\,|p\rangle = 0$

- IBP redundancy can be systematically removed by $U(N) \supset \otimes_{i=1}^{N} U(1)_i$

IBP redundancy $\longleftrightarrow$ Momentum conservation

B. Henning and T. Melia, Phys. Rev. D **100**, no.1, 016015 (2019) doi:10.1103/PhysRevD.100.016015 [arXiv:1902.06754 [hep-ph]].
B. Henning and T. Melia, [arXiv:1902.06747 [hep-th]].

- The amplitude bases are the basis of some special $U(N)$ representations

H. L. Li, Z. Ren, J. Shu, M. L. Xiao, J. H. Yu and Y. H. Zheng, [arXiv:2005.00008 [hep-ph]].

- It be constructed by the computer programs

(**Field theory can not do it!!!**)

# Massless amplitude basis

**Standard Model Effective Field Theory from On-shell Amplitudes**
Teng Ma, Jing Shu, Ming-Lei Xiao
arXiv:1902.06752 [hep-ph].

- Basic structure of massless amplitudes

$$\mathcal{M}(\{p_i, h_i\}) = \underbrace{f(|i], |i\rangle)}_{\text{Massless LG charged}} \underbrace{g(s_{ij})}_{\text{Massless LG neutral}} \underbrace{T^{\{\alpha\}}}_{\text{Gauge structure}}$$

- An amplitude basis just corresponds to the leading interaction of an operator

$$F_{\mu\nu} \to \partial_\mu A_\nu - \partial_\nu A_\mu \qquad\qquad D_\mu \to \partial_\mu$$

- The complete amplitude bases of a scattering process can be obtained by finding all its independent unfactorizable amplitudes allowed by LG, gauge symmetry, and spin statistic

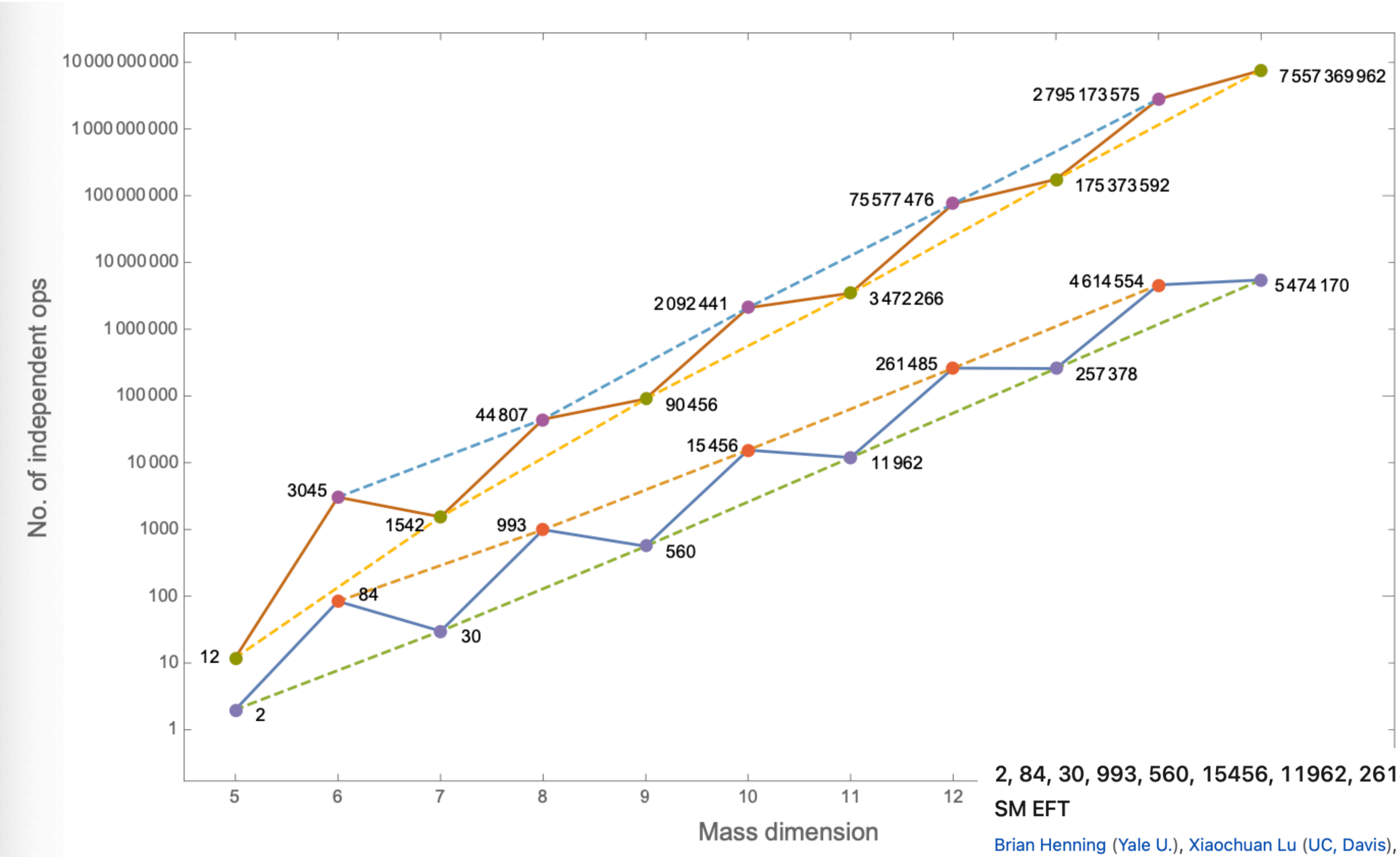
Three gluons

$$\mathcal{M}(G^{A+}G^{B+}G^{C+}) = [12][23][31]f^{ABC} \qquad f^{ABC}G^A_{\mu\nu}G^B_{\nu\rho}G^C_{\rho\mu}$$

$$\mathcal{M}(G^{A-}G^{B-}G^{C-}) = \langle 12\rangle\langle 23\rangle\langle 31\rangle f^{ABC} \qquad f^{ABC}\tilde{G}^A_{\mu\nu}G^B_{\nu\rho}G^C_{\rho\mu}$$

# What is more on > dim 6

- Recent progress: Hilbert series technique.




The number
IS HUGE
with big
mass dim!

2, 84, 30, 993, 560, 15456, 11962, 261485, ...: Higher dimension operators in the SM EFT

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How to systematic generate the independent basis?

# Massless amplitude basis

- Systematically construct the complete amplitude bases of N external massless particles without IBP via $U(N)$ symmetry

Quantum number under $SU(2)_l \otimes SU(2)_r \otimes U(N)$

$$\tilde{\lambda}^k_{\dot{\alpha}} \equiv |k] = (1, 2, N)$$

$$\lambda_{k\alpha} \equiv |k\rangle = (2, 1, \bar{N})$$

- A Semi-standard YoungTableau (SSYT) of a U(N) representation is a polynomial of spinors

$$\left.\begin{array}{|c|}\hline \phantom{i} \\ \hline \phantom{j} \\ \hline\end{array}\right._r \otimes \begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array} = \frac{1}{2!}\epsilon^{\beta\alpha}\left(\tilde{\lambda}^i_{\dot{\alpha}}\tilde{\lambda}^j_{\dot{\beta}} - \tilde{\lambda}^j_{\dot{\alpha}}\tilde{\lambda}^i_{\dot{\beta}}\right) = [ij]$$

B. Henning and T. Melia, Phys. Rev. D **100**, no.1, 016015 (2019) doi:10.1103/PhysRevD.100.016015 [arXiv:1902.06754 [hep-ph]].
B. Henning and T. Melia, [arXiv:1902.06747 [hep-th]].

$$\left.\begin{array}{|c|}\hline \phantom{k} \\ \hline \phantom{k} \\ \hline\end{array}\right._l \otimes \left.\begin{array}{|c|}\hline k_1 \\ \hline \cdot \\ \hline \cdot \\ \hline k_{N-2} \\ \hline\end{array}\right\} {\scriptstyle N-2} = \frac{\left(\epsilon^{ijk_1\ldots k_{N-2}} + \text{anti-sym in } k_1..k_{N-2}\right)}{(N-2)!}\epsilon^{\beta\alpha}\lambda_{i\alpha}\lambda_{j\beta}$$

$$= \langle ij\rangle\epsilon^{ijk_1\ldots k_{N-2}}$$

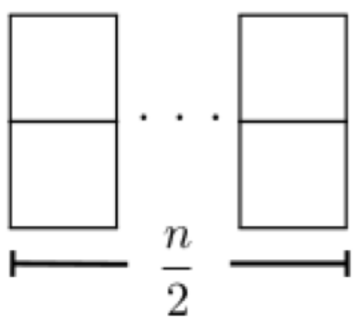
Blue column for left-handed spinor product

