High-precision Drell-Yan Production at N^3LO in QCD

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2107.09085 [Phys.Rev.Lett. 128 (2022)] and 2205.11426

December 21, 2022
SPeCial4Young: SYSU-PKU Collider Physics forum
For Young Scientists

Three 'dark clouds' in particle physics since 2021

- 3.1 σ deviation with SM for lepton universality in beauty-quark decays

Test of lepton universality in beauty-quark decays #1

LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Mar 22, 2021)

Published in: *Nature Phys.* 18 (2022) 3, 277-282 • e-Print: 2103.11769 [hep-ex]

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- 4.2 σ deviation with SM for $g-2$ of μ^+ particle

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm #1

Muon g-2 Collaboration • B. Abi (Oxford U.) et al. (Apr 7, 2021)

Published in: *Phys.Rev.Lett.* 126 (2021) 14, 141801 • e-Print: 2104.03281 [hep-ex]

pdf links DOI cite claim reference search 1,112 citations

- 7 σ deviation with SM for M_W measurement with the CDF II detector

High-precision measurement of the W boson mass with the CDF II detector #1

CDF Collaboration • T. Aaltonen (Helsinki U. and Helsinki Inst. of Phys.) et al. (Apr 8, 2022)

Published in: *Science* 376 (2022) 6589, 170-176

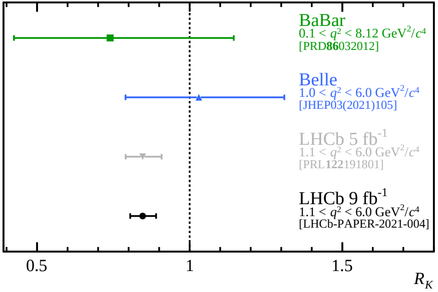
pdf links DOI cite claim reference search 272 citations

Precision measurements and predictions become a key to reveal new physics

Precision measurements and predictions for derived quantities in SM

- With inputs of fundamental parameters, SM can directly predict derived quantities.
- Precision measurements and predictions don't depend on each other

Example, R_K measurments in beauty-quark decays



$$R_H = \frac{\int_{q^2_{\min}}^{q^2_{\max}} \frac{d\mathcal{B}(B\to H\mu^+\mu^-)}{dq^2} dq^2}{\int_{q^2_{\min}}^{q^2_{\max}} \frac{d\mathcal{B}(B\to He^+e^-)}{dq^2} dq^2}$$

Nature Phys. 18 (2022) 3, 277-282

Precision predictions for fundamental parameters in SM

- SM can not directly predict fundamental parameters like M_Z or M_W.
- Predictions in SM for them are based on electroweak fits.
- For example, to predict M_W from electroweak fit, it relys on the relation

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_\mu}\left(1+\Delta r(M_t,\,M_H,\cdots)\right)$$

  as well as other precisely-measured parameters.
- Δr encodes higher order corrections from SM

Precision measurements of fundamental parameters

For processes without neutrino final states, like M_Z measurement

- e^+e^- colliders
  - Control e^+e^- beam energy to produce nearly on-shell Z bosons
- Hadron colliders
  - Reconstruct M_Z from the final state dilepton system

For processes with neutrino final states, like M_W measurement

- e^+e^- colliders
  - Control e^+e^- beam energy to produce a pair of nearly on-shell W^+W^- bosons
- Hadron colliders
  - Template fits to some observable distributions (the only method)

Precision measurement of M_W in hadron colliders

Template fits procedures:

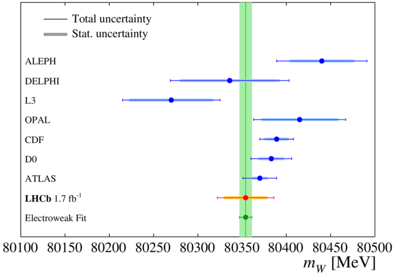
- Measure relavant distributions (data), for example

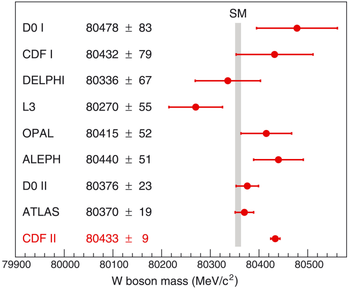
$$p_T^{l/\nu}, m_T^W = \sqrt{2 p_T^l p_T^\nu (1 - \cos\Delta\phi)}$$

- Generate several theoretical templates with different M_W (theory)
- The measured M_W corresponds to the template that best fits to the data

Measured M_W are affected by experimental and theoretical uncertainties

- ATLAS: Powheg + Pythia8 + DYNNLO
- LHCb: Powheg + Pythia8 + DYTurbo
- CDFII: ResBos (NNLL+NLO) + Photos





ATLAS, LHCb, CMS all have on-going measurement of W mass

More in the template used by CDFII

Default template: ResBos [Balazs, Landry, Brock, Nadolsky, Yuan, 97, 2003]

- ResBos uses the CSS resummation formula:

$$\frac{d\sigma}{dQ^2 d^2\vec{p_T} dy} = \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} e^{-S(b)} C \otimes f_1 C \otimes f_2 + Y$$

