UC SANTA BARBARA

Kavli Institute for Theoretical Physics

Hilbert Series, Higgs, and HEFT

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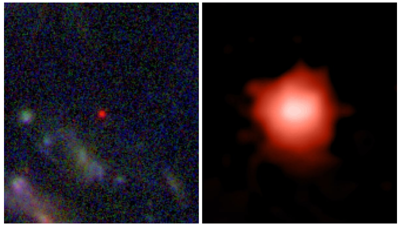
The universe at different scales

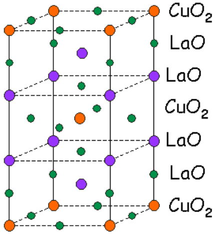
Standard Model

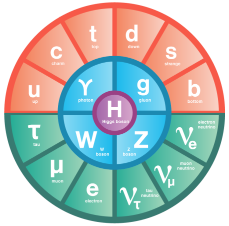
New Particles

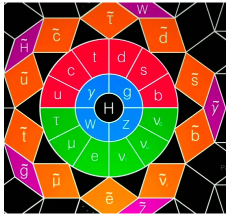
10^{27} 10^{20} 10^{10} 1m 10^{-10} 10^{-15} 10^{-35}

Very big → Very Small









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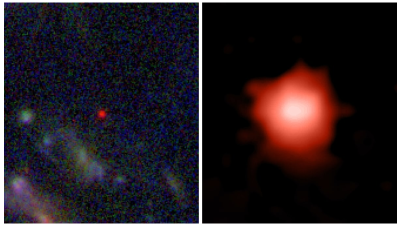
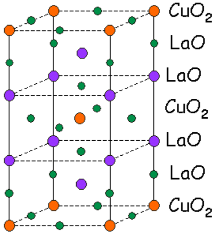
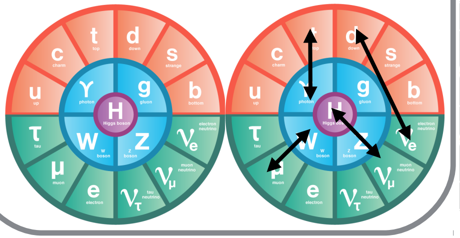




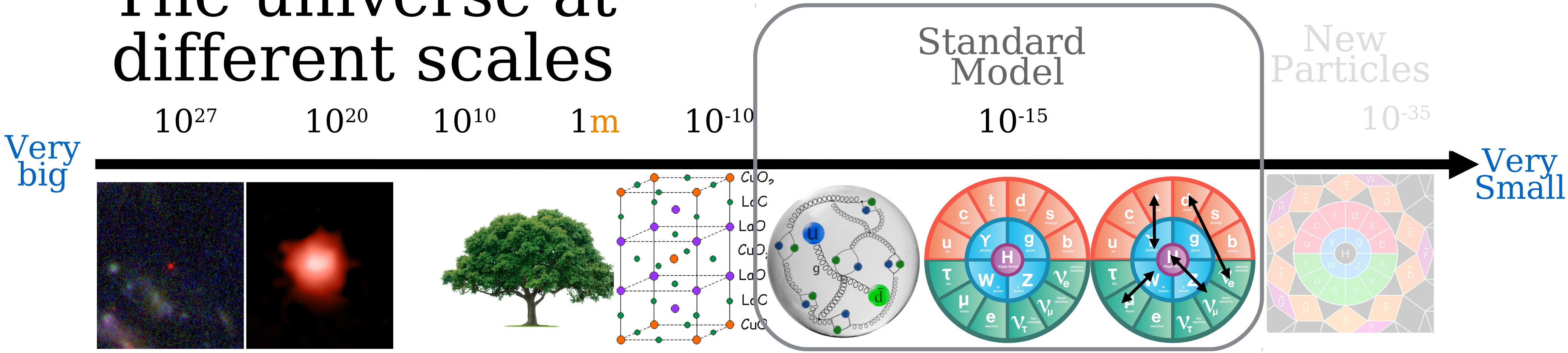


Effective **F**ield **T**heory:

New Interactions

- Model independent
- Exhaustive
- Guide for experiments

The universe at different scales



New Physics under our nose: **QCD**

Effective **F**ield **T**heory:
New Interactions

- Model independent
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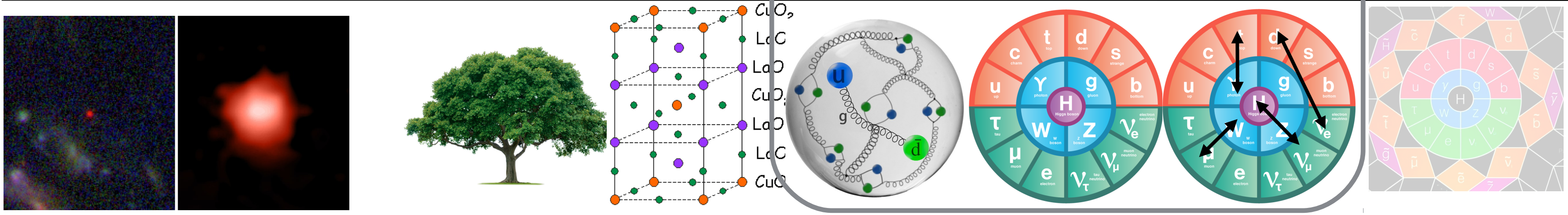
The universe at different scales

Very big — 10^{27} — 10^{20} — 10^{10} — 1m — 10^{-10} — 10^{-15} — 10^{-35} — Very Small

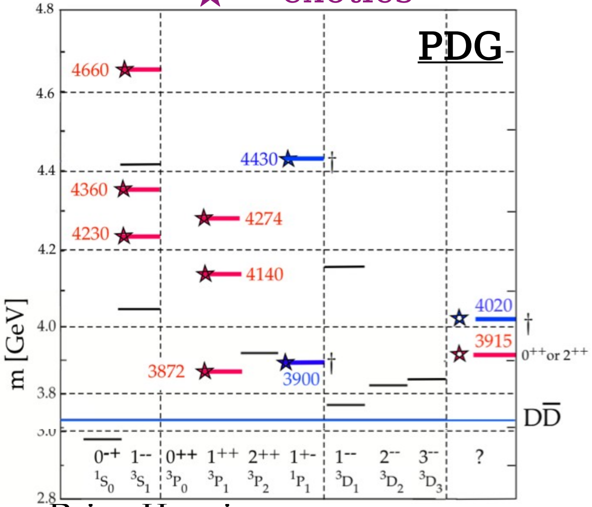
Standard Model

New Particles



Charmonium spectrum

★ = exotics



New Physics under our nose: **QCD**

Numerous quantitative and qualitative mysteries in the strong sector

Effective **F**ield **T**heory:

New Interactions

- Model independent
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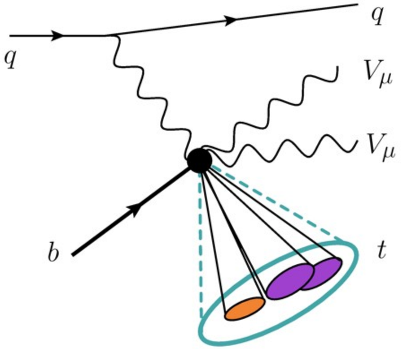
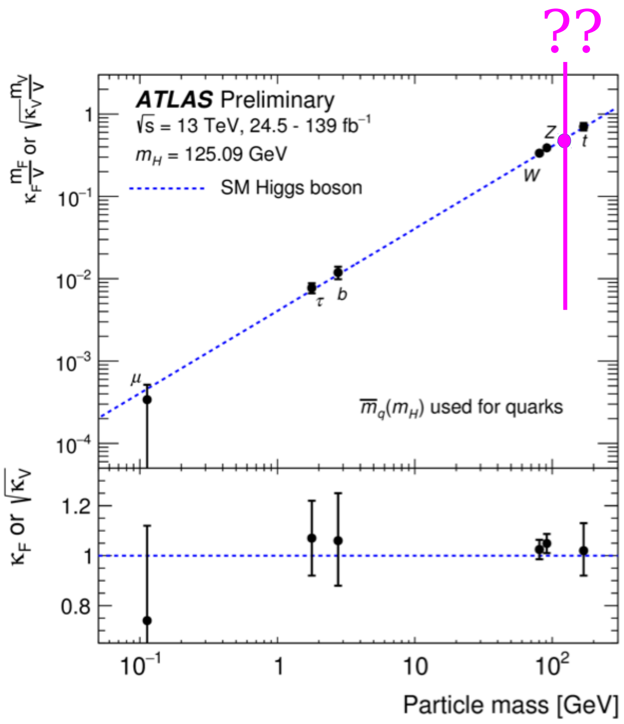


