
Hilbert Series, Higgs, and HEFT



Kavli
Institute
for
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Physics

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The universe at different scales

Very big

10^{27}

10^{20}

10^{10}

1m

10^{-10}

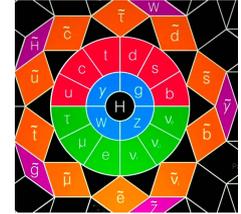
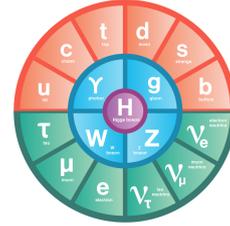
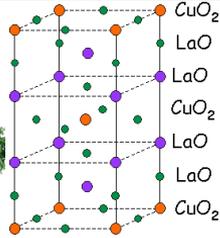
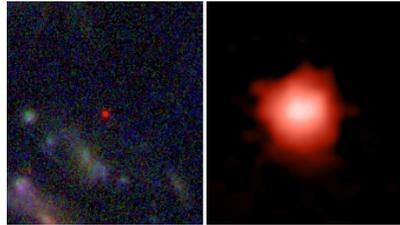
Standard Model

10^{-15}

New Particles

10^{-35}

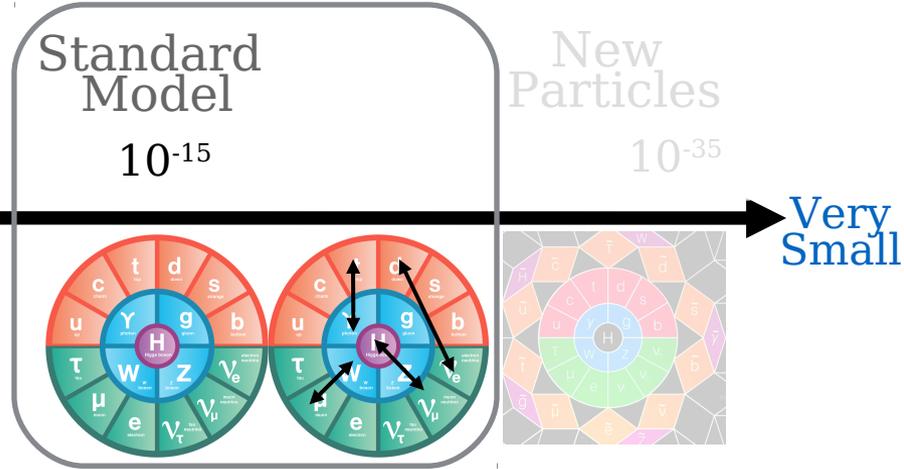
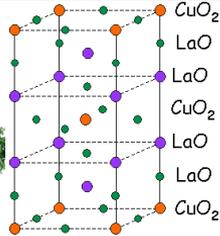
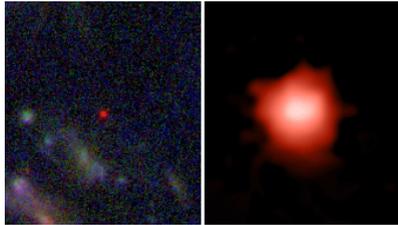
Very Small



The universe at different scales

10^{27} 10^{20} 10^{10} **1m** 10^{-10}

Very big



Very Small

Effective Field Theory:

New Interactions

- Model independent
- Exhaustive
- Guide for experiments

The universe at different scales

Very big

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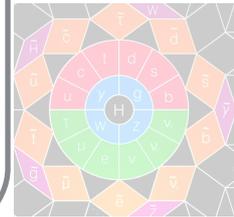
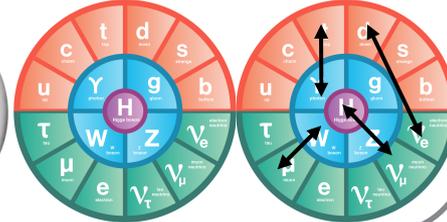
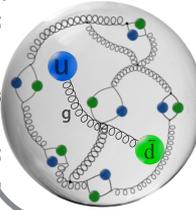
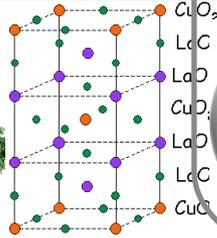
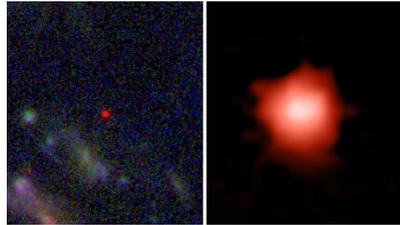
Standard Model

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New Particles

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Very Small



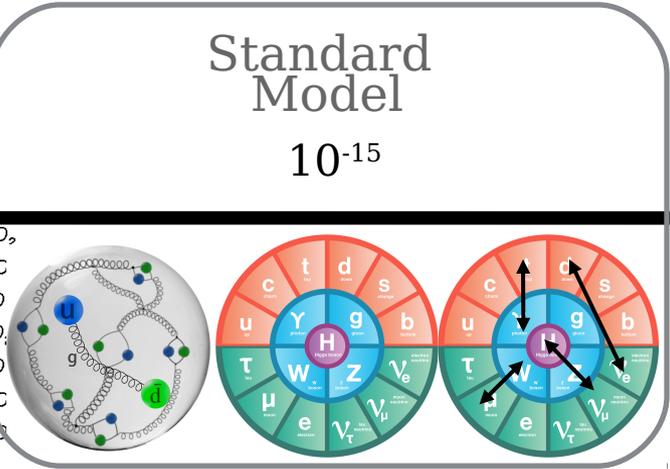
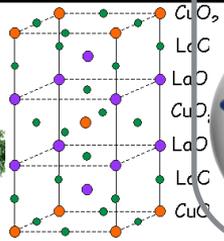
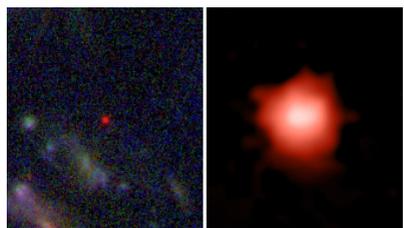
New Physics
under our nose: **QCD**

Effective **F**ield **T**heory:
New Interactions
- Model independent
- Exhaustive
- Guide for experiments

The universe at different scales

10²⁷ 10²⁰ 10¹⁰ 1m 10⁻¹⁰

Very big



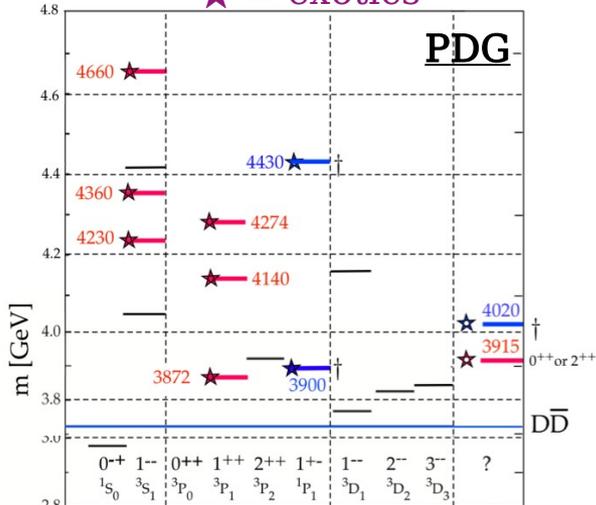
New Particles

10⁻³⁵

Very Small

Charmonium spectrum

★ = exotics



Brian Henning

New Physics under our nose: QCD

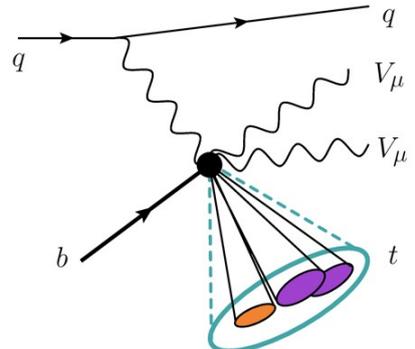
Numerous quantitative and qualitative mysteries in the strong sector

This talk: EFT

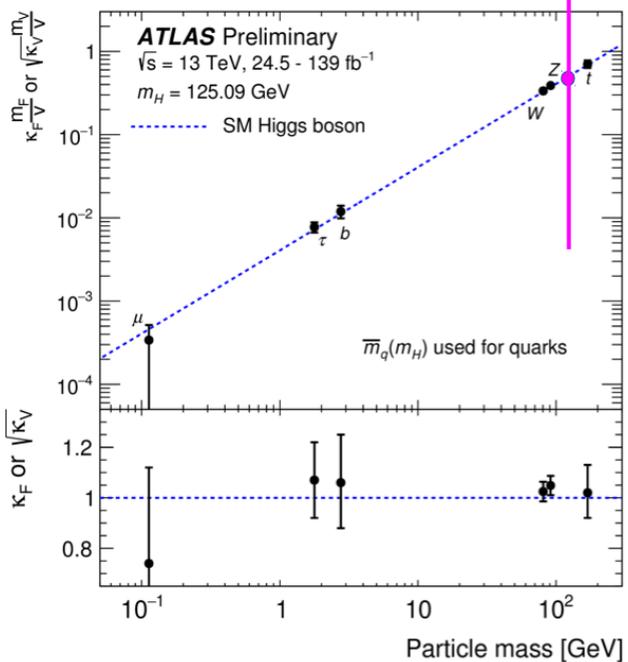
Effective Field Theory:

New Interactions

- Model independent
- Guide for experiments



EFT & the hunt for new physics



ATLAS Exotics Searches* - 95% CL Upper Exclusion Limits

Status: May 2020

ATLAS Preliminary
 $\int \mathcal{L} dt = (3.2 - 139) \text{ fb}^{-1}$ $\sqrt{s} = 8, 13 \text{ TeV}$

| Model | ℓ, γ | Jets [†] | E_T^{miss} | $[\mathcal{L} dt[\text{fb}^{-1}]$ | Limit | Reference | |
|--|--|-------------------------------|-------------------------------|-----------------------------------|--------------------------------------|--|---|
| Extra dimensions | ADD $G_{KK} + g/q$ | $0, e, \mu$ | $1-4j$ | Yes | 36.1 | M_D 7.7 TeV, M_2 8.5 TeV, M_3 8.9 TeV, M_4 8.2 TeV, M_5 9.55 TeV | $n = 2$ $n = 3$ HLZ NLO $n = 6$ $n = 6, M_D = 3 \text{ TeV}$, rot BH $n = 6, M_D = 3 \text{ TeV}$, rot BH |
| | ADD non-resonant $\gamma\gamma$ | 2γ | - | - | 36.7 | | |
| | ADD QBH | - | $2j$ | - | 37.0 | | |
| | ADD BH high Σp_T | $\geq 1, e, \mu$ | $\geq 2j$ | - | 3.2 | | |
| | ADD BH multijet | - | $\geq 3j$ | - | 3.6 | | |
| | RS1 $G_{KK} \rightarrow \gamma\gamma$ | 2γ | - | - | 36.7 | G_{KK} mass 4.1 TeV | $k/\bar{M}_D = 0.1$ |
| | Bulk RS $G_{KK} \rightarrow WW/ZZ$ | multi-channel | - | - | 36.1 | G_{KK} mass 2.3 TeV | $k/\bar{M}_D = 1.0$ |
| | Bulk RS $G_{KK} \rightarrow WV \rightarrow \ell\nu qq$ | $1, e, \mu$ | $2j/1j$ | Yes | 139 | G_{KK} mass 2.0 TeV | $k/\bar{M}_D = 1.0$ |
| | Bulk RS $g_{KK} \rightarrow tt$ | $1, e, \mu$ | $\geq 1, b, \geq 1J, \geq 2J$ | Yes | 36.1 | g_{KK} mass 3.8 TeV | $\Gamma/m = 15\%$ |
| | 2UED / RPP | $1, e, \mu$ | $\geq 2, b, \geq 3j$ | Yes | 36.1 | KK mass 1.8 TeV | Tier (1,1), $2(A^{(1,1)} \rightarrow tt) = 1$ |
| Gauge bosons | SSM $Z' \rightarrow \ell\ell$ | $2, e, \mu$ | - | - | 139 | Z' mass 5.1 TeV | |
| | SSM $Z' \rightarrow \tau\tau$ | 2τ | - | - | 36.1 | Z' mass 2.42 TeV | |
| | Leptophobic $Z' \rightarrow bb$ | - | $2b$ | - | 36.1 | Z' mass 2.1 TeV | |
| | Leptophobic $Z' \rightarrow tt$ | $0, e, \mu$ | $\geq 1, b, \geq 2J$ | Yes | 139 | Z' mass 4.1 TeV | $\Gamma/m = 1.2\%$ |
| | SSM $W' \rightarrow \ell\nu$ | $1, e, \mu$ | - | Yes | 139 | W' mass 6.0 TeV | |
| | SSM $W' \rightarrow \tau\nu$ | 1τ | - | Yes | 36.1 | W' mass 3.7 TeV | |
| | HVT $W' \rightarrow WZ \rightarrow \ell\nu qq$ model B | $1, e, \mu$ | $2j/1j$ | Yes | 139 | W' mass 4.3 TeV | $g_V = 3$ |
| | HVT $V' \rightarrow WV \rightarrow qq qq$ model B | $0, e, \mu$ | $2j$ | - | 139 | V' mass 3.8 TeV | $g_V = 3$ |
| | HVT $V' \rightarrow WH/ZH$ model B | multi-channel | - | - | 36.1 | V' mass 2.93 TeV | $g_V = 3$ |
| | HVT $W' \rightarrow WH$ model B | multi-channel | - | - | 139 | W' mass 3.2 TeV | CERN-EP-2020-073 |
| CI | LRSM $W_R \rightarrow tb$ | multi-channel | - | - | 36.1 | W mass 3.25 TeV | 1807-10473 |
| | LRSM $W_R \rightarrow \mu N_R$ | 2μ | $1j$ | - | 80 | W mass 5.0 TeV | 1904-12679 |
| | CI $qqqq$ | - | $2j$ | - | 37.0 | A 21.8 TeV, A_{LL} 35.8 TeV | 1703-09127 |
| DM | CI $\ell\ell qq$ | $2, e, \mu$ | - | - | 139 | A 35.8 TeV | CERN-EP-2020-066 |
| | CI $tttt$ | $\geq 1, e, \mu$ | $\geq 1, b, \geq 1j$ | Yes | 36.1 | $ C_{ll} = 4\epsilon$ | 1811-02305 |
| | Axial-vector mediator (Dirac DM) | $0, e, \mu$ | $1-4j$ | Yes | 36.1 | $\#_{\text{DM}}$ 1.55 TeV | $g_V = 0.25, g_A = 1.0, m(\chi) = 1 \text{ GeV}$ |
| | Colored scalar mediator (Dirac DM) | $0, e, \mu$ | $1-4j$ | Yes | 36.1 | $\#_{\text{DM}}$ 1.67 TeV | $g_V = 1.0, m(\chi) = 1 \text{ GeV}$ |
| LO | $VV_{\chi\chi}$ EFT (Dirac DM) | $0, e, \mu$ | $1j, \leq 1j$ | Yes | 3.2 | M_{χ} 700 GeV | $m(\chi) < 150 \text{ GeV}$ |
| | Scalar reson. $\phi \rightarrow t\bar{t}$ (Dirac DM) | $0-1, e, \mu$ | $1, b, 0-1j$ | Yes | 36.1 | $\#_{\phi}$ 3.4 TeV | $y = 0.4, \lambda = 0.2, m(\chi) = 10 \text{ GeV}$ |
| | Scalar LO 1 st gen | $1, 2, e, \mu$ | $\geq 2j$ | Yes | 36.1 | LO mass 1.4 TeV | $\beta = 1$ |
| | Scalar LO 2 nd gen | $1, 2, \mu$ | $\geq 2j$ | Yes | 36.1 | LO mass 1.56 TeV | $\beta = 1$ |
| Heavy quarks | Scalar LO 3 rd gen | 2τ | $2b$ | - | 36.1 | $\#(LO_2^{\text{sc}} \rightarrow b\bar{b}) = 1$ | 1902-08103 |
| | Scalar LO 3 rd gen | $0-1, e, \mu$ | $2b$ | Yes | 36.1 | $\#(LO_2^{\text{sc}} \rightarrow \tau\bar{\tau}) = 0$ | 1902-08103 |
| | VLO $TT \rightarrow Ht/Zt/Wb + X$ | multi-channel | - | - | 36.1 | T mass 1.37 TeV | SU(2) doublet |
| | VLO $BB \rightarrow Wt/Zb + X$ | multi-channel | - | - | 36.1 | B mass 1.34 TeV | SU(2) doublet |
| | VLO $T_{33} T_{33} \rightarrow Wt + X$ | $2(\text{SS}) \geq 3, e, \mu$ | $\geq 1, b, \geq 1j$ | Yes | 36.1 | T_{33} mass 1.64 TeV | $\#(T_{33} \rightarrow Wt) = 1, c(T_{33} Wt) = 1$ |
| | VLO $Y \rightarrow Wb + X$ | $1, e, \mu$ | $\geq 1, b, \geq 1j$ | Yes | 36.1 | Y mass 1.85 TeV | $\#(Y \rightarrow Wb) = 1, c_W(Wb) = 1$ |
| Excited fermions | VLO $B \rightarrow Hb + X$ | $0, e, \mu, 2\gamma$ | $\geq 1, b, \geq 1j$ | Yes | 79.8 | B mass 1.21 TeV | $\kappa_B = 0.5$ |
| | VLO $QQ \rightarrow WqWq$ | $1, e, \mu$ | $\geq 4j$ | Yes | 20.3 | Q mass 690 GeV | ATLAS-CONF-2018-024 |
| | Excited quark $q^* \rightarrow qg$ | - | $2j$ | - | 139 | q^* mass 6.7 TeV | only u' and d' , $A = m(q')$ |
| | Excited quark $q^* \rightarrow q\gamma$ | 1γ | $1j$ | - | 36.7 | q^* mass 5.3 TeV | only u' and d' , $A = m(q')$ |
| Other | Excited quark $b^* \rightarrow b\gamma$ | - | $1, b, 1j$ | - | 36.1 | b^* mass 2.6 TeV | 1805-09299 |
| | Excited lepton ℓ^* | $3, e, \mu$ | - | - | 20.3 | ℓ^* mass 3.0 TeV | $A = 3.0 \text{ TeV}$ |
| | Excited lepton ν^* | $3, e, \mu, \tau$ | - | - | 20.3 | ν^* mass 1.6 TeV | 1411-2921 |
| | Type III Seesaw | $1, e, \mu$ | $\geq 2j$ | Yes | 79.8 | N mass 560 GeV | $A = 1.6 \text{ TeV}$ |
| LRSM Majorana ν | 2μ | $2j$ | - | 36.1 | N mass 3.2 TeV | $m(W_R) = 4.1 \text{ TeV}, g_L = g_R$ | |
| Higgs triplet $H^{\pm,0} \rightarrow \ell\ell$ | $2, 3, 4, e, \mu$ (SS) | - | - | 36.1 | $H^{\pm,0}$ mass 870 GeV | 1809-1105 | |
| Higgs triplet $H^{\pm,0} \rightarrow \ell\tau$ | $3, e, \mu, \tau$ | - | - | 20.3 | $H^{\pm,0}$ mass 400 GeV | DV production | |
| Magnetic monopoles | - | - | - | 36.1 | multi-charged particle mass 1.22 TeV | DV production, $\#(H^{\pm,0} \rightarrow \ell\tau) = 1$ | |
| | | | | 34.4 | monopole mass 2.37 TeV | DV production, $ q = 5e$ | |
| | | | | | | DV production, $ g = 1g_D, \text{spin } 1/2$ | |

*Only a selection of the available mass limits on new states or phenomena is shown.

