Karlsruhe Institute of Technology

Institute for Theoretical Particle Physics

# Non-factorizable corrections to Higgs production in Vector Boson Fusion

**Higgs 2023, Beijing, 27 November - 02 December 2023**

Ming-Ming Long | In collaboration with Christian Brønnum-Hansen, Kirill Melnikov and Jérémie Juvin-Quarroz

# Outline


**1. Introduction**
- Higgs production
- Vector Boson Fusion

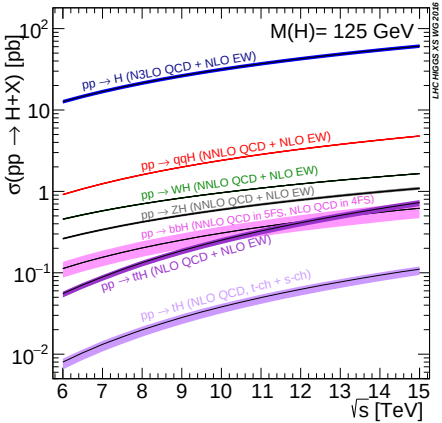
**2. Beyond eikonal**
- One-loop amplitudes
- Two-loop amplitudes

**3. Running coupling effects**
- Fermion-bubble corrections

**4. Summary**

# Higgs production in VBF

- large cross section



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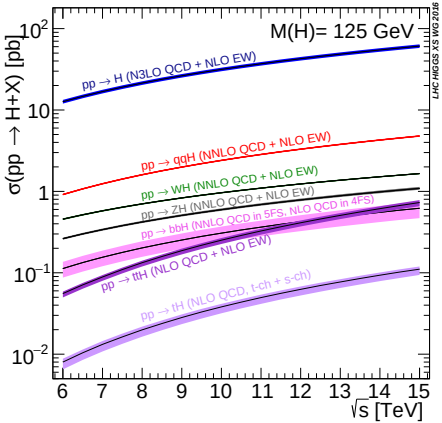


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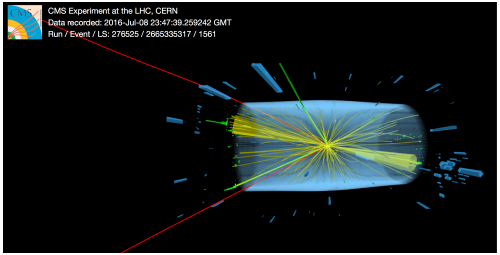

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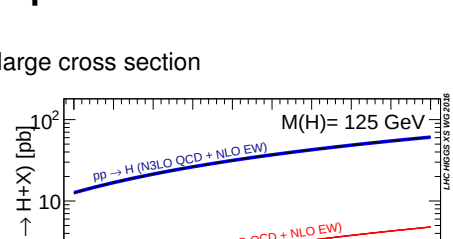


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- HVV (anomalous) couplings; CP properties of Higgs; Higgs decays

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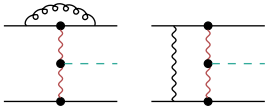

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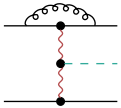

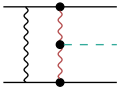

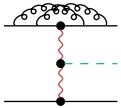

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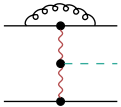

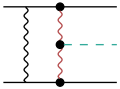

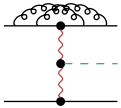

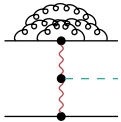

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**N3LO**
- Dreyer, Karlberg 2016

**NF**
- Liu, Melnikov, Penin 2019
- Dreyer, Karlberg, Tancredi 2020
- Gates 2023
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| | $\sigma^{(13\ \mathrm{TeV})}$ [pb] | $\sigma^{(14\ \mathrm{TeV})}$ [pb] | $\sigma^{(100\ \mathrm{TeV})}$ [pb] |
|---|---|---|---|
| LO | $4.099\,^{+0.051}_{-0.067}$ | $4.647\,^{+0.037}_{-0.058}$ | $77.17\,^{+6.45}_{-7.29}$ |
| NLO | $3.970\,^{+0.025}_{-0.023}$ | $4.497\,^{+0.032}_{-0.027}$ | $73.90\,^{+1.73}_{-1.94}$ |
| NNLO | $3.932\,^{+0.015}_{-0.010}$ | $4.452\,^{+0.018}_{-0.012}$ | $72.44\,^{+0.53}_{-0.40}$ |
| N3LO | $3.928\,^{+0.005}_{-0.001}$ | $4.448\,^{+0.006}_{-0.001}$ | $72.34\,^{+0.11}_{-0.02}$ |