- Non-perturbative contributions from PDFs and BLNY form S_{NP}

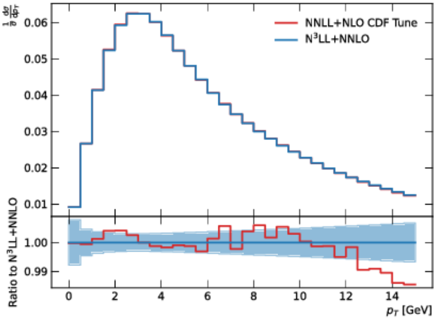
$$S(b) = S_{\text{NP}} S_{\text{Pert}}, S_{\text{NP}} = \Big[-g_1 - g_2 \ln\big(\frac{Q}{2Q_0}\big) - g_1 g_3 \ln(100 x_1 x_2) \Big] b^2$$

CDFII template generation

- Use BLNY fitted values[03] for g_1, g_3
- Modify g_2 by fitting to the p_T^Z data
- α_s tuning to Z data
- Use the obtained $g_1 \cdots g_3, \alpha_s$ to generate templates for $p_T^{l/\nu}$ and m_T^W

ResBos2 (N3LL+NNLO) [Isaacson Ph.D. thesis, 17] is more precise than ResBos

- Mimic the procedure by CDF, perform a pseudoexperiment
- pseudodata: N3LL+NNLO



We determine that the data-driven techniques used by CDF capture most of the higher order corrections, and using higher order corrections would result in a decrease in the value reported by CDF by at most 10 MeV.[Isaacson, Yao, Yuan, 22]

Theoretical predictions can be further improved by including N3LL′ resummation, NNLO mixed QCD-EW and N3LO QCD corrections

QCD factorization

- Parton model

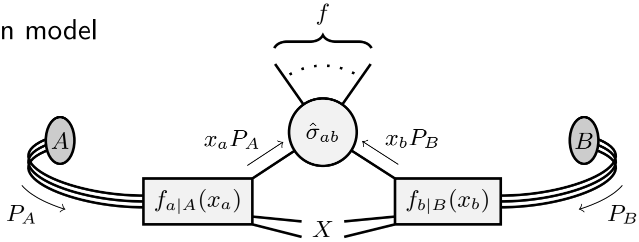


Figure by A. Huss

$$\sigma_{AB} = \sum_{ab} \int_0^1 dx_a \int_0^1 dx_b \, f_{a|A}(x_a) \, f_{b|B}(x_b) \hat{\sigma}_{ab}(x_a, x_b) \big(1 + \mathcal{O}(\Lambda_{\mathrm{QCD}}/Q)\big)$$

$$\hat{\sigma}_{ab} = \hat{\sigma}^{(0)}_{ab} + \alpha_S/(4\pi)\hat{\sigma}^{(1)}_{ab} + (\alpha_S/(4\pi))^2\hat{\sigma}^{(2)}_{ab} + (\alpha_S/(4\pi))^3\hat{\sigma}^{(3)}_{ab} + \mathcal{O}(\alpha_S^4)$$

- $f(x)$ is parton distribution function, fit from experimental data, few %
- $\hat{\sigma}_{ab}$ is partonic cross section, perturbatively calculable, aims at few %

State of the art precision for DY in fixed order calculations

In pure QCD

- NNLO cross section[R. Hamberg et al., 91; R.V. Harlander and W. B. Kilgore, 02]
- Analytic NNLO rapidity distribution[C. Anastasiou, L. J. Dixon, K. Melnikov, and F. Petriello, 03, 04]
- NNLO fully differential distributions implemented in DYNNLO, FEWZ and MATRIX[K. Melnikov and F. Petriello, 06,06; S. Catani, L. Cieri, G. Ferrera, D. Florian and M. Grazzini, 09, 10; R. Gavin, Y.Li, F. Petriello, and S. Quackenbush, 11]
- Analytic N^{3}LO cross section for $\gamma^* + Z$ production and also W production [C. Duhr, F. Dulat, and B. Mistlberger, 20, 20, 21]
- This talk: N^{3}LO precision predictions for γ^* and W productions
- N^{3}LO fiducial predictions for Drell-Yan at the LHC [X. Chen, , T. Gehrmann, E.W.N. Glover, A. Huss, P. Monni, E. Re, L. Rottoli, P. Torrielli, 22]

In mixed QCD-EW

- NNLO mixed QCD-EW $\alpha_S\,\alpha_{\text{EW}}$[S. Dittmaier, A. Huss and C. Schwinn, 16; A. Behring, F. Buccioni, F. Caola, M. Delto, M. Jaquier, K. Melnikov, and R. Rontsch, 20,21; L. Buonocore, M. Grazzini, S. Kallweit, C. Savoini, and F. Tramontano, 21; R. Bonciani, L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, and A. Vicini, 21; F. Buccioni et al, 22]

Q_T subtraction at N^{3}LO

- A direct generalization of Q_T subtraction at NNLO [S. Catani and M. Grazzini, 07]

$$\frac{d^2\sigma_V^{(3)}}{dQ^2dy} = \int_0^{q_T^{\rm cut}} dq_T \frac{d^3\sigma_V^{(3)}}{dq_T dQ^2 dy} + \int_{q_T^{\rm cut}} dq_T \frac{d^3\sigma_{V+J}^{(2)}}{dq_T dQ^2 dy}$$

- Since $q_T^{\text{cut}} > 0$, for the above q_T^{cut} part

$$V_{\text{NLO}} \to (V+J)_{\text{LO}} \cdots , V_{\text{N}^3\text{LO}} \to (V+J)_{\text{NNLO}}$$