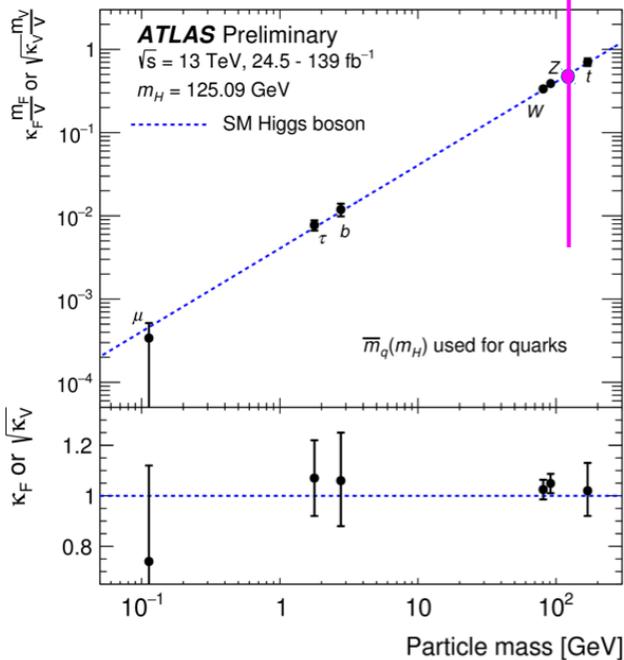
[†]Small-radius (large-radius) jets are denoted by the letter | (J).

πiggσ2U23 5U/1N0V/2U23

we've found this...

...but not these

EFT & the hunt for new physics



Are we sure we're thinking of everything?

ATLAS Exotics Searches* - 95% CL Upper Exclusion Limits

Status: May 2020

ATLAS Preliminary
 $\int \mathcal{L} dt = (3.2 - 139) \text{ fb}^{-1}$ $\sqrt{s} = 8, 13 \text{ TeV}$

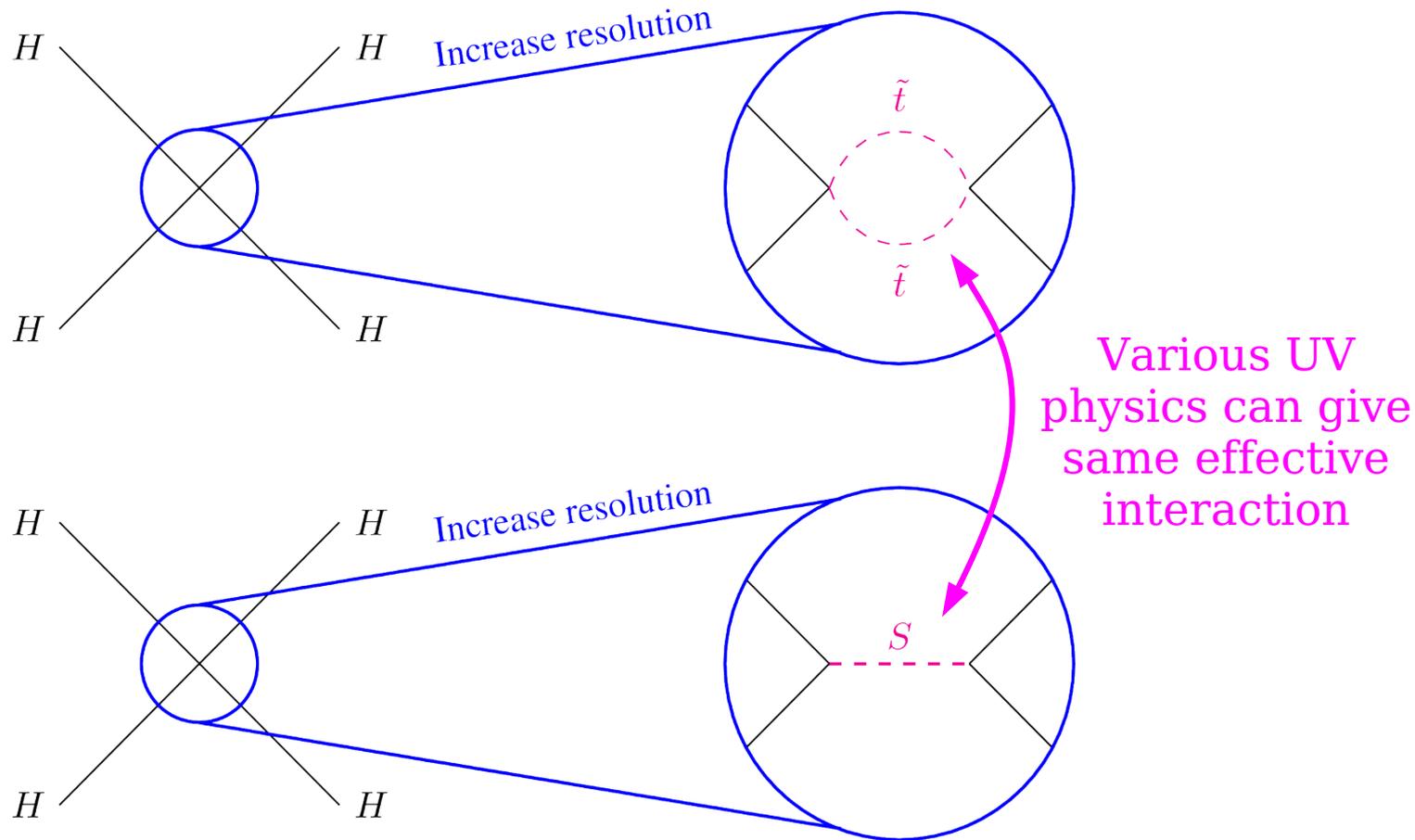
| Model | ℓ, γ | Jets [†] | E_T^{miss} | $[\mathcal{L} dt[\text{fb}^{-1}]$ | Limit | Reference |
|------------------|--|---|------------------------|-----------------------------------|------------------------|---|
| Extra dimensions | ADD $G_{KK} + g/q$ | $0 e, \mu$ | $1-4j$ | Yes | 36.1 | M_D 7.7 TeV $n=2$ |
| | ADD non-resonant $\gamma\gamma$ | 2γ | - | - | 37.0 | M_s 8.5 TeV $n=3 \text{ HLZ NLO}$ |
| | ADD QBH | - | $2j$ | - | 36.0 | M_{*} 8.9 TeV $n=6$ |
| | ADD BH high Σp_T | $\geq 1 e, \mu$ | $\geq 2j$ | - | 3.2 | M_{*} 8.2 TeV $n=6, M_D = 3 \text{ TeV, rot BH}$ |
| | ADD BH multijet | - | $\geq 3j$ | - | 3.6 | M_{*} 9.55 TeV $n=6, M_D = 3 \text{ TeV, rot BH}$ |
| | RS1 $G_{KK} \rightarrow \gamma\gamma$ | 2γ | - | - | 36.7 | $G_{KK} \text{ mass}$ 4.1 TeV $k/\bar{M}_{Pl} = 0.1$ |
| | Bulk RS $G_{KK} \rightarrow WW/ZZ$ | multi-channel | - | - | 36.1 | $G_{KK} \text{ mass}$ 2.3 TeV $k/\bar{M}_{Pl} = 1.0$ |
| | Bulk RS $G_{KK} \rightarrow WV \rightarrow \ell\nu qq$ | $1 e, \mu$ | $2j/1j$ | Yes | 139 | $G_{KK} \text{ mass}$ 2.0 TeV $k/\bar{M}_{Pl} = 1.0$ |
| | Bulk RS $g_{KK} \rightarrow tt$ | $1 e, \mu$ | $\geq 1 b, \geq 1J/2j$ | Yes | 36.1 | $g_{KK} \text{ mass}$ 3.8 TeV $\Gamma/m = 15\%$ |
| | 2UED / RPP | $1 e, \mu$ | $\geq 2 b, \geq 3j$ | Yes | 36.1 | KK mass 1.8 TeV Tier (1,1), $2(A^{(1,1)} \rightarrow tt) = 1$ |
| Gauge bosons | SSM $Z' \rightarrow \ell\ell$ | $2 e, \mu$ | - | - | 139 | Z' mass 5.1 TeV |
| | SSM $Z' \rightarrow \tau\tau$ | 2τ | - | - | 36.1 | Z' mass 2.42 TeV |
| | Leptophobic $Z' \rightarrow bb$ | - | $2 b$ | - | 36.1 | Z' mass 2.1 TeV |
| | Leptophobic $Z' \rightarrow tt$ | $0 e, \mu$ | $\geq 1 b, \geq 2J$ | Yes | 139 | Z' mass 4.1 TeV $\Gamma/m = 12\%$ |
| | SSM $W' \rightarrow \ell\nu$ | $1 e, \mu$ | - | Yes | 139 | W' mass 6.0 TeV |
| | SSM $W' \rightarrow \tau\nu$ | 1τ | - | Yes | 36.1 | W' mass 3.7 TeV |
| | HVT $W' \rightarrow WZ \rightarrow \ell\nu qq$ model B | $1 e, \mu$ | $2j/1j$ | Yes | 139 | W' mass 4.3 TeV |
| | HVT $V' \rightarrow WV \rightarrow qq qq$ model B | $0 e, \mu$ | $2J$ | - | 139 | V' mass 3.8 TeV |
| | HVT $V' \rightarrow WH/ZH$ model B | multi-channel | - | - | 36.1 | V' mass 2.93 TeV |
| | HVT $W' \rightarrow WH$ model B | multi-channel | - | - | 139 | W' mass 3.2 TeV |
| CI | LRSM $W'_K \rightarrow tb$ | multi-channel | - | - | 36.1 | W mass 3.25 TeV |
| | LRSM $W'_K \rightarrow \mu N_R$ | 2μ | $1J$ | - | 80 | W mass 5.0 TeV |
| | CI $qqqq$ | - | $2j$ | - | 37.0 | J mass 21.8 TeV η_{LL} |
| DM | Axial-vector mediator (Dirac DM) | $0 e, \mu$ | $1-4j$ | Yes | 36.1 | \tilde{m}_{DM} 1.55 TeV $g_{\tau} = 0.25, g_{\tau} = 1.0, m(\chi) = 1 \text{ GeV}$ |
| | Colored scalar mediator (Dirac DM) | $0 e, \mu$ | $1-4j$ | Yes | 36.1 | \tilde{m}_{DM} 1.67 TeV $g_{\tau} = 1.0, m(\chi) = 1 \text{ GeV}$ |
| | $VV_{\chi\chi}$ EFT (Dirac DM) | $0 e, \mu$ | $1J, \leq 1j$ | Yes | 3.2 | \tilde{m}_{χ} 700 GeV $m(\chi) < 150 \text{ GeV}$ |
| LO | Scalar reson. $0 \rightarrow \phi \rightarrow 1j$ (Dirac DM) | $0-1 e, \mu$ | $1 b, 0-1J$ | Yes | 36.1 | \tilde{m}_{ϕ} 3.4 TeV $y = 0.4, \lambda = 0.2, m(\chi) = 10 \text{ GeV}$ |
| | Scalar LO 1 st gen | $1, 2 e$ | $\geq 2j$ | Yes | 36.1 | LO mass 1.4 TeV $\beta = 1$ |
| | Scalar LO 2 nd gen | $1, 2 \mu$ | $\geq 2j$ | Yes | 36.1 | LO mass 1.56 TeV $\beta = 1$ |
| | Scalar LO 3 rd gen | 2τ | $2 b$ | - | 36.1 | $\mathcal{B}(LO_3^{\pm} \rightarrow b\tau) = 1$ |
| Heavy quarks | VLO $TT \rightarrow Ht/Zt/Wb + X$ | multi-channel | - | - | 36.1 | T mass 1.37 TeV |
| | VLO $BB \rightarrow Wt/Zb + X$ | multi-channel | - | - | 36.1 | B mass 1.34 TeV |
| | VLO $T_{33} T_{33} \rightarrow Wt + X$ | $2(SS) \geq 3 e, \mu \geq 1 b, \geq 1j$ | Yes | 36.1 | T_{33} mass 1.64 TeV | |
| | VLO $Y \rightarrow Wb + X$ | $1 e, \mu$ | $\geq 1 b, \geq 1j$ | Yes | 36.1 | Y mass 1.85 TeV |
| | VLO $B \rightarrow Hb + X$ | $0 e, \mu, 2 \gamma$ | $\geq 1 b, \geq 1j$ | Yes | 79.8 | B mass 1.21 TeV |
| | VLO $QQ \rightarrow WtWq$ | $1 e, \mu$ | $\geq 4j$ | Yes | 20.3 | Q mass 690 GeV |
| Excited fermions | Excited quark $q^* \rightarrow qg$ | - | $2j$ | - | 139 | q^* mass 6.7 TeV |
| | Excited quark $q^* \rightarrow q\gamma$ | 1γ | $1j$ | - | 36.7 | q^* mass 5.3 TeV |
| | Excited quark $b^* \rightarrow bg$ | - | $1 b, 1j$ | - | 36.1 | b^* mass 2.6 TeV |
| | Excited lepton ℓ^* | $3 e, \mu$ | - | - | 20.3 | ℓ^* mass 3.0 TeV |
| | Excited lepton ν^* | $3 e, \mu, \tau$ | - | - | 20.3 | ν^* mass 1.6 TeV |
| Other | Type III Seesaw | $1 e, \mu$ | $\geq 2j$ | Yes | 79.8 | N^c mass 560 GeV |
| | LRSM Majorana ν | 2μ | $2j$ | - | 36.1 | N mass 3.2 TeV |
| | Higgs triplet $H^{\pm, 0} \rightarrow \ell\ell$ | $2, 3, 4 e, \mu$ (SS) | - | - | 36.1 | $H^{\pm, 0}$ mass 870 GeV |
| | Higgs triplet $H^{\pm, 0} \rightarrow \ell\tau$ | $3 e, \mu, \tau$ | - | - | 20.3 | $H^{\pm, 0}$ mass 400 GeV |
| | Multi-charged particles | - | - | - | 36.1 | multi-charged particle mass 1.22 TeV |
| | Magnetic monopoles | - | - | - | 34.4 | monopole mass 2.37 TeV |
| | | | | | | |

*Only a selection of the available mass limits on new states or phenomena is shown.

[†]Small-radius (large-radius) jets are denoted by the letter j (J).