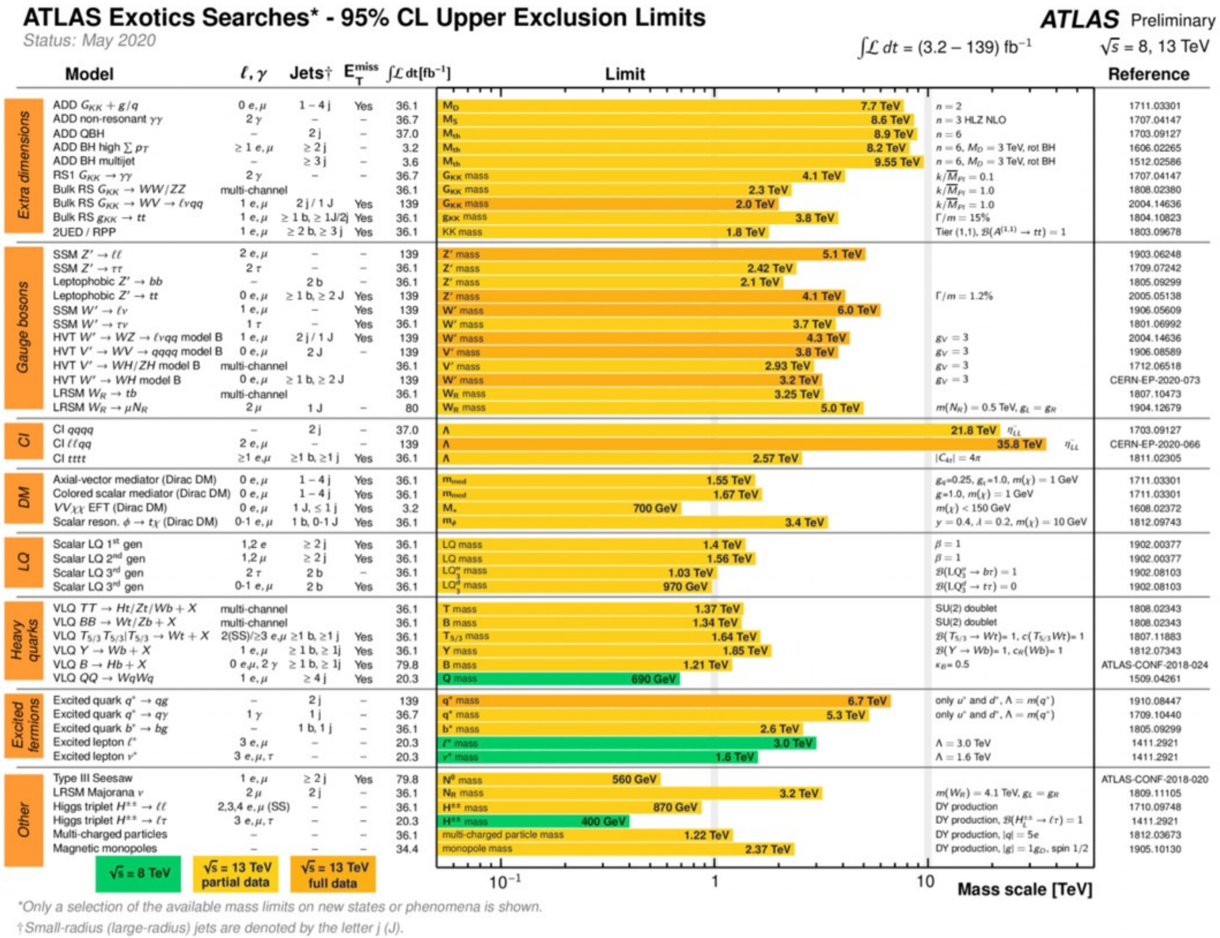
This talk: EFT

EFT & the hunt for new physics

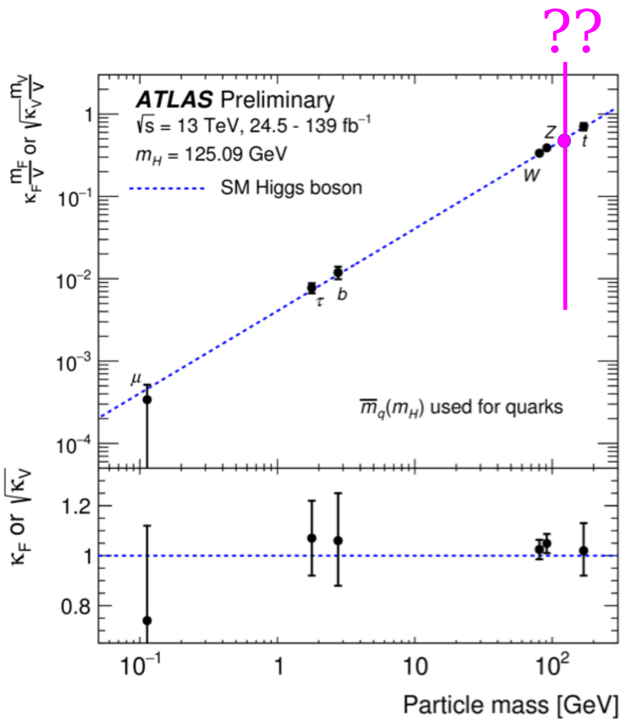


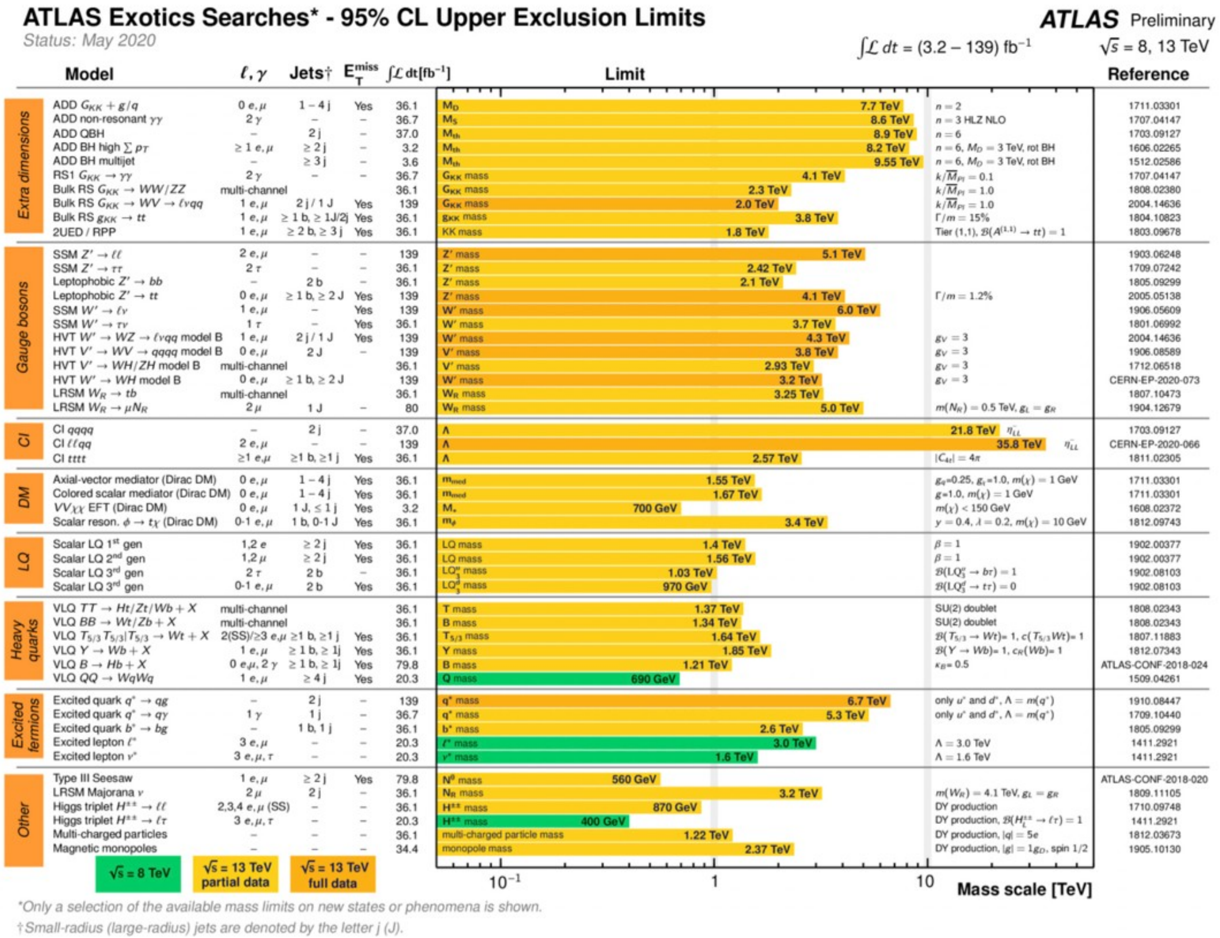
we've found this…

…but not these



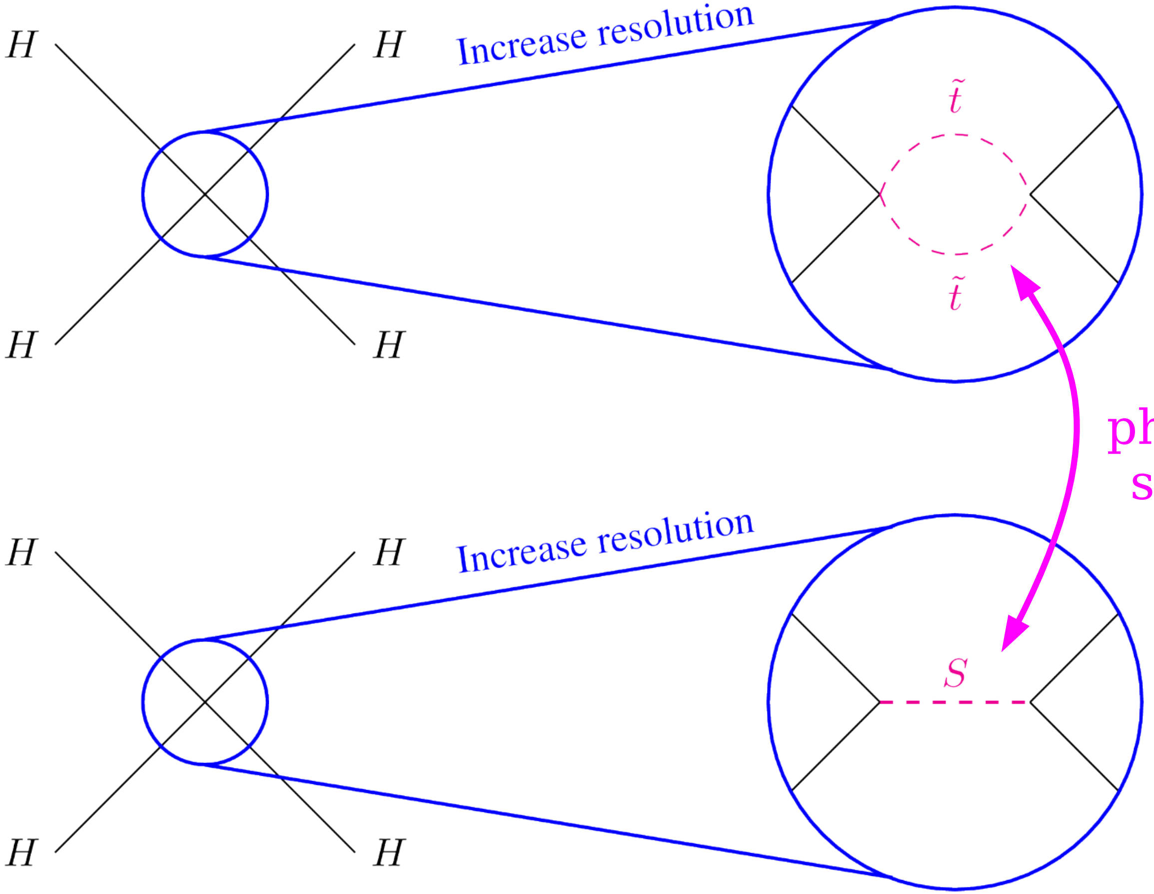
EFT & the hunt for new physics





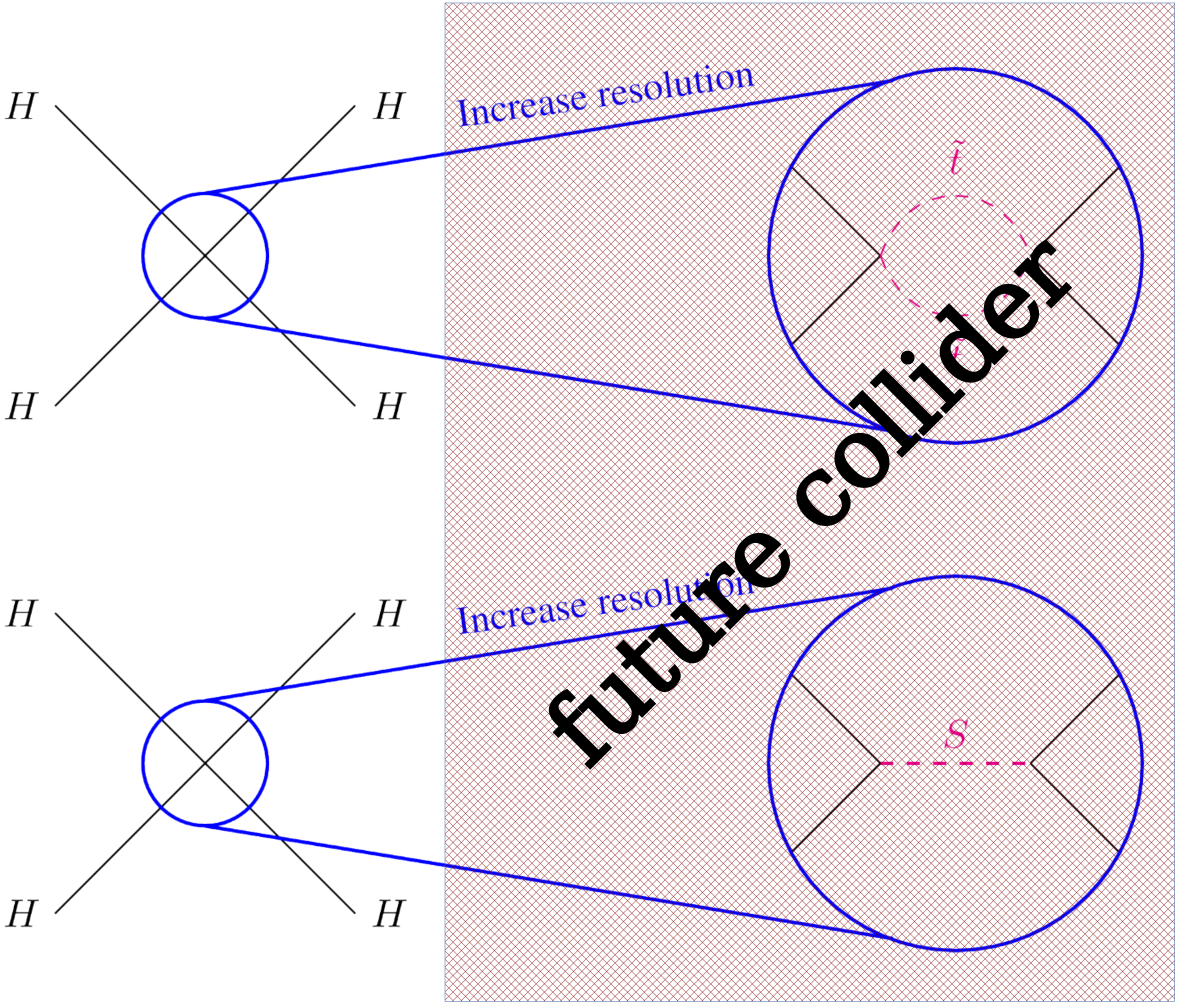
Are we sure we're thinking of everything?

A shift in thinking



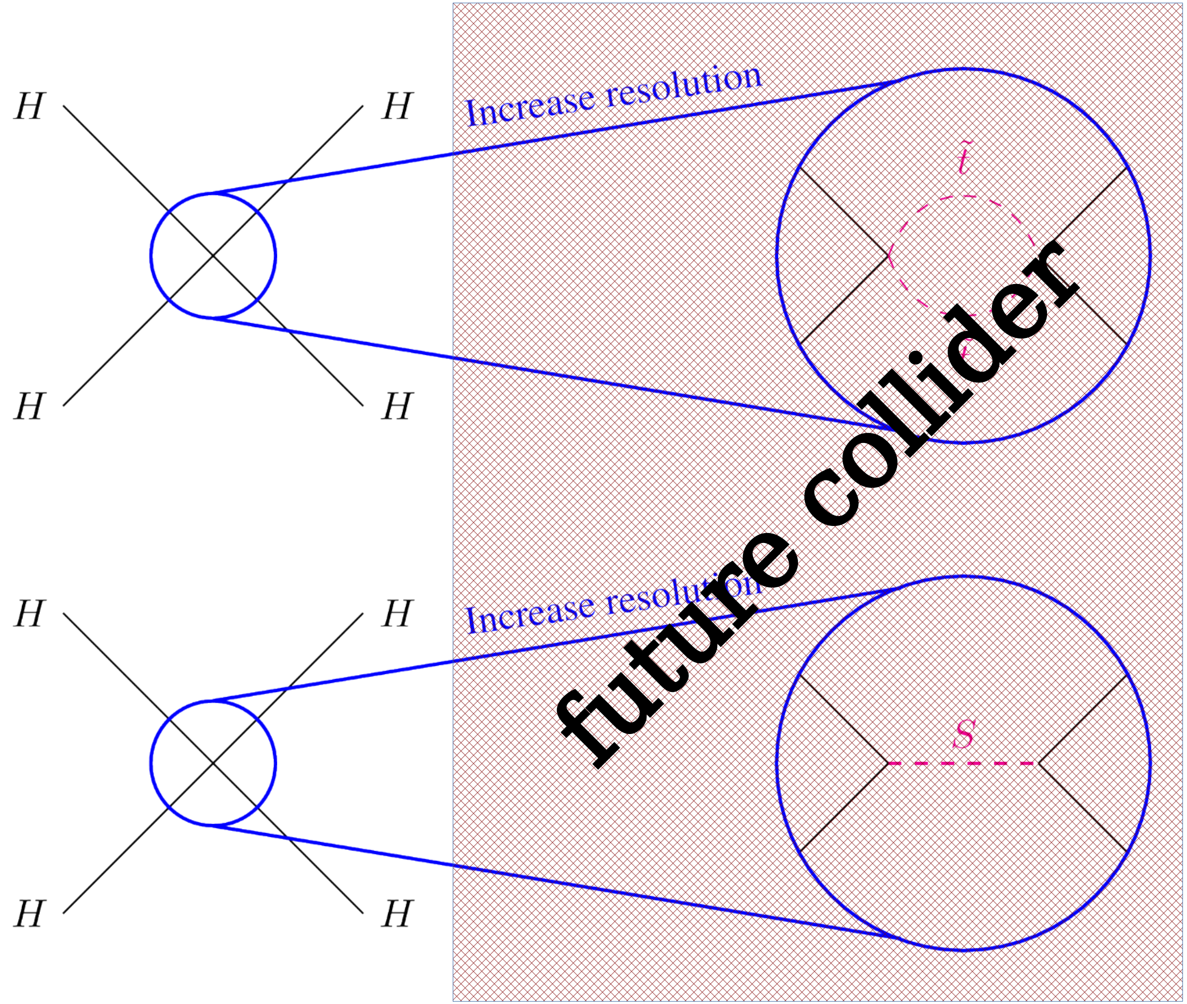
Various UV physics can give same effective interaction

A shift in thinking



What if you're resolution limited?

A shift in thinking



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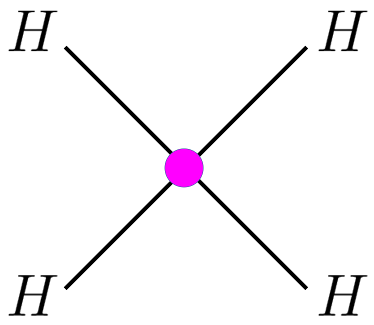
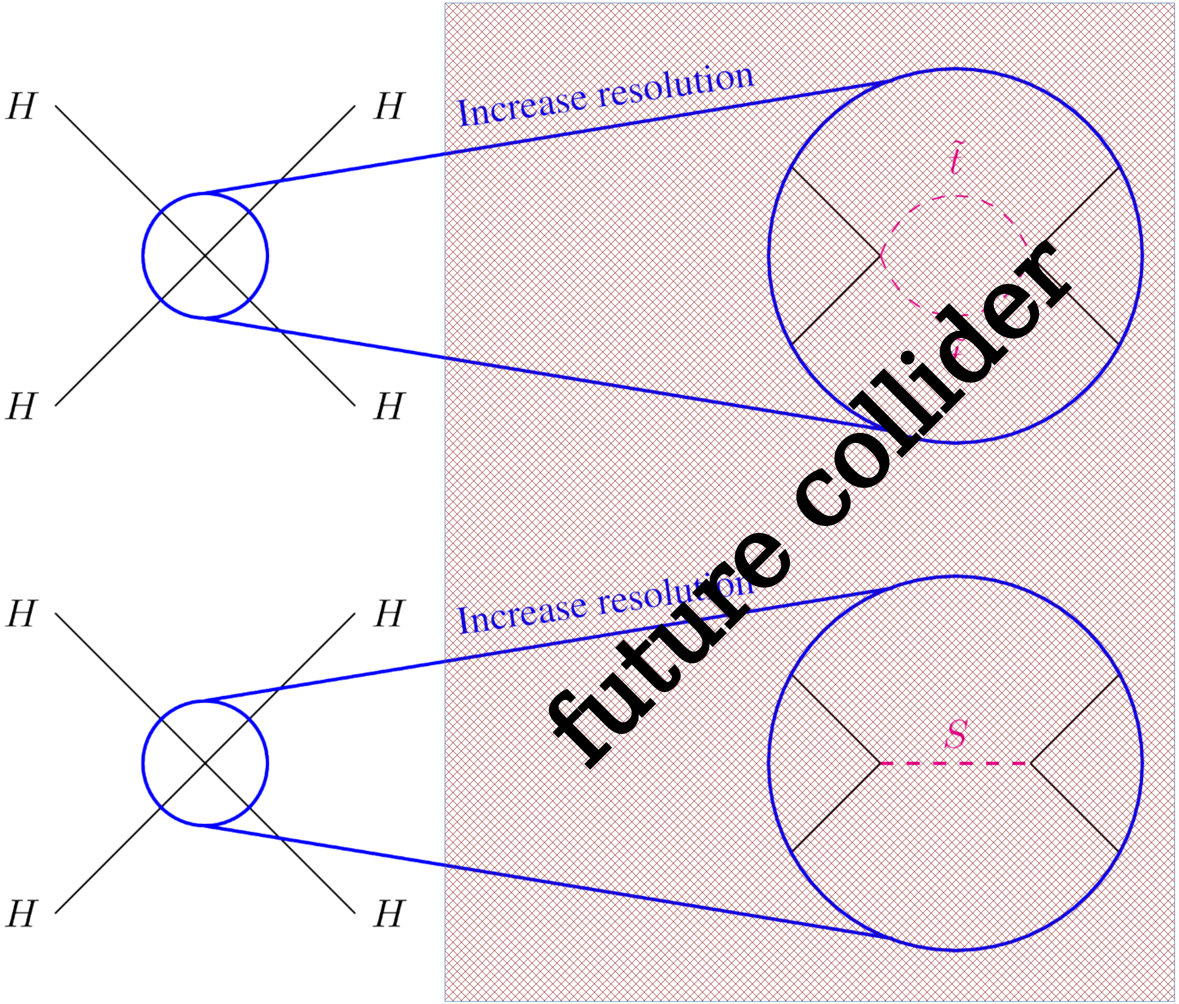
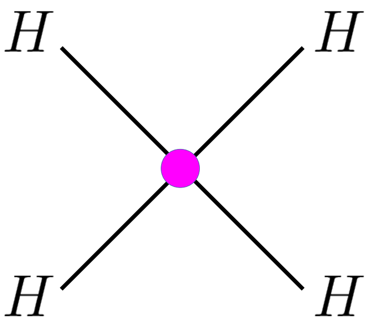