- The above q_T^{cut} part reduces to an NNLO calculation of $V+J$ [A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, A. Huss, and T. A. Morgan, 16; R. Boughezal, J. M. Campbell, R. K. Ellis, C. Focke, W. T. Giele, X. Liu, and F. Petriello, 16, 16]
- We use the event generator NNLOJET to compute the above q_T^{cut} part
- The below q_T^{cut} part can be approximated using the LP TMD factorization

Transverse-momentum-dependent (TMD) factorization

- TMD factorization at leading power (LP) in SCET

$$\frac{d^4\sigma}{dQ^2 d^2\boldsymbol{q}_T dy} = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{-i\boldsymbol{q}_T\cdot\boldsymbol{b}} \sum_q \frac{\sigma_{\rm LO}^V}{E_{\rm CM}^2} \Big[\overbrace{\sum_k \int_{x_1}^1 \frac{dz_1}{z_1} \mathcal{I}_{qk}(z_1,\boldsymbol{b}) f_{k/h_1}(x_1/z_1)}^{B_q}$$

$$\times \sum_j \int_{x_2}^1 \frac{dz_2}{x_2} \mathcal{I}_{\bar{q}_1 j}(z_2,\boldsymbol{b}) f_{j/h_2}(x_2/z_2) \mathcal{S}(\boldsymbol{b}) + (q \leftrightarrow \bar{q}_1) \Big] H_{q\bar{q}_1} \Big(1 + \mathcal{O}(q_T^2/Q^2)\Big)$$

$$x_1 = \sqrt{\tau} e^y, x_2 = \sqrt{\tau} e^{-y}, \tau = (q_T^2 + Q^2)/E_{\rm CM}^2$$

- All ingredients are known to three loops.
- Hard function $H_{q\bar{q}_1}$[P. A. Baikov, K. G. Chetyrkin, A. V. Smirnov, V. A. Smirnov, and M. Steinhauser, 09; R. N. Lee, A. V. Smirnov, and V. A. Smirnov, 10; T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli, and C. Studerus, 10]
- TMD perturbative Soft function $\mathcal{S}(\boldsymbol{b})$[Y. Li and H. X. Zhu, 16]
- Matching kernel $\mathcal{I}_{qi}$[M.-x. Luo, **TZY**, H. X. Zhu, and Y. J. Zhu, 19, 20; M. A. Ebert, B. Mistlberger, and G. Vita, 20]

Above q_T^{cut} part, antenna subtraction at NNLO

A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann, J. Currie, and E. W. N. Glover, 05, 07, 13

Structures of NNLO cross sections

$$
\begin{aligned}
d\sigma^{(2)}_{V+J} = & \int_{d\Phi_{V+J+2J}} \left(d\sigma^{\text{RR}}_{\text{NNLO}} - d\sigma^{S}_{\text{NNLO}}\right) \\
& + \int_{d\Phi_{V+J+J}} \left(d\sigma^{\text{RV}}_{\text{NNLO}} - d\sigma^{T}_{\text{NNLO}}\right) \\
& + \int_{d\Phi_{V+J}} \left(d\sigma^{\text{VV}}_{\text{NNLO}} - d\sigma^{U}_{\text{NNLO}}\right) \\
& + \left[\int_{d\Phi_{V+J+2J}} d\sigma^{S}_{\text{NNLO}} + \int_{d\Phi_{V+J+J}} d\sigma^{T}_{\text{NNLO}} + \int_{d\Phi_{V+J}} d\sigma^{U}_{\text{NNLO}}\right]
\end{aligned}
$$

- Carefully design proper subtraction terms σ^S, σ^T, σ^U
- The first three lines suitable for numerical evaluation at $D = 4$
- Calculate the last line analytically for factorized phase spaces

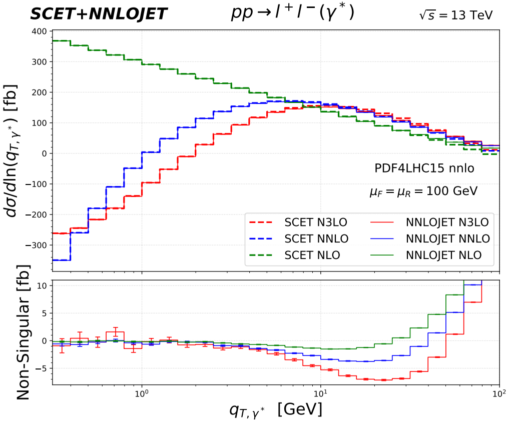
Total cross section and rapidity distributions at LHC

Computational setup

- No fiducial cuts on the decay products
- G_μ EW scheme: values taken from 2020 PDG:
  $M_Z = 91.1876$ GeV, $\Gamma_Z =$2.4952 GeV, $M_W = 80.379$ GeV,
  $\Gamma_W = 2.085$ GeV, $G_F = 1.1663787 \times 10^{-5}$
- $m_e = m_\mu = 0, \alpha_s(m_Z) = 0.118$
- $Q = 100$ GeV for γ^* production (mainly for validation)
- $Q \in [0, \infty]$ for $W^\pm$
- Uncertainties from seven-point scale variations with central scale $\mu_r = \mu_f = Q$
- Unit CKM matrix for LHC process
- Use central values of PDF sets, not including PDF uncertainties