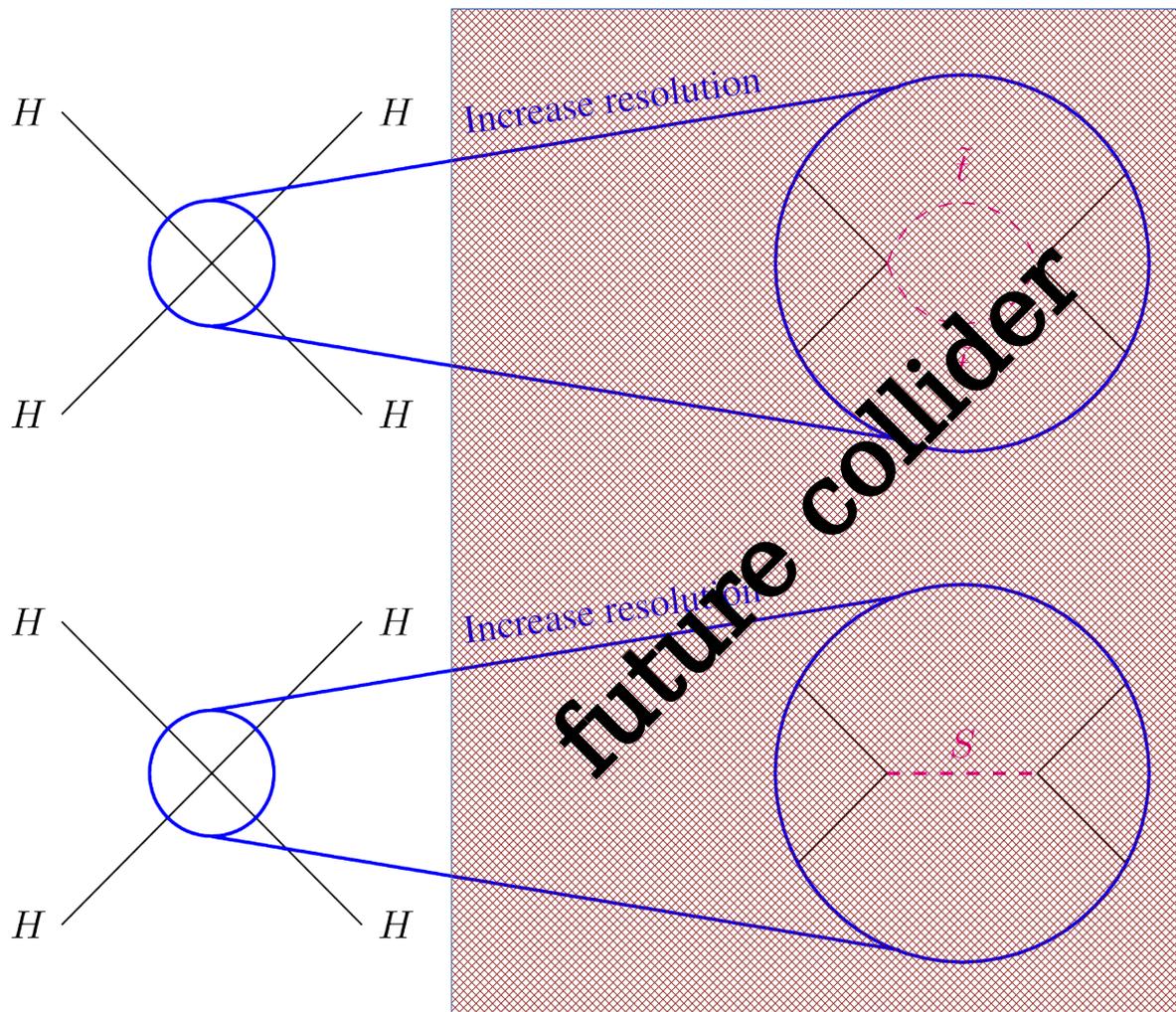
πiggσ2U23 5U/1N0V/2U23

A shift in thinking

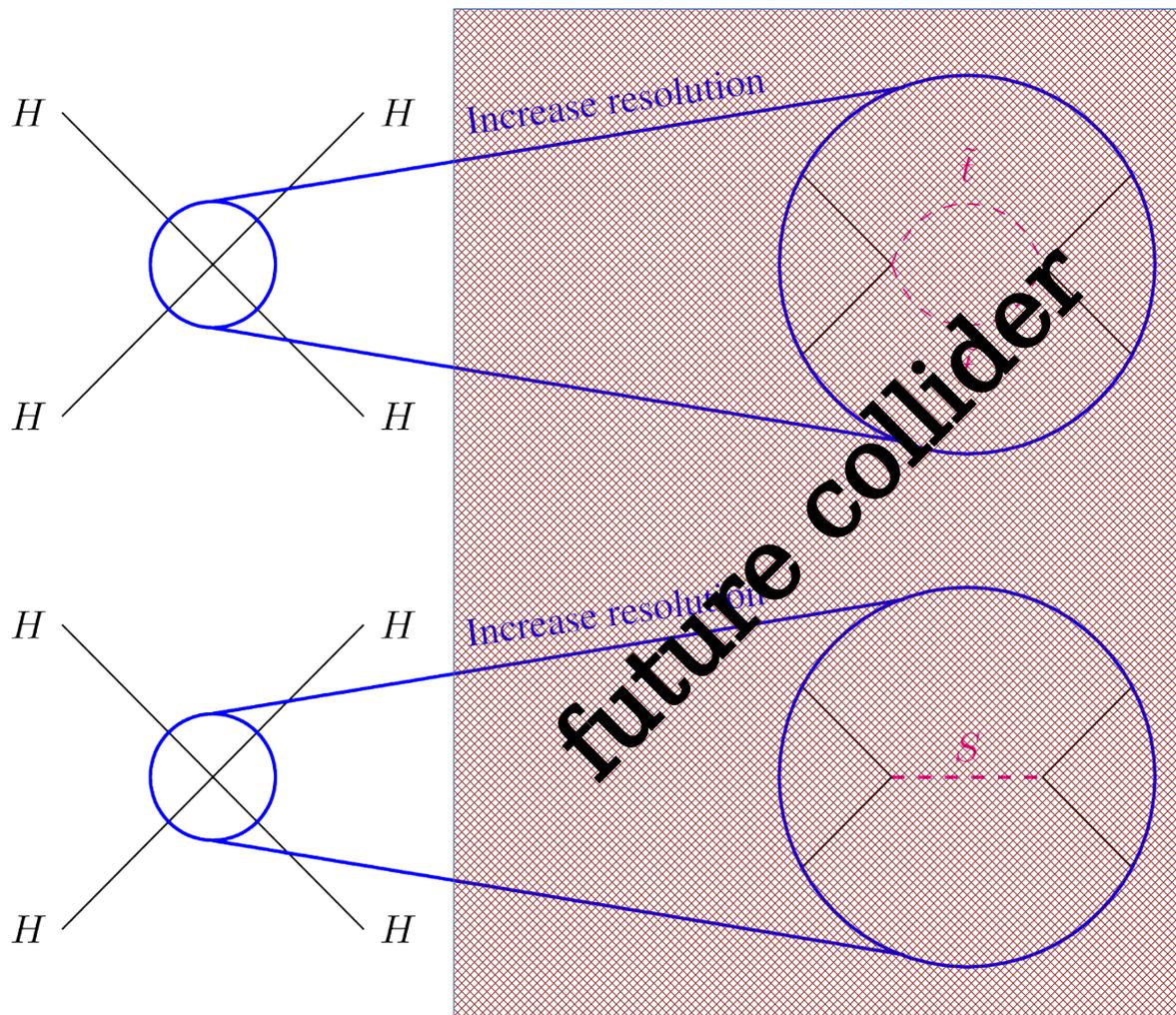


A shift in thinking

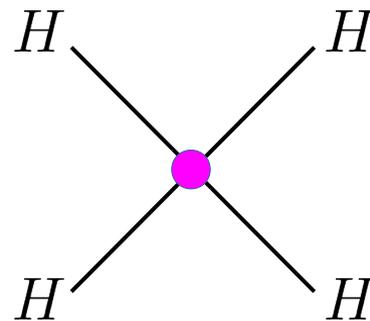
What if you're resolution limited?



A shift in thinking

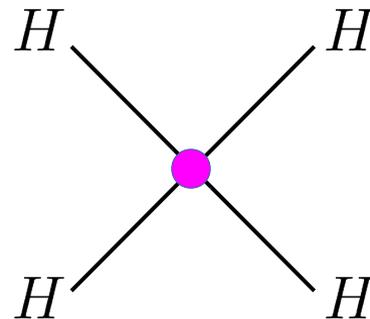
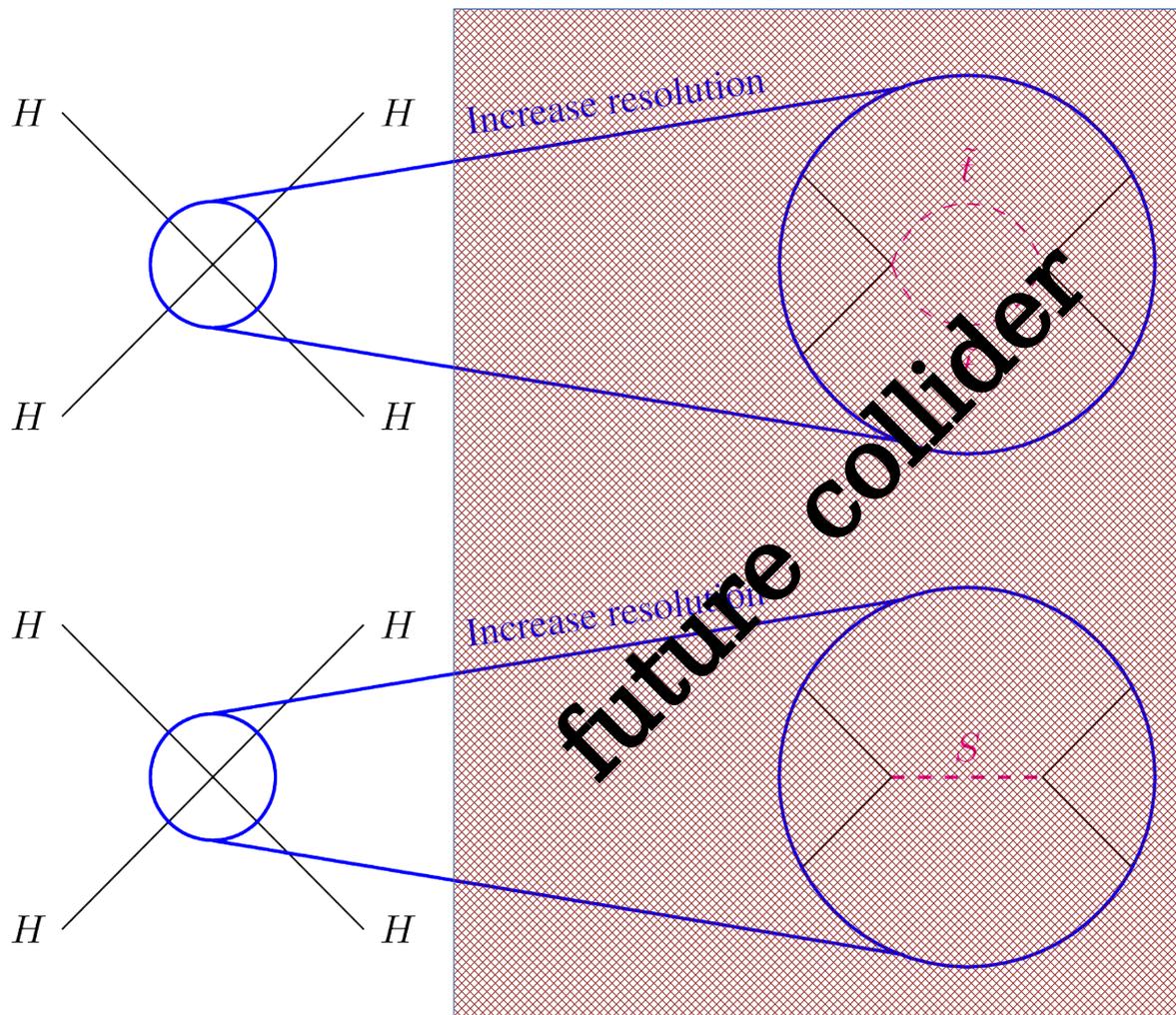


What if you're resolution limited?



Instead parameterize all possible (local) interactions involving particles you can produce

A shift in thinking

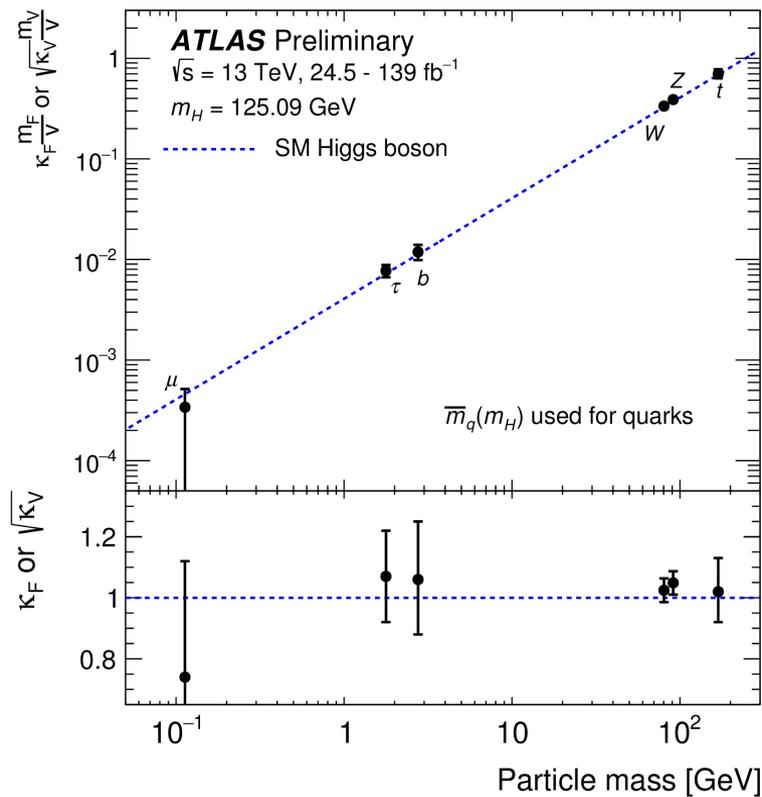


parameterize all possible (local) interactions

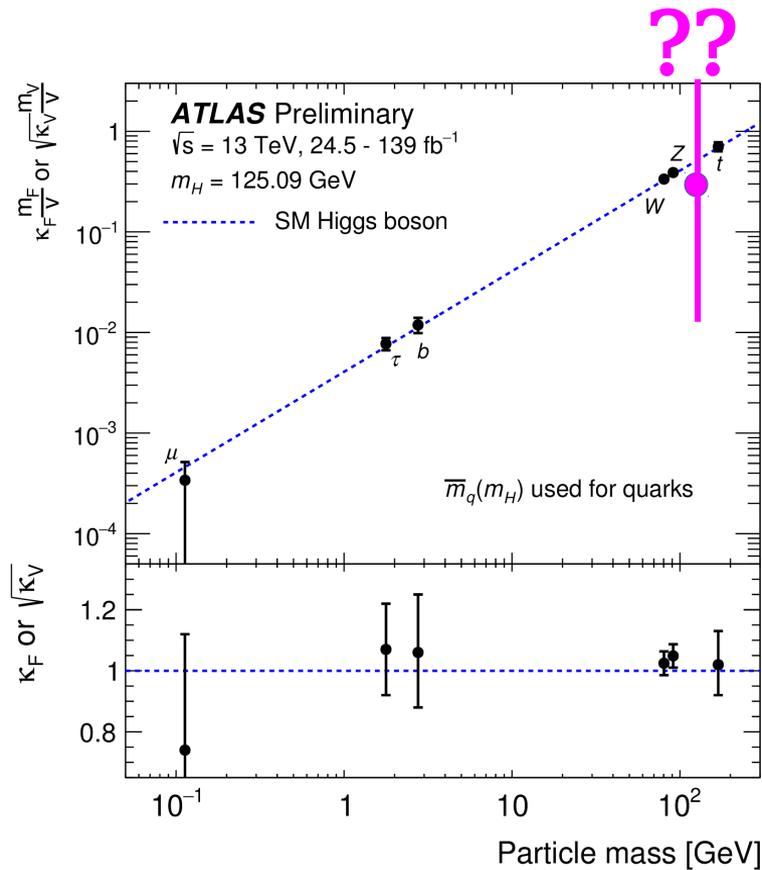


Solved and systematized with Hilbert series methods!