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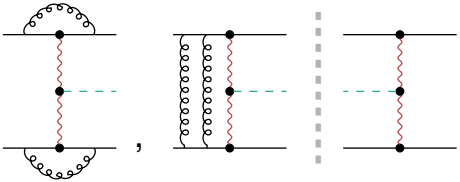

$$\Longrightarrow \frac{(N_c^2-1)^2}{4}, \quad \frac{(N_c^2-1)}{4}$$

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- $\pi^2$ enhancement in non-factorizable contributions [Liu, Melnikov, Penin 2019]

$$\left[\frac{1}{2p_1k+i0}+\frac{1}{-2p_1k+i0}\right]\left[\frac{1}{-2p_2k+i0}+\frac{1}{2p_2k+i0}\right] \Longrightarrow \pi^2$$

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- How to go beyond eikonal approximation?

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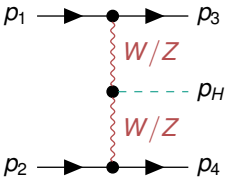
$$q(p_1) + q'(p_2) \to Q(p_3) + Q'(p_4) + H(p_H)$$



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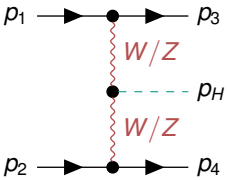


Sudakov decomposition

$$p_i = \alpha_i p_1 + \beta_i p_2 + p_{i,\perp}, \quad i = 3, 4$$

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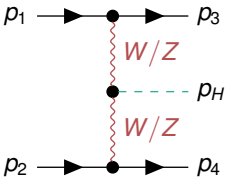


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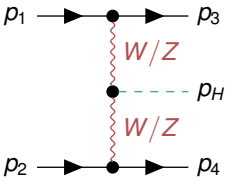


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one has

$$\delta_3\delta_4 \approx \frac{m_H^2 + \mathbf{p}_{H,\perp}^2}{s}, \quad \begin{cases} \delta_3 &= 1 - \alpha_3 \\ \delta_4 &= 1 - \beta_4 \end{cases}$$

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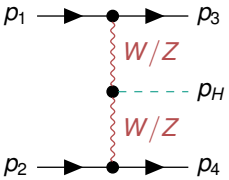


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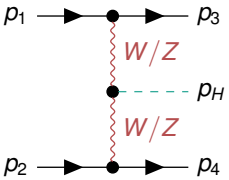


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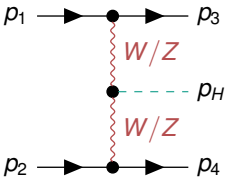


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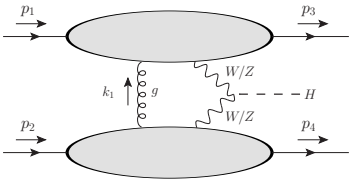
$$\delta_3 \sim \delta_4 \sim \sqrt{\lambda}$$

# One-loop amplitudes




$$\mathcal{M}_1 = g_s^2 g_w^2 g_{VVH} T^a_{i_3 i_1} T^a_{i_4 i_2} \mathcal{A}_1$$

the color-stripped amplitude $\mathcal{A}_1$ reads

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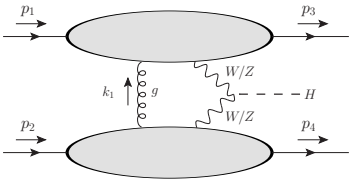


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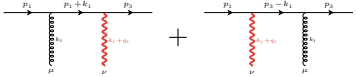

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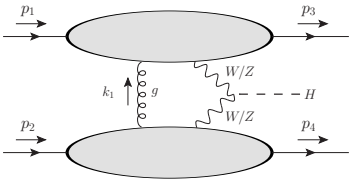


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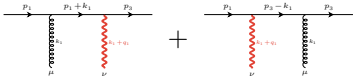