Self-check within q_T subtraction method: q_T distribution

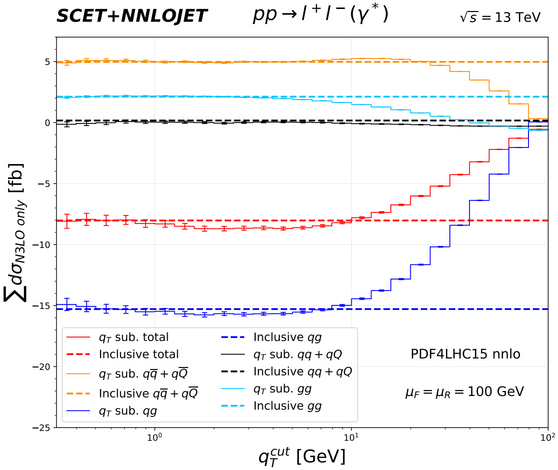
$$Q = 100\text{GeV},\ (d\sigma_V/dq_T)_{\text{N}^3\text{LO}} = Q/q_T\left[\sum_{i=0}^{5} A_i \ln^i(q_T/Q) + \mathcal{O}(q_T/Q)\right]$$



Non-singular contribution tends to zero in the limit of $q_T \to 0$

Validation for γ^*: cross check to the analytic cross section

- Check by different partonic channels, agree well with the analytic results [DDM, 20]
- Large cancellations among different partonic channels



Validation for γ^*: cross check to the analytic cross section

- Large logarithms as a function of q_T^{cut}

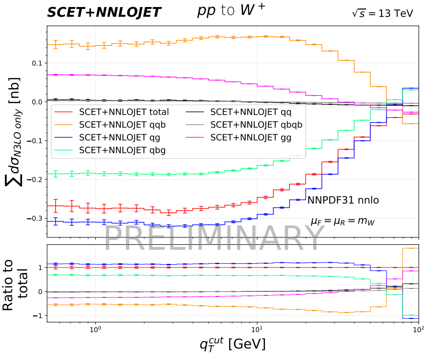
$$\int_0^{q_T^{\text{cut}}} dq_T (d\sigma_V/dq_T) = \sum_{i=0}^{6} B_i \ln^i(q_T^{\text{cut}}/Q) + \mathcal{O}((q_T^{\text{cut}}/Q)^2)$$

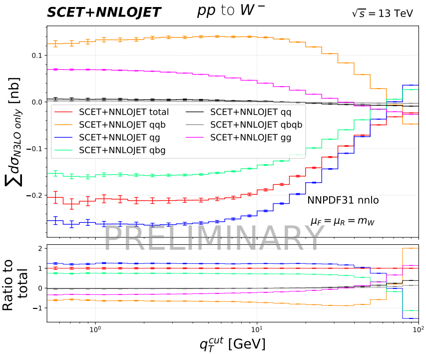
- Total cross section with N^3LO only

q_T^{cut}(GeV)	SCET(fb)	NNLOJET(fb)	combined(fb)	analytic(fb)
0.5	177.37(10)	−185.40(41)	−8.02(42)	-8.03
0.63	169.12(8)	−177.08(35)	−7.96(36)	[DDM]

- Large cancellations between SCET and NNLOJET
- Uncertainties are enhanced by a factor of 20,
  0.25% accuracy $\rightarrow$ 5% fluctuation
- Non-trivial cancellations precisely reproduce the analytic results
- The situation can be improved by including sub-leading power correction

Validation for W production: calculate the cross section

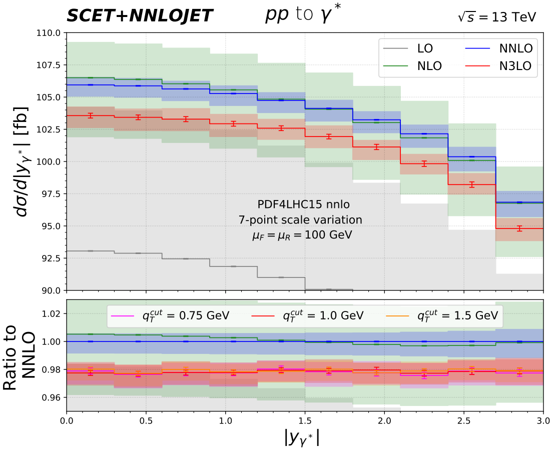




- $0 \leq Q \leq 13\text{TeV}$
- Reaching a plateau when going to a small q_T^{cut}
- Large cancellations among different partonic channels

Rapidity distribution of γ^* production at N^3LO

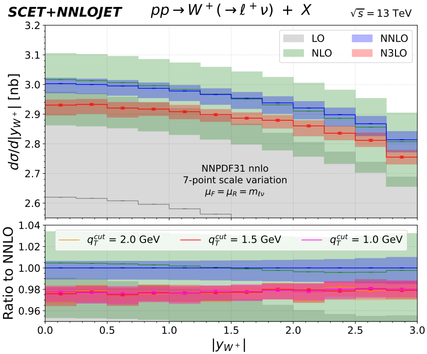
- $Q = 100$ GeV
- q_T^{cut} dependence is smaller than the numerical error
- Large corrections from NNLO to N3LO
- $K_{N^3LO/NNLO} \simeq 0.98$

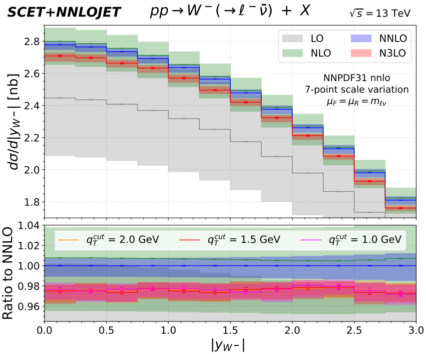


The scale band at N^3LO is outside of the band at NNLO

Due to the missing N^3LO PDF?