Instead parameterize all possible (local) interactions involving particles you can produce

A shift in thinking



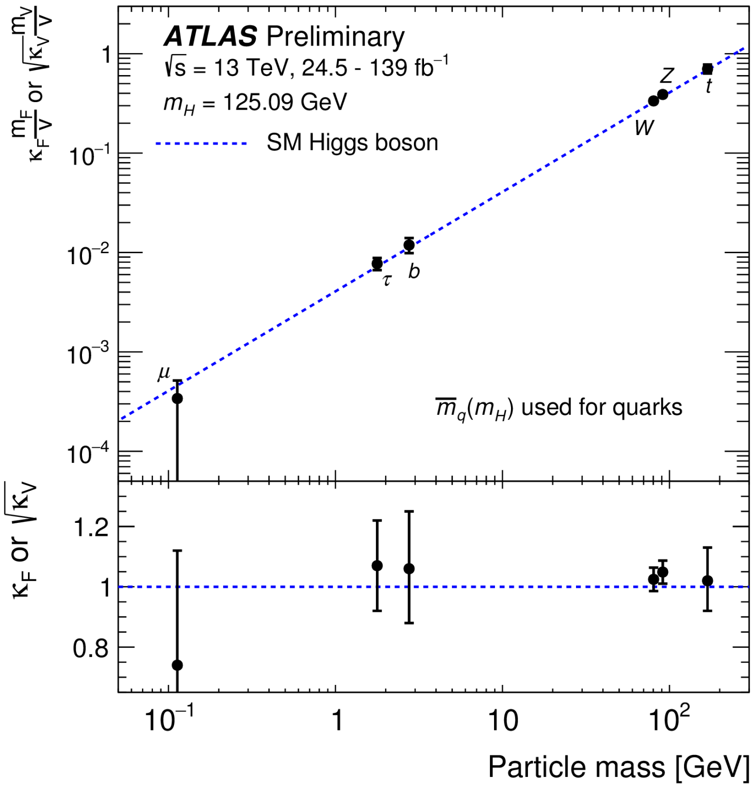


parameterize all possible (local) interactions

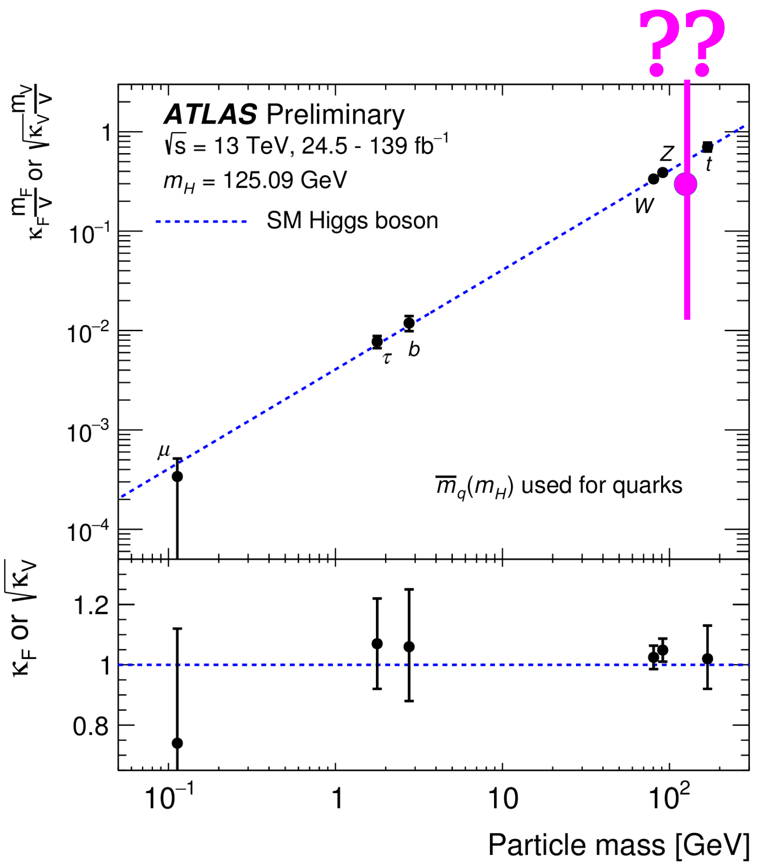
↓

Solved and systematized with Hilbert series methods!

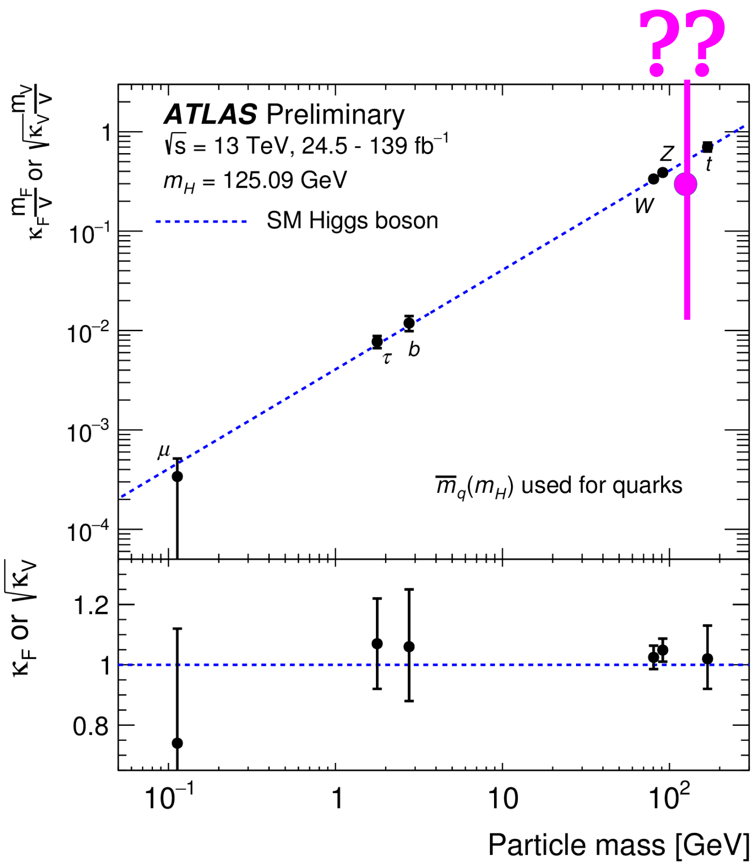
The importance of the Higgs



The importance of the Higgs



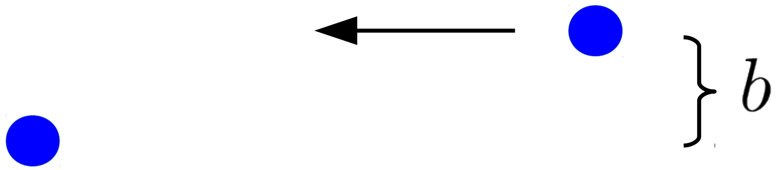
The importance of the Higgs



Deviations in *any* of h couplings leads to unitarity violation

can’t keep growing

$$\frac{y(E)e^{-mr}}{r}$$



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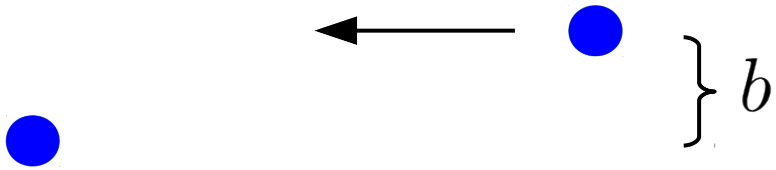

$$ye^{-mb} \text{ maximal for } b = \frac{1}{m}\log y$$

coupling grows at most polynomially with E $\lim_{E\to\infty} y(E) \lesssim E^{\alpha}$

can't keep growing

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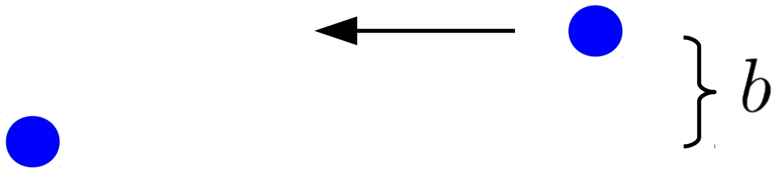

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coupling grows at most polynomially with E $\lim_{E\to\infty} y(E) \lesssim E^{\alpha}$

$$\sigma \sim b^2 \lesssim \frac{\alpha^2}{m^2}\log^2 E \qquad \sigma \sim \frac{1}{E^2}|\mathcal{A}|^2 \Rightarrow \mathcal{A} \lesssim E\log E$$

cross-sections/amplitudes *bounded* in energy! (Froissart bound)

Unitarity violation and EFT

EFT makes all this transparent

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{\Delta_i - 4}} \mathcal{O}_i$$

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single $\mathcal{O}_i$ insertion

$$\mathcal{A}_{\mathcal{O}_i}(E \to \infty) \sim \left(\frac{E}{\Lambda}\right)^{\Delta_i - 4}$$

naughty high-E behavior

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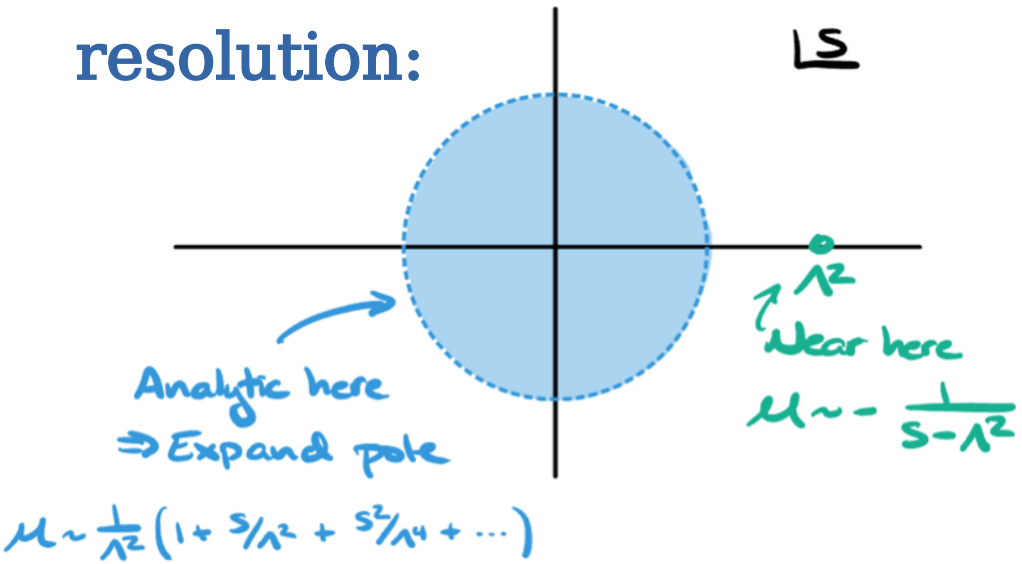
resolution:



Unitarity violation and EFT

$$\sigma_{\text{tot}} \lesssim \log^2 E \qquad \sigma_{\text{tot}} = \sum_X \sigma_{AB\to X}$$

***no* channel can grow too fast**

$\Leftrightarrow$

in EFT *many* channels exhibit E growth

$\Rightarrow$ **multi-boson processes are intimately related!**
(will come back to)

Recent interesting perspectives:
- –Chang & Luty 1902.05556
- –Falkowski & Rattazzi 1902.05936
- –Cohen, Craig, Lu, Sutherland 2108.03240