Expansion by regions

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}, \quad \frac{\mathrm{d}^d k_1}{(2\pi)^d} = -\frac{s}{2} \frac{\mathrm{d}\alpha_1}{2\pi i} \frac{\mathrm{d}\beta_1}{2\pi i} \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{d-2}}$$

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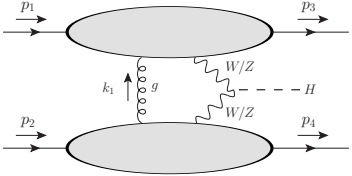


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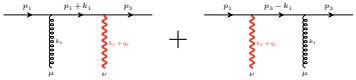


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| | $\alpha_1$ | $\beta_1$ | $\mathbf{k}_{1,\perp}$ | $\mathcal{M}_1$ |
|---|---|---|---|---|
| G | $\lambda$ | $\lambda$ | $\sqrt{\lambda}$ | -2 |
| G-S | $\lambda$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | -2 |
| S | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | -2 |
| C | 1 | $\lambda$ | $\sqrt{\lambda}$ | -3/2 |
| H | 1 | 1 | 1 | 0 |

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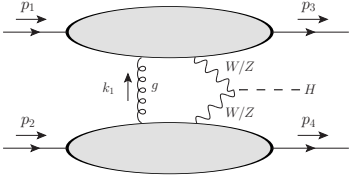


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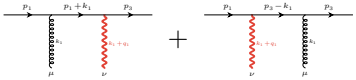
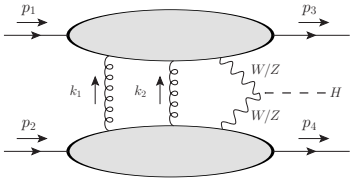



Expansion by regions

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1,\perp}, \quad \frac{\mathrm{d}^d k_1}{(2\pi)^d} = -\frac{s}{2} \frac{\mathrm{d}\alpha_1}{2\pi i} \frac{\mathrm{d}\beta_1}{2\pi i} \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{d-2}}$$

| | $\alpha_1$ | $\beta_1$ | $\mathbf{k}_{1,\perp}$ | $\mathcal{M}_1$ |
|---|---|---|---|---|
| G | $\lambda$ | $\lambda$ | $\sqrt{\lambda}$ | -2 |
| G-S | $\lambda$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | -3/2 |
| S | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | -1 |
| C | 1 | $\lambda$ | $\sqrt{\lambda}$ | 0 |
| H | 1 | 1 | 1 | 0 |

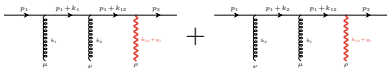
# Two-loop amplitudes




$$\mathcal{M}_2 = -ig_s^4 g_w^2 g_{VVH} \left(\frac{1}{2}\{T^a, T^b\}\right)_{i_3 i_1} \left(\frac{1}{2}\{T^a, T^b\}\right)_{i_4 i_2} \mathcal{A}_2$$

$$\mathcal{A}_2 = \frac{1}{2!} \int \frac{\mathrm{d}^d k_1}{(2\pi)^d} \frac{\mathrm{d}^d k_2}{(2\pi)^d} \frac{1}{d_1 d_2 d_3 d_4} J_{\mu\nu\rho} \tilde{J}^{\mu\nu\rho} \,.$$

with the current $J^{\mu\nu\rho}$

$J^{\mu\nu\rho} =$  $+$ $+$(permu.)

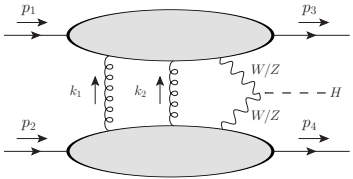
$$k_i = \alpha_i p_1 + \beta_i p_2 + k_{i,\perp},$$

$\alpha_i$: $\lambda$, $\sqrt{\lambda}$, $1$

$\beta_i$: $\lambda$, $\sqrt{\lambda}$, $1$

$k_{i,\perp}$: $\sqrt{\lambda}$, $1$

# Two-loop amplitudes

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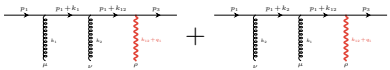
$J^{\mu\nu\rho} =$  $+$ $+$(permu.)