Rapidity distribution of W production at N^{3}LO



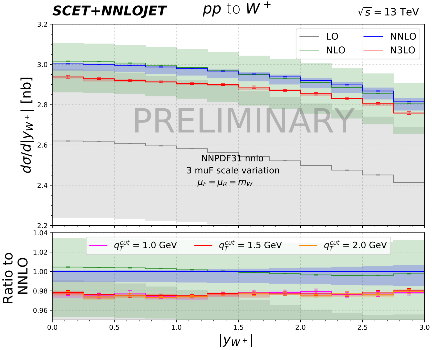


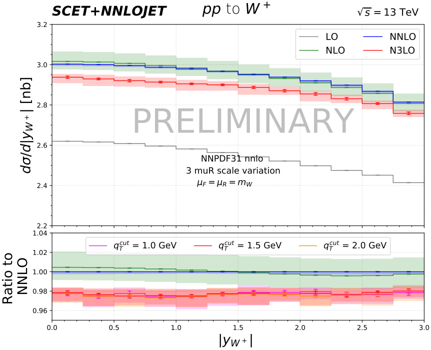
- LO: $u\bar{d} \to W^+$
- $0 \leq Q \leq 13$TeV
- $K_{\text{N}^3\text{LO/NNLO}} \simeq 0.98$

- LO: $d\bar{u} \to W^-$
- $0 \leq Q \leq 13$TeV
- $K_{\text{N}^3\text{LO/NNLO}} \simeq 0.98$

Since $f_u > f_d$, the W^+ bosons tend to be produced at larger $|y|$ compared to W^-.

Explore scale uncertainty for W^+ production at N^{3}LO





- Fixing $\mu_R = Q$, varying μ_F
- μ_F uncertainty reduces

- Fixing $\mu_F = Q$, varying μ_R
- μ_R uncertainty is enhanced from NNLO to N^{3}LO
- Accidentally small uncertainty at NNLO due to large cancellations among different partonic channels

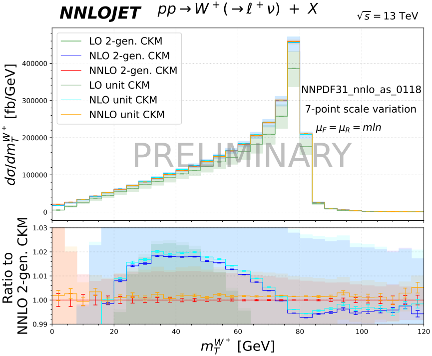
Transverse mass distribution at LHC and Tevatron

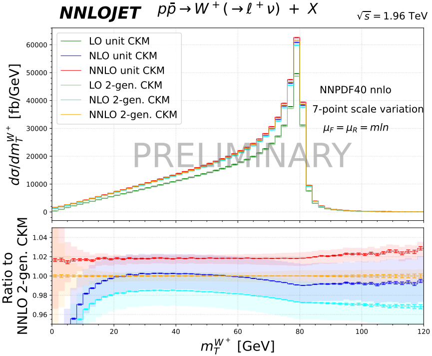
Computational setup

- Inclusive setup: same as rapidity distribution
- CDFII fiducial cuts: $\sqrt{s} = 1.96\,\text{TeV}$, $p_T^W < 15\,\text{GeV}, |\eta_l| < 1$, $p_T^{l/\nu} \in [30, 55]\,\text{GeV}$, $m_T^W \in [60, 100]\,\text{GeV}$
- LHC: unit CKM matrix
- Tevatron: 2-generation quark mixing (Cabbibo mixing)

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.9737 & 0.2245 & 0 \\ 0.221 & 0.987 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Unit CKM vs 2-generation CKM