Two white boxes in a column correspond to a square spinor product

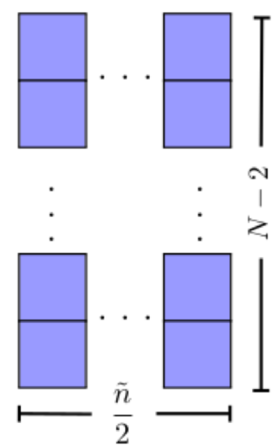
$(N-2)$ blue boxes in a column correspond to a angle spinor product

# Massless amplitude basis

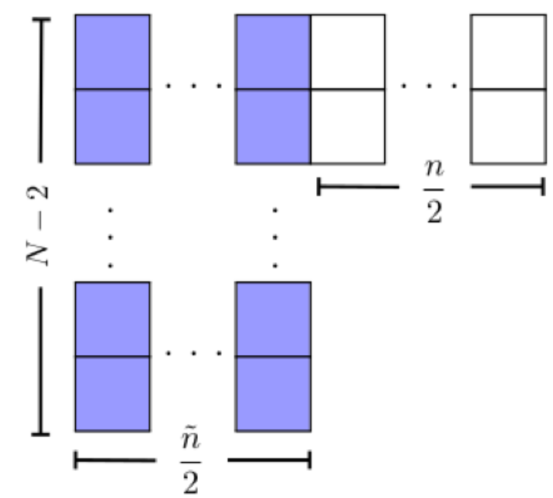
- N-point amplitude bases of massless fields correspond to the bases of the following types of $U(N)$ representations, i.e. SSYTs




Holomorphic bases with $n$ right-handed spinors




Holomorphic bases with $\tilde{n}$ left-handed spinors




Non-holomorphic bases with $n$ right-handed spinor and $\tilde{n}$ left-handed spinors

# Massless amplitude basis

- The amplitude bases contain $n$ right-handed spinors and $\tilde{n}$ left-handed spinors

The white and blue box column number is $n/2$ and $\tilde{n}/2$

- Examples: $N=4$ case:

$\psi_{1R}\psi_{2R}\psi_{3R}\psi_{4R} : (n,\tilde{n}) = (4,0)$ White column number is: $n/2=2$

$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array} \qquad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} = [13][24]+[14][23]$$

$\psi_{1L}\psi_{2L}\psi_{3L}\psi_{4L} : (n,\tilde{n}) = (0,4)$ Blue column number is: $\tilde{n}/2=2$

Blue column length: $N-2=2$

$$\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array} \qquad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} = \langle 13\rangle\langle 24\rangle+\langle 14\rangle\langle 23\rangle$$

$\psi_{1R}\psi_{2R}\psi_{3L}\psi_{4L} : (\tilde{n},n) = (2,2)$ Blue and white column: $n/2=1; \tilde{n}/2=1$

$$\begin{array}{|c|c|}\hline 1 & 1 \\ \hline 2 & 2 \\ \hline \end{array} = \langle 34\rangle[12]$$

# Massive Amplitude Basis

# Massive Effective Field Operator

- EFT of massive fields has wide application in particle physics

Higgs EFT ⊃ SMEFT ⊃ SM

Lower energy QCD

Dark matter EFT

Lower energy EFT

- Massless EFT is not concise in describing physics at EWSB

$W^+ - W^- - Z$

Three massive gauge bosons

Seven massive amplitudes bases

$$\left(W^a_{\mu\nu}\right)^3 |H|^{2n}$$

Infinite massless operator bases

- Massive fields amplitude base construction is very challenge

Redundancy: Equation of motion | Integration by part

Notrivial EOM $p\,|p^I] = m\,|p^I\rangle$

G. Durieux, T. Kitahara, Y. Shadmi and Y. Weiss, JHEP **01**, 119 (2020) doi:10.1007/JHEP01(2020)119 [arXiv:1909.10551 [hep-ph]].
G. Durieux, T. Kitahara, C. S. Machado, Y. Shadmi and Y. Weiss, JHEP **12**, 175 (2020) doi:10.1007/JHEP12(2020)175 [arXiv:2008.09652 [hep-ph]].
A. Falkowski, G. Isabella and C. S. Machado, [arXiv:2011.05339 [hep-ph]].

**How to solve it?**

# Massive amplitude basis

- The scattering amplitude can be factorized in two parts:

$m$ massive

$n$ massless

$$\mathcal{M}^I_{m,n} = \sum_{\{\dot\alpha\}} \mathcal{A}^I_{\{\dot\alpha\}}\left(\{\epsilon_{s_i}\}\right) G^{\{\dot\alpha\}}\left(|j], |j\rangle, p_i\right)$$

Massive LG charged

Massless LG charged

- Massive LG tensor structure (MLGTS) $\mathcal{A}^I(\{\epsilon_{s_i}\})$ is required to be the holomorphic function of $|i^I]$s

(EOM $|i^I\rangle = p_i|i^I]/m_i$ )

Linear in massive polarization tensor

$$\epsilon_{s_i} \equiv |i]^{\{I_1}_{\dot\alpha_1}, \ldots, |i]^{I_{2s_i}\}}_{\dot\alpha_{2s_i}} \in (2s_i+1, 2s_i+1) = SU(2)_l \otimes SU(2)_r$$

$\mathcal{A}^I(\{\epsilon_{s_i}\})$ can not be EOM and IBP redundant!

- Massive LG neutral structure (MLGNS) $G(|j], |j\rangle, p_i)$ is the function of massless spinors $|j], |j\rangle$ and massive momentum $p_i$

$G(|j], |j\rangle, p_i)$ can be both EOM and IBP redundant!

# Massive amplitude basis

$m$ massive
$n$ massless

$$\mathcal{M}^I_{m,n} = \sum_{\{\dot\alpha\}} \mathcal{A}^I_{\{\dot\alpha\}}\left(\{\epsilon_{s_i}\}\right) G^{\{\dot\alpha\}}\left(|j], |j\rangle, p_i\right)$$

**Constructing on-shell operator basis for all masses and spins**
Zi-Yu Dong, Teng Ma, Jing Shu
arXiv:2103.15837 [hep-ph].

- The general framework to construct a complete set of massive amplitude bases

$SU(2)_r$ Young tabular

A complete set of $\mathcal{A}^I(\{\epsilon_{s_i}\})$ bases

$U(N)$ Young tabular

A complete set of $G(|j], |j\rangle, p_i)$ bases

Contraction

A complete set of $\mathcal{M}^I_{m,n}$ bases

# Massive amplitude basis

- MLGTS basis $\mathscr{A}^I(\{\epsilon_{s_i}\})$ is the linear function of polarization tensor $\epsilon_{s_i}$

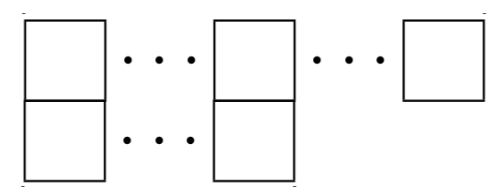
$$\epsilon_{s_i} \equiv |i]^{\{I_1}_{\dot\alpha_1}, \dots, |i]^{I_{2s_i}\}}_{\dot\alpha_{2s_i}} \sim \underbrace{\boxed{i}\cdots\boxed{i}_r}_{(2s_i)}$$

$(2s_i + 1)\ SU(2)_r$
Rep of $\epsilon_{s_i}$

- Any MLGTS basis belongs to the out product of all $\epsilon_{s_i}$'s $SU(2)_r$ Reps

$SU(2)_r$ Rep of $\epsilon_{s_i}$

$$\mathscr{A}^I(\{\epsilon_{s_i}\}) \subset \otimes_{i=1}^{m} \underbrace{\boxed{i}\cdots\boxed{i}_r}_{(2s_i)} =$$



$\oplus \cdots$

- A complete set of $\{\mathscr{A}^I_{\dot\alpha}\}$ bases can be constructed by finding all the $SU(2)_r$ irreducible representations from the out product of all $\epsilon_{s_i}$'s $SU(2)_r$ Reps

# Massive amplitude basis

For example: $\psi\psi' Z h$

$SU(2)_r$ Reps of polarization tensors for external legs

$$\psi \sim \boxed{1} \quad \psi' \sim \boxed{2} \quad Z \sim \boxed{3\,|\,3} \quad h \sim \bullet$$

$$\mathcal{A}^I_{\{\dot\alpha\}}(\{\epsilon_{s_i}\}) \subset \boxed{1} \times \boxed{2} \times \boxed{3\,|\,3} \times \bullet$$
$$= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 3 \\ \hline \end{array} \oplus \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 3 \\ \cline{1-1} \end{array} \oplus \begin{array}{|c|c|c|}\hline 1 & 3 & 3 \\ \hline 2 \\ \cline{1-1} \end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 3 \\ \hline \end{array}$$

- The MLGTS can be read from above SSYTs. For the first one SSYT

$$\mathcal{A}^I_{[2,2]} \equiv \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 3 \\ \hline \end{array} = (|1^I]_{\dot\alpha}|2^J]_{\dot\beta}|3^{K_1}]_{\dot\gamma_1}|3^{K_2}]_{\dot\gamma_2} + \text{perms in } SU(2)_r \text{ indices})$$
$$= [1^I 3^{\{K_1}][2^J 3^{K_2\}}].$$