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| $\alpha_i$ | $\beta_i$ | $k_{i,\perp}$ |
|---|---|---|
| $\lambda$ | $\lambda$ | $\sqrt{\lambda}$ |
| $\sqrt{\lambda}$ | $\sqrt{\lambda}$ | 1 |
| 1 | 1 | |

Only the Glauber and mixed regions contribute!

# Factorization of amplitudes

$$\mathcal{M}_1 = i\frac{g_s^2}{4\pi}\, T^a_{i_3 i_1} T^a_{i_4 i_2}\, \mathcal{M}_0\, \mathcal{C}_1, \quad \mathcal{M}_2 = -\frac{1}{2}\frac{g_s^4}{(4\pi)^2}\left(\frac{1}{2}\{T^a, T^b\}\right)_{i_3 i_1}\left(\frac{1}{2}\{T^a, T^b\}\right)_{i_4 i_2}\mathcal{M}_0\, \mathcal{C}_2$$

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The functions $\mathcal{C}_i$ read

$$\mathcal{C}_1 = 2\int \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{1-2\epsilon}}\frac{(\mathbf{p}^2_{3,\perp}+m_V^2)(\mathbf{p}^2_{4,\perp}+m_V^2)}{\Delta_1\Delta_{3,1}\Delta_{4,1}} \times \Omega_1$$

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with

$$\Omega_i = 1 - \delta_3\left(\frac{m_V^2}{\mathbf{p}^2_{3,\perp}+m_V^2}+\frac{m_V^2}{\Delta_{3,i}}\right) - \delta_4\left(\frac{m_V^2}{\mathbf{p}^2_{4,\perp}+m_V^2}+\frac{m_V^2}{\Delta_{4,i}}\right)$$

# Numerical results

$$\mathrm{d}\hat{\sigma}_{\mathrm{nf}}^{\mathrm{NNLO}} = \frac{N_c^2 - 1}{4N_c^2}\, \alpha_s^2\, \mathcal{C}_{\mathrm{nf}}\, \mathrm{d}\hat{\sigma}^{\mathrm{LO}}, \qquad \mathcal{C}_{\mathrm{nf}} = \mathcal{C}_1^2 - \mathcal{C}_2\,,$$

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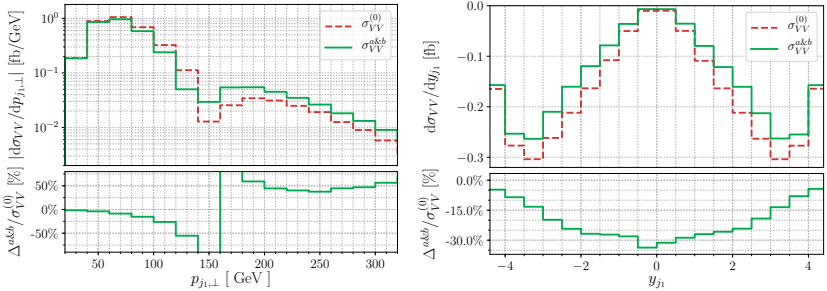

# Scale dependence

- Strong $\mu_R$ dependence

$$\mu_F = \mu_R = \frac{m_H}{2}\left[1 + \frac{4\mathbf{p}_{H,\perp}^2}{m_H^2}\right]^{1/4}$$

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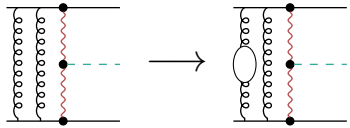
- How to reduce the renormalization scale uncertainties?

- Fermion bubble corrections!



$$\mathcal{C}_{\mathrm{nf}} \to \mathcal{C}_{\mathrm{nf}}(\mu_R)\big|_{N_f \to \beta_0}$$

- It will compensate for the $\mu_R$ dependence of $\alpha_s$.