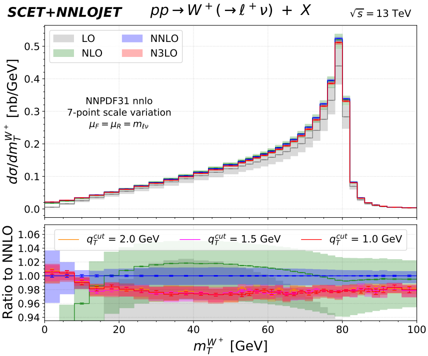




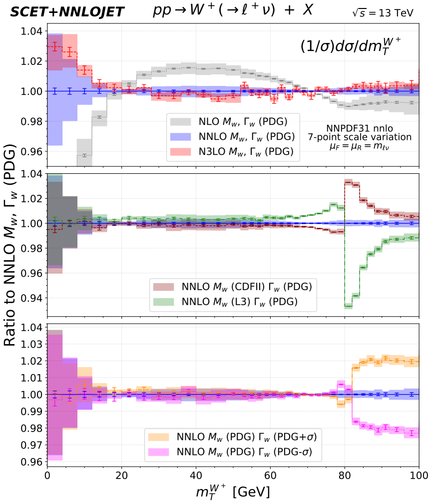
LHC: 0.2% difference

Tevatron: 2% difference

Transverse mass distribution at LHC

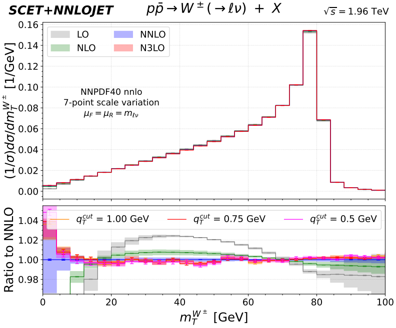


$K_{\mathrm{N3LO/NNLO}} \simeq 0.98$, mostly M_T^W-independent

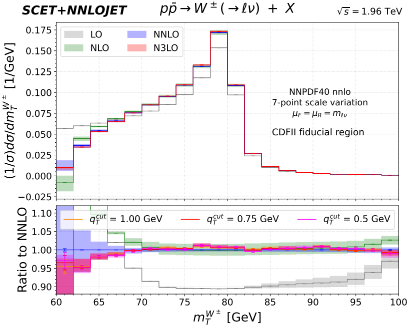


- Small correction for normalized distribution
- NLO-based template fits tend to produce a slightly larger M_W to compenstate missing NNLO corrections

Normalized transverse mass distribution at Tevatron

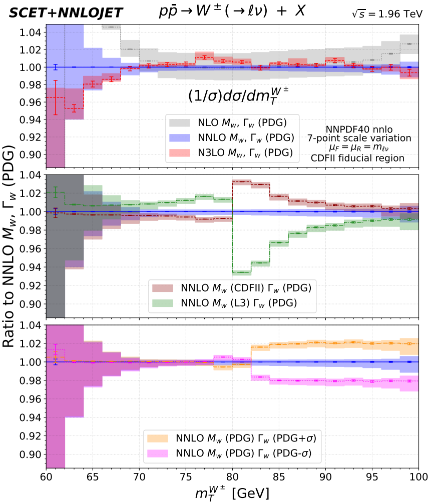


Inclusive setup (bin = 5 GeV)



CDFII fiducial region (bin = 2 GeV)

The sensitivity on M_W to the peak of m_T^W distribution



- The peak region is sensentive on M_W
- Γ_W only affects the distribution above the peak region
- Estimate ΔM_W due to N3LO corrections by the first bin of peak region
- M_W (CDFII) - M_W (PDG)= 80433- 80379 =54 MeV
- NNLO M_W (CDFII) - NNLO M_W (PDG) = 3.25%
- N3LO M_W (PDG) - NNLO M_W (PDG) = 0.6%
- $\Delta M_W(\delta\text{N3LO}) = -54\frac{0.6}{3.25} \simeq -10$ MeV

Summary

- We get the first rapidity and transverse mass distribution for DY at N^3LO in QCD
- The N3LO correction is relevant to a precision of 10 MeV for M_W measurement
- The computation is based on Q_T subtraction generalized to N^3LO
  - The below $q_T^{\rm cut}$ part is computed using the LP TMD factorization
  - The above $q_T^{\rm cut}$ part is computed using NNLOJET
- For validation, we independently recover the known inclusive cross section
- Including the Q_T sub-leading power corrections in the future can further improve the precision and computational efficiency

Thank you!

Backup

Definition of TMD beam function

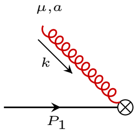
- TMD beam function

$$\mathcal{B}_{q/N}(z, \boldsymbol{b}) = \int \frac{db_-}{2\pi}\, e^{-izb_-\bar{n}\cdot P} \langle N(P)|\bar{\chi}_n(0, b_-, \boldsymbol{b}) \frac{\bar{\not{n}}}{2} \chi_n(0)|N(P)\rangle$$

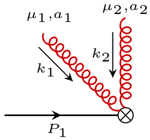
- Collinear Wilson line

$$\chi_n = W_n^\dagger \xi_n \,,\, W_n^\dagger(x) = \overline{\mathcal{P}} \exp\left(-ig_s \int_{-\infty}^{0} ds\bar{n}\cdot A(x+s\bar{n})\right)$$

- Need standard QCD as well as effective eikonal Feynman rules



$\rightarrow \; g_s \, \frac{\bar{n}^\mu}{\bar{n}\cdot k + i0} \, t^a$



$\rightarrow \; \frac{g_s^2 \, \bar{n}^{\mu_1} \, \bar{n}^{\mu_2}}{\bar{n}\cdot k_1 + \bar{n}\cdot k_2 + i0} \sum_{P_{\{1,2\}}} \left[\frac{t^{a_1}\, t^{a_2}}{\bar{n}\cdot k_1 + i0} \right]$

Analytic computation of TMD beam function at N^{3}LO

M.-x. Luo, TZY, H. X. Zhu, Y. J. Zhu [P.R.L. 124 (2020) 9, 092001, JHEP 06 (2021) 115]

- Rapidity divergence and rapidity regulator τ
- Non-standard Feynman propagators and IBP identities

$$0 = \int d^d q \, \frac{\partial}{\partial q^\mu}\left[e^{-b_0 \tau \frac{P\cdot K}{\bar{n}\cdot P}} F(\{\tilde{l}\}) \right]$$

$$= \begin{cases} \int d^d q \, e^{-b_0\tau \frac{P\cdot K}{\bar{n}\cdot P}} \left[-b_0 \tau \frac{P_\mu}{\bar{n}\cdot P} + \frac{\partial}{\partial q^\mu} \right] F(\{\tilde{l}\}) \,, & q = K \,, \\ \int d^d q \, e^{-b_0\tau \frac{P\cdot K}{\bar{n}\cdot P}} \frac{\partial}{\partial q^\mu} F(\{\tilde{l}\}) & q \neq K \,, \end{cases}$$