Adapting EFT into analyses

EFT provides powerful, model-independent way to probe physics

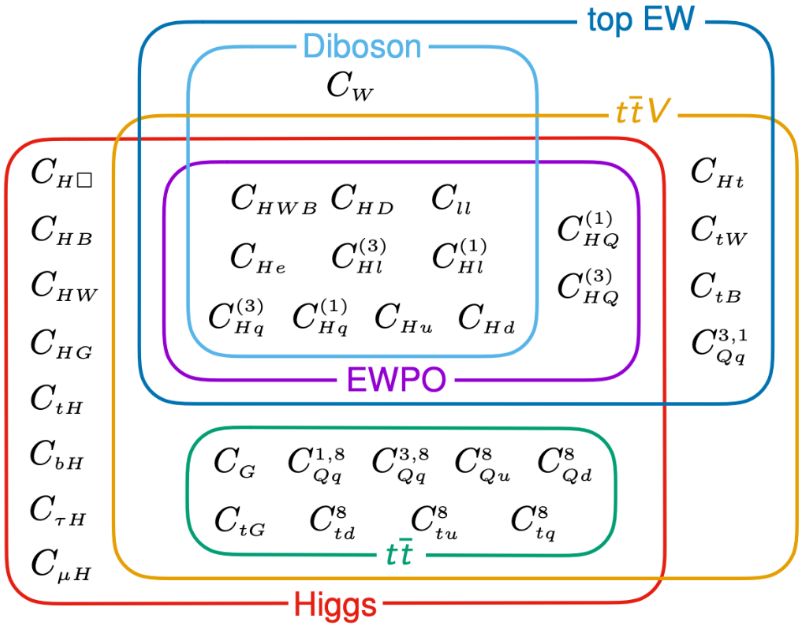
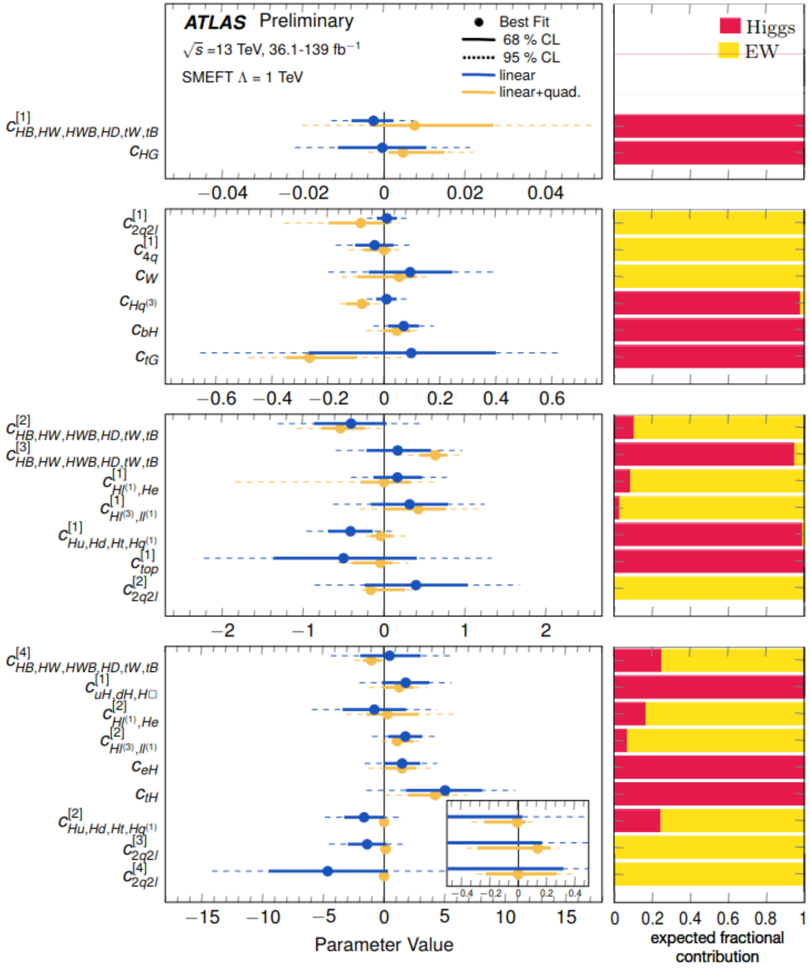
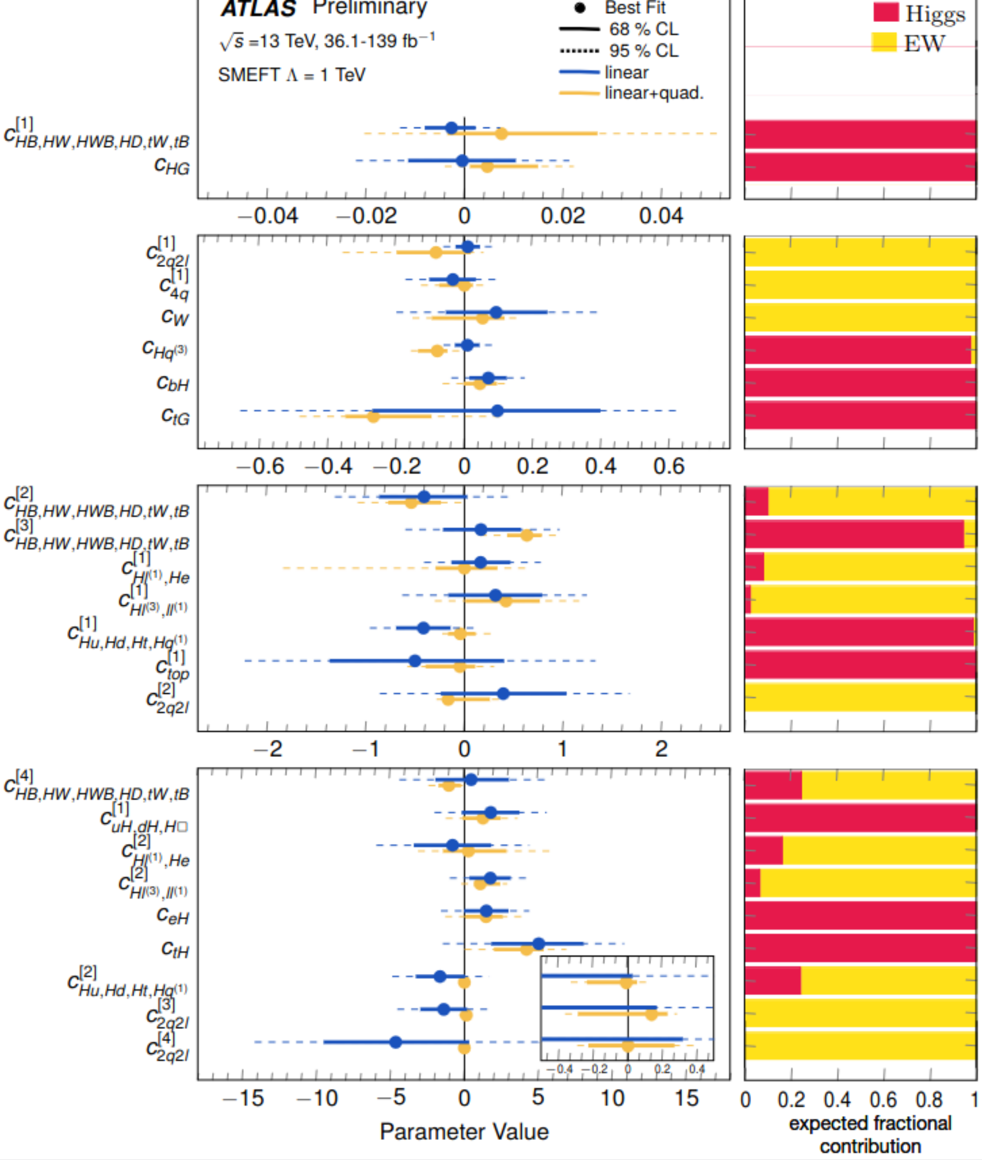


Image credit: Josh McFayden (CMS, Top2022)

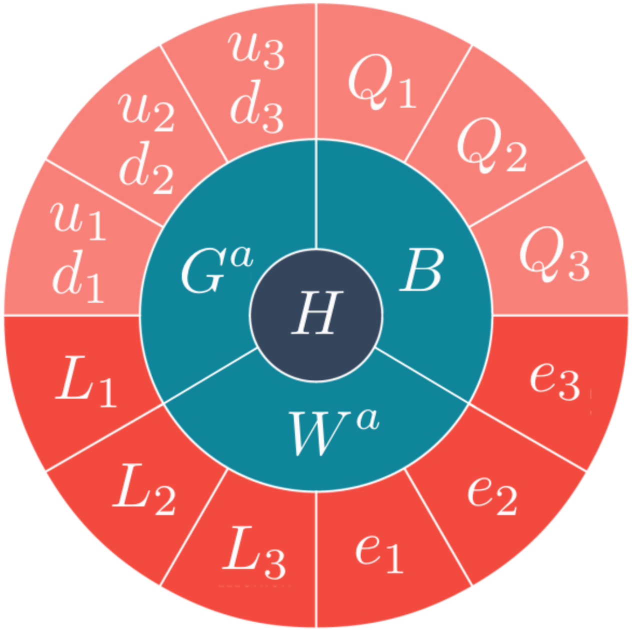


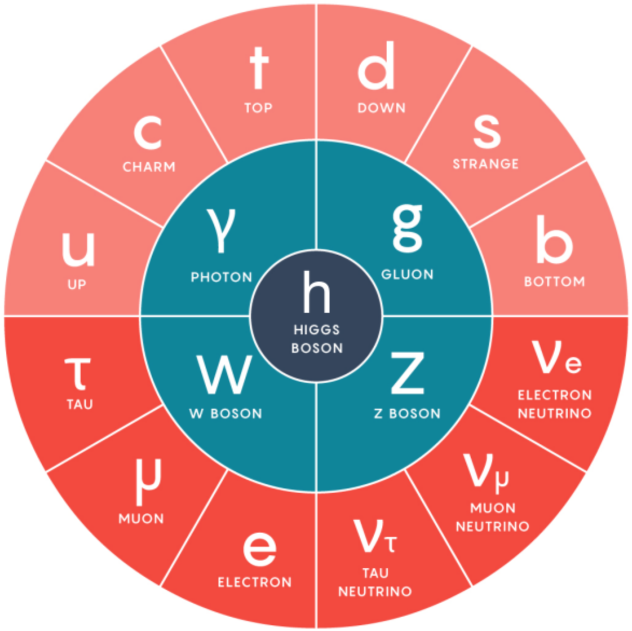
ATLAS PHYS-PUB-2022-037



ATLAS Preliminary
√s =13 TeV, 36.1-139 fb⁻¹
SMEFT Λ = 1 TeV
Best Fit
68 % CL
95 % CL
linear
linear+quad.
Higgs
EW
Parameter Value
expected fractional contribution

Which EFT?





SMEFT ***$SU(2)_L x U(1)_Y$***

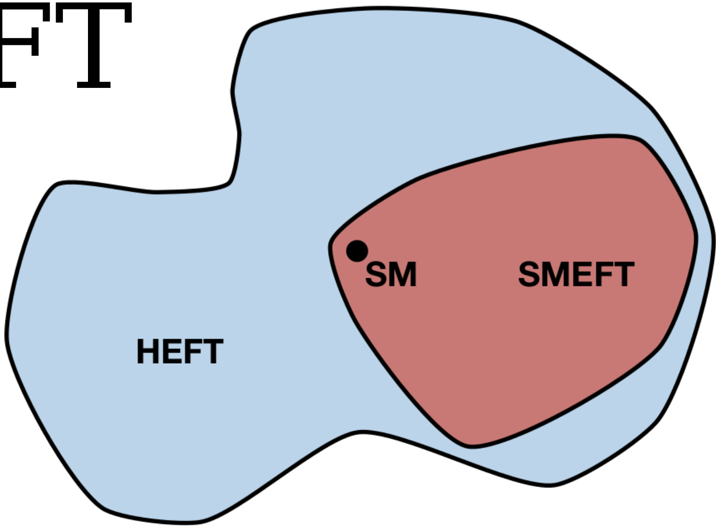
[Weinberg '79, Buchmuller, Wyler '86, …]

HEFT ***$U(1)_{em}$***

[Feruglio '93, Bagger et al. '93, …]

Image credit: N. Craig

SM ⊂ SMEFT ⊂ HEFT



Our world

−obeys E&M at low energies
−EW gauge bosons unified into $SU(2)_L \times U(1)_Y$
−has neutral scalar of mass 125 GeV

⇒MOST general theory: HEFT

Relate the two by field redefinition: $\vec{\phi} = (v+h)\,\vec{n}\,(\pi)\,; \quad \vec{\phi}\cdot\vec{\phi} = (v+h)^2$

SMEFT can always be written as HEFT:

$$\mathcal{L} = \frac{1}{2}A(\vec{\phi}\cdot\vec{\phi})(\partial\vec{\phi}\cdot\partial\vec{\phi}) + \frac{1}{2}B\left(\vec{\phi}\cdot\vec{\phi}\right)(\vec{\phi}\cdot\partial\vec{\phi})^2 - V\left(\vec{\phi}\cdot\vec{\phi}\right)$$
$$= \frac{1}{2}\left[A + (v+h)^2 B\right](\partial h)^2 + \frac{1}{2}(v+h)^2 A(\partial\vec{n})^2 - V$$

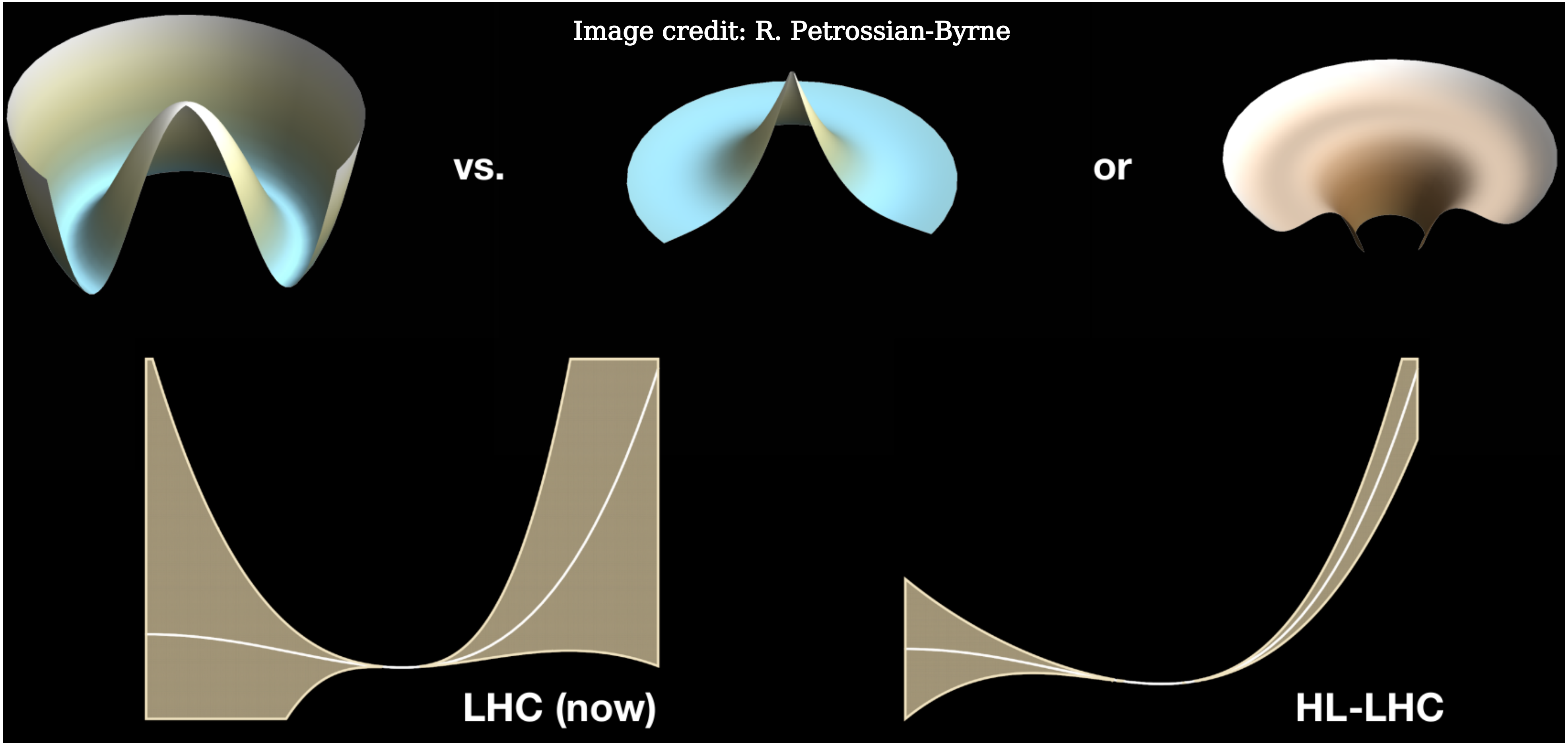
Correlations at every order between h, v

HEFT cannot always be written as SMEFT:

$$\mathcal{L} = \frac{1}{2}[K(h)]^2(\partial h)^2 + \frac{1}{2}[vF(h)]^2(\partial\vec{n})^2 - V(h)$$
$$= \frac{1}{2}\frac{v^2F}{\vec{\phi}\cdot\vec{\phi}}(\partial\vec{\phi})^2 + \frac{1}{2}(\vec{\phi}\cdot\partial\vec{\phi})^2\frac{1}{\vec{\phi}\cdot\vec{\phi}}\left(K^2 - \frac{v^2F^2}{\vec{\phi}\cdot\vec{\phi}}\right) - \tilde{V}(\vec{\phi}\cdot\vec{\phi})$$

Generically non-analytic at the origin

HEFT allows most general parameterization of Higgs potential



What goes wrong by only working with SMEFT?