# Massive amplitude basis

- MLGNS $G(|j], |j\rangle, p_i)$ is the function of massless spinors $|j], |j\rangle$ and massive momentum $p_i$

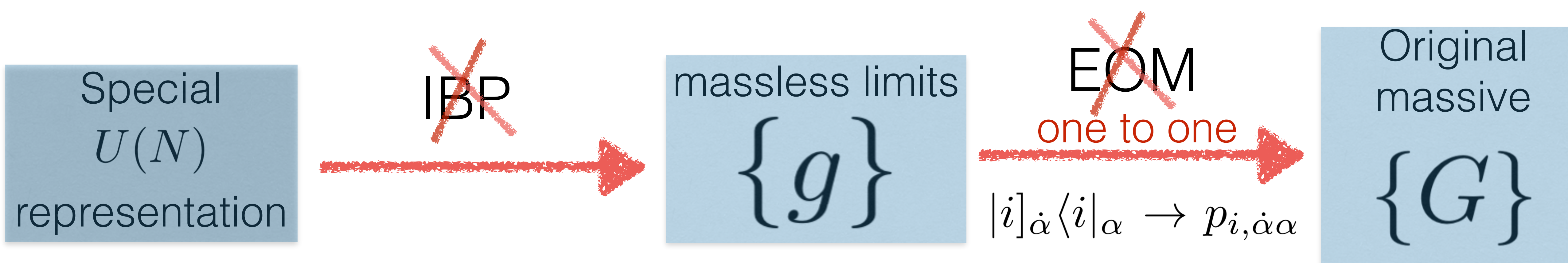
- MLGNS $G(|j], |j\rangle, p_i)$ bases suffer from EOM and IBP redundancy

- EOM redundancy can be removed by first constructing $G(|j], |j\rangle, p_i)$ massless limit bases

Construct massless limits

$$p_{i,\dot{\alpha}\alpha} \to |i]_{\dot{\alpha}} \langle i|_\alpha : G(|j], |j\rangle, p_i) \to g \equiv G(|j], |j\rangle, |i]\langle i|)$$

- IBP redundancy can be removed by $U(N)$ symmetry



# Massive amplitude basis

- The general framework to construct a complete set of massive amplitude bases

$SU(2)_r$ Young tabular

A complete set of $\mathscr{A}^I(\{\epsilon_{s_i}\})$ bases

$U(N)$ Young tabular

A complete set of massless $g(|j], |j\rangle, p_i)$ bases

$|i]_{\dot\alpha}\langle i|_{\alpha} \to p_{i,\dot\alpha\alpha}$

A complete set of $G(|j], |j\rangle, p_i)$ bases

Contraction

A complete set of $\mathscr{M}^I_{m,n}$ bases

The amplitude bases constructed by this procedure are independent without any redundancy !!

# Massive amplitude basis

- Example $W^+ - W^- - Z$ amplitude bases

MLGTS

$$
\begin{aligned}
\mathcal{A}^I_{\{\dot\alpha\}}(\{\epsilon_{s_i}\}) \subset\ & \begin{array}{|c|c|}\hline 1&1\\\hline\end{array} \times \begin{array}{|c|c|}\hline 2&2\\\hline\end{array} \times \begin{array}{|c|c|}\hline 3&3\\\hline\end{array} \\
=\ & \begin{array}{|c|c|c|}\hline 1&1&2\\\hline 2&3&3\\\hline\end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1&1&2&2\\\hline 3&3\\\cline{1-2}\end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1&1&2&3\\\hline 2&3\\\cline{1-2}\end{array} \oplus \begin{array}{|c|c|c|c|}\hline 1&1&3&3\\\hline 2&2\\\cline{1-2}\end{array} \\
\oplus\ & \begin{array}{|c|c|c|c|c|}\hline 1&1&2&2&3\\\hline 3\\\cline{1-1}\end{array} \oplus \begin{array}{|c|c|c|c|c|}\hline 1&1&2&3&3\\\hline 2\\\cline{1-1}\end{array} \oplus \begin{array}{|c|c|c|c|c|c|}\hline 1&1&2&2&3&3\\\hline\end{array} \\
\equiv\ & \mathcal{A}^I_{[3,3]} \oplus \mathcal{A}^I_{[(4,2)^1]} \oplus \mathcal{A}^I_{[(4,2)^2]} \oplus \mathcal{A}^I_{[(4,2)^3]} \\
& \oplus \mathcal{A}^I_{[(5,1)^1]} \oplus \mathcal{A}^I_{[(5,1)^2]} \oplus \mathcal{A}^I_{[6]}
\end{aligned}
$$

- Total seven $W^+ - W^- - Z$ amplitude bases

$$
\begin{aligned}
\mathcal{A}^I_{[3,3]} &: G^{\bullet}_{d=0} = 1 \\
\mathcal{A}^I_{[(4,2)^{1,2,3}]} &: G^{[3]}_{d=2} = \begin{array}{|c|c|c|}\hline 1&2&3\\\hline\end{array} \\
\mathcal{A}^I_{[(5,1)^{1,2}]} &: G^{[6]}_{d=4} = \begin{array}{|c|c|c|c|c|c|}\hline 1&1&2&2&3&3\\\hline\end{array} \\
\mathcal{A}^I_{[6]} &: G^{[9]}_{d=6} = \begin{array}{|c|c|c|c|c|c|c|c|c|}\hline 1&1&1&2&2&2&3&3&3\\\hline\end{array}
\end{aligned}
$$

Reason: **Not valid**

Blue column length:
$N-2=1$

$$\begin{array}{|c|c|}\hline 1&2\\\hline &3\\\cline{2-2}\end{array}$$

**Trivial kinetic**

$$2p_i \cdot p_j = (\epsilon_{ijk} m_k)^2 - m_i^2 - m_j^2$$

G. Durieux, T. Kitahara, Y. Shadmi and Y. Weiss, JHEP **01**, 119 (2020) doi:10.1007/JHEP01(2020)119 [arXiv:1909.10551 [hep-ph]].

# Computer Programs

- Massive EFT operators construction can be automatically done by computer programs: https://github.com/hamiguazzz/Massive

Z.-Y. Dong, T. Ma, J. Shu, and Z.-Z. Zhou, *The New Formulation of Higgs Effective Field Theory*, (2022), arXiv:2211.16515 [hep-ph].

**A.91 Type: $ZZZZ$**

**A.91.1 Dimension = 4, $\mathcal{O}_4^1$**

| Type: $ZZZZ$ $\quad d=4 \quad \mathcal{O}_4^1$ |
|---|
| $Z_\mu Z_\nu Z_\rho Z_\sigma \,\mathrm{Tr}\left(\sigma^\mu \bar\sigma^\rho\right) \mathrm{Tr}\left(\sigma^\nu \bar\sigma^\sigma\right)$ |

**A.91.2 Dimension = 6, $\mathcal{O}_6^{1\sim4}$**

| Type: $ZZZZ$ $\quad d=6 \quad \mathcal{O}_6^{1\sim4}$ |
|---|
| $Z_\mu Z_\nu Z^+_{\rho\sigma} Z^+_{\xi\tau} \,\mathrm{Tr}\left(\sigma^\mu \sigma^\nu \bar\sigma^{\xi\tau} \bar\sigma^{\rho\sigma}\right)$ |
| $Z^-_{\mu\nu} Z_\rho Z_\sigma Z^+_{\xi\tau} \,\mathrm{Tr}\left(\sigma^\rho \sigma^{\mu\nu} \sigma^\sigma \bar\sigma^{\xi\tau}\right)$ |
| $Z^-_{\mu\nu} Z^-_{\rho\sigma} Z_\xi Z_\tau \,\mathrm{Tr}\left(\sigma^\xi \sigma^{\mu\nu} \sigma^{\rho\sigma} \bar\sigma^\tau\right)$ |
| $(D_\mu Z_\rho)\, Z_\sigma\, (D_\nu Z_\xi)\, Z_\tau \,\mathrm{Tr}\left(\sigma^\sigma \bar\sigma^\mu \sigma^\xi \bar\sigma^\tau\right) \mathrm{Tr}\left(\bar\sigma^\nu \sigma^\rho\right)$ |

**A.86 Type: $ZZ\gamma^+\gamma^-$**

**A.86.1 Dimension = 6, $\mathcal{O}_6^1$**

| Type: $ZZ\gamma^+\gamma^-$ $\quad d=6 \quad \mathcal{O}_6^1$ |
|---|
| $Z_\mu Z_\nu \gamma^+_{\rho\sigma} \gamma^-_{\xi\tau} \,\mathrm{Tr}\left(\sigma^\mu \sigma^{\xi\tau} \sigma^\nu \bar\sigma^{\rho\sigma}\right)$ |