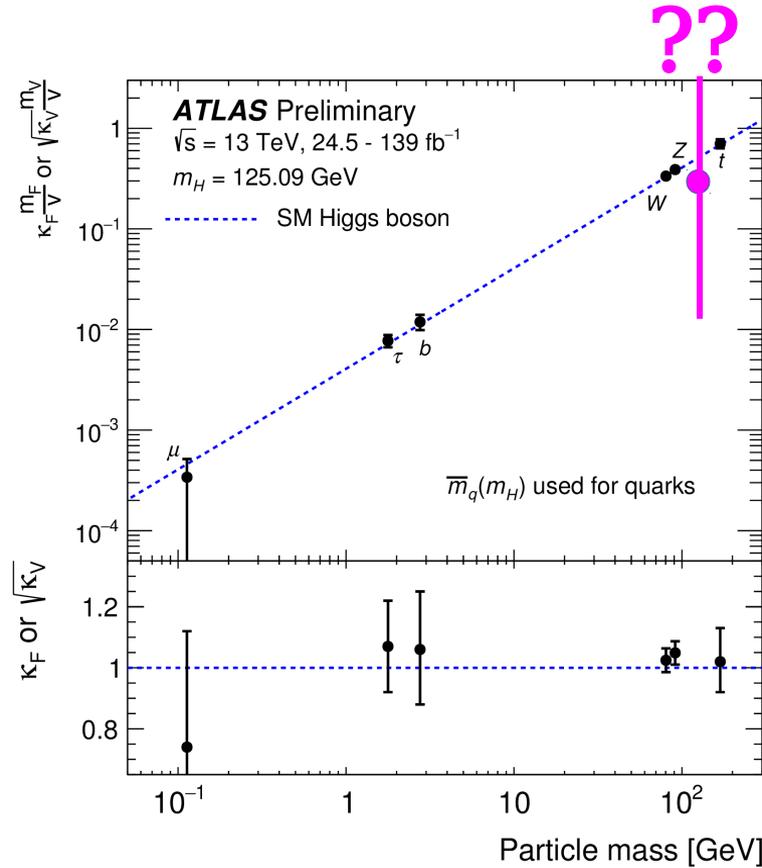
The importance of the Higgs



The importance of the Higgs



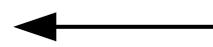
The importance of the Higgs



Deviations in *any* of h couplings leads to unitarity violation

can't keep growing

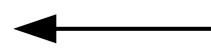
$$\frac{y(E)e^{-mr}}{r}$$



} b

can't keep growing

$$\frac{y(E)e^{-mr}}{r}$$



} b

$$ye^{-mb} \text{ maximal for } b = \frac{1}{m} \log y$$

coupling grows at most polynomially with E $\lim_{E \rightarrow \infty} y(E) \lesssim E^\alpha$

can't keep growing

$$\frac{y(E)e^{-mr}}{r}$$



} b

$$ye^{-mb} \text{ maximal for } b = \frac{1}{m} \log y$$

coupling grows at most polynomially with E $\lim_{E \rightarrow \infty} y(E) \lesssim E^\alpha$

$$\sigma \sim b^2 \lesssim \frac{\alpha^2}{m^2} \log^2 E \quad \sigma \sim \frac{1}{E^2} |\mathcal{A}|^2 \Rightarrow \mathcal{A} \lesssim E \log E$$

cross-sections/amplitudes

***bounded* in energy!** (Froissart bound)

Unitarity violation and EFT

EFT makes all
this transparent

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{\Delta_i - 4}} \mathcal{O}_i$$

Unitarity violation and EFT

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$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{\Delta_i-4}} \mathcal{O}_i$$

single \mathcal{O}_i
insertion



$$\mathcal{A}_{\mathcal{O}_i}(E \rightarrow \infty) \sim \left(\frac{E}{\Lambda} \right)^{\Delta_i-4}$$

naughty high-E behavior

Unitarity violation and EFT

EFT makes all this transparent

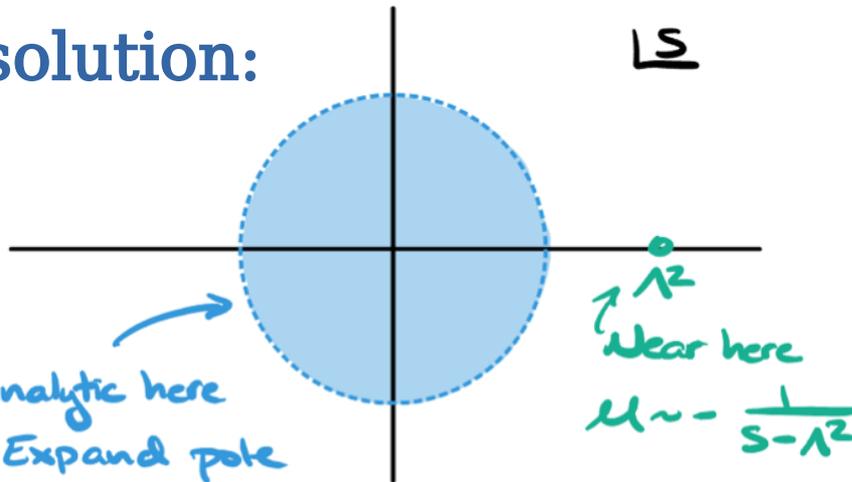
$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{\Delta_i - 4}} \mathcal{O}_i$$

single \mathcal{O}_i insertion

$$A_{\mathcal{O}_i}(E \rightarrow \infty) \sim \left(\frac{E}{\Lambda} \right)^{\Delta_i - 4}$$

naughty high-E behavior

resolution:



Analytic here
 \Rightarrow Expand pole

Near here
 $\mu \sim -\frac{1}{s - \Lambda^2}$

$$\mu \sim \frac{1}{\Lambda^2} \left(1 + \frac{s}{\Lambda^2} + \frac{s^2}{\Lambda^4} + \dots \right)$$

Unitarity violation and EFT

$$\sigma_{\text{tot}} \lesssim \log^2 E \quad \sigma_{\text{tot}} = \sum_X \sigma_{AB \rightarrow X}$$

***no* channel can grow too fast**



in EFT *many* channels exhibit E growth

**⇒ multi-boson processes
are intimately related!
(will come back to)**

Recent interesting perspectives:

- Chang & Luty 1902.05556
- Falkowski & Rattazzi 1902.05936
- Cohen, Craig, Lu, Sutherland 2108.03240

Adapting EFT into analyses

EFT provides powerful, model-independent way to probe physics

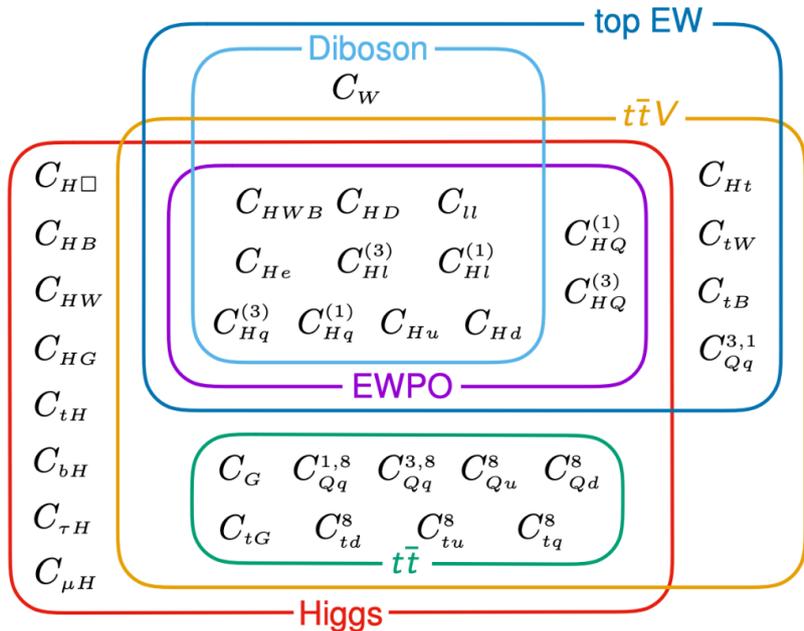
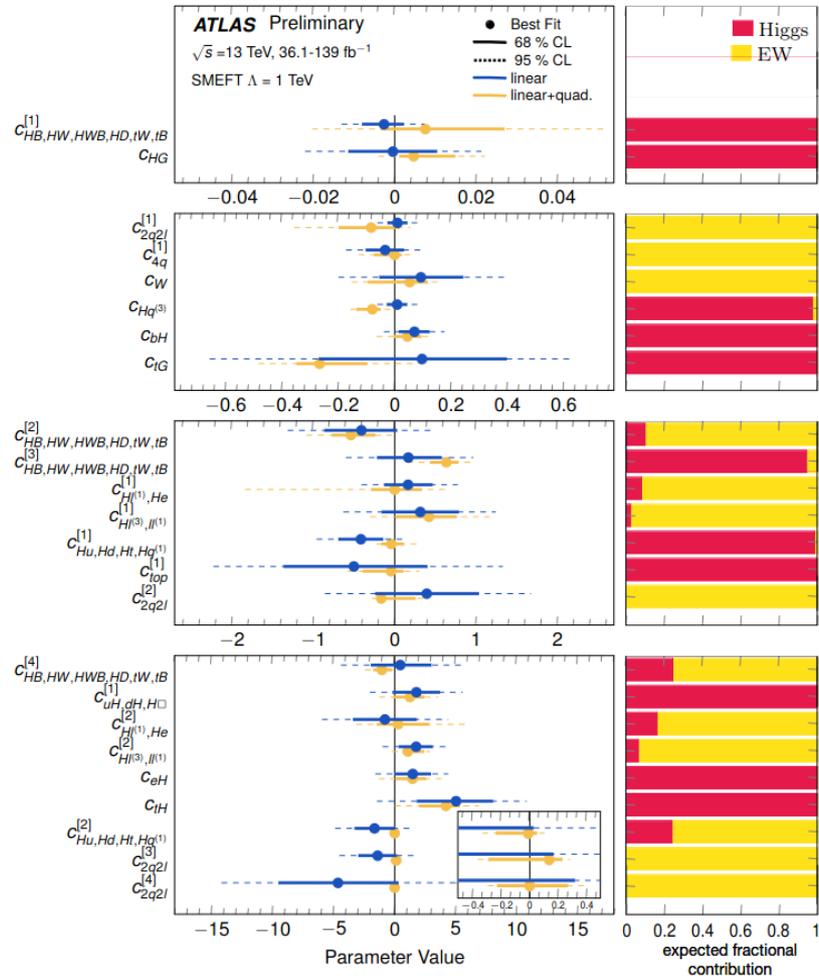
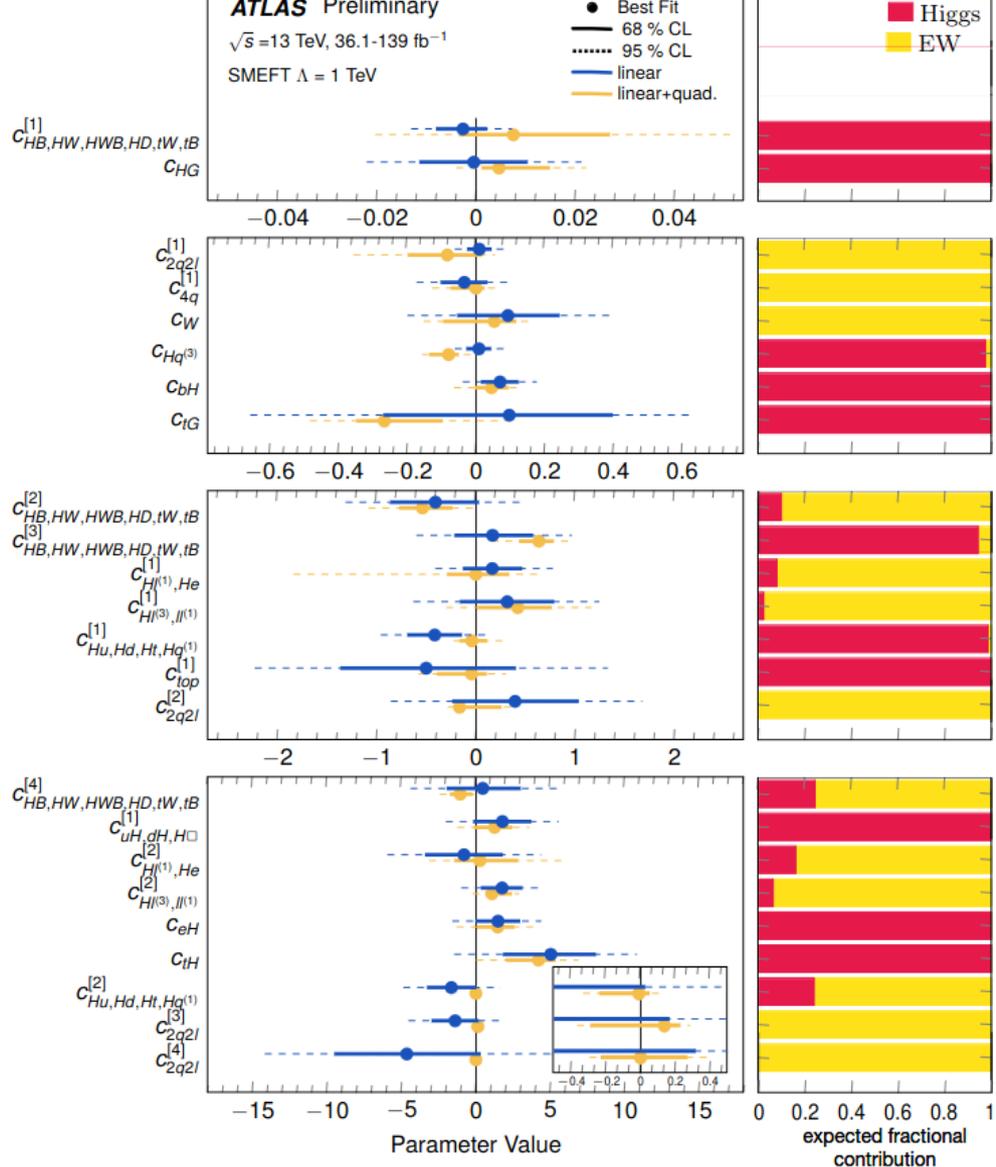
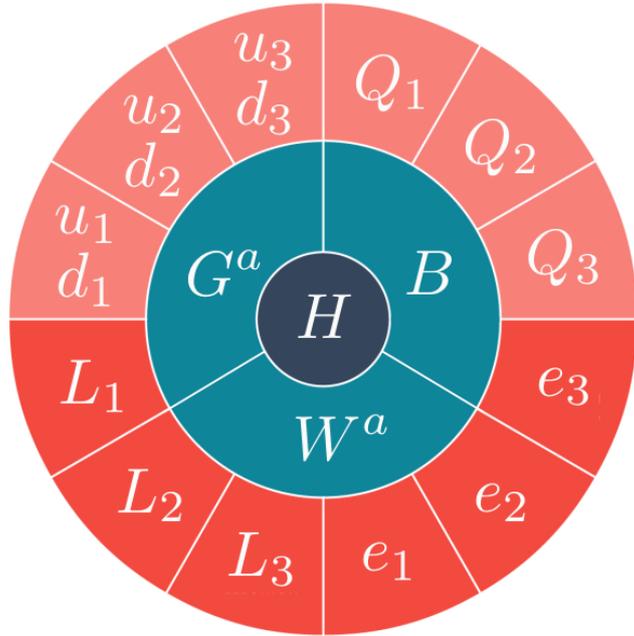


Image credit: Josh McFayden (CMS, Top2022)



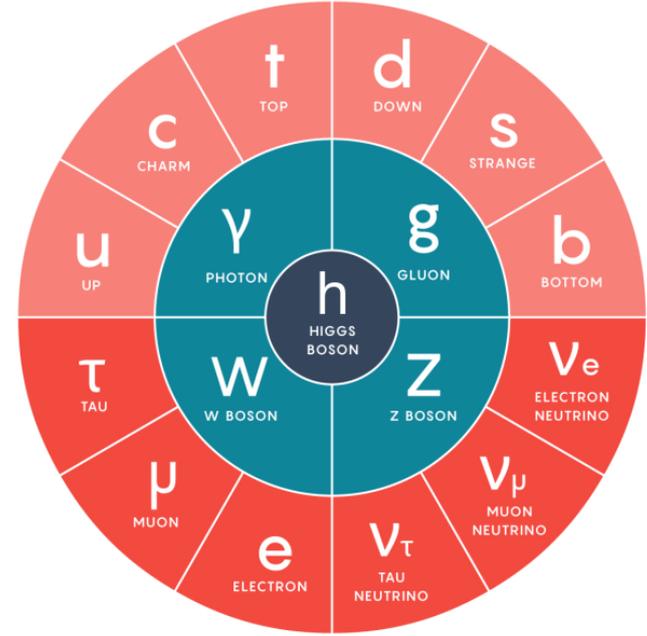


Which EFT?



SMEFT $SU(2)_L \times U(1)_Y$

[Weinberg '79, Buchmuller, Wyler '86, ...]



HEFT $U(1)_{em}$

[Feruglio '93, Bagger et al. '93, ...]