- Expansion of differential equation (DE) in the limit $\tau \to 0$

$$f_i(z,\tau,\epsilon) \overset{\tau\to 0}{=} \sum_j \sum_n \sum_{k=0} g_i^{(j,n,k)}(z,\epsilon) \tau^{j+n\epsilon} \ln^k \tau$$

Single variable DE with momentum fraction z: $d\vec{g}/(dz) = \boldsymbol{A}(z,\epsilon)\vec{g}$

- The final results are in terms of the well-understood harmonic polylogrithms (HPLs)

Computation parameters for γ^* production

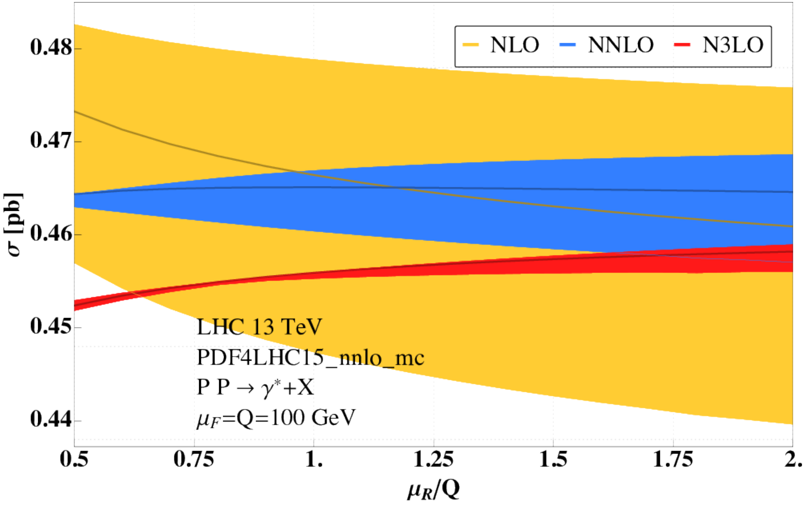
- Focus on the off-shell photon case
- Center of mass energy $\sqrt{s} = 13\text{TeV}$
- Fix invariant mass $Q = 100$ GeV and without including fiducial cuts
- PDF set: PDF4LHC15_nnlo_mc with central member
- Fixed α_{QED} value: $\alpha_{\text{QED}}(0) = 1/137.035999139$
- $\alpha_S(m_Z) = 0.118$ with scale variation values calculated from LHAPDF
- Seven-point scale variations

$$\mu_r = 0.5Q\,, \quad \mu_f = 0.5Q\,, 1.0Q$$
$$\mu_r = 1.0Q\,, \quad \mu_f = 0.5Q\,, 1.0Q\,, 2.0Q$$
$$\mu_r = 2.0Q\,, \quad \mu_f = 1.0Q\,, 2.0Q$$

- Parameters are identical to the DDM study[C. Duhr, F. Dulat, and B. Mistlberger, 2020]

Explore the uncertainties from scale variation

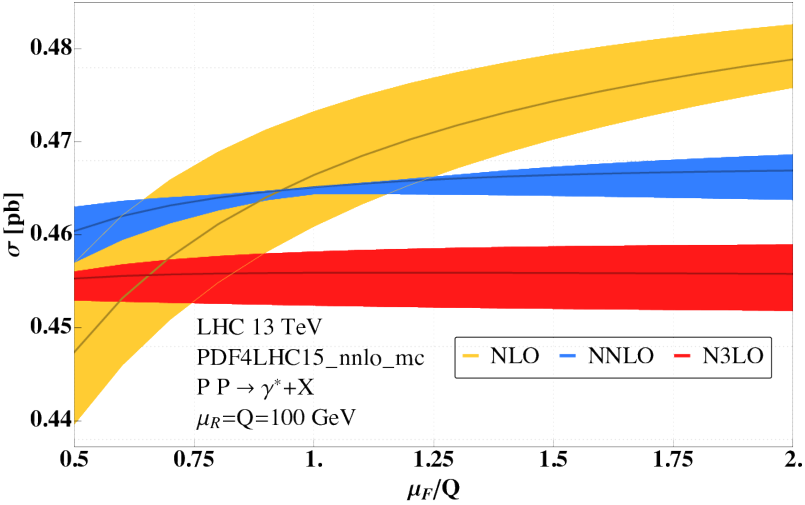
- Fix μ_F, vary the μ_R from $0.5Q$ to $2Q$, the scale uncertainty reduces



[C. Duhr, F. Dulat, and B. Mistlberger, 2020]

Explore the uncertainties from scale variation

- Fix μ_R, vary the μ_F from $0.5Q$ to $2Q$, the scale band becomes wider



[C. Duhr, F. Dulat, and B. Mistlberger, 2020]

Validation for γ^*: cross check to the analytic cross section

- Large logarithms as a function of q_T^{cut}

$$\int_0^{q_T^{\text{cut}}} dq_T (d\sigma_{\gamma^*}/dq_T) = \sum_{i=0}^{6} B_i \ln^i (q_T^{\text{cut}}/Q) + \mathcal{O}((q_T^{\text{cut}}/Q)^2)$$