# Fermion bubble

We only consider the leading eikonal approximation. To include the effects of running $\alpha_s$, replace $\Delta_{1,2}$ in $\mathcal{C}_{1,2}$ [Brodsky, Lepage, Mackenzie 1983]

$$\tilde{\Delta}_i = \Delta_i \left(1 + \frac{\beta_0 \alpha_s}{2\pi} \ln \frac{\mathbf{k}_{i,\perp}^2}{\mu_R^2 e^{5/3}}\right)$$

$$\mathcal{C}_{\rm nf} = 4 \int \frac{\mathrm{d}^2\mathbf{k}_{1,\perp}}{(2\pi)} \frac{\mathrm{d}^2\mathbf{k}_{2,\perp}}{(2\pi)} \frac{\Delta_3\Delta_4}{\tilde{\Delta}_1\tilde{\Delta}_2} \left(\frac{\Delta_3\Delta_4}{\Delta_{3,1}\Delta_{4,1}\Delta_{3,2}\Delta_{4,2}} - \frac{1}{\Delta_{3,12}\Delta_{4,12}}\right)$$

we obtain

$$\mathcal{C}_{\rm nf} = \mathcal{C}_{\rm nf}^{(0)} + \frac{\alpha_s\beta_0}{\pi}\left(\mathcal{C}_{\rm nf}^{(0)} \ln\left(\frac{\mu_R^2 e^{5/3}}{m_V^2}\right) + \mathcal{C}_{\rm nf}^{(1)}\right) + \mathcal{O}(\alpha_s^2\beta_0^2)$$

where

$$C_{\rm nf}^{(0)} = \left(C_1^{(0)}\right)^2 - 2C_1^{(1)}, \qquad C_{\rm nf}^{(1)} = C_1^{(0)}C_1^{(1)} - 3C_1^{(2)} + 2\zeta_3$$

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the auxiliary function

$$C_1(\nu) = -2 \int \frac{\mathrm{d}^2\mathbf{k}_{1,\perp}}{2\pi} \frac{\Delta_3\ \Delta_4\ m_V^{2\nu}}{\Delta_1^{1+\nu} \Delta_{3,1} \Delta_{4,1}}$$

One-dimensional integral representation of $C_1^{(0,1,2)}$

$$C_1^{(0)} = \int_0^1 \mathrm{d}t \, \frac{\Delta_x \Delta_y}{r_{12}^2} \left[ \ln r_2 - 2 \ln r_{12} + \frac{r_2 - r_1}{r_2} \right]$$

$$C_1^{(1)} = \int_0^1 \mathrm{d}t \, \frac{\Delta_x \Delta_y}{r_{12}^2} \left[ \frac{1}{2} \ln^2 r_{12} - \ln r_{12} \left( \frac{r_2 - r_1}{r_2} + \ln \frac{r_2}{r_{12}} \right) \right.$$
$$\left. + 2 \ln \frac{r_2}{r_{12}} + \frac{\pi^2}{6} - \mathrm{Li}_2 \left( \frac{r_1}{r_{12}} \right) \right]$$

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We used

$$\Delta_x = 1 + x$$
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- robust but slow

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$$\left. + 2 \ln \frac{r_2}{r_{12}} + \frac{\pi^2}{6} - \mathrm{Li}_2 \left( \frac{r_1}{r_{12}} \right) \right]$$

$$C_1^{(2)} = \int_0^1 \mathrm{d}t\, \frac{\Delta_x \Delta_y}{r_{12}^2} \left[ -\frac{1}{6} \ln^3 r_{12} + \frac{1}{2} \ln^2 r_{12} \left( \frac{r_2 - r_1}{r_2} + \ln \frac{r_2}{r_{12}} \right) \right.$$
$$+ \frac{\pi^2}{6} \frac{r_2 - r_1}{r_2} + \ln^2 \left( \frac{r_2}{r_{12}} \right) \ln \frac{r_1}{r_{12}} - \ln r_{12} \left( \frac{\pi^2}{6} + 2 \ln \frac{r_2}{r_{12}} - \mathrm{Li}_2 \left( \frac{r_1}{r_{12}} \right) \right)$$
$$\left. - \frac{r_2 - r_1}{r_2} \mathrm{Li}_2 \left( \frac{r_1}{r_{12}} \right) - \ln \frac{r_2}{r_{12}} \left( \frac{\pi^2}{6} - \mathrm{Li}_2 \left( \frac{r_1}{r_{12}} \right) \right) + 2\mathrm{Li}_3 \left( \frac{r_2}{r_{12}} \right) - 2\zeta_3 \right]$$