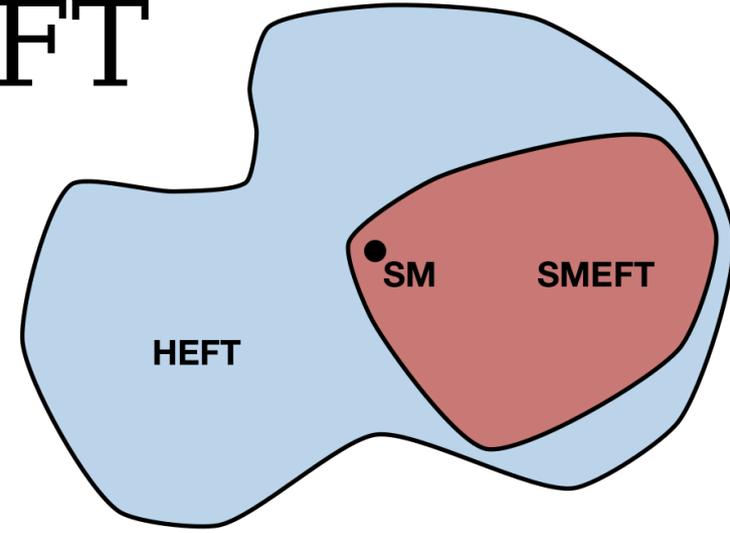
Image credit: N. Craig

SM \subset SMEFT \subset HEFT

Our world

- obeys E&M at low energies
- EW gauge bosons unified into $SU(2)_L \times U(1)_Y$
- has neutral scalar of mass 125 GeV

\Rightarrow MOST general theory: HEFT



Relate the two by field redefinition:

$$\vec{\phi} = (v + h) \vec{n}(\pi); \quad \vec{\phi} \cdot \vec{\phi} = (v + h)^2$$

SMEFT can always be written as HEFT:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} A(\vec{\phi} \cdot \vec{\phi}) (\partial \vec{\phi} \cdot \partial \vec{\phi}) + \frac{1}{2} B(\vec{\phi} \cdot \vec{\phi}) (\vec{\phi} \cdot \partial \vec{\phi})^2 - V(\vec{\phi} \cdot \vec{\phi}) \\ &= \frac{1}{2} [A + (v + h)^2 B] (\partial h)^2 + \frac{1}{2} (v + h)^2 A (\partial \vec{n})^2 - V \end{aligned}$$

Correlations at every order between h, v

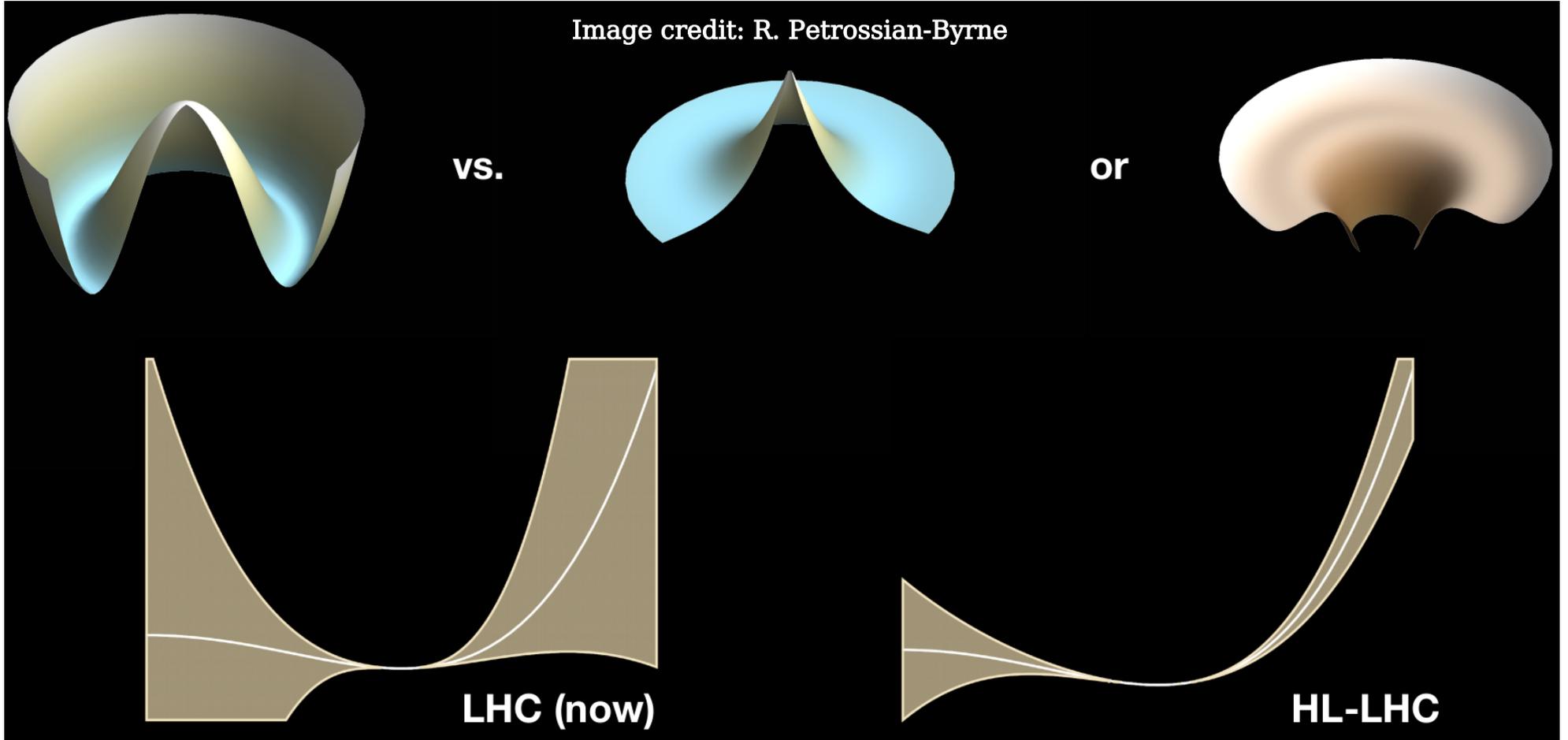
HEFT cannot always be written as SMEFT:

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} [K(h)]^2 (\partial h)^2 + \frac{1}{2} [vF(h)]^2 (\partial \vec{n})^2 - V(h) \\ &= \frac{1}{2} \frac{v^2 F}{\vec{\phi} \cdot \vec{\phi}} (\partial \vec{\phi})^2 + \frac{1}{2} (\vec{\phi} \cdot \partial \vec{\phi})^2 \frac{1}{\vec{\phi} \cdot \vec{\phi}} \left(K^2 - \frac{v^2 F^2}{\vec{\phi} \cdot \vec{\phi}} \right) - \tilde{V}(\vec{\phi} \cdot \vec{\phi}) \end{aligned}$$

Generically non-analytic at the origin

HEFT allows most general parameterization of Higgs potential

Image credit: R. Petrossian-Byrne



What goes wrong by only working with SMEFT?

- Potential errors in interpretation. Say we see a deviation from the SM
 - In SMEFT, $SU(2)$ symmetry typically means deviations are correlated
 - In HEFT this is not necessarily the case
- We should make *all* motivated measurements
 - Just because $2 \rightarrow 2$ might look SM, that does not imply $2 \rightarrow 3$, $2 \rightarrow 4$, ... necessarily are
(see, e.g., Falkowski, Rattazzi 1902.05936; Cohen, Craig, Lu, Sutherland 2108.0324)

HEFT scenarios

General lore: new particles that significantly contribute to EW symmetry breaking are captured by HEFT

...but that lore is not general enough...for example

HEFT scenarios

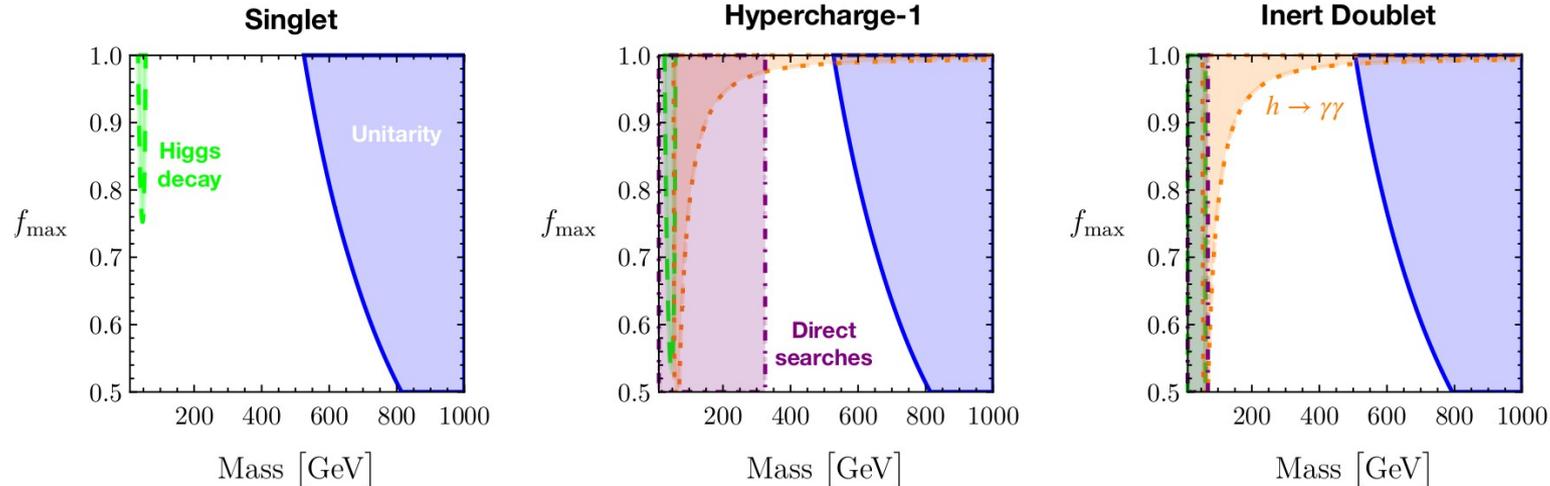
General lore: new particles that significantly contribute to EW symmetry breaking are captured by HEFT

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HEFT required whenever a particle receives more than half its mass from the Higgs

Banta, Cohen, Craig, Lu, Sutherland 2110.02967

Scalars



HEFT scenarios

General lore: new particles that significantly contribute to EW symmetry breaking are captured by HEFT

...but that lore is not general enough...for example

HEFT required whenever a particle receives more than half its mass from the Higgs

Banta, Cohen, Craig, Lu, Sutherland 2110.02967

But perhaps the most motivated place for HEFT* is:

high-multiplicity EW boson processes

*this is also well-motivated for SMEFT, as we'll see

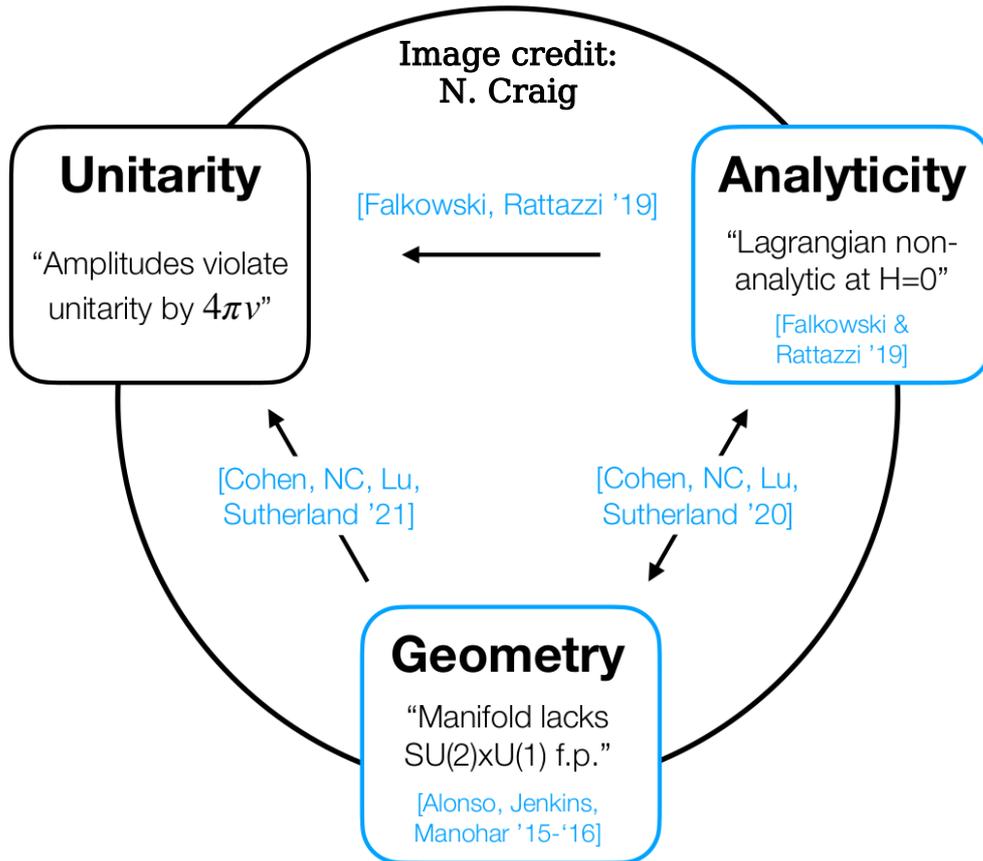
4πv and the LHC

Scale of unitarity violation:

HEFT: $4\pi v \sim 3 \text{ TeV}$

SMEFT: $4\pi \Lambda \sim \text{arbitrary}$

Image credit:
N. Craig



| Process | Unitarity Violating Scale |
|---|---|
| $h^2 Z_L \leftrightarrow h Z_L$ | $66.7 \text{ TeV} / \delta_3 - \frac{1}{3} \delta_4 $ |
| $h Z_L^2 \leftrightarrow Z_L^2$ | $94.2 \text{ TeV} / \delta_3 $ |
| $h W_L Z_L \leftrightarrow W_L Z_L$ | $141 \text{ TeV} / \delta_3 $ |
| $h Z_L^2 \leftrightarrow h Z_L^2$ | $9.1 \text{ TeV} / \sqrt{ \delta_3 - \frac{1}{5} \delta_4 }$ |
| $h W_L Z_L \leftrightarrow h W_L Z_L$ | $11.1 \text{ TeV} / \sqrt{ \delta_3 - \frac{1}{5} \delta_4 }$ |
| $Z_L^3 \leftrightarrow Z_L^3$ | $15.7 \text{ TeV} / \sqrt{ \delta_3 }$ |
| $Z_L^2 W_L \leftrightarrow Z_L^2 W_L$ | $20.4 \text{ TeV} / \sqrt{ \delta_3 }$ |
| $h Z_L^3 \leftrightarrow Z_L^3$ | $6.8 \text{ TeV} / \delta_3 - \frac{1}{6} \delta_4 ^{\frac{1}{3}}$ |
| $h Z_L^2 W_L \leftrightarrow Z_L^2 W_L$ | $8.0 \text{ TeV} / \delta_3 - \frac{1}{6} \delta_4 ^{\frac{1}{3}}$ |
| $Z_L^4 \leftrightarrow Z_L^4$ | $6.1 \text{ TeV} / \delta_3 - \frac{1}{6} \delta_4 ^{\frac{1}{4}}$ |

Unitarity violation involving Higgs trilinear and quartic
Chang, Luty 1902.05556

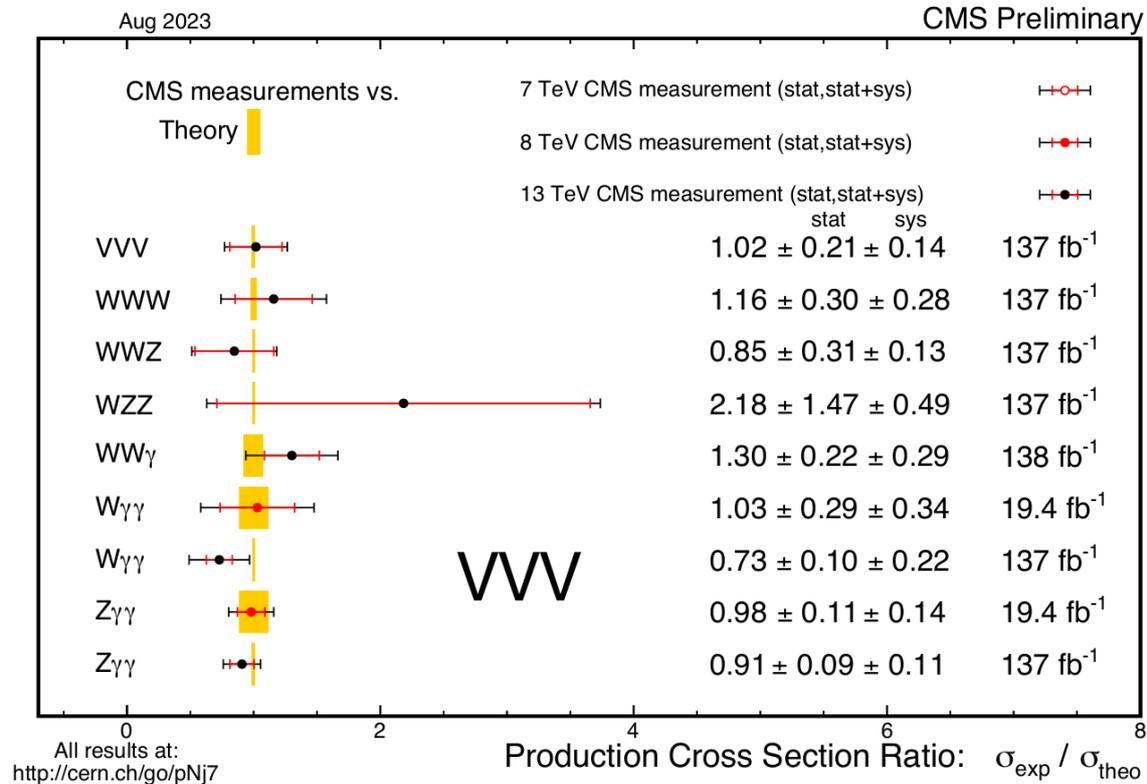
Multi-boson processes

⇒ Unitarity violation in many channels

⇒ Multi-boson processes sensitive probes

⇒ Numerous exciting (and challenging) opportunities

- high-multiplicity
- polarization tagging
- hadronic decays



See Mai Liu's talk at this conference

Goldstones = longitudinals

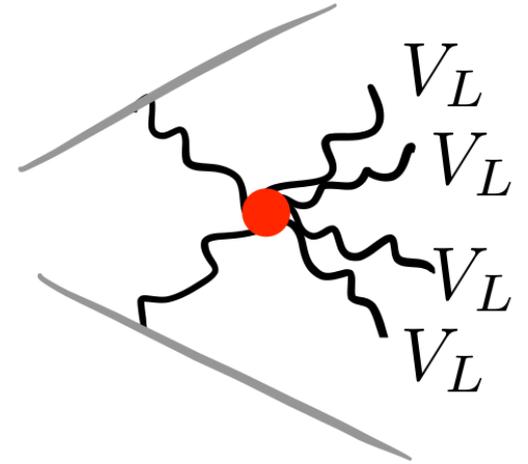
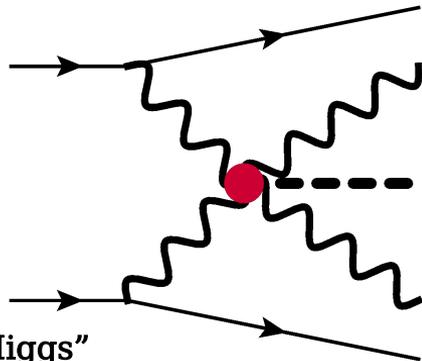
$$|H|^2 \sim (v + h)^2 + \vec{\phi}^2 \quad \rightarrow \quad \text{HC: } |H|^2 \mathcal{O}_{\text{SM}} \supset vh\mathcal{O}_{\text{SM}}$$

$$\text{HwH: } |H|^2 \mathcal{O}_{\text{SM}} \supset \vec{\phi}^2 \mathcal{O}_{\text{SM}}$$

$$|H|^6 \supset vh\phi^4 + \phi^6$$

$$V_L V_L \rightarrow V_L V_L h$$

$$V_L V_L \rightarrow V_L V_L V_L V_L$$



“Higgs without Higgs”
BH, Lombardo, Rimbau, Riva 1812.09299

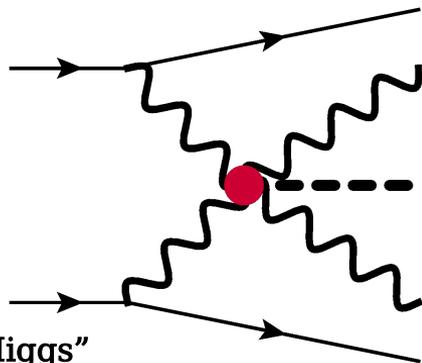
Goldstones = longitudinals

$$|H|^2 \sim (v + h)^2 + \vec{\phi}^2 \quad \longrightarrow \quad \text{HC: } |H|^2 \mathcal{O}_{\text{SM}} \supset vh\mathcal{O}_{\text{SM}}$$