- Potential errors in interpretation. Say we see a deviation from the SM
  - In SMEFT, SU(2) symmetry typically means deviations are correlated
  - In HEFT this is not necessarily the case
- We should make *all* motivated measurements
  - Just because $2 \to 2$ might look SM, that does not imply $2 \to 3$, $2 \to 4$,... necessarily are
    (see, e.g., Falkowski, Rattazzi 1902.05936; Cohen, Craig, Lu, Sutherland 2108.0324)

HEFT scenarios

General lore: new particles that significantly contribute to EW symmetry breaking are captured by HEFT

…but that lore is not general enough…for example

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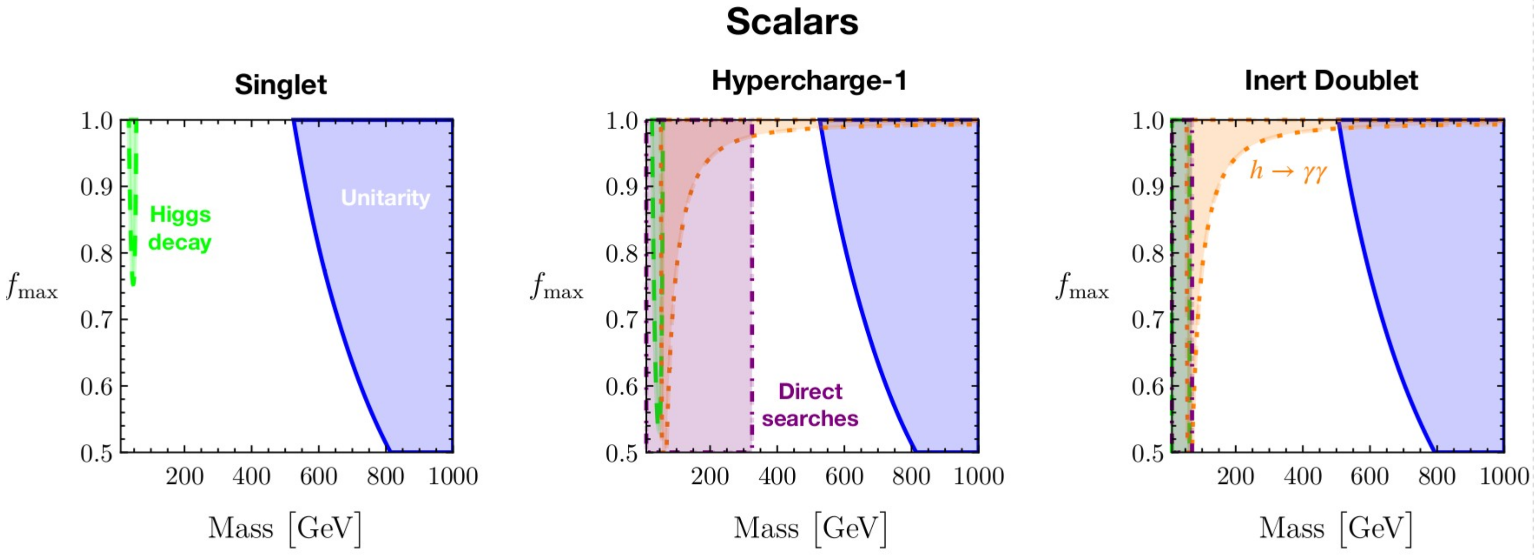
HEFT required whenever a particle receives more than half its mass from the Higgs

Banta, Cohen, Craig, Lu, Sutherland 2110.02967



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HEFT required whenever a particle receives more than half its mass from the Higgs

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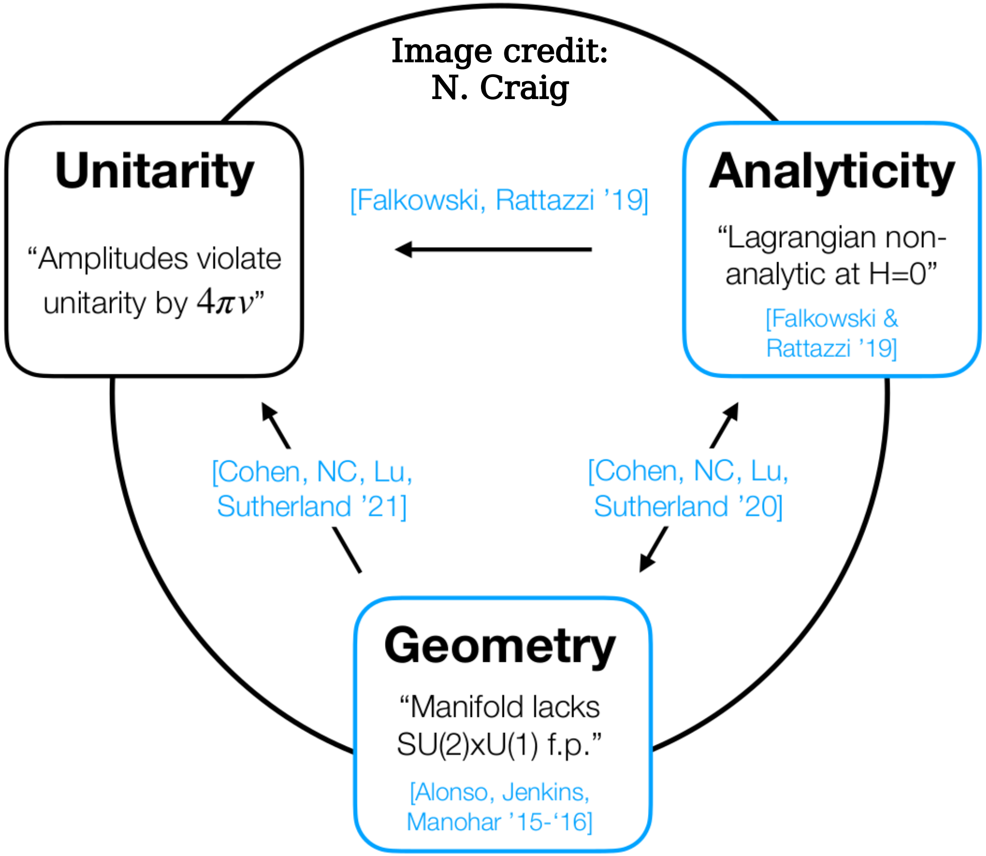
But perhaps the most motivated place for HEFT* is:

high-multiplicity EW boson processes

*this is also well-motivated for SMEFT, as we'll see

4πv and the LHC



Scale of unitarity violation:

HEFT: 4πv ~ 3 TeV

SMEFT: 4πΛ ~ arbitrary

Process	Unitarity Violating Scale
$h^2 Z_L \leftrightarrow hZ_L$	$66.7\ \text{TeV}/\lvert\delta_3 - \frac{1}{3}\delta_4\rvert$
$hZ_L^2 \leftrightarrow Z_L^2$	$94.2\ \text{TeV}/\lvert\delta_3\rvert$
$hW_L Z_L \leftrightarrow W_L Z_L$	$141\ \text{TeV}/\lvert\delta_3\rvert$
$hZ_L^2 \leftrightarrow hZ_L^2$	$9.1\ \text{TeV}/\sqrt{\lvert\delta_3 - \frac{1}{5}\delta_4\rvert}$
$hW_L Z_L \leftrightarrow hW_L Z_L$	$11.1\ \text{TeV}/\sqrt{\lvert\delta_3 - \frac{1}{5}\delta_4\rvert}$
$Z_L^3 \leftrightarrow Z_L^3$	$15.7\ \text{TeV}/\sqrt{\lvert\delta_3\rvert}$
$Z_L^2 W_L \leftrightarrow Z_L^2 W_L$	$20.4\ \text{TeV}/\sqrt{\lvert\delta_3\rvert}$
$hZ_L^3 \leftrightarrow Z_L^3$	$6.8\ \text{TeV}/\lvert\delta_3 - \frac{1}{6}\delta_4\rvert^{\frac{1}{3}}$
$hZ_L^2 W_L \leftrightarrow Z_L^2 W_L$	$8.0\ \text{TeV}/\lvert\delta_3 - \frac{1}{6}\delta_4\rvert^{\frac{1}{3}}$
$Z_L^4 \leftrightarrow Z_L^4$	$6.1\ \text{TeV}/\lvert\delta_3 - \frac{1}{6}\delta_4\rvert^{\frac{1}{4}}$

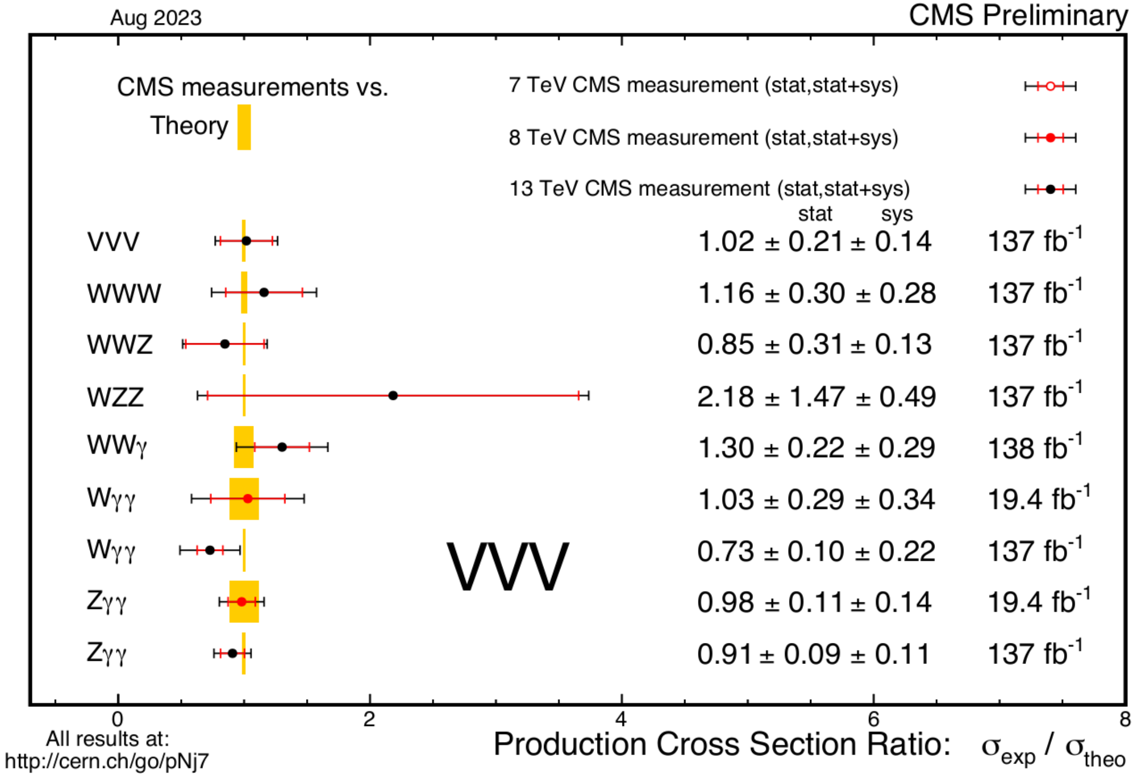
Unitarity violation involving Higgs trilinear and quartic
Chang, Luty 1902.05556

Multi-boson processes

⇒ Unitarity violation in many channels

⇒ Multi-boson processes sensitive probes

⇒ Numerous exciting (and challenging) opportunities
- –high-multiplicity
- –polarization tagging
- –hadronic decays



See Mai Liu's talk at this conference

Goldstones = longitudinals

$$|H|^2 \sim (v+h)^2 + \vec{\phi}^2 \longrightarrow$$

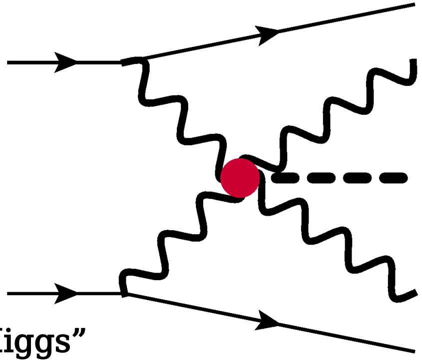
HC: $|H|^2\,\mathcal{O}_{\rm SM} \supset vh\mathcal{O}_{\rm SM}$

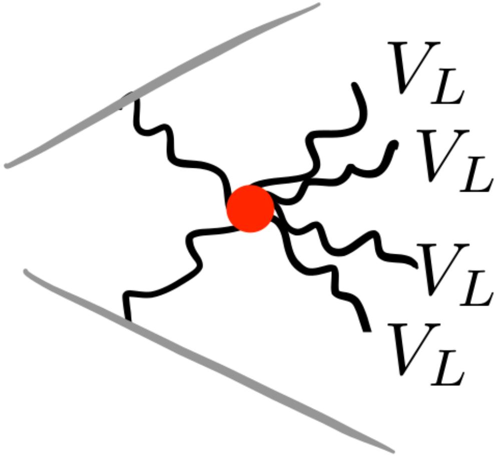
HwH: $|H|^2\,\mathcal{O}_{\rm SM} \supset \vec{\phi}^2\mathcal{O}_{\rm SM}$

$$|H|^6 \supset vh\phi^4 + \phi^6$$

$V_L V_L \to V_L V_L h$

$V_L V_L \to V_L V_L V_L V_L$





"Higgs without Higgs"
BH, Lombardo, Riembau, Riva 1812.09299

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$$|H|^2 \sim (v+h)^2 + \vec{\phi}^2 \longrightarrow$$

HC: $|H|^2 \mathcal{O}_{\text{SM}} \supset vh\mathcal{O}_{\text{SM}}$

HwH: $|H|^2 \mathcal{O}_{\text{SM}} \supset \vec{\phi}^2\mathcal{O}_{\text{SM}}$

$$|H|^6 \supset vh\phi^4 + \phi^6$$

$V_L V_L \to V_L V_L h$

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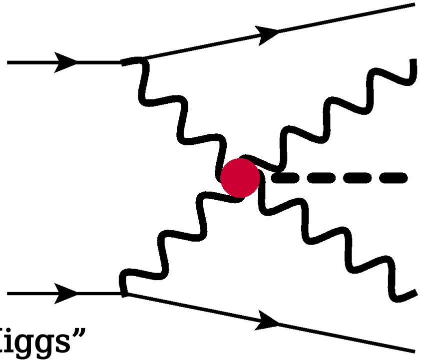
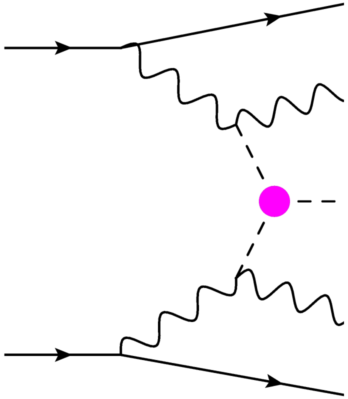


diagram in unitary gauge $\longrightarrow$



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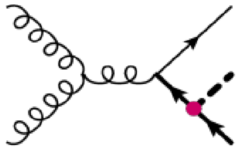
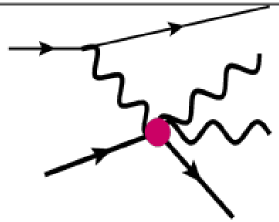
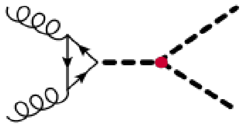
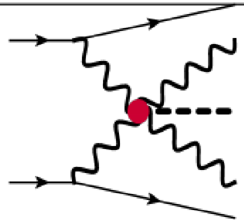
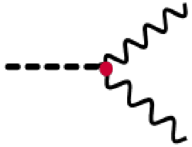
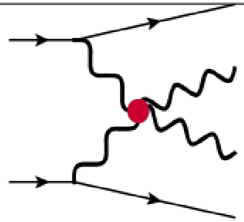
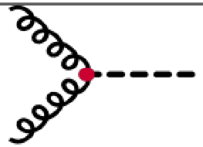
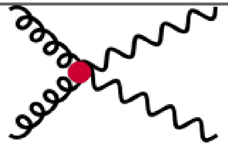
BH, Lombardo, Riembau, Riva
1812.09299

Higgs without Higgs

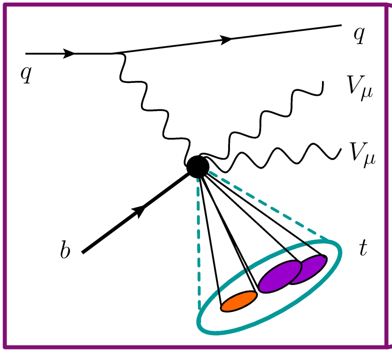
"standard" Higgs couplings (HC) channels