We used

$$\Delta_x = 1 + x$$
$$\Delta_y = 1 + y$$
$$r_1 = xt + y(1-t) - zt(1-t)$$
$$r_2 = 1 + zt(1-t)$$
$$r_{12} = r_1 + r_2$$

and three dimensionless quantities

$$x = \frac{\mathbf{p}_{3,\perp}^2}{m_V^2},\ y = \frac{\mathbf{p}_{4,\perp}^2}{m_V^2},\ z = \frac{\mathbf{p}_{H,\perp}^2}{m_V^2}$$

- robust but slow
- analytic expressions (?)
  - **YES!**

# Numerical results

For 13 TeV at LHC

$$\sigma_{\rm nf}^{\rm LO} = -2.97^{-0.69}_{+0.52}\ \mathrm{fb}$$

$$\sigma_{\rm nf}^{\rm NLO} = -3.20^{-0.01}_{+0.14}\ \mathrm{fb}$$

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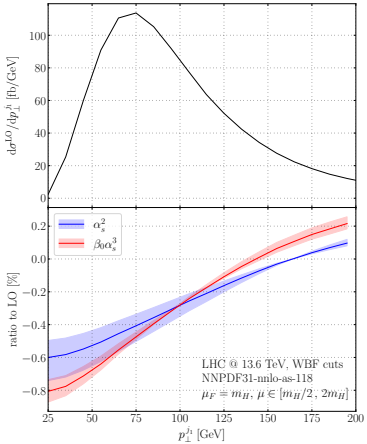


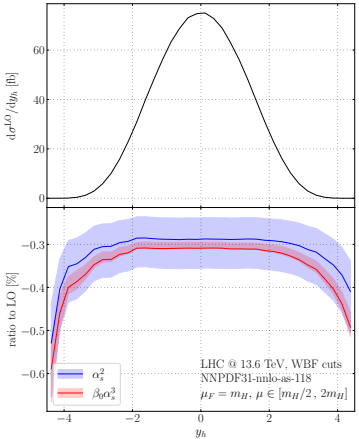

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We have a much better understanding of the NNLO non-factorizable corrections to VBF.

**Thank you for your attention!**

# Setup in Monte Carlo

- PDFs: `NNPDF31-nnlo-as-118`
- VBF cuts

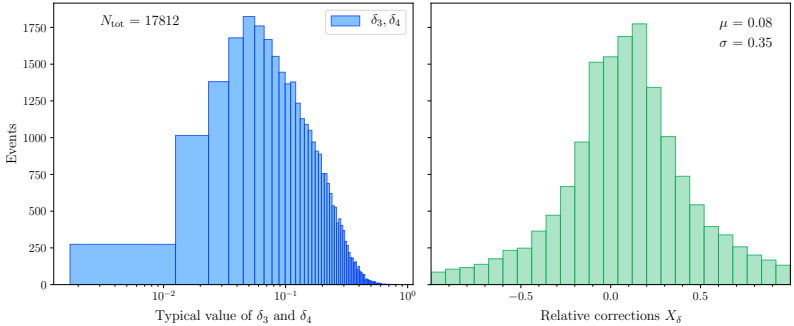
| | |
|---|---|
| anti-$k_t$ | 2 jets, $R = 0.4$ |
| jet transverse momentum | $p_{j,\perp} > 25\ \mathrm{GeV}$ |
| jet rapidity | $\lvert y_j \rvert < 4.5$ |
| jet separation | $\lvert y_{j_1} - y_{j_2} \rvert > 4.5$ |
| invariant mass of jets | $M_{jj} > 600\ \mathrm{GeV}$ |
| separate hemispheres | $y_{j_1} y_{j_2} < 0$ |

Backup slides

# Some checks

- reproduce the leading eikonal approximation [Liu, Melnikov, Penin 2019]
- $\delta_{3,4}$ distribution and one-loop check

$$X_\delta = \frac{\mathcal{A}_1 - \mathcal{A}_1^{\text{G\&G-S}}}{\mathcal{A}_1^{\text{G\&G-S}} - \mathcal{A}_1^{\text{G\&G-S}}\big|_{\delta_{3,4}\to 0}}$$

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# Glauber and mixed regions at one loop