- Total cross section with N^{3}LO only

q_T^{cut}(GeV)	SCET(fb)	NNLOJET(fb)	combined(fb)	analytic(fb)
0.5	177.37(10)	−185.40(41)	−8.02(42)	
0.63	169.12(8)	−177.08(35)	−7.96(36)	-8.03
0.79	152.96(7)	−161.28(30)	−8.31(31)	[DDM]
1	132.15(6)	−140.47(26)	−8.32(27)	

- Large cancellations between SCET and NNLOJET
- Uncertainties are enhanced by a factor of 20,
  0.25% accuracy $\rightarrow$ 5% fluctuation
- Non-trivial cancellations precisely reproduce the analytic results

Contributions from partonic channels at different orders

Use the same parameters as for the main slides

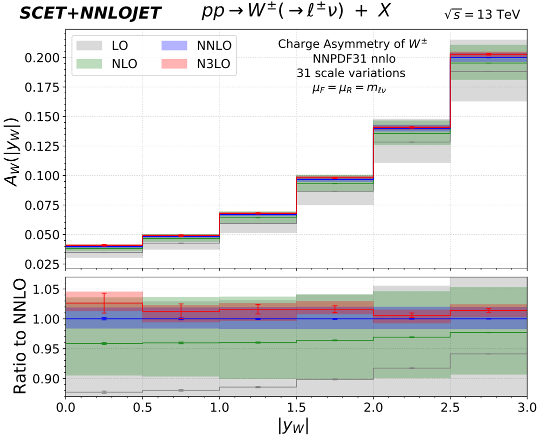
Channels	LO	δNLO	δNNLO	δN^{3}LO
qg	0	-46.43	-29.97	-15.29
$q\bar{q}+q\bar{Q}$	339.62	98.06	25.76	4.97
gg	0	0	2.33	2.12
$qq+qQ$	0	0	0.74	0.17
Total	339.62	51.63	-1.14	-8.03

- Due to large cancellations among different channels, it seems that the perturbation theory is not convergent for total cross sections
- For each partonic channel, the perturbation theory is convergent

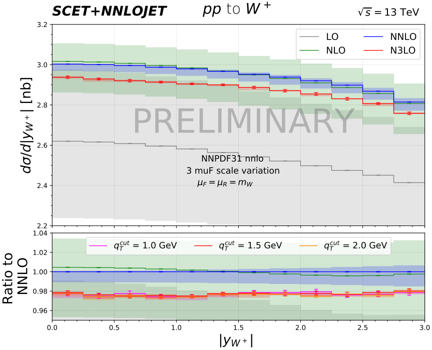
Charge asymmetry of W bosons at N^{3}LO

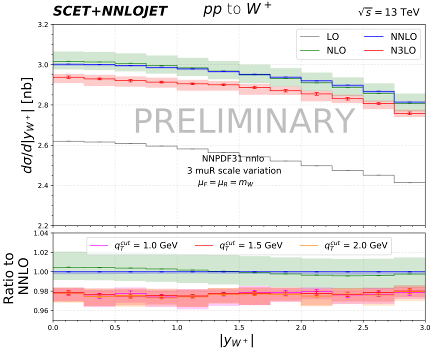
$$A_W(y) = \frac{d\sigma_{W^+}/dy - d\sigma_{W^-}/dy}{d\sigma_{W^+}/dy + d\sigma_{W^-}/dy}$$

Luminosity uncertainty cancels out in the ratio, the experimental measurement for this observable is very precise.



Explore scale uncertainty for W^+ production at N^{3}LO

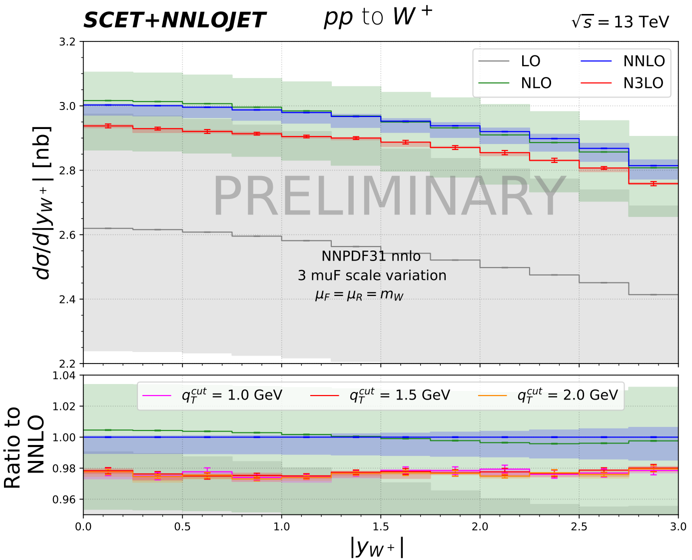




- Fixing $\mu_R = Q$, varying μ_F
- μ_F uncertainty reduces

- Fixing $\mu_F = Q$, varying μ_R
- μ_R uncertainty is enhanced from NNLO to N^{3}LO
- Accidentally small uncertainty at NNLO due to large cancellations among different partonic channels

Factorization scale uncertainty for a fixed μ_R



Renormalization scale uncertainty for a fixed μ_F