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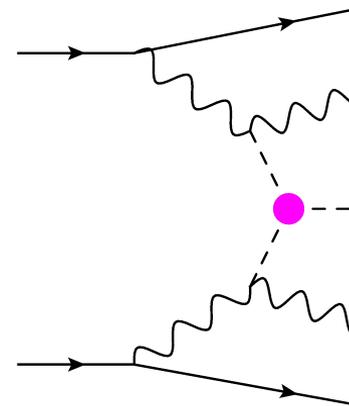
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$$V_L V_L \rightarrow V_L V_L V_L V_L$$

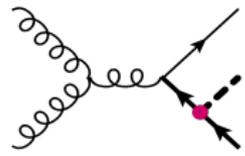
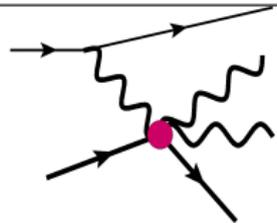
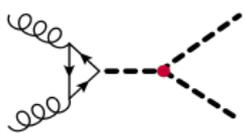
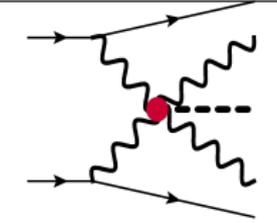
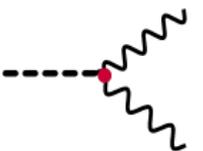
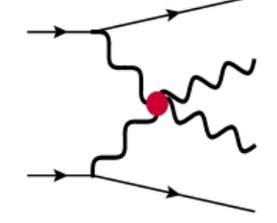
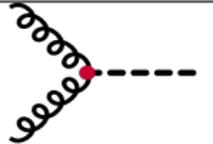
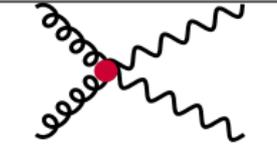
diagram in
unitary gauge



“Higgs without Higgs”
BH, Lombardo, Rimbau, Riva 1812.09299

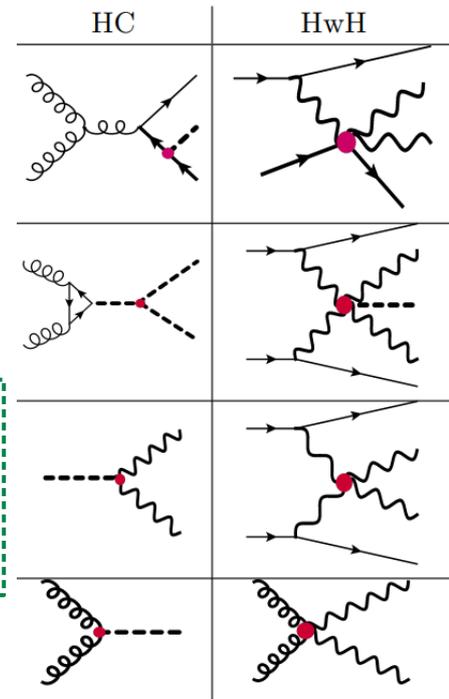
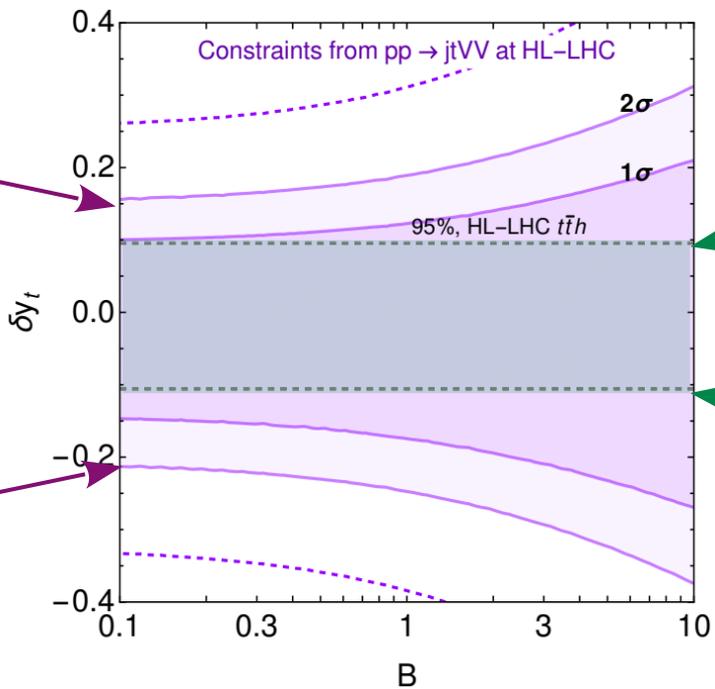
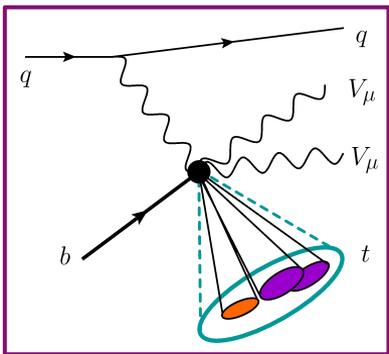
Higgs without Higgs

“standard” Higgs couplings
(HC) channels

| “standard” Higgs couplings (HC) channels | | HC | HwH | Growth |
|---|---|---|--|------------------------------|
| κ_t | \mathcal{O}_{yt} |  |  | $\sim \frac{E^2}{\Lambda^2}$ |
| κ_λ | \mathcal{O}_6 |  |  | $\sim \frac{vE}{\Lambda^2}$ |
| $\kappa_{Z\gamma}$ $\kappa_{\gamma\gamma}$ κ_V | \mathcal{O}_{WW} \mathcal{O}_{BB} \mathcal{O}_r |  |  | $\sim \frac{E^2}{\Lambda^2}$ |
| κ_g | \mathcal{O}_{gg} |  |  | $\sim \frac{E^2}{\Lambda^2}$ |

new—and complementary—
Higgs without Higgs (HwH)
channels

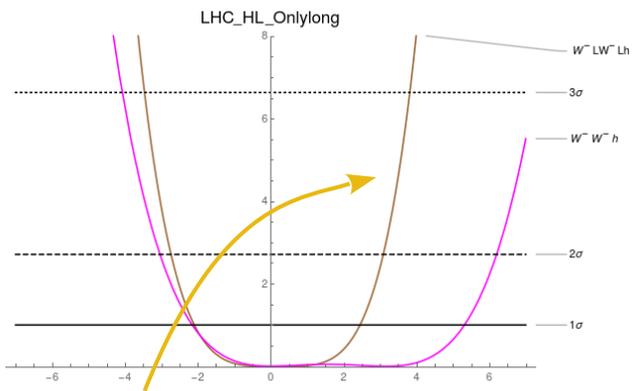
Higgs without Higgs



Experimental opportunities

innovate WITH experimentalists

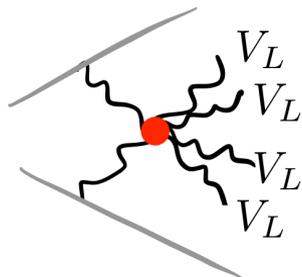
polarization tagging



constraints from only longitudinals are stronger

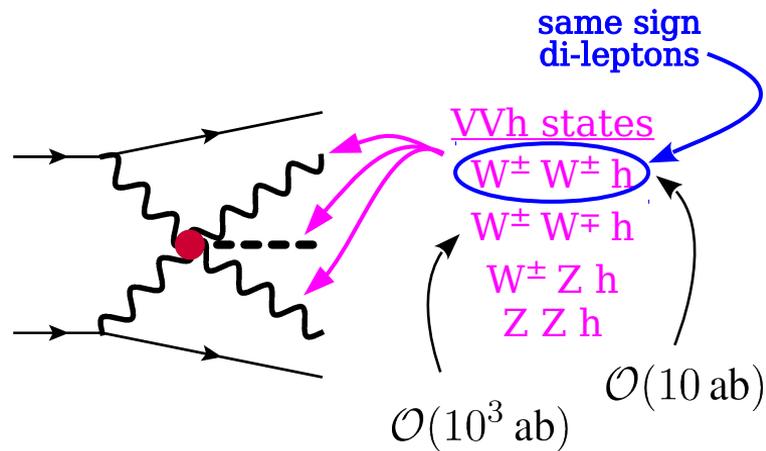
high-multiplicity EW processes

> 4 point massive amplitudes



Challenge analytically AND numerically (e.g. MadGraph)

hadronic decay channels



**Building EFTs, and
knowing you've thought
of everything**

...Or...

how I learned to count

all possible interactions

$$\lambda \mathcal{O}(\phi, \partial\phi) \sim \lambda \phi^k \partial^\mu \phi \Leftrightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} = \lambda \cdot f(p_1, \dots, p_k)$$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^{\Delta_i - 4}} \mathcal{O}_i$$

$$H\psi\psi \sim \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{---}$$

An operator specifies an interaction

$$(\partial_\mu H)^2 \sim \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{---} \quad (p_1 + p_2) \cdot (p_3 + p_4)$$

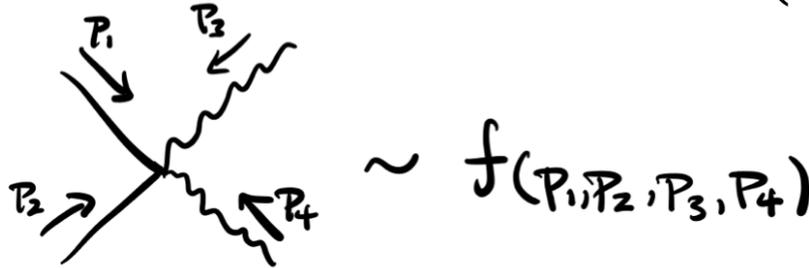
HENCE

$$\text{Tr}(W_{\mu\nu}^3) \sim \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \sim p^3$$

**All possible operators
= all possible interactions**

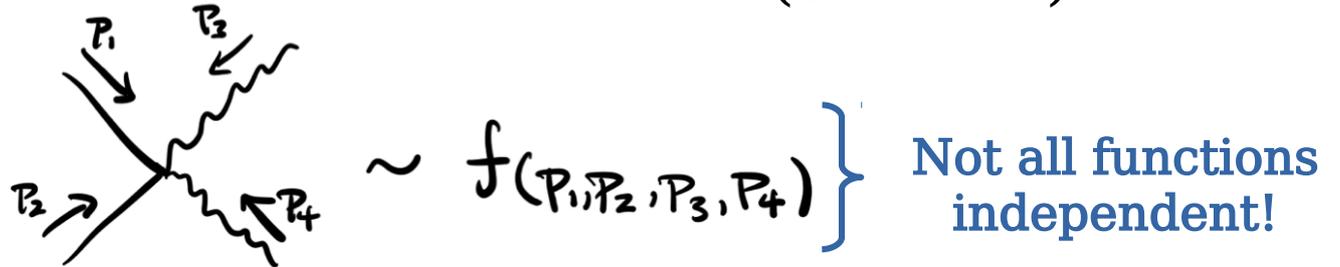
all possible interactions

Experiment sees interactions \Leftrightarrow Measures scattering amps
(S-matrix)



all possible interactions

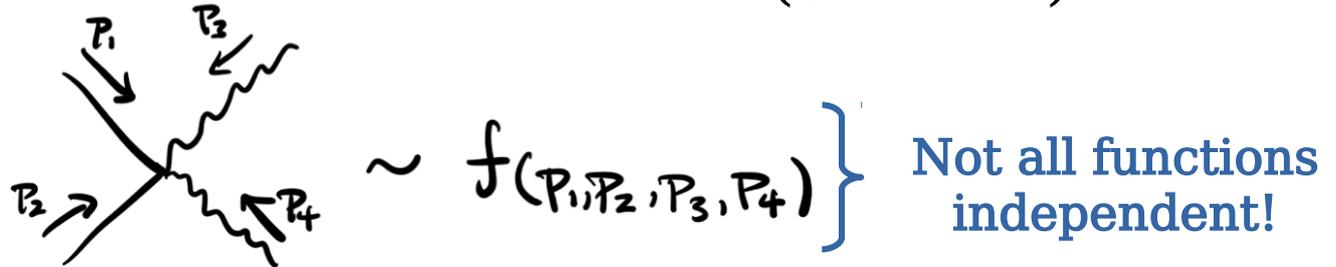
Experiment sees interactions \Leftrightarrow Measures scattering amps (S-matrix)



e.g. $f_2(p_i) = f_1(p_i) + \underbrace{(p_1 + p_2 + p_3 + p_4)}_{= 0 \text{ by momentum conservation}} g(p_i) \simeq f_1(p_i)$

all possible interactions

Experiment sees interactions \Leftrightarrow Measures scattering amps (S-matrix)



e.g. $f_2(p_i) = f_1(p_i) + \underbrace{(p_1 + p_2 + p_3 + p_4)g(p_i)}_{= 0 \text{ by momentum conservation}} \simeq f_1(p_i)$

PROBLEM: determine all independent amplitudes in the SM

CRUCIAL: “independent” \Leftrightarrow rules governing S-mat

EFT operator basis

$$\mathcal{L} = \sum_i c_i \mathcal{O}_i, \quad S = \int d^d x \mathcal{L}(x), \quad Z = \int D\phi e^{iS}$$

spacetime
symmetry!

Lorentz invariance \Leftrightarrow \mathcal{O}_i are Lorentz scalars

Translation invariance \Leftrightarrow can integrate by parts
 $\left(\int dx \partial_\mu \mathcal{O}^\mu(x) = 0 \right)$

On-shell \Leftrightarrow EOM/field redefinitions

**Equivalence relations for operator basis
follow from the S-matrix!**

EFT operator basis

Basic questions: 1) How many ops?
2) What are they?

Find a partition function

operator basis \Leftrightarrow S-matrix

\Rightarrow spacetime symmetry

\Rightarrow can use group theory!

EFT operator basis

Basic questions: 1) How many ops?
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Find a partition function

operator basis \Leftrightarrow S-matrix

\Rightarrow spacetime symmetry

\Rightarrow can use group theory!