Factorization of integrations:

$$\mathcal{A}_1^{\text{G\&G-S}} = -\langle 3|\gamma^\mu|1]\langle 4|\gamma_\mu|2] \int \frac{\mathrm{d}^{d-2}\mathbf{k}_{1,\perp}}{(2\pi)^{d-2}} \frac{1}{\Delta_1 \Delta_{3,1} \Delta_{4,1}} \times \Phi \times \tilde{\Phi}$$

with

$$\Phi = \int \frac{\mathrm{d}\beta_1}{2\pi i} \frac{\Delta_{3,1}}{s\delta_3(\beta_1 - \beta_3) + \Delta_{3,1} + i0} \left[ \frac{1}{\beta_1 + \frac{\Delta_1}{s} + i0} + \frac{1}{-\beta_1 + \frac{\Theta_{3,1}}{s\alpha_3} + i0} \right],$$

$$\tilde{\Phi} = \int \frac{\mathrm{d}\alpha_1}{2\pi i} \frac{\Delta_{4,1}}{-s\delta_4(\alpha_1 + \alpha_4) + \Delta_{4,1} + i0} \left[ \frac{1}{-\alpha_1 + \frac{\Delta_1}{s} + i0} + \frac{1}{\alpha_1 + \frac{\Theta_{4,1}}{s\beta_4} + i0} \right]$$

and

$$\Delta_i = -\mathbf{k}_{i,\perp}^2 \,, \quad \Delta_{3,i} = -(\mathbf{k}_{i,\perp} - \mathbf{p}_{3,\perp})^2 - m_V^2 \,, \quad \Delta_{4,i} = -(\mathbf{k}_{i,\perp} + \mathbf{p}_{4,\perp})^2 - m_V^2 \,,$$

$$\Theta_{3,i} = -\left(\mathbf{k}_{i,\perp}^2 - 2\mathbf{k}_{i,\perp} \cdot \mathbf{p}_{3,\perp}\right) \,, \quad \Theta_{4,i} = -\left(\mathbf{k}_{i,\perp}^2 + 2\mathbf{k}_{i,\perp} \cdot \mathbf{p}_{4,\perp}\right)$$

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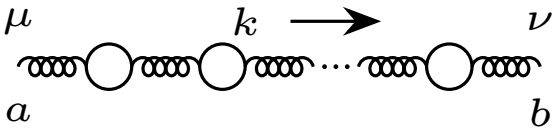
# Dressed gluon propagator




$$G^{\mu\nu}_{ab} = \frac{-ig^{\mu\alpha_1}\delta_{aa_1}}{k^2} Q^{a_1 b_1}_{1,\alpha_1\beta_1} \frac{-ig^{\beta_1\alpha_2}\delta_{b_1 a_2}}{k^2} Q^{a_2 b_2}_{2,\alpha_2\beta_2} \frac{-ig^{\beta_2\alpha_3}\delta_{b_2 a_3}}{k^2} \cdots \frac{-ig^{\beta_{n-1}\alpha_n}\delta_{b_{n-1} a_n}}{k^2} Q^{a_n b_n}_{n,\alpha_n\beta_n} \frac{-ig^{\beta_n\nu}\delta_{b_n b}}{k^2}$$

where the $j$-th bubble reads explicitly

$$Q^{a_j b_j}_{j,\alpha_j\beta_j} = (-1)\, T_f \delta^{a_j b_j} g_s^2 \mu_R^{2\epsilon} \int \frac{d^d l_j}{(2\pi)^d} \frac{\mathrm{Tr}\left[\gamma_{\alpha_j}(\hat{k}+\hat{l}_j)\gamma_{\beta_j}\hat{l}_j\right]}{(k+l_j)^2 l_j^2}, \quad T_f = N_f T_r, \quad T_r = \frac{1}{2}$$

we can get

$$G^{\mu\nu}_{ab} = \frac{-i}{k^2} g^{\mu\nu}\delta_{ab}(-1)^n \left\{ \frac{T_f \alpha_s(\mu_R^2)}{3\pi} \left[ \log\left(\frac{\mu_R^2}{-k^2}\right) + \frac{5}{3} + \mathcal{O}(\epsilon) \right] \right\}^n, \qquad T_f \to -\frac{3}{2}\beta_0$$

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