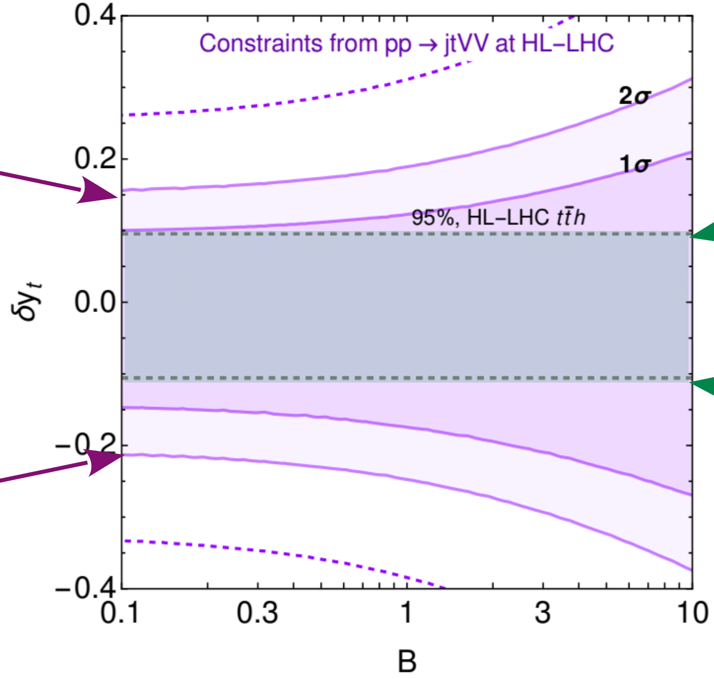
new—and complementary—Higgs without Higgs (HwH) channels

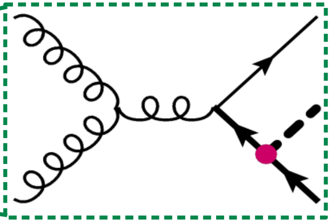
		HC	HwH	Growth
κ_t	$\mathcal{O}_{y_t}$			$\sim \frac{E^2}{\Lambda^2}$
κ_λ	$\mathcal{O}_6$			$\sim \frac{vE}{\Lambda^2}$
$\kappa_{Z\gamma}$ $\kappa_{\gamma\gamma}$ κ_V	$\mathcal{O}_{WW}$ $\mathcal{O}_{BB}$ $\mathcal{O}_r$			$\sim \frac{E^2}{\Lambda^2}$
κ_g	$\mathcal{O}_{gg}$			$\sim \frac{E^2}{\Lambda^2}$

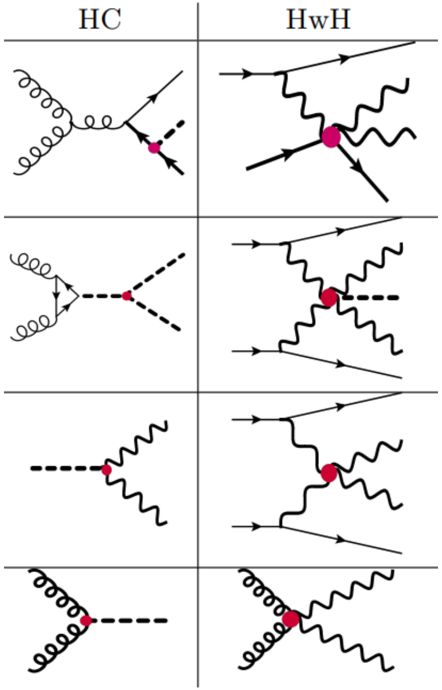
BH, Lombardo, Riembau, Riva
1812.09299

Higgs without Higgs





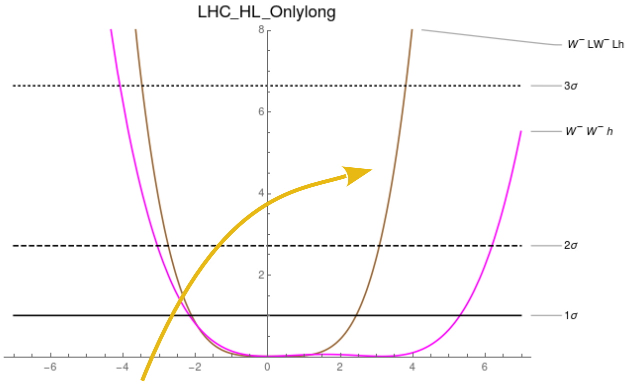




Experimental opportunities

innovate WITH experimentalists

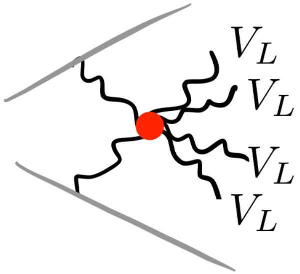
polarization tagging



constraints from only longitudinals are stronger

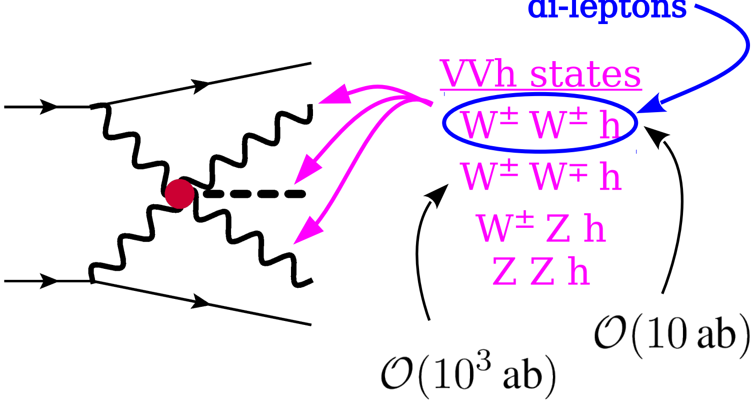
high-multiplicity EW processes

> 4 point massive amplitudes



Challenge analytically AND numerically (e.g. MadGraph)

hadronic decay channels

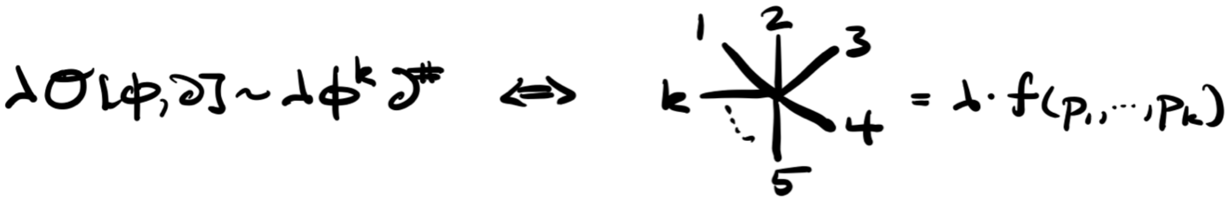


Building EFTs, and knowing you've thought of everything

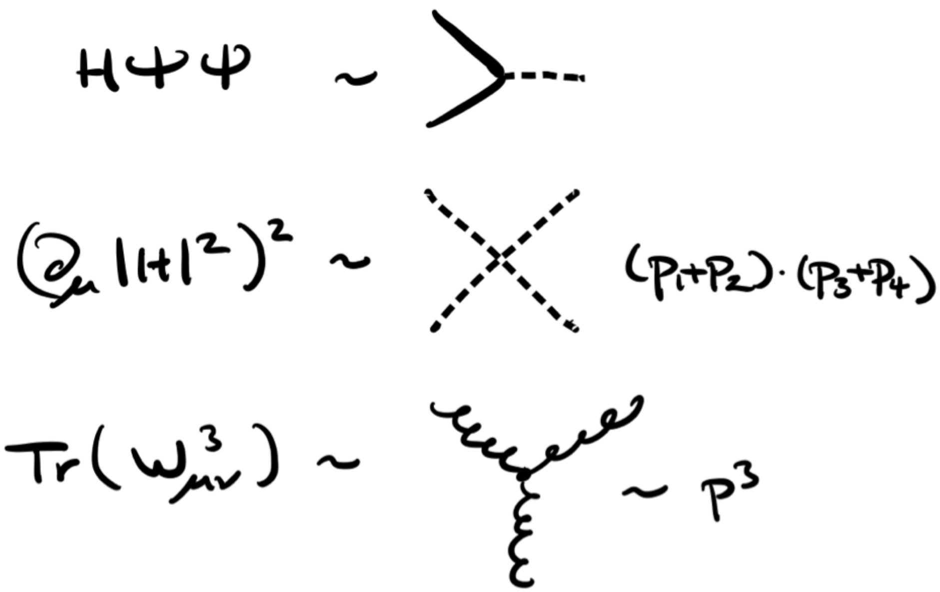
...or...

how I learned to count

all possible interactions



$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^{\Delta_i - 4}} \mathcal{O}_i$$



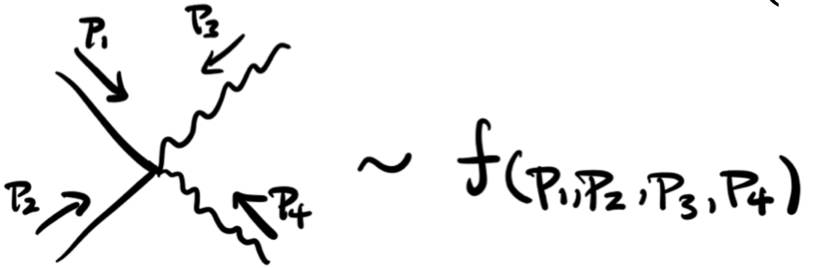
An operator specifies an interaction

HENCE

All possible operators = all possible interactions

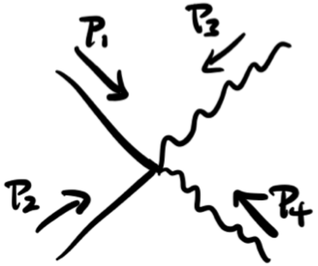
all possible interactions

Experiment sees interactions ⇔ Measures scattering amps (S-matrix)



all possible interactions

Experiment sees interactions $\Leftrightarrow$ Measures scattering amps (S-matrix)

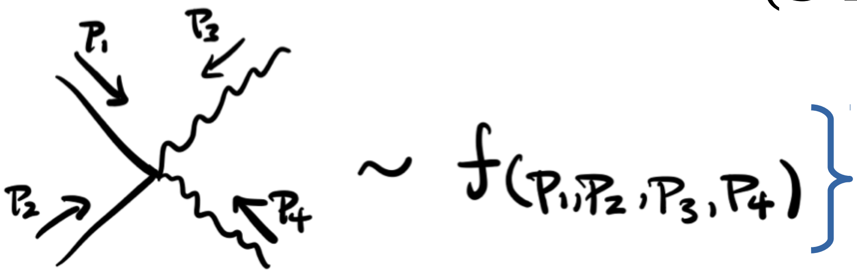


$\sim f(p_1, p_2, p_3, p_4)$ } Not all functions independent!

e.g. $$f_2(p_i) = f_1(p_i) + \underbrace{(p_1 + p_2 + p_3 + p_4)}_{=\,0 \text{ by momentum conservation}} g(p_i) \simeq f_1(p_i)$$

all possible interactions

Experiment sees interactions ⇔ Measures scattering amps (S-matrix)



Not all functions independent!

e.g. $f_2(p_i) = f_1(p_i) + \underbrace{(p_1 + p_2 + p_3 + p_4)}_{=0 \text{ by momentum conservation}} g(p_i) \simeq f_1(p_i)$

= 0 by momentum conservation

PROBLEM: determine all independent amplitudes in the SM

CRUCIAL: "independent" ⇔ rules governing S-mat

BH, Lu, Melia, Murayama 1706.08520

EFT operator basis

$$\mathcal{L} = \sum_i c_i \mathcal{O}_i \ , \quad S = \int d^d x \, \mathcal{L}(x) \ , \quad Z = \int D\phi \, e^{iS}$$

spacetime symmetry!

Lorentz invariance ⇔ $\mathcal{O}_i$ **are Lorentz scalars**

Translation invariance ⇔ **can integrate by parts**

$$\left(\int dx \, \partial_\mu \mathcal{O}^\mu(x) = 0 \right)$$

On-shell ⇔ **EOM/field redefinitions**

Equivalence relations for operator basis follow from the S-matrix!

EFT operator basis

Basic questions: 1) How many ops?

2) What are they?

Find a partition function

operator basis ⇔ S-matrix

⇒ spactime symmetry

⇒ can use group theory!

EFT operator basis

Basic questions: 1) How many ops?
2) What are they?

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operator basis ⇔ S-matrix

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⇒ can use group theory!

"Hilbert series"

$$H = \mathrm{Tr}_{\mathcal{K}}\, \widehat{w} = \sum_{\mathcal{O}\in\mathcal{K}} q^{\Delta(\mathcal{O})} = 1 + c_1 q^1 + c_2 q^2 + \cdots$$

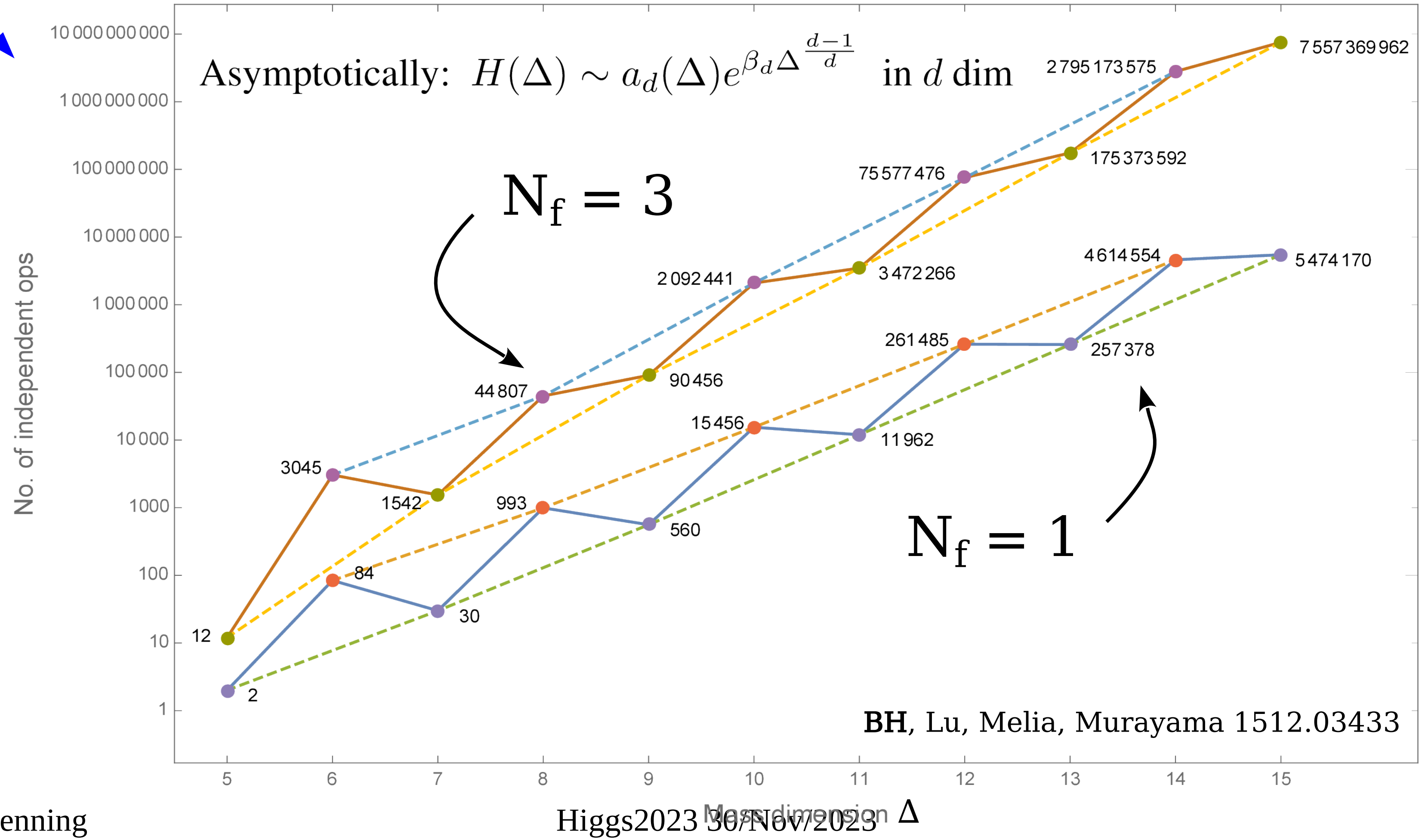
$\mathcal{K}$ = operator basis

c_Δ = # of ops of mass dimension Δ

$$\left[\text{compare } Z = \mathrm{Tr}_{\mathcal{H}}\widehat{U} = \sum_{|i\rangle\in\mathcal{H}} \langle i|e^{-\beta\widehat{H}}|i\rangle = \sum_{\Delta} c_\Delta q^\Delta, \;\; q \equiv e^{-\beta}\right]$$