**A.86.2 Dimension = 8, $\mathcal{O}_8^{1\sim4}$**

| Type: $ZZ\gamma^+\gamma^-$ $\quad d=8 \quad \mathcal{O}_8^{1\sim4}$ |
|---|
| $Z^-_{\mu\nu} Z^+_{\rho\sigma} \gamma^+_{\xi\tau} \gamma^-_{\zeta\eta} \,\mathrm{Tr}\left(\bar\sigma^{\rho\sigma} \bar\sigma^{\xi\tau}\right) \mathrm{Tr}\left(\sigma^{\mu\nu} \sigma^{\zeta\eta}\right)$ |
| $Z^-_{\nu\rho} Z_\sigma \gamma^+_{\xi\tau} \left(D_\mu \gamma^-_{\zeta\eta}\right) \mathrm{Tr}\left(\sigma^\sigma \sigma^{\zeta\eta} \sigma^{\nu\rho} \bar\sigma^\mu \bar\sigma^{\xi\tau}\right)$ |
| $Z_\nu Z^+_{\rho\sigma} \left(D_\mu \gamma^+_{\xi\tau}\right) \gamma^-_{\zeta\eta} \,\mathrm{Tr}\left(\bar\sigma^\mu \sigma^{\zeta\eta} \sigma^\nu\right) \mathrm{Tr}\left(\bar\sigma^{\rho\sigma} \bar\sigma^{\xi\tau}\right)$ |
| $Z_\rho \left(D_\nu Z_\sigma\right) \gamma^+_{\xi\tau} \left(D_\mu \gamma^-_{\zeta\eta}\right) \mathrm{Tr}\left(\sigma^\rho \sigma^\mu \bar\sigma^\nu \sigma^{\zeta\eta} \sigma^\sigma \bar\sigma^{\xi\tau}\right)$ |

# Massless EFT VS Massive EFT

- Map SMEFT bases to HEFT bases

SMEFT bases

$$\mathcal{A}(H^i H_k^\dagger H^j H_l^\dagger) \supset c^+_{HHHH} \frac{s_{13}}{\Lambda^2} T^{+\,ij}_{\;\;\,kl} + c^-_{HHHH} \frac{s_{12} - s_{14}}{\Lambda^2} T^{-\,ij}_{\;\;\,kl}$$

HEFT bases

$$\boldsymbol{\mathcal{M}}(W^+W^-hh) = C^{00}_{6,WWhh} \frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$$

Just bold the spinors

$$\mathcal{A}(G^+G^-hh) = \frac{1}{2}\left(\mathcal{A}(H^1 H_1^\dagger H^2 H_2^\dagger) + \mathcal{A}(H^1 H_1^\dagger H_2^\dagger H^2)\right) = -\frac{c^+_{(H^\dagger H)^2} - 3c^-_{(H^\dagger H)^2}}{2} \frac{s_{12}}{2\Lambda^2}$$

$$-\frac{c^+_{(H^\dagger H)^2} - 3c^-_{(H^\dagger H)^2}}{2} \frac{s_{12}}{2\Lambda^2} \longrightarrow \frac{c^+_{(H^\dagger H)^2} - 3c^-_{(H^\dagger H)^2}}{2} \frac{[\mathbf{12}]\langle\mathbf{12}\rangle}{\Lambda^2}$$

**An EFT hunter's guide to two-to-two scattering: HEFT and SMEFT on-shell amplitudes**

Hongkai Liu (Technion), Teng Ma (Technion and Barcelona, IFAE and BIST, Barcelona), Yael Shadmi (Technion), Michael Waterbury (Technion) (Jan 26, 2023)

Published in: *JHEP* 05 (2023) 241 • e-Print: 2301.11349 [hep-ph]

# Summary

- A complete set of amplitude bases can be efficiently constructed by $SU(2)_l \times SU(2)_r \times U(N)$ representations

- The operators beyond dim=6 can be easily formulated by using the on-shell amplitude methods + the SU(N) Young Tablet.

- Based on our theory,  EFT operators of any massless & massive fields can be automatically constructed by computer programs

- Using on-shell method, SMEFT can be easily mapped into HEFT

Thanks !!

# Back up

# Summary

- The $N$-point scattering amplitude involving massive fields can be factorized in two parts

$$\mathscr{M}^{I}_{m,n} = \sum_{\{\dot{\alpha}\}} \underbrace{\mathscr{A}^{I}_{\{\dot{\alpha}\}}\left(\{\epsilon_{s_i}\}\right)}_{\text{Massive LG charged}} \underbrace{G^{\{\dot{\alpha}\}}\left(|j], |j\rangle, p_i\right)}_{\text{Massless LG charged}}$$

- $\mathscr{A}^{I}$ bases can be constructed by Lorentz subgroup $SU(2)_r$ representations

- $G$ bases can be constructed by global $U(N)$ representations

- The $N$-point amplitude bases involving massive fields can be obtained by contracting $\mathscr{A}^{I}$ and $G$ bases

# Computer Programs

- Massive EFT operators construction can be automatically done by computer programs

4-$Z$ operator bases at dim-4 and 6

**73 Type:** $ZZZZ$

**73.1 Dimension 4 ,** $\mathcal{O}_4^1$

$$Z_\mu Z_\nu Z_\rho Z_\sigma \operatorname{Tr}(\sigma^\mu \bar\sigma^\rho) \operatorname{Tr}(\sigma^\nu \bar\sigma^\sigma)$$

**73.2 Dimension 6 ,** $\mathcal{O}_6^{1\sim 4}$

$$Z_\mu Z^+_{\nu\rho} Z_\sigma Z^+_{\xi\tau} \operatorname{Tr}(\sigma^\mu \bar\sigma^\sigma) \operatorname{Tr}(\bar\sigma^{\nu\rho}\bar\sigma^{\xi\tau})$$

$$Z^-_{\mu\nu} Z_\rho Z_\sigma Z^+_{\xi\tau} \operatorname{Tr}(\sigma^{\mu\nu}\sigma^\sigma) \operatorname{Tr}(\sigma^\rho) \operatorname{Tr}(\bar\sigma^{\xi\tau})$$

$$Z^-_{\mu\nu} Z^-_{\rho\sigma} Z_\xi Z_\tau \operatorname{Tr}(\sigma^{\mu\nu}\sigma^{\rho\sigma}) \operatorname{Tr}(\sigma^\xi \bar\sigma^\tau)$$

$$Z^-_{\nu\rho} Z_\sigma Z_\xi (D_\mu Z_\tau) \operatorname{Tr}(\bar\sigma^\mu \sigma^{\nu\rho}\sigma^\xi) \operatorname{Tr}(\sigma^\sigma \bar\sigma^\tau)$$

4-$W$ operator bases at dim-4

**65 Type:** $W^+W^-W^+W^-$

**65.1 Dimension 4 ,** $\mathcal{O}_4^{1\sim 2}$

$$(W^-)_\mu (W^-)_\nu (W^+)_\rho (W^+)_\sigma \operatorname{Tr}(\sigma^\mu \bar\sigma^\rho) \operatorname{Tr}(\sigma^\nu \bar\sigma^\sigma)$$

$$(W^-)_\mu (W^-)_\nu (W^+)_\rho (W^+)_\sigma \operatorname{Tr}(\sigma^\mu \bar\sigma^\nu) \operatorname{Tr}(\sigma^\rho \bar\sigma^\sigma)$$

# Dimension Reduction of Massive Amplitude Basis

- The $\{\mathscr{A}\,.\,G\}$ bases can not be directly mapped into operator bases due to their dimension mismatch

For example: 4-pt $\psi_1\psi_2\phi_1\phi_2$ has two dim-5 operator bases

$$\text{dim-5} = \left\{[1^I 2^J]\,,\langle 1^I 2^J\rangle\right\}$$

Since the polarization tensors of $\{\mathscr{A}\,.\,G\}$ bases are holomorphic functions of $|i^I]$, $\{\mathscr{A}\,.\,G\}$ can not contain the bases with $|i^I\rangle$s in polarization tensors

Combination of $\{\mathscr{A}\,.\,G\}$

$$\langle 1^I 2^J\rangle = \frac{m_2[1^I 2^J]}{m_1} + \frac{[1^I 322^J]}{m_1 m_2} + \frac{[1^I 422^J]}{m_1 m_2}$$

$\{\mathscr{A}\,.\,G\}$ can not directly contain two polarization tensors of massive gauge boson

$$F^+_{\dot\alpha\dot\beta} = |i^{I_1}]\,|i^{I_2}] \qquad F^0_{\dot\alpha\beta} = mA_\mu = |i^{I_1}\rangle\,|i^{I_2}] \qquad F^-_{\alpha\beta} = |i^{I_1}\rangle\,|i^{I_2}\rangle$$

- The key problem: spinor $|i^I\rangle$ in polarization tensor is expressed by $p_i|i^I]$ in $\{\mathscr{A}\,.\,G\}$ that makes their dimension higher

# Dimension Reduction of Massive Amplitude Basis

- We should find a complete set of lowest dimensional amplitude bases that can directly map into physical operators

- Lowest dimensional amplitude bases means that their dimension can not be reduced further by EOM $p_i|i^I]/m_i = |i^I\rangle$

**How ?**

**Constructing Generic Effective Field Theory for All Masses and Spins**
Zi-Yu Dong, Teng Ma, Jing Shu, Yu-Hui Zheng
arXiv:2202.08350 [hep-ph].