“Hilbert series”

$$H = \text{Tr}_{\mathcal{K}} \hat{w} = \sum_{\mathcal{O} \in \mathcal{K}} q^{\Delta(\mathcal{O})} = 1 + c_1 q^1 + c_2 q^2 + \dots$$

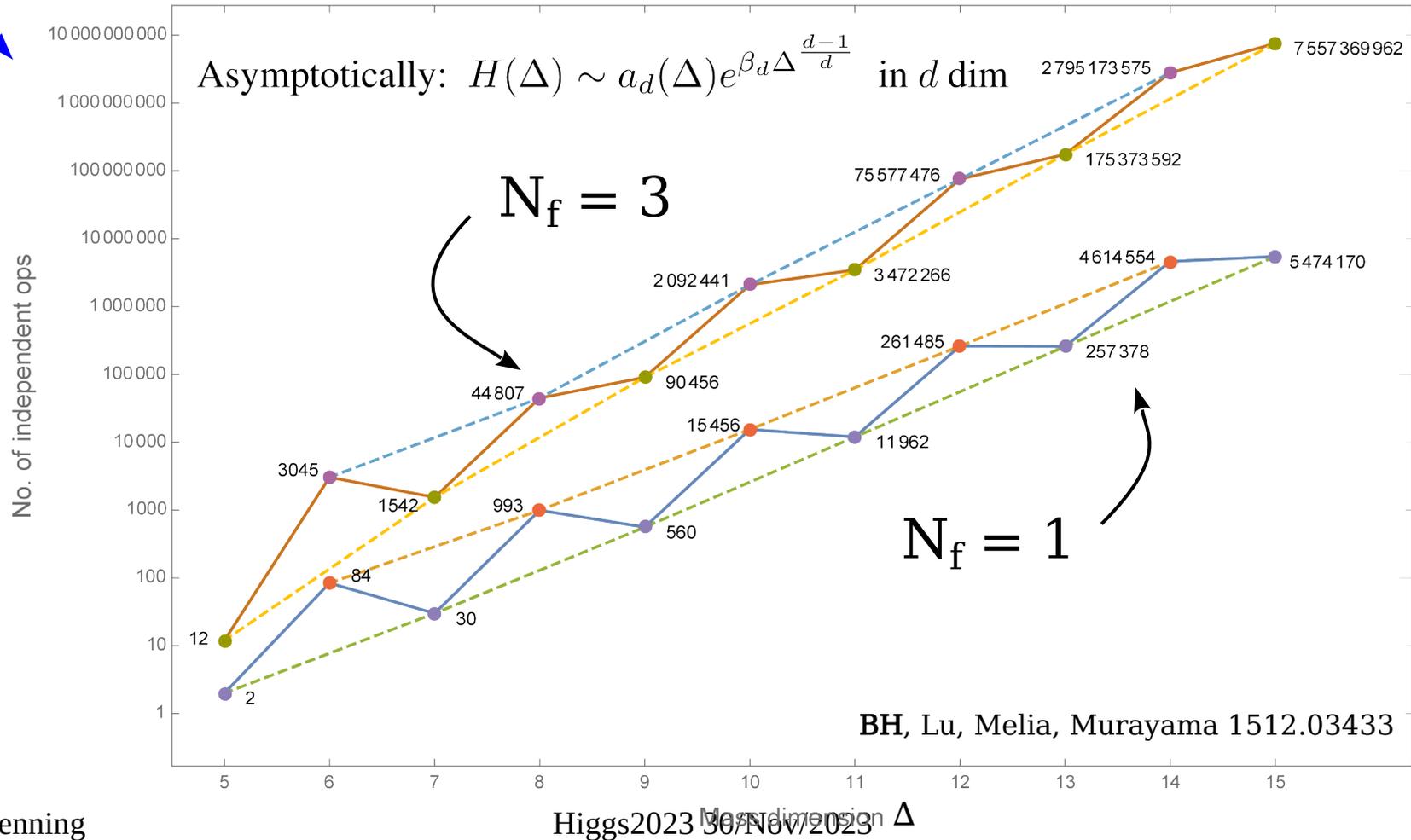
\mathcal{K} = operator basis

c_{Δ} = # of ops of mass dimension Δ

$$\left[\text{compare } Z = \text{Tr}_{\mathcal{H}} \hat{U} = \sum_{|i\rangle \in \mathcal{H}} \langle i | e^{-\beta \hat{H}} | i \rangle = \sum_{\Delta} c_{\Delta} q^{\Delta}, \quad q \equiv e^{-\beta} \right]$$

Hilbert series for SMEFT

ops of dimension Δ in SMEFT



Hilbert series for HEFT

LHC copiously produces W's and Z's

- massive particles
- charged under $U(1)_{EM}$
- $SU(2)_L \times U(1)_Y$ realized non-linearly

Our world: $SU(2)_L \times U(1)_Y / \overbrace{U(1)_{EM}}^{\text{what we see in the IR}}$ W^\pm, Z massive

Typical treatments: $SU(2)_L \times U(1)_Y / SU(2)_D \oplus$ spurions



Hilbert series for HEFT

LHC copiously produces W's and Z's

- massive particles
- charged under $U(1)_{EM}$
- $SU(2)_L \times U(1)_Y$ realized non-linearly

Our world:

$$SU(2)_L \times U(1)_Y / \overbrace{U(1)_{EM}}^{\text{what we see in the IR}} \quad \mathbf{W^\pm, Z \text{ massive}}$$

Typical treatments:

$$SU(2)_L \times U(1)_Y / SU(2)_D \oplus \text{spurions}$$

Hilbert series developed for both pictures

New ingredients: 1) massive particles

2) spurions

3) "form factors" $\mathcal{O} \sim f(h)\mathcal{O}$

Graf, BH, Lu, Melia, Murayama 2211.06275
Sun, Wang, Yu 2211.11598

Construction of NLO and NNLO HEFT ops:
Sun, Xiao, Yu 2206.07722, 2210.14939

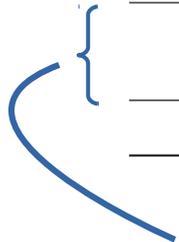
Brian Henning

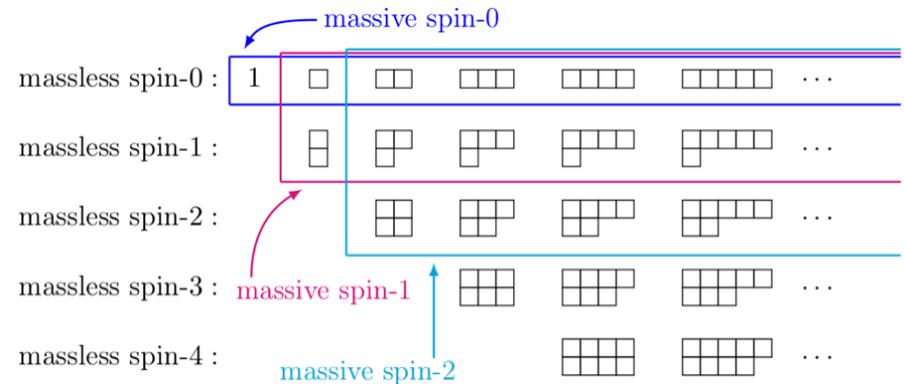
Higgs2023 30/Nov/2023

Hilbert series ingredients

| Field | Lorentz Group | $SU(3)_C$ | $U(1)_{EM}$ | dim |
|--------------------|--------------------------------------|-----------|----------------|---------------|
| u_L, u_R | | 3 | $\frac{2}{3}$ | |
| d_L, d_R | $(\frac{1}{2}, 0), (0, \frac{1}{2})$ | 3 | $-\frac{1}{3}$ | $\frac{3}{2}$ |
| $\nu_L, (\nu_R)$ | | 1 | 0 | |
| e_L, e_R | | 1 | -1 | |
| G_L, G_R | | 8 | 0 | |
| W_L^\pm, W_R^\pm | $(1, 0), (0, 1)$ | 1 | ± 1 | 2 |
| Z_L, Z_R | | 1 | 0 | |
| A_L, A_R | | 1 | 0 | |
| V^\pm | $(\frac{1}{2}, \frac{1}{2})$ | 1 | ± 1 | 1 |
| V^z | | 1 | 0 | 1 |
| h | $(0, 0)$ | 1 | 0 | 1 |

Everything fixed upon specifying particle content and their representations


massive W^\pm, Z
 Can split into longitudinal and transverse components
 ⇒ Higgs mechanism!



HEFT results

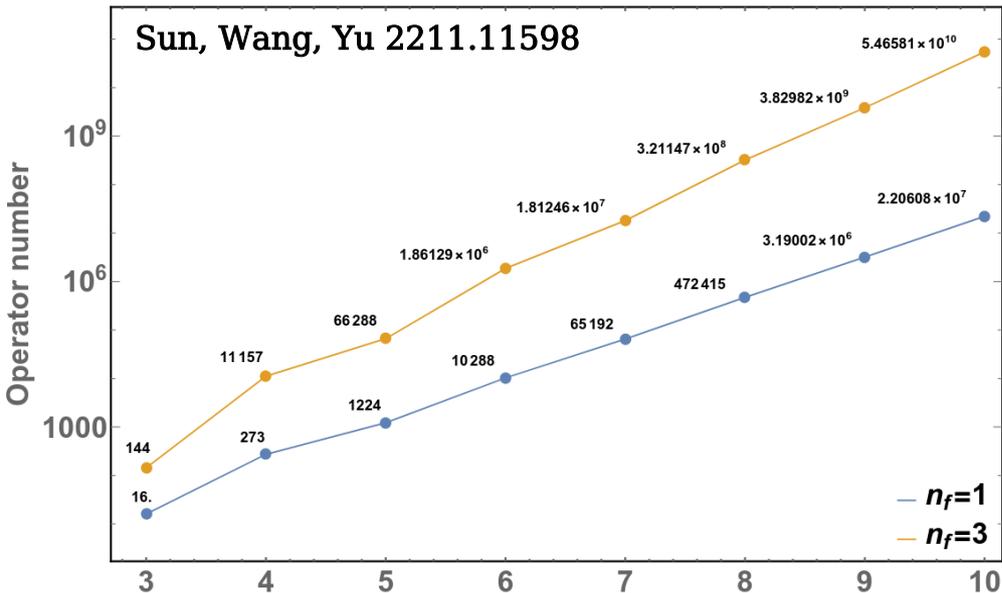
Agrees with NLO bases in literature ✓

(some minor mistakes found; caveat that $t^3_v \neq Q$)

Buchalla, Cata, Krause; Brivio, Gonzalez-Fraile, Gonzalez-Garcia, Merlo; Sun, Xiao, Yu

Easily go to $N^k\text{LO}$, $k > 1$

For $N^2\text{LO}$, see Sun, Xiao, Yu 2210.14939



$SU(2)_L \times U(1)_Y / U(1)_{EM}$
Graf, BH, Lu, Melia, Murayama 2211.06275

$SU(2)_L \times SU(2)_R / SU(2)_V$
+ $T = \text{spurion}$

small caveat

$$T = U\sigma_3U^\dagger$$

$$\langle T \rangle : SU(2)_V \rightarrow U(1)_V$$

$$U(1)_V = Q - \frac{1}{2}(B - L) \neq U(1)_Q$$

Not the same as $U(1)_Q$

(coincides in $B-L = 0$ sector)

| Class | Detailed Class | dim | HEFT |
|--------|-----------------------|-----|------|
| | V^4 | 4 | 5 |
| | hDV^3 | 5 | 4 |
| D^4 | $h^2\mathcal{D}^2V^2$ | 6 | 4 |
| | $h^3\mathcal{D}^3V$ | 7 | 1 |
| | $h^4\mathcal{D}^4$ | 8 | 1 |
| D^2X | V^2X | 4 | 8 |
| X^2 | | 4 | 10 |
| X^3 | | 6 | 6 |

| Class | Detailed Class | dim | (ν)HEFT |
|--------|-----------------------|-----|------------------|
| | V^4 | 4 | $2 + 2T^2 + T^4$ |
| | hDV^3 | 5 | $2T + T^2 + T^3$ |
| D^4 | $h^2\mathcal{D}^2V^2$ | 6 | $2 + 2T^2$ |
| | $h^3\mathcal{D}^3V$ | 7 | T |
| | $h^4\mathcal{D}^4$ | 8 | 1 |
| D^2X | V^2X | 4 | $2 + 4T + 2T^2$ |
| X^2 | | 4 | $6 + 2T + 2T^2$ |
| X^3 | | 6 | $4 + 2T$ |

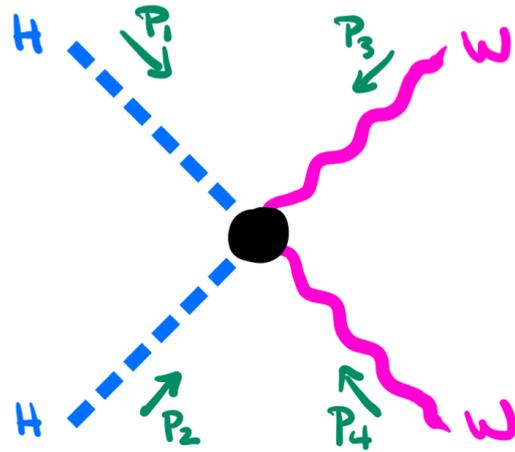
Power counting from Hilbert series

The “natural” expansion of Hilbert series is in

$$k_1 + k_2 + k_3 + k_4 = k$$
$$\partial^{k_1} H \partial^{k_2} H \partial^{k_3} W \partial^{k_4} H$$

(1) Scaling dimension
(powers of energy)

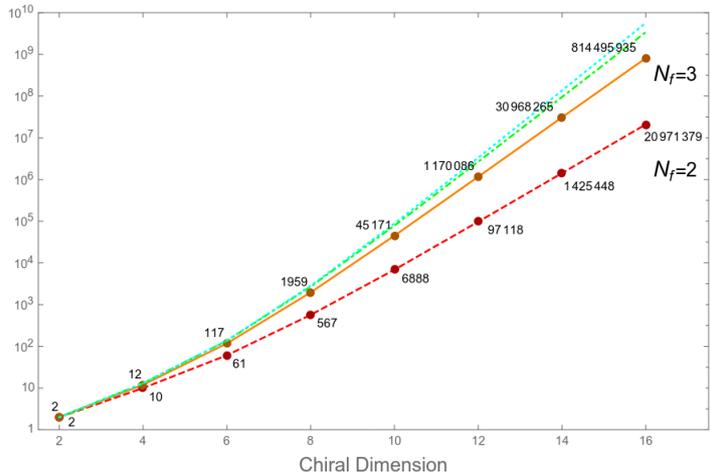
(2) Number of fields



Precisely the organizational scheme
used for “primary observables”

(Chang, Chen, Liu, Luty 2212.06215)

Hilbert goes to town



⇒ **Chiral lagrangians**
 (BH, Lu, Melia, Murayama 1706.08520, ibid + Gráf 2009.01239)

⇒ **HEFT**
 (Gráf, BH, Lu, Melia, Murayama 2211.06275; Sun, Wang, Yu 2211.11598)

⇒ **1d QFT**
 (BH, Lu, Melia, Murayama 1507.07240)

⇒ **Gravity**
 (Rudorfer, Serra, Weiler 1908.08050)

⇒ **NRQCD and HQET**
 (Kobach and Pal 1704.00008, 1804.01534, 1810.0236)

⇒ **N=1 SUSY**
 (Delgado, Martin, Wang 2212.02551, 2305.01736)

⇒ **O(n) NLSM**
 (Bijnens, Gudnason, Yu, Zhang 2212.07901)

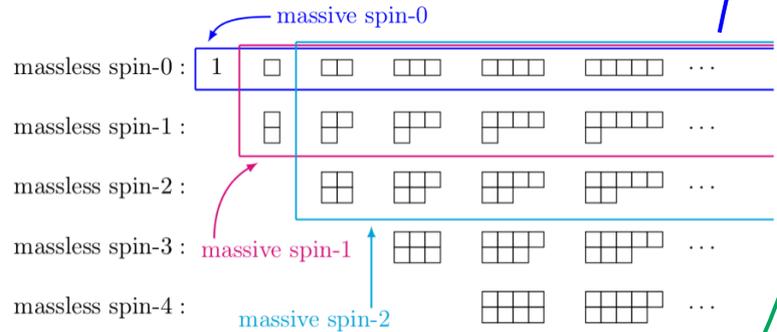
⇒ **CP and SMEFT**
 (Kondo, Murayama, Okabe 2212.02413)

⇒ **BSM model building, asymptotics, algorithms, ...**

$$\mathcal{R}_{UVCS} \sim \mathcal{C}_{UVCS} \oplus \mathcal{R}_{UV} \oplus \mathcal{R}$$

\mathcal{C}_{UVCS} : WEYL TENSOR $\square \oplus \square$
 \mathcal{R}_{UV} : RICCI TENSOR \square
 \mathcal{R} : RICCI SCALAR 1

$$\mathcal{R}_A \sim \begin{pmatrix} 1 \\ \partial_\mu \\ \partial_\mu^2 \\ \vdots \end{pmatrix} (1 - \partial^2)(1 - \partial^4) A_\mu$$



$$\begin{pmatrix} \psi^\dagger \\ D_\perp \psi^\dagger \\ D_\perp^2 \psi^\dagger \\ \vdots \end{pmatrix} \otimes \begin{pmatrix} \mathcal{E} \\ D_\perp \mathcal{E} \\ D_\perp^2 \mathcal{E}, D_t \mathcal{E} \\ \vdots \end{pmatrix} \otimes \begin{pmatrix} \mathbf{B} \\ D_\perp \mathbf{B} \\ D_\perp^2 \mathbf{B}, D_t \mathbf{B} \\ \vdots \end{pmatrix} \otimes \dots \otimes \begin{pmatrix} \psi \\ D_\perp \psi \\ D_\perp^2 \psi \\ \vdots \end{pmatrix} = \begin{pmatrix} \mathcal{O} \\ D_\perp \mathcal{O} \\ D_\perp^2 \mathcal{O}, D_t \mathcal{O} \\ \vdots \end{pmatrix} + \dots$$