Hilbert series for SMEFT

 # ops of dimension Δ in SMEFT



Hilbert series for HEFT

LHC copiously produces W's and Z's
- massive particles
- charged under $U(1)_{EM}$
- $SU(2)_L \times U(1)_Y$ realized non-linearly

what we see in the IR

Our world: $SU(2)_L \times U(1)_Y \big/ U(1)_{EM}$ W±,Z massive

Typical treatments: $SU(2)_L \times U(1)_Y \big/ SU(2)_D \oplus$ spurions

Hilbert series for HEFT

LHC copiously produces W's and Z's
- **massive particles**
- **charged under $U(1)_{EM}$**
- **$SU(2)_L \times U(1)_Y$ realized non-linearly**

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Our world: $SU(2)_L \times U(1)_Y \big/ U(1)_{EM}$ **$W^\pm$,Z massive**

Typical treatments: $SU(2)_L \times U(1)_Y \big/ SU(2)_D \oplus \text{spurions}$

Hilbert series developed for both pictures

New ingredients:
1) **massive particles**
2) **spurions**
3) **"form factors" $\mathcal{O} \sim f(h)\mathcal{O}$**

Graf, **BH**, Lu, Melia, Murayama 2211.06275
Sun, Wang, Yu 2211.11598

Construction of NLO and NNLO HEFT ops:
Sun, Xiao, Yu 2206.07722, 2210.14939

Hilbert series ingredients

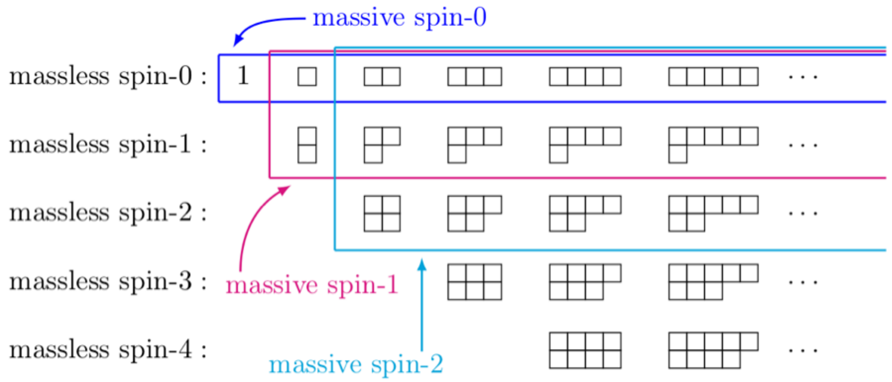
Field	Lorentz Group	$SU(3)_C$	$U(1)_{\text{EM}}$	dim
u_L , u_R	$(\frac{1}{2},0)$, $(0,\frac{1}{2})$	**3**	$\frac{2}{3}$	$\frac{3}{2}$
d_L , d_R		**3**	$-\frac{1}{3}$	
ν_L , (ν_R)		**1**	0	
e_L , e_R		**1**	-1	
G_L , G_R	$(1,0)$, $(0,1)$	**8**	0	2
$W_L^\pm$, $W_R^\pm$		**1**	± 1	
Z_L , Z_R		**1**	0	
A_L , A_R		**1**	0	
$V^\pm$	$(\frac{1}{2},\frac{1}{2})$	**1**	± 1	1
V^z			0	
h	$(0,0)$	**1**	0	1

Everything fixed upon specifying particle content and their representations

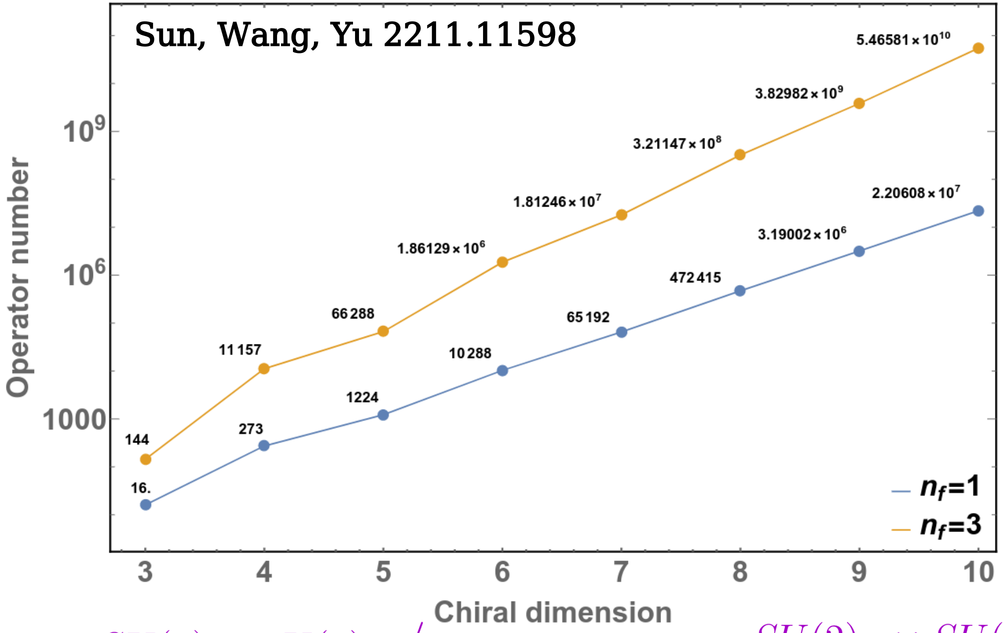
massive $W^\pm$,Z

Can split into longitudinal and transverse components

$\Rightarrow$ Higgs mechanism!



HEFT results



Agrees with NLO bases in literature ✓

(some minor mistakes found; caveat that $t^3_v \neq Q$)

Buchalla, Cata, Krause; Brivio, Gonzalez-Fraile, Gonzalez-Garcia, Merlo; Sun, Xiao, Yu

Easily go to N^{k}LO, k > 1

For N^{2}LO, see Sun, Xiao, Yu 2210.14939

$SU(2)_L \times U(1)_Y / U(1)_{\rm EM}$

Graf, BH, Lu, Melia, Murayama 2211.06275

Class	Detailed Class	dim	HEFT
D^4	V^4	4	5
	$h\mathcal{D}V^3$	5	4
	$h^2\mathcal{D}^2V^2$	6	4
	$h^3\mathcal{D}^3V$	7	1
	$h^4\mathcal{D}^4$	8	1
D^2X	V^2X	4	8
X^2		4	10
X^3		6	6

$SU(2)_L \times SU(2)_R / SU(2)_V$

$+T = \text{spurion}$

Class	Detailed Class	dim	(ν)HEFT
D^4	V^4	4	$2+2T^2+T^4$
	$h\mathcal{D}V^3$	5	$2T+T^2+T^3$
	$h^2\mathcal{D}^2V^2$	6	$2+2T^2$
	$h^3\mathcal{D}^3V$	7	T
	$h^4\mathcal{D}^4$	8	1
D^2X	V^2X	4	$2+4T+2T^2$
X^2		4	$6+2T+2T^2$
X^3		6	$4+2T$

small caveat

$$T = U\sigma_3 U^\dagger$$

$$\langle T\rangle :\ SU(2)_V \to U(1)_V$$

$$U(1)_V = Q - \tfrac{1}{2}(B-L) \neq U(1)_Q$$

Not the same as $U(1)_Q$

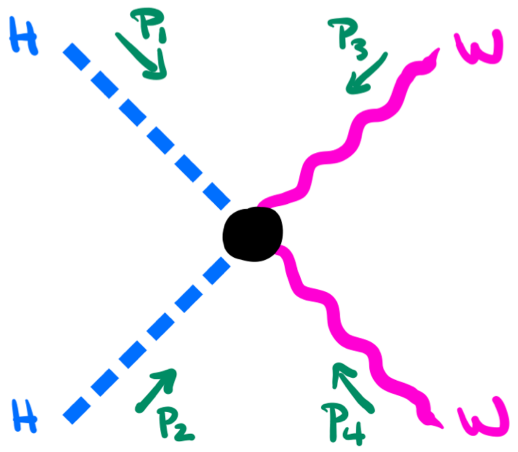
(coincides in B−L = 0 sector)

Power counting from Hilbert series

The "natural" expansion of Hilbert series is in

$$k_1 + k_2 + k_3 + k_4 = k$$

$$\partial^{k_1} H \partial^{k_2} H \partial^{k_3} W \partial^{k_4} H$$



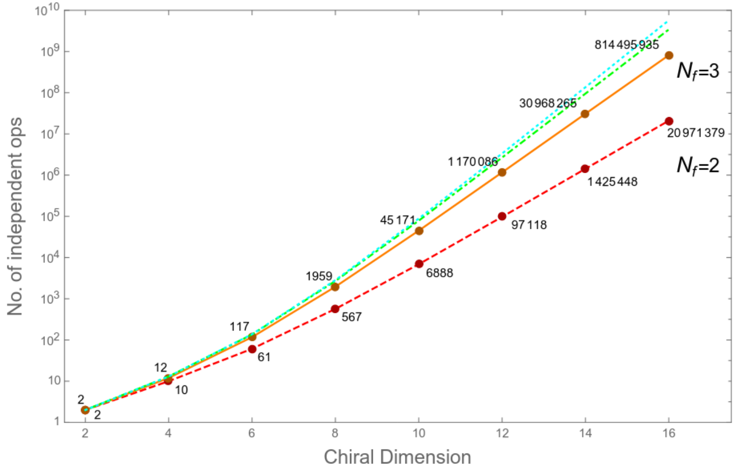
(1) Scaling dimension
(powers of energy)

(2) Number of fields

Precisely the organizational scheme used for "primary observables"

(Chang, Chen, Liu, Luty 2212.06215)

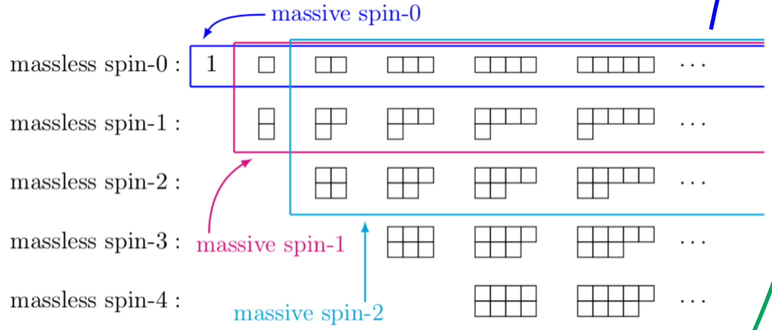
Hilbert goes to town



$R_{\mu\nu\rho\sigma} \sim C_{\mu\nu\rho\sigma} \oplus R_{\mu\nu} \oplus R$

WEYL TENSOR, RICCI TENSOR, RICCI SCALAR 1

$R_A \sim \begin{pmatrix} 1 \\ \partial_\mu \\ \partial_\mu^2 \\ \vdots \end{pmatrix} (1-\partial^2)(1-\partial^\mu) A_\mu$



$$\begin{pmatrix} \psi^\dagger \\ \mathcal{D}_\perp \psi^\dagger \\ \mathcal{D}_\perp^2 \psi^\dagger \\ \vdots \end{pmatrix} \otimes \begin{pmatrix} \mathcal{E} \\ \mathcal{D}_\perp \mathcal{E} \\ \mathcal{D}_\perp^2 \mathcal{E}, \mathcal{D}_t \mathcal{E} \\ \vdots \end{pmatrix} \otimes \begin{pmatrix} \mathbf{B} \\ \mathcal{D}_\perp \mathbf{B} \\ \mathcal{D}_\perp^2 \mathbf{B}, \mathcal{D}_t \mathbf{B} \\ \vdots \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} \psi \\ \mathcal{D}_\perp \psi \\ \mathcal{D}_\perp^2 \psi \\ \vdots \end{pmatrix} = \begin{pmatrix} \mathcal{O} \\ \mathcal{D}_\perp \mathcal{O} \\ \mathcal{D}_\perp^2 \mathcal{O}, \mathcal{D}_t \mathcal{O} \\ \vdots \end{pmatrix} + \cdots$$

⇒ Chiral lagrangians
(**BH**, Lu, Melia, Murayama 1706.08520, ibid + Gráf 2009.01239)

⇒ HEFT
(Gráf, **BH**, Lu, Melia, Murayama 2211.06275; Sun, Wang, Yu 2211.11598)

⇒ 1d QFT
(**BH**, Lu, Melia, Murayama 1507.07240)

⇒ Gravity
(Rudorfer, Serra, Weiler 1908.08050)

⇒ NRQCD and HQET
(Kobach and Pal 1704.00008, 1804.01534, 1810.0236)

⇒ N=1 SUSY
(Delgado, Martin, Wang 2212.02551, 2305.01736)

⇒ O(n) NLSM
(Bijnens, Gudnason, Yu, Zhang 2212.07901)

⇒ CP and SMEFT
(Kondo, Murayama, Okabe 2212.02413)

⇒ BSM model building, asymptotics, algorithms, ...