- Step one: construct a redundant and complete set of amplitude bases $\{\mathscr{C}\,.\,F\}$ that can always contain a complete set of lowest dimensional amplitude bases

- Step two: decompose this redundant $\{\mathscr{C}\,.\,F\}$ bases into $\{\mathscr{A}\,.\,G\}$ bases from low to high dimension and remove the linear correlation bases

$$\{\mathscr{C}\,.\,F\} \xrightarrow{\text{Decompose}} \{\mathscr{A}\,.\,G\} \xrightarrow{\cancel{\text{Linear correlation}}} \{\mathcal{O}_{physical}\}$$

# Dimension Reduction of Massive Amplitude Basis

- $\{\mathscr{A}\,.\,G\}$ bases are complicated and long polynomials of spinor products due to horizontal permutations in their SSYTs
- For decomposition convenience, monomial bases $\{\mathscr{B}\,.\,H\}$ can be constructed from $\{\mathscr{A}\,.\,G\}$ SSYTs without horizontal permutations

One to one

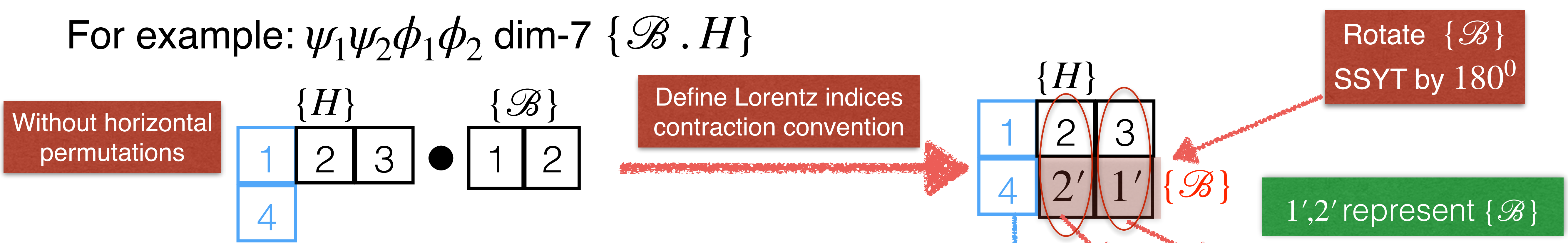
$$\{G\},\{\mathscr{A}\} \longleftrightarrow \{H\},\{\mathscr{B}\}$$

Equivalent

$$\{\mathscr{A}\,.\,G\} \longleftrightarrow \{\mathscr{B}\,.\,H\}$$

- The simplified bases $\{\mathscr{B}\,.\,H\}$ are just monomials of spinor products

For example: $\psi_1\psi_2\phi_1\phi_2$ dim-7 $\{\mathscr{B}\,.\,H\}$

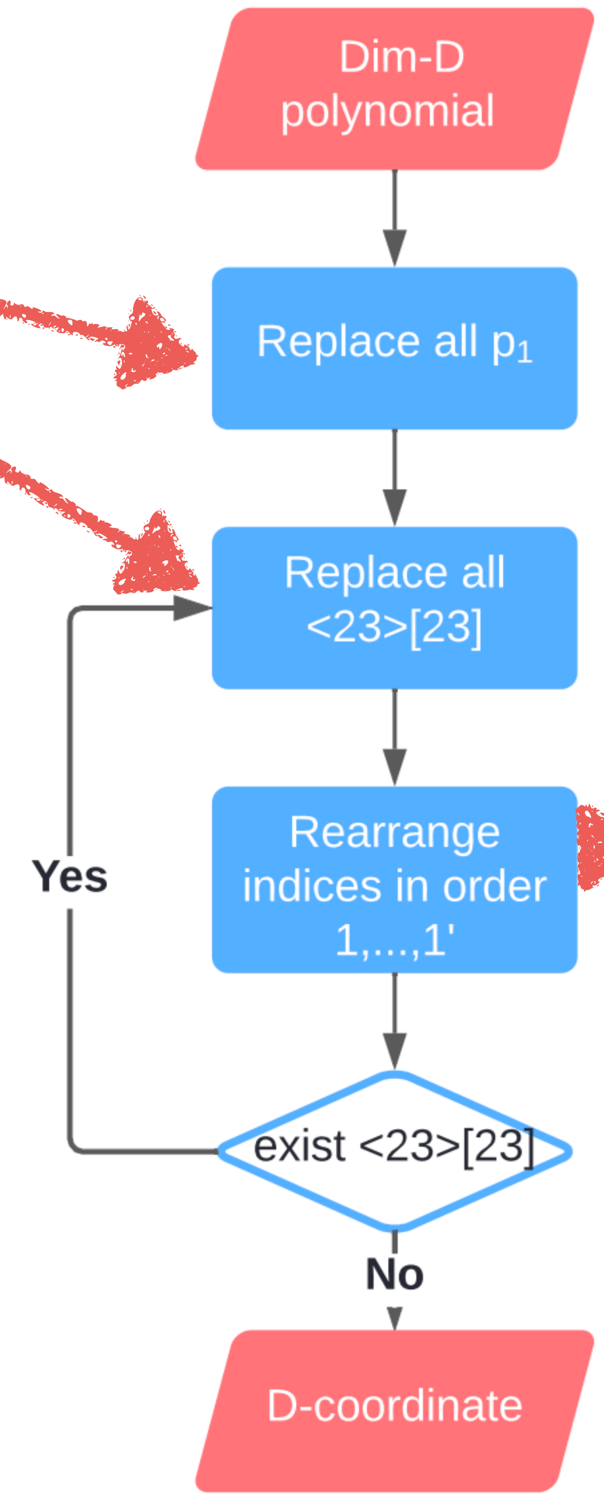



$$= \langle 2_L 3_K\rangle [2^L|_{\dot\alpha}[3^K|_{\dot\beta} \quad \bullet \quad [1|^I_{\dot\gamma}[2|^J_{\dot\delta} \qquad\qquad = \langle 2_L 3_K\rangle [2^L 2^J][3^K 1^I]$$

# Dimension Reduction of Massive Amplitude Basis

- Any polynomials of spinor product can be decomposed into $\{\mathscr{B}\,.\,H\}$ basis space systematically

$\{\mathscr{B}\,.\,H\}$ bases do not contain momentum $p_1$ factors and Mandelstam $s_{23}$




Consistent with SSYT

- Their coordinates in $\{\mathscr{B}\,.\,H\}$ space can be obtained and independence can be determined

# Dimension Reduction of Massive Amplitude Basis

How to systematically construct redundant $\{\mathscr{C}\,.\,F\}$ bases that contain a complete set of lowest dimensional amplitude bases???

- All the possible amplitude bases can be classified by a set of massive polarization tensor configurations $\{\ldots, \epsilon_{s_i}^{l_i}, \epsilon_{s_{l_{i+1}}}^{l_{i+1}}, \ldots\}$

Polarization tensor of massive particle-i with spin-$s_i$

$$\epsilon_{s_i}^{l_i} \equiv |\left(i^I\rangle\right)^{l_i}\left(|i^I]\right)^{2s_i-l_i}, \; 0 \le l_i \le 2s_i$$

Different value of $l_i$ represent different polarization tensor configuration

- A complete set of bases $\{\mathscr{C}\,.\,F\}^{\{\ldots, l_i, l_{i+1}, \ldots\}}$ with one kind of polarization tensor configuration $\{\ldots, \epsilon_{s_i}^{l_i}, \epsilon_{s_{i+1}}^{l_{i+1}}, \ldots\}$ can be constructed by $SU(2)_r \times U(N)$ SSYTs

- $\{\mathscr{C}\,.\,F\}$ bases consist of all sets with different polarization configurations $\{\mathscr{C}\,.\,F\}^{\{\ldots, l_i, l_{i+1}, \ldots\}}, \; \ldots, 0 \le l_i \le 2s_i, \ldots$