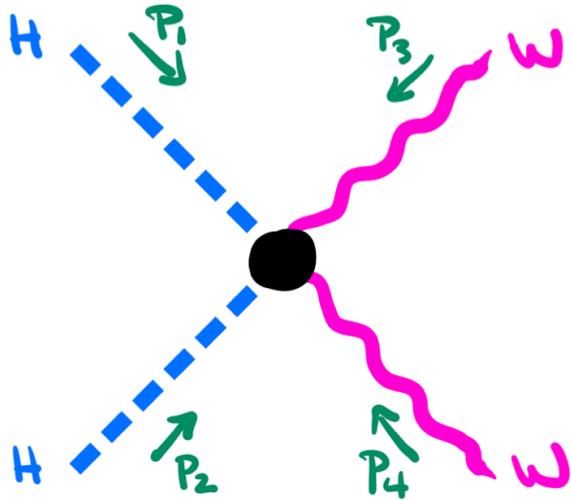
Constructing operators

Example: all ops involving **2 H's**, **2 W's**, and **k derivatives**?

$$k_1 + k_2 + k_3 + k_4 = k$$

degree k polynomial

$$\partial^{k_1} H \partial^{k_2} H \partial^{k_3} W \partial^{k_4} W \Leftrightarrow f(p_1, p_2, p_3, p_4) H(p_1) H(p_2) W(p_3) W(p_4)$$

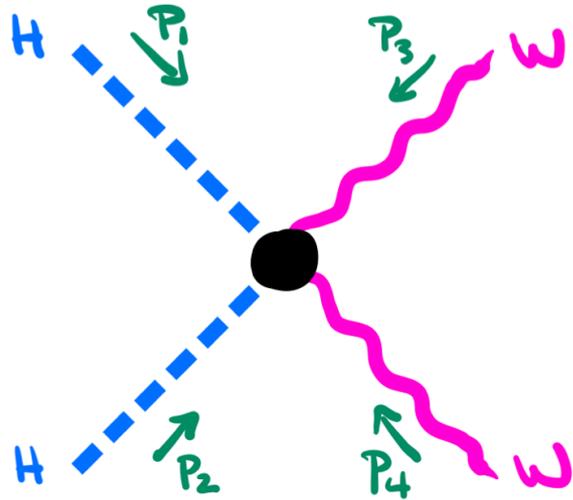


Constructing operators

Example: all ops involving **2 H's**, **2 W's**, and **k derivatives**?

$$k_1 + k_2 + k_3 + k_4 = k$$

$$\partial^{k_1} H \partial^{k_2} H \partial^{k_3} W \partial^{k_4} W \Leftrightarrow \underbrace{f(p_1, p_2, p_3, p_4)}_{\text{degree } k \text{ polynomial}} H(p_1) H(p_2) W(p_3) W(p_4)$$



subject to:

$$\Rightarrow \text{momentum conservation} \quad \sum_i p_i^\mu = 0$$

$$\Rightarrow \text{on-shell} \quad p_i^2 = 0$$

$$\Rightarrow \text{Lorentz invariance} \quad f(\Lambda p_i) = f(p_i) \\ \Lambda \in SO(3, 1)$$

Scalars: BH, Lu, Melia, Murayama 1706.08520
Arbitrary spin: BH, Melia 1902.06747, 1902.06754

See also: Dong, Ma, Shu 2103.15837

Durieux, Kitahara, Machado, Shadmi, Weiss 1909.10551

Constructing operators

BH, T. Melia
1902.06747
1902.06754

- ⇒ momentum conservation
- ⇒ on-shell
- ⇒ Lorentz invariance

constraints define a manifold in phase space

$$\delta(p_1^2) \cdots \delta(p_n^2) \times \delta^4(P^\mu - (p_1^\mu + \cdots + p_n^\mu))$$

use spinors

$$\delta^4(P_{\alpha\dot{\alpha}} - (\lambda^1 \tilde{\lambda}^1 + \cdots + \lambda^n \tilde{\lambda}^n)_{\alpha\dot{\alpha}})$$

Constructing operators

BH, T. Melia
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Want a set of class
functions on the manifold

↳ generalized spherical harmonics



operators ⇔ harmonics on phase space

Constructing operators

BH, T. Melia
1902.06747
1902.06754

- ⇒ momentum conservation
- ⇒ on-shell
- ⇒ Lorentz invariance

constraints define a manifold in phase space

$$\delta(p_1^2) \cdots \delta(p_n^2) \times \delta^4(P^\mu - (p_1^\mu + \cdots + p_n^\mu))$$

use spinors

$$\delta^4(P_{\alpha\dot{\alpha}} - (\lambda^1 \tilde{\lambda}^1 + \cdots + \lambda^n \tilde{\lambda}^n)_{\alpha\dot{\alpha}})$$

$$P_{\alpha\dot{\alpha}} = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} = \begin{pmatrix} |\vec{\lambda}_1|^2 & \vec{\lambda}_1 \cdot \vec{\lambda}_2^* \\ \vec{\lambda}_2 \cdot \vec{\lambda}_1^* & |\vec{\lambda}_2|^2 \end{pmatrix}$$

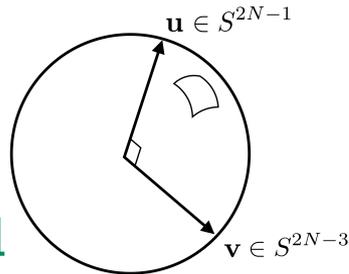
geometry
basically two
orthogonal
spheres

$$r^2 = \vec{v}^2$$

$$r^2 = \vec{u}^2$$

$$0 = \vec{v} \cdot \vec{u}$$

$$G/H = U(N)/U(N-2)$$



“Stiefel manifold”

Grassmannian \subset Stiefel

$$G_{2,N}(\mathbb{C}) = U(N)/U(N-2) \times U(2)$$

Brian Henning

Want a set of class
functions on the manifold

↳ generalized spherical harmonics



operators \Leftrightarrow harmonics on phase space

“conformal - helicity duality”

$SU(2, 2) \times U(N)$ (math world: reductive dual pairs/Howe duality/oscillator representation)

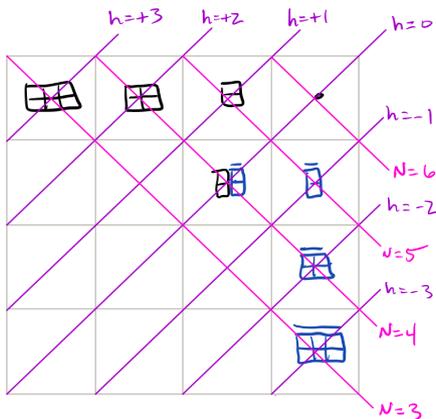
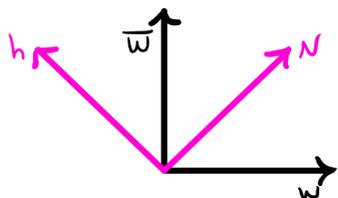
Phase space harmonics

The families of operators
belong to the same
Grassmann harmonic!

BH, T. Melia 1902.06747, 1902.06754

| | | | |
|--|--|--|--|
| | | | |
| | | | |
| | | | |
| | | | |

| | | | |
|---|--|--|--|
| $\begin{matrix} \phi^2\phi \\ \phi\phi\phi \\ F^3 \end{matrix}$ | $\begin{matrix} F^2\phi^2 \\ F\phi^2\phi \\ \phi^4 \end{matrix}$ | $\phi^3\phi^3$ | ϕ^6 |
| | | $\begin{matrix} \phi^4\phi^2 \\ \phi^2\phi\phi^2 \\ \phi^2\phi^2 \end{matrix}$ | $F^2\phi^3$ |
| | | | $\begin{matrix} F^2\phi^2 \\ F\phi^2\phi \\ \phi^4 \end{matrix}$ |
| | | | $\begin{matrix} F^2\phi^2\phi \\ \phi^2\phi\phi \\ F^3 \end{matrix}$ |



Explains structure of EFT
non-renormalization/helicity
selection rules

Cheung & Shen 1505.01844

Azatov, Contino, Machado, Riva 1607.05236

Jiang, Shu, Xiao, Zheng 2001.04481

Method used to construct
dim-8 ops in SMEFT

Li, Ren, Shu, Xiao, Yu 2005.00008, 2012.11615

Dong, Ma, Shu, Zheng 2202.08350

Harmonics and tableaux methods

Massless

Phase space geometry

BH, Melia 1902.06747, 1902.06754;
Larkoski, Melia 2008.06508

SMEFT from on-shell:

Ma, Shu, Xiao 1902.06752;
Jiang, Shu, Xiao, Zheng 2001.04481

Dim-8 SMEFT

Li, Ren, Shu, Xiao, Yu, Zheng 2005.00008, 2012.11615

Massive

On-shell massive amplitudes

Durieux, Kitahara, Shadmi, Weiss 1909.10551;
ibid + Machado 2008.09652;
Falkowski, Isabella, Machado 2011.05339

Tableaux for any mass and spin

Dong, Ma, Shu 2103.15837

HEFT

Sun, Xiao, Yu 2206.07722, 2210.14939;
Dong, Ma, Shu, Zhou 2211.16515

+...

Brian Henning

The powerful methods of group theory, spinor helicity, and on-shell techniques have solved and systematized problems unthinkable just a few years ago

$$\begin{aligned}
\mathcal{A}_{\{\dot{\alpha}\}}^I(\{\epsilon_{s_i}\}) &\subset \boxed{1\ 1} \times \boxed{2\ 2} \times \boxed{3\ 3} \\
&= \boxed{\begin{array}{ccc} 1 & 1 & 2 \\ 2 & 3 & 3 \end{array}} \oplus \boxed{\begin{array}{ccc} 1 & 1 & 2\ 2 \\ 3 & 3 & \end{array}} \oplus \boxed{\begin{array}{ccc} 1 & 1 & 2\ 3 \\ 2 & 3 & \end{array}} \oplus \boxed{\begin{array}{ccc} 1 & 1 & 3\ 3 \\ 2 & 2 & \end{array}} \\
&\oplus \boxed{\begin{array}{ccccc} 1 & 1 & 2 & 2 & 3 \\ 3 & & & & \end{array}} \oplus \boxed{\begin{array}{ccccc} 1 & 1 & 2 & 3 & 3 \\ 2 & & & & \end{array}} \oplus \boxed{1\ 1\ 2\ 2\ 3\ 3} \\
&\equiv \mathcal{A}_{[3,3]}^I \oplus \mathcal{A}_{[(4,2)1]}^I \oplus \mathcal{A}_{[(4,2)2]}^I \oplus \mathcal{A}_{[(4,2)3]}^I \\
&\oplus \mathcal{A}_{[(5,1)1]}^I \oplus \mathcal{A}_{[(5,1)2]}^I \oplus \mathcal{A}_{[6]}^I \quad \text{Image credit:} \\
&\hspace{15em} \text{J. Shu}
\end{aligned}$$

Complete NNLO Operator Bases in Higgs Effective Field Theory

Hao Sun, Ming-Lei Xiao, Jiang-Hao Yu



Comments: 419 pages, 3 tables

Subjects: High Energy Physics - Phenomenology (hep-ph); High Energy Physics - Theory (hep-th)

Cite as: arXiv:2210.14939 [hep-ph]

isomorphic problems

OPERATOR-STATE CORRESPONDENCE(s)

$$|\sigma_{\Delta, \epsilon}\rangle = \sigma_{\Delta, \epsilon}(0)|0\rangle$$

$$|\vec{p}_1 \sigma_1, \dots, \vec{p}_n \sigma_n\rangle = a_{\vec{p}_1}^{\sigma_1} \dots a_{\vec{p}_n}^{\sigma_n} |0\rangle, \quad \phi \sim \int (\epsilon^\sigma(p) a_\sigma^\dagger(p) + \text{h.c.})$$



OPERATOR SPACE

HILBERT SPACE

FEYNMAN RULES &
INTERPOLATING FIELDS

$$\langle T \phi_1 \dots \phi_n \rangle = \int \mathcal{D}\phi e^{iS_0} [e^{iS_{\text{int}}} \phi_1 \dots \phi_n]$$



SCATTERING AMPLITUDES
(S-MATRIX)

LITTLE GROUP SCALINGS

$$\langle \{\vec{p}_i, \sigma_i\}; \text{out} | \{\vec{p}_j, \sigma_j\}; \text{in} \rangle$$



$$\mathcal{M}_{\{\sigma_i\}, \{\sigma_j\}}(\{\vec{p}_i\}, \{\vec{p}_j\})$$

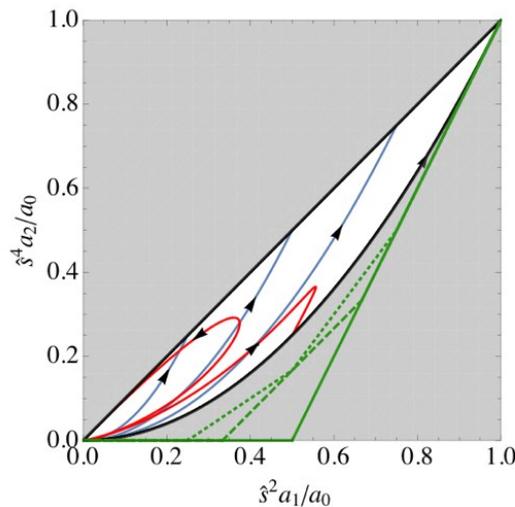
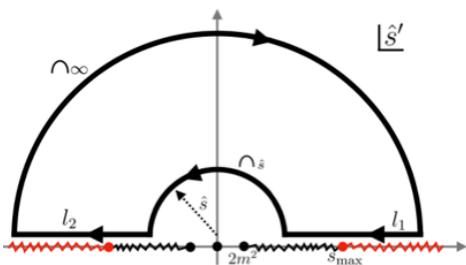


Other applications: EFT

operators/EFT amplitudes

phase space (Grassmannian)
harmonics and EFT positivity

generalize to massive
particles (hard, but useful!)



$$\delta(p_1^2 - m_1^2) \cdots \delta(p_k^2 - m_k^2) \delta^4(P^\mu - \sum_i p_i^\mu)$$

Massive phase space manifold:
Is there a “nice” geometric
formulation?

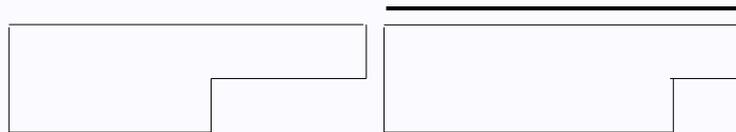
numerous other questions: identical particles (symmeterization); non-renormalization thms;
efficient construction algorithms; amplitudes in $d = 2+1$; ...

THANK YOU!

BACKUP

upshot on Stiefel harmonics

harmonics labeled by Young diagrams
(with at most two rows)



these dictate specific polynomials in the spinors

comments:

- 1) each shape corresponds to operators
- 2) multiple operators belong to same shape
 - a) these involve particles with different spin
- 3) these operators are conformal primaries

Construct states algebraically
e.g.

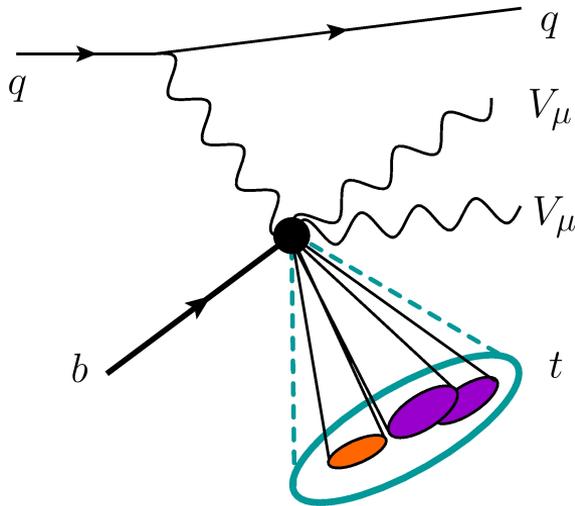
$$|l, \mu = (\mu_1, \dots, \mu_3)\rangle \simeq F^3$$

now apply U(N) lowering op:

$$L_- |l, \mu\rangle \sim |l, \mu'\rangle \simeq \tilde{\psi} F \psi$$

Top Yukawa

HwH: top yukawa



→ boosted top

→ forward jet

$$|\eta_j| > 2.5, p_T^j > 30 \text{ GeV}, E_j > 300 \text{ GeV}$$

→ N_{lep} from vector decays

look for single boosted top + forward jet, then just count leptons

events @ HL-LHC

| Process | 0l | 1l | $l^\pm l^\mp$ | $l^\pm l^\pm$ | 3l(4l) |
|---------------|----------|----------|---------------|---------------|--------|
| $W^\pm W^\mp$ | 3449/567 | 1724/283 | 216/35 | - | - |
| $W^\pm W^\pm$ | 2850/398 | 1425/199 | - | 178/25 | - |
| $W^\pm Z$ | 3860/632 | 965/158 | 273/45 | - | 68/11 |
| ZZ | 2484/364 | - | 351/49 | - | (12/2) |

≥2L: small background

$p_T^t > 250 \text{ GeV} / p_T^t > 500 \text{ GeV}$

Main bkg: $ttjj \rightarrow tW \boxed{bj} j$ large background, but manageable
 $\sim W$