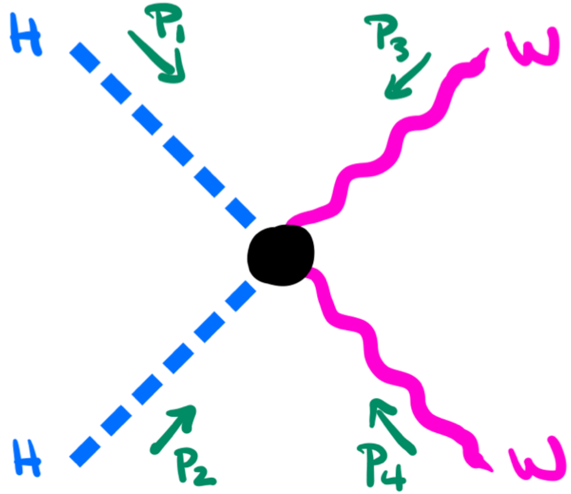
Constructing operators

Example: all ops involving 2 H's, 2 W's, and k derivatives?

$$k_1 + k_2 + k_3 + k_4 = k$$

degree k polynomial

$$\partial^{k_1} H \partial^{k_2} H \partial^{k_3} W \partial^{k_4} W \Leftrightarrow f(p_1, p_2, p_3, p_4) H(p_1) H(p_2) W(p_3) W(p_4)$$



Constructing operators

Example: all ops involving 2 H's, 2 W's, and k derivatives?

$$k_1 + k_2 + k_3 + k_4 = k$$

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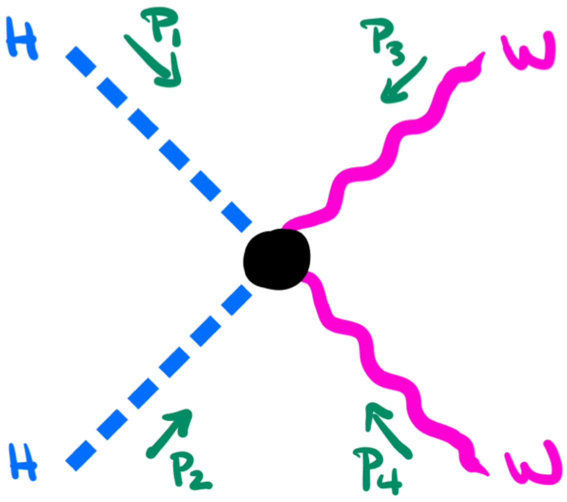
Scalars: **BH**, Lu, Melia, Murayama 1706.08520
Arbitrary spin: **BH**, Melia 1902.06747, 1902.06754
See also: Dong, Ma, Shu 2103.15837
Durieux, Kitahara, Machado, Shadmi, Weiss 1909.10551



subject to:

⇒ **momentum conservation** $\sum_i p_i^\mu = 0$

⇒ **on-shell** $p_i^2 = 0$

⇒ **Lorentz invariance** $f(\Lambda p_i) = f(p_i)$, $\Lambda \in SO(3,1)$

Constructing operators

BH, T. Melia
1902.06747
1902.06754

- ⇒ momentum conservation
- ⇒ on-shell
- ⇒ Lorentz invariance

constraints define a manifold in phase space

$$\delta(p_1^2)\cdots\delta(p_n^2)\times\delta^4\big(P^\mu-(p_1^\mu+\cdots+p_n^\mu)\big)$$

use spinors

$$\delta^4\Big(P_{\alpha\dot\alpha}-(\lambda^1\widetilde\lambda^1+\cdots+\lambda^n\widetilde\lambda^n)_{\alpha\dot\alpha}\Big)$$

Constructing operators

BH, T. Melia
1902.06747
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use spinors →

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Want a set of class functions on the manifold

→ generalized spherical harmonics

⇓

operators ⇔ harmonics on phase space

Constructing operators

BH, T. Melia
1902.06747
1902.06754

⇒ **momentum conservation**
⇒ **on-shell**
⇒ **Lorentz invariance**

constraints define a <u>manifold in phase space</u>

$$\delta(p_1^2)\cdots\delta(p_n^2)\times\delta^4\left(P^\mu-(p_1^\mu+\cdots+p_n^\mu)\right)$$

use spinors →

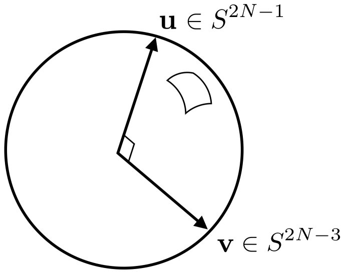
$$\delta^4\left(P_{\alpha\dot\alpha}-(\lambda^1\tilde\lambda^1+\cdots+\lambda^n\tilde\lambda^n)_{\alpha\dot\alpha}\right)$$

$$P_{\alpha\dot\alpha}=\begin{pmatrix}M & 0\\ 0 & M\end{pmatrix}=\begin{pmatrix}\left|\vec\lambda_1\right|^2 & \vec\lambda_1\cdot\vec\lambda_2^*\\ \vec\lambda_2\cdot\vec\lambda_1^* & \left|\vec\lambda_2\right|^2\end{pmatrix}$$

geometry basically two orthogonal spheres

$$r^2=\vec v^2$$
$$r^2=\vec u^2$$
$$0=\vec v\cdot\vec u$$

$$G/H=U(N)/U(N-2)$$



"Stiefel manifold"

Grassmannian ⊂ Stiefel

$$G_{2,N}(\mathbb{C})=U(N)/U(N-2)\times U(2)$$

Want a set of class functions on the manifold

↳ **generalized spherical harmonics**

⇓

operators ⇔ harmonics on phase space

"conformal – helicity duality"

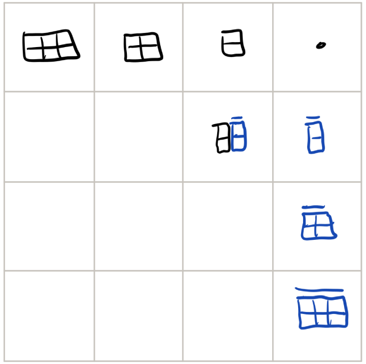
$$SU(2,2)\times U(N)$$

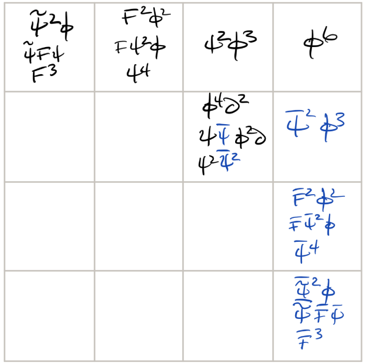
(math world: reductive dual pairs/Howe duality/oscillator representation)

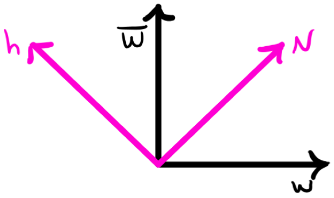
Phase space harmonics

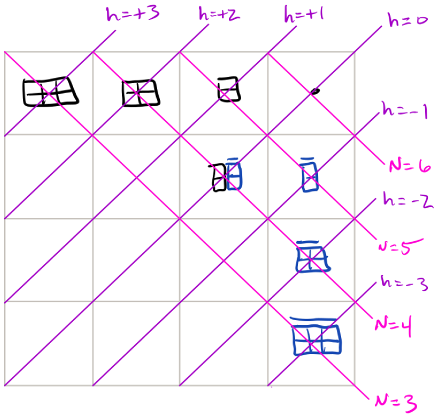
The families of operators belong to the same Grassmann harmonic!

BH, T. Melia 1902.06747, 1902.06754









Explains structure of EFT non-renormalization/helicity selection rules

Cheung & Shen 1505.01844
Azatov, Contino, Machado, Riva 1607.05236
Jiang, Shu, Xiao, Zheng 2001.04481

Method used to construct dim-8 ops in SMEFT

Li, Ren, Shu, Xiao, Yu 2005.00008, 2012.11615
Dong, Ma, Shu, Zheng 2202.08350

Harmonics and tableaux methods

Massless

Phase space geometry
BH, Melia 1902.06747, 1902.06754;
Larkoski, Melia 2008.06508

SMEFT from on-shell:
Ma, Shu, Xiao 1902.06752;
Jiang, Shu, Xiao, Zheng 2001.04481

Dim-8 SMEFT
Li, Ren, Shu, Xiao, Yu, Zheng 2005.00008, 2012.11615

Massive

On-shell massive amplitudes
Durieux, Kitahara, Shadmi, Weiss 1909.10551;
ibid + Machado 2008.09652;
Falkowski, Isabella, Machado 2011.05339

Tableaux for any mass and spin
Dong, Ma, Shu 2103.15837

HEFT
Sun, Xiao, Yu 2206.07722, 2210.14939;
Dong, Ma, Shu, Zhou 2211.16515

+...

The powerful methods of group theory, spinor helicity, and on-shell techniques have solved and systematized problems unthinkable just a few years ago

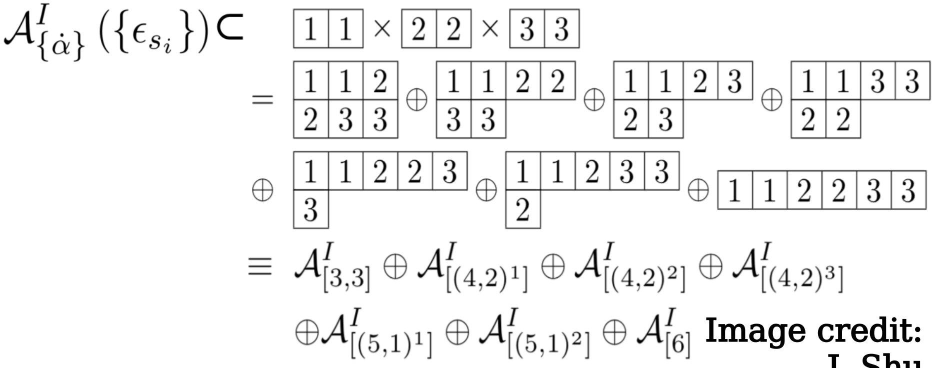


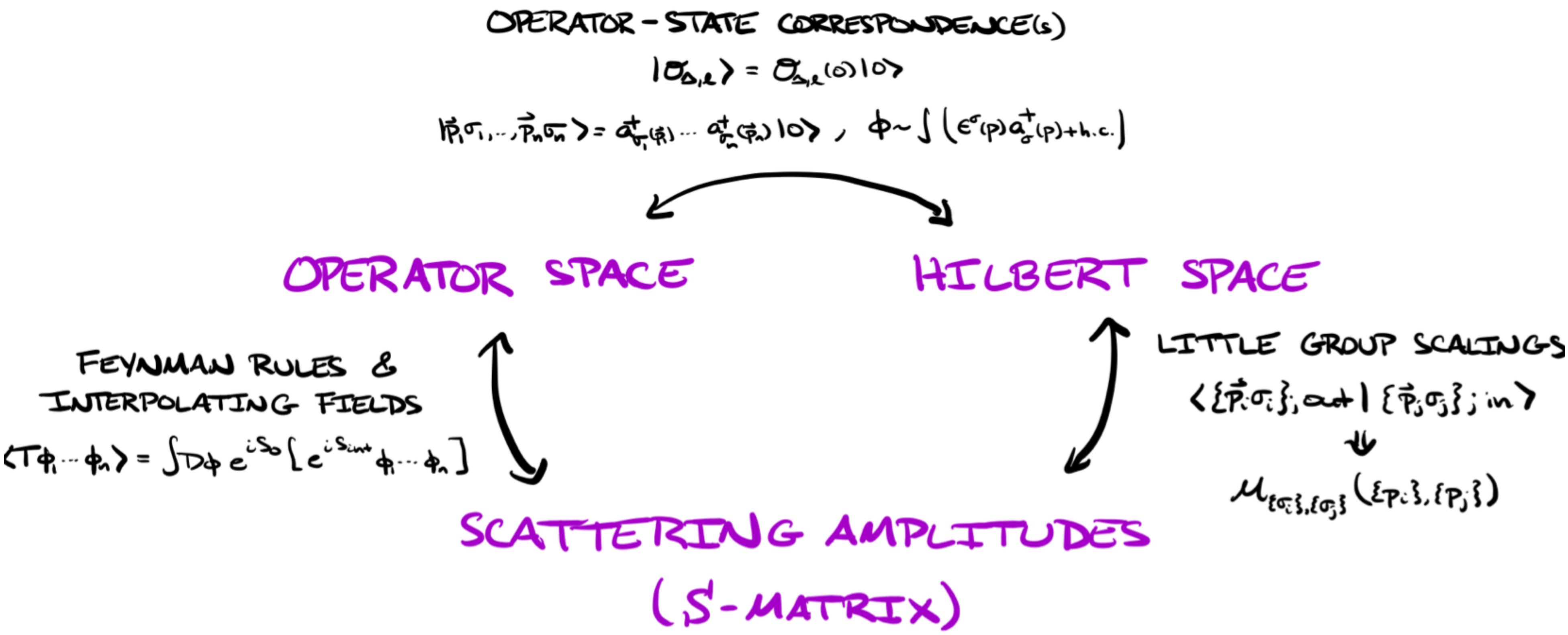
Image credit: J. Shu

Complete NNLO Operator Bases in Higgs Effective Field Theory

Hao Sun, Ming-Lei Xiao, Jiang-Hao Yu

!! Comments: 419 pages, 3 tables
Subjects: High Energy Physics - Phenomenology (hep-ph); High Energy Physics - Theory (hep-th)
Cite as: arXiv:2210.14939 [hep-ph]

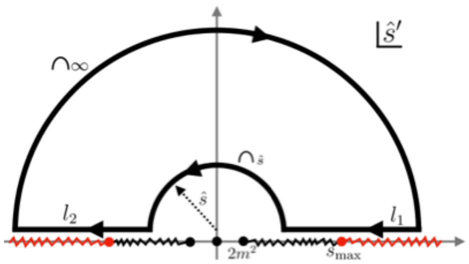
isomorphic problems

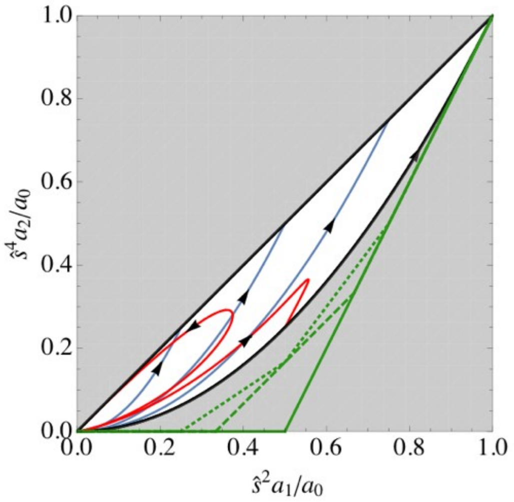


Other applications: EFT

operators/EFT amplitudes

phase space (Grassmannian) harmonics and EFT positivity





generalize to massive particles (hard, but useful!)

$$\underbrace{\delta(p_1^2 - m_1^2)\cdots\delta(p_k^2 - m_k^2)\delta^4(P^\mu - \sum_i p_i^\mu)}$$

Massive phase space manifold: Is there a "nice" geometric formulation?

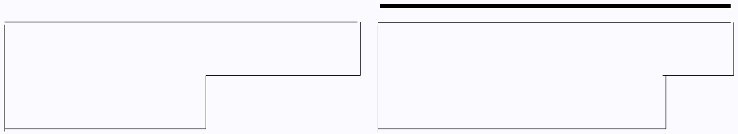
numerous other questions: identical particles (symmeterization); non-renormalization thms; efficient construction algorithms; amplitudes in d = 2+1; ...

THANK YOU!

BACKUP

upshot on Stiefel harmonics

harmonics labeled by Young diagrams
(with at most two rows)



these dictate specific polynomials in the spinors

comments:

1) each shape corresponds to operators

2) multiple operators belong to same shape

a) these involve particles with different spin

3) these operators are conformal primaries

Construct states algebraically
e.g.

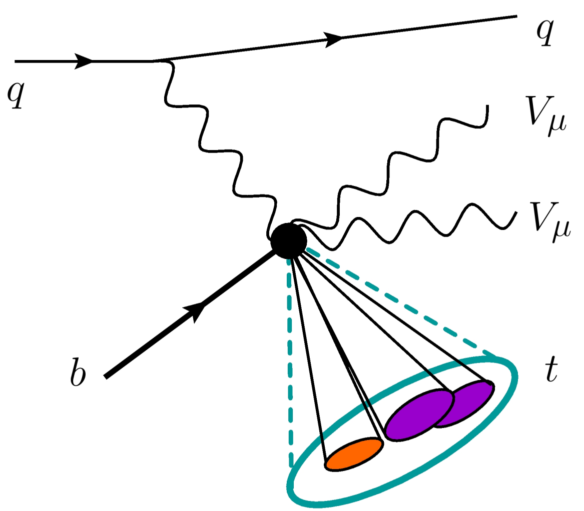
$$|l, \mu = (\mu_1, \ldots, \mu_3)\rangle \simeq F^3$$

now apply U(N) lowering op:

$$L_- \, |l, \mu\rangle \sim |l, \mu'\rangle \simeq \widetilde{\psi} F \psi$$

Top Yukawa

HwH: top yukawa



→ boosted top

→ forward jet

$$|\eta_j| > 2.5,\ p_T^j > 30 \text{ GeV},\ E_j > 300 \text{ GeV}$$

→ N_{lep} from vector decays

look for single boosted top + forward jet, then just count leptons

 # events @ HL-LHC

Process	0ℓ	1ℓ	$\ell^\pm\ell^\mp$	$\ell^\pm\ell^\pm$	$3\ell(4\ell)$
$W^\pm W^\mp$	3449/567	1724/283	216/35	-	-
$W^\pm W^\pm$	2850/398	1425/199	-	178/25	-
$W^\pm Z$	3860/632	965/158	273/45	-	68/11
ZZ	2484/364	-	351/49	-	(12/2)

$p_T^t > 250$ GeV / $p_T^t > 500$ GeV

≥2L: small background

Main bkg: $ttjj \to tW\,bj\,j$ large background, but manageable

~W