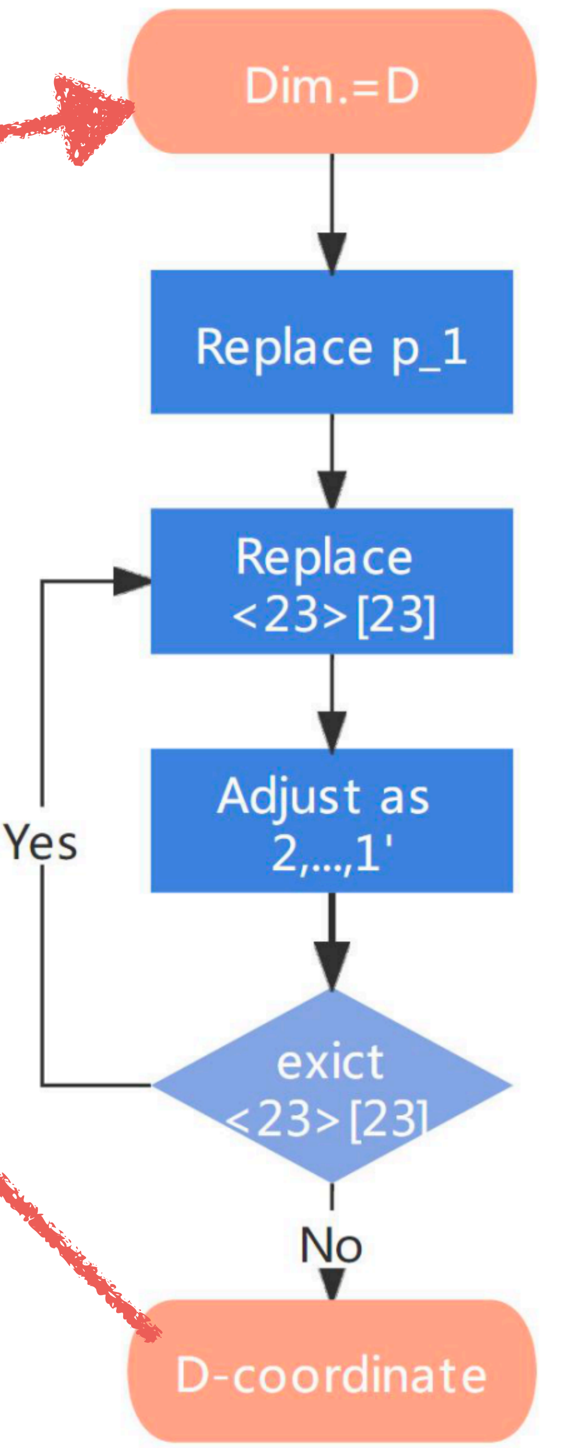
$$\mathscr{C}\,.\,F = \sum_{\{\ldots, l_i, l_j, \ldots\}}^{\ldots, 0 \le l_i \le 2s_i, \ldots} \left\{ \{\mathscr{C}\,.\,F\}^{\{\ldots, l_i, l_j, \ldots\}} \right\}$$

# Dimension Reduction of Massive Amplitude Basis

Decompose the over redundant $\{\mathscr{C}\,.\,F\}$ bases into $\{\mathscr{B}\,.\,H\}$ from low to high dimension

Remove all the linear correlation bases

The complete set of lowest dimensional bases can be obtained



# Example

- For example: $\psi_1\psi_2\phi_1\phi_2$ bases at dim-5, 6, 7

Construct $\{\mathscr{B}\,.\,H\}$ bases at dim-5, 7

$$\{\mathscr{B}\,.\,H\}^{D=5} = \left\{ \begin{array}{|c|}\hline 2' \\ \hline 1' \\ \hline \end{array} \right\} = \{[12]\} \quad \{\mathscr{B}\,.\,H\}^{D=7} = \left\{ \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 2' & 1' \\ \hline \end{array}\,, \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & 2' & 1' \\ \hline \end{array}\,, \begin{array}{|c|c|c|}\hline 1 & 2 & 2' \\ \hline 3 & 4 & 1' \\ \hline \end{array}\,, \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & 2' & 1' \\ \hline \end{array}\,, \begin{array}{|c|c|c|}\hline 1 & 3 & 2' \\ \hline 2 & 4 & 1' \\ \hline \end{array} \right\}$$

$$= \{m_2[132\rangle, m_2[142\rangle, s_{24}[12], [1432], s_{34}[12]\}$$

Construct $\{\mathscr{C}\,.\,F\}$ bases at dim-5, 6, 7

$$\{\mathscr{C}\,.\,F\}_5^{0000},\ \{\mathscr{C}\,.\,F\}_5^{1100}, \{\mathscr{C}\,.\,F\}_6^{0100}, \{\mathscr{C}\,.\,F\}_6^{1000}\ \{\mathscr{C}\,.\,F\}_7^{0000}$$

Remove all the linear correlation bases

Lowest dimension bases at dim-5, 6, 7

$$\{\mathcal{C}\cdot F\}_5 \longrightarrow \{[12], \langle 12\rangle\},$$

$$\{\mathcal{C}\cdot F\}_6 \longrightarrow \{[132\rangle, \langle 132]\},$$

$$\{\mathcal{C}\cdot F\}_7 \longrightarrow \{s_{24}[12], s_{34}[12], s_{34}\langle 12\rangle, \langle 1342\rangle\}$$

# Summary

- A set of $\{\mathscr{C}\,.\,F\}$ bases that can always contain a complete set of lowest dimensional amplitude bases can be constructed by $SU(2)_r \times U(N)$ representations

- The lowest dimensional amplitude bases can be obtained by decomposing $\{\mathscr{C}\,.\,F\}$ into the independent $\{\mathscr{B}\,.\,H\}$ bases

- The lowest dimensional amplitude bases can be directly mapped into EFT operators

- Based on this method, different types of EFT involving massive fields can be constructed by computer programs, like DM EFT, HEFT

# Identical Particles

- For n identical bosons (fermions), the amplitude bases should be totally (ant-) symmetric under these bosons permutation

- For example

For three identical bosons, the amplitude bases should be in $\boxed{1|2|3}$ representation of permutation symmetry $S_3$

- First, find $\boxed{1|2|3}$ representation matrix in the space of amplitude bases $\{\mathcal{O}\}$

$$\mathcal{Y}_{\boxed{1|2|3}}\{\mathcal{O}\} = M_{\boxed{1|2|3}}\{\mathcal{O}\}$$

Young operator

Representation

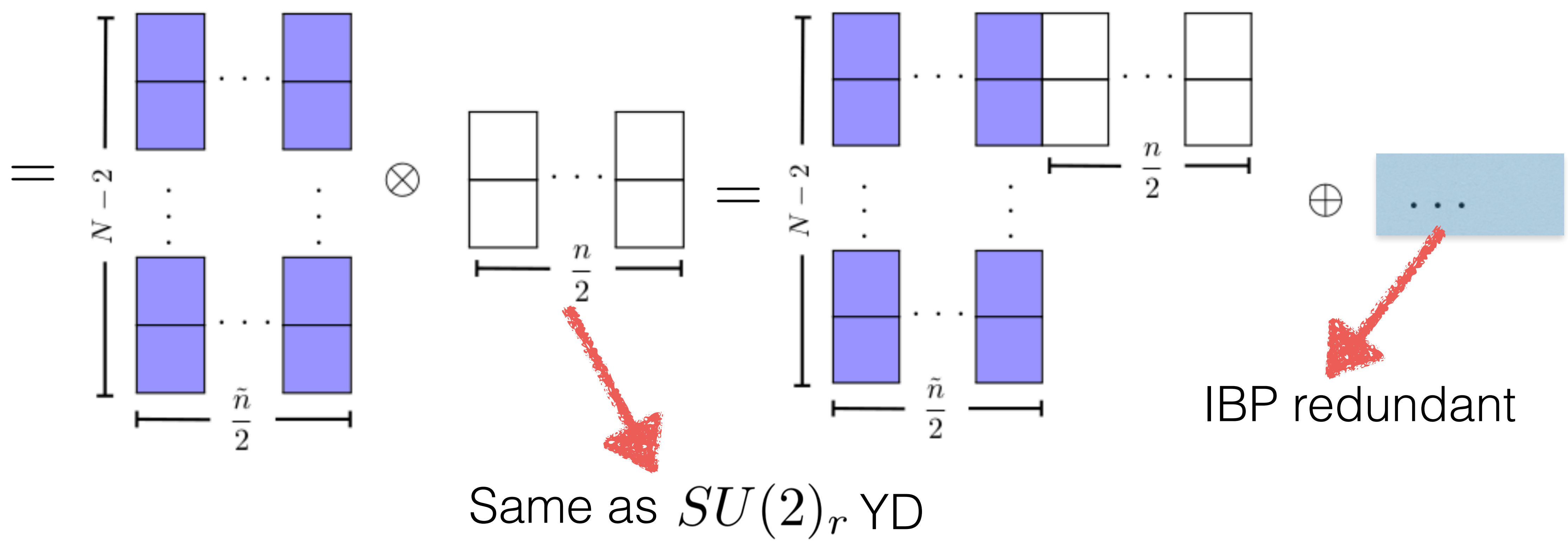
- Find $M_{\boxed{1|2|3}}$ eigenvectors with non-zero eigenvalues and these eigenvectors are the amplitude bases in $\boxed{1|2|3}$ representation

# Lowest dimensional massive amplitude basis

# On-shell amplitude basis

- IBP redundancy can be removed by $U(N) \supset \otimes_{i=1}^{N} U(1)_i$ symmetry

$$f^{(n,\tilde{n})}(\{\lambda,\widetilde{\lambda}\}) = (\overline{g}_{U(N)} \otimes g_{U(N)})$$




- The amplitude basis is the basis of the first U(N) representation

- The computer programs can construct massless basis based on it

# EFT

- Null new physics signals at the detections on ground

$$\Lambda \gg \mathcal{O}(1)\text{TeV} \xrightarrow{\text{precision measurement}} \delta g = g_{exp} - g_{SM} \Leftrightarrow \mathcal{O}_{d\geq 4}$$

- Dark matter detections

$$\mathcal{O}_{DM} \sim ee\phi_{DM}\phi_{DM}$$

$$\{\mathscr{C}\,.\,F\}^{\{l_i\,,\,l_j\,,\,\ldots\}} \equiv \mathscr{C}(|\,i^{\{I_1}]\cdots|\,i^{I_{2s_i-l_i}}]) \cdot F(|\,i^{I_{2s_i-l_i+1}}\rangle\cdots|\,i^{I_{2s_i}\}}\rangle, p_i, |\,j], |\,j\rangle)$$

- Higher spin particles

$$\mathcal{O} \sim (D_{\mu_1}e)\gamma_{\mu_2}\gamma_{\mu_3}e\rho^{\mu_1\mu_2\mu_3}$$

# Massless amplitude basis

- Basic structure of massless amplitudes

T. Ma, J. Shu and M. L. Xiao, [arXiv:1902.06752 [hep-

$$\mathcal{M}(\{p_i, h_i\}) = f(|i], |i\rangle) g(s_{ij}) T^{\{\alpha\}}$$

Massless LG charged

Massless LG neutral

Gauge structure

- An amplitude basis just corresponds to the leading interaction of a operator

- IBP redundancy can be systematically removed by $U(N) \supset \otimes_{i=1}^{N} U(1)_i$

Quantum number under $SU(2)_r \otimes U(N)$

$$\lambda_\alpha^k \equiv |k] = (2, N)$$

Quantum number under $SU(2)_l \otimes U(N)$

$$\tilde{\lambda}_{k\dot{\alpha}} \equiv |k\rangle = (2, \bar{N})$$

$$\begin{array}{|c|}\hline i \\ \hline j \\ \hline\end{array} = (\tilde{\lambda}^i_{\dot{\alpha}} \tilde{\lambda}^j_{\dot{\beta}} - \tilde{\lambda}^j_{\dot{\alpha}} \tilde{\lambda}^i_{\dot{\beta}}) = [ij]$$

$$\begin{array}{|c|}\hline k_1 \\ \hline \cdot \\ \hline \cdot \\ \hline k_{N-2} \\ \hline\end{array} = \frac{(\epsilon^{ijk_1 \ldots k_{N-2}} + \text{anti-sym in } k_1..k_{N-2})}{(N-2)!} \lambda_{i\alpha} \lambda_{j\beta}$$

$$= \langle ij \rangle \epsilon^{ijk_1 \ldots k_{N-2}} \, .$$

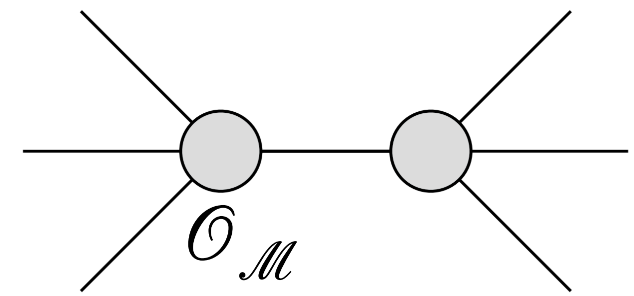
# Problems in massless amplitude basis

- Massless amplitude basis is fail at EWSB phase

G. Durieux, T. Kitahara, Y. Shadmi and Y. Weiss, JHEP **01**, 119 (2020) doi:10.1007/JHEP01(2020)119 [arXiv:1909.10551 [hep-ph]].
G. Durieux, T. Kitahara, C. S. Machado, Y. Shadmi and Y. Weiss, JHEP **12**, 175 (2020) doi:10.1007/JHEP12(2020)175 [arXiv:2008.09652 [hep-ph]].
A. Falkowski, G. Isabella and C. S. Machado,

- Troublesome in calculation at EWSB phase

$$\mathcal{M}_{\mathcal{O}} \to \mathcal{O}_{\mathcal{M}} \xrightarrow{\langle H \rangle \neq 0}$$




- Massless EFT is not concise in describing physics at EWSB

$$W^+ - W^- - Z$$

Three massive gauge bosons

Seven massive amplitudes

$$\left(W^a_{\mu\nu}\right)^3 |H|^{2n}$$

Infinite massless operator bases

- Massive EFT is more useful and convenient at EWSB phase, studying higher spin parties and DM

# Dimension Reduction of Massive Amplitude Basis

How to systematically construct redundant $\{\mathscr{C}\,.\,F\}$ bases that contain a complete set of lowest dimensional amplitude bases???

- All the possible amplitude bases can be classified by polarization tensor configurations $\{\,|\,\left(i^I\rangle\right)^{l_i}\left(\,|\,i^I]\right)^{2s_i-l_i}\}$ of massive particle-i with spin-$s_i$ , $0 \leq l_i \leq 2s_i$
- Construct a complete set of bases $\{\mathscr{C}\,.\,F\}^{\{\ldots,\,l_i,\,l_j,\,\ldots\}}$ with one kind of polarization tensor configuration $\left(\,|\,i^I\rangle\right)^{l_i}\left(\,|\,i^I]\right)^{2s_i-l_i}$ for massive particle-i with spin-$s_i$ by $SU(2)\times U(N)$ SSYTs
- $\{\mathscr{C}\,.\,F\}$ bases consist of all $\{\mathscr{C}\,.\,F\}^{\{\ldots,\,l_i,\,l_j,\,\ldots\}}$ , $0 \leq l_i \leq 2s_i\,, \ldots$

$$\mathscr{C}\,.\,F = \sum_{\{\ldots,\,l_i,\,l_j,\,\ldots\}} \left\{ \{\mathscr{C}\,.\,F\}^{\{\ldots,\,l_i,\,l_j,\,\ldots\}} \right\}$$

# Massive amplitude basis

- Independence proof:

Case I: two bases with two different $\mathscr{A}^I(\{\epsilon_{s_i}\})$ bases

Different massive LG tensor

$$\mathcal{A}^I_{[\lambda]} \cdot G(|j], |j\rangle, p_i) \neq \mathcal{A}^I_{[\lambda']} \cdot G'(|j], |j\rangle, p_i)\,,\ \lambda \neq \lambda'$$

Case II: bases with the same $\mathscr{A}^I(\{\epsilon_{s_i}\})$ bases

$$\mathcal{A}^I_{[\lambda]} \cdot \Big(\sum_{\eta} Z_{[\eta]} G_d^{[\eta]} + \sum_{i,\eta'} Z_{i[\eta']} m_i^2 G_{d-2}^{[\eta']} + \cdots\Big) = 0$$

$$\rightarrow \sum_{\eta} Z_{[\eta]} G_d^{[\eta]} + \sum_{i,\eta'} Z_{i[\eta']} m_i^2 G_{d-2}^{[\eta']} + \cdots = 0$$

$G$ bases are independent

The amplitude bases constructed by this procedure are independent without any redundancy !

# Introduction of on-shell amplitude

- Amplitude structure is determined by little group (LG) and unitarity
- For external massless particle-j with helicity $h_j$ , its amplitude should take $-2h_j$ unit charge of massless LG $U(1)_j$

Polarization tensor $\epsilon_{h_j}$ expression is unique

$$\epsilon_{h_j} = |j]^{2h_j} \left(|j\rangle^{-2h_j}\right)$$

Takes all the LG $U(1)_j$ charge

Linear function of polarization tensor $\epsilon_{h_j}$

$$\mathscr{M}([ij], \langle ij\rangle, \ldots) = \mathscr{M}(\epsilon_{h_j}, p_j, \ldots)$$

- Massive particle-i with spin $s_i$ , the amplitude should be in $2s_i$ indices symmetric representation of massive LG $SU(2)_i$ , i.e. $(2s_i+1)$ representation of $SU(2)_i$

Linear function of polarization tensor $\epsilon_{s_i}$

$$\mathscr{M}^{\{I\}}([ij]^{IJ}, \langle ij\rangle^{IJ}) = \mathscr{M}(\epsilon_{s_i}, |i^I]\langle i_I|, \ldots)$$

$\epsilon_{s_i}$ expression is not unique

$$\epsilon_{s_i} = \left(|i^{\{I}\rangle\right)^{l_i} \left(|i^{I\}}]\right)^{2s_i-l_i} \in (2s_i+1)\,,\ \ l_i \in [0, 2s_i]$$

E. Witten, Commun. Math. Phys. **252**, 189 (2004) doi:10.1007/s00220-004-1187-3 [hep-th/0312171].

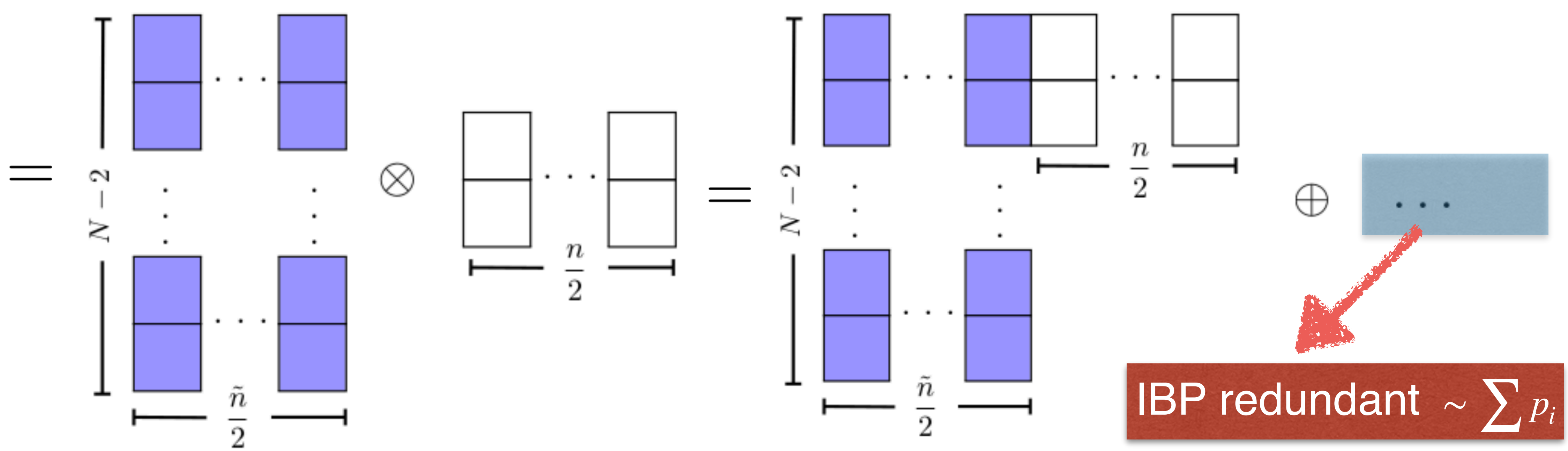
N. Arkani-Hamed, T. C. Huang and Y. t. Huang, arXiv:1709.04891 [hep-th].

# Massless amplitude basis

- IBP redundancy can be removed by $U(N)$ symmetry

Non-holomorphic bases
with $n$ right-handed spinor
and $\tilde{n}$ left-handed spinors

$$f^{(n,\tilde{n})}(\{\lambda, \widetilde{\lambda}\}) = (\overline{g}_{U(N)} \otimes g_{U(N)})$$




- The amplitude bases without IBP are the bases of the first U(N) representation

# Massive amplitude basis

- Two $G$ bases with one massless $g$ are EOM redundant

$$
\begin{tikzcd}
 & G'(|j],|j\rangle,p_i) \arrow[dd, leftrightarrow, "\text{EOM}\quad p_i|i^I] = m_i|i^I\rangle"] \\
g(|j],|j\rangle,|i]\langle i|) \arrow[ur, "|i]'_{\dot\alpha}\langle i|'_{\alpha} \to p_{i,\dot\alpha\alpha}"] \arrow[dr, "|i]_{\dot\alpha}\langle i|_{\alpha} \to p_{i,\dot\alpha\alpha}"'] & \\
 & G(|j],|j\rangle,p_i)
\end{tikzcd}
$$

$$G'(|j],|j\rangle,p_i) - G(|j],|j\rangle,p_i) = \mathcal{O}(m_i^2)$$ **EOM redundant**

- One massless $g$ basis one-to-to corresponds to an independent $G$ basis

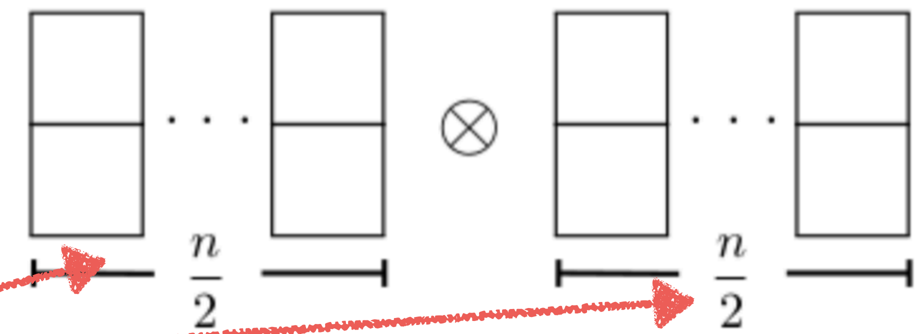
# Massless amplitude basis

- IBP redundancy can be removed by $U(N)$ symmetry

Holomorphic case

Bases with only $n$ right-handed spinors

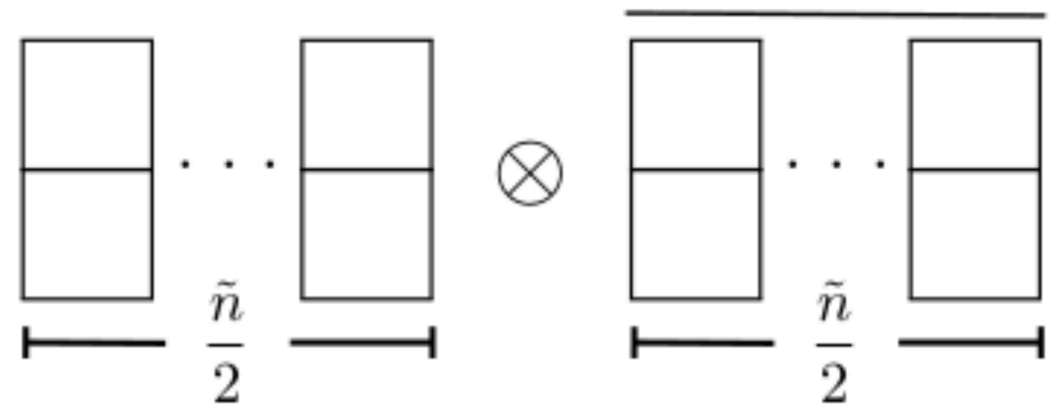
$$f^{(n)}_{\text{hol}}(\{\tilde\lambda\}) = g_{SU(2)_r} \otimes g_{U(N)} =$$



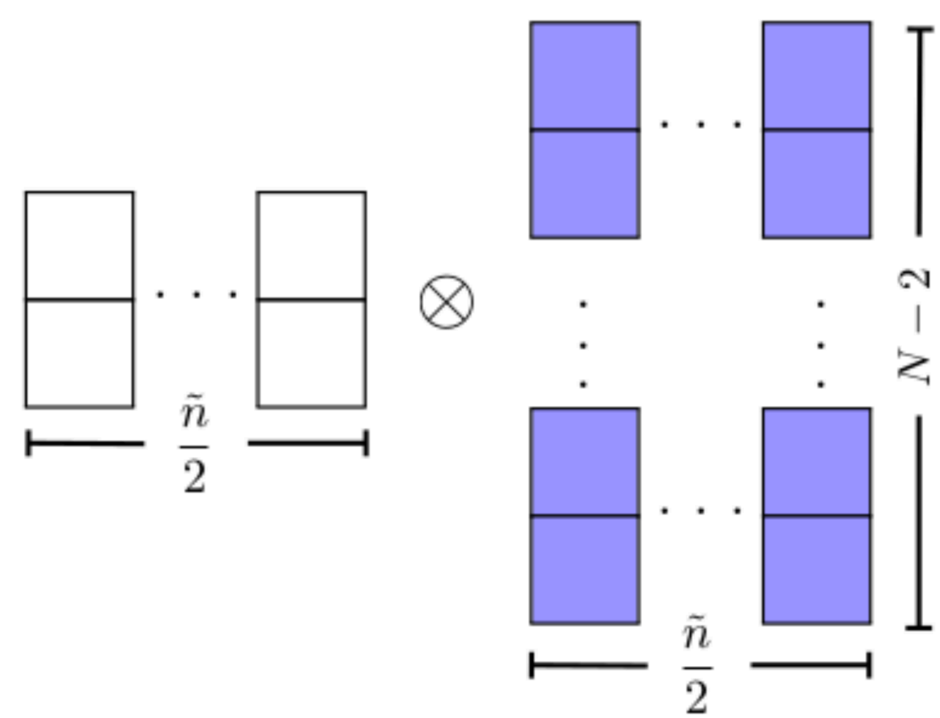
Same shape

Bases with only $\tilde n$ left-handed spinors

$$f^{(\tilde n)}_{\text{anti-hol}}(\{\lambda\}) = g_{SU(2)_l} \otimes \bar g_{U(N)} =$$



$$=$$



A column of white boxes represents the square spinor product $[ij]$

A column of blue boxes represents the angle spinor product $\langle ij\rangle$

- Holomorphic polynomials are not IBP redundant

# Constructing On-shell Operator Basis for All Masses and Spins

**Teng Ma**

**(Technion)**

Based on:

**Arxiv: 2103.15837, 2202.08350**

Collaborators:

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